V. EXERCISES (SOLUTIONS)

1.

Product	Surface Area (in.^{2})	Product	Surface Area (in.^{2})
Baking powder	38.4845	Frosting	53.5635
Cleanser	81.5400	Pineapple juice	116.1133
Coffee	87.6033	Soup	41.6575
Coffee creamer	78.6969	Tomato puree	77.8015

2. Yes, it is possible.

$$\begin{cases} 2\pi r^2 + 2\pi rh = 38.48 & \ldots(1) \\ \pi r^2 h = 17.92 & \ldots(2) \end{cases}$$

From (2),

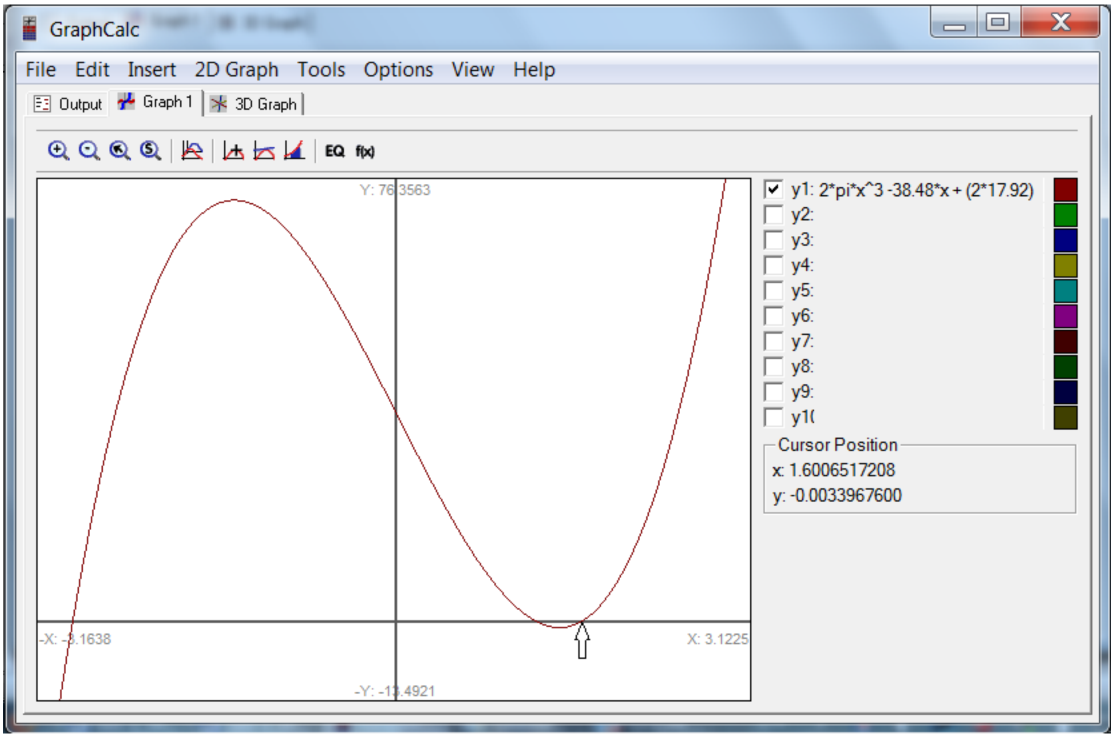
$$h = \frac{17.92}{\pi r^2}$$

Sub. into (1),

$$2\pi r^2 + 2\pi r\left(\frac{17.92}{\pi r^2}\right) = 38.48$$

$$\Rightarrow \quad 2\pi r^3 - 38.48r + 2(17.92) = 0$$

Using GraphCalc to plot the graph $y = 2\pi x^3 - 38.48x + 2(17.92)$



By moving the cursor position to the points on the graph when $y = 0$, we find $r \approx 1.25$ and $r \approx 1.60$ (ignore the negative solution).

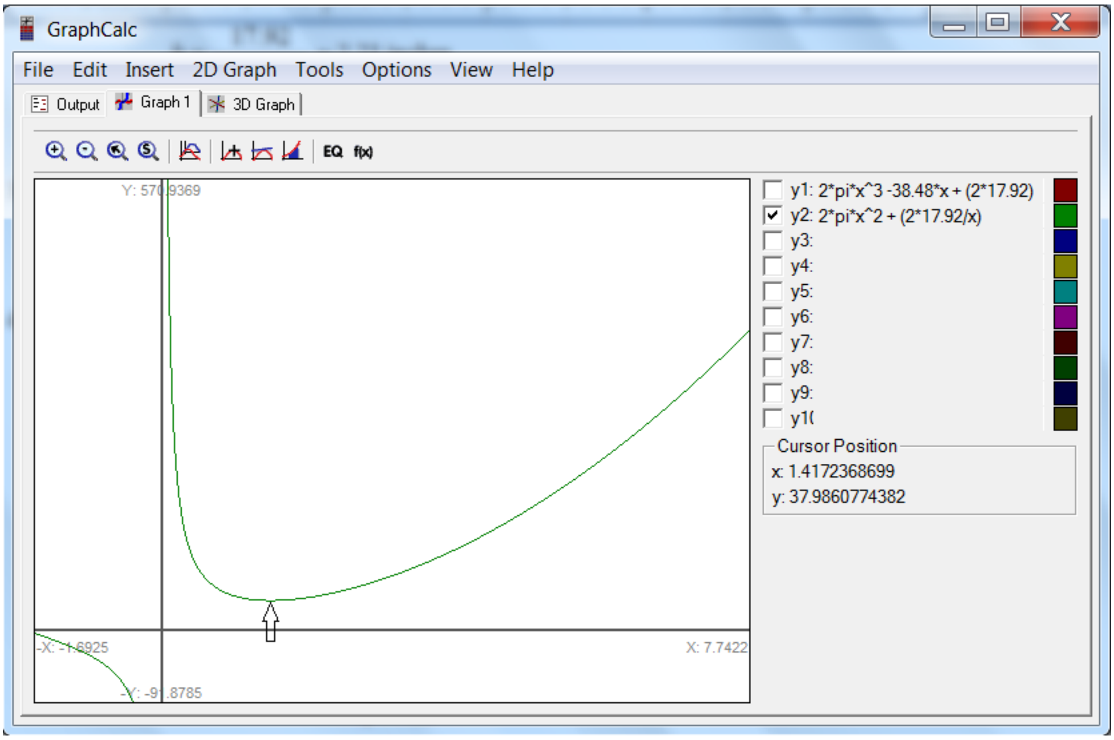
Therefore, the dimension of another container having the same surface area of 38.48 square inches and a volume of 17.92 cubic inches is:

$$r \approx 1.60 \text{ inches}$$
$$h = \frac{17.92}{\pi(1.60)^2} \approx 2.23 \text{ inches}$$

3. From Exercise 2,

$$S = 2\pi r^2 + 2\pi r\left(\frac{17.92}{\pi r^2}\right) = 2\pi r^2 + \frac{2(17.92)}{r}$$

4. Using GraphCalc to plot the graph $y = 2\pi x^2 + \dfrac{2(17.92)}{x}$



By moving the cursor position to the minimum point on the graph, we find surface area is minimized when $r \approx 1.42$ inches.

Using first derivative:

$$\frac{dS}{dr} = 4\pi r - \frac{2(17.92)}{r^2}$$

Solving for critical point,

$$\frac{dS}{dr} = 0$$

$$4\pi r - \frac{2(17.92)}{r^2} = 0$$

$$r = \sqrt[3]{\frac{2(17.92)}{4\pi}} \approx 1.42$$

The surface area is minimized when $r \approx 1.42$ inches, similar as the solution obtained using GraphCalc.

So, the baking powder container given in the Data does not have minimized surface area since it's radius $r = 1.25$ inches is not the solution of the optimization problem solved above.

5. From Exercise 4, $r = \sqrt[3]{\frac{2V}{4\pi}}$

From Exercise 2, $h = \frac{V}{\pi r^2}$

From Exercise 3, $S = 2\pi r^2 + \frac{2V}{r}$

Product	Volume (in.^{3})	Radius (in.)	Height (in.)	Surface Area (in.^{2})
Cleanser	49.54	1.9903	3.9808	74.6711
Coffee	62.12	2.1463	4.2924	86.8297
Coffee creamer	48.42	1.9752	3.9505	73.5412
Frosting	30.05	1.6848	3.3696	53.5054
Pineapple juice	92.82	2.4537	4.9074	113.4865
Soup	20.18	1.4754	2.9509	41.0327
Tomato puree	52.56	2.0300	4.0597	77.6733

6. Some reasons might be that the cylinder with the optimal surface area isn't as pleasing to the eyes, or, in the case of soda cans, do not fit the human hand well as compared to those that doesn't use the optimal surface area.

7. Since

$$\begin{cases} 2\pi r^2 + 2\pi r h = S & \ldots(1) \\ \pi r^2 h = V & \ldots(2) \end{cases}$$

we have

$$V = \pi r^2 \left(\frac{S - 2\pi r^2}{2\pi r} \right) = \frac{1}{2}\left(Sr - 2\pi r^3\right)$$

$$\frac{dV}{dr} = \frac{1}{2}\left(S - 6\pi r^2\right)$$

Solving for critical point,

$$\frac{dV}{dr} = 0$$

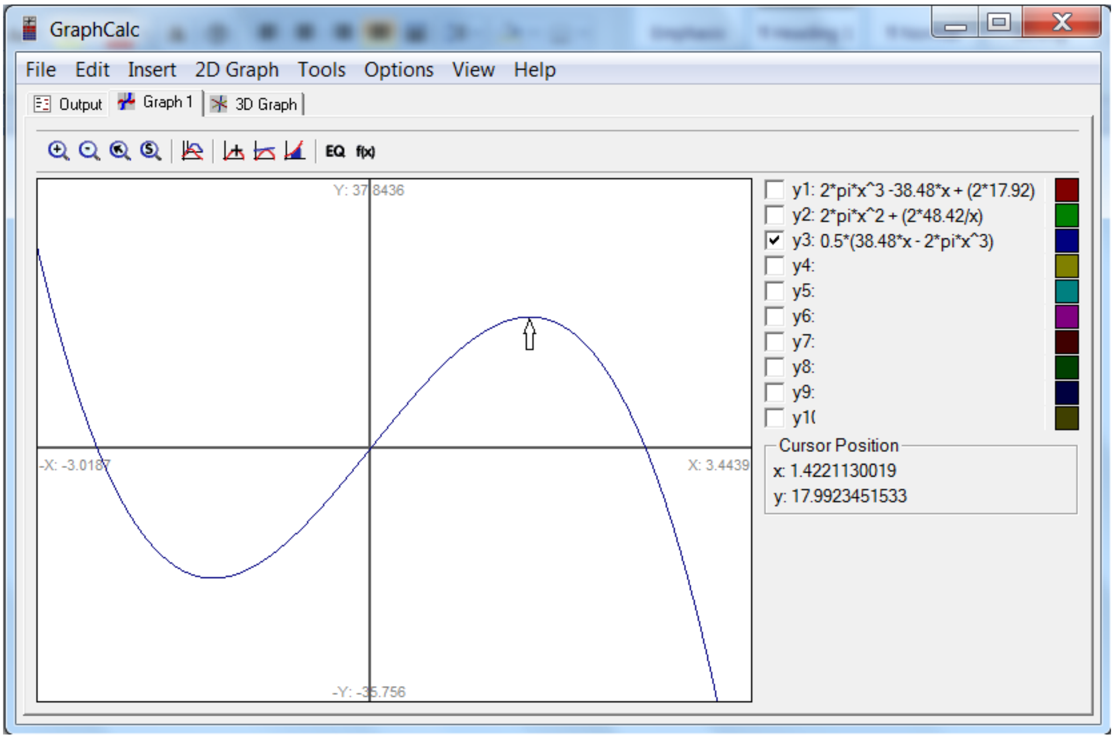
$$\frac{1}{2}\left(S - 6\pi r^2\right) = 0$$

$$r = \sqrt{\frac{S}{6\pi}}$$

8. Given $S = 38.48$,

$$r = \sqrt{\frac{38.48}{6\pi}} \approx 1.429$$

Using GraphCalc to plot the graph $y = \frac{1}{2}\left(Sx - 2\pi x^3\right)$ where $S = 38.48$



By moving the cursor position to the maximum point on the graph, we find volume is maximized when $r \approx 1.42$ inches, consistent with answer in Exercise 4 (ignoring accuracy errors).