1 **(a)**

$$p \rightarrow (q \vee r) \equiv \neg p \vee (q \vee r) \qquad (Theorem)$$
$$\equiv (\neg p \vee q) \vee r \qquad (Associative\ Law)$$
$$\equiv \neg(\neg p \vee q) \rightarrow r \qquad (Theorem)$$
$$\equiv (p \wedge \neg q) \rightarrow r \qquad (De\ Morgan)$$

(b) **(i)**

- R is REFLEXIVE as we must have number of elements in A equal to number of elements in A → A R A
- R is SYMMETRY because if we have A R B, means number of elements in A equal to number of elements in B. Hence, we can also conclude number of elements in B is equal to number of elements in A → B R A also
- R is TRANSITIVE, because if A R B, and B R C, means number of elements in A equal to number of elements in B and also number of elements in B equal to number of elements in C. Hence, number of elements in A is equal to number of elements in C → A R C

(ii) We divide equivalence class based on

[0] = {{}}
[1] = {{1},{2},{3}}
[2] = {{1,2},{2,3},{1,3}}
[3] = {{1,2,3}}

2 **(a)** The characteristic equation is:

$$x^2 = 7x - 10$$
$$x^2 - 7x + 10 = 0$$
$$(x-2)(x-5) = 0$$

Hence, the root of the equation is 2 and 5, so we can write:

$$a_n = A.5^n + B.2^n$$

for some A and B constant. To solve for A and B, we substitute the value for n = 0,1:

$$a_0 = A + B = 2 \;|x2| \qquad 2A + 2B = 4$$
$$a_1 = 5A + 2B = 1 \;|x1| \qquad 5A + 2B = 1 \quad -$$
$$3A = -3$$

Thus, we can solve to get A = -1, and furthermore B = 2-(-1) = 3

Hence, we have that $a_n = 3.2^n - 5^n$

(b) First, we prove for the base case n = 2

We have that $\frac{1}{\sqrt{1}} > \frac{1}{\sqrt{2}}$ and so we have that $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} > \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

So, base case is proven.

Next, for the inductive step, we assume that for n = k, the equation is true, which is:

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{k}} > \sqrt{k}$$

We will prove that for n = k + 1, the equation is also true.

For that case the LHS is $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\cdots+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}$ and based on our assumption we have that:

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\cdots+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}>\sqrt{k}+\frac{1}{\sqrt{k+1}}=\frac{\sqrt{k(k+1)}+1}{\sqrt{k+1}}$$

Observe that $\sqrt{k(k+1)}>k$ and hence equation become

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\cdots+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}>\frac{k+1}{\sqrt{k+1}}=\sqrt{k+1}$$

So, we have that for n = k + 1 it is also true and hence the equation is proven by induction.

3 **(a)** Yes, as we have that the value of function is:

$$f(1)=3, f(2)=2, f(3)=2, f(4)=1$$

Hence, there are no a and b such that if f(a) = f(b) then a≠b, so f is one to one

(b) Yes, as we have that the value of function is:

$$g(1)=2, g(4)=2, g(3)=3, g(4)=1$$

Hence, for every element b in A, there always a in A such that f(a) = b and so f is onto

(c)

$$\begin{aligned} f\ o\ g\ (x) &= f\big(g(x)\big) \\ &= f(2^x\ mod\ 5) \\ &= 3^{2^x}\ mod\ 5 \end{aligned}$$

4 **(a)** We will use the fact that $A-B=A\cap\bar{B}$

So, the equation become:

$$\begin{aligned} (B-A)\cap(C-A) &\equiv (B\ \cap\ \bar{A})\cap(C\ \cap\ \bar{A}) \\ &\equiv (B\ \cap\ C)\cap(\bar{A}\ \cap\ \bar{A}) \\ &\equiv (B\ \cap\ C)\cap\ \bar{A} \\ &\equiv (B\ \cap\ C)-A \end{aligned}$$

(b) To satisfy the requirement, the committee must contain:

- 3 men, 2 women → $\binom{7}{3}.\binom{6}{2}=35.15=525\ ways$
- 4 men, 1 woman → $\binom{7}{4}.\binom{6}{1}=35.6=210\ ways$
- 5 men, 0 woman → $\binom{7}{5}.\binom{6}{0}=21.1=21\ ways$

Put it together, we have 756 ways to choose

5 One of the requirements to have Euler circuit is every vertex must have even degree as in the circuit, we must have number of edge enter the vertex is equal to number of edge out of the vertex, thus making the degree of every vertex is even.

Hence, there are no Euler circuit because vertex b has odd degree (3)

For Hamiltonian circuit, there exists : a-b-c

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