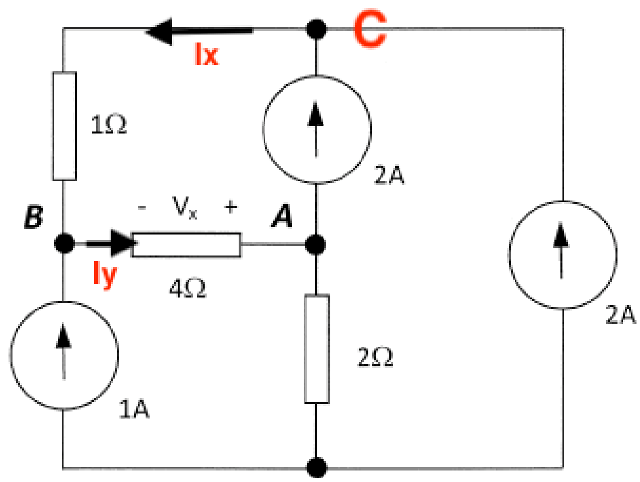
Solver: Fienny Angelina

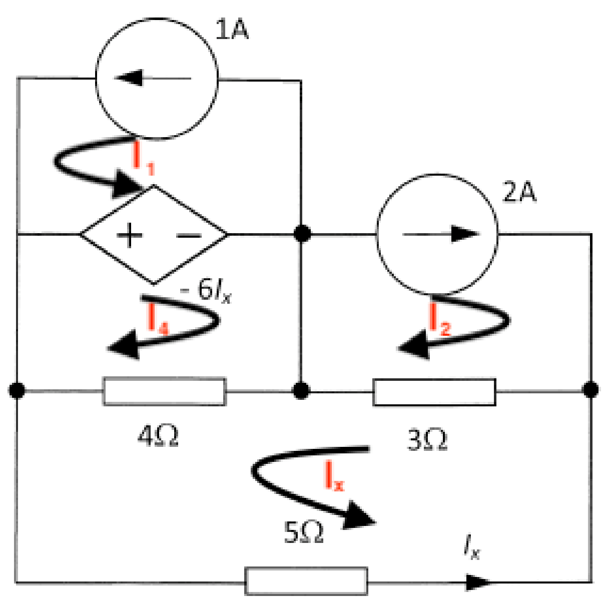
1)

a)



The fastest way is to use node analysis. On node C, $I_x = 2 + 2 = 4$. Furthermore, on node B: $I_y = I_x + 1 = 5\ A$. Thus, $Vx = -I_y * 4 = 20\ V$. (Note that the minus sign is there because the direction of the current is opposed to the direction from A to B. Current flows from higher voltage to lower voltage. Thus, V at B is higher than V at A).

b) Use Mesh Analysis. Number of circuit element = 6, Number of nodes = 3, Number of equations = 6 – 3 + 1 = 4 – 2 (number of known current) = 2.



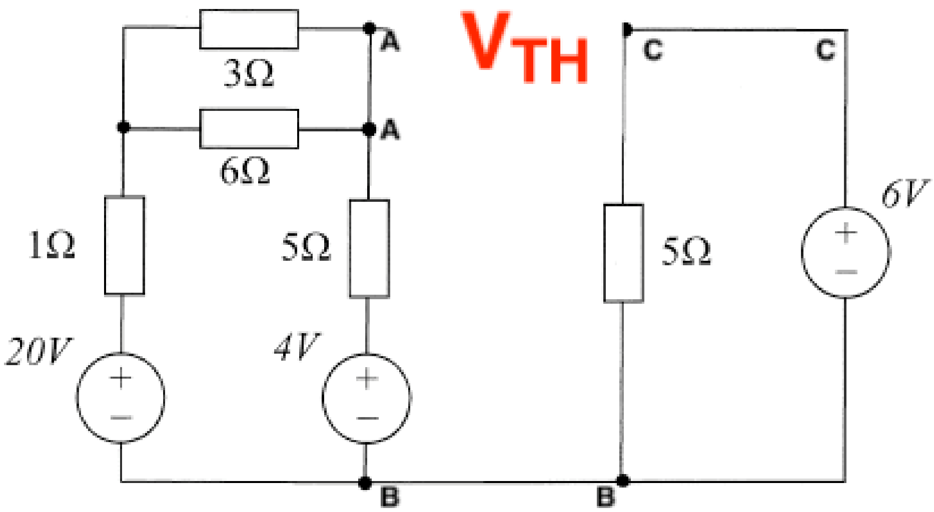
At I_4, $4 * I_4 + (-6 * I_x) = 0$. Thus, $I_4 = 1.5(I_x)$.

At I_x:

$5 * I_x + 3\,(I_x + I_2) + 4\,(I_x + I_4) = 0$

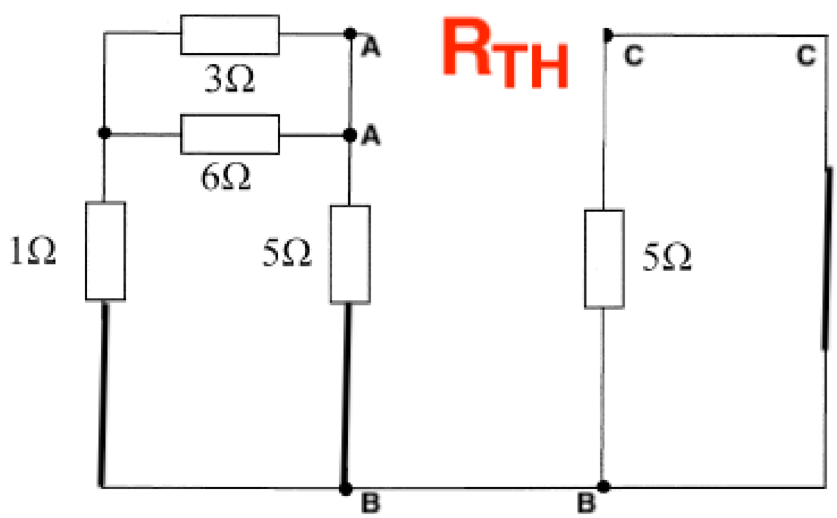
Substituting result from I_4 and the fact that $I_2 = 2$, we will obtain $I_x = -1A$.

c) Use Thevenin Theorem to calculate V_{TH} and R_{TH}.



$V_{TH} = V_{AB} + V_{BC}$
We know from the picture that $V_{BC} = -6V$.
To calculate V_{AB}, we need to know I_{AB}.
$I_{AB}= \frac{V_{Total}}{R_{Total}} = \frac{20-4}{1+3||6+5} = \frac{16}{8} = 2$
$V_{AB} = I_{AB} * 5 + 4 = 14\,V$
$V_{TH=} V_{AB} + V_{BC} = 14 + (-6) = 8\,V.$

To calculate R_{TH}, we will short circuit all voltage source.

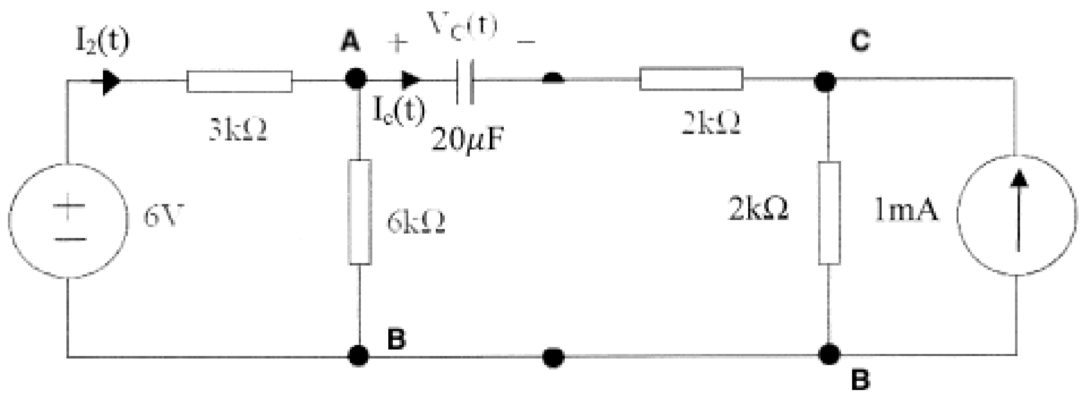


$R_{TH} = R_{AB} + R_{BC}$
$R_{TH} = (3\,||\,6) + 5 + 1 + 0 = 8$
For maximum power transfer,
$R_L = R_{TH} = 8\,ohm$

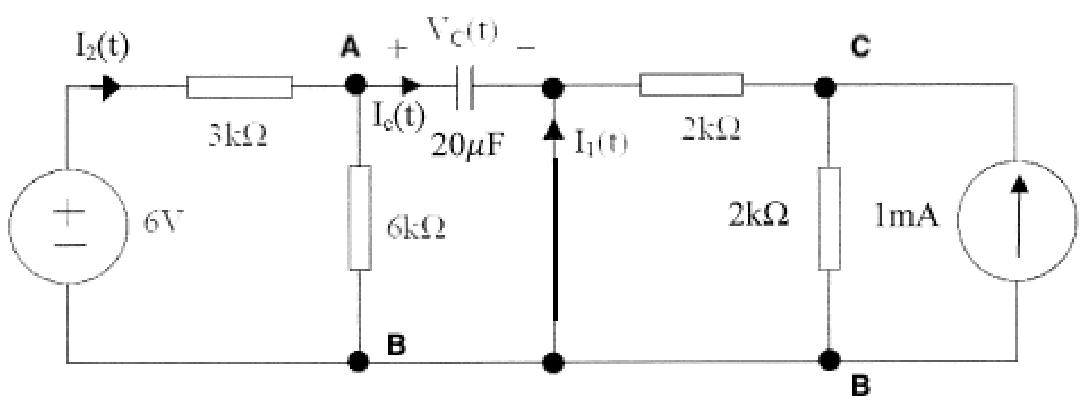
$$P_L = \left(\frac{V_{TH}}{2}\right)^2 \frac{1}{R} = 2\,W$$

2)

a) Below is the drawing of the circuit when t < 0.

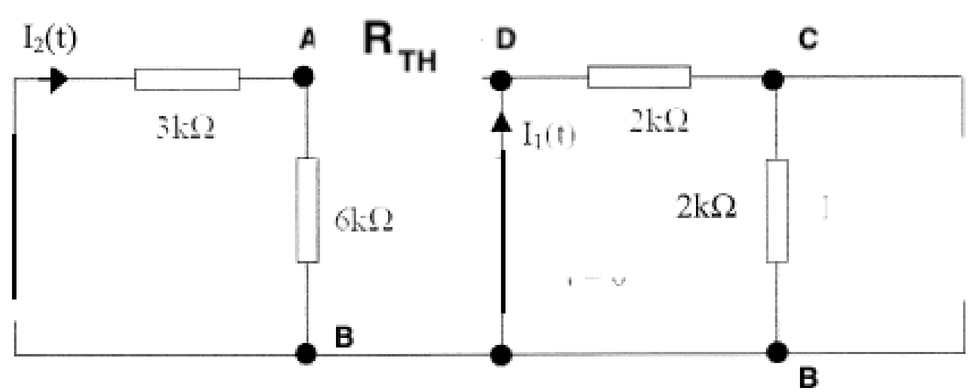


Below is the drawing when t >= 0.



General solution: $V_c(t) = K_1 + K_2\, e^{-t/\tau}$

Find the unknown constants K_1, K_2, τ .
$\tau = R_{TH}C$
Since we only have independent source, we can calculate R_{TH} directly.



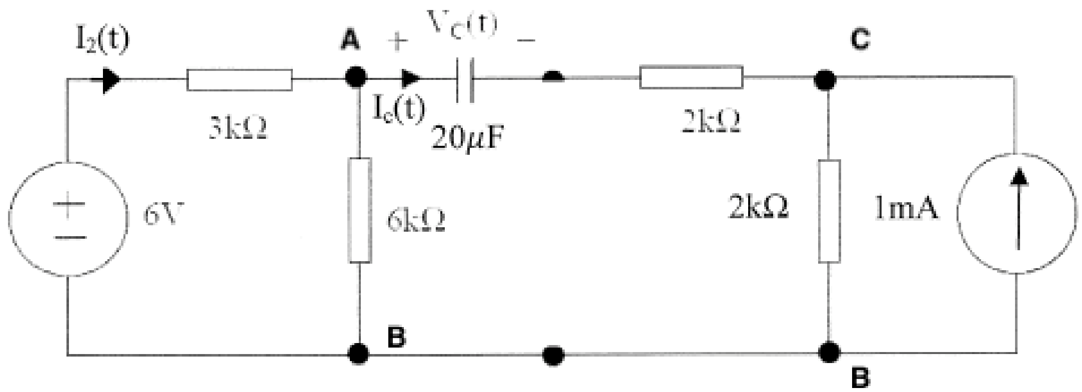
$R_{TH} = 3\,||\,6\ + 0 = 2\ Kohm$

$\tau = R_{TH}C = \ 2000 * 20 * 10^{-6} = 0.04$

Initial condition,

$V_C(0^+) = V_C(0^-)$

Before t = 0, capacitor is fully filled, thus no current pass through the capacitor. Below is again the circuit drawing when t = 0.



$V_C(0^-) = V_{AB} + \ V_{BC}$

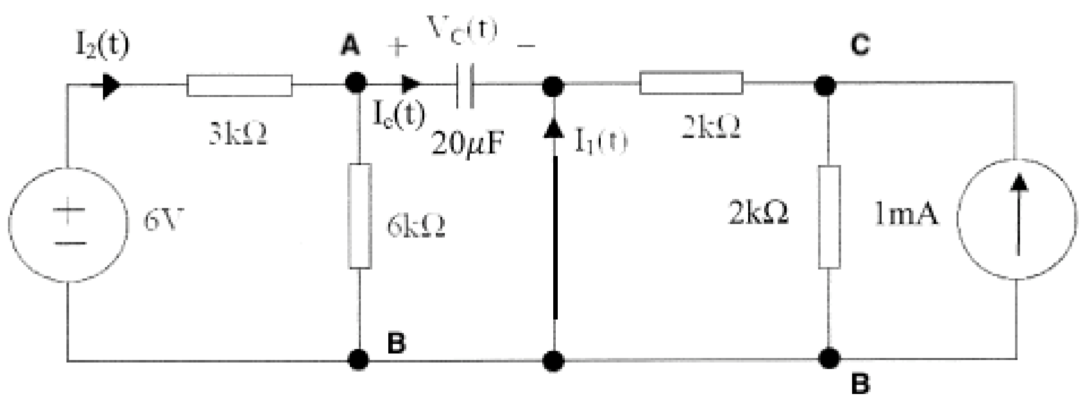
$V_C(0^-) = 6 * \left(\frac{6}{3+6}\right) + \ 2kohm * (-1mA) = 2$

$V_C(0^+) = V_C(0^-) = 2$

Put the result for $V_C(0^+)$ to the general solution,

$V_C(0^+) = K_1 + \ K_2 = \ 2$

Final condition, capacitor is fully filled, thus no current pass through the capacitor.



$V_C(\infty) = V_{AB} + \ V_{BC}$

$V_{AB} = \ 6 * \left(\frac{6}{3+6}\right) = 4V$

$V_{BC} = \ -I_{CB} * \ 2k = \ -0.5mA * 2k = \ -1\,V$

Note that $I_{CB} = \frac{I_{Source}}{2}$ through the current division rule. (The current source supports 2 resistors with same resistance in parallel).

$V_C(\infty) = V_{AB} + \ V_{BC} = 4 - 1 = 3\,V$

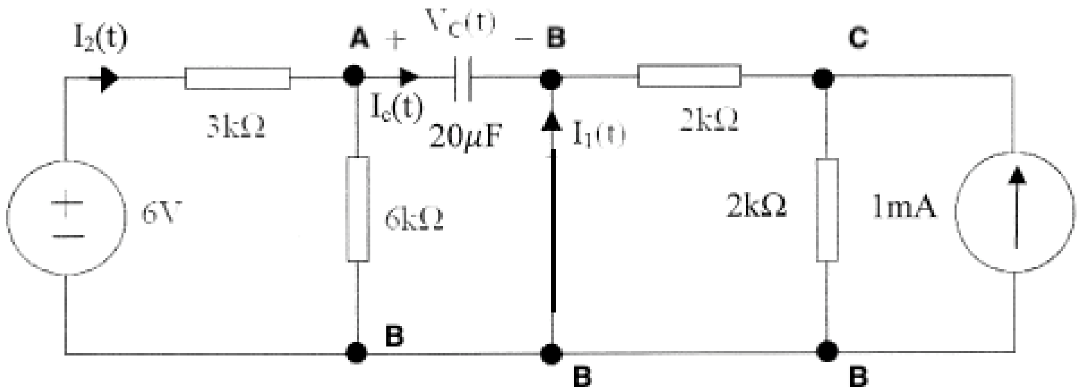
Put this back to the general solution:

$V_C(\infty) = K_1 = 3V$

Thus, we know that $K_2 = 2 - K_1 = -1$

i) Substituting every variable to the general solution, we shall obtain:
$V_c\,(t) = \ -1 + 2\,e^{-t/0.04}$

ii) When t > 0,



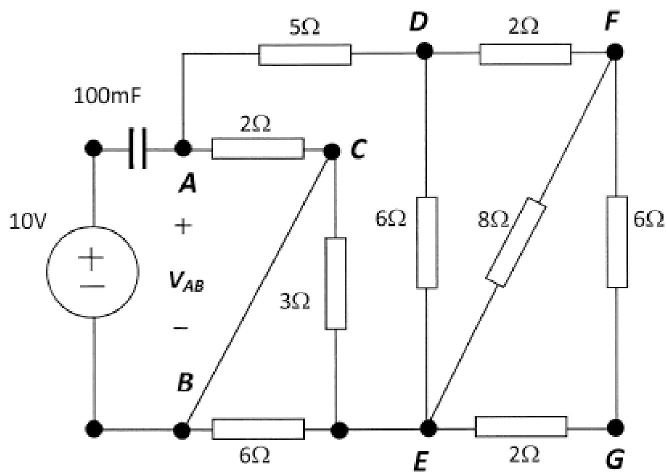
Looking at the loop in the top left corner:
$6 = I_2 * 3k + V_{AB}$
We know that: $V_{AB} = V_C(t)$ when t > 0.
Thus,
$6 = I_2 * 3k + V_C(t)$

$$I_2 = \frac{6 - \ V_c(t)}{3000}A$$

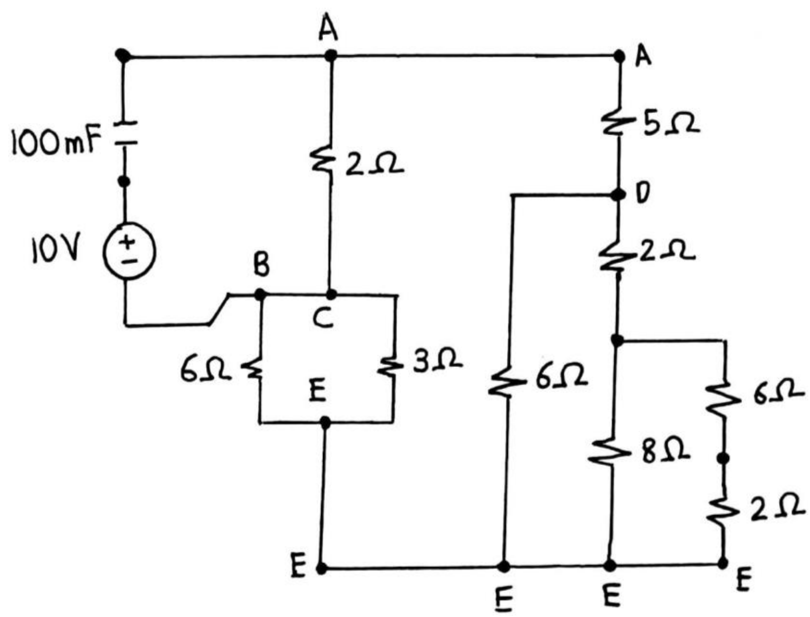
$$I_2 = \frac{6 - (-1 + 2\,e^{-\frac{t}{0.04}})}{3000}A$$

$$I_2 = \frac{7 - 2\,e^{-\frac{t}{0.04}})}{3000}A$$

b) First, let's simplify the circuit.



We can redraw the circuit:



$R_{FGE} = 6 + 2 = 8\ ohm$
$R_{FE} = R_{FGE}\,||\,8 = 8\,||\,8 = 4\ ohm$
$R_{DFE} = 2 + R_{FE} = 2 + 4 = 6\ ohm$
$R_{DE} = 6\,||\,R_{DFE} = 3\ ohm$
$R_{ADE} = 5 + R_{DE} = 8\ ohm$
$R_{CE} = 6\,||\,3 = 2\ ohm$

$$R_{AC/AB} = (R_{CE} + R_{ADE})\,||\,2 = 10\,||\,2 = \frac{5}{3}\,ohm$$

$$\tau = R_{AB} * C = \frac{5}{3} * 0.1 = \frac{1}{6}$$

Assume that the circuit is connected when t = 0, then $V_c(0^+) = 0$.
Moreover, at time t = ∞, the capacitor is fully filled, thus $V_c(\infty) = V_{source} = 10\ V$.
Put the general solution:

$V_c\,(t) = \ K_1 + K_2\ e^{-\frac{t}{\tau}}$
And substitute for t = 0 and infinity, as well as the tau value. We will obtain:
$V_c\,(t) = \ 10 - 10\ e^{-6t}$
Moreover, we know that $V_{AB}(t) = V_{source} - \ V_c\,(t) = \ 10\ e^{-6t}$.
Substitute for t = 1, we will get $V_{AB}(t) = 0.024787\ V$

3)

a)

i) $V_+ = \frac{10}{25}V_2 = \frac{10}{25}(-1.5) = \ -0.6\ V$
From op.amp rule: $V_- = V_+ = \ -0.6\ V$
The current that goes to the 50kOhm resistor: $I_- = \frac{V_1\,-\,V_-}{50K} = \frac{2.1}{50K} = 4.2 * 10^{-5}\ A$
This same current flow to the 75kOhm resistor:
$$I_- = \frac{V_- \ - \ V_O}{75K}$$
$$V_O = V_- \ - 75k * I_-$$
$$V_O = -0.6 \ - 75k * 4.2 * 10^{-5} = \ -3.75\ V$$

ii) To prevent output signal distortion,
$|V_O| \leq 15\ V$
We know that the current that goes to the 50k resistor = The current that goes to the 75k resistor.
$$I_- = \ I_-$$
$$(V_1 - V_-)/50K = (V_- - Vo)\ /\ 75K$$
$$V_o = \frac{5\ V_- - 3\ V_1}{2}$$
$$-15 \leq \ \frac{5\ V_- - 3\ V_1}{2} \leq 15$$
$$-15 \leq \ \frac{5\ V_- - 3 * 1.5}{2} \leq 15$$
$$-30 \leq \ 5\ V_- - 4.5 \leq 30$$
$$-25.5 \leq \ 5\ V_- \leq 34.5$$
$$-5.1 \leq \ V_- \leq 6.9$$
By voltage division rule, we know that
$$V_- = \ \frac{10}{25}V_2$$
$$-5.1 \leq \ V_- \leq 6.9$$
$$-5.1 \leq \frac{10}{25}V_2 \leq 6.9$$
$$-12.75 \leq V_2 \leq 17.25$$

b)

i) $y_1(t)$ is not causal since it depends on input at time $(t+2)$. $y_1(t)$ is memory, because it depends on input other than $x(t)$
$y_2(t)$ is causal. $y_2(t)$ is memoryless.

$y_3(t)$ might be causal depending on the value of the time constant (t_0). If $t_o \geq 5$, it is causal. Otherwise, it is not causal. $y_3(t)$ is memoryless if and only if $t_o = 5$
$y_4(t)$ is causal. $y_4(t)$ is memory.

ii) Yes, because memoryless system means it only depends on the current input value. While the definition of a causal system is that it depends only on the current / past input values. Thus, memoryless system is a causal system

4)

a) Yes. It is because $y_1(t) = g(t) * f(t) = f(t) * g(t) = y_2(t)$

b)

i) $y_o(t) = \int_{-\infty}^{t} x(t)\, dt$

$y(t) = y_o(t) + y_o(t-3)$

$$y(t) = \int_{-\infty}^{t} x(t)\, dt + \int_{-\infty}^{t-3} x(t-3)\, dt$$

We know that when $x(t) = e^{st},\ y(t) = e^{st}H(s)$. (Eigen function).

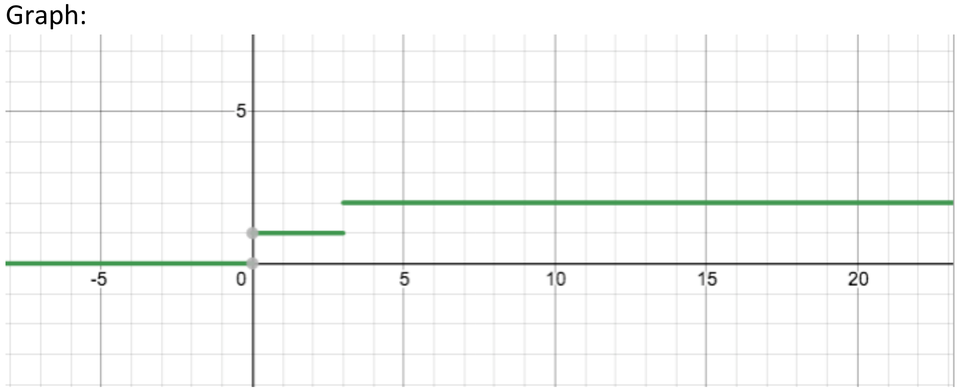
$$y(t) = \int_{-\infty}^{t} e^{st}\, dt + \int_{-\infty}^{t-3} e^{s(t-3)} dt$$

$$e^{st}H(s) = \frac{1}{s}e^{st} + \frac{1}{s}e^{s(t-3)}$$

$$H(s) = \frac{1}{s} + \frac{1}{s}\, e^{-3s}$$

$$h(t) = u(t) + u(t-3)$$

Graph:



ii) $y(t) = \int_{-\infty}^{t} x(t)\, dt + \int_{-\infty}^{t-3} x(t-3)\, dt$

$$y(t) = \int_{-\infty}^{t} \delta(t+2.5)\, dt + \int_{-\infty}^{t-3} \delta(t-0.5)\, dt$$

$$y(t) = u(t+2.5)\Big|_{-\infty}^{t} + u(t-0.5)\Big|_{-\infty}^{t-3}$$

$$y(t) = u(t+2.5) - u(-\infty) + u(t-3.5) - u(-\infty)$$

$$y(t) = u(t+2.5) + u(t-3.5)$$

iii) Apply Laplace transform to signal x(t)

$$x(t) = 0.3\,e^{-0.5t}u(t) \rightarrow X(S) = \frac{0.3}{S+0.5}$$

$$h(t) = u(t) + u(t-3) \rightarrow H(S) = \frac{1}{S} + \frac{e^{-3S}}{S} \ldots (from\ part\ 1)$$

The product of the transfer function and Laplace transform of the input signal:

$$H(S)X(S) = \frac{0.3}{S(S+0.5)} + \frac{0.3}{S(S+0.5)}\ e^{-(3S)}$$

Using partial fraction method, $\frac{0.3}{S(S+0.5)} = \frac{A}{S} + \frac{B}{S+0.5}$

$$A = \left[S\frac{0.3}{S(S+0.5)}\right]_{S\,=\,0} = 0.6$$

$$B = \left[(S+0.5)\frac{0.3}{S(S+0.5)}\right]_{S\,=\,-0.5} = -0.6$$

Thus,

$$H(S)X(S) = \frac{0.6}{S} - \frac{0.6}{S+0.5} + \frac{0.6}{S}\ e^{-(3S)} - \frac{0.6}{S+0.5}e^{-3S}$$

Take the inverse Laplace transform, we get:

$$y(t) = 0.6\,u(t) - 0.6\,e^{-0.5t}u(t) + 0.6\,u(t-3) - 0.6\,e^{-0.5(t-3)}u(t-3)$$

c) Time based system analysis requires two steps (measuring impulse response and convolution). Frequency based needs 3 steps (Transform input signal, find transfer. Function, inverse transform of the product of the first two). However, the advantage is that there is no need to do complex integration

--End of Answers--