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1)

a) We have three points: (1, 2, 3), (4, 5, 6) and (6, 5, 4). Using one of the points with two vectors from it, we could easily define the desired plane as such:

x(u, v) = 1 + (4-1)u + (6-1)v

= 1 + 3u + 5v

y(u, v) = 2 + (5-2)u + (5-2)v

= 2 + 3u + 3v

z(u, v) = 3 + (6-3)u + (4-3)v

= 3 + 3u + v

$u, v \in [0, 1]$

b)

i) $r = 1.2\sin(2\alpha - 0.5\pi),\ \alpha \in [0, 2\pi]$

As the shape is full (and not an arc), the equation would be:

$$x(\alpha) = rcos(\alpha)$$

$$y(\alpha) = rsin(\alpha)$$

$u \in [0, 1]$, hence $\alpha = 2\pi u$:

$$x(u) = 1.2\sin(4\pi u - 0.5\pi)\cos(2\pi u)$$

$$y(u) = 1.2\sin(4\pi u - 0.5\pi)\sin(2\pi u)$$

ii) Adding v into the previous function:

$$x(u, v) = (0.5 + 0.7v)[1.2\sin(4\pi u - 0.5\pi)\cos(2\pi u)]$$

$$y(u, v) = (0.5 + 0.7v)[1.2\sin(4\pi u - 0.5\pi)\sin(2\pi u)]$$

$$u, v \in [0, 1]$$

Perform translation to (1.2, 1.2):

$$x(u, v) = (0.5 + 0.7v)[1.2\sin(4\pi u - 0.5\pi)\cos(2\pi u)] + 1.2$$

$$y(u, v) = (0.5 + 0.7v)[1.2\sin(4\pi u - 0.5\pi)\sin(2\pi u)] + 1.2$$

$$u, v \in [0, 1]$$

c) First, we have to determine which plane the base circle is on – it could either be on XZ plane and sweeps from 90 degrees to 360 degrees or be on YZ plane and sweeps from 0 degree to 270 degrees.

So, putting it on XZ plane:

$$x(\alpha)=0.4\cos(40\alpha)+2$$

$$y(\alpha)=0$$

$$z(\alpha)=0.4\sin(40\alpha)$$

$$\alpha\in[0,2\pi]$$

And perform rotational sweeping:

$$x(\alpha) = (0.4\cos(40\alpha)+2)\cos(0.5\pi+0.75\alpha)$$

$$y(\alpha) = (0.4\cos(40\alpha)+2)\sin(0.5\pi+0.75\alpha)$$

$$z(\alpha)=0.4\sin(40\alpha)$$

Then convert alpha to u:

$$u\in[0,1],\text{ hence }\alpha=2\pi u:$$

$$x(u) = (0.4\cos(80\pi u)+2)\cos(0.5\pi+1.5\pi u)$$

$$y(u) = (0.4\cos(80\pi u)+2)\sin(0.5\pi+1.5\pi u)$$

$$z(u)=0.4\sin(80\pi u)$$

2)

a) The function could be roughly plotted as shown in Figure 1.

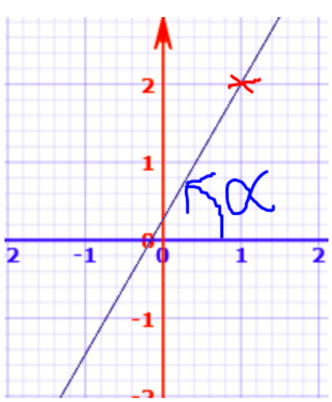

Figure 1

Using the formula y = mx + c, and m = tan(α), we could write the function as $y = \sqrt{3}x + c$. Substituting (1, 2) into the equation, we get $y = \sqrt{3}x + 2 - \sqrt{3}$.

b)

i) Defining pyramid:

$$f_A(x,y,z) = \frac{x}{-2} + \frac{y}{1} + \frac{z}{2} - 1 \geq 0$$

$$f_B(x,y,z) = \frac{x}{2} + \frac{y}{1} + \frac{z}{2} - 1 \geq 0$$

$$f_C(y) = y \geq 0$$

$$f_D(z) = z \geq 0$$

$$f_{pyramid}(x,y,z) = \min(-f_A, -f_B, f_C, f_D) \geq 0$$

Defining cylinder:

$$f_E(x,z) = x^2 + z^2 - 0.5^2 \geq 0$$

$$f_F(x,z) = x^2 + z^2 - 0.25^2 \geq 0$$

$$f_G(y) = y - 2 \geq 0$$

$$f_{cylinder}(x,y,z) = \min(-f_E, f_F, f_C, f_D, -f_G) \geq 0$$

ii) $f_{final}(x,y,z) = \max\left(f_{pyramid}, f_{cylinder}\right) \geq 0$

c) Similar to 1(c), our base shape could be on XZ plane or on XY plane. Here we choose to put it on XY plane.

Define the base circle surface:

$$x(u,v) = (0.4 + 0.4v)(0.8\cos(2\pi u)) + 4$$

$$y(u,v) = (0.4 + 0.4v)(0.8\sin(2\pi u)) + 2$$

$$z(u,v) = 0$$

$$u, v \in [0, 1]$$

Perform rotational sweeping:

$$x(u,v,w) = (0.4 + 0.4v)(0.8\cos(2\pi u)) + 4$$

$$y(u,v,w) = [(0.4 + 0.4v)(0.8\sin(2\pi u)) + 2]\cos\,(0.5\pi + 8\pi w)$$

$$y(u,v,w) = [(0.4 + 0.4v)(0.8\sin(2\pi u)) + 2]\sin\,(0.5\pi + 8\pi w)$$

$$w \in [0, 1]$$

Perform translational sweeping (by observation, the range of center of tube is [-4, 4]):

$$x(u, v, w) = (0.4 + 0.4v)(0.8\cos(2\pi u)) + 4 - 8w$$

3)

a) First convert the texture function to parametric function A. Then convert the target surface to parametric function B. Lastly define the mapping between the parameters of A and B.

b) $A_1(-1, 1) \rightarrow B_1(3, 2)$, $A_4(-3, 2) \rightarrow B_4(2, 2)$, $A_5(-3, 1) \rightarrow B_5(1, 0)$

x' = ax + by + m

y' = cx + dy + n

For A_1 to B_1:

3 = -a + b + m -------(1)

2 = -c + d + n -------(2)

For A_4 to B_4:

2 = -3a + 2b + m -------(3)

2 = -3c + 2d + n -------(4)

For A_1 to B_1:

1 = -3a + b + m -------(5)

0 = -3c + d + n -------(6)

(5) – (1): -2 = -2a => a = 1

(6) – (2): -2 = -2c => c = 1

(3) – (1): -1 = -2a + b => b = 1

(4) – (2): 0 = -2c + d => d = 2

(1): 3 = -1+1 + m => m = 3

(2): 2 = -1 + 2 + n => n = 1

The affine transformation matrix would be $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

c) [not so confident in this answer]

Since the two points of the line (50, -40, 30) and (-100, 80, -60) are scalar to each other, the origin lies on this line. And the graph would look like Figure 2:

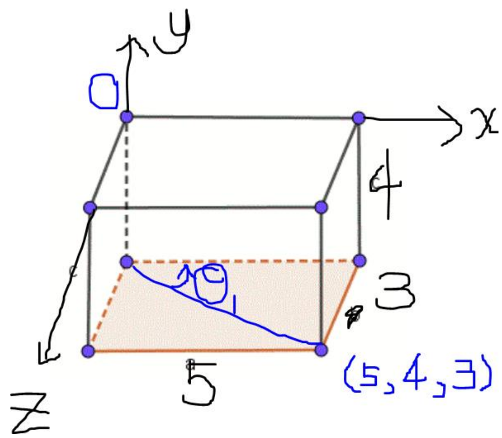

Figure 2

As shown in Figure 2, we rotate the line along the y-axis with $\tan(\theta_1)$ = 3/5.

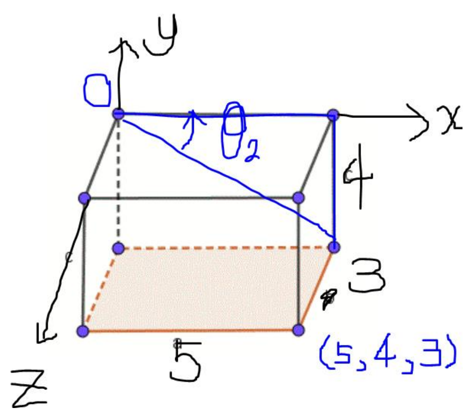

Figure 3

Then we rotate the line along z-axis with $\tan(\theta_2)$ = 4/5 as shown in Figure 3.

So the transformation matrixes for rotating the line towards x-axis would be:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} cos\theta_2 & -sin\theta_2 & 0 & 0 \\ sin\theta_2 & cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta_1 & 0 & sin\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ -sin\theta_1 & 0 & cos\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} cos\theta_2 cos\theta_1 & -sin\theta_2 & cos\theta_2 sin\theta_1 & 0 \\ sin\theta_2 cos\theta_1 & cos\theta_2 & sin\theta_2 sin\theta_1 & 0 \\ -sin\theta_1 & 0 & cos\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Perform reflection:

$$\begin{bmatrix} x'' \\ y'' \\ z'' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta_2 cos\theta_1 & -sin\theta_2 & cos\theta_2 sin\theta_1 & 0 \\ sin\theta_2 cos\theta_1 & cos\theta_2 & sin\theta_2 sin\theta_1 & 0 \\ -sin\theta_1 & 0 & cos\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x'' \\ y'' \\ z'' \\ 1 \end{bmatrix} = \begin{bmatrix} cos\theta_2 cos\theta_1 & -sin\theta_2 & cos\theta_2 sin\theta_1 & 0 \\ -sin\theta_2 cos\theta_1 & -cos\theta_2 & -sin\theta_2 sin\theta_1 & 0 \\ sin\theta_1 & 0 & -cos\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotate the line back to its original orientation:

$$\begin{bmatrix} x''' \\ y''' \\ z''' \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} cos\theta_1 & 0 & -sin\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ sin\theta_1 & 0 & cos\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta_2 & sin\theta_2 & 0 & 0 \\ -sin\theta_2 & cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta_2 cos\theta_1 & -sin\theta_2 & cos\theta_2 sin\theta_1 & 0 \\ -sin\theta_2 cos\theta_1 & -cos\theta_2 & -sin\theta_2 sin\theta_1 & 0 \\ sin\theta_1 & 0 & -cos\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x''' \\ y''' \\ z''' \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} cos\theta_1 cos\theta_2 & cos\theta_2 & -sin\theta_1 & 0 \\ -sin\theta_2 & cos\theta_2 & 0 & 0 \\ sin\theta_1 cos\theta_2 & sin\theta_1 sin\theta_2 & cos\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta_2 cos\theta_1 & -sin\theta_2 & cos\theta_2 sin\theta_1 & 0 \\ -sin\theta_2 cos\theta_1 & -cos\theta_2 & -sin\theta_2 sin\theta_1 & 0 \\ sin\theta_1 & 0 & -cos\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$sin\theta_1 \approx 0.5145, cos\theta_1 \approx 0.8575$

$sin\theta_2 \approx 0.6247, cos\theta_2 \approx 0.7809$

$$\begin{bmatrix} x''' \\ y''' \\ z''' \\ 1 \end{bmatrix} = \begin{bmatrix} -0.2346 & -1.0281 & 0.4592 & 0 \\ -0.8366 & -0.2195 & -1.2373 & 0 \\ 0.4950 & -0.5097 & -0.5517 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

For the three points (10, 0, 0), (0, 10, 0), (0, 0, 10), applying the transformation matrix, the resulting coordinates are: (-2.346 -8.366, 4.950), (-10.281, -2.195, -5.097), (4.592, -12.373, -5.517).

4)

a) Texture mapping would not modify the geometry of the sphere. The sphere generated would hence have a smooth surface with chessboard image on top of it.

Displacement mapping would modify the geometry of the sphere. The sphere generated would have a rough surface of chessboard pattern as the white tiles generates peak and the black tiles generate trough.

b) Equation of shape A:

$x_A(u, v) = \mathrm{v} * \cos(2\pi u) - 2$

$y_A(u, v) = \mathrm{v} * \sin(2\pi u) + 2$

$u, v \in [0, 1]$

Equation of shape B:

$x_B(u, v) = 1 + 3u + v$

$y_B(u, v) = 1 + 2v$

$u, v \in [0, 1]$

Define morphing:

$x'(u, v) = (1 - t)x_A + (t)x_B$

$y'(u, v) = (1 - t)y_A + (t)y_B$

where $t = \sin\left(\frac{\pi}{2} * \frac{k-1}{100-1}\right), 1 \leq k \leq 100$

c)

i) $f(y, z) = y^2+z^2-2z = 0$
$f(y, z) = y^2 + (z+1)^2 - 1 = 0$
Let P (0, 0, 2),

a*PL = [0 0 18]
$18^2 = a^2$
a = 18
PL = [0 0 1]

b*PO = [27 0 9]
$27^2+9^2 = b^2$
$810 = b^2$

b = $\frac{9}{\sqrt{10}}$

PO = $\frac{1}{\sqrt{10}}$[3 0 1]

ii) $k_{diffuse} = 0.4, I_{source} = 1$, and the normal PN is [0 0 1], as for a cylinder its normal is the radius towards the particular point.
Hence, $\cos(\theta) = PN \cdot PL = [0\ 0\ 1] \cdot [0\ 0\ 1] = 1.$

$$I_{diffuse} = I_{source} * k_{diffuse} * \cos(\theta) = 0.4$$

iii) The observer, the point being observed, and the light source lies on the same plane, which is plane x in this case, and we know that the specular reflection would be maximum when the incident angle is same as the reflection angle.

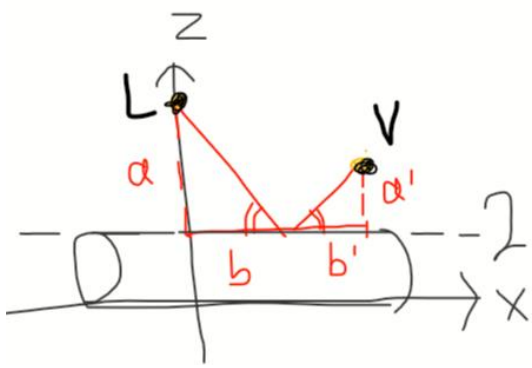

Figure 4

And then we could draw two triangles that are similar, as shown in Figure 4. According to similar triangle's properties, a'/a would be equivalent to b'/b, and hence:

Let P(x, 0, 2) be the point of maximum specular reflection to the observer,

$$\frac{b'}{b} = \frac{a'}{a}$$

$$\frac{27 - x}{x - 0} = \frac{11 - 2}{20 - 2}$$

$$27 - x = \frac{1}{2}x$$

$$x = \frac{2}{3} * 27 = 18$$

Hence the point P's coordinate is (18, 0, 2).

--End of Answers--