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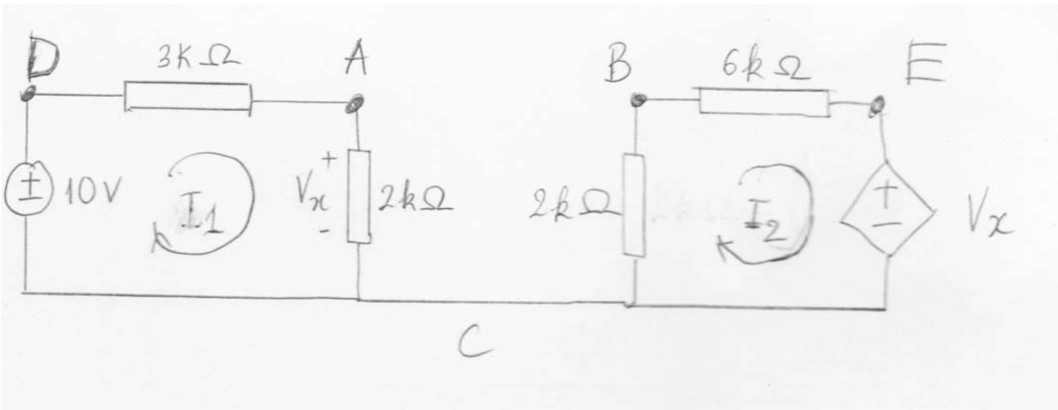
1. (a)
   - For Loop CABC (I_2):
     $V_{CA} + V_{AB} + V+ = 0$
     => $(I_2 - I_1)*2 + 3.6 + I_2*2 = 0$ (1)
   - $I_1 = 1A$ (2)
   - (1) and (2) => $I_2 = -0.4A$ => $V_0 = -I_2*2 = 0.8V$

   (b)

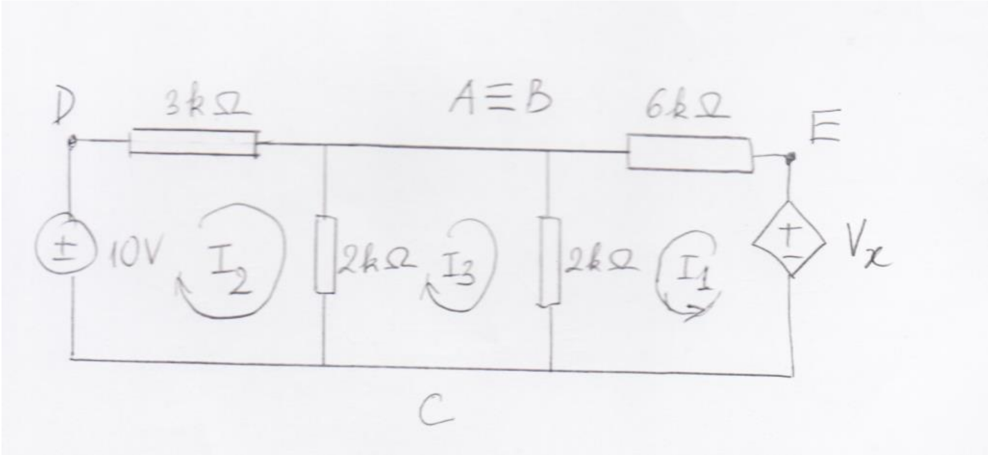
   Find $V_{TH} = V_{AB}$ (open circuit)



   - Loop I_1:
     $V_{AC} + V_{CD} + V_{DA} = 0$
     ⇨ $I_1 * 2K + I_1 * 3K = 0$
     ⇨ $I_1 = 2mA$
     ⇨ $V_X = V_A = 4V = V_{EC}$ (1)
   - Loop I_2:
     $V_{BE} + V_{EC} + V_{CB} = 0$
     ⇨ $I_2 * 6K + 4 + I_2 * 2K = 0$
     ⇨ $I_2 = -0.5$ mA
     ⇨ $V_{CB} = -1V$ => $V_B = -1V$ (2)

     (1) and (2) => $V_{AB} = V_{TH} = V_A - V_B = 5V$

Find $I_{TH} = I_{AB}$ (closed circuit)



- $V_X = V_{AC} = (I_2 – I_3)*2K$
- Loop I_1:
  $E_{VB} + V_{BC} + V_{CE} = 0$
  $=> I_1 * 6K + (I_1 + I_3) * 2K - V_X = 0$
  $=> I_1 * 6K + (I_1 + I_3) * 2K – (I_2 – I_3)*2K = 0$
  $=> 8I_1 – 2I_2 + 4I_3 = 0$ (1)
- Loop I_2:
  $V_{BC} + V_{CA} = 0$
  $=> (I_1 + I_3) * 2K + (I_3 – I_2)*2K = 0$
  $=> 2I_1 – 2I_2 + 4I_3 = 0$ (2)
- Loop I_3:
  $V_{DA} + V_{AC} + V_{CD} = 0$
  $=> I_2 * 3K + (I_2 – I_3) * 2K – 10 = 0$
  $=> 5I_2 – 2I_3 = 10$ (3)

  From (1), (2) and (3) $=> I_1 = 0A, I_2 = 2.5A, I_3 = 1.25A => I_{TH} = I_3 = 1.25$ A

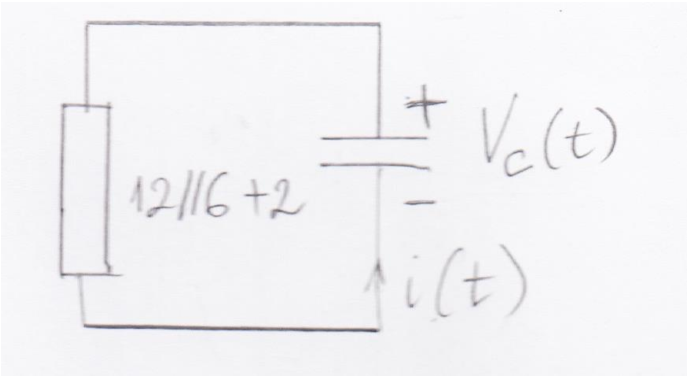
Find R_{TH}

- $R_{TH} = V_{TH}/I_{TH} = 4\Omega$
- $P_{MAX} = V_{TH}^2/4R_{TH} = 1.5625W$

2.

(a)

(i)



- R_{TH} = 12//6 + 2 = 6 Ω

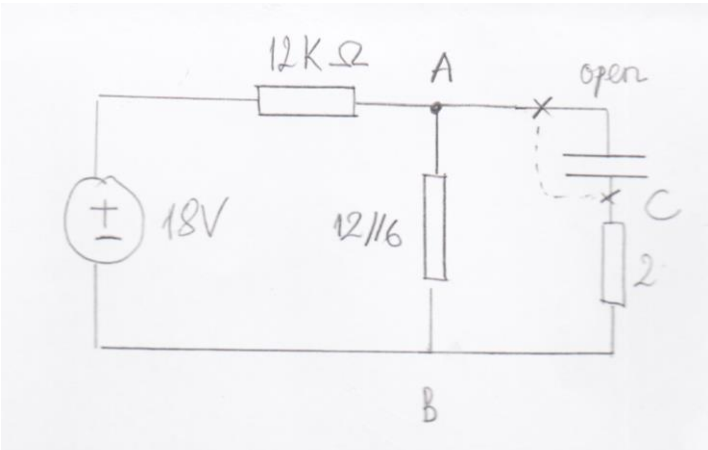
- $C\frac{dv(t)}{dt} + \frac{v(t)}{R} = 0$

$$=> 100 * 10^{-6} * \frac{dv(t)}{d(t)} + \frac{v(t)}{6000} = 0$$

$$=> \frac{dv(t)}{dt} + \frac{5}{3}v(t) = 0$$

(ii)

- Solution form: $i(t) = K_1 + K_2e^{-t/\tau}$
- $\tau = C * R_{TH} = 0.6$
- $K_1 = i(\propto) = 0$
- ***Find*** $\underline{\boldsymbol{K_2 = i(0+)}}$



Notice R_{AB} = 12//6 = 4 Ω

$v_C(0-) = v_{AB}(0-) = \frac{(12//6)}{12+12//6} * 18 = \frac{4}{12+4} * 18 = 4.5V$

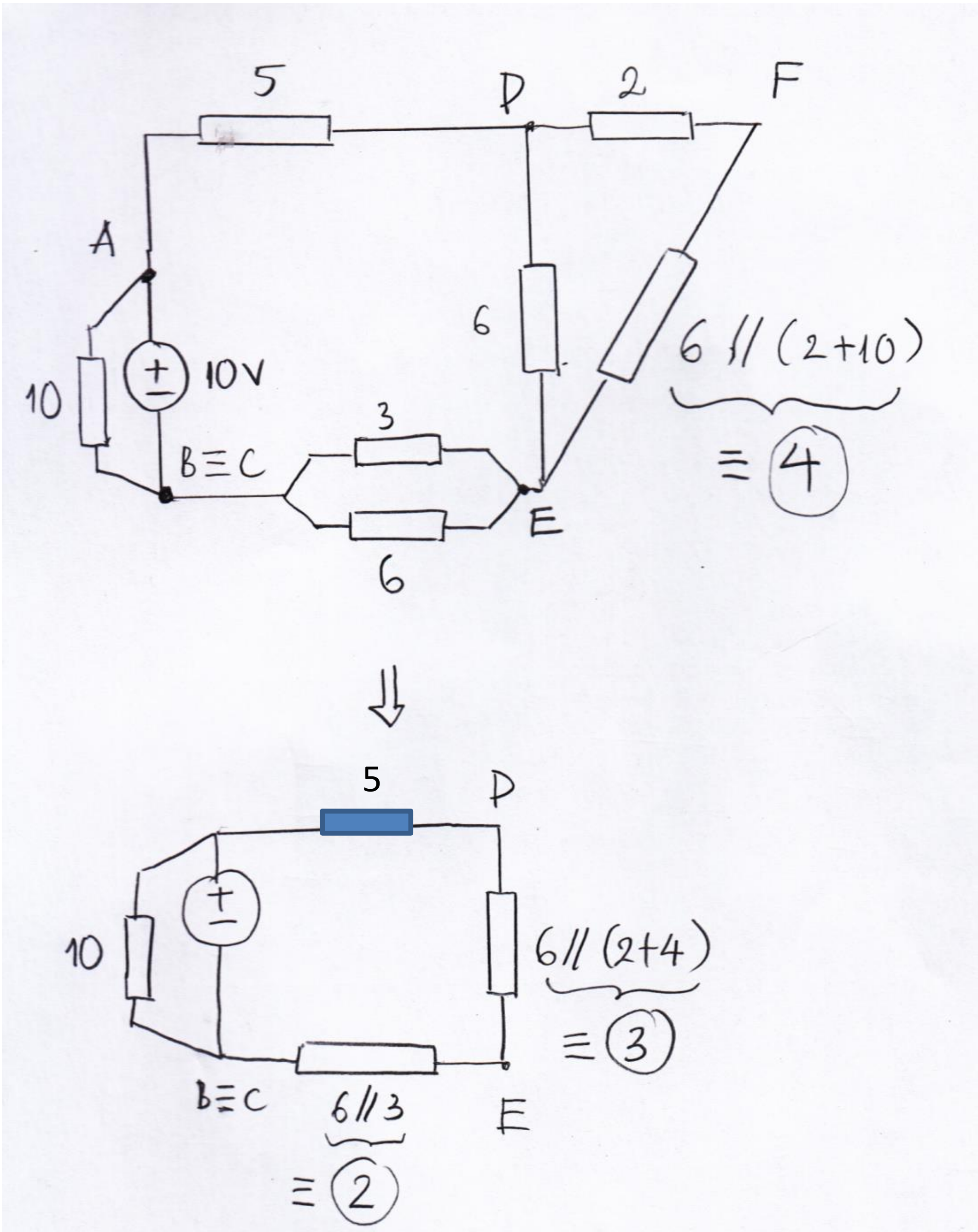
At t = 0+ => R_C = (12//6) + 2 = 4 + 2 = 6 Ω

$v_C(0+) = v_C(0-) = 4.5V => i_C(0+) = \frac{v_c(0+)}{R_C} = \frac{4.5}{6} = 0.75\,A$

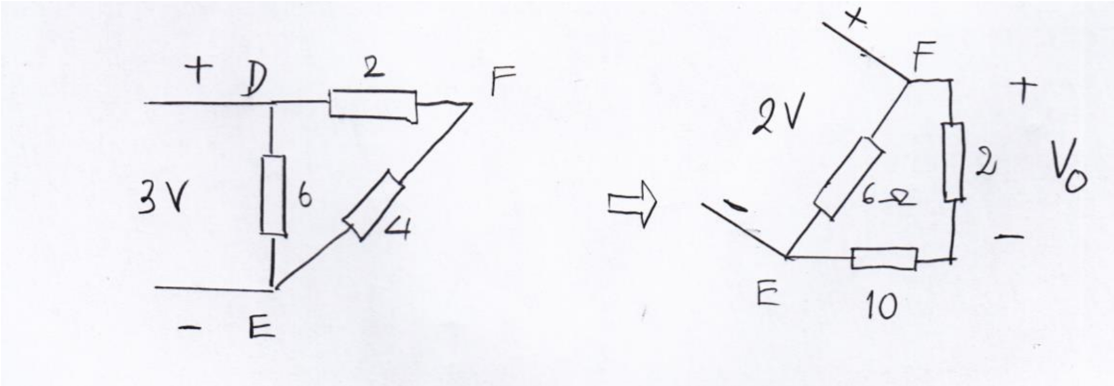
Conclusion: $i(t) = 0.75\, e^{-t/0.6}\, A$

(b)

- Redraw the circuit as follow:



- From this can see that: $V_{DE} = V_{source}\frac{3}{3+2+5} = 3V$



- Circuit on the left $=> V_{FE} = V_{ED}\frac{4}{4+2} = 2V$
- Circuit on the right $=> V_0 = V_{EF}\frac{2}{2+10} = \frac{1}{3}V$

3.

(a) Using superposition:

$$V1\ acting\ alone: V_{+1} = V_1 * \frac{15}{15+10} = \frac{3}{5}V_1$$

$$V2\ acting\ alone: V_{+2} = V_2 * \frac{10}{15+10} = \frac{2}{5}V_2$$

$$=> V_{IN} = V_+ = \frac{3}{5}V_1 + \frac{2}{5}V_2$$

$$=> \frac{V_{OUT}}{V_{IN}} = 1 + \frac{100}{R_A} = \frac{3V_1 + 2V_2}{\frac{1}{5}(3V_1 + 2V_2)} = 5$$

$$=> R_A = \frac{100}{4} = 25\ \Omega$$

(b)

$$y_1(t) \to x_1(t) = \frac{dy_1(t)}{dt} + 7y_1(t) + 8$$

$$y_2(t) \to x_2(t) = \frac{dy_2(t)}{dt} + 7y_2(t) + 8$$

$$y_3(t) = y_1(t) + y_1(t) \to x_3(t) = \frac{d(y_1(t) + y_2(t))}{dt} + 7(y_1(t) + y_2(t)) + \mathbf{8}$$

$$x_1(t) + x_2(t) = \frac{d(y_1(t) + y_2(t))}{dt} + 7(y_1(t) + y_2(t)) + \mathbf{16}$$

$$=> x_1(t) + x_2(t)\ is\ diffrent\ from\ x_3(t)$$

=> Not linear

(c) Non-causal, since y(t) depends on x(t+1)

(d)

- $x(t) = te^{-2t}u(t)$

$=> X(s) = \frac{1}{(s+2)^2}$

- $Y(s) = \frac{2}{(s+2)^3}$
- $H(s) = \frac{Y(s)}{X(s)} = \frac{2}{s+2}$

$=> h(t) = 2e^{-2t}u(t)$

4. (a)

$$y(t) = x(t) * h(t) = \int_{-inf}^{+inf} u(i)u(t-i)di = \int_{0}^{t} 1\, di = t$$

(b)

Transform the original equation, obtain: $s^2Y(s) + 2sY(s) - 8Y(s) = 2X(s)$

$=> H(s) = \frac{Y(s)}{X(s)} = \frac{2}{(s-2)(s+4)}$

Transform input signal, obtain: $X(s) = e^{-2s}/s$

$=> Y(s) = H(s)X(s) = e^{-2s}\frac{2}{(s-2)(s+4)s} = e^{-2s}(-\frac{1}{4s} + \frac{1}{6(s-2)} + \frac{1}{12(s+4)})$

$=> y(t) = (-\frac{1}{4} + \frac{1}{6}e^{2t} + \frac{1}{12}e^{-4t})\, u(t-2)$

(c)

$$I(t) = C\frac{dV_c(t)}{dt} = C\frac{d(V_c(t) - I(t)R)}{dt} = \frac{d(V_c(t) - 10I(t))}{dt}$$

Apply Laplace transform, obtain:

$I(s) = sV(s) - 10s\, I(s)$

$=> I(s) = \frac{2}{1+10s} = \frac{1}{5}\frac{1}{\frac{1}{10}+s}$

$=> I(t) = \frac{1}{5}e^{-\frac{1}{10}t}u(t)$

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Thank you and all the best for your exams! ☺