20th CSEC – Past Year Paper Solution *2018-2019 Sem 2*
MH 1812 – Discrete Mathematics

1)

a) $\neg(p \to q) \lor (p \land q) \equiv p$

i)

p	q	$p \to q$	$\neg(p \to q)$	$p \land q$	$\neg(p \to q) \lor (p \land q)$
T	T	T	F	T	T
T	F	F	T	F	T
F	T	T	F	F	F
F	F	T	F	F	F

$\therefore \neg(p \to q) \lor (p \land q) \equiv p$ Hence, proved

ii) $\neg(p \to q) \lor (p \land q) \equiv p$

$LHS \equiv \neg(\neg p \lor q) \lor (p \land q)$ [Conversion Theorem]

$\equiv (p \land \neg q) \lor (p \land q)$ [DeMorgan's]

$\equiv p \land (\neg q \lor q)$ [Distributivity]

$\equiv p \land T$ [Tautology]

$\equiv p \equiv RHS$

$\therefore$ LHS = RHS

b)

i) 1 5 0 5 2

5 slots, 1 repeat (5 x2)

5 4 3 2 1 options

Distinguishable permutations $= \frac{5!}{2!} = \mathbf{60}$

ii) Last slot is fixed 0 or 2, hence it has 2 options $\Rightarrow 2!$

Remaining slots has 4 options $\Rightarrow \frac{4!}{2!}$ (5 repeated)

$\therefore$ Number of even permutations $= \frac{4!}{2!} \times 2! = 4! = \mathbf{24}$

iii) For simplicity,

(>2019) can be taken as Total (=60) – (<2019) to reduce the number of cases to consider

$\therefore$ To find number of permutations less than 2019.

The 1st slot is fixed to 0 (this must happen because 2019 is 4 digits)

0 _ _ _ _

The 2nd slot can only be 1 then

0 1 _ _ _

The remaining slots can then take any other number as this number is already less than 2019 by having the thousand digit as 1.

$\therefore No. of\ permutations = 1 \times 1 \times \frac{3!}{2!} = 3$ (5 repeated)

$\therefore No. of\ permutation\ greater\ than\ 2019 = 60 - No. of\ permutations\ less\ than\ 2019$
$= 60 - 3 = \mathbf{57}$

2)

a)

i) R is reflexive (1, 1), (2, 2), (3, 3)

ii) R is not symmetric (1, 3) but no (3, 1)

iii) R is not anti-symmetric (1, 2), (2, 1)

iv) {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 3)}
R is already its own transitive closure.

b) $f: z \rightarrow z$ F is the set of f.
i.e. any junction belonging in F is injective.
The question basically defines 2 functions and asks us to check if they are injective, i.e. one-one

i) $g(x) = 7x - 2$
Given that x is an integer, subtraction and multiplication on an integer will result in an integer. $\Rightarrow g \in F$
[Formal proof:]
Let $g(x_1) = g(x_2)$
$7x_1 - \cancel{2} = 7x_2 - \cancel{2}$
$x_1 = x_2$
$\therefore$ g is injective $\Rightarrow g \in F$

ii) $h(x) = x^2 - 5x$
Let $h(x_1) = h(x_2)$
$\Rightarrow x_1^2 - 5x_1 = x_2^2 - 5x_2$
$x_1^2 - x_2^2 = 5x_1 - 5x_2$
$(x_1 - x_2)(x_1 + x_2) = 5(x_1 - x_2)$
$x_1 + x_2 = 5$
$x_1 \neq x_2$ as can be seen if $x_1 = 5, x_2 = 5 - x_1 = 0 \neq x_1$
$\therefore$ h(x) is not injective $\Rightarrow h \notin F$

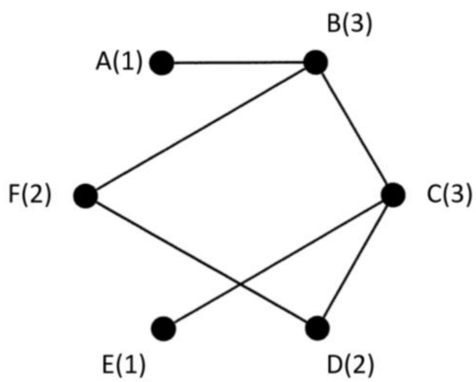
iii) We have already shown that g(x) is one-one.
If g(x) is surjective, it is invertible.
Let $g(x) = 7x - 2 = y$
$\Rightarrow x = \frac{y+2}{7}$ which may not be an integer as can be seen when $g(x) = 10 \in z, x = \frac{12}{7} \notin z$
$\Rightarrow$ It is **false** that for every $x \in z$ there exist a $y \in z$
$\therefore$ g(x) is not invertible

3) $a_n = a_{n-1} + 6a_{n-2}$
Characteristic Equation:
$x^2 = x + 6$
$\Rightarrow x^2 - x - 6 = 0$
$\Rightarrow x^2 - 3x + 2x - 6 = 0$
$\Rightarrow x = 3 \; or \; x = -2$ [2 distinct roots]
$s_1 = 3, s_2 = -2$
$a_n = u(3)^n + v(-2)^n$

$a_0 = 1 \Rightarrow 1 = u(3)^0 + v(-2)^0$
$u + v = 1$ --- (1)
$a_1 = 8 \Rightarrow 3u - 2v = 8$ --- (2)
Solving (1) & (2)
(1) x2: $2u + 2v = 2$ --- (3)
(2) & (3): $5u = 10$
$\Rightarrow u = 2$ --- (4)
Subst (4) into (1): $2 + v = 1$
$\Rightarrow v = -1$
$\therefore \boldsymbol{a_n = 2(3)^n + (-1)(2)^n}$

4)

a)



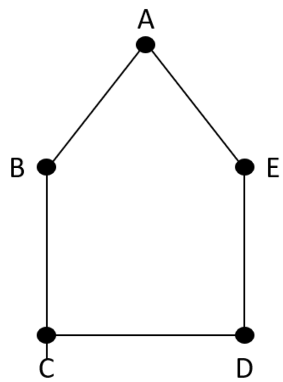
i) Conditions for Euler path – walk along each edge only once.
Clearly, by observation of nodes A and E, it is **not possible to have an Euler path** in the given graph because although it is possible to traverse the graph from A to E, the BC would be missed out if traversed as ABFDCE or ECDFBA.

ii) Conditions for Hamiltonian path – walk along each vertex only once.
Again, we can observe the graph and see that on traversing the vertices in the order of ABFDCE we **obtain a Hamiltonian path**.

iii) Conditions for Euler circuit – All nodes must have even degree.
This condition is not satisfied. Hence, the graph **does not have an Euler circuit**.

b) The vertices are all connected such that every set of 3 adjacent vertices are all connected by at most (exactly here) 2 edges.

∴ The above graph satisfies the given condition.



5)

i) $P(n) = \sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}$

$P(1) = \frac{1}{(2-1)(2+1)} = \frac{1}{3}$ [LHS]

$$RHS = \frac{1}{2(1)+1} = \frac{1}{3} = LHS$$

$\therefore P(1)\ is\ true.$

$$P(k) = \sum_{i=1}^{k} \frac{1}{(2i-1)(2i+1)} = \frac{k}{2k+1}$$

Let P(k) be true

Prove $P(k+1) = \sum_{i=1}^{k+1} \frac{1}{(2i-1)(2i+1)} = \frac{k+1}{2(k+1)+1}$ is true.

$$LHS = \sum_{i=1}^{k+1} \frac{1}{(2i-1)(2i+1)}$$

$$= \sum_{i=1}^{k} \frac{1}{(2i-1)(2i+1)} + \frac{1}{(2(k+1)-1)(2(k+1)+1)}$$

$$= \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$$

$$= \frac{k(2k+3)+1}{(2k+1)(2k+3)}$$

$$= \frac{2k^2+3k+1}{(2k+1)(2k+3)}$$

$$= \frac{(2k+1)(k+1)}{(2k+1)(2k+3)}$$

$$= \frac{k+1}{2k+3}$$

$$= \frac{k+1}{2(k+1)+1} = RHS$$

$\therefore P(k+1)$ is true $\Rightarrow P(n)$ is true for all $n \geq 1$.

Since P(1) is true and P(k+1) is true if P(k) is true, by mathematical induction,

ii) $\sum_{k=13}^{37} \frac{1}{(2k-1)(2k+1)} = \sum_{k=1}^{37} \frac{1}{(2k-1)(2k+1)} - \sum_{k=1}^{12} \frac{1}{(2k-1)(2k+1)}$

$$= \frac{37}{2(37)+1} - \frac{12}{2(12)+1}$$

$$= \frac{37}{75} - \frac{12}{25}$$

$$= \mathbf{\frac{1}{75}}$$

--End of Answers--

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