Solver: Fuhank Buntara

Email Address: fbuntara002@e.ntu.edu.sg

1. (a)

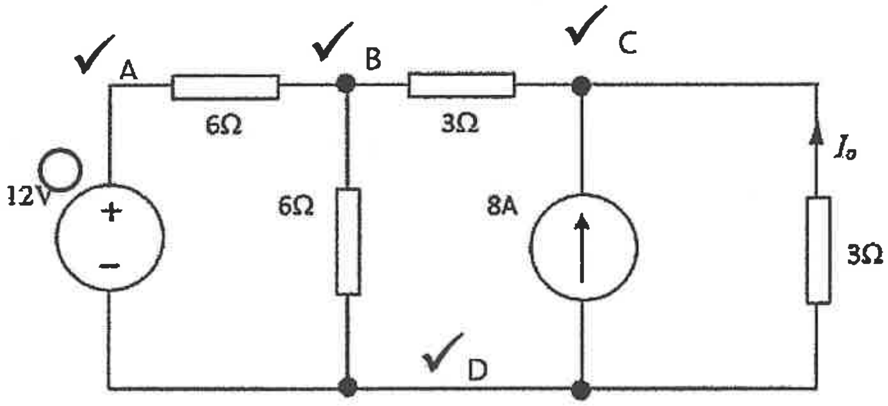


Figure Q1a

Note: ✓ = Node; O = Voltage Source

This question can be solved using node (KCL) or mesh (KVL) analysis. For this solution, node analysis is used.

Step 1 : Total number equation needed is: Total Node –Total Voltage Source – 1
4 – 1 – 1 = 2 Equations.

Step 2 : Decide the ground (0 V). For this circuit, Node D is set as ground because it has more branches (4 branches) than the other nodes.

Step 3 : From the voltage source and branch AD, the difference between node A and node D is 12 V. As D is set as ground, Node A voltage value is 0 V + 12 V = 12 V.

Step 4 : Find 2 equations from 2 nodes which voltages are still unknown.

Equation 1 (Node B):

$$\frac{V_{BA}}{R_{BA}} + \frac{V_{BD}}{R_{BD}} + \frac{V_{BC}}{R_{BC}} = 0\,A$$

$$\frac{V_B - V_A}{R_{BA}} + \frac{V_B - V_D}{R_{BD}} + \frac{V_B - V_C}{R_{BC}} = 0\,A$$

$$\frac{V_B - 12\,V}{6\Omega} + \frac{V_B - 0\,V}{6\Omega} + \frac{V_B - V_C}{3\Omega} = 0\,A$$

$$4V_B - 2V_C = 12\,V \ldots\ldots\ldots 1)$$

Equation 2 (Node C):

$$\frac{V_{CB}}{R_{CB}} + \frac{V_{CD}}{R_{CD}} - I_{in} = 0\,A$$

$$\frac{V_C - V_B}{R_{CB}} + \frac{V_C - V_D}{R_{CD}} - I_{in} = 0\,A$$

$$\frac{V_C - V_B}{3\Omega} + \frac{V_C - 0\,V}{3\Omega} - 8\,A = 0\,A$$

$$-V_B + 2V_C = 24\,V \ldots\ldots\ldots 2)$$

From 1) and 2) by elimination, $V_B = 12\,V$, $V_C = 18\,V$

Step 5 : The current $I_o = \frac{V_D - V_C}{R_{DC}} = \frac{0\,V - 18\,V}{3\Omega} = -6\,A$; Minus sign here shows that the actual current direction is the opposite of I_o.

(b)

→The first thing to do is find the open circuit voltage (V_{CD}). Both node and mesh analysis can be used, however again node analysis is used for this part.

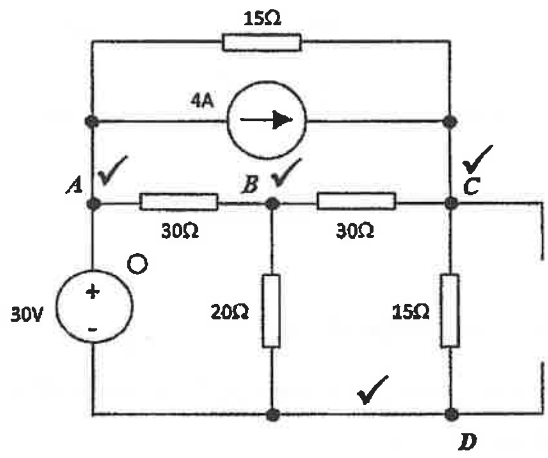


Figure Q1b

Note: ✓ = Node; O = Voltage Source

Step 1 : Total number equation needed is: Total Node –Total Voltage Source – 1
4 – 1 – 1 = 2 Equations.

Step 2 : Decide the ground (0 V). For this circuit, Node D is set as ground because it has more branches (4 branches) than the other nodes and node D is connected to a voltage source.

Step 3 : From the voltage source and branch AD, the difference between node A and node D is 30 V. As D is set as ground, Node A voltage value is 0 V + 30 V = 30 V.

Step 4 : Find 2 equations from 2 nodes which voltages are still unknown.
Equation 1 (Node B):

$$\frac{V_{BA}}{R_{BA}} + \frac{V_{BD}}{R_{BD}} + \frac{V_{BC}}{R_{BC}} = 0\,A$$

$$\frac{V_B - V_A}{R_{BA}} + \frac{V_B - V_D}{R_{BD}} + \frac{V_B - V_C}{R_{BC}} = 0\,A$$

$$\frac{V_B - 30\,V}{30\Omega} + \frac{V_B - 0\,V}{20\Omega} + \frac{V_B - V_C}{30\Omega} = 0\,A$$

$$7V_B - 2V_C = 60\,V \ldots\ldots\ldots 1)$$

Equation 2 (Node C):

$$\frac{V_{CB}}{R_{CB}}+\frac{V_{CA}}{R_{CA}}+\frac{V_{CD}}{R_{CD}}-I_{in}=0\,A$$
$$\frac{V_C-V_B}{R_{CB}}+\frac{V_C-V_A}{R_{CA}}+\frac{V_C-V_D}{R_{CD}}-I_{in}=0\,A$$
$$\frac{V_C-V_B}{30\Omega}+\frac{V_C-30\,V}{15\Omega}+\frac{V_C-0\,V}{15\Omega}-4\,A=0\,A$$
$$-V_B+5V_C=180\,V\,\ldots\ldots\ldots.2)$$

From 1) and 2) by elimination, $V_B = 20\,V$, $V_C = 40\,V$

Step 5 : The open voltage $V_{CD} = V_C - V_D = 40\,V - 0\,V = 40\,V$............3)

→The next thing to do is find the Thévenin's resistance of CD.

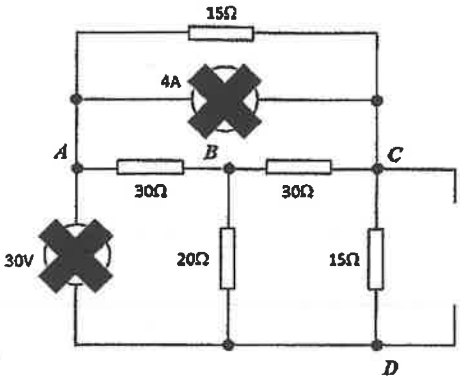


Figure Q1b

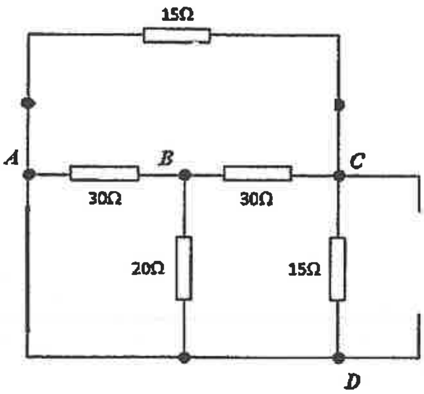
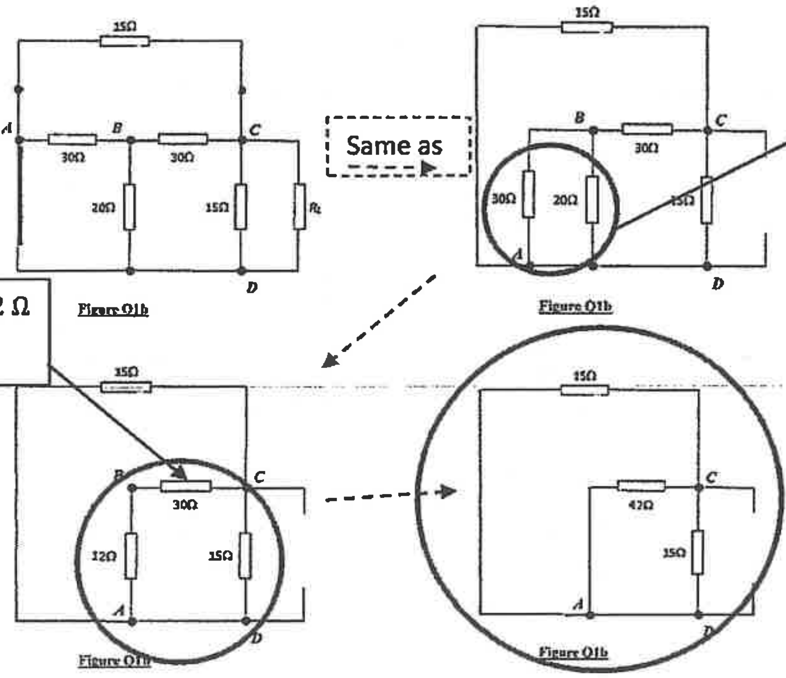


Figure Q1b

As this circuit doesn't consist of dependent sources, the Thévenin's resistance CD is the same as the equivalent resistance of CD if all of voltage sources are short-circuited and all current sources are open circuited.

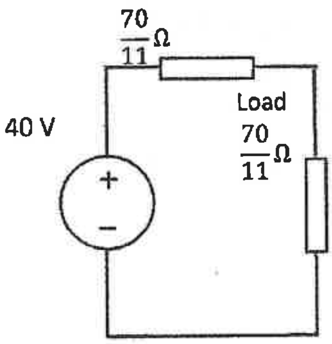


$$\frac{1}{30\Omega}+\frac{1}{20\Omega}=\frac{1}{R_{AB}}$$
$$R_{AB}=12\,\Omega$$

$R_{AC} = 30\Omega + 12\,\Omega$
$= 42\Omega$

$$\frac{1}{15\Omega}+\frac{1}{15\Omega}+\frac{1}{42\Omega}=\frac{1}{R_{CD}}$$
$$R_{CD}=\frac{70}{11}\Omega\,\ldots.4)\text{(Ans)}$$

→R_{CD} is Thévenin's resistance. As the open source voltage (3) and Thévenin's resistance (4) are known, the new Thévenin's circuit is shown below. To produce maximum power in load, the load resistance must be the same as Thévenin's resistance ($\frac{70}{11}\Omega$). In this series circuit, As the Thévenin's resistance is the same as the load's, the voltage is distributed evenly between these two ($V_{load} = 20\ V$). From the data above, the maximum power transfer of the load is:



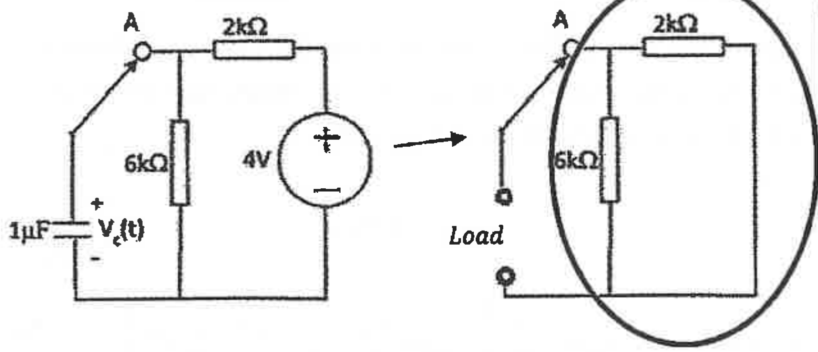
$$P = \frac{(V_{load})^2}{R_{load}}$$

$$P = \frac{(20\ V)^2}{\frac{70}{11}\Omega} = \frac{440}{7}W\ (62.85714\ W)$$

2. (a)

(i) To find the time constant, $\tau = R_{th}C$, Thévenin's resistance of the capacitor must be found with the same method as finding usual Thévenin's resistance. Remember that all voltage sources have to be short circuited.

Case 1: If A is connected



$$\frac{1}{2k\Omega} + \frac{1}{6k\Omega} = \frac{1}{R_{th}}$$

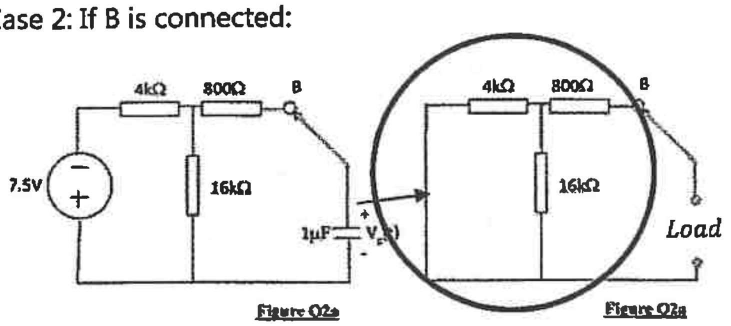
$$R_{th} = 1.5\ k\Omega$$

$$\tau = R_{th}C$$

$$\tau = 1{,}5\ k\Omega\,.\,1\mu F = 1.5\ ms$$

(ans)

Case 2: If B is connected:



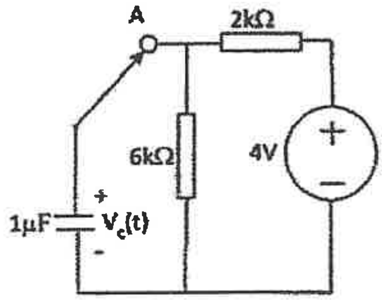
$$R_{th} = 0.8\ k\Omega + \frac{4k\Omega \times 16\ k\Omega}{4\ k\Omega + 16\ k\Omega}$$

$$R_{th} = 4\ k\Omega$$

$$\tau = R_{th}C$$

$\tau = 4\ k\Omega\,.\,1\mu F = 4\ ms$ (ans)

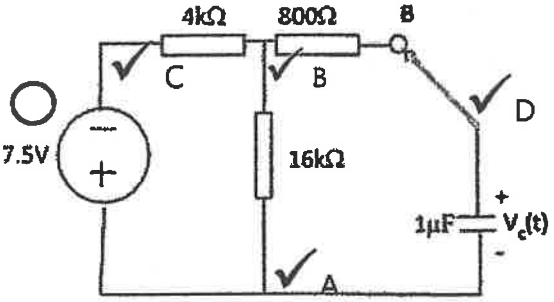
(ii) To use step by step method, t(0+), t(∞), τ must be known. and At t=0, the switch changes direction from A to B. Before t = 0 s, the capacitor has reached steady state which means no current flows around the capacitor and the capacitor voltage at t<0, t(0-), and t(0+) are the same.



→At t<0 and t(-0):

As capacitor can be considered as an open circuit for steady current, no current flows in capacitor. However as capacitor and resistor 6kΩ has parallel relationship, the voltage in 6kΩ and capacitor are the same.

$$V_c(t<0) = V_{6k\Omega} = \frac{6\ k\Omega}{6k\Omega + 2k\Omega} \times 4V = 3\ V$$



→At t(0+):

The voltage at the capacitor is 3 V now (t(0-) =t(0+)) . There's current flows through the capacitor.

→At t(∞):

Capacitor becomes "open circuited again", hence no current crosses through the capacitor. From 3 V, the capacitor's voltage changes until it reaches a stable point that no current pass through it anymore. Again, I use node analysis to find this voltage.

Note: ✓ = Node; O = Voltage Source

Step 1 : Total number equation needed is:
=Total Node –Total Voltage Source – node with no current – 1
=4 – 1 – 1 – 1 = 1 Equation.

Step 2 : Decide the ground (0 V). For this circuit, Node A is set as ground because it has more branches (3 branches) than the other nodes and it sticks with voltage supply.

Step 3 : From the voltage source and branch AD, the difference between node A and node C is -7.5 V. As A is set as ground, Node C voltage value is 0 V + (-7.5 V) = -7.5 V.

Step 4 : Find 1 equation from 1 node which voltage is still unknown.
Equation (Node B):

$$\frac{V_{BA}}{R_{BA}} + \frac{V_{BD}}{R_{BD}} + \frac{V_{BC}}{R_{BC}} = 0\ A;\ \frac{V_{BD}}{R_{BD}} = 0\ (no\ current)$$

$$\frac{V_B - 0\ V}{16\ k\Omega} + \frac{V_B - (-7.5\ V)}{4\ k\Omega} = 0\ A$$

$$V_B = -6V$$

Step 5 : As no current flows through capacitor(node B to node D), the voltage between node B and D are the same, which is -6 V. The capacitor's voltage is:

$$V_{capacitor} = V_D - V_A = 0\ V - 6\ V = -6V\ (ans)$$

From all of the information above, the equation can be found.

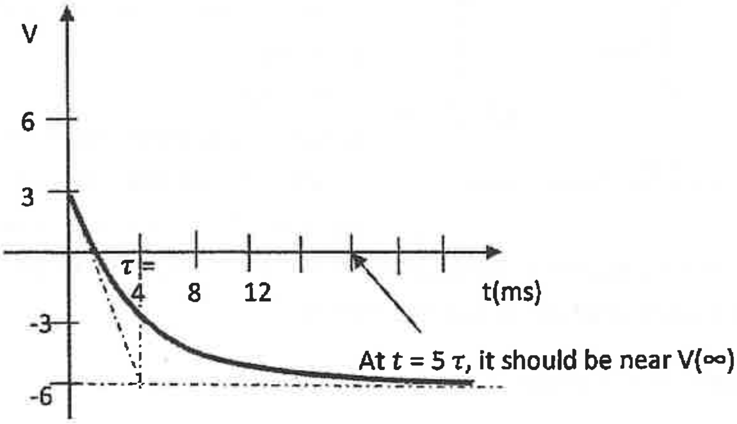
- The general equation: $V_{cap} = K_1 + K_2 \times e^{-t/\tau}$
- When t = 0 : $V_{cap}(0+) = K_1 + K_2 \times e^{-\frac{0}{\tau}}$

$$3\,V \quad = K_1 + K_2 \ldots\ldots\ldots 1)$$

- When t = ∞ : $V_{cap}(\infty) = K_1 + K_2 \times e^{-\frac{\infty}{\tau}}$; remember $e^{-\infty}$ equals 0.

$$-6V = K_1 \ldots\ldots\ldots\ldots\ldots\ldots 2)$$

- Substitute 2 into 1, $K_2 = 3\,V + 6\,V = 9\,V$.
- The general eq: $V_{cap} = -6\,V + 9\,V \times e^{-\frac{t}{4ms}} (ans)$



(b)
This circuit below can be transformed:

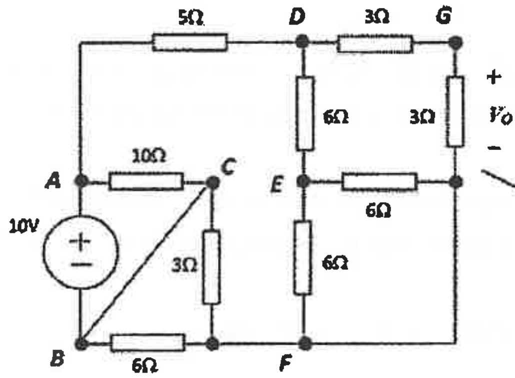


Figure Q2b

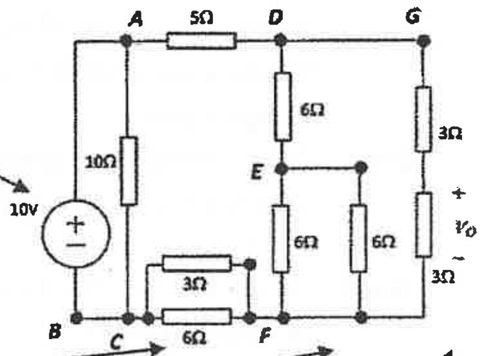


Figure Q2b

$\frac{1}{3\Omega} + \frac{1}{6\Omega} = \frac{1}{R_{CF}}$ $R_{CF} = 2\Omega$	$R_{DF} = 6\Omega + \frac{6\Omega \times 6\Omega}{6\Omega + 6\Omega}$ $R_{DF} = 9\Omega$	$R_{GF} = 3\Omega + 3\Omega$ $R_{GF} = 6\Omega$

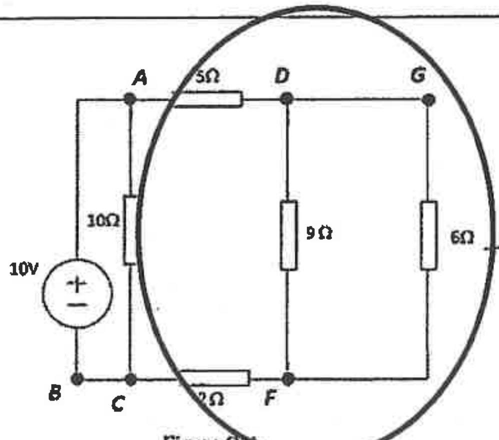


Figure Q2b

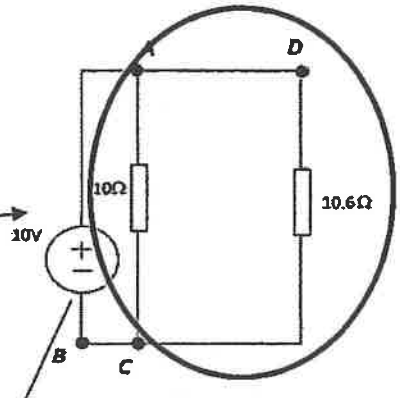


Figure Q2b

$$R_{th} = 5\Omega + \frac{9\Omega \times 6\Omega}{9\Omega + 6\Omega} + 2\Omega$$

$$R_{th} = 10.6\Omega$$

$$R_{th} = \frac{10\Omega \times 10.6\Omega}{10\Omega + 10.6\Omega} = 5.15\,\Omega$$

Figure above shows that
$V_{AC} = V_{ADC} = 10\ V$

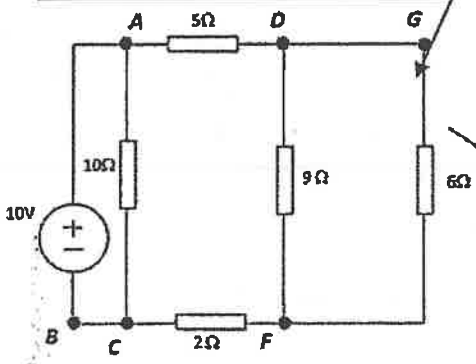


Figure Q2b

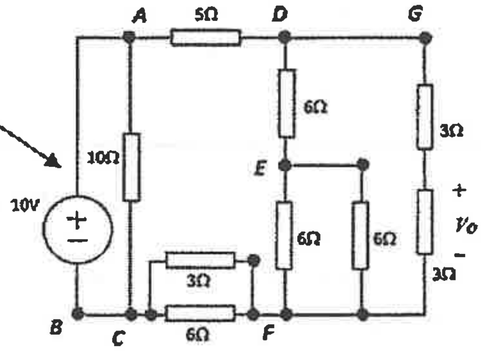


Figure Q2b

If $V_{ADFC} = 10\ V$, then $V_{DF} =$
$V_{DGF} = 10\ V \times \frac{\frac{9\Omega \times 6\Omega}{9\Omega + 6\Omega}}{10.6\ V} =$
$3.3962\ V$.

If $V_{DGF} = 3.3962\ V$, then
$V_{DF} = 3.3962\ V \times \frac{3\Omega}{3\Omega + 3\Omega} =$
$1.698\ V\ (1.7\ V)(ans)$

3. (a)

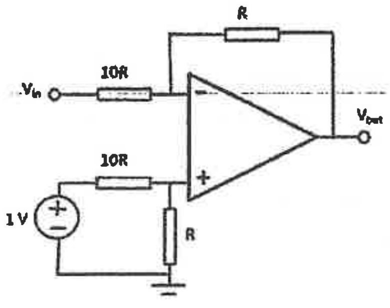


Figure Q3

From the configuration, this is an inverted op-amps. Every op amps follows two principles: 1) Voltages at (+) and (–) must be the same and 2) No current passes through (+) -> (–). As V_{out} depends on voltage at (–), voltage at (+) must be found.

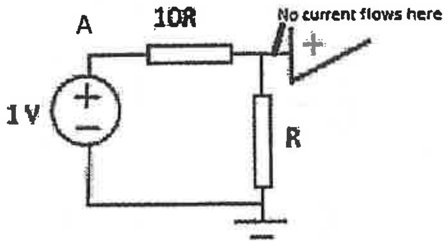
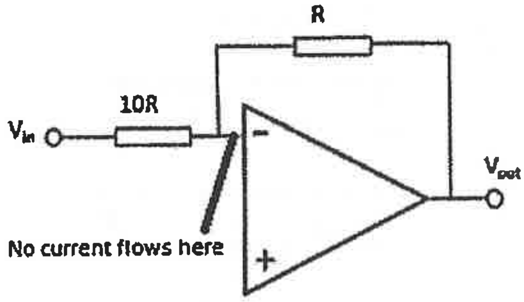


Figure Q3

To find V_{out}, again loop analysis is used. $\perp$ means, that place is 0 V, hence V_A is 1 V. To find V_+:

$$\frac{V_+ - V_A}{10\,R} + \frac{V_+ - 0\,V}{R} = 0\,A$$
$$\frac{V_+ - 1V}{10\,R} + \frac{V_+}{R} = 0\,A$$
$$V_+ = \frac{1}{11}\,V$$

Based on 1) principle, V_- is also $\frac{1}{11}V$. Now, the V_{out} and V_{in} relation is (using virtual ground principle):



$$\frac{V_{in} - V_-}{10\,R} = \frac{V_- - V_{out}}{R}$$
$$\frac{V_{in} - \frac{1}{11}\,V}{10\,R} = \frac{\frac{1}{11}\,V - V_{out}}{R}$$
$$\frac{V_{in} - \frac{1}{11}\,V}{10\,R} = \frac{\frac{1}{11}\,V - V_{out}}{R}$$
$$V_{out} = \frac{1}{10}\,V - \frac{1}{10}V_{in} \dots\dots (ans)$$

(b)

$$y(t) = \sin[x(t-1)]$$

We have to prove the system above is linear or time invariant or both.

To prove the system above is linear:

Step 1: Suppose $y_1(t) = \sin[x_1(t-1)]$

Step 2: Suppose $y_2(t) = \sin[x_2(t-1)]$

Step 3: To test the system is linear: If the input signal is the addition of $x_1(t)$and $x_2(t)$, the output must be $y_1(t) + y_2(t)$.

$$y_{x_1+x_2}(t) = \sin[x_1(t-1) + x_2(t-1)]$$
$$y_1(t) + y_2(t) = \sin[x_1(t-1)] + \sin[x_2(t-1)]$$
$$\sin[x_1(t-1)] + \sin[x_2(t-1)] \neq \sin[x_1(t-1) + x_2(t-1)]$$
$$y_1(t) + y_2(t) \neq y_{x_1+x_2}(t)$$

Hence the system is not linear.

To prove the system above is time invariant:

Step 1: Suppose $y(t) = \sin[x(t-1)]$

Step 2: System is time invariant if ***delayed signal output*** $y(t-t_0)$ is equal to signal with ***delayed input*** $x_d = x(t-t_0)$. Suppose $y_d(t)$ is signal with delayed **input**.

$$y_d(t) = \sin[x_d((t-1)] = \sin[x(t-t_0) - 1]$$

Step 3: $y(t) = \sin[x(t-1)]$, then the delayed **output**:

$$y(t-t_0) = \sin[x(t-t_0) - 1]$$
$$y(t-t_0) = y_d(t)$$

Hence the system is time invariant.

(c)

Impulse response $h(t) = u(t)$

Input signal : $x(t) = (t + 1)\, u(t + 1)$

Output system $x(t) * h(t)$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t - \tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} (\tau + 1)\, u(\tau + 1)\, u(t - \tau)\, d\tau$$

If $\tau < -1$, the function's result will be 0. Also, if $\tau > t$, it also turns to 0. Both domain can be ignored as it doesn't have any effect to the integration result. Hence, if the integration's domain is changed, the equation above:

$$= \int_{-1}^{t} (\tau + 1)\, d\tau; \qquad [in\ this\ domain, u(t - \tau) = u(\tau + 1) = 1]$$

$$= \left[\frac{1}{2}\tau^2 + \tau\right]_{-1}^{t}$$

$$= \left(\frac{1}{2}t^2 + t\right) - \left(\frac{1}{2} - 1\right)$$

$$= \frac{1}{2}t^2 + t + \frac{1}{2}$$

However, this output signal appears in range $-1 < \tau < t$. Hence, the signal output is:

$$= \left(\frac{1}{2}t^2 + t + \frac{1}{2}\right) u(t + 1) \ldots\ldots (ans)$$

4. (a)

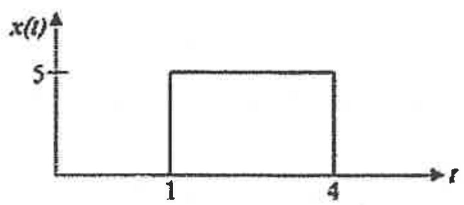


Figure Q4a

The figure beside shows the signal with combinations of unit step signal u(t). To find its Laplace transform, the signal equation with the respect to time must be found. The equation of figure on the left is:

$$x(t) = 5[u(t - 1) - u(t - 4)]$$

The Laplace transform of $u(t - a)$ is $\frac{e^{-as}}{s}$, hence the Laplace transform of $x(t)$:

$$x(s) = 5\left[\frac{e^{-1s}}{s} - \frac{e^{-4s}}{s}\right] \ldots\ldots (ans)$$

(b)

Impulse response $h(t) = e^{-4t}u(t)$

Input signal : $x(t) = u(t-2)$

First step: Find the Laplace transform of both impulse response and input signal

$h(t) = e^{-4t}u(t)$	$e^{at}u(t) \rightarrow \frac{1}{s-a}$	$h(s) = \frac{1}{s+4}$
$x(t) = u(t-2)$	$u(t-a) \rightarrow \frac{e^{-as}}{s}$	$x(s) = \frac{e^{-2s}}{s}$

Second step: The output signal in Laplace domain is the multiplication of impulse response and input signal.

$$y(s) = x(s)h(s) = \frac{1}{s+4} \times \frac{e^{-2s}}{s} = e^{-2s} \times \frac{1}{(s)(s+4)}$$

Third step: Converse the output signal from Laplace domain back to the time domain (real signal). To ease the conversion, it is necessary to use partial fraction.

$$\frac{1}{(s)(s+4)} = \frac{A}{s} + \frac{B}{s+4}$$

$$\frac{1}{(s)(s+4)} = \frac{A(s+4)}{s} + \frac{B(s)}{s+4}$$

$$\frac{1}{(s)(s+4)} = \frac{A(s+4)}{(s)} \frac{B(s)}{(s+4)}$$

$$1 = A(s+4) + B(s)$$

$$0 = A + B$$

$$4A = 1; A = \frac{1}{4}$$

$$B = -A = -\frac{1}{4}$$

Therefore:

$$e^{-2s} \times \frac{1}{(s)(s+4)} = e^{-2s} \times \left(\frac{\frac{1}{4}}{s} - \frac{\frac{1}{4}}{s+4}\right)$$

The inverse Laplace transform of $\frac{1}{s-a}$ is $e^{at}u(t)$. So:

$$e^{-2s} \times \left(\frac{1}{4} \times \frac{1}{s} - \frac{1}{4} \times \frac{1}{s+4}\right) \rightarrow e^{-2s} \times \left(\frac{1}{4}u(t) - \frac{1}{4} \times e^{-4t}u(t)\right)$$

In time domain, multiply e^{-bs} to a signal means delay the signal for 'b' seconds (delay properties). Hence:

$$e^{-2s} \times \left(\frac{1}{4}u(t) - \frac{1}{4} \times e^{-4t}u(t)\right) = \left(\frac{1}{4}u(t-2) - \frac{1}{4} \times e^{-4(t-2)}u(t-2)\right) \ldots\ldots\ldots (Ans)$$

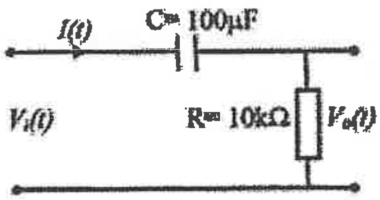
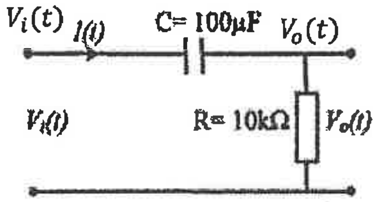


Figure Q4c

(c)

To find $V_o(t)$, the input signal and impulse response must be known. In this question, the impulse response is still unknown. Therefore, the first thing to do is find the impulse response.



Assume this is 0 V
Figure Q4c

Using node analysis again, the equation of the system above is:

$$\frac{V_o(t) - 0\,V}{R} + C\frac{d(V_o(t) - V_i(t))}{dt} = 0\,A$$
$$\frac{V_o(t)}{10\,k\Omega} + C\frac{dV_o(t)}{dt} - C\frac{dV_i(t)}{dt} = 0\,A$$
$$\frac{V_o(t)}{10\,k\Omega} + (\frac{1}{10k}F)\frac{dV_o(t)}{dt} = (\frac{1}{10k}F)\frac{dV_i(t)}{dt}$$

The figure above is an LTI system. In LTI system, one of the characteristic is: if the input signal is e^{st} then the output signal is $e^{st}H(s)$ with H(s) is the impulse response in Laplace domain (Eigenvalues). If inside the equation e^{st} is inserted:

$$\frac{(e^{st}H(s))}{10\,k\Omega} + (\frac{1}{10k}F)\frac{d(e^{st}H(s))}{dt} = (\frac{1}{10k}F)\frac{d(e^{st})}{dt}$$
$$\frac{e^{st}H(s)}{10\,k\Omega} + (\frac{1}{10k}F)(s)(e^{st}H(s)) = (\frac{1}{10k}F)(s)(e^{st})$$
$$H(s) + s\,H(s) = s$$
$$H(s) = \frac{s}{1+s}$$

The input signal is u(t).

$x(t) = u(t)$	$x(s) = \frac{1}{s}$

The output signal in Laplace domain is the multiplication of impulse response and input signal.

$$y(s) = x(s)h(s) = \frac{1}{s} \times \frac{s}{1+s} = \frac{1}{1+s}$$

Converse the output signal from Laplace domain back to the time domain (real signal).

The inverse Laplace transform of $\frac{1}{s-a}$ is $e^{at}u(t)$. Therefore:

$$\frac{1}{1+s} \to e^{-t}u(t) \ldots\ldots\ldots.(ans)$$

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Thank you and all the best for your exams! ☺