20th CSEC – Past Year Paper Solution *2015-2016 Sem 2*
MH 1812 – Discrete Mathematics

1)

a) $5^1 mod 7 = 5$
$5^2 mod 7 = 25 mod 7 = 4$
$5^3 mod 7 = 125 mod 7 = 6$
$5^4 mod 7 = 2$
$5^5 mod 7 = 3$
$5^6 mod 7 = 1$
$5^{2016} \equiv (5^6)^{336} \equiv 1^{336} (mod 7) \equiv 1$

b) Last row is the critical row, it shows that when $\neg q$ and $p \to q$ are true, $\neg p$ is also true.

p	q	$\neg q$	$p \to q$	$\neg p$
T	T	F	T	F
T	F	T	F	F
F	T	F	T	T
F	**F**	**T**	**T**	**T**

c) 1, (hole 1) 2, (hole 2) 3, (hole 3) 4, …, (hole n-1) n
There are r number, and all numbers are positive. So, we need to choose r-1 from n-1 holes.
Thus, # of solutions = $(n-1)C_{r-1}$

d)

i) $\forall x \in A, \exists y \in B, P(x, y)$
<u>Proof</u>
when $x = 1, y = 4$,
$5|(x + y)$
$\therefore P(x, y)$ is true.
When $x = 3, y = 2$
$5|(x + y)$
$\therefore P(x, y)$ is true.

ii) $\exists y \in B, \forall x \in A, P(x, y)$
<u>Proof</u>
Only when $x = 1, y = 4$ or $x = 3, y = 2$, will fulfill $P(x, y)$
$\therefore There\ is\ no\ intersection\ between\ them, so\ the\ statement\ \exists y \in B, \forall x \in A, P(x, y)$ is false.

2)

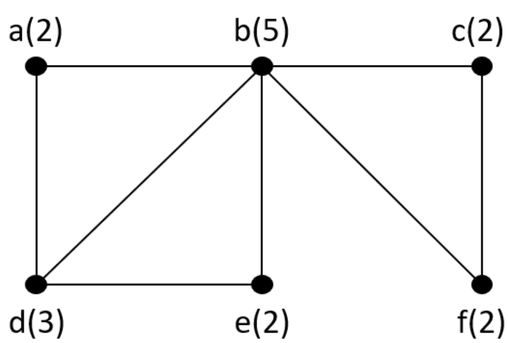
a) $x^2 = 4x - 4$
$x^2 - 4x + 4 = 0$
$(x - 2)^2 = 0$
$x = 2$
$\therefore a_n = nC_1(2^n) + C_2 2^n$
Substitute $a_1 = 2$
$2 = C_1(2) + 2(C_2)$
$1 = C_1 + C_2$ --- (1)

Substitute $a_2 = 8$
$8 = 2(C_1)(2^2) + 2^2 C_2$
$2 = 2C_1 + C_2$ --- (2)
(2) – (1) $\Rightarrow C_1 = 1, C_2 = 0$
$\therefore a_n = n2^n$

b) Proof by induction
When $n = 1$
$LHS = 1 \cdot 2 = 2$
$$RHS = \frac{1(1+1)(1+2)}{3} = 2$$
$LHS = RHS$
When $n = k$
Assume $1(2) + 2(3) + \cdots k(k+1) = \frac{k(k+1)(k+2)}{3}$
When $n = k + 1$
$$LHS = 1(2) + 2(3) + \cdots k(k+1) + (k+1)(k+2) = \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$$
$$= (k+1)(k+2)\left(\frac{k}{3} + 1\right)$$
$$= \frac{(k+1)(k+2)(k+3)}{3} = RHS$$

3)
a) Proof by set identity
$RHS = (B \cup C) - A$
$= (B \cup C) \cap \overline{A}$
$= (B \cap \overline{A}) \cup (C \cap \overline{A})$
$= (B - A) \cup (C - A)$ [Proven]

b) It's not Euler circuit, as degree of node B and D are odd number.
It's Euler path, as there are 2 nodes with odd degree
It's not Hamilton circuit as node B is the only node connect left-right graph together. It's impossible start and end at the same node without passing any node twice.



4)

a) It's not reflexive, as $(b, b), (c, c), (d, d) \notin R$
It's not symmetric, as $(a, b) \in R, but\ (b, a) \notin R$
It's not transitive, as $(a, b) \in R \wedge (b, c) \in R, but\ (a, c) \notin R$

b)

i) $f(x) = x^2 + 2x + 3$
$= (x^2 + 2x + 1) + 2$
$= (x + 1)^2 + 2$
As $x \leq -1$,
range of $(x + 1)^2$ is $(0, \infty)$
∴ Range of f(x) is $(2, \infty)$

ii) $f\left(f^{-1}(x)\right) = x$
$f^{-1}(x)^2 + 2f^{-1}(x) + 3 = x$
$(f^{-1}(x) + 1)^2 + 2 = x$
$f^{-1}(x) = \sqrt{x - 2} - 1$

--End of Answers--

Solver: Gan Hsien Yan