Information and Uncertainty

Midterm Examination

SPRING 2021

1. Write down the definitions (10 points each)

   (a) Axioms of von Neumann and Morgenstern's expected utility theory

   (b) Lemon's problem

   (c) Moral hazard problem

   (d) Revelation principle

2. (Second price auction)

   (a) (10 points) Describe the second price auction as a Bayesian game, in which each player's valuation of the object is private information, and the valuation is drawn independently.

   (b) (20 points) If you eliminate weakly dominated strategies from each player's strategy space, what is the remaining set of strategies of each player?

3. Suppose that a public good is provided to a group of people if at least one person is willing to pay the cost of the good. Assume that the people differ in their valuations of the good, and each person knows only her own valuation. Let v_i be the valuation of player i, which is drawn from the same distribution $F(v) = \mathsf{P}(v_i \leq v)$ for every player. Assume that $c > 0$ is sufficiently small so that $\mathsf{P}(v_i > c) > 0$. If no one contributes, no public good is offered and no player can realize v_i. If player i does not contribute, but someone else does, then his payoff is v_i. If player i contributes, his payoff is $v_i - c$.

   (a) (20) Calculate a symmetric Nash equilibrium in which player i contributes if and only if his valuation v_i is less than some threshold value.

(b) (10) As the number of the people in the society increases, does the probability (according to the above Nash equilibrium) that a particular player contributes increase or decrease? Justify your answer.

(c) (30) As the number of the people in the society increases, does the probability that the public good is offered to the society (according to the above Nash equilibrium) increase or decrease? Justify your answer.

4. (Beauty contest: 30) Consider a game with n players, where $n \geq 10$. Each player $i \in \{1, \ldots, n\}$ has to choose integer $k_i \in \{0, 1, \ldots, 20\}$. The winners are those players who choose an integer that is closest to $\frac{3}{4}$ of the average.

   Let $k_1, \ldots, k_n$ be the integers selected by players. The average is

$$\frac{1}{n} \sum_{i=1}^{n} k_i.$$

   We then consider

$$d_i = \left| k_i - \frac{3}{4} \frac{1}{n} \sum_{i=1}^{n} k_i \right|.$$

   If

$$d_i = \min(d_1, \ldots, d_n),$$

   then player i is a winner. Note that there can be multiple winners.

   Calculate all Nash equilibria of this game.