# 20th CSEC – Past Year Paper Solution *2016-2017 Sem 2*
# CZ/CE 1011 – Engineering Mathematics I

1)

a) $\frac{1+j}{1-j}=\frac{(1+j)^2}{1-j^2}=j$

$$\frac{(1+j)^{16}}{(1-j)^{15}}=(1+j)\left(\frac{1+j}{1-j}\right)^{15}=(1+j)\cdot j^{15}=(1+j)\cdot(-j)=1-j$$

b) $z=a+jb$

$z=e^z=e^{a+jb}=e^a(\cos b+j\sin b)=e^a cosb+je^a\sin b$

Hence, we have

$$a=e^a\cos b\,,b=e^a\sin b\Rightarrow a=\frac{b\cos b}{\sin b}$$

Substitute a by above equation, we get

$$b=e^{\frac{b\cos b}{\sin b}}\sin b$$

c) $\boldsymbol{AB}=(0,3,3),\boldsymbol{BC}=(4,1,-1)$

$\boldsymbol{AB}\cdot\boldsymbol{BC}=0+3-3=0\Rightarrow$ **AB** is perpendicular to **BC**

Since **AB** is perpendicular to **BC**:

$$Area\ of\ \Delta ABC=\frac{1}{2}\times|\boldsymbol{AB}|\times|\boldsymbol{BC}|=9$$

d) $Let\ \boldsymbol{u}=(a,b)\ where\ |\boldsymbol{u}|=a^2+b^2=1\ and\cos\theta=a,\sin\theta=b$

$Let\ \boldsymbol{v}=(c,d)\ where\ |\boldsymbol{v}|=c^2+d^2=1\ and\cos x=c,\sin x=d$

$\boldsymbol{u}\cdot\boldsymbol{v}=|\boldsymbol{u}|\cdot|\boldsymbol{v}|\cdot\cos<\boldsymbol{u},\boldsymbol{v}>=\cos(\theta-x)$

And we also have $\boldsymbol{u}\cdot\boldsymbol{v}=ac+bd=\cos\theta\cos x+\sin\theta\sin x$

Hence, we get $\cos(\theta-x)=\cos\theta\cos x+\sin\theta\sin x$

2)

a) $\det(A-3I_2)=\begin{vmatrix}-2 & 3\\ 4 & -6\end{vmatrix}=0$

Thus, the equation $(A-3I_2)x=0$ has infinite number of solutions.

b) Let A = B + C where

$$B=\begin{bmatrix}a & b & c\\ b & d & e\\ c & e & f\end{bmatrix},C=\begin{bmatrix}0 & -g & -h\\ g & 0 & -i\\ h & i & 0\end{bmatrix},B+C=\begin{bmatrix}a & b-g & c-h\\ b+g & d & e-i\\ c+h & e+i & f\end{bmatrix}$$

We can easily get

$$a=2,b=2,c=\frac{5}{2},d=1,e=\frac{3}{2},f=2,g=-1,h=\frac{3}{2},i=-\frac{1}{2}$$

Hence, $A=\begin{bmatrix}2 & 2 & \frac{5}{2}\\ 2 & 1 & \frac{3}{2}\\ \frac{5}{2} & \frac{3}{2} & 2\end{bmatrix}+\begin{bmatrix}0 & 1 & -\frac{3}{2}\\ -1 & 0 & \frac{1}{2}\\ \frac{3}{2} & -\frac{1}{2} & 0\end{bmatrix}$

c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ -1 & -5 & 5 \end{bmatrix} \cdot \begin{bmatrix} \sin\alpha \\ \cos\beta \\ \tan\gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Thus, $\alpha = 0\ or\ \pi, \beta = \frac{\pi}{2}, \gamma = 0\ or\ \pi$

d) False

3)

a)

i) Mean is 7.
Mode is 6.
1st quartile = 25% percentile = 6
2nd quartile = 50% percentile = 7
3rd quartile = 75% percentile = 9

ii) $IQR = 3rd\ quartile - 1st\ quartile = 3$
$Upper\ Inner\ Fence = 9 + 1.5 * 3 = 13.5$
$Lower\ Inner\ Fence = 9 - 1.5 * 3 = 1.5$
$Upper\ Adjacent = 13, Lower\ Adjacent = 4$
Based on above data, the box plot can be easily drawn.

iii) Positive skew.

b)

i) Binomial distribution.

ii) $P = \frac{C(9,3)}{C(10,3)} = \frac{7}{10}$

iii) $P_3 = 0.99^3$

iv) $P = P(all\ are\ not\ defective \mid 3\ are\ defective) = \frac{0.99^{10}}{0.99^3} = 0.99^7$

4)

a)

i) $P(X > 0) = 1 - P(X = 0) = 1 - e^{-\lambda}$
$P(X = 1) = \lambda e^{-\lambda}$
$$\frac{\lambda e^{-\lambda}}{1 - e^{-\lambda}} = \lambda \Rightarrow \lambda = \ln 2$$

ii) Normal distribution.

b)

i) Sample mean is 147.5 mmol/L.
Sample standard deviation is 11.37 mmol/L.
To obtain a 98% confidence interval for the true mean:

$$L = 147.5 - 0.5040 \times \frac{11.37}{\sqrt{10}} = 145.69\ mmol/L$$
$$R = 147.5 + 0.5040 \times \frac{11.37}{\sqrt{10}} = 149.31\ mmol/L$$

Hence, the interval is (145.69,149.31) mmol/L.

ii) Based on the 98% confidence interval, one cannot claim that the patient has hyponatremia.

c) The first one is bisection method. The method will not work when there's only one root and all values are greater than 0 or less than 0.
Another method is Newton's method. The method will not work if the first guess (or any guess thereafter) is a point at which there is a horizontal tangent line, then this line will never hit the x-axis, and Newton's Method will fail to locate a root.

--End of Answers--

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