Suggested Solution to CE2004 Circuit and Signal Analysis

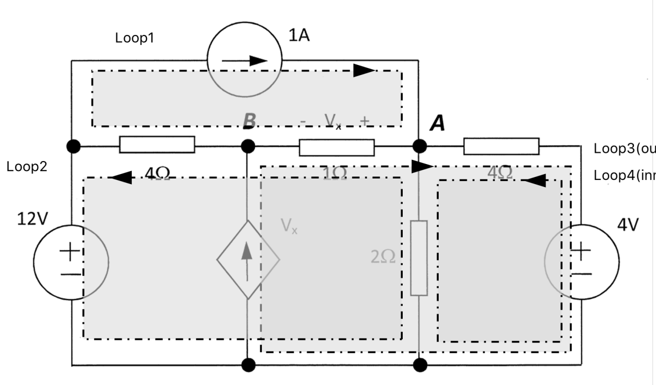
Provider: LI Yuanming YLI056@e.ntu.edu.sg

1.

Solve.

(a)

By loop analysis, we set voltage of each nodes as shown on the graph:



We need to solve for Loop2, 4.

Set Loop2, 4 have the current of I_2, I_4.

By KVL,

Loop2: $4(I_2+1)+(1+I_2-V_x)+2(I_2-I_4)+12=0$,

Loop4: $4(I_4-V_x)+2(I_4-I_2)=4$.

There is one more equation for V_x: $V_x=(1+I_2-V_x)$

Solve linear equation system (1), (2), (3),

We have $V_x=-\frac{28}{31}$

We can also apply other loops/meshes to obtain the result. Node analysis can be applied for verify the answer. However, as required, we shall not apply node analysis for solving.

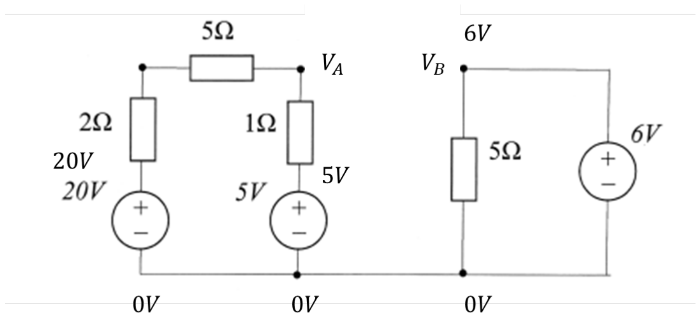
(b)

By Thevenin's Theorem, we transfer the circuit to its Thevenin equivalence by

i) Obtaining equivalent voltage

ii) Obtaining equivalent resistance

We open the R_L, and find the voltage between the load resistor, or $V_B - V_A$.

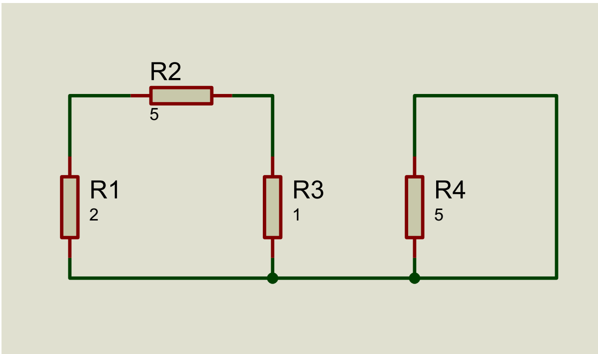


Obviously, $V_B = 6V$.

By Voltage Divider Rule, $V_A = 5 + \frac{1}{1+5+2} \cdot (20-5) = 6.875(V)$.

Therefore, $V_{TH} = 6.875V - 6V = 0.875(V)$

There is no dependent source. We simply short all the voltage source,



$$R_{TH} = (5+2)//1 + 0 = \frac{7}{8}\Omega$$

The maximum power happens when $R_L = R_{TH}$. Therefore, $R_L = 0.875\Omega$.

The maximum power $P_{max} = \frac{V_{TH}^2}{4R_{TH}} = 0.219W$.

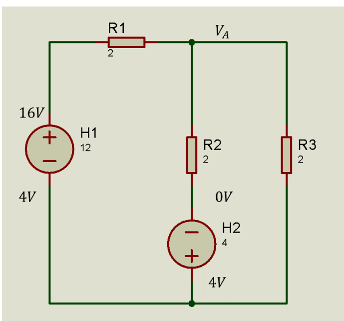
End of Solution.

2.

Solve.

(a)

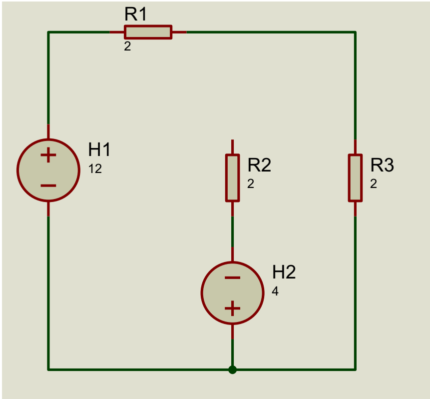
At $t \to 0^-$, the inductor acts as a short circuit. The current passing through L_1: $\frac{V_A - 4}{R_3}$.



$$\frac{16 - V_A}{2} = \frac{V_A}{2} + \frac{V_A - 4}{2}$$

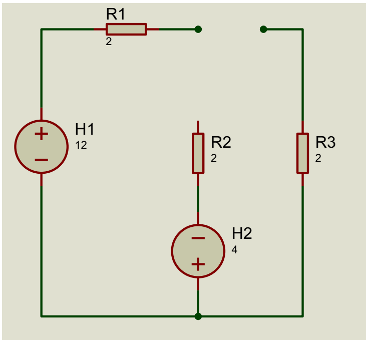
Thus, $V_A = \frac{20}{3}V, i_{0^-} = i_{0^+} = \frac{4}{3}A$

At $t \to \infty$,



$$i_\infty = \frac{12}{2+2} = 3(A)$$

We treat the inductor as a load, by Thevenin's Theorem,



$$V_{TH} = 12V, R_{TH} = 4\Omega$$
$$\tau = R_{TH}/L = 2s$$

Using step-by-step approach,

we have the general solution to the current passing through L_1

$$i(t) = K_1 + K_2 \cdot \exp\left(-\frac{t}{\tau}\right)$$

Solve for K_1, K_2,

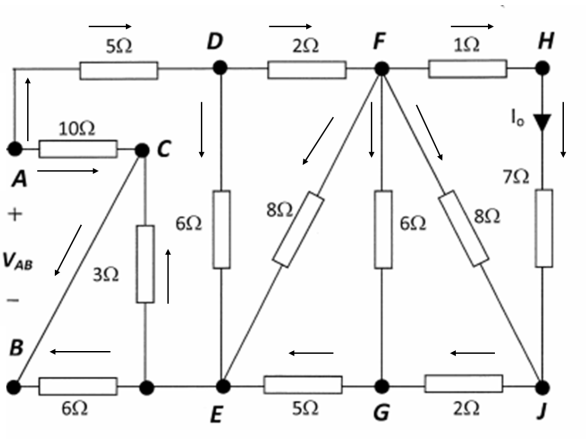
$$K_1 = 3, K_2 = -\frac{5}{3}$$

$$V_0(t) = i_t \cdot R_3 = 6 - \frac{10}{3} \cdot exp(-t/2)$$

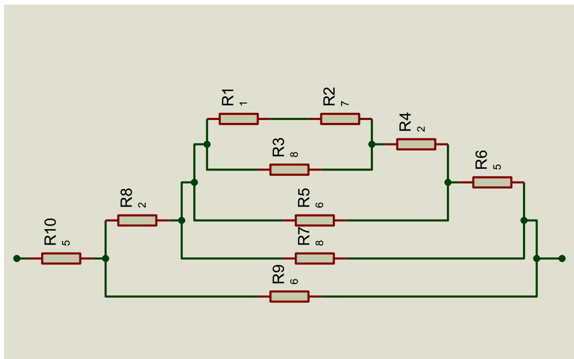
(b)

We would like to find the Thevenin's equivalence between node A and node B if a voltage of V_{AB} is applied.

The circuit is not complicated, as the current flow is clear. We would like to find R_{TH} by analyzing the current.

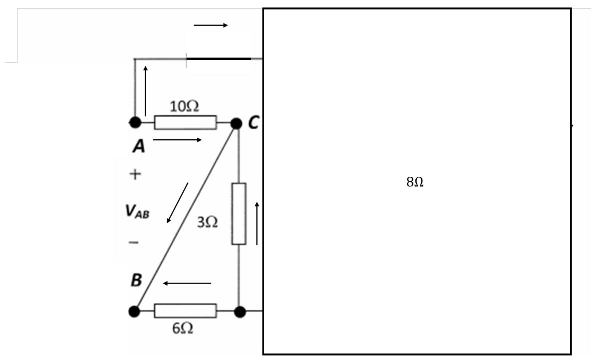


First, we find equivalent resistance between node A and E:



$$\mathrm{R_{AE}} = \left(\left(\left(((1+7)//8+2)//6+5\right)//8+2\right)//6+5 = 8(\Omega)\right.$$

Now we deal with $\mathrm{R_{AB}}$:



As B and C have the same potential, $\mathrm{R_{TH}} = \mathrm{R_{AB}} = \mathrm{R_{AC}}$.

$$\mathrm{R_{AC}} = (8 + 3//6)//10 = 5(\Omega)$$

Therefore, the time constant $\tau = R_{TH}C = 50\mu s$, $V_{AB} = 10 \cdot \exp(-1) = 3.679V$

End of Solution.

3.

Solve.

(a)

(i)

Assume $\mathrm{X_1(t)}$ is periodical. Therefore, it has the relationship $\mathrm{X_1(t) = X_1(t+T)}$.

$$\sin(3\omega t+\sigma)=\sin(3\omega(t+T)+\sigma)$$
$$3\omega t+\sigma=3\omega(t+T)+\sigma+2k\pi$$
$$T=\frac{2k\pi}{3\omega}$$

We let period to be the minimum positive T:

$$T=\frac{2\pi}{3\omega}$$

$X_1(t)$ is a periodic function.

(ii)

$X_2(t)$ to be an odd function, $X_2(0)=0$.

Therefore, $AX_1(0)+0=\sin(0+\sigma)=0$. σ should be 2kπ.

Moreover, $X_2(t)=-X_2(-t)$, which is $-AX_1(t)-AX_1(-t)=2|t|$

LHS is less than 2A, a constant. As t increases, LHS cannot be equal to RHS.

Therefore, there is no such combination of ω and σ, such that $X_2(t)$ is an odd function.

(b)

(1)

$0.3t \le t$ is always satisfied for $t \ge 0$. Therefore, the output of the system always depends on the past/current input. The system is causal.

(2)

Let $y_1(t)=x_1(0.3t), y_2(t)=x_2(0.3t)$.

For additivity, we have

$$y(t)=\big(x_1(0.3t)+x_2(0.3t)\big)=y_1(t)+y_2(t)$$

For homogeneity, we have

$$y(t)=k\big(x_1(0.3t)\big)=ky_1(t)$$

Therefore, the system is linear.

(3)

For t = 1, $y(1)=x(0.3)$. The system depends on the past input. It is memory.

(c)

$$y(t)=h_1(t)*x(t)+h_2(t)*x(t)$$
$$=\int h_1(\tau)x(t-\tau)dt+\int h_2(\tau)x(t-\tau)dt$$
$$=\int\big(h_1(\tau)+h_2(\tau)\big)x(t-\tau)dt$$
$$=\big(h_1(t)+h_2(t)\big)*x(t)$$

Therefore, the system described in (a) is the same as (b).

(d)

Supply $x(t)=e^{st}$, and let $y(t)=h(t)*e^{st}$. By Laplace transform,

$$\mathcal{L}[LHS]=2.5H(s)e^{st}+3sH(s)e^{st}+0.5s^2H(s)e^{st}$$

$$\mathcal{L}[\mathrm{RHS}]=0.5e^{st}+1.5se^{st}$$

Solve the equation for H(s):

$$\mathrm{H(s)} = \frac{0.5 + 1.5\mathrm{s}}{2.5 + 3\mathrm{s} + 0.5\mathrm{s}^2} = \frac{3\mathrm{s} + 1}{(\mathrm{s} + 1)(\mathrm{s} + 5)}$$

$$\mathrm{X(s)} = \mathcal{L}[4\mathrm{e}^{-3\mathrm{t}}u(\mathrm{t})] = 4\frac{1}{\mathrm{s} + 3}$$

Therefore, $Y(s) = H(s)X(s) = \frac{4(3s+1)}{(s+1)(s+5)(s+3)} = -\frac{1}{1+\mathrm{s}} + \frac{8}{3+\mathrm{s}} - \frac{7}{5+\mathrm{s}}$

We do inverse Laplace Transform,

$$y(t) = \mathcal{L}^{-1}[Y(s)] = -7e^{-5t} + 8e^{-3t} - e^{-t}$$

End of Solution.

4.

Solve.

(a)

The applied negative feedback can improve its performance (gain stability, linearity, frequency response, step response) and reduces sensitivity to parameter variations due to manufacturing or environment. (source from https://en.m.wikipedia.org/wiki/Negative-feedback_amplifier) Only inverting op amp has negative feedback.

(b)

Apply superposition, we let R_2 and R_3 grounded. Therefore, V_- is grounded. The gained voltage for non-inverting op amp is $V_1 = \frac{R_f}{R_1}V_{\text{input}}$. Similarly, we have

$$V_2 = \frac{R_f}{R_2}\mathrm{V}_{\text{input}}, V_3 = \frac{R_f}{R_3}\mathrm{V}_{\text{input}}$$

$$V_{\text{output}} = V_1 + V_2 + V_3 = (R_1^{-1} + R_2^{-1} + R_3^{-1})R_f V_{\text{input}}$$

(c)

(i) For no distortion, $V_{\text{output}} \in [V_{\text{EE}}, V_{\text{CC}}]$, which is

$$3V_{\text{input}} \in [-15\mathrm{V}, 15\mathrm{V}]$$

as the relationship of $R_{1,2,3}$ and R_f.

The condition of V_{input} should be $V_{\text{input}} \in [-5\mathrm{V}, 5\mathrm{V}]$.

(ii) When $R_{2,3}$ are grounded, the voltage gain is unity. Such op amp can act as a voltage buffer to transfer a voltage source from one impedance level to another without affecting the current, no matter how the impedance between the input circuit and output circuit.

The input impedance is $Z_{\text{in}} = R_2//R_3//Z_{original\ in} = \frac{R_2R_3}{R_2+R_3}$, the output impedance is $Z_{\text{out}} = R_f$

End of Solution.

End of Paper.