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1) a)

i)

$x = 3\cos^3(t)$ $\qquad$ $y = 3\sin^3(t)$

$\left(\frac{x}{3}\right)^{\frac{2}{3}} = \cos^2(t)$ $\qquad$ $\left(\frac{y}{3}\right)^{\frac{2}{3}} = \sin^2(t)$

Using the trigonometric identity, $\cos^2(t) + \sin^2(t) - 1 = 0$: $\left(\frac{x}{3}\right)^{\frac{2}{3}} + \left(\frac{y}{3}\right)^{\frac{2}{3}} - 1 = 0$

ii) $-1 < \cos^3(t) < 1$

$-3 < 3\cos^3(t) < 3$

$-3 < x < 3$

The same concept is applied to y, i.e. $-1 < \sin^3(t) < 1$ and $-3 < y < 3$

The domains of x and y are [-3, 3]

b) Define a circular surface with radius 2 and centre (0,0):

$x = 2u\cos(2\pi v)$

$y = 2u\sin(2\pi v) \qquad u, v \in [0, 1]$

The sampling resolution is [6, 6].

c)

i) From the equation, $r = 0.9\cos(5\alpha) = 0.9\cos(5(2\pi u)) = 0.9\cos(10\pi u)$.

A circular curve is defined by $= r\cos(2\pi u)$, $\ y = r\sin(2\pi u)$, $\ u \in [0.1]$.

Substitute $r = 0.9\cos(10\pi u)$ into the equations:

$x = 0.9\cos(10\pi u)\cos(2\pi u)$

$y = 0.9\cos(10\pi u)\sin(2\pi u)$

ii) $x = 0.9\cos(10\pi u)\cos(2\pi u)$

$y = 0.9\cos(10\pi u)\sin(2\pi u)$

$z = 1.5v - 0.5 \qquad u, v \in [0, 1]$

2) a)

i) The plane intersects the three axes at (4, 0, 0), (0, -2, 0) and (0, 0, $-\frac{4}{3}$).

One method to solve is by using the formula for a bilinear surface,

$$P = P_1 + u(P_2 - P_1) + v[P_3 - P_1 + u(P_4 - P_3 - (P_2 - P_1))]$$

Let $P_4 = P_3$: $P = P_1 + u(P_2 - P_1) + v[P_3 - P_1 + u(P_1 - P_2)]$

$x = 4 + u(0 - 4) + v[0 - 4 + u(4 - 0)] = 4 - 4u + v(-4 + 4u)$

$y = 0 + u(-2 - 0) + v[0 - 0 + u(0 - (-2))] = -2u + 2uv$

$z = 0 + u(0 - 0) + v\left[-\frac{4}{3} - 0 + u(0 - 0)\right] = -\frac{4}{3}v$

ii) The domains of u and v are [0, 1].

b)

i) $x = 0.5u(1-w),\ y = 0.8v(1-u),\ z = 0.5w(1-v)$
$u, v, w \in [0, 1]$

ii) The slanted surface is defined by

$$\frac{x}{0.5} + \frac{y}{0.8} + \frac{z}{0.5} = 1$$

$$2x + \frac{5}{4}y + 2z - 1 = 0$$

The area below the slanted surface is $1 - 2x - \frac{5}{4}y - 2z \geq 0$

Hence, the implicit function is $f(x, y, z) = \min\left(1 - 2x - \frac{5}{4}y - 2z, x, y, z\right) \geq 0$

c)

i) The circular disk can be defined by performing rotational sweeping on the line $x = 0.1u + 0.1$, followed by translation along x-axis. The functions for the final result are:
$x = (0.1u + 0.1)\cos(2\pi v) + 1$
$y = (0.1u + 0.1)\sin(2\pi v)$
$u, v \in [0, 1]$

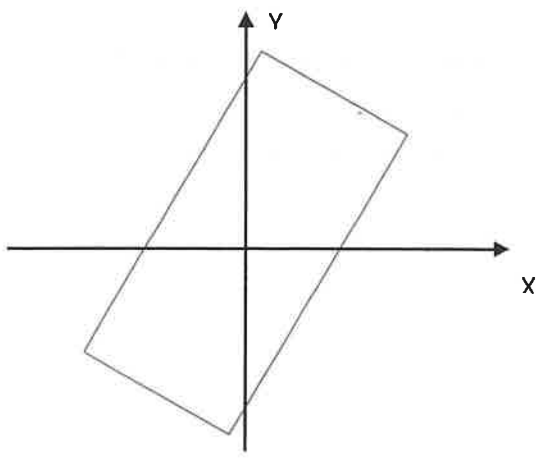
ii) The transformations are (1) rotational sweeping along y-axis, in anticlockwise direction for π rad. and (2) translational sweeping along y-axis for 0.5 units. The transformation matrix is:

$$\begin{bmatrix} \cos(\pi w) & 0 & \sin(\pi w) & 0 \\ 0 & 1 & 0 & 0.5w \\ -\sin(\pi w) & 0 & \cos(\pi w) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

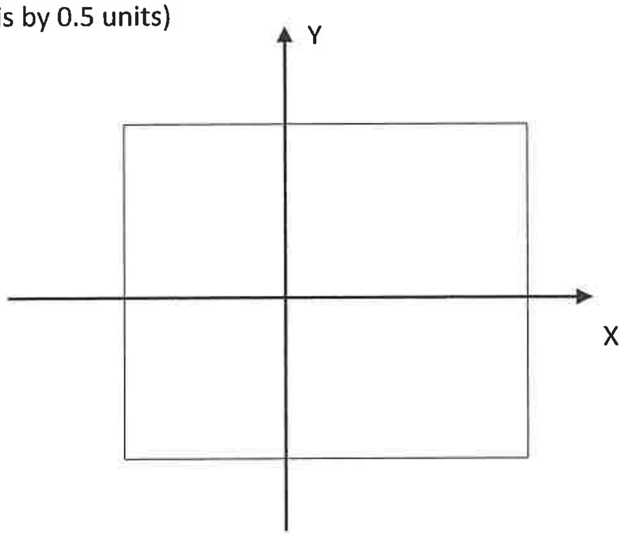
Perform matrix multiplication on x, y and z functions (z function is z = 0) and you will get:
$x = [(0.1u + 0.1)\cos(2\pi v) + 1]\cos(\pi w)$
$y = (0.1u + 0.1)\sin(2\pi v) + 0.5w$
$z = [(0.1u + 0.1)\cos(2\pi v) + 1](-\sin(\pi w))$
$u, v, w \in [0, 1]$

3) a)

i) (The transformation is rotation along z-axis in anticlockwise direction)



ii) (The transformation is scaling along y-axis with scale factor of 2, and translation along x-axis by 0.5 units)



b)

i)

$$x = \left(1 - \frac{t}{10}\right)\cos\theta + \frac{t}{10}(2\cos\theta) = \left(1 + \frac{t}{10}\right)\cos\theta$$
$$y = \left(1 - \frac{t}{10}\right)(2\sin\theta) + \frac{t}{10}(1 + \sin\theta) = (2 - \frac{t}{10})\sin\theta + \frac{t}{10}$$
$$t \in [0, 10], \quad \theta \in [0, 2\pi]$$

ii) Substitute the following into the equations:

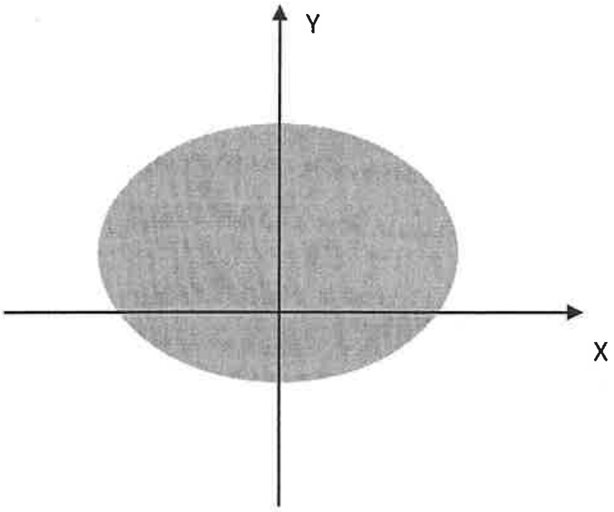
$$\frac{t}{10} = \sin\left[\frac{\pi}{2}\left(\frac{k-1}{98}\right)\right], \qquad k \in [1, 99]$$

$$\therefore\ x = \left(1 + \sin\left[\frac{\pi}{2}\left(\frac{k-1}{98}\right)\right]\right)\cos\theta$$
$$y = (2 - \sin\left[\frac{\pi}{2}\left(\frac{k-1}{98}\right)\right])\sin\theta + \sin\left[\frac{\pi}{2}\left(\frac{k-1}{98}\right)\right]$$

To find the shape at 50th frame, substitute k = 50.

$$x = \left(1 + \sin\left(\frac{\pi}{4}\right)\right)\cos\theta = 1.707\cos\theta$$
$$y = \left(2 - \sin\left(\frac{\pi}{4}\right)\right)\sin\theta + \sin\left(\frac{\pi}{4}\right) = 1.293\sin\theta + 0.707$$

The shape is an ellipse with radii 1.707 and 1.293 and centre (0, 0.707).



c)

i)

$$L = \begin{bmatrix}3\\2\\1\end{bmatrix} - \begin{bmatrix}1\\1\\0\end{bmatrix} = \begin{bmatrix}2\\1\\1\end{bmatrix}$$

ii) Perform the transformation in the following order:

(1) Translate by (-1, -1, 0)

(2) Rotate along x-axis in anticlockwise direction, by $\frac{\pi}{4}$rad.

(3) Rotate along y-axis in anticlockwise direction, by 0.4636rad. (which is $\tan^{-1}\left(\frac{1}{2}\right)$).

(4) Reflect along x-axis

(5) Rotate along y-axis in clockwise direction, by 0.4636rad.

(6) Rotate along x-axis in clockwise direction, by $\frac{\pi}{4}$rad.

(7) Translate by (1, 1, 0)

(8) Reflect along XY plane

The transformation matrices are:

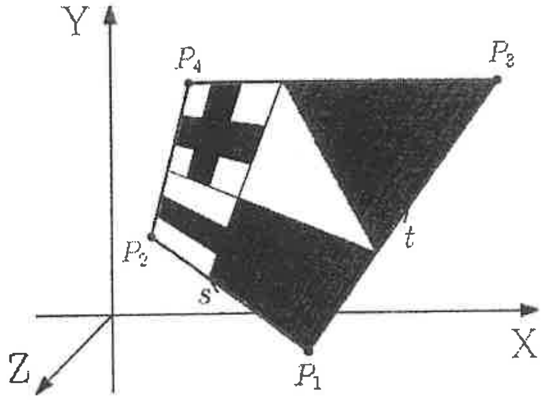
$$\underbrace{\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&1\end{bmatrix}}_{(8)}\underbrace{\begin{bmatrix}1&0&0&1\\0&1&0&1\\0&0&1&0\\0&0&0&1\end{bmatrix}}_{(7)}\underbrace{\begin{bmatrix}1&0&0&0\\0&\cos\left(-\frac{\pi}{4}\right)&-\sin\left(-\frac{\pi}{4}\right)&0\\0&\sin\left(-\frac{\pi}{4}\right)&\cos\left(-\frac{\pi}{4}\right)&0\\0&0&0&1\end{bmatrix}}_{(6)}\underbrace{\begin{bmatrix}\cos(-0.4636)&0&\sin(-0.4636)&0\\0&1&0&0\\-\sin(-0.4636)&0&\cos(-0.4636)&0\\0&0&0&1\end{bmatrix}}_{(5)}$$

$$\underbrace{\begin{bmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&1\end{bmatrix}}_{(4)}\underbrace{\begin{bmatrix}\cos(0.4636)&0&\sin(0.4636)&0\\0&1&0&0\\-\sin(0.4636)&0&\cos(0.4636)&0\\0&0&0&1\end{bmatrix}}_{(3)}\underbrace{\begin{bmatrix}1&0&0&0\\0&\cos\left(\frac{\pi}{4}\right)&-\sin\left(\frac{\pi}{4}\right)&0\\0&\sin\left(\frac{\pi}{4}\right)&\cos\left(\frac{\pi}{4}\right)&0\\0&0&0&1\end{bmatrix}}_{(2)}\underbrace{\begin{bmatrix}1&0&0&-1\\0&1&0&-1\\0&0&1&0\\0&0&0&1\end{bmatrix}}_{(1)}$$

4)

a) For a point light source, the position and the intensity of light source need to be provided. For an ambient light source, only the intensity of light source is needed.

b)

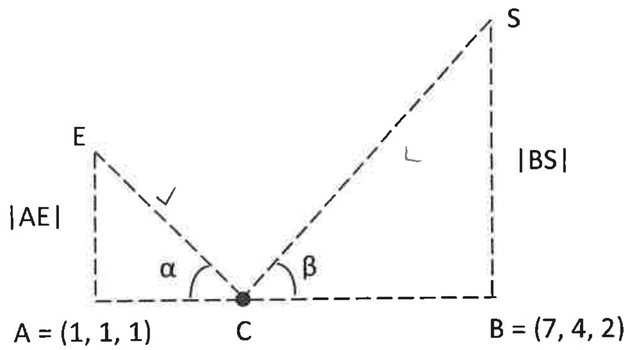


c)

i) $I = k_a I_a + k_d I_s \cos\theta + k_s I_s \cos^n \phi$
$\quad = k_a I_a + k_d I_s (N.L) + k_s I_s (V.R)^n$

ii) For maximum diffuse reflection, $\cos\theta$ needs to be = 1 (maximum). i.e. The angle between vectors N and L, θ is equal to 0. Hence, the coordinates of the point with max diffuse reflection is (7, 4, 2).

iii) For maximum specular reflection, $\cos\phi$ needs to be = 1 (maximum). i.e. The angle between vectors V and R, ϕ is equal to 0. Let point C be the point with max specular reflection. You can find C by illustrating the angles as follows:



where |BS| = 2|AE| and α = β. Hence, |AC| = 2|CB| and |AB| = 3|AC|

$$C = \frac{1}{3}\left(\begin{bmatrix}7\\4\\2\end{bmatrix} - \begin{bmatrix}1\\1\\1\end{bmatrix}\right) + \begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}3\\2\\4/3\end{bmatrix}$$

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