20th CSEC – Past Year Paper Solution *2016-2017 Sem 1*
CZ/CE 1011 – Engineering Mathematics I

1)

a) The effect on the modulus: No effect. Modulus remains the same.
The effect on the argument: The angle is rotated counterclockwise by 90 degrees.
$z = je^{1+j} = je \cdot e^j = je(\cos 1 + j\sin 1) = je\cos 1 - e\sin 1$
Modulus = 2.718
Argument = 2.57

b) $\left(1 - j\sqrt{3}\right)^{27}(-2-j2)^8 = \left(2\left(e^{-\frac{j\pi}{3}}\right)^{27}\right)\left(2\sqrt{2}\left(e^{-\frac{j3\pi}{4}}\right)\right)^8 = 2^{39}e^{j\pi} = -2^{39}$

c) $AB = (-2,-1,-1)$
$BC = (1,-1,3)$
$\boldsymbol{n} \cdot AB = \boldsymbol{n} \cdot BC = 0$
$\boldsymbol{n} = \left(1, -\frac{5}{4}, \frac{3}{4}\right)$

d)

i) $(1,2-2) \cdot (2,x,0) = 2 + 2x = 3 \cdot \sqrt{4+x^2} \cdot \frac{\sqrt{2}}{2}$
$x = 2, x = 14$

ii) $(3,0,1) \cdot \frac{(1,2,-2)}{\left(\sqrt{1+4+4}\right)^2} = \frac{1}{9}$

2)

a)

i) *Editor's comment:* Answer is not given by solver, but a general proof of why every row of an orthogonal matrix is a row vector of length 1 is required.

ii) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$ so it is orthogonal.

Editor's comment: For the second part, we can use $AA^T = I_n$ and multiply the inverse of A on the left of both sides of the equation to obtain $A^T = A^{-1}$.

b) Rewrite the linear systems

$$\begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 3 & 3 & 2 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & a & 4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 2 \\ b \end{bmatrix}$$

i) $a = 2, b \neq 4$

ii) $a = 2, b = 4$

c) $adj(A) = \begin{bmatrix} 6 & -2 \\ -1 & 3 \end{bmatrix}$
$\det(A) = 3 \cdot 6 - 1 \cdot 2 = 16$

$$A^{-1} = \frac{1}{\det(A)} adj(A) = \frac{1}{16}\begin{bmatrix} 6 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{3}{8} & -\frac{1}{8} \\ -\frac{1}{16} & \frac{3}{16} \end{bmatrix}$$

3)

a)

i) A bar chart is required.

ii) Mean = 6.15
Mode = 8
Median = 6.5
20th percentile = 4.8
Sample Variance = 2.43

iii) Only 2 marks here. Few lines of comments relating to the distribution will be enough.

b)

i) $P = \frac{1}{26^8}$

ii) $P = \frac{1}{C_{26}^8}$

c) For 3 people:

$$P = \frac{12 \cdot 11 \cdot 10}{12^3} = \frac{55}{72}$$

For n people:

$$P = \frac{12 \cdot 11 \cdot 10 \cdot \ldots \cdot (12 - n + 1)}{12^n}$$

If n > 12:
Not valid, because at least 2 of them should be born in the same month

4)

a)

i) Firstly, we observe that integral of f(t) from 0 to positive infinity is 1, so that we don't worry about any negative values of t.

$$P = 1 - \frac{\int_0^{30} f(t)}{\int_0^{\infty} f(t)} = 0.548$$

ii) $Sample\ size\ N = 20$

$$P(t > 60) = 1 - \frac{\int_0^{60} f(t)}{\int_0^{\infty} f(t)} = 0.3$$

Therefore, $delta = 20 \cdot 0.3 = 6$
The distribution of X is a binomial distribution.
$P(x = 0) = 0.7^{20}$
$P(x = 1) = 0.7^{19} \cdot 0.3$

$P(x = 2) = 0.7^{18} \cdot 0.3^2$

…

$P(x = 20) = 0.3^{20}$

b)

i) Null hypothesis: $\mu = 100$
Alternative hypothesis: $\mu \neq 100$
Significance level: $\alpha = 0.05$
Because $N \geq 30$, consider it as a normal distribution:

$$Z = \frac{96 - 100}{\frac{16.6}{\sqrt{30}}} = -1.32$$

$p = 0.1868 > 0.05$
Do no reject H_0.

ii) 85% confidence interval = 0.075 on each side of a standard distribution.
$Z = 1.44$
$CI = (91.635,100.364)$

c) Can be found in lab manual.
Newton's method is a method for finding successively better approximations to the roots (or zeroes) of a real-valued function. The Newton method in one variable is implemented as follows:
The method starts with a function f defined over the real numbers x, the function's derivative f', and an initial guess x0 for a root of the function f. If the function satisfies the assumptions made in the derivation of the formula and the initial guess is close, then a better approximation x1 is

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Geometrically, (x1,0) is the intersection of the x-axis and the tangent of the graph of f at (x0,f(x0)).
The process is repeated as

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

until a sufficiently accurate value is reached.

--End of Answers--

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