(milgoram setting.
+ information uncertaint).

Lecture 4. Order Flow and the Probability of Informed Trading

어느쪽으로가 informed & uninformed traders
인지 파악하기 위함.

1. The Distribution of Buys and Sells

Assume that conditional on V, the probability of a current order direction q_t (i.e., a buy or a sell) does not depend on the direction of prior orders: i.e.,

$$\Pr(q_t|V,q_{t-1},q_{t-2},\ldots) = \Pr(q_t|V).$$

buy order가 들어올 확률

Let p denote the conditional probability of incoming a buy order given V. Then one sees that the conditional distributions of b buys given n trades and V are binomial:

$$\Pr(b|n,V) = \begin{pmatrix} n \\ b \end{pmatrix} p^b(1-p)^{n-b}. \tag{1.1}$$

- Extending the previous Glosten and Milgrom sequential trading model, we compute p as:

$V \left\langle \begin{matrix} \overline{V} \\ \underline{V} \end{matrix} \right.$

$$\begin{aligned}
\Pr(Buy|\underline{V}) &= \frac{\Pr(Buy,\underline{V})}{\Pr(\underline{V})} \\
&= \frac{\overset{0}{\Pr(Buy,\underline{V},I)} + \overset{\delta(1-\mu)/2}{\Pr(Buy,\underline{V},U)}}{\Pr(\underline{V})} \\
&= \frac{0+\delta(1-\mu)/2}{\delta} \\
&= \frac{1-\mu}{2}
\end{aligned}$$

and

$$\Pr(Buy|\overline{V}) = \frac{1+\mu}{2},$$

both of which are "constant" over successive trades.

Remark 1.1. Advancing (1.1), one sees that the number of buys conditional only on n trades is a mixture of binomials:

$p \to P(b|\underline{V}) = \frac{1-\mu}{2}.$

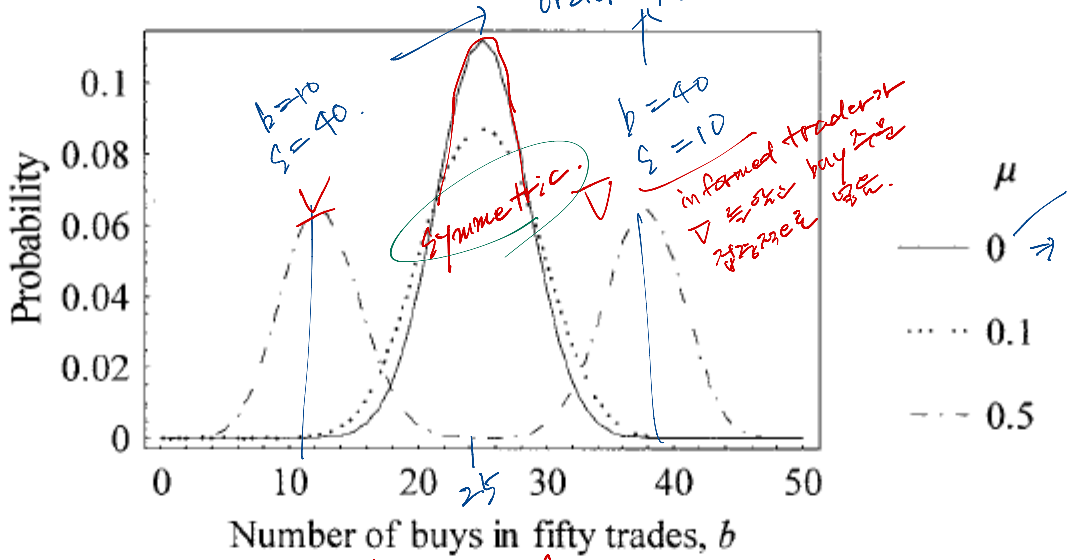
$P(b|V)$

mixture distribution
2 over binomial.

$$\begin{aligned}
\Pr(b|n) &= \Pr(V=\underline{V})\times\Pr(b|n,\underline{V}) + \Pr(V=\overline{V})\times\Pr(b|n,\overline{V}). \\
&= \delta\begin{pmatrix} n \\ b \end{pmatrix}\left(\frac{1-\mu}{2}\right)^b\left(1-\frac{1-\mu}{2}\right)^{n-b} \\
&\quad +(1-\delta)\begin{pmatrix} n \\ b \end{pmatrix}\left(\frac{1+\mu}{2}\right)^b\left(1-\frac{1+\mu}{2}\right)^{n-b}.
\end{aligned} \tag{1.2}$$

For $n = 50$ and $\delta = 0.5$, the following plot shows (1.2) for different μ. With no informed trading (i.e., $\mu = 0$), the distribution is close to normal. With $\mu = 0.5$, the distribution is bimodal; that is to say, "one-sided" order flow (i.e, a preponderance of buys or sells) emerges if a proportion of informed traders is high. Put differently, the data exhibiting distinct one-sided order flow may indicate a high proportion of informed traders.

one sided order flow ⇒ ∵ informed는 $\overline{V}$ 일때 무조건 b. $\underline{V}$ 일때 무조건 s.



one sided order flow
: informed trader가 존재한다는 반증.
본인이 갖고 있는 정보로 일방적인 주문을 넣은 것.

2. Easely and O'Hara (1992)

Easely and O'Hara (1992) extend the basic sequential trade model by introducing *information event uncertainty*. Assume that the number of arriving traders in any finite interval follows a Poisson arrival process; specifically, the Poisson arrival intensities for informed and uninformed traders are μ and ε, respectively.

μ: 일정하지 X. 정보가 매번 들어오는 것이 아님. 정보 발생이 불확실해야 한다. sequential : 정보 발생 확률 1로 봤음.

α: 정보 발생 확률. r.v. p(μ|α). α 확률로 정보 발생하여야 Informed 거래 함. 1-α 정보 발생X → I: 거래X.

- Informed buyers arrive with intensity μ only if $\overline{V}$ occurs, and informed sellers arrive with intensity μ only if $\underline{V}$ occurs. In all states, uninformed buyers and sellers arrive with intensity ε.

$\overline{V}$ ⟨ buyer : μ, seller : 0
$\underline{V}$ ⟨ buyer : 0, seller : μ.

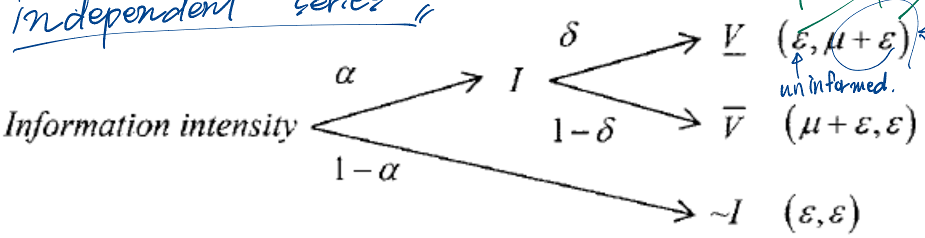
Proposition 2.1. *Let $N_1(t)$ and $N_2(t)$ denote the number of type I and type II events, respectively, occurring in $[0,t]$. Assume that $N_1(t)$ and $N_2(t)$ follow Poisson arrival processes with λ_1 and λ_2, respectively. Suppose that $N_1(t)$ and $N_2(t)$ are independent. Define $N(t) = N_1(t) + N_2(t)$. Then, $N(t)$ follows a Poisson arrival process with $\lambda_1 + \lambda_2$.*

수리통계로 증명해보기!

ε: uninformed buy sell 동일 비율.

- If $\underline{V}$ occurs, the total arrival intensity of buyers is ε and that of sellers is $\mu + \varepsilon$. If $\overline{V}$ occurs, the total arrival intensity of buyers is $\mu + \varepsilon$ and that of sellers is ε.

independent series



1-μ-ε.

I: informed seller: μ
II: uninformed seller: ε
1-J
I+II : N(t).

statistics.
통계: toy

I: buy
II: sell } indep.
↑ assumption.

α

μ

p(μ|α) "
: 통합하거나 복수 나눠서

Given that an information event occurs with the probability of α, the unconditional number of buys and sells on any day are jointly distributed as a Poisson mixture:

intensity.

$$
\begin{aligned}
\Pr(b,s) &= \Pr(b,s|\text{No information event}) + \Pr(b,s|\text{Information event}, \underline{V}) \\
&\quad + \Pr(b,s|\text{Information event}, \overline{V}) \\
&= (1-\alpha)\Pr(b;\varepsilon)\Pr(s;\varepsilon) + \alpha\delta\Pr(b;\varepsilon)\Pr(s;\mu+\varepsilon) \\
&\quad + \alpha(1-\delta)\Pr(b;\mu+\varepsilon)\Pr(s;\varepsilon),
\end{aligned} \tag{2.1}
$$

where $\Pr(n;\lambda)$ denotes the probability of n arrivals when λ is the intensity parameter.

- The joint probability $\Pr(b,s)$ can be used to construct a sample likelihood function, and one can estimate all parameters of $\alpha, \delta, \mu, \varepsilon$ by maximum likelihood estimation. Specifically, one writes (2.1) as

$$
\begin{aligned}
L(b,s|\theta) &= (1-\alpha)e^{-\varepsilon t}\frac{(\varepsilon t)^b}{b!}e^{-\varepsilon t}\frac{(\varepsilon t)^s}{s!} + \alpha\delta e^{-\varepsilon t}\frac{(\varepsilon t)^b}{b!}e^{-(\varepsilon+\mu)t}\frac{((\varepsilon+\mu)t)^s}{s!} \\
&\quad + \alpha(1-\delta)e^{-(\varepsilon+\mu)t}\frac{((\varepsilon+\mu)t)^b}{b!}e^{-\varepsilon t}\frac{(\varepsilon t)^s}{s!}.
\end{aligned}
$$

  Over h independent days, the likelihood of observing the data $M = (b_i, s_i)_{i=1}^h$ is the product of the daily likelihoods

$$
L(M|\theta) = \prod_{i=1}^{h} L(b_i, s_i|\theta). \quad \Leftarrow \ \because \text{independent}.
$$

Remark 2.2. The *probability of informed trading* (PIN) is defined as the unconditional probability that a randomly chosen trader is informed on a randomly chosen day: i.e.,

analytic (X).
numeric (O).

$$
\begin{aligned}
\text{PIN} &= \frac{E[\text{Informed order arrival intensity}]}{E[\text{Total order arrival intensity}]} \\
&= \frac{\alpha(\delta\mu + (1-\delta)\mu)}{(1-\alpha)(\varepsilon+\varepsilon) + \alpha\left[\underset{\underline{V}}{\delta}(\underset{b}{\varepsilon}+\underset{s}{\mu+\varepsilon}) + \underset{\overline{V}}{(1-\delta)}(\underset{b}{\mu+\varepsilon}+\underset{s}{\varepsilon})\right]} \\
&= \frac{\alpha\mu}{\alpha\mu + 2\varepsilon}.
\end{aligned}
$$

$\frac{\partial \text{PIN}}{\partial \mu} > 0.$

As μ increases, PIN increases. Recall that high μ corresponds to a strong tendency of one-sided order flow. Therefore, the observation suggests that order flow tends to be one-sided on a trading day with high PIN.

PIN. : feild 에서 실제 계산함
⇒ trading strategy 수립.