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1. (a)

Accuracy is the degree of closeness of the measured value to the true (actual) value of the variable being measured. It indicates the maximum error that can be expected from a measurement within the operating range of the transducer.
Precision refers to the degree of agreement within a number of measurements. It is defined as the probability of a large number of readings falling within the cluster of closeness.
Precision doesn't guarantee accuracy as the sensor can have high precision while all the measurements have a certain offset from the true value, making the sensor inaccurate.

(b)

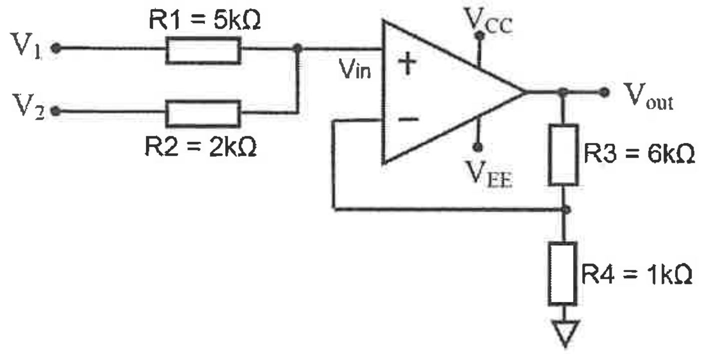
active transducers convert physical (non-electrical) signal into an electrical signal. They do not require external excitation (power supply) to operate. One example is the Quartz crystal that makes use of piezoelectric property.

(c)

New sensitivity of the temperature sensor at 40°C is 0.05 + 0.01 = 0.06mA/°C
The difference in output current is 14 – 10 = 4mA, therefore the difference in temperature between the 2 measurements are 4mA / 0.06mA/°C = 66.67°C. so the unknown temperature is 166.67°C

(d)

A non-inverting summation amplifier can achieve the desired output. The diagram is illustrated as below



calculation:

V_{in} can be expressed using voltage divider rule of V_1 and V_2 (assume V_2 is lower):

$$V_{in} = V_2 + (V_1 - V_2)\frac{R_2}{R_1 + R_2} = \frac{R_2 V_1 + R_1 V_2}{R_1 + R_2}$$

The non-inverting op-amp amplifies the input voltage:

$$V_{out} = (1 + \frac{R_3}{R_4})V_{in}$$

Thus

$$V_{out} = (R_2 V_1 + R_1 V_2)\frac{1 + \frac{R_3}{R_4}}{R_1 + R_2}$$

If we choose the 4 resistors' values such that $1+\frac{R_3}{R_4}=R_1+R_2$
then $V_{out}=R_2V_1+R_1V_2$

To minimize loading effect of the transducers and overpower unwanted noise, the resisters should be in the kΩ range. To get the desired output, R2 = 2kΩ, R1 = 5kΩ. Thus 1+R3/R4 = 7, set R4 = 1kΩ, thus R3 = 6kΩ. The suitable resistor values are reflected in the diagram.

2. (a)
   2 factors need to be considered:
   (1) output saturation
   the output voltage is constraint by the Vcc and Vee input of the op-amp, so the amplified input signal should not be higher than Vcc or lower than Vee.
   Maximum Vin = 0.01V
   The close-loop gain of this inverting op-amp = -Rf / R1
   Thus |0.01*(-Rf/R1)| ≤ 15
   Rf ≤ 1500kΩ
   (2) gain bandwidth
   frequency of the input signal f_{CL}= ω/2π = 8000/2π = 4000/π
   gain bandwidth product = |(-Rf/R1)*4000/π| ≤ 1MHz
   Rf ≤ 785.40kΩ

   Thus the maximum possible value for Rf is 785.40kΩ

   (b)
   The frequency range of input signal without distortion is:
   ω ≤ SlewRate / V_{out}(max)
   thus SlewRate ≥ ω * V_{out}(max)
   we can get from the question that ω = 10^4 and V_{in}(max) = 0.7
   V_{out}(max) = V_{in}(max) * |Av| = 0.7 * (180k/10k) = 12.6V
   thus SlewRate ≥ 10^4 * 12.6 = 126000V/s = 0.126V/μs

   (c)
   An active first-order low-pass Butterworth filter is needed to achieve the requirement.
   The gain formula of this filter is: $\left|\frac{v_O}{v_{in}}\right|=\frac{A_F}{\sqrt{1+(f/f_H)^2}}$, where A_F is the pass-band gain.
   According to the question, 20*$\log_{10}(A_F)$ = 10dB → A_F = 10^(1/2)
   also, at 25KHz: 20 * $\log_{10}$(Vo/Vin) = -40dB → Vo/Vin = 1/100
   thus $\frac{1}{100}=\frac{10^{1/2}}{\sqrt{1+(25KHz/f_H)^2}}$
   $1+(25KHz/f_H)^2=10^5$
   f_H = $25KHz/\sqrt{10^5-1}$ = 79.06Hz, which is the cut-off frequency.

(d)
The dynamic range for each bit of the ADC is 6.02dB, therefore the 12-bit ADC has a dynamic range of 72.24dB. To reduce the noise amplitude to within 1 LSB, the desired SNR should be 72.24dB.
The original SNR of the signal is $20*\log_{10}(Vsignal/Vnoise)$ = 32.04dB, thus the filter only need to achieve additional 72.24 – 32.04 = 40.20dB of SNR.
We choose 2.5kHz as the cut-off frequency, thus the transition range is $\log_{10}(15kHz/2.5KHz)$ = 0.778 decade
Thus the attenuation rate required = 40.20dB / 0.778 decade = 51.67 dB/decade
n-th order filter has attenuation rate of 20n dB → minimum: 3rd order filter with f_H=2.5KHz

3. (a)

(i) Assume that *y* is z-transformable and the z-transform of y(k) is Y(z). set the sampling interval to 1.
u(k) is a unit step function that shifted to the right by 1. The Z-transform of unit step function is z/(z-1), so Z[u(k)] = 1/(z-1)
Apply shifting theorem, note that y(0) = y(1) = 0:
$y(k+2) = z^2 * [Y(z) - y(0) - z^{-1} y(1)] = z^2 Y(z)$
$y(k+1) = z * [Y(z) - y(0)] = z Y(z)$
so the difference equation can be rewritten as:
$z^2 * Y(z) - 3/4 * z Y(z) + 1/8 Y(z) = 1/(z-1)$

$$Y(z) = \frac{1}{(z-1)(z^2 - \frac{3}{4}z + \frac{1}{8})}$$

(ii) Apply final value theorem:
$y(kT)_{k\to\infty} = \lim_{z\to1}(1-z^{-1})Y(z)$

$$= \lim_{z\to1} \frac{(1-z^{-1})z^{-1}}{(1-z^{-1})(z^2 - \frac{3}{4}z + \frac{1}{8})}$$ (divide by z on both denominator and numerator)

$= \lim_{z\to1} z^{-1} / (z^2 - 3/4 * z + 1/8)$
$= 1 / (1 - 3/4 + 1/8)$
$= 8/3$

(iii) Using the difference equation, note that u(0) = 0, u(k) = 1 for *k* in positive integers, y(0) = y(1) = 0
When k=0, the equation is: y(2) – 3/4 * y(1) + 1/8 * y(0) = 0, thus y(2) = 0
When k=1, the equation is: y(3) – 3/4 * y(2) + 1/8 * y(1) = 1, thus y(3) = 1
When k=2, the equation is: y(4) – 3/4 * y(3) + 1/8 * y(2) = 1, thus y(4) = 7/4

(b)
Ziegler-Nichols Tuning Method is to determine the appropriate proportional, integral and derivative gains for a PID controller. It starts off by switching off the integral and derivative control of the PID controller, then increase the proportional gain until the PID controller response oscillates with constant amplitude. Note down the gain value as K_u and the period of sustained oscillation as P_u, thus the PID gains are $0.6K_u$, $0.5P_u$ and $0.125P_u$ respectively.

/4.

Increase in Proportional gain results in decreased steady state offset and increased oscillation. Increase in integral gain doesn't affect steady state offset but increases oscillation. Increase in Derivative gain results in decreased oscillation and improved stability, but it's sensitive to noise.

4. (a)

The close-loop transfer function of this control system is:

$$\frac{K\frac{0.368z+0.264}{z^2-1.368z+0.368}}{1+K\frac{0.368z+0.264}{z^2-1.368z+0.368}}$$

normalize to one-level fraction: $\frac{(z^2-1.368z+0.368)K(0.368z+0.264)}{(z^2-1.368z+0.368)+K(0.368z+0.264)}$

by definition, the denominator of the transfer function is the characteristic equation of the system, rearrange the equation: $z^2-(1.368-0.368K)z+0.368+0.264K$ --(1)

Substitute $\zeta = 0.76$, $\omega_n = 10.6$ and $T = 0.05$ into standard characteristic equation

$$z^2-2\cos\left(\omega_n T\sqrt{1-\varsigma^2}\right)e^{-\varsigma\omega_n T}z+e^{-2\varsigma\omega_n T}$$

, we know the desired characteristic equation is: $z^2-1.25836z+0.44682K$ --(2)

Compare equation (1) and (2), we know that $\begin{cases}1.368-0.368K=1.25836\\0.368+0.264K=0.44682\end{cases}$

Solve K from both equations independently to check if the calculation is correct (will have minute differences due to rounding in earlier steps): *K = 0.298*

(b)

Equation (1) from question (a) is the characteristic equation of the system:

$$P(z)=z^2-(1.368-0.368K)z+0.368+0.264K$$

So $a_0 = 1$, $a_1 = -1.368+0.368K$, $a_2 = 0.368+0.264K$

1. $a_0 > 0$, thus Jury stability test can be applied
2. $|a_2| < a_0$
   $\rightarrow 0.368 + 0.264K < 1 \rightarrow K < 2.39$
3. $P(1) > 0$
   $\rightarrow 1 - 1.368 + 0.368K + 0.368 + 0.264K > 0 \rightarrow K > 0$
4. $P(-1) > 0$ for even $n = 2$
   $\rightarrow 1 + 1.368 - 0.368K + 0.368 + 0.264K > 0 \rightarrow K < 26.31$
5. As maximum n=2, Jury stability table is not needed

In conclusion, for the close-loop system to be stable, the range of K is $0 \le K < 2.39$, when K=0, the system is marginally stable.

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Thank you and all the best for your exams! ☺