1 **(a)** $[(p \wedge q) \rightarrow r] \leftrightarrow [p \rightarrow (q \rightarrow r)]$

$\equiv [[(p \wedge q) \rightarrow r] \rightarrow [p \rightarrow (q \rightarrow r)]] \wedge [[p \rightarrow (q \rightarrow r)] \rightarrow [(p \wedge q) \rightarrow r]]$
(Definition Biconditional)

$\equiv [\sim[(p \wedge q) \rightarrow r] \vee [p \rightarrow (q \rightarrow r)]] \wedge [\sim[p \rightarrow (q \rightarrow r)] \vee [(p \wedge q) \rightarrow r]]$
(Conversion Theorem)

$\equiv [\sim[\sim(p \wedge q) \vee r] \vee [\sim p \vee (\sim q \vee r)]] \wedge [\sim[\sim p \vee (\sim q \vee r)] \vee [\sim(p \wedge q) \vee r]]$
(Conversion Theorem)

$\equiv [[(p \wedge q) \wedge \sim r] \vee [\sim p \vee (\sim q \vee r)]] \wedge [[p \wedge \sim(\sim q \vee r)] \vee [\sim(p \wedge q) \vee r]]$
(De Morgan's Theorem)

$\equiv [[(p \wedge q) \wedge \sim r] \vee [\sim p \vee (\sim q \vee r)]] \wedge [[p \wedge (q \wedge \sim r)] \vee [(\sim p \vee \sim q) \vee r]]$
(De Morgan's Theorem)

$\equiv [[p \wedge q \wedge \sim r] \vee [\sim p \vee \sim q \vee r]] \wedge [[p \wedge q \wedge \sim r] \vee [\sim p \vee \sim q \vee r]]$
(Commutativity)

$\equiv [p \wedge q \wedge \sim r] \vee [\sim p \vee \sim q \vee r]$
(Idempotent Law)

$\equiv [(p \wedge q) \wedge \sim r] \vee [\sim(p \wedge q) \vee \sim(\sim r)]$

$\equiv [(p \wedge q) \wedge \sim r] \vee \sim[(p \wedge q) \wedge \sim r]$
(De Morgan's Theorem)

$\equiv$ **T**

(b) **(i)** $S \cap T = \{ a \mid a \text{ is divisible by } 21, a \in Z \}$
$\forall x, \forall y \in S \cap T$
Let $z = x + y$
$z = 21k + 21p \qquad (k, p \in Z)$
$z = 21(k + p)$
$z = 21q \qquad (q \in Z)$
$\therefore z \in S \cap T$
$\Rightarrow S \cap T$ is closed under addition

(ii) $S \cup T = \{ a \mid a \text{ is divisible by 7 or 3 or both}, a \in Z \}$
$\forall x, \forall y \in S \cup T$
Let $z = x * y$
Since x and y were either divisible by 7 or by 3 or by both,
their product must also be divisible by either 7 or 3 or by both
$\therefore z \in S \cup T$

$\Rightarrow$ S ∪ T is closed under multiplication

2 **(a)** As A is the set of integers mod 4

A = { 0, 1, 2, 3 }

For f : A→ A f (x) = ax + b a, b ∈ A

Since f is injective and the size of domain is the same as the size of range,
f must also be surjective.

Also, f is a strictly increasing function as a,b are positive constants
with a≠0 as then f(x) = b, which is not injective.

There exists only one function that is all injective, surjective and increasing at the same time:
f(x) = x , with the mapping f : { (0,0) , (1,1) , (2,2) , (3,3) }

∴ Cardinality of the required set is One.

(b) Let (x,y) ∈ $A \times (B \cap C)$

$x \in A\ and\ y \in (B \cap C)$

$x \in A\ and\ y \in B\ and\ y \in C$

$x \in A\ and\ y \in B\quad and\quad x \in A\ and\ y \in C$

$x \in A\ and\ y \in B\quad and\quad x \in A\ and\ y \in C$

$(x, y) \in A \times B\quad and\quad (x, y) \in A \times C$

$(x, y) \in (A \times B) \cap (A\ \times C)$

$\therefore A \times (B \cap C) \subseteq (A \times B) \cap (A\ \times C)$

Let (x,y) ∈ $(A \times B) \cap (A\ \times C)$

$(x, y) \in A \times B\quad and\quad (x, y) \in A \times C$

$x \in A\ and\ y \in B\quad and\quad x \in A\ and\ y \in C$

$x \in A\ and\ y \in B\ and\ y \in C$

$x \in A\ and\ y \in (B \cap C)$

(x,y) ∈ $A \times (B \cap C)$

$\therefore (A \times B) \cap (A\ \times C) \subseteq A \times (B \cap C)$

$\Longrightarrow\ (A \times B) \cap (A\ \times C) =\ A \times (B \cap C)$

3 **(a)** Since $A \cap A = A\ and\ A \cap A \neq \emptyset$,

$(A, A) \notin R$

$\therefore R\ is\ not\ Reflexive$

(b) $\forall A, \forall B \in P(S)$
$A\ R\ B \implies A \cap B = \emptyset \implies B \cap A = \emptyset \implies B\ R\ A$
$\therefore R\ is\ Symmetric$

(c) $\forall A, \forall B \in P(S)\ with\ A \neq B,$
$A\ R\ B \implies A \cap B = \emptyset \implies B \cap A = \emptyset \implies B\ R\ A$
Since both $(A, B) \in R\ and\ (B, A) \in R\ with\ A \neq B,$
R is not Antisymmetric

(d) $\exists A, \exists B, \exists C \in P(S)\ where\ A \subset C$, we have
$A\ R\ B \implies A \cap B = \emptyset$
$B\ R\ C \implies B \cap C = \emptyset$
But $A \cap C \neq \emptyset \implies (A, C) \notin R$
Thus, it is possible $(A, B) \in R\ and\ (B, C) \in R\ but\ (A, C) \notin R$
$\therefore R\ is\ not\ Transitive$

4 No, the graph does not contain a Euler path as it has more than two vertices with an odd degree. (The graph consists of 4 vertices with an odd degree)

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