CSEC 20th – Past Year Paper Solution 2017-2018 Semester2

CZ2003 – Computer Graphics & Visualisation

Question 1

1)

a) $P = P_1 + u(P_2 - P_1) + v(P_3 - P_1)$

$x = 2 + u(4 - 2) + v(0 - 4)$

$x = 2 + 2u - 4v$

$y = 2 + u(0 - 2) + v(3 - 0)$

$y = 2 - 2u + 3v$

$z = 0 + u(3 - 0) + v(4 - 3)$

$z = 3u + v$

Parametric:

$$x = 2 + 2u - 4v$$
$$y = 2 - 2u + 3v$$
$$z = 3u + v$$

b) $x(u) = ((1.5 - 0.3)u + 0.3)\cos(2\pi(-2u) + \frac{\pi}{2})$

$= (1.2u + 0.3)\cos\left(-4\pi u + \frac{\pi}{2}\right)$

$y(u) = ((1.5 - 0.3)u + 0.3)\sin(2\pi(-2u) + \frac{\pi}{2})$

$= (1.2u + 0.3)\sin(-4\pi u + \frac{\pi}{2})$

c)

i) $r = 1 + \cos\alpha\sin3\alpha$

$r\cos\alpha = \cos\alpha(1 + \cos\alpha\sin3\alpha)$

$x(u) = \cos(2\pi u)(1 + \cos(2\pi u)\sin(6\pi u))$

$r\sin\alpha = \sin\alpha(1 + \cos\alpha\sin3\alpha)$

$y(u) = \sin(2\pi u)(1 + \cos(2\pi u)\sin(6\pi u))$

ii) $x(u, v) = \cos(2\pi u)(1 + \cos(2\pi u)\sin(6\pi u))$

$y(u, v) = \sin(\pi v) + \sin(2\pi u)(1 + \cos(2\pi u)\sin(6\pi u))$

$z(u, v) = -2 + \big(2 - (-2)\big)v$

$= -2 + 4v$

Question 2

2)

a) Plane:

$x + y + z = 0$

Sphere:

$x^2 + y^2 + z^2 = 1$

Ellipsoid:

$\left(\frac{x}{2}\right)^2 + y^2 + z^2 = 1$

Cylinder:

$1 - x^2 - y^2 = 0$

Cone:

$z^2 - x^2 - y^2 = 0$

b)

i) Square polygon:

$f_1(x, y, z) = \min(y + 1, 1 - y, x + 1, 1 - x) \geq 0$

Triangular polygon:

$f_2(x, y, z) = \min(-y - 1, y - x + 2, y + x + 2) \geq 0$

Half-elliptical disk:

$f_3(x, y, z) = \min\left(1 - x^2 - \left(\frac{y - 1}{0.5}\right)^2, y - 1\right) \geq 0$

Circular disk:

$f_4(x, y, z) = 0.5^2 - x^2 - (y + 1)^2 \geq 0$

ii) Final object:

$f(x, y, z) = \min(\max(f_1, f_2, f_3), -f_4) \geq 0$

c)

i)
$$x(\mathrm{u}, v) = 0.6 + 0.2u$$
$$y(\mathrm{u}, v) = 0.2(1 - u) - 0.4(1 - u)v$$
$$= (0.2 - 0.4v)(1 - u)$$

ii)
$$x(\mathrm{u}, v) = (0.6 + 0.2u)\sin\left(3\pi w + \frac{\pi}{2}\right)$$
$$y(\mathrm{u}, v) = (0.2 - 0.4v)(1 - u) - 0.75 + 1.5w$$
$$z(\mathrm{u}, v) = (0.6 + 0.2u)\cos\left(3\pi w + \frac{\pi}{2}\right)$$

Question 3

3)

a) $\tau = \sin\left(\frac{\pi}{2}\frac{k-1}{99}\right)$ where k is the frame index, $k \in [1,100]$

b) $A = (1\,,4)$ $A = (1\,,4)$

$B = (3\,,5)$ $E = (2\,,2)$

$C = (6\,,5)$ $F = (2\,,-1)$

$D = (2\,,3)$ $G = (0\,,3)$

Let the transformation matrix be $\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$

$a + 4b + c = 1$

$d + 4e + f = 4$

$3a + 5b + c = 2$

$3d + 5e + f = 2$

$6a + 5b + c = 2$

$6d + 5e + f = -1$

$2a + 3b + c = 0$

$2d + 3e + f = 3$

Solving it,

Transformation matrix: $\begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & 5 \\ 0 & 0 & 1 \end{bmatrix}$

c) In this case, scale first then translate, then rotate

Scaling:

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation:

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation:

$$\begin{bmatrix} \cos\left(\frac{\pi}{4}\right) & 0 & \sin\left(\frac{\pi}{4}\right) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\left(\frac{\pi}{4}\right) & 0 & \cos\left(\frac{\pi}{4}\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & & 0 \\ 0 & \cos(4.7124) & -\sin(4.7124) & 0 \\ 0 & \sin(4.7124) & \cos(4.7124) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

All the transformation:

$$\begin{bmatrix} \cos\left(\frac{\pi}{4}\right) & 0 & -\sin\left(\frac{\pi}{4}\right) & 0 \\ 0 & 1 & 0 & 0 \\ \sin\left(\frac{\pi}{4}\right) & 0 & \cos\left(\frac{\pi}{4}\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & & 0 \\ 0 & \cos(4.7124) & -\sin(4.7124) & 0 \\ 0 & \sin(4.7124) & \cos(4.7124) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \dots$$

$$\dots \begin{bmatrix} \cos\left(\frac{\pi}{4}\right) & 0 & \sin\left(\frac{\pi}{4}\right) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\left(\frac{\pi}{4}\right) & 0 & \cos\left(\frac{\pi}{4}\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.0090 & -2.1212 & -2.4775 & -0.26589 \\ 1.4142 & 0.02503 & 3.5354 & 4.2335 \\ -0.99099 & -2.1212 & 2.5225 & 1.73411 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where A is the transformation matrix

$$A \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2.39 \\ 4.26 \\ -0.387 \\ 1 \end{bmatrix}$$

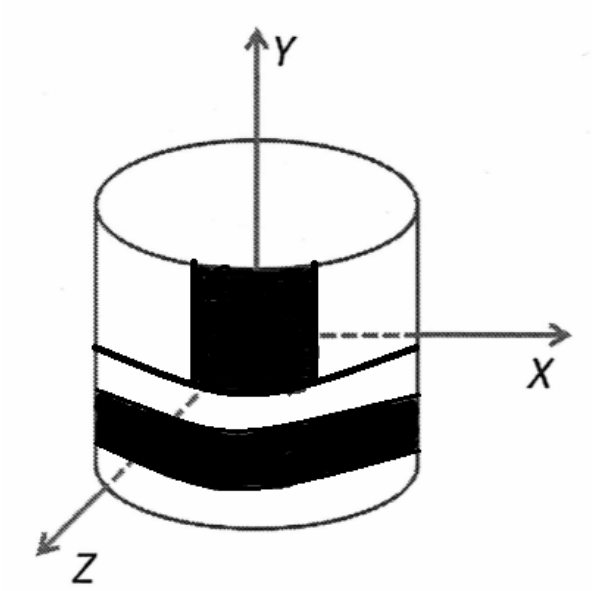
Coordinates of the final location = (-2.39,4.26,-0.387)

Question 4

4)

a) (b) is ambient reflection only while (a) is created by the combination of ambient, diffuse and specular reflections.
(b) only shows the silhouette of the object which is due to ambient reflection where the intensity at each point on the surface is constant.
(a) shows the different intensity of reflected light at different points on the surface of the object which is mainly caused by diffuse and specular reflections which is highly dependent on the position of the light source and the normal vector of the surface at each point.

b)



c)

i) $N = 2(x-1), 2(y-1), 2z = 2(-2-1), 2(1-1), 2(0) = (-6,0,0)$

$\hat{N} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$

$L = \begin{pmatrix} -11 \\ 13 \\ 0 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -9 \\ 12 \\ 0 \end{pmatrix}$

$\hat{L} = \frac{1}{15}\begin{pmatrix} -9 \\ 12 \\ 0 \end{pmatrix}$

$I_d = K_d I_s \cos\theta = K_d I_s(\hat{N} \cdot \hat{L})$

$I_d = (0.8)(1)\frac{1}{15}[\begin{pmatrix} -9 \\ 12 \\ 0 \end{pmatrix}\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}]$ = 0.48

ii) Maximum specular reflection intensity when V=R

R = 2(N·L)N-L

$= 2\left(\frac{1}{\sqrt{2}}\right)\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} - \frac{1}{15}\begin{pmatrix} -9 \\ 12 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} -0.814 \\ -0.8 \\ 0 \end{pmatrix}$

So $\begin{pmatrix} -0.814 \\ -0.8 \\ 0 \end{pmatrix}$ is the direction vector to where the viewer should be at.

Hence the location of the viewer $= \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -0.814 \\ -0.8 \\ 0 \end{pmatrix} = \begin{pmatrix} -2.814 \\ 0.2 \\ 0 \end{pmatrix}$