Question 1

1)

a) x = u + 1
y = u – 2
z = 2u + 1

Sub u = 0,
x = 1, y = -2, z = 1
A = (1, -2, 1)

Sub u = 1,
x = 2, y = -1, z = 3
B = (2, -1, 3)

AB = (2-1, -1-(-2), 3-1) = (1, 1, 2)

Implicit Equation of the plane,
X + y + 2z – D = 0
Sub (1, 2, 3) into the equation,
D = 1 + 2 + (2*3)
= 9

X + y + 2z – 9 = 0

b) Sub u = 0 into all 3 sets of equation,
x = 1, y = 1, z =1
x =3, y = 2, z = 0
z = 2, y = 3, z = 4

<u>OR</u>

Sub u = 1 into all 3 sets of equation,
x = 3, y = 2, z = 0
x = 2, y = 3, z = 4
x = 1, y = 1, z = 1

Both are fine

P1 = (1, 1, 1)
P2 = (3, 2, 0)
P3 = (2, 3, 4)

x(u,v) = 1+ u(3-1) + v(2-1) = 1 + 2u + v
y(u,v) = 1 + u(2-1) + v(3-1) = 1 + u +2v
z(u,v) = 1 + u(0-1) + v(4-1) = 1 – u + 3v

c) The sinusoidal curve, $\sin(2 \pi u)$
Sinusoidal curve with amplitude 0.25, $0.25\sin(2 \pi u)$
Sinusoidal curve with 10 periodic oscillations, $0.25\sin(20 \pi u)$
The equation of the circle :
$x(u) = 0.75 \cos(\pi u)$
$y(u) = 0.75 \sin(2 \pi u)$

Sub the sinusoidal curve equation in as the radius to the circle equation,
$x(u) = (0.25\sin(20 \pi u) + 0.75)*\cos(\pi u)$
$y(u) = (0.25\sin(20 \pi u) + 0.75) *\sin(\pi u)$

Question 2

2)

a) $x(u) = w * 4 \cos (2 \pi u) + 1$
$y(u) = w * 5 \sin (2 \pi u)\cos (\pi v)+ 2$
$z(u) = w * 6 \sin (2 \pi u)\sin(\pi v) + 3$

b)

i) Sphere
$f(x,y,z) = (0.1)^2 - x^2 - (y-1.1)^2 - z^2 \geq 0$

Pyramid
$f(x,y,z) = \min(1-x-y, 1-z-y, 1+x-y, 1+z-y,y) \geq 0$

Cylinder,
$f(x,y,z) = (0.25)^2 - x^2 - (y)^2 \geq 0$

Final equation,
$f(x,y,z) = \min(\max((0.1)^2 - x^2 - (y-1.1)^2 - z^2, \min(1-x-y, 1-z-y, 1+x-y, 1+z-y,y)), -((0.25)^2 - x^2 - (y)^2)) \geq 0$
$f(x,y,z) = \min(\max(Sphere,Pyramid), -Cylinder) \geq 0$

ii) Coordinates of centre = (0, 0.6, 0)
Size = [2, 1.2, 2]

c) The sinusoidal curve,
$x(u) = (u + 5)$ // 0.5 unit away from the origin
$y(u) = 0.2\sin(3 \pi u)$ //amplitude of 0.2 and a period of 3π

Rotational sweeping of 1.25 clockwise, with this set of equation rotational sweeping starts on the x-axis.
$x=(u+0.5)*\cos(1.25 \pi v)$
$y=0.2*\sin(3 \pi u)$
$z=(u+0.5)*\sin(1.25 \pi v)$

To set the rotational sweeping to start on the z-axis.
$x=(u+0.5)*\cos(1.25 \pi v + pi/2)$
$y=0.2*\sin(3 \pi u)$
$z=(u+0.5)*\sin(1.25 \pi v + pi/2)$

For translational sweeping in y-axis,
x=(u+0.5)*cos(1.25 π v + pi/2)
y=0.2*sin(3 π u) +(-0.5 + 1.5 * w)
z=(u+0.5)*sin(1.25 π v + pi/2)

Question 3

3)

a) f(x,y) + (g(x,y) – f(x,y))t ≥ 0
   f(x,y)(1-t) + g(x,y)t ≥ 0
   Revise the model,
   f(x,y)(1-s) + g(x,y)s ≥ 0
   s = sin (0.5* π(k-1)/(120-1)) , k = 1,2,….,120

b) Translate x Scale =

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 4 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 4 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2\cos(2\pi u) \\ \sin(2\pi u) \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2\cos(2\pi u) + 2 \\ 3\sin(2\pi u) \\ -1 \\ 1 \end{bmatrix}$$

   x(u) = 2cos(2πu) + 2
   y(u) = 3sin(2πu)
   z(u) = -1

c) Coordinate 1,
   (-1,1) -> (1,0)
   1 = -a +b + m
   0 = -c + d + n

   Coordinate 2,
   (-9,3) -> (5,2)
   5 = -9a + 3b + m
   4 = -9c + 3d + n

   Coordinate 3,
   (-13,1) -> (5,4)
   5 = -13a + b + m
   4 = -13c + d + n

   Solving the simultaneous equations ,
   a= -1/3 , b =2/3, c = -1/3, m = 0, n = 0

The affine matrix

$$\begin{bmatrix} -1/3 & 2/3 & 0 \\ -1/3 & -1/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Find the new coordinates for (-12, -2),

$$\begin{bmatrix} -1/3 & 2/3 & 0 \\ -1/3 & -1/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -12 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 8/3 \\ 14/3 \\ 1 \end{bmatrix}$$

Find the new coordinates for (-12, 0),

$$\begin{bmatrix} -1/3 & 2/3 & 0 \\ -1/3 & -1/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -12 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix}$$

Question 4

4)

a) A point light source requires the specification on the location of the light source while the directional light source requires the specification of the direction.

b) x = (90 – t)cos(s)
y = (90 – t) sin(s)
z = t
s = [0,2π] , t =[0,60]

image of 351 x 143,
u = [0,350] v = [0, 142]

(s-0)/ (2π-0) = (u-0)/(350-0)
s = πu/175

(t-0)/ (60-0) = (v-0)/(350-0)
t = 30v/71

x(u,v) = $(90 - \frac{30}{71}v)\cos(\frac{\pi u}{175})$
y(u,v) = $(90 - \frac{30}{71}v)\sin(\frac{\pi u}{175})$
z(u,v) = $\frac{30}{71}v$

c)

i)

Find the light vector L,
L = [(0, 9, 13) – (0,0,1)]/ ||[(0,9,13) – (0,0,1)]||
= [0,3/5,4/5]

Find the normal vector N,
N = [0, 0 ,1]

Find the reflected light direction R,
R = 2(LN)N – L
= [0, -3/5, 4/5]

Find the viewing vector V,

$V = [(-6, 0, 9) – (0, 0, 1)]/||[(-6,0,9) – (0, 0, 1)]||$

$= [-3/5, 0, 4/5]$

LN = 4/5 = 0.8

RV = 16/25 = 0.64

$I = (0.2*0.4) + (0.6*0.6*0.6) + (0.5 * 0.6 * 0.64^2)$

$= 0.491$ (3dp)

ii) I = (0.2*0.4) = 0.08

Point light source is coming from behind.

== End of Answers ==

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