Solver: Wong Yuan Neng

1) *//Based on PYP observations, Q1 is always the morale-killing question and is not worth the time. Skip this shit if you don't know how to do and don't be demoralized.*

   a) Given $\lim_{x\to 0}\frac{sinx}{x}=1$ and $\frac{d}{dx}\log_a x=\frac{1}{xlna}$, find the limits below

   i) $\lim_{x\to 0}\frac{x+xcosx}{sinx\times cosx}=\lim_{x\to 0}\frac{x}{sinx\times cosx}+\lim_{x\to 0}\frac{xcosx}{sinx\times cosx}$

   $=\lim_{x\to 0}\frac{2x}{2\times sinx\times cosx}+\lim_{x\to 0}\frac{x}{sinx}$

   $=\lim_{x\to 0}\frac{(2x)}{\sin(2x)}+\lim_{x\to 0}\left(\frac{sinx}{x}\right)^{-1}$

   $=\lim_{x\to 0}\left(\frac{\sin(2x)}{(2x)}\right)^{-1}+\lim_{x\to 0}\left(\frac{sinx}{x}\right)^{-1}$

   $=(\lim_{x\to 0}\frac{(2x)}{\sin(2x)})^{-1}+(\lim_{x\to 0}\frac{x}{\sin x})^{-1}$

   $=(1)+(1)$ [When x→0, 2x→0, so both limit expressions are =1.]

   $=2$

   ii) $\lim_{x\to\infty}\frac{2^x-3^x}{3^x-4^x}=\lim_{x\to\infty}\frac{\frac{2^x}{4^x}-\frac{3^x}{4^x}}{\frac{3^x}{4^x}-\frac{4^x}{4^x}}=\lim_{x\to\infty}\frac{(\frac{2}{4})^x-(\frac{3}{4})^x}{\left(\frac{3}{4}\right)^x-1}=\frac{0-0}{0-1}=0$

   //One way to handle exponents is to divide by largest denominator (just like handling polynomials) so that each base is now between 0 and 1. By doing so, when x is large then the large power of a base between 0 and 1 will return 0. Not sure how to go about using the given info of $\frac{d}{dx}(log_a x)=\frac{1}{xlna}$ in this solution, but the one I gave would be a good choice as it can be applied to any fraction-related exponent questions with any power (x+k).

   b) Differentiate the following functions wrt x

   i) $y=\frac{4}{x}+6sinx$

   By differentiating on both sides,

   $\frac{dy}{dx}=\frac{0(x)-(1)(4)}{x^2}+[(0)(sinx)+(cosx)(6)]$ *//quotient and product rule*

   $\frac{dy}{dx}=\frac{-4}{x^2}+6cosx$

   $\frac{dy}{dx}=6cosx-\frac{4}{x^2}$

   ii) $y=\frac{cosx}{1-cosx}$

   By differentiating on both sides,

   $\frac{dy}{dx}=\frac{(-sinx)(1-cosx)-(sinx)(cosx)}{(1-cosx)^2}$

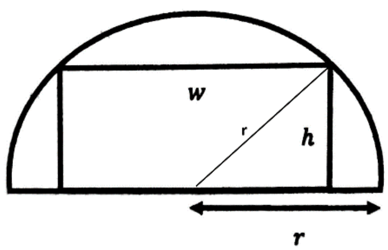
   $\frac{dy}{dx}=\frac{-sinx+sinxcosx-sinxcosx}{(1-cosx)^2}$

   $=\frac{-sinx}{(1-cosx)^2}$

$$\frac{dy}{dx} = -\frac{sinx}{(1-cosx)^2}$$

c) The function $y = ax^2 + bx + c$ passes through the point (2,9) and its tangent at the point (1,2) is $y = 4x - 2$. Find a,b,c.
Using (2,9), 9 = a$(2)^2$ + b(2) + c ➔ 4a + 2b + c = 9 [eqn 1]
Using (1,2), 2 = a$(1)^2$ + b(1) + c ➔ a + b + c = 2 [eqn 2]
Take differentiation on both sides to obtain dy/dx:
$y = ax^2 + bx + c$ ➔ $\frac{dy}{dx} = 2ax + b$
The given tangent has gradient 4, so 4 = 2ax+b
Given the point of the tangent is (1,2), thus 4 = 2a(1) +b ➔ 2a+b=4 [eqn 3]
Using Graphing calculator to solve equations 1,2 and 3, *//yes use GC to save time*
Thus a=3, b=-2, c=1.

d) Determine dimensions (h and w) of the largest rectangle **<u>in terms of r</u>** that can be inscribed in a semicircle of radius r.



Area of rectangle R = hw
Since we want largest rectangle in terms of r then we need to find $\frac{dR}{dw}$ or $\frac{dR}{dh}$.
First, we need to find an eqn relating w,h,r.
Notice that 0.5w, h and r gives a right-angled triangle. ie. $r^2 = h^2 + (0.5w)^2$.
Thus, $h = \sqrt{r^2 - 0.25w^2}$ (reject negative root because h>0)

$R = hw = \left(\sqrt{r^2 - 0.25w^2}\right) * w$
$= w(r^2 - 0.25w^2)^{0.5}$

Differentiating implicitly on both sides wrt w, (treating r as a constant because r is fixed)
$\frac{dR}{dw} = (r^2 - 0.25w^2)^{0.5} + 0.5(r^2 - 0.25w^2)^{-0.5}(-0.5w)$
$= (r^2 - 0.25w^2)^{0.5} - \frac{0.25w}{(r^2 - 0.25w^2)^{0.5}}$

Since R is max, then point is stationary ie. $\frac{dR}{dw} = 0$
ie. $(r^2 - 0.25w^2)^{0.5} - \frac{0.25w}{(r^2-0.25w^2)^{0.5}} = 0$
➔ $(r^2 - 0.25w^2)^{0.5} = \frac{0.25w}{(r^2-0.25w^2)^{0.5}}$
Multiplying on both sides by $(r^2 - 0.25w^2)^{0.5}$,
➔ $(r^2 - 0.25w^2)^1 = 0.25w$

➔ $0.25w^2 + 0.25w - r^2$=0

➔ $w = \frac{-0.25 \pm\sqrt{0.25^2-4(0.25)(-r^2)}}{2(0.25)}$

Thus, $w = \frac{-0.25 \pm\sqrt{0.0625+r^2}}{0.5} = -0.5 \pm 2\sqrt{0.0625+r^2}$

➔ $w = -0.5 + 2\sqrt{0.0625+r^2}$ since w>0

We then verify if this value for w maximises R for an arbitrary value of r (use a calculator to calculate):

w	$\left(-0.5+2\sqrt{0.0625+r^2}\right)^-$	$-0.5+2\sqrt{0.0625+r^2}$	$\left(-0.5+2\sqrt{0.0625+r^2}\right)^+$
Eg.	$2\sqrt{0.0625+r^2}-1$	$-0.5+2\sqrt{0.0625+r^2}$	$2\sqrt{0.0625+r^2}$
dR/dw	+	0	-
slope	/	-	\

//In general, if there is only 1 value for w, it WILL maximize/minimize according to the qn. This tabular check is usually a "process" to show you care to spend 5min verifying your answer.

Recall that $h = \sqrt{r^2 - 0.25w^2} = \sqrt{r^2 - 0.25\left(-0.5+2\sqrt{0.0625+r^2}\right)^2}$

$= \sqrt{r^2 - 0.25\left(4(0.0625+r^2) - 2\sqrt{0.0625+r^2} + 0.25\right)}$

$= \sqrt{r^2 - \left((0.0625+r^2) - 0.5\sqrt{0.0625+r^2} + 0.0625\right)}$

$= \sqrt{r^2 - 0.125 + r^2 + 0.5\sqrt{0.0625+r^2}} = \sqrt{2r^2 - 0.125 + 0.5\sqrt{0.0625+r^2}}$

2)

a) Evaluate $\int_1^4 \sqrt{x+1}\,dx$

= $\int_1^4 (x+1)^{0.5}\,dx$ = $[\frac{1}{1.5} * (x+1)^{1.5}]_1^4$

= $\frac{2}{3} * [(4+1)^{1.5} - (1+1)^{1.5}]$

= $\frac{2}{3} * [5^{1.5} - 2^{1.5}]$ = 5.57 (3sf)

//Tip: Do spend 10s checking answer using GC: use MATH→9→fnInt((x+1)$^{0.5}$,x,1,4)

//further details do check online.

b) Evaluate $\int cos^2x\,dx$ (Compound angle not necessary to eval it).

Using compound angle (not necessary but should be accepted):

$\int cos^2x\,dx$

= $0.5 * \int 2cos^2x - 1 + 1\,dx$

= $0.5 * \int cos2x + 1\,dx$

= $0.5 * (0.5 * sin2x + x) + c$

= $0.25sin2x + 0.5x + c$, where $c \epsilon R$

//can't think of any other non-compound angle methods...

c) Find dy/dx if $y = \left(\int_0^x (t+1)^2 dt\right)^2$

Differentiating on both sides wrt x, thus

$$\frac{dy}{dx} = \frac{d}{dx}\left(\int_0^x (t+1)^2 dt\right)^2$$

$$= 2 * \left(\int_0^x (t+1)^2 dt\right) * \frac{d}{dx}\left(\int_0^x (t+1)^2 dt\right)$$

$$= 2 * \left[\frac{1}{3}(t+1)^3\right]_0^x * \frac{d}{dx}\left(\left[\frac{1}{3}(t+1)^3\right]_0^x\right)$$

$$= 2 * \left[\frac{1}{3}(x+1)^3 - \frac{1}{3}(0+1)^3\right] * \frac{d}{dx}\left[\frac{1}{3}(x+1)^3 - \frac{1}{3}(0+1)^3\right]$$

$$= \left[\frac{2}{3}(x+1)^3 - \frac{2}{3}\right] * \frac{d}{dx}\left[\frac{1}{3}(x+1)^3 - \frac{1}{3}\right]$$

$$= \left[\frac{2}{3}(x+1)^3 - \frac{2}{3}\right] * \left[(x+1)^2\right]$$

$$= \frac{2}{3}(x+1)^2\left[(x+1)^3 - 1\right]$$

//you can shorten the working by using fundamental theorem 1 (see lecture notes)
//in my working I proved this theorem at the same time that's why it's long.

d) What values of a and b maximize the value of $\int_a^b (1 + x - x^2)\, dx$?
Maximising the value = maximizing the area BETWEEN graph AND x-axis.

Thus we find x-intercepts of the curve $y = 1 + x - x^2$.
When y=0, $x^2 - x - 1 = 0$ ➔ $(x - 0.5)^2 - 0.25 - 1 = 0$ (Complete the square method)
$\therefore (x - 0.5)^2 = 1.25$
➔ $x - 0.5 = \pm\sqrt{1.25}$
➔ $x = 0.5 \pm \sqrt{1.25}$
Hence, lower-bound $a = 0.5 - \sqrt{1.25}$ and upper-bound $b = 0.5 + \sqrt{1.25}$

e) Temperature of food court is 25. Initially egg's temperature is at 95. After 2 min, egg's temperature is 50. How much time will it take before Yew Lee can eat the egg at 40?
//suspicious that no model function is given…

So assume relationship between temperature T and time t can be modelled by an exponential curve as shown below.
ie. $T = ce^{-kt} + s$
At t=∞, T=25 (temp of food court).
ie. $25 = \lim_{t\to\infty}(ce^{-kt} + s)$
➔ $25 = c * \lim_{t\to\infty}\left(\frac{1}{e^{kt}}\right) + s$

Since $e = 2.718$,

so 25 = c*($\frac{1}{\infty}$)+s

➔ s=25.

At t=0, T=95.

ie. 95 = ce$^{-k(0)}$ + 25

→ c = 70

At t=2, T=50.

ie. 50 = 70e$^{k(2)}$ + 25

→ e^{2k} = 5/14 → k= 0.5*ln(5/14)

Thus when T = 40,

40 = 70e$^{(0.5*\ln(5/14))(t)}$ + 25

→ 0.5t*ln(5/14) = ln(15/70)

→ t = 2 $* \frac{\ln\left(\frac{3}{14}\right)}{\ln\left(\frac{5}{14}\right)}$

t = 2.99min.

3)

a) Find a formula for the general term a_n of the sequence assuming that the pattern continues. Find the limit of the sequence.

Formula:

Notes:

(1) Remember alternate negative sign involves $(-1)^{n+1}$
(2) Numerator is of course increment by 1.
(3) Denominator is always even number so maybe pattern may involve multiplication by 2?
(4) Notice that denominator is always close to multiple of 3, so maybe power of 3?

sign	+	-	+	-	+	-	+	-
Value	5/2	6/4	7/10	8/28	9/82	10/244	11/730	12/2188
Pattern for denominator	3^0+1	3^1+1	3^2+1	3^3+1	3^4+1	3^5+1	3^6+1	3^7+1

So it can be deduced that the pattern is $a_n = (-1)^{n+1}\frac{4+n}{3^{n-1}+1}$

Limit of the sequence: Not exist because sequence is alternating.

[Applying exponent rule to an alt sequence will give ln(-ve value) = undefined]

**this can be proved using divergence criterion test (not covered in lectures):*

Using subsequence n=2x-1,

$\lim_{n\to\infty}\left[(-1)^{n+1}\frac{4+n}{3^{n-1}+1}\right]=\lim_{2x-1\to\infty}\left[(-1)^{2x}\frac{4+(2x-1)}{3^{2x-2}+1}\right]=\lim_{2x-1\to\infty}\left[\frac{4+(2x-1)}{3^{2x-2}+1}\right]=\lim_{2x-1\to\infty}\left[\frac{4+(2x-1)}{3^{2x-2}+1}\right]$

Using subsequence n=2x,

$\lim_{n\to\infty}\left[(-1)^{n+1}\frac{4+n}{3^{n-1}+1}\right]=\lim_{2x\to\infty}\left[(-1)^{2x+1}\frac{4+2x}{3^{2x-1}+1}\right]=\lim_{2x\to\infty}\left[-\frac{4+2x}{3^{2x-1}+1}\right]=-\lim_{2x\to\infty}\left[\frac{4+2x}{3^{2x-1}+1}\right]$

Since $\lim_{2x-1\to\infty}\left[\frac{4+(2x-1)}{3^{2x-2}+1}\right] \neq -\lim_{2x\to\infty}\left[\frac{4+2x}{3^{2x-1}+1}\right]$*, ie. sign itself is already different, thus divergent, ie. limit does not exist.*

b) Use Squeeze theorem to find the limit of the following sequence.

$$a_n = \frac{\sin n + 3}{5^n + n^2 + n + 1}$$

We know $-1 \leq \sin n \leq 1$, so we know $\frac{2}{5^n+n^2+n+1} \leq a_n \leq \frac{4}{5^n+n^2+n+1}$

$\lim_{n\to\infty} \frac{2}{5^n+n^2+n+1} = \frac{2}{\infty} = 0$, likewise, $\lim_{n\to\infty} \frac{4}{5^n+n^2+n+1} = \frac{4}{\infty} = 0$

Since $\lim_{n\to\infty} \frac{2}{5^n+n^2+n+1} = \lim_{n\to\infty} \frac{4}{5^n+n^2+n+1} = 0$, thus $\lim_{n\to\infty} \frac{\sin n+3}{5^n+n^2+n+1} = 0$.

Hence, a_n is convergent by squeeze theorem and limit = 0.

[You can verify it using https://www.symbolab.com/solver/limit-calculator]

c) Determine and show whether this series converges or diverges:

$\sum_{n=1}^{\infty} \frac{6^n}{(5n+1)^n}$ ➔ $\frac{6^n}{(5n+1)^n} < \frac{6^n}{(5n)^n}$

Apply ratio test:

$$\lim_{n\to\infty} \left|\frac{\frac{6^{n+1}}{(5(n+1))^{n+1}}}{\frac{6^n}{(5n)^n}}\right| = \lim_{n\to\infty} \left|\frac{6^{n+1}}{(5n+5)^{n+1}} * \frac{(5n)^n}{6^n}\right| = \lim_{n\to\infty} \left|\frac{6}{5n+5} * (\frac{5n}{5n+5})^n\right| = \lim_{n\to\infty} \left|\frac{6}{5n+6} * (1 - \frac{5}{5n+6})^n\right| = \left(\lim_{n\to\infty} \left|\frac{6}{5n+6}\right|\right) * \left(\lim_{n\to\infty} \left|\left(1 - \frac{5}{5n+6}\right)^n\right|\right) = (0) * (\ldots) = 0$$

[can just ignore 2nd part since first part =0. Don't bother wasting time to evaluate.]

Since $\sum_{n=1}^{\infty} \frac{6^n}{(5n)^n}$ is convergent series, hence $\sum_{n=1}^{\infty} \frac{6^n}{(5n+1)^n}$ is converges as well.

d) Determine and show whether this series converges or diverges

$\sum_{n=1}^{\infty} \frac{2^n n^4 (2n+1)!}{(n!)^4}$ ➔ With factorials and exponents involved, we should attempt ratio test first.

$$\lim_{n\to\infty} \left|\frac{\frac{2^{n+1}(n+1)^4(2(n+1)+1)!}{\left((n+1)!\right)^4}}{\frac{2^n n^4(2n+1)!}{(n!)^4}}\right| = \lim_{n\to\infty} \left|\frac{(n!)^4}{[(n+1)(n!)]^4} * \frac{2 * 2^n (n+1)^4 (2n+3)!}{2^n n^4 (2n+1)!}\right|$$

$$= \lim_{n\to\infty} \left|\frac{(n!)^4}{(n+1)^4 (n!)^4} * \frac{2(n+1)^4 * 2^n (2n+3)(2n+2)[(2n+1)!]}{2^n n^4 (2n+1)!}\right|$$

$$= \lim_{n\to\infty} \left|\frac{1}{(n+1)^4} * \frac{2(n+1)^4 (2n+3)(2n+2) * 2^n (2n+1)!}{n^4 * 2^n (2n+1)!}\right|$$

$$= \lim_{n\to\infty} \left|\frac{2(2n+3)(2n+2)}{n^4}\right|$$

$$= \lim_{n\to\infty} \left|\frac{8n^2 + 20n + 12}{n^4}\right|$$

$$= \lim_{n\to\infty} \left|\frac{\frac{8}{n^2} + \frac{20}{n^3} + \frac{12}{n^4}}{1}\right| = 0 < 1$$

Thus, (absolutely) convergent.

e) Find the radius of convergence for the following series.

$\sum_{n=1}^{\infty} \frac{2^n (x-4)^n}{n^5-2n+1} = \sum_{n=1}^{\infty} \frac{2^n}{n^5-2n+1} * (x-4)^n$

➔ $c_n = \frac{2^n}{n^5-2n+1}$

Radius of convergence $R = \lim_{n\to\infty} \left|\frac{c_n}{c_{n+1}}\right| = \lim_{n\to\infty} \left|\frac{\frac{2^n}{n^5-2n+1}}{\frac{2^{n+1}}{(n+1)^5-2(n+1)+1}}\right|$

$$= \lim_{n\to\infty} \left|\frac{2^n}{2^{n+1}} * \frac{(n+1)^5-2n-1}{n^5-2n+1}\right|$$

$$= \lim_{n\to\infty} \left|\frac{1}{2} * \frac{\left(1+\frac{1}{n}\right)^5 - \frac{2}{n^4} - \frac{1}{n^5}}{1-\frac{2}{n^4}+\frac{1}{n^5}}\right|$$

$$= 0.5 * \frac{(1+0)^5-0-0}{1-0+0}$$

$= 0.5(1)$

$= 0.5.$

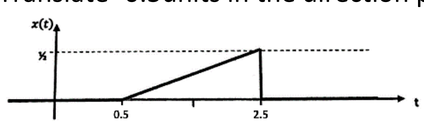
4) GG 20m Fourier

a) Sketch y(t) = -2x(-2t+1)+0.5 given x(t).

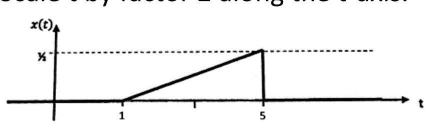
We attempt to derive a graphic transformation process from x(t) to y(t). [In general, order of transformation is $T_x S_x S_y T_y$]

y(t) = -2x(-2(t-0.5)) + 0.5

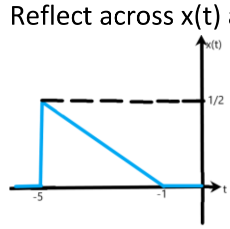
(1) Translate -0.5units in the direction parallel to positive t-axis.



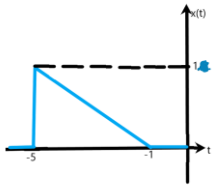
(2) Scale t by factor 2 along the t-axis.



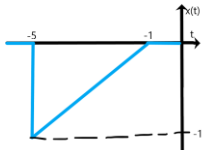
(3) Reflect across x(t) axis.



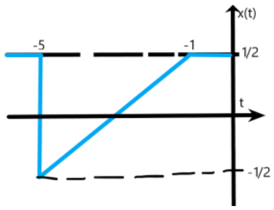
(4) Scale x(t) by factor 2 along x(t)-axis



(5) Reflect across t-axis.



(6) Translate +0.5units in the direction parallel to positive x(t)-axis.



This is the resulting sketch (**Your answer only needs to have this 1 final sketch**).

b) Given y(t), determine fundamental frequency Ω_0 of the signal and hence list all C_k that are nonzero.

$$y(t) = 1 + 2\cos\left(2\pi 3t + \frac{\pi}{3}\right) + 3e^{+j\left(2\pi 4t - \frac{\pi}{4}\right)}$$

$$= 1 + 2\left[\frac{e^{+j\left(2\pi 3t + \frac{\pi}{3}\right)} + e^{-j\left(2\pi 3t + \frac{\pi}{3}\right)}}{2}\right] + 3e^{+j\left(2\pi 4t - \frac{\pi}{4}\right)}$$

$$= 1 + e^{+j\left(2\pi 3t + \frac{\pi}{3}\right)} + e^{-j\left(2\pi 3t + \frac{\pi}{3}\right)} + 3e^{+j\left(2\pi 4t - \frac{\pi}{4}\right)}$$

Fundamental frequency $\Omega_0 = 2\pi * \text{GCD}(3,4) = 2\pi$ $\quad$ *//see $2\pi 3t$ and $2\pi 4t$ ie. $2\pi F t$*

Perform CTFS Analysis to obtain a value for C_k:
First, find T_0, which is the period for the summed signal y(t).

$$T_0 = \frac{1}{F} = \frac{1}{1} = 1$$

$$A_0 = \frac{1}{2(0.5)} * \int_{-0.5}^{0.5} y(t)\, dt$$

$$A_n = \frac{1}{(0.5)} * \int_{-0.5}^{0.5} y(t)\cos(2\pi(1)tn)\, dt = 2\int_{-0.5}^{0.5} y(t)\cos(2\pi tn)\, dt$$

$$= 2\int_{-0.5}^{0.5} y(t) * \frac{e^{j(2\pi tn)} + e^{-j(2\pi tn)}}{2}\, dt = \int_{-0.5}^{0.5} y(t) * e^{j(2\pi tn)} + y(t) * e^{-j(2\pi tn)}\, dt$$

$$B_n = \frac{1}{(0.5)} * \int_{-0.5}^{0.5} y(t)\sin(2\pi(1)tn)\, dt = 2\int_{-0.5}^{0.5} y(t)\sin(2\pi tn)\, dt$$

$$= 2\int_{-0.5}^{0.5} y(t) * \frac{e^{j(2\pi tn)} - e^{-j(2\pi tn)}}{2}\, dt = \int_{-0.5}^{0.5} y(t) * e^{j(2\pi tn)} - y(t) * e^{-j(2\pi tn)}\, dt$$

$$\int_{-0.5}^{0.5} y(t) * e^{j(2\pi tn)}\, dt = \int_{-0.5}^{0.5} \left[1 + e^{+j\left(2\pi 3t+\frac{\pi}{3}\right)} + e^{-j\left(2\pi 3t+\frac{\pi}{3}\right)} + 3e^{+j\left(2\pi 4t-\frac{\pi}{4}\right)}\right] * e^{j(2\pi tn)}\, dt$$
$$= \int_{-0.5}^{0.5} \left[e^{j(2\pi tn)} + e^{+j\left(2\pi 3t+\frac{\pi}{3}\right)+j(2\pi tn)} + e^{-j\left(2\pi 3t+\frac{\pi}{3}\right)+j(2\pi tn)} + 3e^{+j\left(2\pi 4t-\frac{\pi}{4}\right)+j(2\pi tn)}\right] dt$$
$$= \int_{-0.5}^{0.5} \left[e^{j(2\pi tn)} + e^{+j\left(2\pi(3+n)t+\frac{\pi}{3}\right)} + e^{-j\left(2\pi(3-n)t+\frac{\pi}{3}\right)} + 3e^{+j\left(2\pi(4+n)t-\frac{\pi}{4}\right)}\right] dt$$
$$= \left[\frac{1}{2\pi jn} * e^{j(2\pi tn)} + \frac{1}{2\pi j(3+n)} * e^{+j\left(2\pi(3+n)t+\frac{\pi}{3}\right)} + \frac{1}{-2\pi j(3-n)} * e^{-j\left(2\pi(3-n)t+\frac{\pi}{3}\right)} + \frac{1}{2\pi j(4+n)} * 3e^{+j\left(2\pi(4+n)t-\frac{\pi}{4}\right)}\right]_{-0.5}^{0.5} = \cdots$$

$$\int_{-0.5}^{0.5} y(t) * e^{-j(2\pi tn)}\, dt = \int_{-0.5}^{0.5} \left[1 + e^{+j\left(2\pi 3t+\frac{\pi}{3}\right)} + e^{-j\left(2\pi 3t+\frac{\pi}{3}\right)} + 3e^{+j\left(2\pi 4t-\frac{\pi}{4}\right)}\right] e^{-j(2\pi tn)}\, dt$$
$$= \int_{-0.5}^{0.5} \left[e^{-j(2\pi tn)} + e^{+j\left(2\pi 3t+\frac{\pi}{3}\right)-j(2\pi tn)} + e^{-j\left(2\pi 3t+\frac{\pi}{3}\right)-j(2\pi tn)} + 3e^{+j\left(2\pi 4t-\frac{\pi}{4}\right)-j(2\pi tn)}\right] dt$$
$$= \int_{-0.5}^{0.5} \left[e^{-j(2\pi tn)} + e^{+j\left(2\pi(3-n)t+\frac{\pi}{3}\right)} + e^{-j\left(2\pi(3+n)t+\frac{\pi}{3}\right)} + 3e^{+j\left(2\pi(4-n)t-\frac{\pi}{4}\right)}\right] dt$$
$$= \left[\frac{1}{-2\pi jn} * e^{-j(2\pi tn)} + \frac{1}{2\pi j(3-n)} * e^{+j\left(2\pi(3-n)t+\frac{\pi}{3}\right)} + \frac{1}{-2\pi j(3+n)} * e^{-j\left(2\pi(3+n)t+\frac{\pi}{3}\right)} + \frac{1}{2\pi j(4-n)} * 3e^{+j\left(2\pi(4-n)t-\frac{\pi}{4}\right)}\right]_{-0.5}^{0.5} = \cdots$$

Therefore $A_0 = \cdots, A_n = \cdots, B_n = \cdots$
//you will be able to get these 3 coefficients in terms of n. Working too long, shall not do.

c)

i) Find the FT $X(j\Omega)$ of the above signal.
CTFT analysis:

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}\, dt$$
$$= \int_{0}^{1} (-2t + 2)e^{-j\Omega t}\, dt$$
$$= \int_{0}^{1} -2te^{-j\Omega t}\, dt + \int_{0}^{1} 2e^{-j\Omega t}\, dt$$
$$= -\int_{0}^{1} t * 2e^{-j\Omega t}\, dt + \int_{0}^{1} 2e^{-j\Omega t}\, dt$$

Integrate by parts:

let $u = t \; and \frac{dv}{dt} = 2e^{-j\Omega t}$
Thus, $du = dt$
$and\; v = \int 2e^{-j\Omega t}\, dt = \frac{2}{-j\Omega} e^{-j\Omega t}$.

$$\int_0^1 t * 2e^{-j\Omega t}\, dt = \left[t\left(\frac{2}{-j\Omega} e^{-j\Omega t}\right)\right]_0^1 - \int_0^1 \frac{2}{-j\Omega} e^{-j\Omega u} du$$

$$= \left(\frac{2}{-j\Omega} e^{-j\Omega}\right) - \left[\frac{2}{(j\Omega)^2} e^{-j\Omega u}\right]_0^1$$

$$= \frac{2}{-j\Omega} e^{-j\Omega} - \frac{2}{(j\Omega)^2} e^{-j\Omega} + \frac{2}{(j\Omega)^2}$$

$$= e^{-j\Omega}\left(\frac{2}{-j\Omega} - \frac{2}{(j\Omega)^2}\right) + \frac{2}{(j\Omega)^2}$$

$$X(j\Omega) = -\int_0^1 t * 2e^{-j\Omega t}\, dt + \int_0^1 2e^{-j\Omega t}\, dt$$

$$= -\left(e^{-j\Omega}\left(\frac{2}{-j\Omega} - \frac{2}{(j\Omega)^2}\right) + \frac{2}{(j\Omega)^2}\right) + \left[\frac{2}{-j\Omega} e^{-j\Omega t}\right]_0^1$$

$$= e^{-j\Omega}\left(\frac{2}{j\Omega} + \frac{2}{(j\Omega)^2}\right) - \frac{2}{(j\Omega)^2} + \left(\frac{2}{-j\Omega} e^{-j\Omega} - \frac{2}{-j\Omega}\right)$$

$$= e^{-j\Omega}\left(\frac{2}{j\Omega} + \frac{2}{(j\Omega)^2} - \frac{2}{j\Omega}\right) - \frac{2}{(j\Omega)^2} + \frac{2}{j\Omega}$$

$$= e^{-j\Omega}\left(\frac{2}{(j\Omega)^2}\right) - \frac{2}{(j\Omega)^2} + \frac{2}{j\Omega}$$

$$= \frac{2}{(j\Omega)^2}\left(e^{-j\Omega} - 1\right) + \frac{2}{j\Omega}$$

ii) Evaluate $X(j0)$ and explain what this value represents.

$$X(j0) = \frac{2}{(j0)^2}\left(e^{-j0} - 1\right) + \frac{2}{j0}$$

$$= \frac{2}{0}(1-1) + \frac{2}{0}$$

$$= zero\ division\ error$$

When $\Omega = 0$, this means angular frequency is 0, which also means frequency F=0.
Hence, the signal will not exist.

Remarks: This paper is somewhat difficult. But for such difficult math papers (or math papers in general), it is crucial to grab as many method marks as possible by showing the examiner what you know even if you cannot provide a solution.

--End of Answers--