CE2004 – Circuit and Signal Analysis

Semester 2 Examination 2017-18

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1. (a) Use nodal analysis and take D as the reference point.

No. of equations to be derived $= 4 - 1 = 3$.

$V_A = 12V$ (1)

$V_B = 6V$ (2)

Applying KCL at node C:

Sum of currents entering node C $= 0$

$\frac{V_B - V_C}{2k} + \frac{V_A - V_C}{8k} + \frac{V_D - V_C}{4k} = 0$ (3)

Substitute (1) and (2) into (3),

$\frac{6 - V_C}{2k} + \frac{12 - V_C}{8k} + \frac{(-V_C)}{4k} = 0$

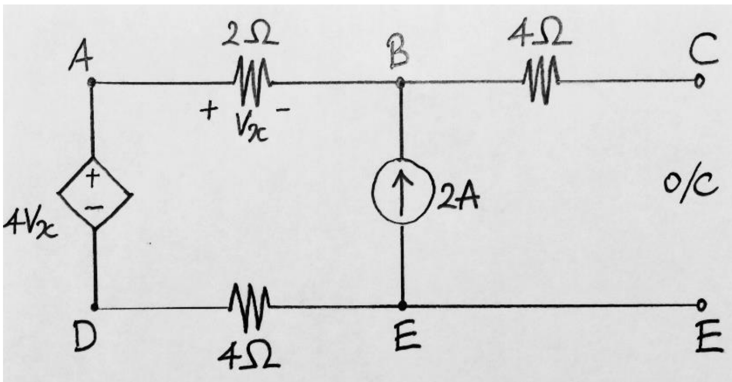
$4(6 - V_C) + (12 - V_C) - 2V_C = 0$

$7V_C = 36$

$V_C = \frac{36}{7}V$

$I_0 = \frac{V_C}{4k} = \frac{36/7}{4k} = \frac{9}{7}mA$

(b) To find R_{TH} we need to find the voltage across resistor R_L open-circuit and the current flowing through R_L short-circuit.



$V_{CE}(o/c)$ $= V_{BE}$ ($V_C = V_B$ since no current flows from B to C)

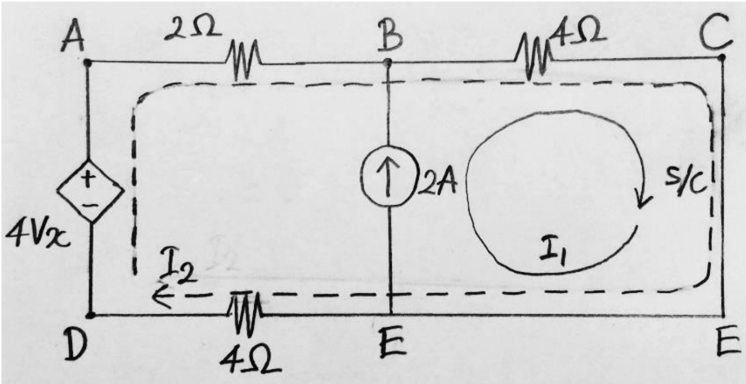
$= V_{BA} + V_{AD} + V_{DE}$

$= (2A)(2\Omega) + 4V_x + (2A)(4\Omega)$

$= 4 + 4(-2)(2) + 8$

$= -4V$

Use loop analysis to find $I_{CE}(s/c)$.



For Loop 1: $I_1 = 2A$

For Loop 2: $(2\Omega)I_2 + (4\Omega)(I_1 + I_2) + (4\Omega)I_2 - 4V_x = 0$

$2I_2 + 8 + 4I_2 + 4I_2 - 4(2I_2) = 0$

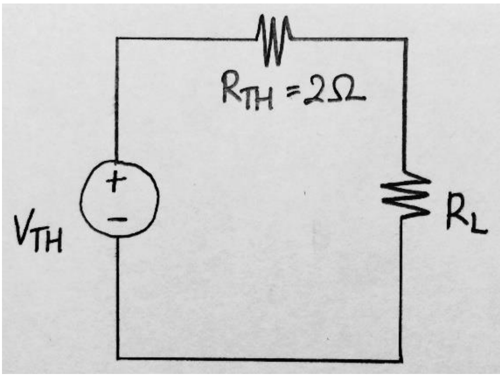
$2I_2 = -8$

$I_2 = -4A$

$$I_{CE}(s/c) = I_1 + I_2$$
$$= 2 + (-4)$$
$$= -2A$$

$$R_{TH} = \frac{V_{CE}(o/c)}{I_{CE}(s/c)}$$
$$= \frac{-4}{-2}$$
$$= 2\Omega$$

Thevenin's equivalent circuit:



To achieve maximum power transfer to the load, $R_L = R_{TH} = 2\Omega$.

The maximum power transfer to the load $= I^2R$

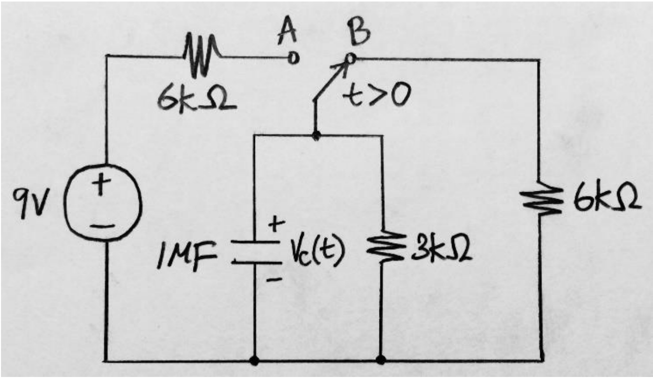
$= (V_{TH}/2R)^2R$

$= {V_{TH}}^2/(4R)$

$= (-4)^2/(4 \times 2)$

$= 2W$

2. (a) (i) General solution: $V_C(t) = K_1 + K_2 e^{-t/\tau}$



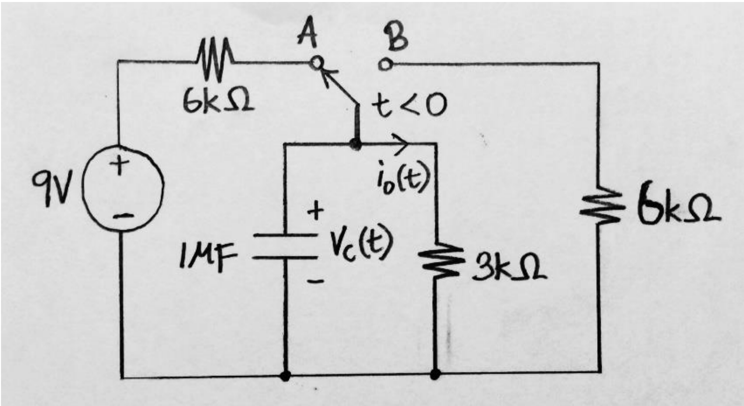
To find R_{TH} across the capacitor for $t > 0$, short-circuit the 9V voltage source.

Also, no current flows from the capacitor to A for t > 0 since it is open-circuit.

$R_{TH} = 3k//6k$

$R_{TH} = 2k\Omega$

$\tau = CR_{TH} = (1\mu F)(2k\Omega) = 0.002s$



At $t = 0-$, capacitor acts like an open-circuit since it is fully charged.

$V_C(0-) = Voltage\ across\ the\ 3k\Omega\ resistor$ (parallel)

$V_C(0-) = \frac{3k}{6k+3k} \times 9V = 3V$ (potential divider rule)

$V_C(0+) = V_C(0-) = 3V$ (voltage continuity property)

Therefore, $V_C(0) = K_1 + K_2 e^{-(0/\tau)}$

$3 = K_1 + K_2$

At $t = \infty$, capacitor will be fully discharged.

$V_C(\infty) = 0$

$0 = K_1 + K_2 e^{-(\infty/\tau)}$

$0 = K_1 + K_2(0)$

$K_1 = 0$

$K_2 = 3 - 0 = 3$

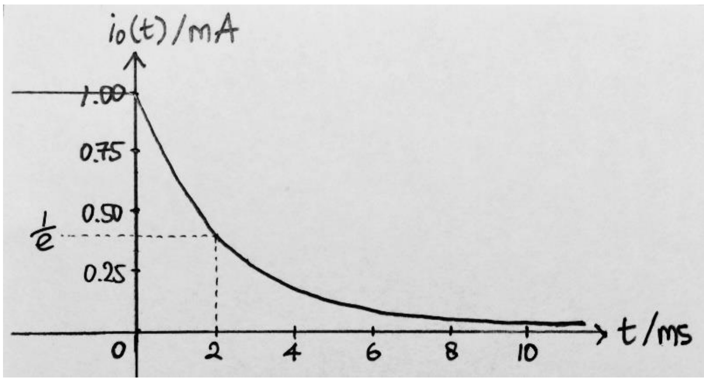
Hence, $V_C(t) = 3e^{-t/0.002}\ V$.

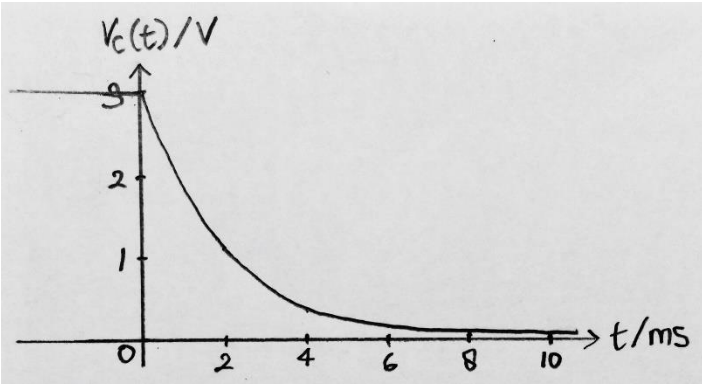
$i_0(t) = \frac{V_C(t)}{3k\Omega}$

$i_0(t) = \frac{3e^{-t/0.002}}{3k}$

$i_0(t) = e^{-t/0.002}\ mA$

(a) (ii)





(b) Capacitance across CD

= [(Capacitance across the parallel 3µF, 4µF and 3µF capacitors) in series with 10µF] // 1µF // 4µF

$= 1/[1/(3+4+3)\ +\ 1/10] + 1 + 4$

$= 5 + 1 + 4$

$= 10\mu F$

$\tau = R \times C = 1k\Omega\ \times 10\mu F = 10ms$

3. (a) (i) $V_2 = -\frac{R_F}{R_1} \times V_{in} = -\frac{20k}{15k} \times 1 = -\frac{4}{3}V$

(ii) By potential divider rule,

$V_- = \frac{5k}{10k+5k} \times V_{out}$

$V_- = \frac{1}{3}V_{out}$

Applying op-amp voltage rule: $V_+ \approx V_-$

$V_+ = \frac{1}{3}V_{out}$

$V_{out} = 3V_+$

Use superposition to derive V_+.

Considering V_2 source alone, $V_+ = \frac{20k}{10k+20k} \times V_2$

$$V_+ = \frac{2}{3}V_2$$

Considering 4V source alone, $V_+ = \frac{10k}{10k+20k} \times 4V$

$$V_+ = \frac{4}{3}V$$

Hence, $V_+ = \frac{2}{3}V_2 + \frac{4}{3}$ (by superposition)

$$V_+ = \frac{2}{3}(-\frac{4}{3}) + \frac{4}{3}$$

$$V_+ = \frac{4}{9}V$$

$V_{out} = 3V_+ = 3\left(\frac{4}{9}\right) = \frac{4}{3}V.$

(iii) No, there is no voltage distortion in the built circuit because all the values of the output voltage from the two op-amps are within the range of [-15V, 15V].

(b) (i) The system is memoryless because y(t_0) depends only on input at t = t_0.

(ii) The system is causal because y(t_0) depends on input only for t ≤ t_0.

(iii) $x_1(t) \rightarrow y_1(t) = x_1(t)\sin(2t)\cos(5t/6)$ (1)

Let $x_2(t) = x_1(t - t_0)$

$x_2(t) \rightarrow y_2(t) = x_2(t)\sin(2t)\cos(5t/6)$

$= x_1(t - t_0)\sin(2t)\cos(5t/6)$

From (1), $y_1(t - t_0) = x_1(t - t_0)\sin[(2(t - t_0)]\cos[5(t - t_0)/6]$

Since $y_2(t) \neq y_1(t - t_0)$, $y(t)$ is not time-invariant.

(iv) $x_1(t) \rightarrow y_1(t) = x_1(t)\sin(2t)\cos(5t/6)$

$x_2(t) \rightarrow y_2(t) = x_2(t)\sin(2t)\cos(5t/6)$

$x_1(t) + x_2(t) \rightarrow [x_1(t) + x_2(t)]\sin(2t)\cos(5t/6)$

$= x_1(t)\sin(2t)\cos(5t/6) + x_2(t)\sin(2t)\cos(5t/6)$

$= y_1(t) + y_2(t)$

Hence, the system is additive.

$x_1(t) \rightarrow \ y_1(t) = x_1(t)\sin(2t)\cos(5t/6)$

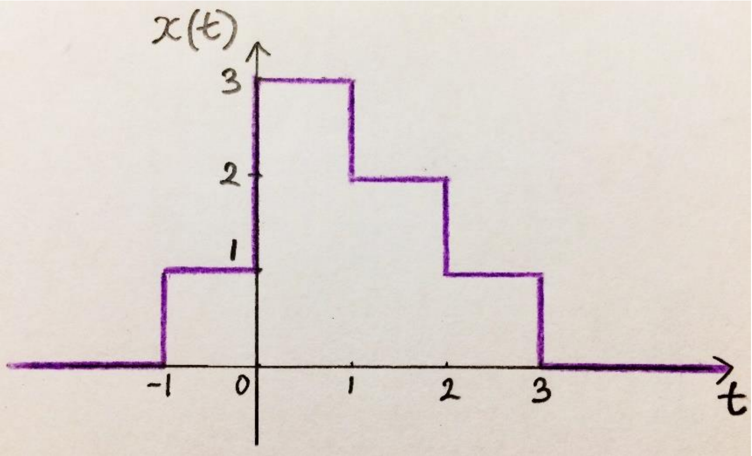
$a.x_1(t) \rightarrow \ a.x_1(t)\sin(2t)\cos(5t/6)$

$= a.y_1(t)$

Hence, the system is also homogeneous.

Therefore, the system is linear since it is additive and homogeneous.

4. (a) (i) $x(t) = u(t+1) + 2u(t) - u(t-1) - u(t-2) - u(t-3)$



(ii) $x(t)$ is not an even nor an odd function.

$x(t) \neq x(-t)$, thus $x(t)$ is not an even function.

$x(t) \neq -x(-t)$, thus $x(t)$ is not an odd function.

(b) $h(t) = u(t-3) - u(t),\ x(t) = u(t-2)$

Using convolution integral,

$y(t) = h(t) * x(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{+\infty} h(\tau)x(t-\tau)d\tau$

$$
\begin{aligned}
y(t) \quad &= \int_{-\infty}^{+\infty} h(\tau)x(t-\tau)d\tau \\
&= \int_{-\infty}^{+\infty} [u(\tau-3) - u(\tau)][u(t-\tau-2)]d\tau \\
&= \int_{-\infty}^{+\infty} -[u(\tau) - u(\tau-3)][u(t-\tau-2)]d\tau \\
&= -\int_0^3 u(t-2-\tau)d\tau
\end{aligned}
$$

$$
y(t) = \begin{cases} -3,\ for\ t > 5 \\ -t+2,\ for\ 2 < t \leq 5 \\ 0,\ for\ t \leq 2 \end{cases}
$$

(c) (i) $y''(t) + 5y'(t) + 6y(t) - x'(t) - x(t) = 0$

$y''(t) + 5y'(t) + 6y(t) = x'(t) + x(t)$

Let $x(t) = e^{st},\ y(t) = H(s).e^{st}$.

$$s^2H(s)e^{st} + 5s\,H(s)e^{st} + 6H(s)e^{st} = s\,e^{st} + e^{st}$$

$$H(s)e^{st}(s^2 + 5s + 6) = e^{st}(s + 1)$$

$$H(s) = \frac{s+1}{s^2+5s+6}$$

$$H(s) = \frac{s+1}{(s+2)(s+3)}$$

(ii) $x(t) = 2.4e^{-t}u(t)$

$X(s) = \frac{2.4}{s+1}$ (see Appendix B)

$$H(s) = \frac{s+1}{(s+2)(s+3)}$$

$$Y(s) = H(s).X(s)$$

$$= \frac{s+1}{(s+2)(s+3)} \cdot \frac{2.4}{s+1}$$

$$= 2.4[\frac{1}{(s+2)(s+3)}]$$

$$= 2.4[\frac{1}{(s+2)} - \frac{1}{(s+3)}]$$

$$= \frac{2.4}{(s+2)} - \frac{2.4}{(s+3)}$$

$y(t) = (2.4e^{-2t} - 2.4e^{-3t})u(t)$ (see Appendix B)