CSEC 20th – Past Year Paper Solution 2016-2017 Sem2

CZ2003 – Computer Graphics & Visualisation

Question 1

1)

a) $x = r\sin\theta\sin\varphi$

$y = r\cos\theta$
$z = r\sin\theta\cos\varphi$

b) $x(u) = u$
$y(u) = \sqrt{u}\sin(8\pi u)$

c) Convert equations of the rose from polar to cartesian coordinates
$x = r\cos\alpha = 0.5\sin(4\alpha)\cos\alpha$
$y = r\sin\alpha = 0.5\sin(4\alpha)\sin\alpha$

Since the rose is shaded,
$x = 0.5u\sin(8\pi v)\cos(2\pi v)$
$y = 0.5u\sin(8\pi v)\sin(2\pi v)$
$z = 0$

The above equations is an origin-centred rose, we need to translate it to be centred at (1, -1).
$x = 0.5u\sin(8\pi v)\cos(2\pi v) + 1$
$y = 0.5u\sin(8\pi v)\sin(2\pi v) - 1$
$z = 0$

A rotation about y-axis is done and translation in y-axis:
$x = (0.5u\sin(8\pi v)\cos(2\pi v) + 1)\sin(1.5\pi w + \pi/2)$
$y = 0.5u\sin(8\pi v)\sin(2\pi v) - 1 + 2w$
$z = (0.5u\sin(8\pi v)\cos(2\pi v) + 1)\cos(1.5\pi w + \pi/2)$

Note that we need to add $\pi/2$ to the rotation because the rotation starts from x axis.
Alternative solution: make the rotation about y-axis anticlockwise.

Question 2

2)

a) Implicit equation: $1-\left(\frac{x}{3}\right)^2-\left(\frac{y}{2}\right)^2=0$

Polar coordinate: $1-\left(\frac{\mathrm{rcos}\alpha}{3}\right)^2-\left(\frac{\mathrm{rsin}\alpha}{2}\right)^2=0$

$$1=r^2\left(\frac{\cos^2\alpha}{9}+\frac{\sin^2\alpha}{4}\right)$$
$$r=\frac{1}{\sqrt{\frac{\cos^2\alpha}{9}+\frac{\sin^2\alpha}{4}}}$$

b) The pyramid is defined by 5 half space planes which can be defined by implicit equation of intercept:

$$f(x,y,z)=\min(1-x-y,1+x-y,1-z-y,1+z-y,1+y)$$

c) $P=P_1+u(P_2-P_1)+v(P_3-P_1)$
$x=2+u(3-2)+v(2-2)$
$x=2+u$

$y=1+u(2-1)+v(6-1)$
$y=1+u+5v$

$z=1+u(2-1)+v(4-1)$
$z=1+u+3v$

Parametric:

$$x=2+u$$
$$y=1+u+5v$$
$$y=1+u+5v$$

Implicit: $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$

$$P_1P_2=\begin{pmatrix}3\\2\\2\end{pmatrix}-\begin{pmatrix}2\\1\\1\end{pmatrix}=\begin{pmatrix}1\\1\\1\end{pmatrix}$$
$$P_1P_3=\begin{pmatrix}2\\6\\4\end{pmatrix}-\begin{pmatrix}2\\1\\1\end{pmatrix}=\begin{pmatrix}0\\5\\3\end{pmatrix}$$
$$n=\begin{pmatrix}1\\1\\1\end{pmatrix}\times\begin{pmatrix}0\\5\\3\end{pmatrix}=\begin{pmatrix}-2\\-3\\5\end{pmatrix}$$

$$\therefore -2(x-3)+(-3)(y-2)+5(z-2)=0$$

$$-2x-3y+5z=0$$

Question 3

3)

a) $x = \cos(s)$
$y = \sin(s)$
$z = 5t$

Map (u,v) to (t,s)

$t \to u$: $t = u$

$s \to v$: $\dfrac{s-0}{\pi-0} = \dfrac{v-0}{10-0}$

$s = \dfrac{\pi v}{10}$

$\therefore x = \cos\left(\dfrac{\pi}{10} \quad v\right)$
$y = \sin\left(\dfrac{\pi}{10}v\right)$
$z = 5u$

b) For affine transformation,

$$x(\tau) = ax + by + l$$
$$y(\tau) = cx + dy + m$$

Solving $x(\tau)$:

$$a(1) + b(0) + l = 7$$
$$a(2) + b(1) + l = 8$$
$$a(3) + b(0) + l = 11$$

We get $a = 2$, $b = -1$, $l = 5$.

Solving $y(\tau)$:

$$c(1) + d(0) + m = -1$$
$$c(2) + d(1) + m = 1$$
$$c(3) + d(0) + m = 1$$

We get $c = 1$, $d = 1$, $m = 5 - 2$.

$\therefore$ 3x3 matrix is $\begin{bmatrix} 2 & -1 & 5 \\ 1 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

c)

i) The order is always SCALE first, then ROTATION, finally TRANSLATION
$\therefore$ scale -> translation

$$\underset{T}{\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \underset{S}{\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

ii) $$\begin{bmatrix}1&0&0&0\\0&1&0&1\\0&0&1&2\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&\frac{3}{5}&\frac{4}{5}&0\\0&-\frac{4}{5}&\frac{3}{5}&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}\frac{5}{\sqrt{50}}&0&\frac{5}{\sqrt{50}}&0\\0&1&0&0\\-\frac{5}{\sqrt{50}}&0&\frac{5}{\sqrt{50}}&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}\cos(1.57)&-\sin(1.57)&0&0\\\sin(1.57)&\cos(1.57)&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$\begin{bmatrix}5/\sqrt{50}&0&-5/\sqrt{50}&0\\0&1&0&0\\5/\sqrt{50}&0&5/\sqrt{50}&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&3/5&-4/5&0\\0&4/5&3/5&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&5\\0&1&0&4\\0&0&1&3\\0&0&0&1\end{bmatrix}$$

We use the method of "aligning a vector to the z-axis.

Question 4

4)

a) Bump mapping. The surface geometry of Fig Q4a is not modified, only the surface normal is modified (fake normal) as if the surface has been modified. This is evident as the surface is smooth.

b) Path : $(x, y) = (4\pi\tau, 4\cos(2\pi\tau)) \qquad \tau \in [0,1]$

For any $\tau \in [0,1]$, the center if the unit disk is on path.

$$\therefore\ center(4\pi\tau, 4\cos(2\pi\tau))$$

$$f(x,y,z) = \min\ \left((x-4\pi\tau)^2 + \left(y-4\cos(2\pi\tau)\right)^2 - 1\right) \geq 0$$

$$\tau = \sin\left(\frac{\pi}{2}\frac{k-1}{99}\right)$$

Where k is the frame index $k \in [1,100]$.

c)

i) $I_d = K_d I_s \cos\theta = K_d I_s(\hat{N} \cdot \hat{L})$

$$N = \begin{pmatrix}0\\2y\\2z\end{pmatrix} = \begin{pmatrix}0\\2\\2\sqrt{3}\end{pmatrix} \qquad \hat{N} = \frac{1}{\sqrt{4+12}}\begin{pmatrix}0\\1\\\sqrt{3}\end{pmatrix}$$

$$L = \begin{pmatrix}8\\22\\0\end{pmatrix} - \begin{pmatrix}8\\1\\\sqrt{3}\end{pmatrix} = \begin{pmatrix}0\\21\\-\sqrt{3}\end{pmatrix} \qquad \hat{L} = \frac{1}{\sqrt{441+3}}\begin{pmatrix}0\\21\\-\sqrt{3}\end{pmatrix} = \frac{1}{\sqrt{444}}\begin{pmatrix}0\\21\\-\sqrt{3}\end{pmatrix}$$

$$I_d = 0.6(0.8)\frac{2}{4\sqrt{444}}\left[\begin{pmatrix}0\\1\\\sqrt{3}\end{pmatrix}.\begin{pmatrix}0\\21\\-\sqrt{3}\end{pmatrix}\right]$$

$$= 0.205$$

ii)

Note: Light source , Observer & point must lie on same plane

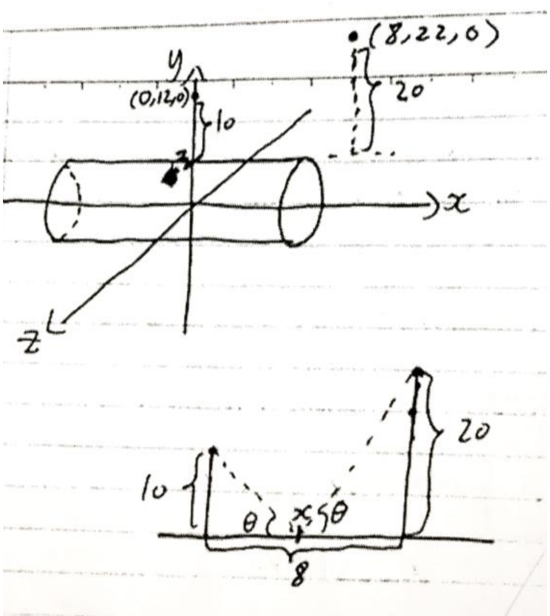

Figure1: Visualisation of the problem

$\tan\theta = \frac{10}{x}$ $\qquad$ $\tan\theta = \frac{20}{8-x}$

$\frac{10}{x} = \frac{20}{8-x}$

$x = \frac{8}{3}$

$\therefore P(\frac{8}{3}, 2, 0)$

--End of Answers—

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