CZ1012 Engineering Mathematics-2, NTU
Semester-2, Examination 2017/18

Ans 1. a) $\lim_{x\to\infty} \frac{\lfloor x \rfloor}{x^2}$

by definition, $\lfloor x \rfloor = x - \{x\}$ $\qquad 0 \leq \{x\} < 1$

$$x-1 < \lfloor x \rfloor \leqslant x$$

also, $\lim_{x\to\infty} \frac{(x-1)}{x^2} = 0$ and $\lim_{x\to\infty} \frac{x}{x^2} = 0$

Thus, by Sandwich Theorem, $\lim_{x\to\infty} \frac{\lfloor x \rfloor}{x^2} = 0$

Hence Proved.

b) We can write $y = \sin(e^x) . \sin x$

so, applying the product rule

$$\frac{dy}{dx} = \cos(e^x) . e^x . \sin x + \cos x . \sin(e^x)$$

c) $f(x) = x^3 + ax^2 + bx + 1$

$f'(x) = 3x^2 + 2ax + b$

for both max. and min. $f'(x) = 0$

so, $f'(1) = f'(3) = 0$

$3(1)^2 + 2a(1) + b = 0$ .... eq 1

$3(3)^2 + 2a(3) + b = 0$ .... eq 2

Subtracting eq 1 from eq 2,

$(27 + 6a + b) - (3 + 2a + b) = 0$

$24 + 4a = 0$

$\boxed{a = -6}$

putting the value in eq 1,

$\boxed{b = 9}$

d)

$$e^2 e^x x^2 y^2 = 1$$

$$\frac{d(1)}{dx} = \frac{d(e^2 e^x x^2 y^2)}{dx}$$

$$0 = e^2 \left\{ \frac{d(e^x)}{dx} x^2 y^2 + \frac{d(x^2 . y^2)}{dx} e^x \right\}$$

$$0 = e^2 \left\{ e^x x^2 y^2 + \left(2xy^2 + 2y \frac{dy}{dx} x^2\right) e^x \right\}$$

$$0 = x^2 y^2 + 2xy^2 + 2y \frac{dy}{dx} x^2$$

$$0 = xy^2 + 2y^2 + 2yx \frac{dy}{dx}$$

$$0 = xy + 2y + 2x . \frac{dy}{dx}$$

so, $\boxed{\frac{dy}{dx} = \frac{-y(x+2)}{2x}}$

Ans 2.

a) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \underbrace{\left(\frac{1}{n}\right)}_{\to \Delta x} \underbrace{\cos\left(\frac{i}{n}\right)}_{\to f(x)}$ $\Rightarrow$ $\frac{i}{n} = a + i\left(\frac{1}{n}\right)$ $\quad \frac{1}{n} = \frac{b-a}{n}$

$a = 0 \qquad b - a = 1$

$b = 1$

$$= \int_a^b f(x) . dx$$

$$= \int_0^1 \cos(x)\, dx$$

$$= \left[\sin(x)\right]_0^1$$

$$= \sin(1) - \sin(0)$$

b) $\int x^2 e^x \, dx$

integrating by parts,

$$x^2 \int e^x dx - \int 2x \left(\int e^x dx \right) dx \quad + C$$

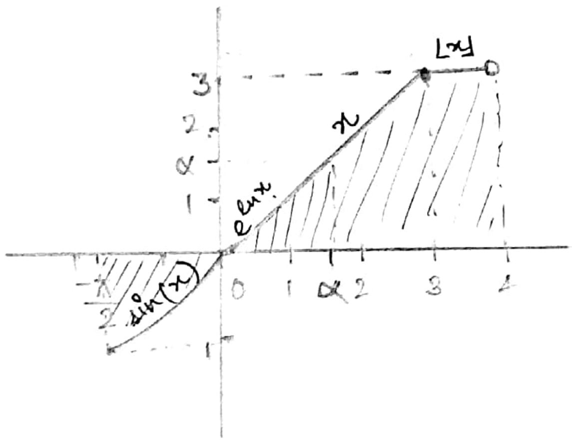
$$x^2 e^x - 2 \int x e^x \, dx \quad + C$$

$$x^2 e^x - 2 \left[x \int e^x dx - \int 1 . e^x dx \right] + C$$

$$x^2 . e^x - 2 \left[x e^x - e^x \right] + C$$

$$x^2 . e^x - 2 x e^x + 2 e^x + C$$

c)



(as $e^{\ln(x)} = x$, the second and third part are essentially the same for the given intervals.)

d) Area bounded $= \int_0^4 f(x) \, dx$

$$= \int_0^{\alpha} e^{\ln(x)} dx + \int_{\alpha}^{3} x \, dx + \int_3^4 \lfloor x \rfloor \, dx$$

$$= \int_0^3 x \, dx \quad + \quad 3 \left[x \right]_3^4$$

$$= \frac{1}{2} \left[x^2 \right]_0^3 + 3$$

$$= \frac{9}{2} + 3$$

$= 7.5$ sq. units.

Ans 3.

a) By observing,

$n!_0 + n$

b)

Ans 4. c) we know $C_k = \frac{1}{T_0} \int x(t) e^{-ikw_0 t} dt$

by looking at the graph,

$$T_0 = 4 \quad , \quad w = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$= \frac{1}{4}\left(\int_1^2 -1\, e^{-ikw_0 t} dt + \int_{-1}^{+1} 0 \cdot e^{-ikw_0 t} dt + \int_{-2}^{-1} 1\, e^{-ikw_0 t} dt \right)$$

$$= \frac{1}{4}\left\{ \left(\frac{e^{-ikw_0 t}}{-ikw_0 t}\right)_1^2 \times -1 + \left(\frac{e^{-ikw_0 t}}{-ikw_0 t}\right)_{-2}^{-1} \times 1 \right\}$$

$$= \frac{1}{4}\left\{ -1 \times \left(\frac{e^{-2ikw_0}}{-2ikw_0 t} - \frac{e^{-ikw_0}}{-ikw_0}\right) + 1 \times \left(\frac{e^{ikw_0}}{ikw_0} - \frac{e^{2ikw_0}}{2ikw_0}\right) \right\}$$

$$= \frac{1}{4}\left\{ -1 \times \left(\frac{e^{-ikw_0}}{ikw_0} - \frac{e^{-2ikw_0}}{2ikw_0}\right) + \left(\frac{e^{ikw_0}}{ikw_0} - \frac{e^{2ikw_0}}{2ikw_0}\right) \right\}$$

$$= \frac{1}{4}\left\{ \frac{-2e^{-ikw_0} + e^{-2ikw_0}}{2ikw_0} + \frac{2e^{ikw_0} - e^{2ikw_0}}{2ikw_0} \right\}$$

$$= \frac{1}{4}\left\{ \frac{2\left(e^{ikw_0} - e^{-ikw_0}\right) + \left(e^{-2ikw_0} - e^{2ikw_0}\right)}{2ikw_0} \right\}$$

$$= \frac{1}{4}\left\{ \frac{2\sin(kw) - 2\sin(2kw)}{2ikw_0} \right\}$$

$$= \frac{1}{4(ikw_0)}\left(\cancel{2}\sin\left(\frac{k\pi}{2}\right) - \sin(k\pi) \right\}$$

$$= \boxed{\frac{1}{2ik\pi}\left(\sin\frac{k\pi}{2} - \sin k\pi\right)}$$

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