milgrom : 한 번 거래후 빠짐.
strategic : side에 대한 전략. 시장에 계속 머물러 있음

Lecture 5. Strategic Trade Models

1. The Kyle (1995) Model

model setting
Dealer : 1명(?). uninformed : many. informed : 1

1,000주. 한번에 사는게 아니라 50, 80, ... 반복적으로 거래.

There is a single informed trader who trades repeatedly. She strategically (a) sets "trade size" taking into account the adverse price concession associated with larger quantities and (b) splits her orders and spreads out them over time to minimize the "trade impact". price concession. 시장가에 영향을 안 주면서 쪼개서 거래.

Remark 1.1. In the sequential trade model, there are "many" informed agents and each trader can trade "only once" when she is drawn as an arriving trader. Trading behaviors are not strategic in that the trader does not care about the outcome of his order size on subsequent trades.

sequential trading model. : μ 만큼의 proportion (informed). one time 거래하고 빠지는 사람.
kyle : strategic informed trader 거래 반복적으로 하는 사람.

2. Single-Period Case

2.1. Assumptions

informed trader 존재

$x>0$: net buy
$x<0$: net sell

Assume that (a) security value is $v \sim N(p_0, \sigma_v^2)$, (b) an informed trader knows v in advance and submits a net order flow x; i.e., $x>0$ means a buy order, and $x<0$ means a sell order, (c) a noise trader submits a net order flow $u \sim N(0, \sigma_u^2)$, independent of v, (d) a dealer observes a total net order flow $y = x + u$ and sets a price, p, and (e) all trades are cleared at p.

exogenous
BID-ASK X. 간단히 p에서 거래!

2.2. The Informed Trader's Decision

informed, uninformed. but dealer는 total만 알 수 있음!
milgrom : dealer 존재. assymetric 하에서 dealer가 어떻게 BID Ask quoting을 하게 되는지에 관심을 맞춤!

trading : uninformed
① profit ← informed
② liquidity
③ hedging
④ sensation.

Since the informed trader wants to trade aggressively (e.g., buying a large quantity if $v>p$), the dealer uses a linear price rule, i.e.,

$v>p \rightarrow$ informed $x\uparrow \rightarrow y\uparrow \rightarrow p\uparrow$

$$p = \lambda y + \mu, \tag{2.1}$$

p가 오르면서, dealer 입장에서 adverse selection risk 을 당함. risk of taking picked-off.

where $\lambda (>0)$ and μ are constant. The pricing rule (2.1) implies that the dealer protects himself by setting a price that is increasing in the informed trader's net order flow.

- The informed trader knows the dealer's pricing rule. So, the informed trader's profit is

$$\begin{aligned} \pi &= (v-p)x \\ &= [v - \lambda(x+u) - \mu]x. \end{aligned}$$

$v>p$, $x>0$ ⊕ ; $v<p$, $x<0$ ⊕.

Since v is "constant" for the informed trader, her expected profit is

informed trader 입장에서 v는 상수 (정확히 알고 있음) μ만 r.v. → $E(\mu)=0$.

$$\begin{aligned} E[\pi] &= (v - \lambda x - \mu)x, \\ &= (v-\mu)x - \lambda x^2. \end{aligned} \tag{2.2}$$

Then, the informed trader strategically chooses her order flow x to maximize $E[\pi]$ in (2.2): i.e.,

$$\max_x (v-\mu)x - \lambda x^2.$$

$p = \lambda x + \mu$

- The first-order condition (FOC) is

$$x = \frac{v - \mu}{2\lambda}, \tag{2.3}$$

which means that *the informed trader's optimal decision about the order flow x is endogenous of the dealer's decision about setting the values of λ and μ.*

dealer decision에 내생적이다.

dealer's decision 따라 x를 결정.

2.3. The Dealer's Decision

Theorem 2.1. *Suppose that X and Y are bivariate normal random variables with means μ_X and μ_Y, variances σ_X^2 and σ_Y^2, and covariance σ_{XY}. Then, the conditional expectation of Y given X is*

$$E[Y|X] = \mu_Y + \frac{\sigma_{XY}}{\sigma_X^2}(X - \mu_X).$$

The dealer cannot directly observe the informed trader's order flow x; instead he conjectures that the informed order flow is linear in v, i.e.,

$u \sim N(0, \sigma_u^2)$

$v \sim N(p_0, \sigma_v^2)$

$x = \alpha + \beta v \sim N(\alpha + \beta p_0, \beta^2 \sigma_v^2)$

$$x = \alpha + \beta v, \tag{2.4}$$

where $\beta > 0$. For the total order flow $y = x + u$ with $x = \alpha + \beta v$, the dealer, who does not know v, obtains that

$$\begin{aligned} E[y] &= E[\alpha + \beta v + u] \\ &= \alpha + \beta p_0, \end{aligned}$$

$y = x + u$
$= \alpha + \beta v + u$
$\sim N(\alpha + \beta p_0, \sigma_u^2 + \beta^2 \sigma_v^2)$.

$$\begin{aligned} Var[y] &= Var[\alpha + \beta v + u] \\ &= \beta^2 Var[v] + Var[u] + 2\beta Cov[u, v] \\ &= \beta^2 \sigma_v^2 + \sigma_u^2, \end{aligned}$$

and

$$\begin{aligned} Cov[y, v] &= Cov[\alpha + \beta v + u, v] \\ &= \beta Cov[v, v] + Cov[u, v] \\ &= \beta \sigma_v^2. \end{aligned}$$

- Since $y \sim N(\alpha + \beta p_0, \beta^2 \sigma_v^2 + \sigma_u^2)$ and $v \sim N(p_0, \sigma_v^2)$, it shows

dealer:
y를 바탕으로
v를 추정하고 싶음.

$$\begin{aligned} E[v|y] &= E[v] + \frac{Cov[v, y]}{Var[y]}(y - E[y]) \\ &= p_0 + \frac{\beta \sigma_v^2}{\beta^2 \sigma_v^2 + \sigma_u^2}(y - \alpha - \beta p_0) \\ &= \left(\frac{\beta \sigma_v^2}{\beta^2 \sigma_v^2 + \sigma_u^2}\right) y + \left(\frac{-\alpha \beta \sigma_v^2 + p_0 \sigma_u^2}{\beta^2 \sigma_v^2 + \sigma_u^2}\right). \ // \end{aligned}$$

① Informed trader : λ, μ에 내생적.
② dealer : x에 내생적. (α, β)
⇓
서로의 movement에 의존.
⇓
의사결정.
서로가 전략적으로 game 중.

Since all trades clear at an informationally efficient price, it must hold $E[v|y] = p$: i.e.,

$$\left(\frac{\beta\sigma_v^2}{\beta^2\sigma_v^2+\sigma_u^2}\right)y+\left(\frac{-\alpha\beta\sigma_v^2+p_0\sigma_u^2}{\beta^2\sigma_v^2+\sigma_u^2}\right)=\lambda y+\mu,$$

dealer의 pricing decision.

yielding

$$\lambda=\frac{\beta\sigma_v^2}{\beta^2\sigma_v^2+\sigma_u^2}\text{ and }\mu=\frac{-\alpha\beta\sigma_v^2+p_0\sigma_u^2}{\beta^2\sigma_v^2+\sigma_u^2}. \tag{2.5}$$

α, β : dealer가 생각하는 informed order flow.
dealer's optimal pricing decision : x에 내생적. (α, β)

- From (2.5), one knows that *the dealer estimates the informed order flow x (i.e., α and β) and then strategically decides his price rule p (i.e., λ and μ)*. That is, the dealer's optimal decision about the price is endogenous of the informed trader's order flow x.

Remark 2.2. The dealer knows the informed trader's optimization process, given in (2.3), and solves for α and β in (2.4) as

informed trader의 전략.

$$\frac{v-\mu}{2\lambda}=\alpha+\beta v$$

dealer
∥
x

$\boxed{\alpha, \beta, \lambda, \mu.}$ unknown.
계산 가능. ⇐ 식 4개.

for all v, yielding that

$$\alpha=-\frac{\mu}{2\lambda}\text{ and }\beta=\frac{1}{2\lambda}. \tag{2.6}$$

Solving (2.5) and (2.6) leads to

$$\alpha=-\frac{p_0\sigma_u}{\sigma_v},\ \beta=\frac{\sigma_u}{\sigma_v},\ \lambda=\frac{\sigma_v}{2\sigma_u},\text{ and }\mu=p_0.$$

2.4. Equilibrium

In equilibrium where the informed trader acts strategically in response to the dealer's strategic movement, the informed trader's expected profit is given by

$$\begin{aligned}E[\pi] &= (v-\lambda x-\mu)x\\ &= \left(v-\lambda\left(\frac{v-\mu}{2\lambda}\right)-\mu\right)\left(\frac{v-\mu}{2\lambda}\right)\\ &= \frac{(v-\mu)^2}{4\lambda}\\ &= \frac{(v-p_0)^2\sigma_u}{2\sigma_v}.\end{aligned}$$

결론 상태에서 informed trader의 기대 수익.

So, $E[\pi]$ is increasing in (a) the divergence between the true value v and the mean p_0 and (b) the variance of noise trading, i.e., σ_u^2.

위장.

- Interestingly, (b) implies that the noise trading provides "camouflage" for the informed trader; e.g., all else equal, the informed trader trading on her private information can make more money in widely held and frequently traded stock.

dealer가 속해 있는 informed trader가 이익을 보는 구조
→ informed trader는 uninformed trader의 주문 뒤에 숨어서 본색을 identify하지 못하게 한다.

y가 정확한 signal을 주는 dealer의 pricing setting 정확해 짐.
σ_u ↑ → 사자 팔자 반복 → y가 x에 대해 정확한 signal을 받지 못하고 credit이 떨어짐.
→ V에 대한 signal 부정확 ⇒ pricing 정확은 ↓.
→ E[π] ↑ (∵ |v−p0| ↑).

dealer, informed trader : 전략적 game.

transaction to transaction. evenly spaced duration (X).

3. Multiple Rounds of Trading

거래. 가정.

Consider the case of N auctions that are equally spaced over a unit interval of time; i.e., the time interval between auctions is $\Delta t = 1/N$. The noise trader's order flow arriving at the nth auction, denoted by Δu_n, follows $\Delta u_n \sim N(0, \sigma_u^2 \Delta t)$. Brownian motion.

- In equilibrium, Kyle (1995) shows that (a) the informed trader's demand is $\Delta x_n = \beta_n (v - p_{n-1})\Delta t$, where β_n is trading intensity per unit time, (b) the price at the nth auctions is $p_n = p_{n-1} + \lambda_n(\Delta x_n + \Delta u_n)$, and (c) market efficiency requires $p_n = E[v|y_n]$, where y_n is the cumulative order flow over time until the nth auction.

$\lambda_n \geq 0$.

누적된 주문량. information set.

Remark 3.1. The informed trader tends to trade on the same side of the market; i.e., she buys on average if information is favorable but sells on average if information is negative. So, the informed trader's order flow is positively correlated, i.e.,

informed order flow : positive autocorrelation.

$$Cov[\Delta x_i, \Delta x_j] > 0$$

for any pair of auctions i and j. The noise trader's order flow is serially uncorrelated, however.

↳ ∵ exogenous shock.

Proposition 3.2. *In equilibrium, the conditional expectation of the incremental total order flow, $E[\Delta y_n|y_{n-1}]$, is zero, where $\Delta y_n = y_n - y_{n-1}$ and $y_n = x_n + u_n$. That is, the total order flow series $\{y_n\}_{n=1}^N$ is serially uncorrelated.*

Proof. It shows

$$\begin{aligned}
E[\Delta y_n|y_{n-1}] &= E[\Delta x_n + \Delta u_n|y_{n-1}] \\
&= E[\Delta x_n|y_{n-1}] \\
&= E[\beta_n(v - p_{n-1})\Delta t|y_{n-1}] \\
&= \beta_n E[v - p_{n-1}|y_{n-1}]\Delta t \\
&= \beta_n (E[v|y_{n-1}] - p_{n-1})\Delta t \\
&= 0,
\end{aligned}$$

$E(\Delta y_n | y_{n-1})$
$= E(y_n - y_{n-1} | y_{n-1})$
$= E(y_n | y_{n-1}) - y_{n-1} = 0$.
$E(y_n | y_{n-1}) = y_{n-1}$.
⇒ y_n is unpredictable. (uncorrelated).

Δx_n : (+)
Δu_n : X.
결합 → Δy_n : X.

since market efficiency implies $E[v|y_{n-1}] = p_{n-1}$. Since $E[\Delta y_n|y_{n-1}] = 0$ implies $E[y_n|y_{n-1}] = y_{n-1}$, one sees that the best forecast of the total order flow at the nth auction is the total order flow at the $(n-1)$th auction; put differently, y_n is unpredictable. □

- The total order flow is "uncorrelated," but the informed order flow is "positively correlated." This suggests that *the informed trader can hide behind the uninformed order flow*. That is, the informed trader trades over time so that the dealer cannot predict (on the basis of the total order flow) what she will do next. In equilibrium, the Kyle model implies that *the informed trader splits her order flow, distributing it over the N auctions*, which corresponds to the real-world practice of splitting orders.

milgrom 처럼 informed 따라 uninformed 따라) dealer 입장. 중간가격 결정.

만주 → informed trader 한번에 눈치 챔.
→ 마치 uninformed 인 것처럼 50주 단위로 전개해서 거래. (order splitting).

informed : correlated
uninformed : unco
total order (dealer) : unco

→ informed trader ⇒ 단번에 숨을 수 있다.

그렇게 하다보면 본인의 identity를 숨겨서 본인의 profit이 증가 드러나면 가격을 올리기 때문