Information and Uncertainty

Final Examination

Spring 2021

Follow the convention of the lectures if the description of the problem is incomplete, or the notation is not clear.

1. (Second price auction: 20 points) Consider IPV framework and the second price auction. Construct a Nash equilibrium which does not use the truthful bidding strategy, and show that the constructed Nash equilibrium outcome is not efficient.

2. (Rubinstein: 30 points) Consider the alternating offer bargaining model of Rubinstein [1982] between a buyer and a seller, who are negotiating how to split 1 unit of surplus. Let δ_b and δ_s be the discount factors for the buyer and the seller. It is possible that $\delta_b \neq \delta_s$. Construct a subgame perfect equilibrium. (You do not need to prove that it is a subgame perfect equilibriium.)

3. (Grove's scheme: 30 points) There are N agents in this economy whose preference is

$$\theta_i x + t_i$$

where θ_i is the marginal utility of public project x, which is 1, if initiated, and 0, otherwise; and t_i is the transfer payment from the government to agent i. If $t_i > 0$, then one can interpret t_i as a subsidy to agent i, and if $t_i < 0$, then t_i can be interpreted as tax changed to agent i. θ_i ($i = 1, \ldots, N$) is observed only by agent i, but everyone including the government knows that θ_i is selected from the standard normal distribution over $\Re$. The government asks each agent about θ_i. Let $\hat{\theta}_i$ be the "reported" type of agent i. The government will initiate the public project if and only if

$$\sum_{i=1}^{N} \hat{\theta}_i \geq c$$

where c is the cost of the public project, and choose transfer payment to agent i as

$$t_i = \left[\sum_{j \neq i} \hat{\theta}_j\right] - c.$$

Prove that $\hat{\theta}_i = \theta_i$ (i.e., truthful revelation) is a weakly dominant strategy of agent i. You can show that the truthful revelation is a best response against any profile of strategies of the other players.

4. (Dividing a dollar with changes) Consider an elaboration of the dividing the dollar game. There are two players. Player i submit his demand $d_i \in [0, 1]$. If $d_1 + d_2 \le 1$, then player i receives d_i. In the first version, if $d_1 + d_2 > 1$, then both players receive 0. Each player is trying to maximize his income.

   If $d_1+d_2 > 1$, and $d_1 \neq d_2$, then the party who submits the smaller portion receives $\min(d_1, d_2)$ and the other party receives $1 - \min(d_1, d_2)$. If $d_1 + d_2 > 1$ and $d_1 = d_2$, then each party receives one half of the dollar.

   Calculate the Nash equilibrium of the second version of the game as follows:

   (a) (10 points) Prove that if $d_1 + d_2 > 1$ and $d_1 \neq d_2$, then (d_1, d_2) cannot be a Nash equilibrium. (Note that either d_1 or d_2 must be larger than 0.5.)

   (b) (10 points) Prove that if $d_1 + d_2 > 1$ and $d_1 = d_2$, then (d_1, d_2) cannot be a Nash equilibrium. (Note that in this case, $d_1 = d_2 > 0.5$.) Combining the first two steps, you can conclude that if (d_1, d_2) is a Nash equilibrium, then $d_1 + d_2 \le 1$.

   (c) (10 points) Prove that if $d_1 + d_2 < 1$, then (d_1, d_2) cannot be a Nash equilibrium. (Note that d_1 or d_2 is less than 0.5)

   (d) (10 points) We now know that in a Nash equilibrium, $d_1+d_2 = 1$. Prove that $d_1 = d_2$.