20th CSEC – Past Year Paper Solution *2018-2019 Sem 1*
MH 1812 – Discrete Mathematics

1)

a) The inverse of $\neg p \rightarrow \neg q$ is $p \rightarrow q$

p	q	$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$	$p \rightarrow q$
T	T	F	F	T	T
T	F	F	T	**T**	**F**
F	T	T	F	**F**	**T**
F	F	T	T	T	T

As we can see, the truth table for $\neg p \rightarrow \neg q$ and $p \rightarrow q$ is different. Therefore, we conclude that for $\neg p \rightarrow \neg q$ and $p \rightarrow q$ are not logically equivalent.

b) $\big(q \wedge (p \rightarrow \neg q)\big) \rightarrow \neg p$

$= \big(q \wedge (\neg p \vee \neg q)\big) \rightarrow \neg p$	[Conversion Theorem]
$= \big((q \wedge \neg p) \vee (q \wedge \neg q)\big) \rightarrow \neg p$	[Distributivity]
$= \big((q \wedge \neg p) \vee (False)\big) \rightarrow \neg p$	[Contradiction]
$= (q \wedge \neg p) \rightarrow \neg p$	[Unity]
$= \neg(q \wedge \neg p) \vee \neg p$	[Conversion Theorem]
$= (\neg q \vee p) \vee \neg p$	[DeMorgan's]
$= \neg q \vee (p \vee \neg p)$	[Commutative; Associative]
$= True$	[Tautology]

2) Direct Proof:

$$\sum_{j=n}^{2n-1} (2j+1) = (2n+1) + (2(n+1)+1) + \cdots + (2(2n-1)+1)$$

$= (2n+1) + (2n+3) + \cdots + \big(2n + (2n-1)\big)$

$= (n \times 2n) + (1 + 3 + \cdots + 2n - 1)$

$= 2n^2 + n^2$ $\quad [Since\ 1 + 3 + \cdots + (2n-1) = n^2]$

$= 3n^2$

Proof by induction:

For $n = 1$

$$LHS: \sum_{j=n}^{2n-1} (2j+1) = (2 \times 1) + 1 = 3$$

$RHS: 3n^2 = 3 \times 1 = 3$

$\Rightarrow LHS = RHS$

For the purpose of induction, assume that for $n = k \rightarrow \sum_{j=k}^{2k-1}(2j+1) = 3k^2$

For $n = k+1$, we want to proof $\sum_{j=k+1}^{2(k+1)-1}(2j+1) = 3(k+1)^2$, for the induction to be complete

$$\sum_{j=k+1}^{2(k+1)-1} 2j+1 = \left[\sum_{j=k}^{2k-1}(2j+1)\right] - (2k+1) + (2(2k)+1) + (2(2k+1)+1)$$

$= 3k^2 - 2k - 1 + 4k + 1 + 4k + 3$

$= 3k^2 + 6k + 3$

$= 3(k^2 + 2k + 1) = 3(k + 1)^2$

3) $a_n = a_{n-1} + 2n + 1 = a_{n-2} + 2(n-1) + 1 + 2n + 1$
$= a_{n-3} + 2(n-2) + 1 + \cdots + 2n + 1$
$= a_{n-4} + 2(n-3) + 1 + \cdots + 2n + 1$
…
$= a_{n-k} + 2(n-k+1) + 1 + \cdots + 2n + 1$
$= a_0 + 2(1) + 1 + 2(2) + 1 + \cdots + 2n + 1$
$= 2 + 2(1 + 3 + \cdots + n) + (1 \times n)$
$\therefore a_n = n^2 + 2n + 2$

4)

a) $\sum_{i=1}^{k} x_i = n \rightarrow x_1 + x_2 + \cdots + x_k = n$
Let * denotes n
The question is similar to partition n into k different groups. Consider T as a barrier for each group.
T*…****T****T**
As we can see, there are n-1 available slots to put the barrier (T) between groups. Moreover, to divide n into k different groups, we only need k-1 barriers. Therefore, there are $\binom{n-1}{k-1}$

b) This problem can be solved similarly as above, the number of distinct tuples is

$$\binom{n+k-1}{k-1}$$

c) Consider X as 01. Therefore, we have 4 X and 5 1's. Number of possible combinations:

$$\frac{9!}{4!\,5!}$$

5) $\overline{(A-B) \cup (B-A)} = (A \cap B) \cup (\bar{A} \cap \bar{B})$

$\boldsymbol{A}$	$\boldsymbol{B}$	$\boldsymbol{A-B}$	$\boldsymbol{B-A}$	$\overline{\boldsymbol{(A-B) \cup (B-A)}}$	$(\boldsymbol{A \cap B})$	$(\bar{\boldsymbol{A}} \cap \bar{\boldsymbol{B}})$	$(\boldsymbol{A \cap B}) \cup (\bar{\boldsymbol{A}} \cap \bar{\boldsymbol{B}})$
F	F	F	F	T	F	T	T
F	T	F	T	F	F	F	F
T	F	T	F	F	F	F	F
T	T	F	F	T	T	F	T

6)

a) R is symmetric because $3^x \equiv 3^y \ mod\ 5$ implies $3^y \equiv 3^x \ mod\ 5$
Proof: $3^x = 5k + 3^y \rightarrow 3^y = 5(-k) + 3^x \rightarrow 3^y \equiv 3^x \ mod\ 5$

R is reflexive because $3^x \equiv 3^x \ mod\ 5$. Obviously because $3^x = (5 \times 0) + 3^x$

R is transitive. Proof:

$$3^x \equiv 3^y \ mod\ 5, 3^y \equiv 3^z \ mod\ 5$$
$$3^x = 5k_1 + 3^y \ and \ 3^y = 5k_2 + 3^z$$

$$3^x = 5(k_1 + k_2) + 3^z$$
$$3^x = 3^z \ mod\ 5$$

Therefore, we have x R y and y R z implies x R z

Since R is reflexive, transitive, and symmetric, R is an equivalence relation.

b) $[0] = [0,4,8]$
$[1] = [1,5,9]$
$[2] = [2,6]$
$[3] = [3,7]$

7)

a) Range of the function = $\{5, 6, 9, 14, 21\}$

b) $y = x^2 + 5 \rightarrow x^2 = y - 5 \rightarrow x = -\sqrt{y-5}$
$\therefore f^{-1}(x) = -\sqrt{x-5}$

--End of Answers--

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