Market Microstructure – Final Exam

Junwoo Yang

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Problem 1. In the case of N auctions, Kyle (1985) shows

$$E[y_n - y_{n-1} \mid y_{n-1}] = 0$$

where y_n denotes the total order flow at the nth auction. Based on this, provide the implication for the informed trader's order submission strategy.

Answer. In Glosten and Milgrom sequential trade models, informed traders are out of the market after a single transaction, while in Kyle's strategic trade model, informed trader splits order and remain in the market. Like trade impact, the current orders affect the subsequent traders, which Kyle considered. The informed trader tends to trade on the same side of the market; i.e., she buys on average if information is favorable but sells on average if information is negative. So, the informed trader's order flow is positively correlated, i.e., $\mathrm{Cov}[\Delta x_i, \Delta x_j] > 0$ for any pair of auctions i and j. The noise trader's order flow is serially uncorrelated, however. $E[y_n - y_{n-1} \mid y_{n-1}] = 0$ implies that the best forecast of the total order flow at the nth auction is the total order flow at the $(n-1)$th auction; put differently, y_n is unpredictable. The total order flow is uncorrelated, but the informed order flow is positively correlated. This suggests that the informed trader can hide behind the uninformed order flow. That is, the informed trader trades over time so that the dealer does not change current quoting because the dealer cannot predict (on the basis of the total order flow) what she will do next. If the security value based on information is higher than the price quoted by dealer, the informed trader wants to trade as much as possible with aggressive orders. However, if a large quantity is submitted as a single order, the dealer may notice that it is an order from informed trader. In equilibrium, the Kyle model implies that the informed trader splits her order flow, distributing it over several auctions, which corresponds to the real-world practice of splitting orders. □

Problem 2. It is said that incoming orders change the price-time priority of existing limit orders in the book. Specify (a) a case in which the price priority of the existing limit buy order increases and (b) a case in which the time priority of the existing limit sell order increases.

Answer. (a) Suppose that there are 10 limit buy orders at price L and one at $L - k$ ($k > 0$) in current order book. If subsequent market sell orders execute 10 limit buy orders at L, then the price priority of the limit buy order at $L - k$ increases. (b) Suppose that there are 10 limit sell orders at L in the book. If subsequent market buy orders execute 9 of the limit sell orders at L, then the

time priority of the remaining 1 limit sell order at L increases. This is due to the first-in first-out principle. Namely, the price priority of the existing limit buy order increases if subsequent market sell orders execute the existing orders at better prices, and the time priority of the existing limit sell order increases if subsequent market buy orders execute the existing limit sell orders at the same price. □

Problem 3. The Parlour (1998) model predicts that the longer queue at the ask will induce more limit buy orders. Explain why.

Answer. Long queue at the ask can make a limit sell order unattractive to a subsequent trader due to lower execution probability. Therefore, the subsequent sellers use market sell order rather than limit sell order because of the crowding out of limit sell order. Accordingly, limit buy orders at bid are executed and probability of execution of limit buy orders increase. As a result, buyers submit more limit buy order than market buy order. □

Problem 4. Explain the difference between a market order and a limit order. Under what circumstances would a trader prefer to use a market order instead of a limit one?

Answer. The trader can submit either a market order or a limit order. The market order ensures immediate execution at the unfavorable price. The limit order can be executed at the favorable price but is subject to adverse-selection risk as well as nonexecution risk. This is because limit prices are fixed over time and thus limit orders can become mispriced when new public information arrives. This is because limit orders cannot be revised under the Foucault model. Therefore, more risk-averse traders who hate to be exposed to both risks would prefer to use a market order. Furthermore, if the queue at the same direction limit order gets longer, the market order will be relatively preferred due to the crowding-out mechanism under the Parlour model. When asset volatility is lower, limit order traders are less exposed to adverse selection risk and tend to submit more aggressive orders to ask for a smaller compensation; consequently, the cost of market order trading decreases. More traders find it optimal to carry their trades using market orders. □

Problem 5. The Glosten and Milgrom (1985) model implies

$$(A - E[V|U, Buy]) \times Pr(U|Buy) = (E[V|I, Buy] - A) \times Pr(I|Buy).$$

What is the economic intuition of it?

Answer. Glosten and Milgrom model은 asymmetric information model로서 informed trader와 uninformed trader가 공존한다고 설정한다. informed trader의 proportion μ와 security payoff V가 낮아질 확률 δ (높아질 확률; $1-\delta$)를 이용하여 여러 event의 조건부확률을 계산한다. ask price에 대한 산식에 law of iterated expectation을 이용하여 전개하면 문제의 등식을 얻을 수 있다. 좌변의 $A - E[V|U, Buy]$는 uninformed trader들로 부터 얻은 gain을 의미하고, 우변의 $E[V|I, Buy] - A$는 informed trader로 부터 잃은 loss를 의미한다. informed trader로 부터 잃은 손실이 uninformed trader로 부터 얻은 이익으로 cover된다는 의미이다. 즉, dealer는 uninformed trader에서 informed trader로 profit을

transfer 시키는 역할을 할 뿐 dealer는 다른 profit을 얻지 않는다(현실은 그렇지 않지만). dealer는 informed trader와의 거래에서 항상 손실을 보게 되기 때문에, informed trader의 비율이 높아질수록 bid-ask spread를 높임으로써 자신을 protect하게 된다. 따라서 기본적으로 이 Glosten and Milgrom model은 dealer가 어떻게 ask와 bid를 설정해야하는지에 초점이 맞춰져 있다. 거래가 진행되면서 정보가 축적될수록 A를 더 정확하게 추정하게 된다. □