Solver: Ng Jing Nee

1)

a)

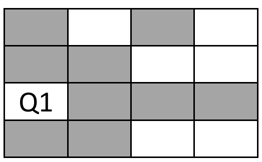
i) True
ii) False
iii) True
iv) False
v) True

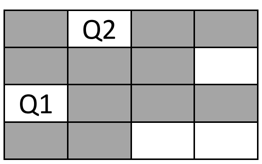
b) Max (h1, h2) is better.
Let C* be the true cost to goal. Assuming g1 == g2,

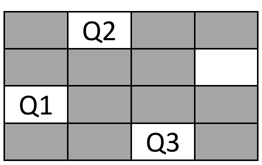
$f1 \geq f2$ if $h1 \geq h2$

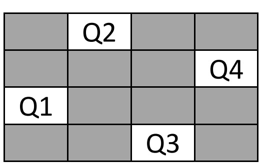
∴ f1 is closer to C* than f2, and the set of nodes where f1 < C* is ≤ the set of nodes where f2 < C*.
In the 3 choices given, only max (h1, h2) gives the highest value of h.

c)





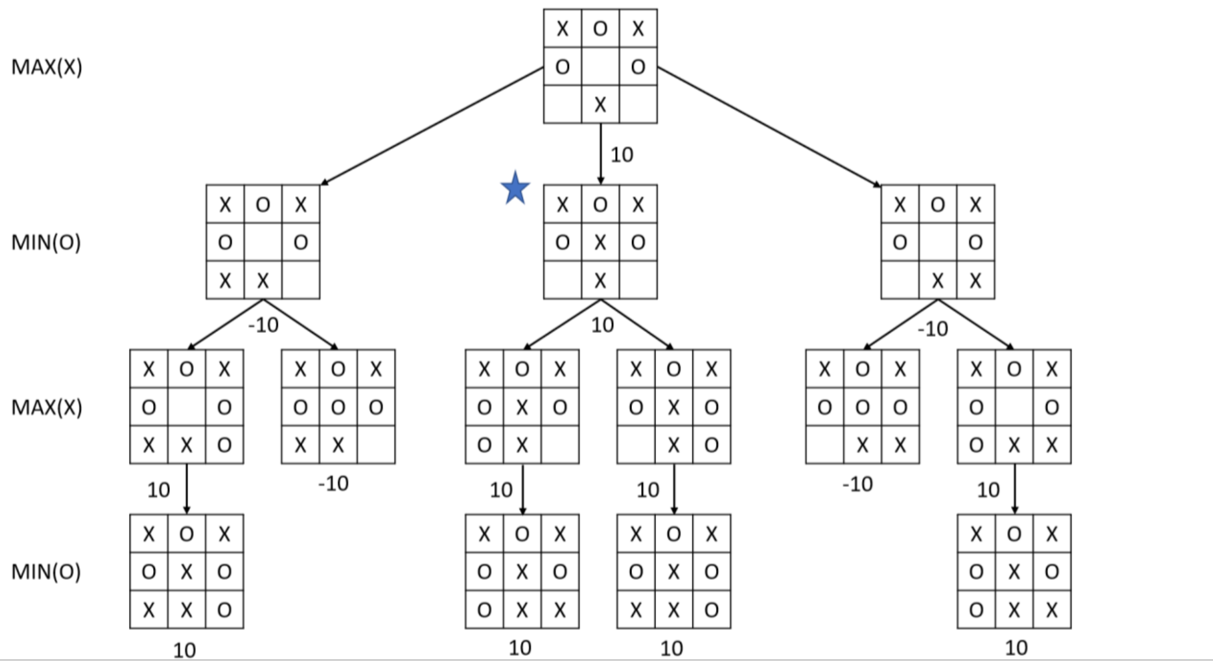




Candidate values: Q1 – 3, Q2 – 1, Q3 – 4, Q4 – 2

 → cannot place Q

d) ★ : optimal move



2)

a) Final Path: SDG [Expansion does not change the parent of the nodes in the frontier]
Expansion List: S, A, B, D, C, G

b) Final Path: SACG
Expansion List: S, A, C, G

c) Final Path: SDCG

Node to be expanded	**Frontier**
	S (0)
S	B (1), A (2), D (4)
B	A (2), D (4)
A	D (4), C (6)
D	C (**5**), G (9)
C	G (**7**)
G	

d) Final Path: SDG
Expansion List: S (5), D (2), G (0)

e) Final Path: SDCG

Node to be expanded	**Frontier**
	S (0 + 5 = 5)
S	A (2 + 3 = 5), B (1 + 4 = 5), D (4 + 2 = 6)
A	B (5), D (6), C (6 + 1 = 7)

B	D (6), C (7)
D	C (**5 + 1 = 6**), G (9 + 0 = 9)
C	G (**7 + 0 = 7**)
G	

3)

a)

i) $P \Rightarrow Q \Leftrightarrow \neg Q \Rightarrow \neg P$

P	**Q**	$\boldsymbol{P \Rightarrow Q}$	$\boldsymbol{\neg Q \Rightarrow \neg P}$	$\boldsymbol{P \Rightarrow Q \Leftrightarrow \neg Q \Rightarrow \neg P}$
F	F	T	T	T
F	T	T	T	T
T	F	F	F	T
T	T	T	T	T

∴ Hence, the equivalence holds.

ii) $\neg(P \Leftrightarrow Q) \Leftrightarrow (P \Leftrightarrow \neg Q)$

LHS:

$\neg(P \Leftrightarrow Q) \quad \equiv \neg\big((P \Rightarrow Q) \wedge (Q \Rightarrow P)\big)$

$\equiv \big(\neg(\neg P \vee Q)\big) \vee \big(\neg(\neg Q \vee P)\big)$

$\equiv (P \wedge \neg Q) \vee (Q \wedge \neg P)$

RHS:

$(P \Leftrightarrow \neg Q) \quad \equiv (P \Rightarrow \neg Q) \wedge (\neg Q \Rightarrow P)$

$\equiv (\neg P \vee \neg Q) \wedge (Q \vee P)$

$\equiv \big((\neg P \vee \neg Q) \wedge Q\big) \vee \big((\neg P \vee \neg Q) \wedge P\big)$

$\equiv \big((\neg P \wedge Q) \vee \cancel{(\neg Q \wedge Q)}\big) \vee \big(\cancel{(\neg P \wedge P)} \vee (\neg Q \wedge P)\big)$

$\equiv (\neg P \wedge Q) \vee (\neg Q \wedge P)$

$\equiv (P \wedge \neg Q) \vee (Q \wedge \neg P)$

Since LHS ≡ RHS, LHS ⇒ RHS and RHS ⇒ LHS holds.

∴ $\neg(P \Leftrightarrow Q) \Leftrightarrow (P \Leftrightarrow \neg Q)$

b) $P \vee Q$ (1)

$P \Rightarrow R$ (2)

$Q \Rightarrow S$ (3)

Assume $\neg(S \vee R)$ (4)

From (4), $\neg(S \vee R) \vDash \neg S \wedge \neg R$ (5)

From (5), $\neg S \wedge \neg R \vDash \neg R$ (6)

From (2), $P \Rightarrow R \vDash \neg P \vee R$ (7)

From (6) + (7), $\neg P \vee R, \neg R \vDash \neg P$ (8)

From (1) + (8), $P \vee Q, \neg P \vDash Q$ (9)

From (3), $Q \Rightarrow S \vDash \neg Q \vee S$ (10)

From (9) + (10), $\neg Q \vee S, Q \vDash S$ (11)

From (5), $\neg S \wedge \neg R \vDash \neg S$ (12)

From (11) + (12), contradiction

∴$KB \vDash S \vee R$

c)

i) Let A be the statement "A works hard", B be the statement "B is happy", C be the statement "C is happy", and D be the statement "D is happy".
$A \Rightarrow B \vee C$ (1)
$B \Rightarrow \neg A$ (2)
$D \Rightarrow \neg C$ (3)

ii) Prove $A \Rightarrow \neg D \equiv \neg A \vee \neg D$ (4)
Prove by refutation, assuming $\neg(\neg A \vee \neg D)$ (5)
From (5), $\neg(\neg A \vee \neg D) \equiv A \wedge D$ (6)
From (6), $A \wedge D \vDash A$ (7)
and $\vDash D$ (8)
From (3) + (8), $D \Rightarrow \neg C, D \vDash \neg C$ (9)
From (2), $B \Rightarrow \neg A \vDash \neg B \vee \neg A$ (10)
From (7) + (10), $A, \neg A \vDash \emptyset$
Since there is a contradiction, $A \Rightarrow \neg D$ is true.

4)

a) $\exists x \forall y, x \neq y, singapore(x) \Rightarrow \neg singapore(y)$

b) $\exists x\big(P(x) \Rightarrow Q(x)\big)$
$\equiv \big(\exists x \neg P(x)\big) \vee \big(\exists x\, Q(x)\big)$
$\equiv \neg\neg\big(\exists x \neg P(x)\big) \vee \big(\exists x\, Q(x)\big)$
$\equiv \neg\big(\forall x P(x)\big) \vee \big(\exists x\, Q(x)\big)$
$\equiv \forall x P(x) \Rightarrow \exists x\, Q(x)$

c)

i) $\forall x, InNTU(x) \Rightarrow Undergrad(x) \vee Postgrad(x)$ (1)
$\exists x, InNTU(x) \Rightarrow Talent(x)$ (2)
$Talent(John) \wedge \neg Postgrad(John)$ (3)
$InNTU(John) \Rightarrow Undergrad(John)$ (4)
From (1), $\forall x, \neg InNTU(x) \vee Undergrad(x) \vee Postgrad(x)$ (5)
From (2), $\exists x, \neg InNTU(x) \vee Talent(x)$ (6)
From (4), $\neg InNTU(John) \vee Undergrad(John)$ (7)

ii) Assume $\neg InNTU(John)$. (8)
From (7) + (8), $\neg InNTU(John) \vee Undergrad(John), \neg Undergrad(John) \vDash \neg InNTU(John)$ (9)
From (3), $Talent(John) \wedge \neg Postgrad(John) \vDash Talent(John)$ (10)
From (6) + (10), $x = John, \neg InNTU(John) \vee Talent(John), Talent(John) \vDash InNTU(John)$ (11)
From (9) + (11), contradiction
$\therefore InNTU(John) \Rightarrow Undergrad(John)$ holds.

--End of Answers--