Lecture 3. Sequential Trade Models

Trader: Informed / uninformed.
Dealer

1. Asymmetric Information Models

The Roll model is a "symmetric" information model; i.e., everyone possesses the same information. In realistic cases, however, some traders are informed than others; i.e., information asymmetry exists. The *asymmetric information models* reflect this reality.

정보의 비대칭 모델.

- In the *sequential trade model*, randomly selected traders (either informed or uninformed) arrive sequentially and independently (e.g., Copeland and Galai 1983; and Glosten and Milgrom 1985). In the *strategic trader model*, a single informed agent can trade at multiple times (e.g., Kyle 1985; and O'Hara 1995).

public / private.

문제점: private information 이 가격에 반영.

Remark 1.1. As shall be shown later, the asymmetric information models predict (a) that a trade reveals private information; e.g., buy orders are initiated by traders who have private positive information, (b) that dealers set their quotes accordingly in response to the incoming orders, and (c) that trades can engender a permanent impact on subsequent prices.

Buy → permanent / temporary

2. Glosten and Milgrom (1985)

노벨상. (option theory).

2.1. Trader's Decision

random variable.

There is one security with a value V that is either low ($\underline{V}$) with probability of δ or high ($\overline{V}$) with probability of $1-\delta$. A trader is drawn randomly from the population of informed (I) and uninformed (U) traders, where the proportion of informed traders is μ and the proportion of uninformed traders is $1-\mu$.

- Informed traders know the value V that is revealed publicly after the market closes. So, the informed trader buys if $V=\overline{V}$ and sells if $V=\underline{V}$: i.e.,

$$\Pr(Buy|V=\overline{V}) = \Pr(Sell|V=\underline{V}) = 1$$
$$\Pr(Sell|V=\overline{V}) = \Pr(Buy|V=\underline{V}) = 0.$$

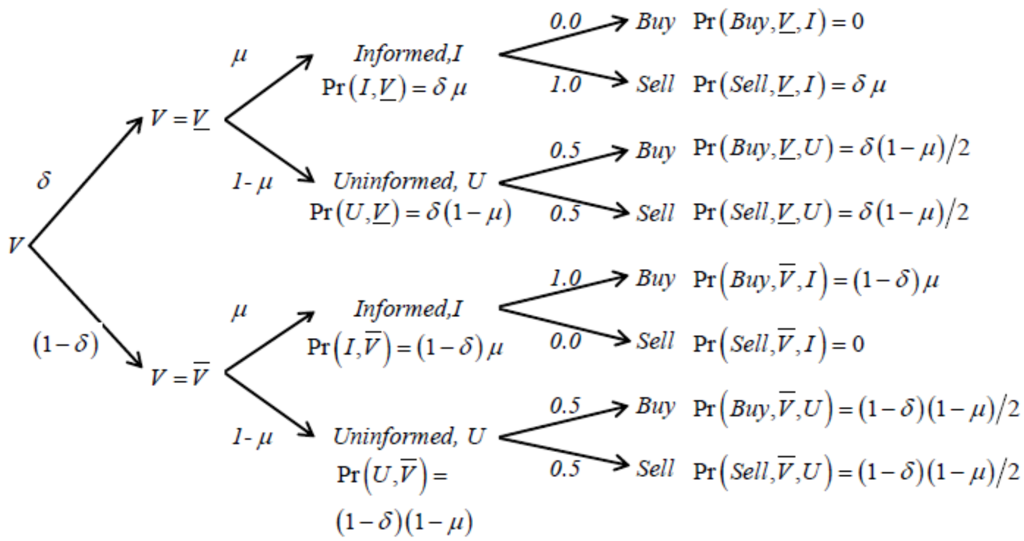
In comparison, uninformed traders buy or sell with equal probability, which is independent of V: i.e.,

uncondition.

$$\Pr(Buy) = \Pr(Sell) = 0.5$$

The trader's decision is described as the form of an event tree with the probability that each node occurs. The event tree is given by

$Pr(I\underline{V}) = Pr(I)Pr(\underline{V})$



- For instance, the probability that the informed trader buys the security when the value is low is given by

$$\begin{aligned}\Pr(Buy,\underline{V},I) &= \Pr(V=\underline{V})\times\Pr(\text{Informed trader arrives})\times\Pr(Buy|V=\underline{V})\\ &= \delta\times\mu\times 0\\ &= 0\end{aligned}$$

Pr(Buy).
uninformed의 경우 V와 independent.

and the probability that the uninformed trader buys the security when the value is low is

$$\begin{aligned}\Pr(Buy,\underline{V},U) &= \Pr(V=\underline{V})\times\Pr(\text{Uninformed trader arrives})\times\Pr(Buy|V=\underline{V})\\ &= \delta\times(1-\mu)\times 0.5.\end{aligned}$$

Remark 2.1. The unconditional probabilities of buy and sell are

$$\begin{aligned}\Pr(Buy) &= \Pr(Buy,\underline{V},I)+\Pr(Buy,\underline{V},U)+\Pr(Buy,\overline{V},I)+\Pr(Buy,\overline{V},U)\\ &= \frac{1+\mu(1-2\delta)}{2}\end{aligned}$$

$f(x)=\int_{-\infty}^{\infty} f(x,y)\,dy$; marginal.

$f(x) = \iint f(x,y,z)\,dz\,dy$

and

$$\begin{aligned}\Pr(Sell) &= \Pr(Sell,\underline{V},I)+\Pr(Sell,\underline{V},U)+\Pr(Sell,\overline{V},I)+\Pr(Sell,\overline{V},U)\\ &= \frac{1-\mu(1-2\delta)}{2}. = 1-Pr(Buy).\end{aligned}$$

2.2. *Dealer's Decision*

Assume (a) that a buy occurs at the dealer's ask price A and a sell occurs at the dealer's bid price B, (b) that the dealer does "not" know whether the trader is informed or uninformed, (c) that the competition among dealers drives expected profits to zero, and (d) that the dealer cannot cross-subsidize buys with sells or vice versa. By the assumption (d), it suffices to analyze buys and sells separately. When setting bid and ask quotes, the dealer attempts to "infer" from submitted buy and sell orders whether V is either $\underline{V}$ or $\overline{V}$ for his profit maximization.

Buy ⇆ matching.
Ask ⇆

Bayesian updating.

V

Ask.

informed. Buyer

⇓

Buy.

adverse selection risk.

Buy — Ask

Sell. — BID

one side quating 분석하므로.
other side is mirror image

dealer가 막연히 $\underline{V}$로 생각하고 있는데 Buy order가 들어왔을 때.

Buy order가 들어왔을 때 $\underline{V}$의 확률

- Given that the trade is a buy order, his revised belief that V equals to $\underline{V}$ is given by

나눠짐

$$
\begin{aligned}
\Pr(\underline{V}|Buy) &= \frac{\Pr(\underline{V},Buy)}{\Pr(Buy)} \\
&= \frac{\Pr(Buy,\underline{V},I)+\Pr(Buy,\underline{V},U)}{\Pr(Buy)} \\
&= \left(0+\frac{\delta(1-\mu)}{2}\right)\times\left(\frac{2}{1+\mu(1-2\delta)}\right) \\
&= \frac{\delta(1-\mu)}{1+\mu(1-2\delta)}. \qquad (2.1)
\end{aligned}
$$

dealer는 정확한 정보(I)와 noise(U)를 갖지만 누구의 order인지 모른다.

For $0<\mu<1$ and $0<\delta<1$, $\partial\Pr(\underline{V}|Buy)/\partial\mu<0$. That is, the dealer more strongly believes that V is less likely to be $\underline{V}$, provided that the trade is a buy order, as more informed traders in the population.

I 비중이 높아질수록, Pr($\underline{V}$|Buy)이 낮아짐. ; Buy order가 informed trader의 order일 가능성 ↑ : dealer가 정확한 정보를 가졌을 확률 ↑.

- Given that the trade is a sell order, his revised belief that V equals to $\underline{V}$ is given by

Dealer가 생각하는 확률.

$$
\begin{aligned}
\Pr(\underline{V}|Sell) &= \frac{\Pr(\underline{V},Sell)}{\Pr(Sell)} \\
&= \frac{\Pr(Sell,\underline{V},I)+\Pr(Sell,\underline{V},U)}{\Pr(Sell)} \\
&= \left(\delta\mu+\frac{\delta(1-\mu)}{2}\right)\times\left(\frac{2}{1-\mu(1-2\delta)}\right) \\
&= \frac{\delta(1+\mu)}{1-\mu(1-2\delta)}. \qquad (2.2)
\end{aligned}
$$

It shows $\partial\Pr(\underline{V}|Sell)/\partial\mu>0$. I가 많아질수록 정확한 정보의 확률↑.

- Using (2.1) and (2.2), one has

initial quat는 없는 상태.

$$
\begin{aligned}
\Pr(\overline{V}|Buy) &= 1-\Pr(\underline{V}|Buy) \\
&= \frac{(1-\delta)(1+\mu)}{1+\mu(1-2\delta)} \qquad (2.3)
\end{aligned}
$$

t=1. t=2.

A_t B_t 0 → quating. by dealer.

and

$$
\begin{aligned}
\Pr(\overline{V}|Sell) &= 1-\Pr(\underline{V}|Sell) \\
&= \frac{(1-\mu)(1-\delta)}{1-\mu(1-2\delta)} \qquad (2.4)
\end{aligned}
$$

선 후.

It shows $\Pr(\underline{V}|Sell)>\Pr(\underline{V}|Buy)$.

Trader → Dealer

Buy/Sell 선택 BID/Ask 조정

V → { informed : sell ; uninformed : sell + buy.

A, B quating :

The assumption (c) implies that the dealer sets quotes to make the expected profit (π) on each transaction zero; i.e.,

Ask(dealer가 판 가격).

그 주식의 가치 기대값.

$$E[\pi|Buy] = A - E[V|Buy] = 0$$

$$E[\pi|Sell] = E[V|Sell] - B = 0.$$

BID.
dealer가 지불하는 가격 (산).

- In particular, the dealer's ask is

$$\begin{aligned} A &= E[V|Buy] \\ &= \underline{V} \times \Pr(\underline{V}|Buy) + \overline{V} \times \Pr(\overline{V}|Buy) \\ &= \underline{V}\left(\frac{\delta(1-\mu)}{1+\mu(1-2\delta)}\right) + \overline{V}\left(\frac{(1-\delta)(1+\mu)}{1+\mu(1-2\delta)}\right) \\ &= \frac{\underline{V}\delta(1-\mu) + \overline{V}(1-\delta)(1+\mu)}{1+\mu(1-2\delta)} \end{aligned} \tag{2.5}$$

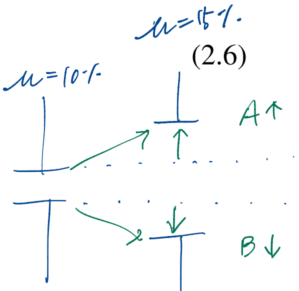
when $\mu = 1$
$A = \overline{V}$
$B = \underline{V}$

dealer 입장
adver selection risk
당할 확률이 높아짐.
(V>A) Buy. A↑

and the dealer's bid is

informed trader
: V>A Buy
V<B Sell.

$$\begin{aligned} B &= E[V|Sell] \\ &= \underline{V} \times \Pr(\underline{V}|Sell) + \overline{V} \times \Pr(\overline{V}|Sell) \\ &= \frac{\underline{V}\delta(1+\mu) + \overline{V}(1-\mu)(1-\delta)}{1-\mu(1-2\delta)}. \end{aligned} \tag{2.6}$$



- From (2.5) and (2.6), one computes the bid-ask spread as

=A−B.
↳ 이득 손해는 무관.

$$A - B = \frac{4(1-\delta)\delta\mu(\overline{V} - \underline{V})}{1-(1-2\delta)^2\mu^2}.$$

Consider $\delta = 1/2$ for simplicity. Then, $A - B = (\overline{V} - \underline{V})\mu$, which means that the bid-ask spread increases with the proportion of informed traders.

adver selection risk.

Remark 2.2. Using the law of iterated expectations (i.e., $E[X] = E[E[X|Y]]$), one writes

$$\begin{aligned} A &= E[V|Buy] \\ &= E[V|U, Buy]\Pr(U|Buy) + E[V|I, Buy]\Pr(I|Buy). \end{aligned}$$

V가 A보다 커서 informed trader가 Buy order를 내서 Ask에 거래되면, dealer 입장에서 V가 아닌 더 낮은 가격인 A(<V)에 판 입장이 됨.
이 risk를 피하기 위해 Ask 높임.

Using $\Pr(U|Buy) + \Pr(I|Buy) = 1$, it shows

$$A\left(\Pr(U|Buy) + \Pr(I|Buy)\right) = E[V|U, Buy]\Pr(U|Buy) + E[V|I, Buy]\Pr(I|Buy)$$

or

$$\underbrace{(A - E[V|U, Buy])}_{\text{Gain from the uninformed}}\Pr(U|Buy) = \underbrace{(E[V|I, Buy] - A)}_{\text{Loss from the informed}}\Pr(I|Buy). \tag{2.7}$$

(2.7) implies that the dealer sets the ask quote to cover his losses to the informed traders with his

informed로 부터 발생한 손실을 uninformed로 부터 얻은 수익으로 메꾼다.
dealer는 informed에 거래에서 항상 손해.
dealer는 이익을 transfer 시킬 뿐 이익을 얻지 않음.

실제 그렇지는 않음.

Roll model에서는 외생적으로 BID Ask가 결정되지만
milgrom model은 내생적으로 결정
↳ dealer가 전략적으로 반응함.
↳ 실제에 더 유사.

gains from uninformed traders; i.e., there is a net wealth transfer from uninformed to informed traders. The same argument is applied to setting the bid quote.

2.3. *Market Dynamics: Bid and Ask Quotes over Time*

After the initial trade, the dealer updates his beliefs about V and posts new quotes. The next trader arrives, and the process repeats. Let δ_k denote the probability of $\underline{V}$ given δ_{k-1} and the direction of the kth trade (i.e., Buy_k or $Sell_k$), with the unconditional probability being $\delta_0 = \delta$. Then, (2.1) and (2.2) can be generalized as

정보가 많아질수록
정보가 정확해지고
시간이 지나
V에 대해
정확한 값
생각을 가짐.

$$\delta_k(Buy_k;\delta_{k-1}) = \frac{\delta_{k-1}(1-\mu)}{1+\mu(1-2\delta_{k-1})} \tag{2.8}$$

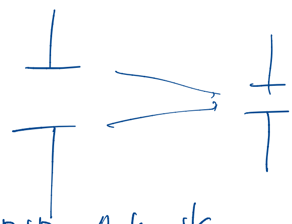
$$\delta_k(Sell_k;\delta_{k-1}) = \frac{\delta_{k-1}(1+\mu)}{1-\mu(1-2\delta_{k-1})}. \tag{2.9}$$

- Using (2.8) and (2.9), one computes the dealer's ask and bid prices given the kth trade as

μ : fixed.

$k=1$ . . . .

$$\begin{aligned} A_k &= E[V|Buy_k;\delta_{k-1}] \\ &= \underline{V}\times\delta_k(Buy_k;\delta_{k-1}) + \overline{V}\times(1-\delta_k(Buy_k;\delta_{k-1})) \\ &= \underline{V}\left(\frac{\delta_{k-1}(1+\mu)}{1-\mu(1-2\delta_{k-1})}\right) + \overline{V}\left(1-\frac{\delta_{k-1}(1+\mu)}{1-\mu(1-2\delta_{k-1})}\right) \\ &= \frac{\underline{V}\delta_{k-1}(1-\mu)+\overline{V}(1-\delta_{k-1})(1+\mu)}{1+\mu(1-2\delta_{k-1})} \end{aligned}$$



경쟁.
Competition 이기 때문에
BID-Ask 가 줄어듦
많은 정보가 누적되어 V값이 정확해짐.
aggressive.

BID-Ask ↓
∵ dealer가 정보가 축적되어 정확해짐.

and

q_k < −1 Sell, 1 Buy.

$$\begin{aligned} B_k &= E[V|Sell_k;\delta_{k-1}] \\ &= \underline{V}\times\delta_k(Sell_k;\delta_{k-1}) + \overline{V}\times(1-\delta_k(Sell_k;\delta_{k-1})) \\ &= \frac{\underline{V}\delta_{k-1}(1+\mu)+\overline{V}(1-\mu)(1-\delta_{k-1})}{1-\mu(1-2\delta_{k-1})}, \end{aligned}$$

$E(q_k|\Omega_{k-1}) = 0$
⇒ symmetric

⇒ 장초기와 장마감기로 비슷한 흐름.

both of which are *varying* over time. With each trade, the dealer estimates the conditional probabilities more precisely, and hence uncertainty is reduced. As a result, the spread will "decline" over time toward the closing of markets.

한 방향 주문만 계속 들어옴.

V < informed : Buy, uninformed : Sell & Buy → Buy! order flow invalance! 실제 시장에서는 많음.

Remark 2.3. Glosten and Milgrom (1985) show that the informed traders always trade in the same direction although they are drawn randomly; consequently, the order flow is not symmetric, i.e., $E[q_k|\Omega_{k-1}] \neq 0$, and the orders are serially correlated.

Remark 2.4. *Price impact* refers to the effect of an incoming order on subsequent prices. In Glosten and Milgrom model, a buy (sell) order on the kth trade causes a downward (upward) revision in the conditional probability of having $\underline{V}$, which in turn leads to an increase (decrease) in the ask price. So, the kth trade has a "permanent" price impact in that it changes the subsequent transaction price.

closed form solution의 존재.

milgrom : 지정가 주문시장에서는 중요!

시장가 주문시장에서는 구분됨.

지정가 주문시장

make mkt : 유동성 공급 (dealer), 지정가 주문자
take mkt : 유동성 수요 (trader) 시장가 주문자
→ 지정가, 시장가 동일 인물 : 최적화 복잡. (closed form solution 거의 없음).

Dealer : Buy 들어오면 A < V 로 생각. → A↑ : price impact.

< temparary
permanent : milgrom model에서는 (assymetric) permanent price impact* 가 있다고 생각. ∵ informed trader 가 존재하기 때문.

Roll model 은 symmetric 이기 때문에 E(q) = 0. * 가 없음.