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1)

a)

i) Required output: $V_{out} = 4V_1 + V_2$

$$Vout = \left(1 + \frac{R_f}{R_1}\right) \times Vin$$

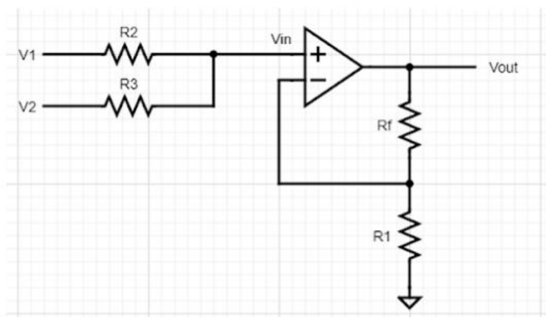

Figure 1. Non-Inverting Amplifier

$$Vin = \left(\frac{R_3}{R_2+R_3}\right) \times V_1 + \left(\frac{R_2}{R_2+R_3}\right) \times V_2$$

$$Vin = \left(\frac{R_3V_1+R_2V_2}{R_2+R_3}\right)$$

$$Vin = \left(\frac{R_3V_1+R_2V_2}{R_2+R_3}\right)$$

$$\therefore Vout = \left(\frac{R_3V_1+R_2V_2}{R_2+R_3}\right)\left(1 + \frac{R_f}{R_1}\right)$$

$$Vout = (R_3V_1 + R_2V_2)\left(\frac{1+\frac{R_f}{R_1}}{R_2+R_3}\right)$$

In order to obtain $V_{out} = 4V_1 + V_2$:

1) Let $R_3 = 4k\Omega,\ R_2 = 1k\Omega$
2) Let $\left[\frac{\left(1+\frac{R_f}{R_1}\right)}{R_2+R_3}\right] = 1 \Rightarrow \left(1 + \frac{R_f}{R_1}\right) = (R_2 + R_3) \Rightarrow \left(1 + \frac{R_f}{R_1}\right) = 5k\Omega$

$Rf = 4k\Omega,\ R1 = 1k\Omega$

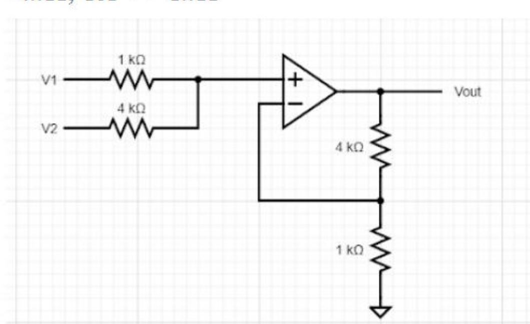

Figure 2. Non-Inverting Amplifier with Resistance value filled

ii) It was clarified during the exam that part i. & ii. can be done together, hence part ii.'s requirement is fulfilled in part i.

b) Cut-off Frequency of Low Pass Filter f_{CL} = *80Hz*
Cut-off Frequency of High Pass Filter f_{CH} = *0.5Hz*
Since Magnitude Response shows a Band-pass response with a gain of 20dB at pass-band, an Active Low Pass Filter & Active High Pass Filter is connected in series.

$20log_{10}A_{FT} = 20dB$

$log_{10}A_{FT} = 1dB$

$A_{FT} = 10^1 = 10$

Total Gain achieved from Active Low Pass & High Pass Filter in series = 10.

Gain of Active Low Pass & High Pass Filter would be $A_{FL} = A_{FH} = \sqrt{10} = 3.162$

Using Non Inverting Amplifier, $3.162 = 1 + \frac{R_{F1}}{R_1} = 1 + \frac{R_{F2}}{R_2}$

Let R_1=R_2 = 1kΩ, R_{F1}=R_{F2} = 2.162kΩ

Given $0.1\mu F$ capacitors, cut-off frequency is found using the following formula:

$f_C = \frac{1}{2\pi RC}$

$f_{CH} = 0.5Hz = \frac{1}{2\pi R_H(0.1\mu F)}$

$R_H = 3.183M\Omega$

$f_{CL} = 80Hz = \frac{1}{2\pi R_L(0.1\mu F)}$

$R_L = 19.894k\Omega$

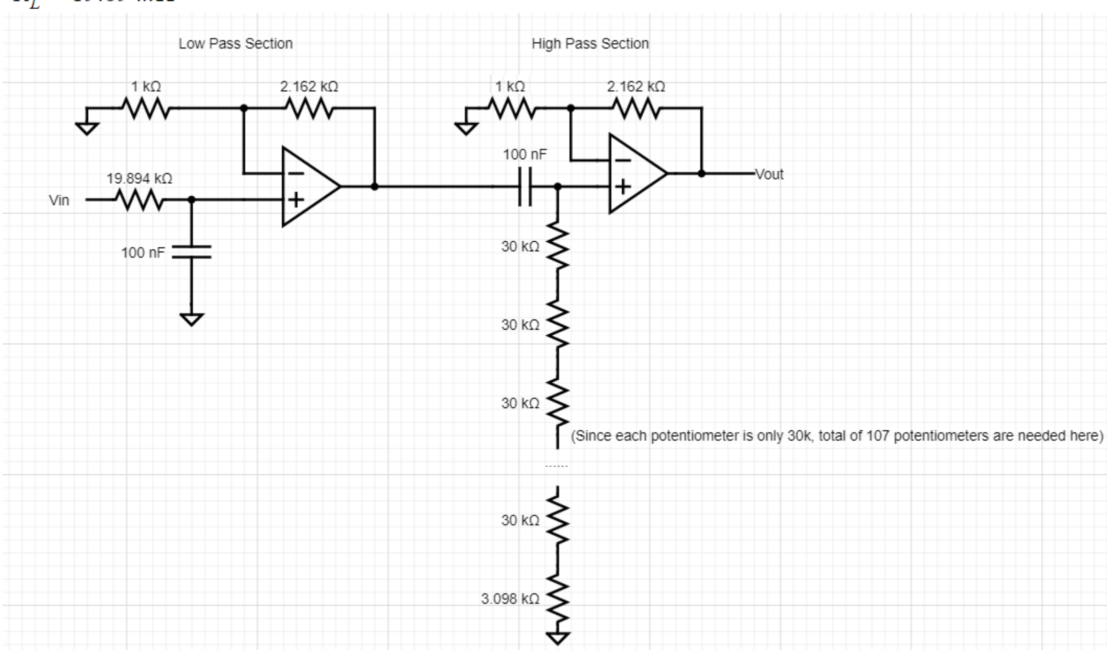

Figure 3. Active Band Pass Filter with a pass band of 0.5Hz - 80Hz, and a total gain of 20dB.

c) Ultrasound Pulse Generator sends RF Electrical pulses to transducer, thereby activating the transducer. The transducer then generates ultrasound RF pulses (inverse piezoelectric effect), and all reflected ultrasound waves vibrate the same transducer which converts the vibrations into electrical pulses (piezoelectric effect), thereby forming the ultrasonic medical image.

2)

a)

i) Any term with subscript of 'd' refers to signal (differential mode input), and any term with a subscript of 'c' refers to noise (common mode input).

$CMRR(dB) = 70dB$

$CMRR = 20log_{10}\frac{A_d}{A_c} = 70$

$log_{10}\frac{A_d}{A_c} = 3.5$

$\frac{A_d}{A_c} = 10^{3.5}$

Using Output SNR Formula: $\frac{V_{od}}{V_{oc}} = CMRR \times \frac{V_{id}}{V_{ic}}$

$$\frac{V_{od}}{1mV} = 10^{3.5} \times \frac{0.005V}{200mV}$$

$$\frac{V_{od}}{1mV} = 79.057$$

$$V_{od} = 0.079V$$

$$Maximum\ Amplification = \frac{0.079}{0.005} = 15.81$$

ii) Slew Rate Formula to ensure that there is no distortion: $2\pi fK \leq SR$, where

$K = V_{od} = V_{id} \times A_d$

Given $SR = 0.4V/\mu s = 400\,000 V/s$

Input Signal $V_{ba} = 0.005\sin(400 \times 10^6 \pi t)$

$\alpha sin 2\pi ft = 0.005\sin(400 \times 10^6 \pi t)$

$2\pi fK = (400 \times 10^6) \times 0.079$

$2\pi fK = 31\,600\,000\ V/s$

Since $2\pi fK \geq SR$, output will be distorted.

iii) Given in question that $R_1 = R_4, R_3 = R_2$ we can exploit the special case of Difference Amplifier: $V_{out} = \left(\frac{R_b}{R_a}\right) \times V_{ba}$

Let $R_b = R_1 = R_4, R_a = R_3 = R_2$

$$\frac{R_b}{R_a} = 15.81 \rightarrow Letting\ R_a = R_3 = R_2 = 1k\Omega, R_b = R_1 = R_4 = 15.81k\Omega$$

b)

$Dynamic\ Range\ (14\ bit) = Desired\ SNR(dB) = 20log_{10}2^{14} = 84.288dB$

$Dynamic\ Range\ (12\ bit) = Desired\ SNR(dB) = 20log_{10}2^{12} = 72.247dB$

$Current\ SNR(dB) = 20log_{10}\frac{200mV}{5mV} = 32.04dB$

$Additional\ SNR\ Required\ (14\ bit)\ (dB) = 84.288 - 32.04 = 52.248dB$

$Additional\ SNR\ Required\ (12\ bit)\ (dB) = 72.247 - 32.04 = 40.207dB$

Select between 3kHz ~ 15kHz, f_c = 3.2kHz

Number of decades between 3.3kHz to 15kHz $log_{10}\frac{15kHz}{3.2kHz} = 0.671\ Decade$

Order of Filter (14 Bit): $Ceiling(\frac{52.248dB}{20\times0.671dB}) = 4th\ Order\ Filter$

Order of Filter (12 Bit): $Ceiling(\frac{40.207dB}{20\times0.671dB}) = 3rd\ Order\ Filter$

∴ Minimum order of anti-aliasing filter changes from 4 to 3.

(Note: Answer may vary depending on the value of f_c chosen, but the method is the same)

3)

a)

i) **True**. The stated expression is a unit-step function. Performing a z-transform followed by applying geometric series formula on the expression will give you 11-z-1or zz-1

ii) **False**. One-sided z-transform starts from k=0, not k=-∞

iii) **False**. When examining the poles, it is said to be stable if all the poles fall within the unit circle.

iv) **False**. A system is critically stable if there is only 1 pole on the unit circle. Multiple poles on the unit circle indicates that the system is unstable.

v) **False**. The concept to determine the stability of a closed-loop system is Jury Stability Test. Ziegler Nichols method is heuristic PID tuning method.

b) i)

$y(k-2)-0.4y(k-1)+0.03y(k)=u(k-2)$

Assuming Sampling Interval T=1, applying Shifting Theorem to the respective terms:

$y(k-2) \Rightarrow z^{-2}Y(z)$

$0.4y(k-1) \Rightarrow 0.4z^{-1}Y(z)$

$0.03y(k) \Rightarrow 0.03Y(z)$

$u(k-2) \Rightarrow z^{-2}U(z)$

Hence, $Z[y(k-2)-0.4y(k-1)+0.03y(k)]=Z[u(k-2)]$

$z^{-2}Y(z)-0.4z^{-1}Y(z)+0.03Y(z) = z^{-2}U(z)$

$Y(z)(z^{-2}-0.4z^{-1}+0.03) = z^{-2}U(z)$

Transfer function is expressed in terms of output over input, $\frac{Y(z)}{U(z)}$.

$\frac{Y(z)}{U(z)} = \frac{z^{-2}}{z^{-2}-0.4z^{-1}+0.03}$

Multiplying Numerator & Denominator by z^2,

$\frac{Y(z)}{U(z)} = \frac{1}{1-0.4z+0.03z^2}$

ii)

$Y(z) = \frac{1}{1-0.4z+0.03z^2}U(z)$

Given that $u(k)$ is a unit step function, $U(z) = \frac{z}{z-1} = \frac{1}{1-z^{-1}}$

$Y(z) = \frac{1}{1-0.4z+0.03z^2}\left(\frac{1}{1-z^{-1}}\right)$

Using Initial Value Theorem: $y(0) = \lim_{z\to\infty} Y(z)$

$y(0) = \frac{1}{1-0.4\infty+0.03(\infty)^2}\left(\frac{1}{1-\frac{1}{\infty}}\right)$

$y(0) = \frac{1}{1-0.4\infty+0.03(\infty)^2}\left(\frac{1}{1-0}\right)$

$y(0) = \frac{1}{\infty} = 0$

Initial value = 0

iii) Transfer function: $\frac{Y(z)}{U(z)} = \frac{1}{1-0.4z+0.03z^2}$

Solving for the poles by finding the roots after letting denominator

$0.03z^2-0.4z+1=0$

$z=10$ **or** $z=3.33$

Since both poles ≥ 1, the system is unstable.

4)

a)

Let Transfer Function of System $= G(z)$

Transfer function of Digital Control System $\frac{y}{y^*} = \frac{KG(z)}{1+KG(z)}$

$$\frac{y}{y^*} = \frac{K\left[\frac{1}{(z-0.1)(z-0.2)}\right]}{1+K\left[\frac{1}{(z-0.1)(z-0.2)}\right]}$$

$$\frac{y}{y^*} = \frac{K}{(z-0.1)(z-0.2)+K}$$

$$\frac{y}{y^*} = \frac{K}{z^2-0.3z+0.02+K}$$

Characteristic equation: $z^2 - 0.3z + 0.02 + K$

Comparing it with the standard characteristic equation:

$$z^2 - 2cos(\omega_n T\sqrt{1-\zeta^2})e^{-\zeta\omega_n T}z + e^{-2\zeta\omega_n T}$$

We can see that:

$$e^{-2\zeta\omega_n T} = 0.02 + K$$

To achieve $\zeta = 0.5,\ \omega_n = 4$ given $T = 0.1$,

$$e^{-2(0.5)(4)(0.1)} = 0.02 + K$$

$$K = 0.6703 - 0.02$$

$$K = 0.6503$$

b)

Characteristic equation: $z^2 - 0.3z + 0.02 + K$

Since Characteristic Equation is a 2nd Order Polynomial, highest term is a_2

$a_0 = 1$, $a_1 = -0.3$, $a_2 = 0.02 + K$

Criterion 1: $|a_n| < a_0$

$$|a_2| < a_0$$

$$|0.02 + K| < 1$$

$$-1.02 < K < 0.98$$

Criterion 2: $P(1) > 0$

$$(1)^2 - 0.3(1) + 0.02 + K > 0$$

$$(1)^2 - 0.3(1) + 0.02 + K > 0$$

$$K > -0.72$$

Criterion 3: $P(-1) > 0 \text{ if } n \text{ is even},\ P(-1) < 0 \text{ if } n \text{ is odd}$

Since n = 2, $P(-1) > 0$

$$(-1)^2 - 0.3(-1) + 0.02 + K > 0$$

$$K > -1.32$$

Combining the inequalities, $-0.72 < K < 0.98$

--End of Answers--