HFT. 일본: arrow project.

Lecture 7. Dealers and Their Inventories

1. Inventory Control Models

A dealer must accommodate randomly arriving traders and replenish his inventory by buying (at the bid price B) from and selling (at the ask price A) to these traders. In general, theoretical models predict that the dealer will have a "preferred" inventory level.

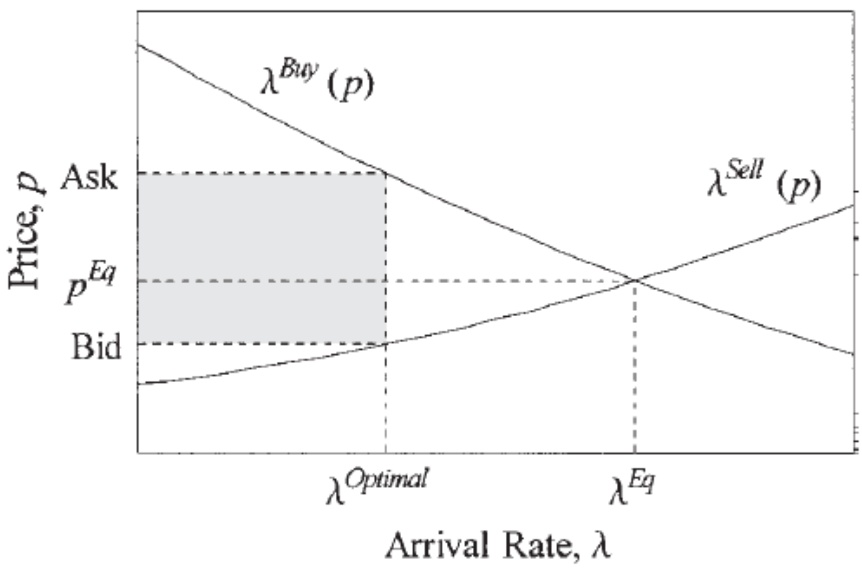
1.1. Garman (1976)

Buyers and sellers arrive randomly in continuous time, and transact a single quantity. The arrival intensities for buyers and sellers are $\lambda^{Buy}(p)$, a function of the price they pay, and $\lambda^{Sell}(p)$, a function of the price they receive, respectively. A dealer quotes an ask price, A, to the buyers and a bid price, B, to the sellers.

- The dealer is supposed to set quotes to balance supply and demand "on average," i.e., $\lambda^{Buy}(A) = \lambda^{Sell}(B)$. So, the dealer's average profit on every unit turned over is

$$\begin{aligned} \pi &= A \cdot \lambda^{Buy}(A) - B \cdot \lambda^{Sell}(B) \\ &= (A - B)\lambda^{Buy}(A) \\ &= (A - B)\lambda^{Sell}(B). \end{aligned}$$

A wide bid-ask spread increases the profit but at the same time depresses the rate of arrivals (which in turn deceases the profit). Specifically, as A increases, $\lambda^{Buy}(A)$ decreases; or as B decreases, $\lambda^{Sell}(B)$ decreases. The dealer's problem is to set the bid and ask to maximize the shaded rectangle.



To accommodate the asynchronous buying and selling, the dealer should maintain stock and cash inventories. The key constraint is that *the dealer's inventories should not drop below certain levels*.

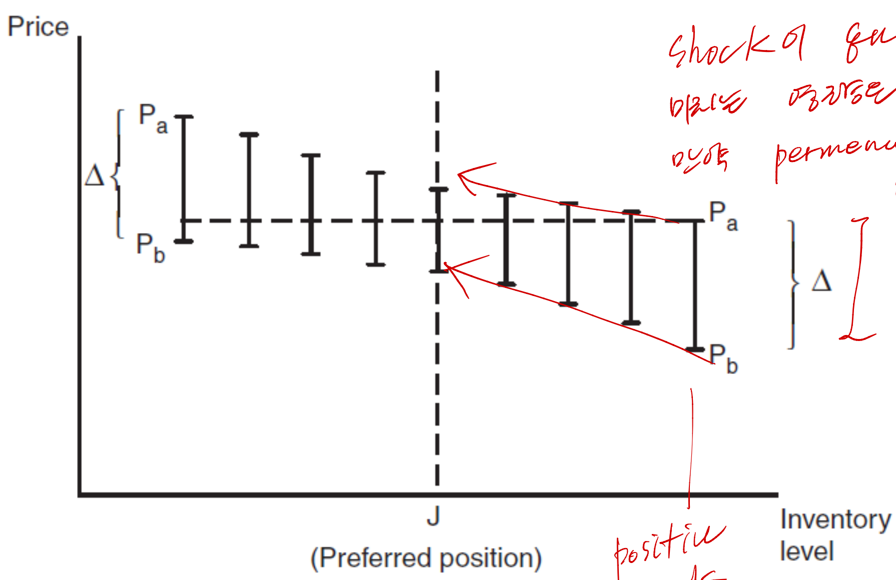
- If $\lambda^{Buy}(A) = \lambda^{Sell}(B)$, Garman (1976) shows that stock holdings follow a zero-drift random walk and, as a result, the dealer will be ruined with probability one, since the zero-drift random walk will eventually hit any "finite" barrier with probability one.

Remark 1.1. Regarding the expected ruin, Garman (1976) notes, "*The order of magnitude makes it clear that the specialists [dealer] must pursue a policy of relating their prices to their inventories in order to avoid failure*." Put differently, dealers must "continuously" change their bid and ask quotes to elicit an expected imbalance of buy and sell orders to push their inventories in the direction of their preferred long-run position.

1.2. Amihud and Mendelson (1980)

Buyer and sellers arrive asynchronously, and their arrival intensities depend on the posted quotes. A dealer's inventory is constrained to lie between upper and lower bounds. The dealer maximizes the expected profit per unit time. In this model, an extreme inventory position depresses the expected profit of a risk-neutral dealer and, as a result, the dealer is subject to aversion to extreme positions.

- Amihud and Mendelson (1980) predict (a) that the dealer has a "preferred" inventory level; for instance, as the inventory nears the upper boundary, the dealer lowers the bid and ask, thereby encouraging a buy order and discouraging a sell order, (b) that bid and ask prices are monotone decreasing functions of inventory level, (c) that the bid-ask spread is positive, (d) that the spread is increasing in distance from preferred position, and (e) the quotes are not set symmetrically about the true value.



shock이 quoting에
미치는 영향은 temporary.
만약 permenant 하다면
BID-Ask
이 수준이
유지됨.

positive
shock.

milgrom
: uninformed trader의
거래할 확률이 높아지면
adverse.
BID-Ask spread 높여버림. (∵ Risk-aversion).

risk-neutral : mean 으로만 결정.
vol 와 관계 X.

1.3. Stoll (1978)

A risk-averse dealer has an exponential utility function $U(W) = -e^{-\alpha W}$ for $\alpha > 0$. Suppose that the dealer can buy or sell n shares of a security (with $n < 0$ corresponding to a short position) at a price P prior to quoting. The security has random payoff $X \sim N(\mu_X, \sigma_X^2)$. With zero initial endowments of shares and cash, the dealer's wealth is $W = n(X - P) \sim N(n(\mu_x - P), n^2\sigma_X^2)$.

quoting 전에.

Theorem 1.2. *For $Z \sim N(\mu, \sigma^2)$, it shows*

moment generating ft.

$$E[e^Z] = \exp\left(\mu + \frac{\sigma^2}{2}\right).$$

1) $n>0$ (buy) $X>P \rightarrow W>0$.
2) $n<0$ (sell) $X<P \rightarrow W>0$.

- Since $-\alpha W \sim N(-\alpha n(\mu_x - P), \alpha^2 n^2 \sigma_X^2)$, the dealer's expected utility $E[U(W)]$ is given by

$$\begin{aligned} E[U(W)] &= E[-e^{-\alpha W}]. \\ &= -E[e^{-\alpha W}] \\ &= -\exp\left(-\alpha n(\mu_x - P) + \frac{\alpha^2 n^2 \sigma_X^2}{2}\right). \end{aligned}$$

Definition 1.3. The *certainty equivalent* (*CE*) of a random payoff x is defined by a certain amount that has a utility level equal to the expected utility of x. In other words, the CE of x is the value c satisfying

$$U(c) = E[U(x)].$$

- For $W \sim N(\mu_W, \sigma_W^2)$, the CE of W is the value c satisfying

$-\alpha\left(\mu_W - \frac{\alpha\sigma_W^2}{2}\right)$

$$-\exp(-\alpha c) = -\exp\left(-\alpha\mu_W + \frac{\alpha^2\sigma_W^2}{2}\right).$$

That is, one obtains

$$c = \mu_W - \frac{\alpha\sigma_W^2}{2}.$$

max $E(U(W))$
$\Updownarrow$
min $\exp(-\alpha c)$
$\Updownarrow$
max c (CE)

From $E[U(W)] = -\exp(-\alpha c)$, one sees that maximizing $E[U(W)]$ is equivalent to minimizing $e^{-\alpha c}$ or equally maximizing c (since $\alpha > 0$).

The optimal holding of shares, denoted by n^*, is obtained by maximizing $E[U(W)]$ or maximizing the CE of W over n:

↳ 선호하는 inventory 수준.

$$\max_n n(\mu_X - P) - \frac{\alpha n^2 \sigma_X^2}{2}.$$

- One computes n^* by solving the FOC:

$$\mu_X - P - \alpha n^* \sigma_X^2 = 0,$$

which leads to

$$\boxed{n^* = \frac{\mu_X - P}{\alpha\sigma_X^2}.} \tag{1.1}$$

dealer가 quoting 전에 보유 utility를 maximize하는 optimal한 inventory 수준. quoting 하는 순간 이 수준에서 벗어나게 됨.

Now, suppose that the dealer holds n^* shares, which means that his expected utility is optimal prior to bidding. The dealer posts a bid B for one share. After bidding, the dealer's terminal wealth will be

$$W = \begin{cases} n^* X & \text{if the bid is not hit} \\ (n^*+1)X - B & \text{if the bid is hit.} \end{cases}$$

B에서 1주 늘어남. ⟷ expected return maximize.

- The dealer is indifferent to execution if the CE is the same whether or not the bid is hit: i.e.,

  무차별로.

  $$n^*\mu_X - \frac{\alpha n^{*2}\sigma_X^2}{2} = (n^*+1)\mu_X - B - \frac{\alpha(n^*+1)^2\sigma_X^2}{2},$$

  which implies

  $$\boxed{B = \mu_X - \frac{\alpha(2n^*+1)\sigma_X^2}{2}.} \tag{1.2}$$

dealer

The bid B leaving the dealer indifferent between execution and remaining at his optimum is obtained by substituting (1.1) for n^* in (1.2) as

$$\begin{aligned} B &= \mu_X - \alpha n^* \sigma_X^2 - \frac{\alpha\sigma_X^2}{2} \\ &= \mu_X - \alpha\left(\frac{\mu_X - P}{\alpha\sigma_X^2}\right)\sigma_X^2 - \frac{\alpha\sigma_X^2}{2} \\ &= \boxed{P - \frac{\alpha\sigma_X^2}{2}.} \end{aligned}$$

→ n^* : optimal 수준에서 n^*+1 로 벗어난 대가

(∵ risk-averse 이기 때문에 optimal에서 벗어나길 싫어하는 것).

$\sigma_X^2 \uparrow \to B \downarrow$

- A dealer has aversion to extreme positions (because he is risk averse) and thus considers the possibility of a shock (i.e., σ_X^2) to his portfolio when setting a bid quote. If the dealer is initially at his portfolio optimum, he will set the bid price to impound compensation for being pulled off his optimum, if the bid is hit.

volatility 가 크면 optimal inventory 에서 벗어나길 더 싫어함. 그럼에도 벗어나면 더 큰 보상 요구. ∴ 더 싼 Bid에 매입을 요구. 반대로, σ_X^2 가 작아지면 보상을 덜 요구하고 BID를 올려 줌.

Remark 1.4. When the dealer posts an ask A, then the dealer's terminal wealth is

$$W = \begin{cases} n^* X & \text{if the ask is not hit} \\ (n^*-1)X + A & \text{if the ask is hit.} \end{cases}$$

Ask 받고 1주 팔아서 inventory 감소

So, the dealer is indifferent if

$$n^*\mu_X - \frac{\alpha n^{*2}\sigma_X^2}{2} = (n^*-1)\mu_X + A - \frac{\alpha(n^*-1)^2\sigma_X^2}{2},$$

which implies

$$A = \mu_X - \frac{\alpha(2n^*-1)\sigma_X^2}{2}. \tag{1.3}$$

check!

n^* 대입해서 정리해보기

$\sigma_X^2 \uparrow \to A \uparrow$ 도출.

(1.2) and (1.3) imply that the bid-ask spread is

$$A - B = \alpha\sigma_X^2,$$

$A = P + \frac{\alpha\sigma_X^2}{2}$

dealer는 원하는 inventory 수준이 있음. n^* ← ∵ expected return) $E(u(W))$ 최대화 해주므로.

risk-averse 이므로 벗어나길 싫어한다.

∴ inventory 에 대한 불확실성이 커지면 Ask는 높이고 BID는 낮춤으로써 보상을 요구.

BID-Ask spread ↑

dealer는 BID, Ask 를 조정하면서 preferred position으로 가고 싶어함.

$n^* \to n^*+1$. ($B\downarrow$): 사기 싫으면 seller 들이 거래 싫은 B↓.

$n^* \leftarrow n^*-1$ ($A\uparrow$) 면 buyer 〃

$A\uparrow$: buyer 떠나라.

1~3 중요함.
: 원하는 inventory level이 있고, 유지하려고 노력한다. (Inventory Control)
BID, Ask 조정함으로써.

A 높여서 buyer 차단했는데 Buyer 들어 들어서 $x^*-1 \to x^*$로 돌아오면 A가 높은 수준을 유지하지 않고 높여 수준으로 낮아짐.
그렇지 않으면 다른 dealer 와 거래하게 됨.
BID도 높여줌.

so that the dealer sets the bid-ask spread wider as the uncertainty of the security value increases.

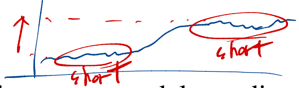
2. Empirical Analysis of Dealer Inventories

은둔 10년 10년 뒤
prefered position 일정 가정.
⇒ but data say that it's change.

The inventory control models assert that the inventory series, denoted by x_t, is mean-reverting (i.e., $\lim_{s\to\infty} \partial x_{t+s}/\partial \varepsilon_t = 0$), which in turn means that x_t is "stationary." To check the validity of the models, therefore, one may apply "unit tests" to determine whether the inventory series x_t is stationary.

non-stationary process.

- Regarding the NYSE specialist inventories, Hasbrouck and Sofianos (1993) find (a) that inventories are sometimes negative, (b) that there is no obvious drift or divergence, and (c) that inventory is a mean-reverting series. Importantly, the authors claim that the specialist may be pursuing short-term inventory, but toward preferred positions that are "slowly" time-varying.



When a trader purchased from a dealer, the inventory models predict that the dealer would raise the ask price to discourage another incoming buy order. The previous asymmetric models also predict the same short-run relation for the different reason that buyers maybe informed. To differentiate the two effects, one needs to consider the long-run behavior in that the information effects are "permanent", while the inventory effects are "temporary".

- For NYSE data, Hasbrouck and Sofianos (1993) find that most of quote dynamics are attributable to trades, with inventories contributing little explanatory power. The inventory positions are not apparently managed by adjustment of publicly quoted bids and asks, which is inconsistent with the intuition of inventory control models.

Remark 2.1. Even if inventory management is not effected with publicly quoted bids and asks, the dealers can adjust their inventories with (a) selective, nonpublic quoting, (b) interdealer markets, and (c) market-specific rules that allow the dealer to participate in trades without public quoting.

Time series //.
dealer inventory. $\{x_t\}$.
$\lim_{s\to\infty} \frac{\partial x_{t+s}}{\partial \varepsilon_t} = 0$.
t 시점 shock이 미래 inventory 수준에 미치는 영향이 감소를 줄이는.

milgrom
: uninformed trader에 거래할 확률이 높아지면 BID-Ask spread 높여버림. (∵ risk-aversion). adverse.

shock 발생. (positive).
→ i) Inventory.
: BID-Ask 줄어듦. (t→0).
ii) assymetric.
: Informed trader가 들어옴을 의미.
⇒ BID-Ask 유지됨.

ii). mendel. says.
Ask, BID I → I 낮춰서
⇒ mean-reverting.
⇒ $\boxed{\text{stationary process.}}$

주어진 data가 stationary냐 non-stationary냐.
⇒ unit-root test.
unit-root process 면 non-stationary.
⇒ inventory가 돌아오지 않고 머물러 있음 ⇒ dealer가 inventory control 하지 않고 있다는 것을 의미.
⇒ 기존 모델이 틀렸음을 의미.

main 요점.
Hasbrouck 말하길.
BID. Ask 영향.
① Adverse-selection risk.
② Inventory control.
어느 것이 dominate?
① > ②. 일정기간 유지 실제의 가까움. (B-A spread).
② > ①. temparary.

주요인은 adverse-selection risk에 의해
BID-Ask spread가 결정된다.

주식 A가 B보다 BID-Ask가 크다 ⇒ "A가 information assymetric이 더 심하다"로 해석함.

결론

BID-Ask spread는 주로 adverse selection risk로 결정된다.

Hasbrouck

: 1~3 이론 전부 부정하나? No.

dealer는 ① non-public.
② interdealer market.
③ market-specific rule.

inventory rule이 틀리지 않을 수 있다.

↳ inventory control이 가능하다. Dealer 입장

① Adver selection risk. (수동적)
milgrom
kyle.
└ 손해 피하기 위함
└ 손해볼 수 밖에 없음 (또는 해결책이 없으므로).
└ permanent).
informed trader 빠져나갈때 까지.

② Inventory control motivation (능동적)
garman
mandelson
스톨
└ 원래 원하는 수준.
└ 빨리 제자리 돌아옴.
└ temporary).

①이 크지만 결국 ② 무시할순 X.
여전히 inventory 관리하는 설이 많음.
(non-public quoting, ... 가능하다.)