20th CSEC – Past Year Paper Solution *2015-2016 Sem 1*
MH 1812 – Discrete Mathematics

1)

a) $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ (by definition)
$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$ (by definition)
$\equiv [(\neg p \vee q) \wedge \neg q] \vee [(\neg p \vee q) \wedge p]$ (DeMorgan's)
$\equiv [(\neg p \wedge \neg q) \vee (q \wedge \neg q)] \vee [(\neg p \wedge p) \vee (p \wedge q)]$ (DeMorgan's)
$\equiv (\neg p \wedge \neg q) \vee (p \wedge q)$
$\equiv p \wedge q \vee \neg p \wedge \neg q$ (Operator precedence)

b) $\neg\big(\exists x \in X, \forall y \in Y, P(x,y)\big) \equiv \big(\forall x \in X, \exists y \in Y, P(x,y)\big)$

X can only be 2 or 3:
① When $x = 2, y = 7$:
$x \equiv y \bmod 5 = 2$
That is, P (2,7) is true.

② When $x = 3, y = 8$:
$x \equiv y \bmod 5 = 3$
That is, P (3,8) is true.

So $\forall x \in X, \exists y \in Y, P(x,y)$ is true, which means $\neg\big(\exists x \in X, \forall y \in Y, P(x,y)\big)$ is true.

c) A valid argument satisfies:If the premises are true, then the conclusion is true.

$p \wedge q$	[premise 1]
∴ p is true, and q is true	[i]
$p \rightarrow \neg r$ and p is true	[premise 2 and i]
∴ $\neg r$ is true	[ii]
$q \rightarrow \neg s$ and q is true	[premise 3 and i]
∴ $\neg s$ is true	[iii]

So from (ii) and (iii), $\neg r \wedge \neg s$ is true.
According to the definition, this argument is valid.

2)

a)

i) Let $y \in f(S \cup T)$ arbitrary.
Then there exists $x \in (S \cup T)$ such that $f(x) = y$.
Since $x \in (S \cup T), x \in S$ or $x \in T$, then $f(x) \in f(S)$ or $f(x) \in f(T)$.
That is, $f(x) \in f(S) \cup f(T)$
Thus, $f(S \cup T) \subseteq f(S) \cup f(T)$ [1]

Let $y \in f(S) \cup f(T)$ arbitrary.
Then there exists $x \in S$ or $x \in T$ such that $f(x) = y$.
Since $x \in S$ or $x \in T$, $x \in (S \cup T)$, then $f(x) \in f(S \cup T)$
Thus, $f(S) \cup f(T) \subseteq f(S \cup T)$ [2]

From [1] and [2], $f(S \cup T) = f(S) \cup f(T)$.

ii) Disprove:
Suppose set A = {x1, x2, x3}, set Y = {y1, y2}, $f(x1) = f(x3) = y1, f(x2) = y2$.
If $S = \{x1, x2\}, T = \{x2, x3\}, then\ S \cap T = x2, f(x2) = y2$.
However, $f(S) = \{y1, y2\}, f(T) = \{y1, y2\}, f(S) \cap f(T) = \{y1, y2\}$, which is not equal to y2.

b) Yes, we can use membership table to solve this question. We should notice that premise is $A \oplus B = B \oplus C$, and we should get the conclusion that $A = C$.

$A \oplus B$	$B \oplus C$	B	A	C
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	1	1
1	0	0	1	1
1	0	1	0	0
1	1	0	1	1
1	1	1	0	0

And we can get the conclusion that $A = C$.

3) We can use matrix representation to solve this question.
Since the relation should be reflexive, which means every element of A is related to itself, the matrix's diagonal entries should all be true while the others can be true or false.

$$\begin{bmatrix} T & TorF & TorF & TorF \\ TorF & T & TorF & TorF \\ TorF & TorF & \ddots & TorF \\ TorF & TorF & TorF & T \end{bmatrix}$$

So we can construct 2^{n*n-n} distinct matrixes. That is, there are 2^{n*n-n} distinct reflexive relations on a set with n elements.

4)

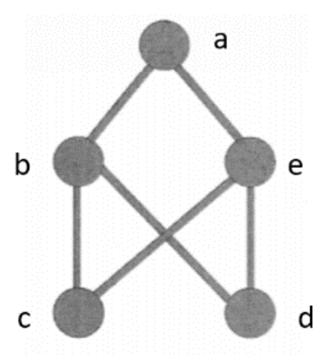

Figure 1: Graph

a) Yes, this graph is bipartite. Bipartite graph is a graph whose vertices can be partitioned into 2 (disjoint) subsets V and W such that each edge only connects a $v \in V$ and a $w \in W$.
Suppose $V = \{a, c, d\}$, $W = \{e, b\}$, then each edge only connects a node in V and a node in W. Thus, this graph is bipartite.

b) An Euler path (Eulerian trail) is a walk on the edges of a graph which uses each edge in the original graph exactly once. (The beginning and end of the walk may or not be the same vertex).
If we follow this order: e→c→b→d→e→a→b, we can walk on the edges of the graph which use each edge exactly once.

--End of Answers--

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