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1a

Loop: $-5\text{ V} + I_3(1\Omega) + (I_3 + I_1)1\Omega + (I_3 + I_2)2\Omega = 0\text{ V}$

Mesh: $(I_2 - I_1)(2\Omega) + (I_3 + I_2)2\Omega - 3.6\text{V} = 0\text{ V}$

$I_0 = I_2 + I_3$

$I_1 = 1\text{A}$

From the equations above, we can get:

$I_3 = 0.4\text{ V}$

$I_2 = 1.2\text{ V}$

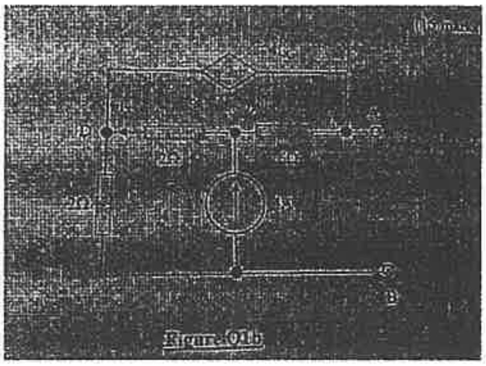
$I_0 = 1.6\text{ V}$

Hence, $V_0 = I_0 \,.\, 2\Omega = 3.2\text{ V}$

1b.

$Ix = I_1 + I_2$

$I_2 = 4\text{A}$



Mesh 1: $-4\,Ix + 6\,I_1 + 2Ix = 0$

$-4(4 + I_1) + 6I_1 + 2(4+I_1) = 0$

$12\,I_1 = 8$

$I_1 = 2/3\text{ A}$

$Ix = 4\ 2/3\text{ A}$

VAB $= \text{VAD} + \text{VDB}$

$= 4\,Ix + 4(2)$

$= 26\ 2/3\text{ V}$

Mesh: $-v + (I_1 + I_2)\,2 + I_2(2) = 0$

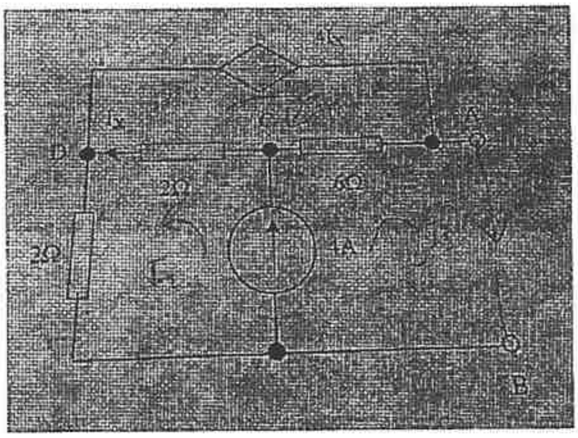
Mesh: $-v + (I_3 - I_1)\,6 = 0$

Mesh: $-4(I_1+I_2) + 6(I_1-I_3) + 2(I_1+I_2) = 0$

Solving the above equations,

$I_3 = 8/14$ A

RTh = (26 2/3) / (8/14) = 46 2/3 Ω



2a.i.

Using step by step approach:

$Vc(t) = k1 + k2\, e^{-t/T}$

$\tau = C\, RTh$

Short Circuiting independent voltage source and open circuiting independent current source.

RTh = 3//6 = 2 Ω

$\tau = 50(10^{-6})\,(2) = 10^{-4}$

$Vc(0+) = k1 + k2 = Vc(0-)$

$6\text{ V} = I_2(6+3)$

$I_2 = -2/3$ mA

VAB = 2/3 (6) = 4V

VCD = 2mA (1KΩ) = 2V

Vc(0-) = VAB + VDC = 2V

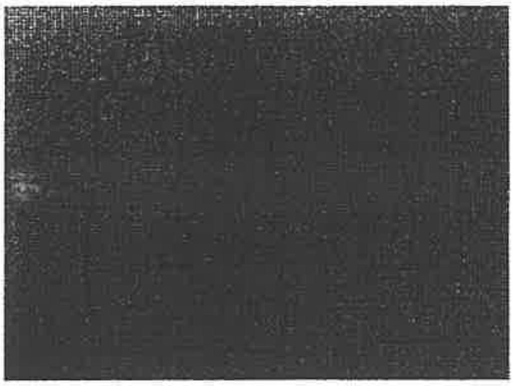
At ∞, capacitor acts like open circuit.

Vc(∞) = k1 = 4 V

K2 = 2 V – 4V = -2 V

$Vc(t) = 4 - 2e^{-10000t}$

2a.ii



Mesh 1:

$6\ V + (I_2(t)+Ic(t))\ (6K\Omega) + I_2(t)\ (3K\Omega) = 0$

Mesh 2:

$Vc(t) + (I_2(t)+Ic(t))\ (6K\Omega) = 0$

Subtracting equations of mesh 2 into the one of mesh 1:

$6\ V + I_2(t)\ (3K\Omega) - Vc(t) = 0$

$I_2(t) = (-6 + Vc(t))/3$ mA

$I_2(t) = (-6+4-2e^{-10000t})/3$ mA

Using equation from mesh 2:

$Vc(t) + (I2(t)+Ic(t))\ (6K\Omega) = 0$

$Ic(t) = (12 - 3Vc(t))/6$ mA

$Ic(t) = (12 - 3(4-2e^{-10000t}))/6$ mA

$Ic(t) = e^{-10000t}$ mA

2b.

$R_{PQ}= ((2+2//4)+2)//2 + 2 = 3\ 1/3\ K\Omega$

$R_{PR} = ((2+2)//4+2)//2 = 4/3\ K\Omega$

$R_{PS} = (((2+2)//4)+2)//2 + 1 = 2\ 1/3\ K\Omega$

$R_{QR} = ((2+2)//4)//4 + 2 = 3\ 1/3\ K\Omega$

$R_{QS} = (((2+2)//4)//\ (2+2)) + 2 + 1 = 4\ 1/3\ K\Omega$

$R_{RS} = 1\ K\Omega$

Max load current = $12\ V /(1k\ \Omega) = 12mA$

3a.i.

y(t) = cos(2t)x(t)

y1(t) = cos(2t)x1(t)

x2(t) = x1(t-t0)

y2(t) = cos(2t)x2(t)

= cos(2t) x1(t-t0) i

y1(t-t0) = cos(2(t-t0)) x1(t-t0) ii

i is not the same as ii. Therefore, y(t) is time variant.

3.a.ii.

x1(t) -> cos(2t)x1(t)

x2(t) -> cos(2t)x2(t)

x1(t) + x2(t) -> cos(2t)(x1(t) + x2(t)) i

y1(t) + y2(t) = cos(2t)(x1(t) + x2(t)) ii

i is similar to ii. Therefore, it satisfies the additivity properties.

x1(t) -> cos(2t)x1(t)

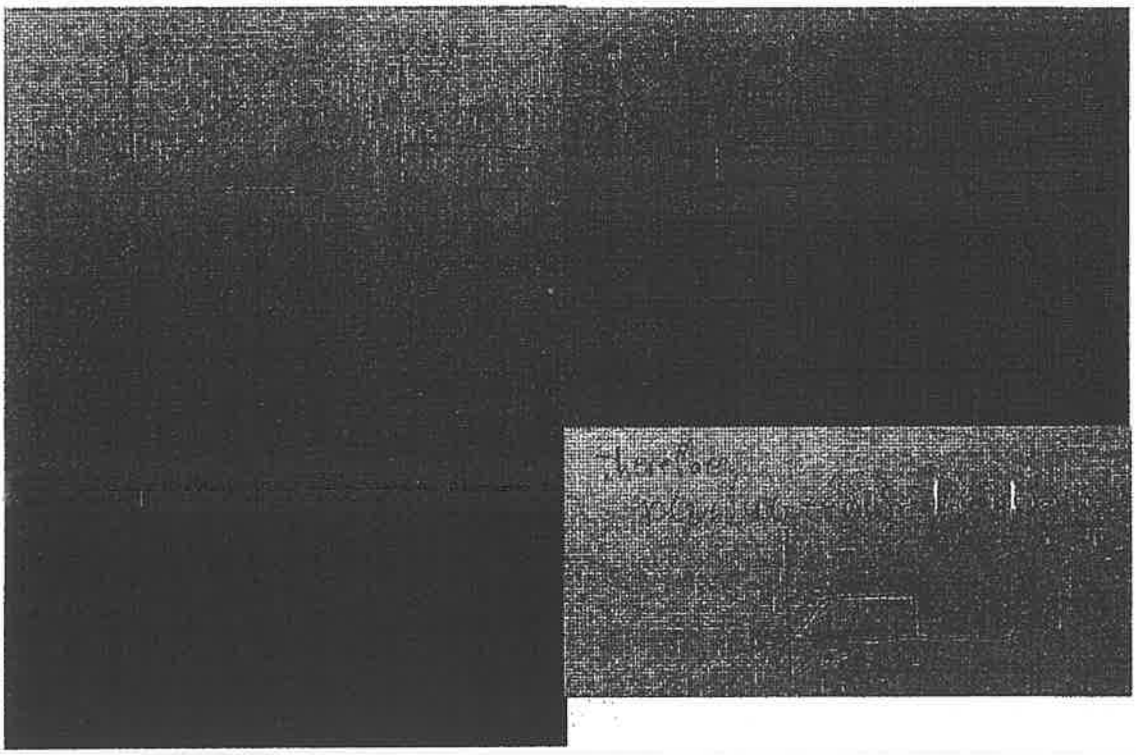
a x1(t) -> a cos(2t)x1(t) i

a y1(t) = a cos(2t)x1(t) ii

i is similar to ii. Therefore, it satisfies the homogeneity properties.

It can be concluded that y(t) is linear.

3b. $p(t) = r(2t)\,u(-t+0.5) + (u(t-0.5) - u(t-2))$



3c.

$y(t) = \int_{-\infty}^{\infty} x(t-T)h(T)\,dT$

$t<0 \quad y(t) = 0$

$0<t<0.5 \quad y(t) = \int_0^t (2)(2)\,dT$

$= \int_0^t (2)(2)\,dT$

$= 4t$

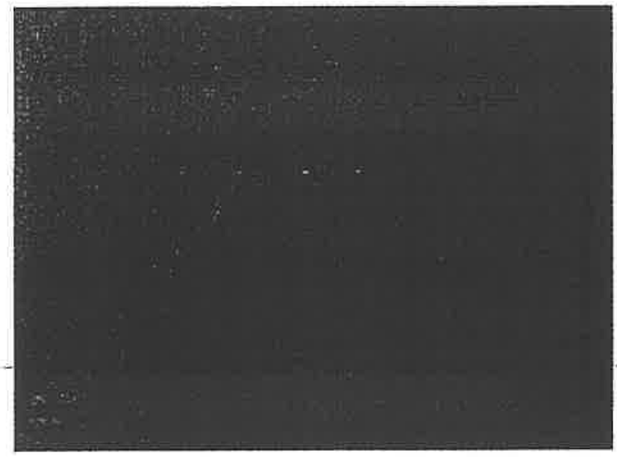
$0.5<t<2 \quad y(t) = \int_{t-0.5}^{t} (2)(2)\,dT$

$= 2$

$2<t<2.5 \quad y(t) = \int_{t-0.5}^{2} (2)(2)\,dT$

$= 10 - 4t$

$t>2.5 \quad y(t) = 0$



Rough Sketch is enough for the exam

4a

$Y(t) = e^{-3t}\,[u(t+1) + u(t-2)]$

$F(y(t)] = \int_{-\infty}^{\infty} e^{-3t}\, u(t+1)\, e^{-jwt} dt + \int_{-\infty}^{\infty} e^{-3t}\, u(t-2)\, e^{-jwt} dt$

$= \int_{-1}^{\infty} e^{(-3-jw)t} dt + \int_{2}^{\infty} e^{(-3-jw)t} dt$

$= \frac{1}{-3-jw}\, e^{3+jw} + \frac{1}{-3-jw}\, e^{-(6+2jw)}$

4b

Y(s) = X(s) H(s)

X(s) = 1/s+4

$H(s) = 5(1/s\, e^{-2s})$

$Y(s) = 5\,\frac{e^{-2s}}{s(s+4)}$

$$\frac{1}{s(s+4)} = \frac{A}{s} + \frac{B}{(s+4)}$$

A = ¼ , B = - ¼

$Y(s) = {}^{5}/_{4}\,\frac{e^{-2s}}{s} - {}^{5}/_{4}\,\frac{e^{-2s}}{s+4}$

$= 5/4\; u(t-2) - 5/4\; e^{-4(t-2)}\; u(t-2)$

4c

$y''(t) - y'(t) - 6y(t) = 4x(t)$

$s^2\, y(s) - s\, y(s) - 6\, y(s) = 4\, x(s)$

$(s^2-s-6)\, y(s) = 4x(s)$

$(s^2-s-6)\, y(s) = 4(1/s\, e^{-s})$

$y(s) = \frac{4e^{-s}}{(s-3)(s+2)(s)}$

$\frac{4}{(s-3)(s+2)(s)} = \frac{A}{(s-3)} + \frac{B}{(s+2)} + \frac{C}{(s)}$

A = -4/15, B = 2/5, C = -2/3

$y(s) = \frac{-4\, e^{-s}}{15(s-3)} + \frac{2\, e^{-s}}{5(s+2)} - \frac{e^{-s}}{3s}$

$y(t) = -4/15\; e^{3(t-1)}\; u(t-1) + 2/5\; e^{-2(t-1)} u(t-1) - 2/3\; u(t-1)$

Good Luck for Your Examinations!!! Don't forget to take rest!