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1.

a) i) To prove that the given signal is WSS, we will find the mean and autocorrelation of y(t):

$$M_y = \int_0^{2\pi} 3\cos\left(100\pi t + \frac{\pi}{3} + \theta\right)\frac{d\theta}{2\pi} = 0$$

$$R_y(\tau) = E\big(Y(t)Y(t+\tau)\big) = E\left(9\cos\left(100\pi t + \frac{\pi}{3} + \theta\right)\cos\left(100\pi t + 100\pi\tau + \frac{\pi}{3} + \theta\right)\right)$$

$$= \frac{9}{2}E\left(\cos\left(200\pi t + 100\pi\tau + \frac{2\pi}{3} + 2\theta\right)\right) + \frac{9}{2}E(\cos(100\pi\tau))$$

$$= \frac{9}{2}\int_0^{2\pi}\cos\left(200\pi t + 100\pi\tau + \frac{2\pi}{3} + 2\theta\right)\frac{d\theta}{2\pi} + \frac{9}{2}\cos(100\pi\tau)$$

$$= \frac{9}{2}\cos(100\pi\tau)$$

Since M_y does not depend on t and $R_y(\tau)$ depends only on τ, y(t) is WSS

Power of the signal is: $P = R_y(0) = 4.5$

ii) if θ is constant, then M_y will depend on t and y(t) will not be WSS

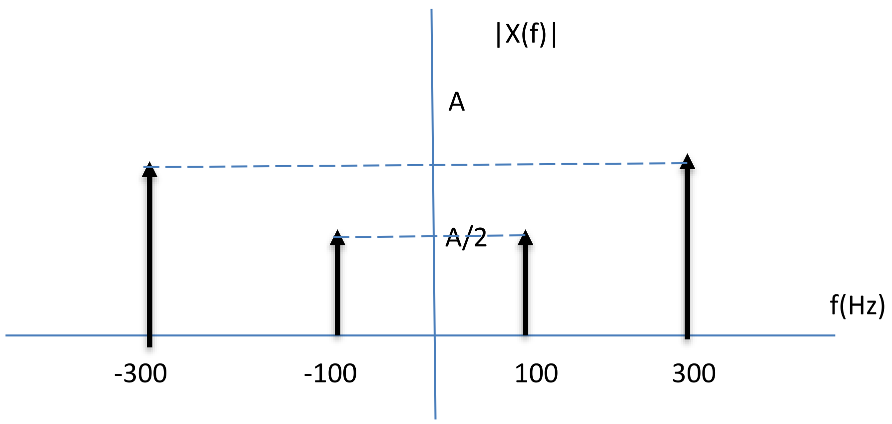
b) i) The filter will remove the frequency that is more than half the sampling rate.

Since $h(t) = Bsinc(300t)$, the maximum frequency of the filtered signal is 150 Hz. Therefore, the appropriate sampling rate is $2 \times 150 = 300\ Hz$

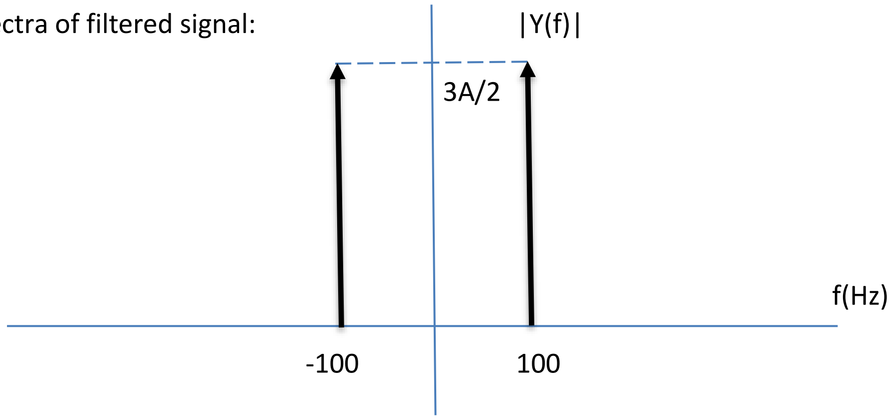
ii) $h(t) = Bsinc(300t) \therefore H(f) = \frac{B}{300} \times rect(\frac{f}{300})$

To have magnitude gain = 3, $\frac{B}{300} = 3 \therefore B = 900$

Magnitude spectra of input signal:



Magnitude spectra of filtered signal:



iii) The sampling frequency is 300Hz, so there will be 300 samples per second

The signal is quantized to 10 levels, so there will be 4 bits per sample (since $2^3 < 10 < 2^4$)

Therefore, number of bits per minute is $300 \times 4 \times 60 = 72000$

2.

a) i) We need to minimize the following error:

$$\epsilon = E\left[\left(Ax(n-1) + Bx(n-3) - x(n)\right)^2\right]$$

$$= A^2E\left(x^2(n-1)\right) + B^2E\left(x^2(n-3)\right) + E\left(x^2(n)\right) + 2ABE\left(x(n-1)x(n-3)\right) - 2AE\left(x(n-1)x(n)\right) - 2BE\left(x(n-3)x(n)\right)$$

$$= A^2R_x(0) + B^2R_x(0) + R_x(0) + 2ABR_x(2) - 2AR_x(1) - 2BR_x(3)$$

$$= 2A^2 + 2B^2 + 2 - \frac{4A}{e}$$

Therefore,

$$\frac{\partial\epsilon}{\partial A} = 0 \therefore 4A - \frac{4}{e} = 0 \therefore A = \frac{1}{e}$$

$$\frac{\partial\epsilon}{\partial B} = 0 \therefore 4B = 0 \therefore B = 0$$

Substitute A and B to find minimum mean square error:

$$\epsilon = \frac{2}{e^2} + 0 + 2 - \frac{4}{e^2} = 2 - \frac{2}{e^2}$$

ii) We can observe that the mean square error of DPCM always have R(0) term. For white Gaussian noise, R(0) is infinite, which make DPCM becomes ineffective with very high error.

b) i) The average bit energy is $E = \frac{E_b + 0}{2} = 1.5 \times 10^{-9} (J)$

The bit error probability is:

$$P_e = \frac{1}{2} \exp\left(-\frac{E}{2N_0}\right) = 0.005544$$

ii) With the same error for coherent receiver:

$$Q\left(\left(\frac{E}{N_0}\right)^{\frac{1}{2}}\right) = 0.005544 \therefore \frac{E}{N_0} = 6.45 = 8.10(dB)$$

The old SNR is $\frac{E}{N_0} = 9.00 = 9.54\ dB$, so the gain in term of SNR is 1.44 dB

c) Advantages of FSK:

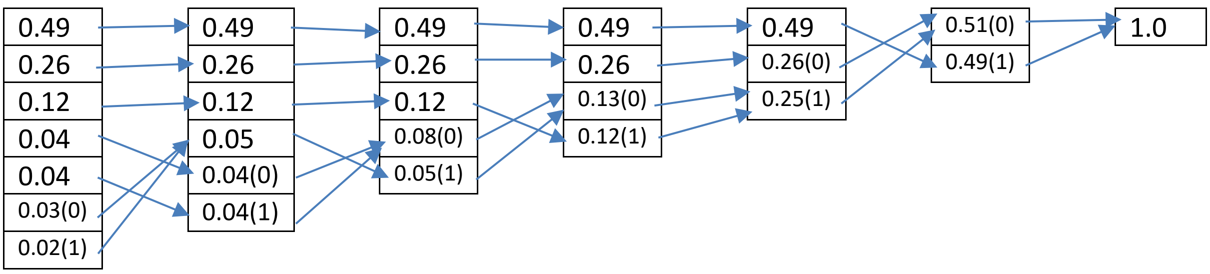
- Less affected by noise and signal amplitude variation
- Linear amplifiers are not required in the transmitter, hence easier implementation

Disadvantages of FSK:

- Sideband of a frequency modulated transmission extend to infinity theoretically. Filters therefore are used but will have some distortion to the signal
- More complicated demodulator
- High bandwidth, hence lower bit rates

3.

a) i)



X	1	2	3	4	5	6	7
Probability	0.49	0.04	0.12	0.03	0.04	0.02	0.26
Code word	1	01000	011	01010	01001	01011	00
Length	1	5	3	5	5	5	2

ii) The expected code length:

$$L = 0.49 \times 1 + 0.04 \times 5 + 0.12 \times 3 + 0.03 \times 5 + 0.04 \times 5 + 0.02 \times 5 + 0.26 \times 2 = 2.02$$

Entropy of X is:

$$H(X) = -(0.49\lg(0.49) + 0.04\lg(0.04) + 0.12\lg(0.12) + 0.03\lg(0.03) + 0.04\lg(0.04) + 0.02\lg(0.02) + 0.26\lg(0.26)) = 2.0128$$

So, the coding efficiency for this coding is: $\frac{H(x)}{L} = \frac{2.0128}{2.02} = 99.6\%$

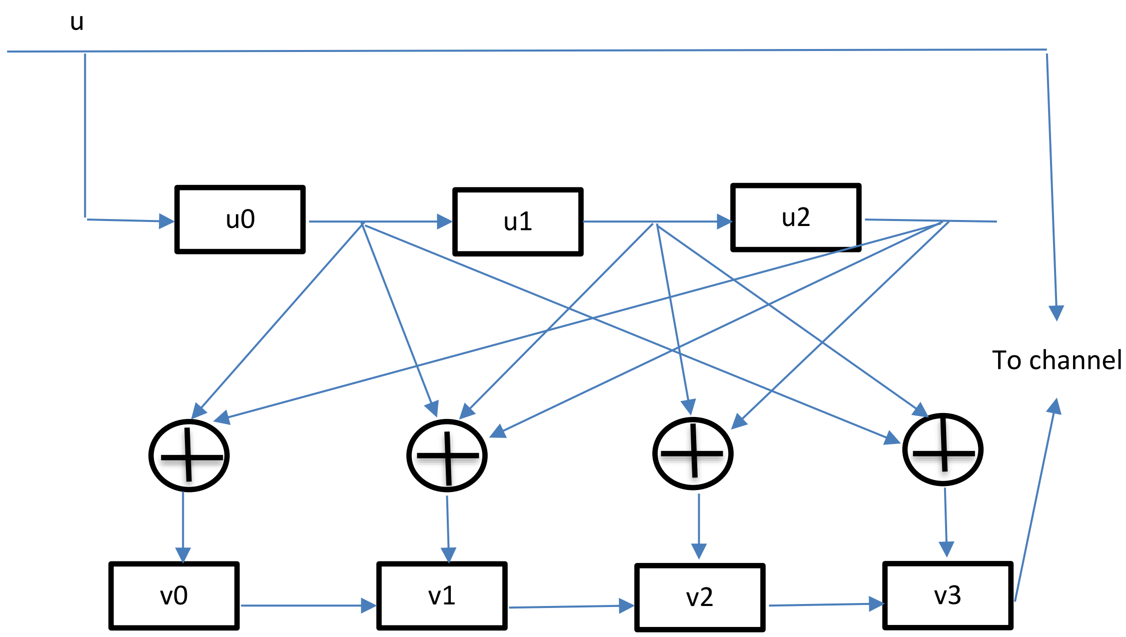
iii) In order to show that the obtained code is uniquely decodable, we only need to show that Kraft's inequality holds:

$$\sum 2^{-l_i} = 0.5 + 0.25 + 0.125 + 4 \times 0.03125 \leq 1$$

b) i) First, we convert G into its systematic form:

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The circuit for G is as follow:



ii) The 8 possible words is:

0000000	1110001	0111010	1001011
1101100	0011101	1010110	0100111

Minimum Hamming distance is 4, so the random error correction ability is 1

The upper bound of the probability of error is:

$$P(E) = \sum_{i=2}^{7} 7Ci\ 0.02^{i} 0.98^{7-i} = 0.007857$$

4.

a) First, we calculate the syndrome of the received word: [1 0 1 0]

Now, let's find syndrome of some correctable error patterns:

000000000 => 0000 (1)

100000000 => 0100 (2)

010000000 => 0110 (3)

001000000 => 0010 (4)

000100000 => 1100 (5)

000010000 => 1111 (6)

000001000 => 0011 (7)

000000100 => 1000 (8)

000000010 => 1001 (9)

000000001 => 0001 (10)

None of these corresponds to 1010, so we will try sum 2 of them

The calculated syndrome 1010 can be summed by (3) and (5) (notice that (4) and (8) also satisfies but (3) comes before (4))

Therefore, the coset leader of the received word is 010100000

b) i) First, we will divide $x^4 + x^6 + x^7 + x^8$ by g(x), the remainder is 1

Therefore, the resulting code word is 1 0 0 0 1 0 1 1 1 0

ii) The received code word polynomial is $x^2 + x^3 + x^4 + x^6 + x^8$

To obtain the syndrome, we divide this polynomial by g(x) and obtain the remainder:

$$r(x) = 1 + x^3 \ \ (quotient\ is\ 1 + x + x^2 + x^3 + x^4)$$

The syndrome obtained by the error pattern is computed by divide e(x) by g(x):

$$r(x) = 1 + x^3 \ (quotient\ is\ x^2 + x^3)$$

We can see that they give the same result

iii) The syndrome computation circuit for g(x) is as follow: