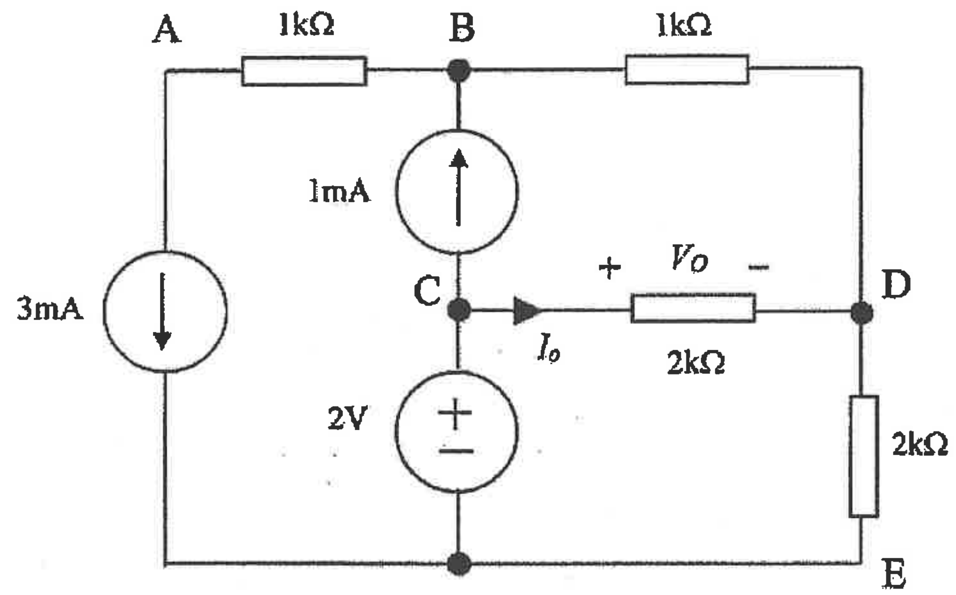
Solver: Hans Albert Lianto

Email Address: HANS0032@e.ntu.edu.sg

1 (a)



Given this diagram, let's use Node Analysis with Node E as the reference node.

We know that, $V_C = 2\ V$.

At Node B,

$$3m + (-1m) + \frac{V_B - V_D}{1k} = 0$$

$$\frac{V_D - V_B}{1k} = 2\ mA$$

At Node D,

$$\frac{V_D - V_C}{2k} + \frac{V_D - V_B}{1k} + \frac{V_D}{2k} = 0$$

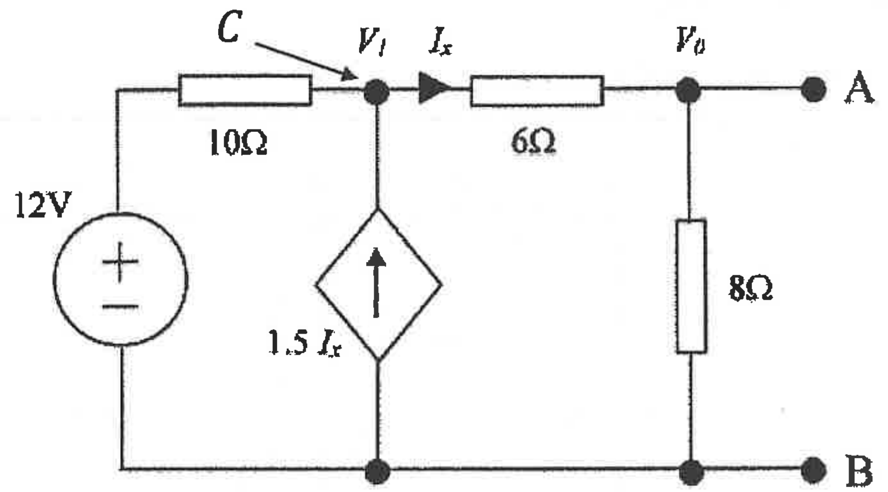
Substitute $V_C = 2\ V$ and $\frac{V_D - V_B}{1k} = -2\ mA$ to the equation in Node D.

$$\frac{V_D - 2}{2k} + 2m + \frac{V_D}{2k} = 0$$

$$V_D = -1\ V$$

$$V_O = V_C - V_D = 3\ V$$

(b)



We are using node B as a reference to find $V_{AB}(o/c)$, which is equal to V_0.

At Node C (indicated by the arrow),

$$I_x - 1.5I_x + \frac{V_1 - 12}{10} = 0$$

At Node A,

$$-I_x + \frac{V_0}{8} = 0$$

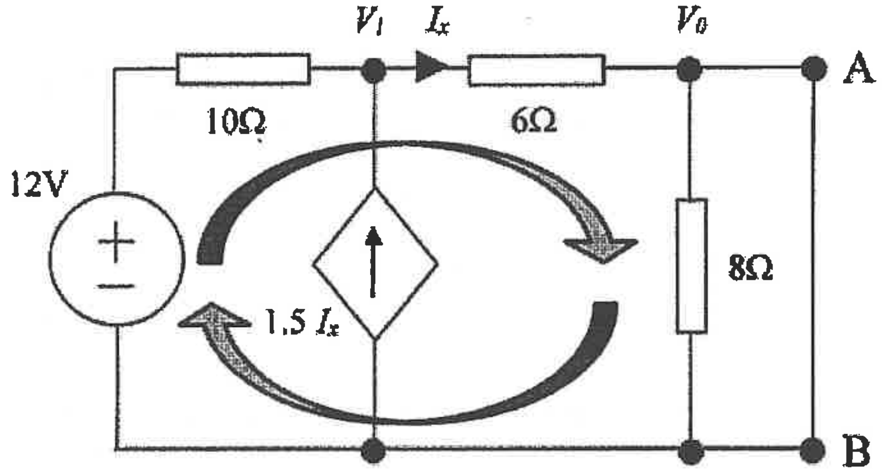
We know that:

$$I_x = \frac{V_1 - V_0}{6}$$

Solve for V_0 using these three equations to find:

$$V_0 = \frac{32}{3}\,V \approx 10.67\,V = V_{AB}(o/c)$$

To find the Thevenin resistance, we must first find $I_{AB}(s/c)$.



Looking at the short-circuit created, we can infer that $I_x = I_{AB}(s/c)$.

We can infer that the current across the 10 Ω resistor is $0.5I_x$, which in the context of the diagram, flows from right to left.

So just use Loop Analysis on the loop indicated above:

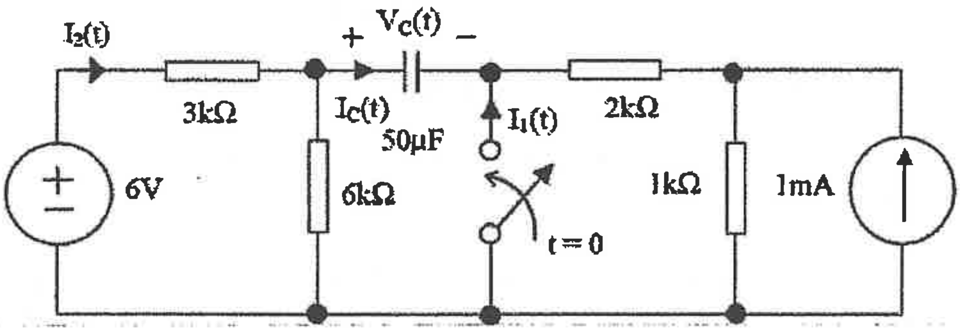
$$-12 + (-0.5I_x)(10) + (I_x)(6) + (0)(8) = 0$$

$$I_x = 12\,A = I_{AB}(s/c)$$

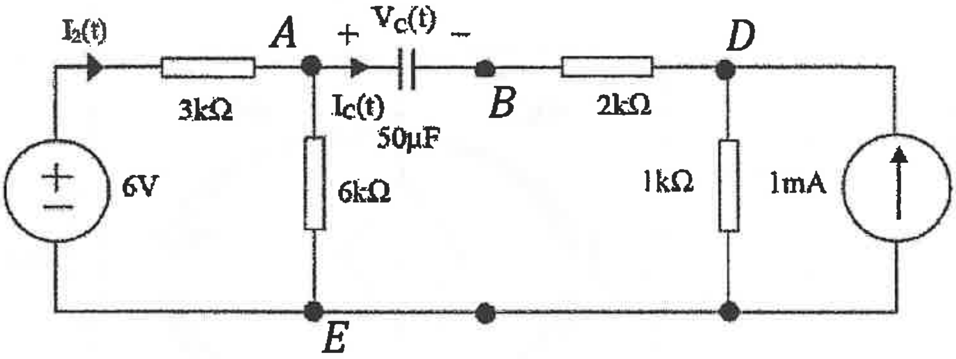
Thevenin resistance across A and *B* is:

$$R_{TH} = \frac{V_{AB}(o/c)}{I_{AB}(s/c)} = \frac{\frac{32}{3}\,V}{12\,A} = \frac{8}{9}\,\Omega \approx 0.89\,\Omega$$

2 (a) (i)



We will use the step-by-step approach for this.
Try to find V_C just right before the switch is closed ($V_C(t = 0-)$).



Since the capacitor at this point is at steady-state, it follows that the capacitor is like an open circuit. Let's now declare three nodes A, B, and D.

Inferring by logic, $V_B = V_D$ (since no current flows from B to D) and $V_C(t = 0-) = V_A - V_B$.

So, using Node Analysis on Node A with Node E as the reference node,

$$\frac{V_A - 6}{3k} + \frac{V_A}{6k} = 0$$

$$V_A = 4\,V$$

Also use it on Node D with Node E as the reference node,

$$-1m + \frac{V_D}{1k} = 0$$

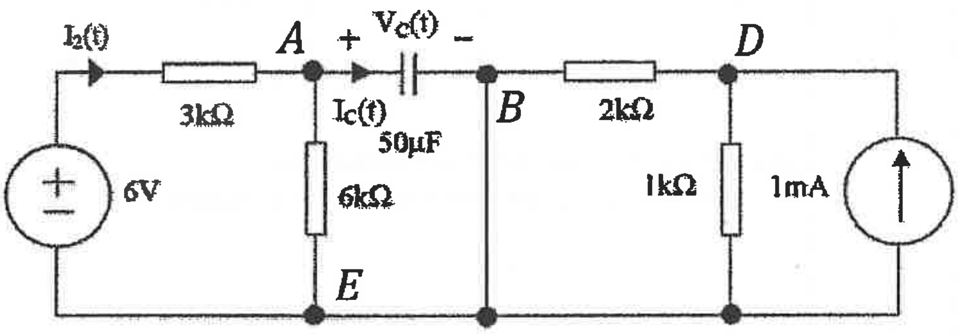
$$V_D = V_B = 1\,V$$

So,

$$V_C(t = 0-) = 3\,V$$

Since we're talking about a capacitor here, $V_C(t = 0+) = V_C(t = 0-) = 3\,V$.

Now, let's consider the case where the switch has been closed for a long time and find $V_C(t = \infty)$.



Again, referring to the diagram in the previous page, the capacitor is again at a steady state, so the capacitor is like an open circuit (no current flows through it).

There is no need to find V_A again as the circuit on the left side of the diagram remains the same. (V_A is still $4\ V$)

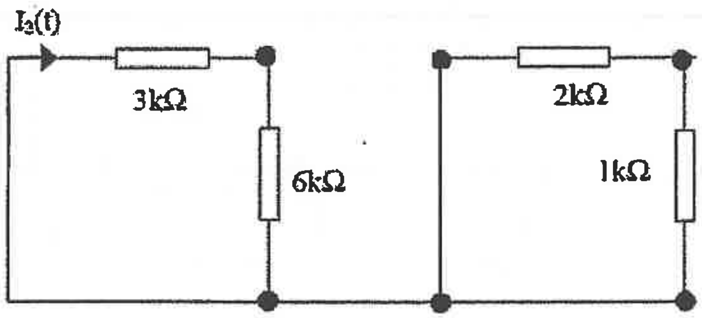
Now, notice, that, looking at the diagram, that $V_B = V_E$ and since E is the reference node, $V_B = 0\ V$. So,

$$V_C(t = \infty) = 4\ V$$

The last step is to find τ, which is equal to the Thevenin resistance across the capacitor with independent voltage sources being short circuits and independent current sources being open circuits times the capacitance of the capacitor.

$$\tau = CR_{TH}$$

Finding R_{TH}:



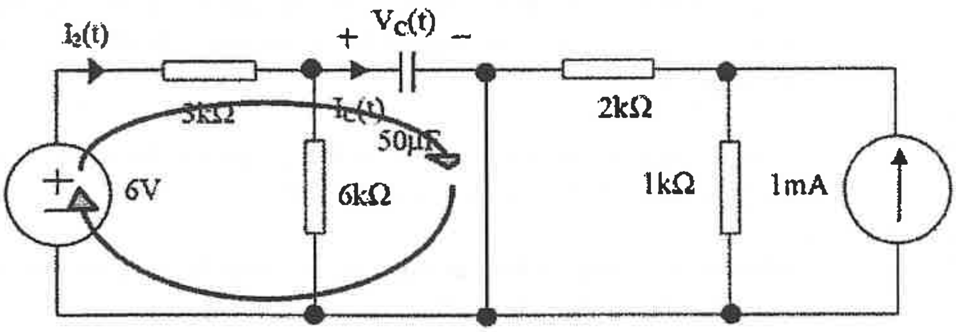
$$R_{TH} = \frac{1}{\frac{1}{3k} + \frac{1}{6k}} = 2\ k\Omega$$

$$\tau = (2\ k\Omega)(50\ \mu\text{F}) = 0.1\ \text{s}$$

So, the equation of $V_c(t)$ for $t > 0$ is:

$$V_c(t) = 4 - e^{-10t}$$

(ii) Refer to the next page diagram.

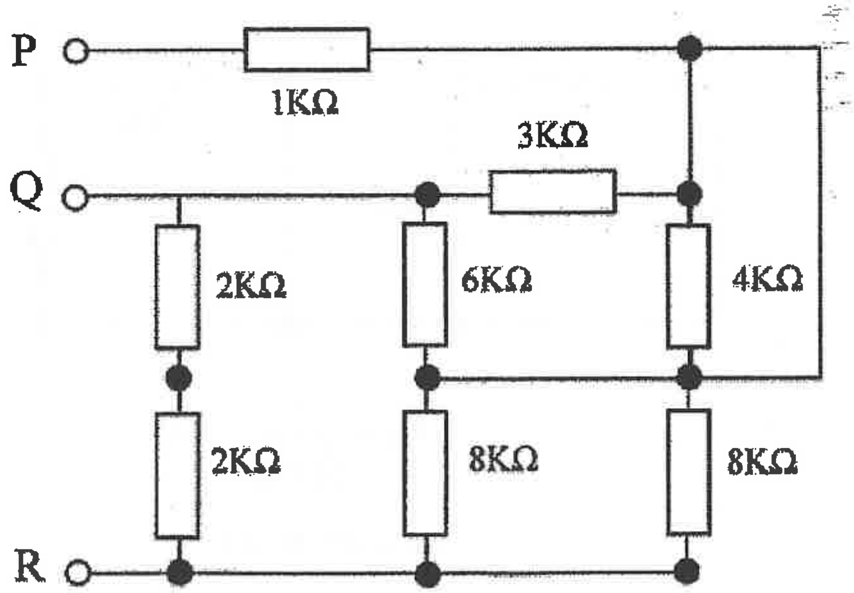


To find an expression for $I_2(t)$, use Loop Analysis on the loop indicated above when $t > 0$:

$$-6 + 3k(I_2(t)) + V_c(t) = 0$$
$$3k(I_2(t)) = 2 + e^{-10t}$$

$$I_2(t) = \frac{1}{3}(2 + e^{-10t})\ mA$$

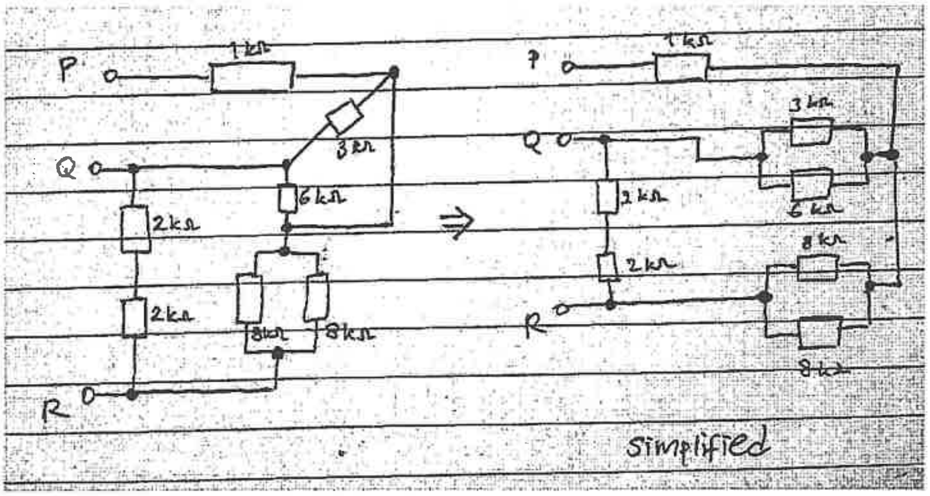
(b) This is nasty.



Let's start easy and find R_{QR}.

$$R_{QR} = \frac{1}{\frac{1}{2k + 2k} + \frac{1}{\frac{1}{\frac{1}{6k} + \frac{1}{3k + 4k}} + \frac{1}{\frac{1}{8k} + \frac{1}{8k}}}} \approx 2.57\ k\Omega$$

If you notice, the short circuit renders the 4 $k\Omega$ resistor moot (useless), so now, let's find the resistance between P and Q.



$$R_{PQ} = 1k + \cfrac{1}{\cfrac{1}{\cfrac{1}{\frac{1}{3k}+\frac{1}{6k}}} + \cfrac{1}{\cfrac{1}{\frac{1}{8k}+\frac{1}{8k}} + 2k + 2k}} = 2.6\ k\Omega$$

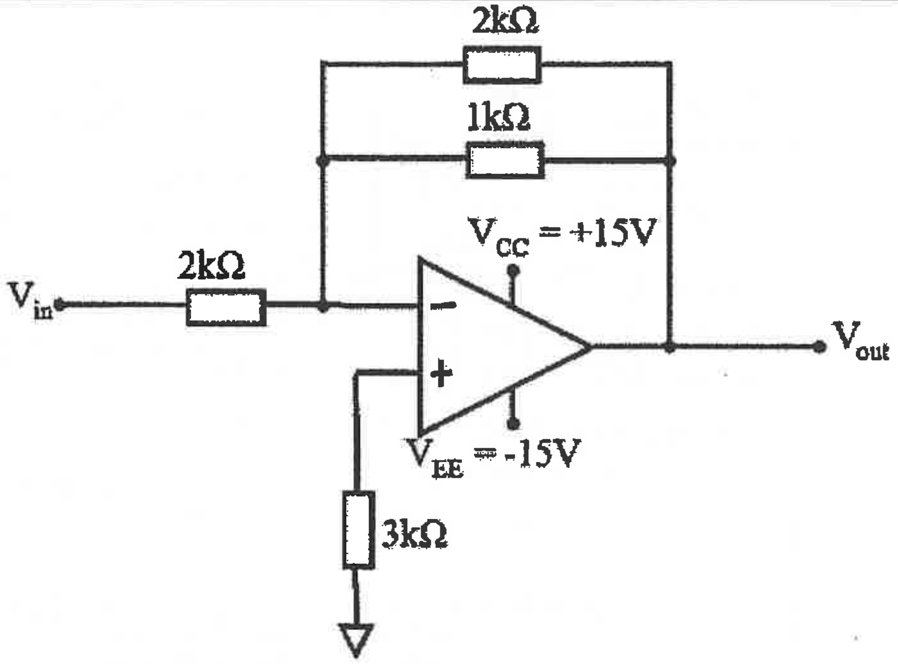
$$R_{PR} = 1k + \cfrac{1}{\cfrac{1}{\cfrac{1}{\frac{1}{8k}+\frac{1}{8k}}} + \cfrac{1}{\cfrac{1}{\frac{1}{3k}+\frac{1}{6k}} + 2k + 2k}} = 3.4\ k\Omega$$

The maximum load current that can be obtained is if the $12\ V$ source is connected across PQ (which has a lone $1\ k\Omega$ resistor which technically has 'all' the current) and has the least resistance among PQ and PR. Don't consider QR as all the currents are already split in all their resistors, whereas in PQ and PR, the $1\ k\Omega$ resistor right beside terminal P has all the current.

So, maximum load current is:

$$I_{PQ(1k\Omega)} = \frac{12\ V}{2.6\ k\Omega} \approx 4.62\ mA$$

3 (a) (i)



First things first, this is an inverting amplifier. Second, the op-amp's (assuming it is ideal) infinite input impedance makes the current going through the op-amp, and hence the 3 $k\Omega$ resistor to be zero. Hence, $V_+ = 0\ V$.

Third, the 2 $k\Omega$ and 1 $k\Omega$ resistors are in parallel, so they are equivalent to a $\frac{2}{3}$ $k\Omega$ resistor. So, we can derive V_{out} in terms of V_{in} based on these resistor values.

$$V_{out} = -\frac{\frac{2}{3}}{2} * V_{in} = -\frac{1}{3}V_{in}$$

So, if V_{in} is 9 V, then $V_{out} = -3\ V$.

(ii)

So that there is no distortion,

$$-15\ V \leq -\frac{1}{3}V_{in} \leq 15\ V$$

$$-45\ V \leq V_{in} \leq 45\ V$$

Done!

(b) (i)

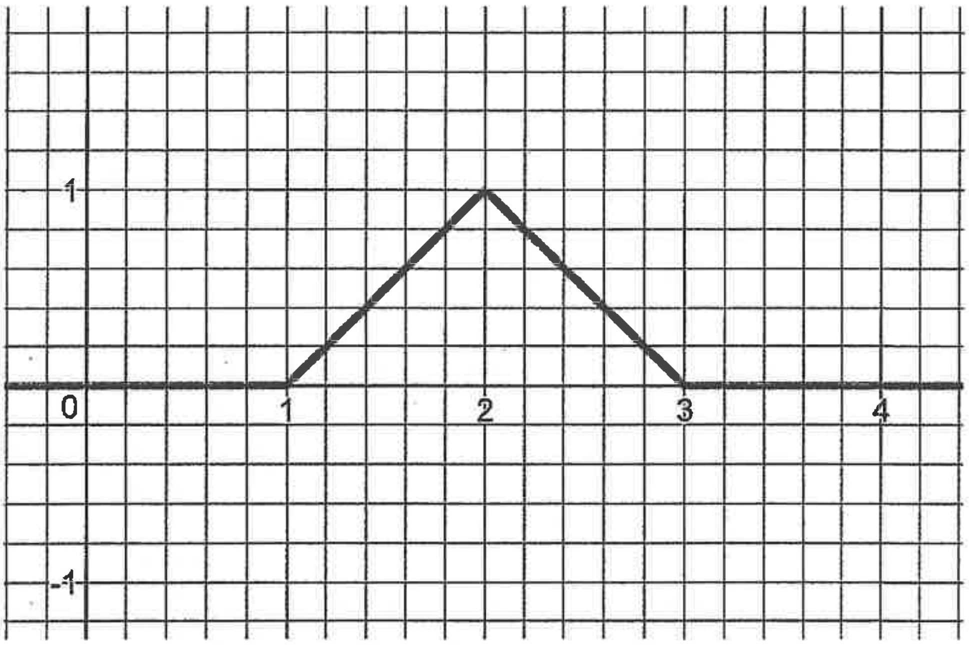
Odd

A signal is odd if, for all t, $f(t) = -f(-t)$.

$$f(3) = 0, \qquad f(-3) = -4, \qquad f(3) \neq -f(-3)$$

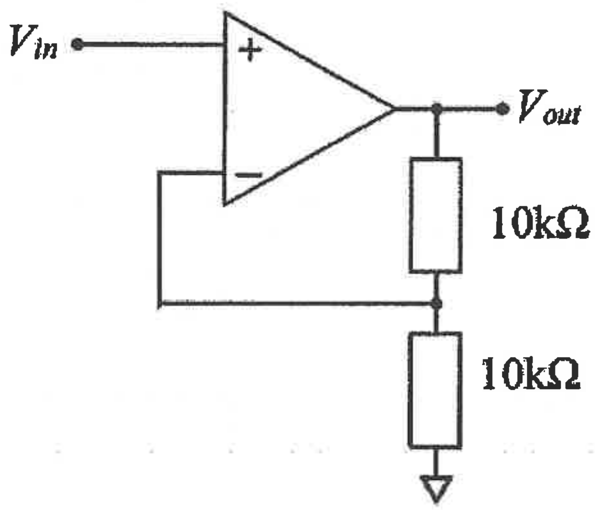
We've found a contradiction. So, the signal is not odd.

Periodic
The signal is also not periodic. If you try to sketch the signal, it does not have a period T in where $f(t) = f(t + T)$ for all t.



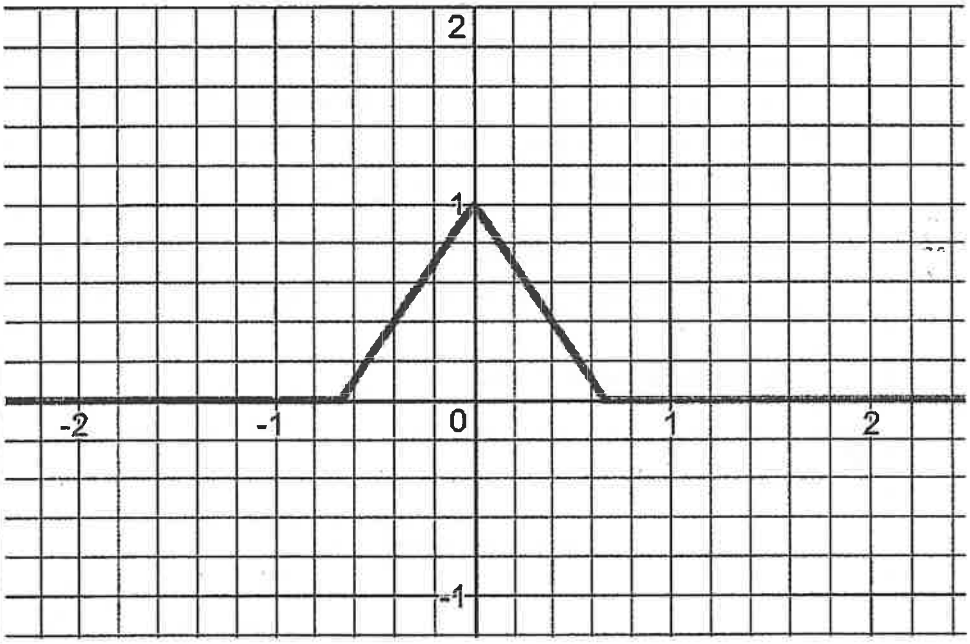
This signal is not periodic.

(ii)



Notice that that is a non-inverting amplifier with a gain of $1 + \frac{10\ k\Omega}{10\ k\Omega} = 2$.

So, $V_{out} = 2V_{in} = 3f(-1.5t + 2) - 2$.

			If $$f(t) = \begin{cases} 1 - \lvert t-2 \rvert, & 1 < t < 3 \\ 0, & otherwise \end{cases}$$ Then $$3f(-1.5t+2) - 2 = \begin{cases} 3(1 - \lvert -1.5t + 2 - 2 \rvert) - 2, & 1 < -1.5t + 2 < 3 \\ 0, & otherwise \end{cases}$$ $$3f(-1.5t+2) - 2 = \begin{cases} 1 - \lvert -1.5t \rvert, & -\frac{2}{3} < t < \frac{2}{3} \\ 0, & otherwise \end{cases}$$ And then we can now sketch: 
4	(a)	(i)	The system is not memoryless as it depends on values of x at other times other than t when y is at time t. ($y(t)$ depends on $x(b)$ where $b \neq t$) For example, at $t = \frac{5}{3}$, $y(\frac{5}{3})$ depends on a past value of x, which is $x(1)$.
		(ii)	The system is not causal because in some cases, y depends on future values of x. For example, at $t = 3$, $y(3)$ depends on a future value of x, which is $x(5)$.
		(iii)	The system is not time-invariant. For a displaced input $x(t - t_0)$, it does not cause an output $y(t - t_0)$.

		(iv)	The system is linear. $Ay_1(t) + By_2(t) = Ax_1(3t-4) + Bx_2(3t-4) = Ax_1(3t-4) + Bx_2(3t-4)$
	(b)	(i)	$y''(t) + 4y'(t) + 3y(t) = x'(t) + 2x(t)$ To find the transfer function, let $x(t) = e^{st}$ and $y(t) = H(s)e^{st}$, where $H(s)$ is the transfer function. Substitute this into the LTI system equation. $s^2H(s)e^{st} + 4sH(s)e^{st} + 3H(s)e^{st} = se^{st} + 2e^{st}$ $H(s) = \frac{s+2}{(s+1)(s+3)}$
		(ii)	Find the Laplace transform of $x(t) = 2e^{-2t}u(t)$. $X(s) = \mathcal{L}[x(t)] = \frac{2}{s+2}$ $H(s)X(s) = \frac{2}{(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3} = \frac{(A+B)s + (3A+B)}{(s+1)(s+3)}$ $A + B = 0$ $3A + B = 2$ $A = 1$ $B = -1$ $H(s)X(s) = \frac{1}{s+1} - \frac{1}{s+3}$ $y(t) = \mathcal{L}^{-1}[H(s)X(s)] = (e^{-t} - e^{-3t})u(t)$