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1 a) It is not memoryless as the output at time t_0, $y(t_0)$, depends on input other than $t = t_0$ such as $t = t_0 - 2$ and $t = 3 - t_0$.
It is not causal as the output at time t_0, $y(t_0)$, does not only depend on input for $t \le t_0$. At $t_0 = 0$, $x(3 - t_0) = x(3)$. This indicates that the input depends 3 seconds before the current time. Hence, it is not causal as it depends on future information.

b)
$$h(t) = u(t) - u(t-4)$$
$$x(t) = e^{-2t}u(t)$$
$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau$$
$$= \int_{-\infty}^{\infty} \big(u(\tau) - u(\tau-4)\big)e^{-2(t-\tau)}u(t-\tau)\,d\tau$$
$$= \int_{-\infty}^{\infty} u(\tau)e^{-2(t-\tau)}u(t-\tau)\,d\tau - \int_{-\infty}^{\infty} u(\tau-4)e^{-2(t-\tau)}u(t-\tau)\,d\tau$$
$$= \int_{0}^{t} e^{-2t+2\tau}\,d\tau - \int_{4}^{t} e^{-2t+2\tau}\,d\tau$$
$$= e^{-2t}\left(\frac{1}{2}e^{2\tau}|_0^t - \frac{1}{2}e^{2\tau}|_4^t\right)$$
$$= e^{-2t}\left(\frac{1}{2}e^{2t}u(t) - \frac{1}{2}u(t) - \frac{1}{2}e^{2t}u(t-4) + \frac{1}{2}e^{8}u(t-4)\right)$$
$$= \boldsymbol{\frac{1}{2}u(t) - \frac{1}{2}e^{-2t}u(t) - \frac{1}{2}u(t-4) + \frac{1}{2}e^{8-2t}u(t-4)}$$

c)
$$h(t) = e^{-2t}u(t)$$
$$x(t) = u(t) - u(t-4)$$
$$H(s) = \frac{1}{s+2}$$
$$X(s) = \frac{1}{s} - e^{-4s}\left(\frac{1}{s}\right)$$
$$Y(s) = H(s)X(s)$$
$$= \frac{1}{s(s+2)}(1 - e^{-4s})$$
$$\frac{1}{s(s+2)} = \frac{k_1}{s} + \frac{k_2}{s+2}$$
$$k_1 = \left[s \cdot \frac{1}{s(s+2)}\right]_{s=0} = \frac{1}{2}$$
$$k_2 = \left[(s+2) \cdot \frac{1}{s(s+2)}\right]_{s=-2} = -\frac{1}{2}$$
$$Y(s) = \left(\frac{1}{2s} - \frac{1}{2(s+2)}\right)(1 - e^{-4s})$$
$$= \boldsymbol{\frac{1}{2}u(t) - \frac{1}{2}e^{-2t}u(t) - \frac{1}{2}u(t-4) + \frac{1}{2}e^{8-2t}u(t-4)}$$

2 a) $x(t) = x(t+T)$ for all t. T is a constant and must be more than 0.

b)
$$Z_{in} = \infty$$
$$Z_{out} = 0$$
$$A_0 = \infty$$

c) i] Yes, there is negative feedback, R_3 provides the negative feedback.

ii] Using superposition, first consider only V_1 and set $V_2 = 0$

$$V_{IN} = \frac{R_1}{R_1 + R_2} V_1$$

$$A_v = 1 + \frac{R_3}{R_4} = \frac{R_3 + R_4}{R_4}$$

$$V_{OUT} = \left(\frac{R_3 + R_4}{R_4}\right)\left(\frac{R_1}{R_1 + R_2}\right) V_1$$

Then, consider only V_2 and set $V_1 = 0$

$$V_{IN} = V_2$$

$$A_V = -\frac{R_3}{R_4}$$

$$V_{OUT} = -\frac{R_3}{R_4} V_2$$

$$\therefore V_{OUT} = \boldsymbol{\left(\frac{R_3 + R_4}{R_4}\right)\left(\frac{R_1}{R_1 + R_2}\right) V_1 - \frac{R_3}{R_4} V_2}$$

iii]
(1) Inverting amplifier – $V_1 = 0$
(2) Non-inverting amplifier – $V_2 = 0$
(3) Unity gain amplifier – $V_2 = 0$, $R_3 = 0$, $R_4 = \infty$