19th CSEC – Past Year Paper Solution 2018-2019 Sem 1
CE/CZ 1011 – Engineering Mathematics I

1) a) i) $P(z) = z^3 - z^2 + z + 1 + a$

By factor theorem, $P(-j) = 0 \Rightarrow (-j)^3 - (-j)^2 + (-j) + 1 + a = 0$

$\Rightarrow j + 1 - j + 1 + a = 0$

$a = -2$

ii) For $a = -2, P(z) = z^3 - z^2 + z - 1$

$= (z-1)z^2 + (z-1)$

$= (z-1)(z^2+1)$

$P(z)\ has\ 3\ roots: 1, j, -j$

b) $z_1 = 2 - j2, z_2 = a + jb$

$|z_1 z_2| = |(2-j2)(a+jb)| = |2a + 2b - 2aj + 2bj|$

$|2a + 2b - 2aj + 2bj| = 16$

$\sqrt{(a+b)^2 + (b-a)^2} = 8$

$\sqrt{(a^2+b^2)2} = 8$ --- (1)

$\arg\left(\frac{z_2}{z_1}\right) = \frac{\pi}{2} \Rightarrow \arg\left(\frac{a - b + aj + bj}{4}\right) = \frac{\pi}{2}$

$\begin{cases} a - b = 0 \\ a + b > 0 \end{cases} \Rightarrow a = b > 0$ --- (2)

(1), (2): $a = b = 4$

$z_2 = 2 + 4j$

$|z_2| = 4\sqrt{2}, \arg(z_2) = \frac{\pi}{4}$

c) Assume P(x,y,z) is a point on L, $x = 3 + 2\lambda, y = \lambda + 4, z = -3\lambda + 1$

$OP = \sqrt{x^2 + y^2 + z^2} = \sqrt{(3+2\lambda)^2 + (\lambda+4)^2 + (-3\lambda+1)^2} = \sqrt{14\lambda^2 + 14\lambda + 26}$

Coordinates of P such that OP is closest to the origin.

OP min: $\sqrt{14\lambda^2 + 14\lambda + 26}\ min \rightarrow 14\lambda^2 + 14\lambda + 26\ min$

As $14\lambda^2 + 14\lambda + 26 = 14\left(\lambda + \frac{1}{2}\right)^2 + \frac{45}{2} \geq \frac{45}{2} \Rightarrow$

$P\ is\ closest\ to\ the\ origin\ if\ and\ only\ if\ \lambda = -\frac{1}{2}$

With $\lambda = -\frac{1}{2}, P = \left(2, \frac{7}{2}, -\frac{1}{2}\right)$

Editor's comment: Alternatively, at point P which is closest to origin, we know that OP must be perpendicular to L. As such, we have the dot product of **OP** and direction vector of **L** to be 0.

$$(3 + 2\lambda, 4 + \lambda, 1 - 3\lambda) \cdot (2, 1, -3) = 0 \Longrightarrow 7 + 14\lambda = 0$$

We recover the same result that $\lambda = -\frac{1}{2}$.

d) $\overrightarrow{AB} = -2i + 7j, \overrightarrow{BC} = 6i + 4j, D: xi + yj$

ABCD is a parallelogram: $\overrightarrow{AB} = \overrightarrow{DC}, \overrightarrow{BC} = \overrightarrow{AD}$

$\begin{cases} 7 - x = -2 \\ 7 - y = 7 \end{cases} \Rightarrow \begin{cases} x = 9 \\ y = 0 \end{cases} \Rightarrow D: 9i + 0j$

2) a)

$$B^T = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}, A^T = \begin{bmatrix} 2 & 3 \\ x & 1 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 4+x & 7 \\ 2+4x & 7 \end{bmatrix}, C = \begin{bmatrix} 3x+2 & 7 \\ 7-x & 7 \end{bmatrix}$$

$$\begin{cases} 4+x = 3x+2 \\ 2+4x = 7-x \end{cases} \Rightarrow x = 1$$

b)

$$AP = Q \Rightarrow \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6k+1 \\ 14k+1 \end{bmatrix}$$

$$\begin{cases} 2x+y = 6k+1 \\ 4x+3y = 14k+1 \end{cases} \Rightarrow \begin{cases} x = 2k+1 \\ y = 2k-1 \end{cases}$$

c)

$$\begin{cases} 2x - y + 2z = 1 \\ x + y - 2z = 2 \\ x - 2y + 4z = -1 \end{cases}$$

$Let\ y - 2z = u, we\ have$

$$\begin{cases} 2x - u = 1 \\ x + u = 2 \end{cases} \Rightarrow \begin{cases} x = 1 \\ u = 1 \end{cases}$$

$Let\ z = t, x = 1, y = 1 + 2t, z = t$

Solutions lie on a line as there is 1 degree of freedom (the parameter t)

d)

$x + y = 1, y + 2 = 0, x + cz = 1$

Plane (x+y=1) meets plane (y+z)=0 at line (1,-1,1)t + (1,0,0)

If 3 planes met a point → $line\ (1,-1,1)t + (1,0,0)$ must intersect plane $x + cz = 1$

Let P(a,b,d) a point as an intersection of line and plane above

$$\begin{cases} a = t_1 + 1 \\ d = t_1 \\ a + cd = 1 \end{cases} \Rightarrow (c+1)t_1 = 0$$

(If c=-1 → many intersections between a line and a plane.)

So $c \neq -1$

Editor's comment: Alternatively, we can do a simple row reduction.

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & c & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & c+1 & 0 \end{array}\right)$$

Note that for the solution to be a point, we require 3 pivot points. Thus it suffices to ensure that c + 1 is not zero, so $c \neq -1$.

e)

A: 6x4, A^T:4x6, B^T:nxm → m=4, n=2, **True**

3) a) i)

Number of students: 50
Mode: 11
Mean: 9.82
Median: 10
25th percentile: 7.25
75th percentile: 12
Trimean: $\frac{mean+mode+median}{3} = 10.27$
Range: $max - min = 16 - 2 = 14$
Variance: 10.23

ii) Almost normal, positive, unimodal

b) i) $P(A \vee B \vee C) = 0.775$

Both teams independently report:

$P(A \vee B) \times P(C) = P\big((A \vee B) \wedge C\big)$

$P(A \vee C) = 0.55, P(B \vee C) = 0.475$

We have

$P(A \vee B \vee C) = P\big((A \vee B) \vee C\big)$

$= P(A \vee C) + P(B \vee C) - P\big((A \wedge B) \vee C\big)$

As only one person in each team can report:

$P(A \wedge B) = 0$

$P\big((A \wedge B) \vee C\big) = P(C)$

$P(C) = 0.55 + 0.475 - 0.775 = 0.25$

$P(A \vee C) = P(A) + P(C) - P(A \wedge C)$

$= P(A) + 0.25 - P(A)P(C)$

$0.55 = 0.75P(A) + 0.25 \Rightarrow P(A) = 0.4$

$P(B \vee C) = P(B) + P(C) - P(B \wedge C)$

$= P(B) + 0.25 - P(B)P(C)$

$0.475 = 0.75P(B) + 0.25 \Rightarrow P(B) = 0.3$

ii) $P(A \vee B) = P(A) + P(B) - P(A \wedge B) = P(A) + P(B) - 0$

$= 0.4 + 0.3 - 0 = 0.7$

$P(C \vee D) = P(C) + P(D) = P(C \wedge D)$

$= 0.25 + 0.35 - 0 = 0.6$

$P(A \vee B \vee C \vee D) = P(A \vee B) + P(C \vee D) - P\big((A \vee B) \wedge (C \vee D)\big)$

$= 0.7 + 0.6 - \ P(A \vee B)P(C \vee D)$

$= 0.7 + 0.6 - 0.7(0.6)$

$= 0.88$

iii) As 2 teams independently report, probability of reporting from team 2 does not affect the probability that guard A reports

$$P(A|C \vee D) = P(A) = 0.4$$

4 a) i) $F(X) = \int_0^{x_0} \frac{1}{10} dx = \frac{x_0}{10}\ \ x_0 \in [0,10]$

$$F(X) = \begin{cases} 0\ for\ x < 0 \\ \dfrac{x}{10}, 0 \le x \le 10 \\ 1, x > 10 \end{cases}$$

ii) Mean waiting time of this sample: 5 mins

Variance of waiting time of this sample: $Var(\overline{x}) = \frac{1}{34}\ Var(x_i)$

Variance of waiting time of individual person

$$= \int_0^{10} \frac{1}{10} x^2 dx - \left(\int_0^{10} \frac{1}{10} x dx\right)^2$$
$$= \frac{1000}{30} - 5^2 = \frac{25}{3}$$
$$Var(\overline{x}) = \frac{1}{34} \times \frac{25}{3} = \frac{25}{102}$$
$$P(5 \leq \overline{x} < 6) = P(\overline{x} < 6) - \frac{1}{2} = P\left(\frac{6-5}{\sqrt{\frac{25}{102}}}\right) - \frac{1}{2} = 0.9783 - 0.5$$
$$= 0.4783$$

b) i) H_0: average monthly utility bills for 4-room HDB flat is \$63
H_A: average monthly utility bills for 4-room HDB flat is not \$63

Let $\overline{x}$ be the average monthly utility bills for 4 month flat

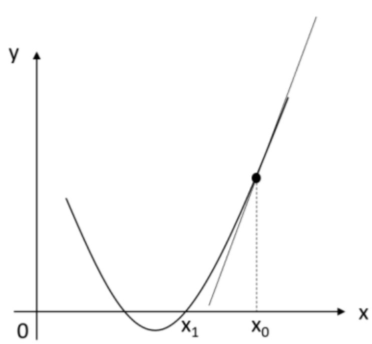
p-value = $2xP\left(\frac{63-65}{\frac{16.38}{\sqrt{50}}}\right) = 0.237 = 23.7\%$

Null hypothesis can't be rejected.

ii) 90.1% confidence interval: 4.95% → 95.05%

$$\left(65 - \frac{11.98}{\sqrt{50}}(1.65), 65 + \frac{11.98}{\sqrt{50}}(1.65)\right)$$
$$(62.204, 67.795)$$

c)



Starts with an initial guess which is reasonably close to the true root (x_0), then the function is approximated by its tangent line to achieve the new guess (x_1): $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$. This process is continuously kept until the guess x_k is close enough to the actual root.

This method has troubles when the guess is exactly the local minimum of the function.

--End of Answers--

Solver: Anonymous