1) $p \wedge q \to p \vee q$;
Therefore, the first clause is true. Now the validity of the argument depends on the validity of r only.
$r \to s$;
No other information about s is given, we cannot infer the validity of r from this argument.
$(\neg r \to q) = r \wedge q$;
Since we know q is true, this argument does not imply the validity of r.
$p \vee r$;
Since we know p is true, this argument does not imply the validity of r as well.
Hence the validity of r is undecidable from the premises given.
<u>The validity of the argument $(p \vee q) \wedge r$ is undecidable.</u>

2)

a) Prove that $S \cup T \subseteq U$
Suppose $x \in S \cup T$, then:
x is divisible by both 2 and 3.
Since 1cm (2,3) = 6,
x is divisible by 6
Therefore $x \in U$

Prove that $U \subseteq S \cup T$
Suppse $y \in U$, then
y is divisible by 6
Since 6 = 1cm (2,3),
y is divisible by both 2 and 3.
Therefore $x \in S \cup T$

b) The proposition $\neg(\forall x \in U, \exists y \in T, x \cdot y \notin S)$ is true
It is equivalent to $(\forall x \in U, \forall y \in T, x \cdot y \in S)$
Suppose $x \in U, y \in T$, then
x is divisible by 6, y is divisible by 3
Since x is divisible by 6, x must be divisible by 2.
$x \cdot y$ must be divisible by 2
Therefore for all x in U, for all y in T, $x \cdot y$ belongs to $S(\forall x \in U, \forall y \in T, x \cdot y \in S)$.
$\therefore \neg(\forall x \in U, \exists y \in T, x \cdot y \notin S)$ is true

3)

a) From $a_n = 7a_{n-1} - 12a_{n-2}$, we have the characteristics equation:
$$s^2 - 7s + 12 = 0$$
Solving the equation, we have
$$s_1 = 3, s_2 = 4$$
$$\Rightarrow a_n = \alpha s_1^n + \beta s_2^n$$
$$a_0 = 2, a_1 = 3$$
$$\begin{cases} \alpha + \beta = 2 \\ 3\alpha + 4\beta = 3 \end{cases}$$

Solving for α and β, we have $\alpha = 5, \beta = -3$

$$\therefore a_n = 5 \times 3^n - 3 \times 4^n$$

b) It is easily verifiable that $D_2 = (2-1)(D_1 + D_0) = n! \sum_{k=0}^{2} \frac{(-1)^k}{k!} = 1$

$$\therefore D_n = n! \sum_{k=0}^{n} \frac{(-1)^k}{k!}$$

holds for $n = 2$. Suppose that

$$D_{p-2} = (p-2)! \sum_{k=0}^{p-2} \frac{(-1)^k}{k!} \ and \ D_{p-1} = (p-1)! \sum_{k=0}^{p-1} \frac{(-1)^k}{k!}$$

holds for some integer p

$$D_p = (p-1)\left(D_{p-1} + D_{p-2}\right)$$

$$D_p = (p-1)\left((p-1)! \sum_{k=0}^{p-1} \frac{(-1)^k}{k!} + (p-2)! \sum_{k=0}^{p-2} \frac{(-1)^k}{k!} \right)$$

$$D_p = (p-1) \times (p-1)! \sum_{k=0}^{p-1} \frac{(-1)^k}{k!} + (p-1)! \sum_{k=0}^{p-2} \frac{(-1)^k}{k!}$$

$$D_p = (p-1) \times (p-1)! \left(\frac{(-1)^{p-1}}{p-1!} \right) + p! \sum_{k=0}^{p-2} \frac{(-1)^k}{k!}$$

$$D_p = (p-1) \times (-1)^{p-1} + p! \sum_{k=0}^{p-2} \frac{(-1)^k}{k!}$$

$$D_p = (-1)^p + p(-1)^{p-1} + p! \sum_{k=0}^{p-2} \frac{(-1)^k}{k!}$$

$$D_p = p! \left(\frac{(-1)^p}{p!} + \frac{(-1)^{p-1}}{(p-1)!} \right) + p! \sum_{k=0}^{p-2} \frac{(-1)^k}{k!}$$

$$D_p = p! \sum_{k=0}^{p} \frac{(-1)^k}{k!}$$

Therefore, since $D_0 = 1, D_1 = 0$

$$D_p = p! \sum_{k=0}^{p} \frac{(-1)^k}{k!}$$

holds for all $n \geq 2$.

4)

a)

i) Is R reflexive? No. A counterexample can be given:

$$1 \not\equiv 1^3 - 1 (mod\ 3)$$

ii) Is R symmetric? No. A counterexample can be given:

$$0 \equiv 2^3 - 2(mod\ 3)$$

However,

$$2 \not\equiv 0^3 - 0(mod\ 3)$$

iii) Is R transitive? No. A counterexample can be given:

$$0 \equiv 2^3 - 2(mod\ 3)$$
$$2 \equiv 8^3 - 8(mod\ 3)$$

However,

$$0 \not\equiv 8^3 - 8(mod\ 3)$$

b)

i) The cardinality of the set T of all functions $f: S \to S$ is

$$|T| = n^n$$

ii) The cardinality of the set U is equal to the number of permutations for n symbols:

$$|U| = A_n^n = n!$$

5)

a)

i) X has a Euler path since it has exactly 2 vertices with odd degree.
Y has a Euler path by the same reasoning.

ii) Neither X nor Y have a Euler circuit since they have odd degree vertices.

iii) X and Y both have Hamilton circuit (starting from the vertex in the center of each hexagon. Go to any other vertex that it is linked to. Go around the hexagon in such a way that the 3rd vertex of the path is not connected to the center. Finally, go back to the center vertex).

b) No. X and Y have different numbers of edges.

--End of Answers--

Solver: Li Shanlan