課題6　1/23　521 22305　岩崎 [illegible]

6-1

$$\langle f, g\rangle = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \sin\omega_0 t \sin 2\omega_0 t \, dt$$

$$= \frac{1}{T}\left(\int_{-\frac{T}{2}}^{\frac{T}{2}} \cos\omega_0 t \, dt - \int_{-\frac{T}{2}}^{\frac{T}{2}} \cos 3\omega_0 t \, dt\right)$$

$$= \frac{1}{T}\left(\frac{1}{\omega_0}\left[\sin\omega_0 t\right]_{-\frac{T}{2}}^{\frac{T}{2}} - \frac{1}{3\omega_0}\left[\sin 3\omega_0 t\right]_{-\frac{T}{2}}^{\frac{T}{2}}\right)$$

$$= \frac{1}{T}\left(\frac{1}{\omega_0}\sin\frac{\omega_0 T}{2} + \frac{1}{\omega_0}\sin\frac{\omega_0 T}{2} - \frac{1}{3\omega_0}\sin\frac{3\omega_0 T}{2} - \frac{1}{3\omega_0}\sin\frac{3\omega_0 T}{2}\right.$$

$$= \frac{1}{T}\left(\frac{2}{\omega_0}\sin\frac{\omega_0 T}{2} - \frac{2}{3\omega_0}\sin\frac{3\omega_0 T}{2}\right)$$

$$= \frac{1}{T}\left(\frac{2}{\omega_0}\sin\pi - \frac{2}{3\omega_0}\sin 3\pi\right)$$

$$= \frac{1}{T}\left(\frac{2}{\omega_0}\cdot 0 - \frac{2}{3\omega_0}\cdot 0\right) = \underline{0}$$

6-2

$$\langle f, g\rangle = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \sin\omega_0 t \sin\omega_0 t \, dt$$

$$= \frac{1}{T}\cdot\int_{-\frac{T}{2}}^{\frac{T}{2}} \cos 0 \cdot t \, dt - \frac{1}{T}\cdot\int_{-\frac{T}{2}}^{\frac{T}{2}} \cos 2\omega_0 t \, dt$$

$$= \frac{1}{T}\left[t\right]_{-\frac{T}{2}}^{\frac{T}{2}} - \frac{1}{T}\cdot\frac{1}{2\omega_0}\cdot\left[\sin 2\omega_0 t\right]_{-\frac{T}{2}}^{\frac{T}{2}}$$

$$= 1 - \frac{1}{2\omega_0 T}\left(\sin\omega_0 T + \sin\omega_0 T\right)$$

$$= 1 - 0$$

$$= \underline{1}$$

6-3

$$\sin nx \sin mx = \frac{1}{2}\{\cos(n-m)x - \cos(n+m)x\}$$

(解)

$$\sin nx \sin mx = \cos(n-m)x - \cos nx \cos mx$$

$$\sin nx \sin mx = \cos(n-m)x - (\cos(n+m)x + \sin nx \sin mx)$$

$$2\sin nx \sin mx = \cos(n-m)x - \cos(n+m)x$$

$$\underline{\sin nx \sin mx = \frac{1}{2}\{\cos(n-m)x - \cos(n+m)x\}}$$

6-4

$$\|f\| = \sqrt{\langle f,f\rangle} = \sqrt{\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f^2(t)\,dt}$$

$$= \sqrt{\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \sin^2 w_0 t\,dt}$$

$$= \sqrt{\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{1-\cos 2w_0 t}{2}\,dt}$$

$$= \sqrt{\frac{2}{T}\left(\left[\frac{1}{2}t\right]_{-\frac{T}{2}}^{\frac{T}{2}} - \frac{1}{2}\int_{-\frac{T}{2}}^{\frac{T}{2}} \cos 2w_0 t\,dt\right)}$$

$$= \sqrt{\frac{2}{T}\left(\frac{T}{2} - \frac{1}{2}\cdot\frac{1}{2w_0}\left[\sin 2w_0 t\right]_{-\frac{T}{2}}^{\frac{T}{2}}\right)}$$

$$= \sqrt{\frac{2}{T}\left(\frac{T}{2} - \frac{1}{4w_0}\sin w_0 \underline{T} - \frac{1}{4w_0}\sin w_0 T\right)}$$

$\frac{2\pi}{w_0}$

$$= \sqrt{\frac{2}{T}\left(\frac{T}{2} - 0 - 0\right)} = \sqrt{1} = \underline{1}$$

$$\|g\| = \sqrt{\langle g,g\rangle} = \sqrt{\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} g^2(t)\,dt}$$

$$= \sqrt{\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \sin^2 2w_0 t\,dt}$$

$$= \sqrt{\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{1-\cos 4w_0 t}{2}\,dt}$$

$$= \sqrt{\frac{2}{T}\left(\frac{T}{2} - \frac{1}{2}\int_{-\frac{T}{2}}^{\frac{T}{2}} \cos 4w_0 t\,dt\right)}$$

$$= \sqrt{\frac{2}{T}\left(\frac{T}{2} - \frac{1}{2}\cdot\frac{1}{4w_0}\left[\sin 4w_0 t\right]_{-\frac{T}{2}}^{\frac{T}{2}}\right)}$$

$$= \sqrt{\frac{2}{T}\left(\frac{T}{2} - \frac{1}{8w_0}\sin 2w_0 T - \frac{1}{8w_0}\sin 2w_0 T\right)}$$

$$= \sqrt{\frac{2}{T}\left(\frac{T}{2} - 0 - 0\right)}$$

$$= \underline{1}$$