EC4331 Notes

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Contents

1 A Classical Monetary Model

Assumptions for Classical Model:

1. Perfect competition in goods and labor markets
2. Flexible prices and wages
3. No capital accumulation
4. No fiscal sector
5. Closed economy

1.1 Households

Representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \cdot U(C_t, N_t)$$

$$\text{s.t. } P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + D_t$$

Consumers face 2 tradeoffs, C_t vs N_t and C_t vs C_{t+1}.

Implied optimality conditions:

1. C_t vs N_t
$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi}$$
$$\Leftrightarrow w_t - p_t = \sigma c_t + \varphi n_t$$
This is the labor supply curve function, where LHS is real wage.
L_S is increasing with n_t, if wage is higher, they will give up leisure to earn more (substitution effect).

   If c_t increases, L_S shifts up. At the same level of wage, n_t is lower. This is because consumption increases utility. To remain the same utility level, consumer can work less (wealth effect).

2. C_t vs C_{t+1}
$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}$$
This is the Euler Equation which is Consumers' choice that maximizing utility.
This equation from a more general form $Q_t = \beta E \left\{ \frac{u_{c,t+1}}{u_{c,t}} \frac{P_t}{P_{t+1}} \right\}$, which also implies $u_{c,t} = \beta E \left\{ u_{c,t+1} \frac{P_t}{Q_t} \frac{1}{P_{t+1}} \right\}$

From this Euler Equation, we can obtain another representation:

$$c_t = E_t(c_{t+1}) - \frac{1}{\sigma}\left[i_t - E_t(\pi_{t+1}) - \rho\right]$$

- c_t is positively related to $E_t(c_{t+1})$ and $E_t(\pi_{t+1})$
- c_t is negatively related to i_t and r_t since $r_t = i_t - E_t(\pi_{t+1})$
- This equation shows the relationship between consumption and interest rate

$$i_t = \sigma E_t(c_{t+1} - c_t) + E_t(\pi_{t+1}) + \rho$$

- This equation let us know in steady state,, $i = \pi + \rho$ since $E_t(c_{t+1} - c_t) = 0$
- Then in steady state, $i = \pi + \rho$, $r = \rho$

1.2 Firms

Firms have production function which takes the form:

$$Y_t = A_t N_t^{1-\alpha}$$

$$y_t = a_t + (1-\alpha)n_t$$

Here a_t follows an exogenous stationary process:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

In classical model, perfect competitive market is assumed, hence firms can only decide n_t, p_t and w_t are taken from the market.

Thus, firms solve

$$\max_{\{N_t\}} \; P_t Y_t - W_t N_t$$

$$\text{s.t. } Y_t = A_t N_t^{1-\alpha}$$

Optimality Condition:

$$\frac{W_t}{P_t} = (1-\alpha)A_t N_t^{-\alpha}$$

$$w_t - p_t = log(1-\alpha) + a_t - \alpha n_t$$

This equation is in the form of real wage = MPL.

If a_t increases, L_D shifts up because MPL is higher, workers are more productive, hence firms desire more workers.

1.3 Equilibrium

To achieve equilibrium we have 4 conditions:

1. Goods market clearing: $y_t = c_t$
2. Labor market clearing: $L_S = L_D$
   $\sigma c_t + \varphi n_t = a_t - \alpha n_t + log(1-\alpha)$
3. Assets market clearing: $B_t = 0$ because we have only 1 representative agent in the economy.
   $r_t = i_t - E_t(\pi_{t+1}) = \rho + \sigma E_t(\Delta c_{t+1})$
4. Production function: $y_t = a_t + (1-\alpha)n_t$

Solving the model means we express endogenous variables as a function of exogenous variables. For each endogenous variable, we can find that

1. $n_t = \psi a_t + \psi_n$
2. $y_t = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} a_t$
3. $r_t = \rho - \sigma\psi_{ya}(1-\rho_a)a_t$
4. $w_t - p_t = \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} a_t + \psi_w$

Discussion:

1. Monetary non-neutrality:
   Money is neutral. Because y_t, r_t, $w_t - p_t$ is independent of m_t. Thus, money doesn't affect these real variables
2. $\frac{\partial y_t}{\partial a_t} > 0$, $\frac{\partial w_t}{\partial a_t} > 0$, how about $\frac{\partial n_t}{\partial a_t}$?
   If a_t increases, the productivity is higher, thus the production $y_t = c_t$ will increase. Since c_t increases, HH are less willing to work to maintain the same level of utility (wealth effect). Therefore, L_S shifts left

   If wealth effect is small, $n^+ > n$
   If σ is big, wealth effect is big, $n^+ < n$
   If $\sigma = 1$, $n^+ = n$

1.4 2 Examples of Monetary Policy

1.4.1 Example 1

$$i_t = i + v_t$$

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

$$\Rightarrow E_t(\pi_{t+1}) = \rho + \pi + v_t - r_t$$

$$\Rightarrow E_t(\hat{\pi}_{t+1}) = v_t - \hat{r}_t$$

$$\Rightarrow \hat{\pi}_{t+1} = v_t - \hat{r}_t + \xi_{t+1}$$

In this example, CB has nothing to do to counter inflation because i_t follows an exogenous path.

1.4.2 Example 2

$$i_t = \rho + \pi + \phi_\pi(\hat{\pi}_t) + v_t$$

$$\Rightarrow \phi_\pi \hat{\pi}_t = E_t(\hat{\pi}_{t+1}) + \hat{r}_t - v_t$$

Let $\hat{\pi}_t > 0$, then inflation is expected,

1. $\phi_\pi = 1$: raise i_t by the same level of inflation
2. $\phi_\pi > 1$: raise i_t more than π_t deviation from π
3. $\phi_\pi < 1$: raise i_t less than π_t deviation from π

ϕ_π needs to be greater than 1 to rule out sunspot shock (clear inflation more effectively).

1.5 Summary

- In classical model, the equilibrium condition requires:
  1. goods market clearing
  2. labour market clearing
  3. asset market clearing
  4. production function
- Money is neutral, it does not affect n_t, r_t, $w_t - p_t$.
- Productivity/technology, a_t, is the main driver of economy in classical model (supply schock):
  1. $a_t \uparrow \Rightarrow y_t \uparrow$
  2. $a_t \uparrow \Rightarrow w_t \uparrow$
  3. $a_t \uparrow \Rightarrow n_t \uparrow$ if substitution effect dominates
  4. $a_t \uparrow \Rightarrow n_t \downarrow$ if wealth effect dominates
  5. $a_t \uparrow \Rightarrow \bar{n}_t$ if both effects are the same
- $\phi_\pi > 1$, in the MP rule, to rule out sunspot shock, or to counter inflation more effectively (Taylor Principle).

2 New Keynesian Model

Assumptions:

1. The market is monopolistic competition, ie. firms produce different goods
2. With probability (1-θ), firms can reset price
3. With probability θ, firms cannot reset price

2.1 Households

HH aggregate consumption level at time t is

$$C_t = \left[\int_0^1 (C(j))^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

- ε: elasticity of substitution goods
- If $\varepsilon \to \infty$, $C_t = \int_0^1 C_j dj$
- This means perfect substitution $\Rightarrow$ perfect competition
- In general, $\varepsilon \uparrow \Rightarrow$ competition $\uparrow \Rightarrow$ markup $\downarrow$

Thus, there is a new optimization problem for HH

$$\max_{\{C_t(j)\}} \; C_t = \left[\int_0^1 (C(j))^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\text{s.t.} \int_0^1 P_t(j) \cdot C_t(j) dj = Z_t$$

By doing FOC and some rearrangement, we can achieve the following:

$$\left(\frac{C_t(i)}{C_t(j)}\right)^{-\frac{1}{\varepsilon}} = \frac{P_t(i)}{P_t(j)}$$

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t \tag{1}$$

Interpretation of (1):
The demand of goods i depends

1. negatively on $P_t(i)$, because people will buy less if it's more expensive
2. positively on P_t, because goods i is cheaper compared to other goods
3. positively on C_t, because...

2.2 Firms

2.2.1 Static Problem

Nominal Cost Function:

$$\Psi(Y) = W \cdot N = W \left(\frac{Y}{A}\right)^{\frac{1}{1-\alpha}}$$

Nominal Marginal Cost Function:

$$\Psi'(Y) = \frac{\partial\ \Psi(Y)}{\partial\ Y} = \frac{1}{1-\alpha}\frac{W}{A}\left(\frac{Y}{A}\right)^{\frac{\alpha}{1-\alpha}}$$

Firms' static problem is

$$\max_{P(j)}\ P(j)\ Y(j) - \Psi(Y(j))$$

$$\text{s.t.}\quad Y(j) = \left(\frac{P(j)}{P}\right)^{-\varepsilon} \cdot Y$$

FOC yields

$$P^*(j) = \frac{\varepsilon}{\varepsilon - 1} \cdot \Psi'(Y(j))$$

- $M \equiv \frac{\varepsilon}{\varepsilon-1}$, markup
- $\varepsilon \uparrow \Rightarrow M \downarrow \because$ more competitive

Since the probability of setting price at any time t is $1-\theta$ is the degree of flexibility, and θ is the degree of price rigidity, we have 2 results due to θ

1. Result 1: $E(\text{duration of a price}) = \frac{1}{1-\theta}$
   - $\theta = 0 \Rightarrow E(durat) = 1 \Rightarrow$ price stay 1 period only $\because$ not rigid
   - $\theta = 1 \Rightarrow E(durat) = \infty \Rightarrow$ price never change $\because$ very rigid

2. Result 2: Aggregate Price Dynamics (ie. how P_t, π_t evolve over time)
   - $P_t^{1-\varepsilon} = (1-\theta)P_t^{*(1-\varepsilon)} + \theta P_{t-1}^{1-\varepsilon}$
     - $1-\theta$ of the firms can set price while θ of the firms can only follow the price in the previous period. P_t is the weighted average of these prices.
     - From this equation, we can find P_t by solving for $P_t^{*(1-\varepsilon)}$ from the maximization problem. We want to know P_t because it determines inflation.
   - $\pi_t = (1-\theta)(P_t^* - P_{t-1})$

- Inflation = fraction of adjusters $(1-\theta)$ $\times$ the changes in prices that these firms make

- $P_t = \theta P_{t-1} + (1-\theta) P_t^*$
  - Price today is weighted by adjusters' and non-adjusters' average price.

  $\therefore$ to find P_t, we already have P_{t-1}, now we need P_t^*

2.2.2 Dynamic Problem

For a firm who can reset price, it will do so to maximize the value of the firm

$$\max_{P_t^*(i)} V_t(i) = E_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+k} \frac{\text{Profit}_{t+k}(i)}{P_{t+k}} \right]$$

$$\Leftrightarrow \max_{\{P_t^*\}} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} \frac{1}{P_{t+k}} \left[P_t^* Y_{t+k|t} - \Psi(Y_{t+k|t}) \right] \right\}$$

- $Y_{t+k|t}$ is the demand conditional on price being last reset at time t (if price is P_t what is the demand?)

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k}$$

- $$\Lambda_{t,t+k} = \beta^k \frac{U_{c,t+k}}{U_{c,t}}$$

  This is the stochastic discount factor.
  Interpretation: C_{t+k} units of consumption at $t+k$ is equivalent to $\Lambda_{t,t+k} \cdot C_{t+k}$ units of consumption today, in terms of today's utility $\beta^k \frac{U_{c,t+k}}{U_{c,t}} \cdot C_{t+k}$

In summary, the firms maximize

$$\max_{\{P_t^*\}} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} \frac{1}{P_{t+k}} \left[P_t^* Y_{t+k|t} - \Psi(Y_{t+k|t}) \right] \right\}$$

$$\text{s.t.} \quad Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k}$$

FOC:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} \frac{Y_{t+k|t}}{P_{t+k}} \left(P_t^* - \frac{\varepsilon}{\varepsilon - 1} \cdot \Psi'_{t+k|t} \right) \right\} = 0$$

Discussion:

1. On average, $P_t^* = \frac{\varepsilon}{\varepsilon-1} \Psi'_{t+k|t} \Rightarrow$ the average markup $= \frac{\varepsilon}{\varepsilon-1}$
   But actual markup $\neq \frac{\varepsilon}{\varepsilon-1} \because \Psi'_{t+k|t} \neq \Psi'_{t+k}$

2. If $\theta = 0$ (flexible price), the FOC collapse to $P_t^* = \frac{\varepsilon}{\varepsilon - 1}\Psi_t'$

New notations:

1.
$$\text{real MC}_t = \frac{\Psi_t'}{P_t}$$

2.
$$\text{Markup}_t = \frac{P_t}{\Psi_t'}$$

3.
$$\therefore \; MC_t = \frac{1}{\text{Markup}_t}$$
$$MC = \frac{1}{M} \text{ in s.s.}$$
$$\Rightarrow \; mc = -\mu$$

4.
$$\log \Psi' = \psi_{t+k}, \text{ log nominal MC}$$

Take FOC, loglinearize it,

$$P_t^* = -mc + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t(\psi_{t+k|t}), \text{ where } -mc = \mu, \; \psi_{t+k|t} = mc_{t+k|t} + p_{t+k}$$

Discussion:

1. P_t^* is desired markup (μ) + weighted average of current and future nominal MC
2. The further away the future ψ_{t+k} has a smaller weight $(\beta\theta)^k$
3. That weight $\downarrow$ faster if $\theta \downarrow \because$ more flexible prices

2.2.3 NK Model

$$P_t^* = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t(\psi_{t+k|t})$$
$$\Leftrightarrow P_t^* = \beta\theta E_t(P_{t+1}^*) + (1-\beta\theta)\hat{MC}_t + (1-\beta\theta)P_t$$

Implication: Marginal Cost $\uparrow \Rightarrow$ Markup $\downarrow \Rightarrow$ Price $\uparrow \Rightarrow$ inflation $\uparrow$

Question: Why $P_{t+k} = P_t$ affects ψ_{t+k}?
Answer: Because, generally, price affects Y, Y affects MC

From the NK Model above, we can derive that

$$\pi_t = \beta E_t(\pi_{t+1}) - \lambda \hat{\mu}_t \text{ , where } \lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

Interpretations:

1. $\pi_t \uparrow$ if $\hat{MC}_t \uparrow$ or $\hat{\mu} \downarrow$
   - Higher MC $\Rightarrow$ expensive cost $\Rightarrow$ firms increase price $\Rightarrow$ inflation increase
   - Markup less than s.s. markup $\Rightarrow$ firms increase price to earn more because current markup not enough to maximize profit $\Rightarrow$ inflation increase
2. $\pi_t \uparrow$ if $E(\pi_{t+1}) \uparrow$
   - If firms expect $E(\pi_{t+1}) \uparrow$, those who can reset the price today will anticipate by $P_t^* \uparrow$, because they worried that at $t+1$, they might not be able to increase price
3. $\pi_t = -\lambda \sum_{k=0}^{\infty} \beta^k E_t(\hat{\mu}_t)$
   - This is the NKPC after solving forward
   - ultimately, what matters for π_t is markup

2.3 Equilibrium

Some conditions for equilibrium:

1. Goods market clearing:
$$y_t = c_t$$
2. Euler Equations:
$$y_t = E_t(y_{t+1}) - \frac{1}{\sigma}(i_t - E_t(\pi_{t+1}) - \rho)$$
3. Labor Market Clearing (derived from first solving y_t because NK model is demand driven):
$$n_t = \frac{1}{1-\alpha}(y_t - a_t) + d_t \text{ , where } d_t \equiv log\left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} di\right]$$

- d_t is distortion in the market, some resources that is wasted and not going into production
- It can be shown that $d_t \simeq 0 + \kappa \cdot \text{var}(p(i)) > 0$
- $var(p(i))$ is the dispersion in price.
- No dispersion means no firms can adjust price
- price dispersion $\Rightarrow$ consumers buy cheaper goods $\Rightarrow Y_t$ not maximised because we assume $C_t(i) = C_t(j) \Rightarrow$ deviation from that creates distortion
- Labor market clearing equation also implies $y_t = a_t + (1-\alpha)n_t - d_t$

Discussion:

1. If price were flexible, $P_t(i) = P_t(j)\ \forall\ i, j, t$
   $\Rightarrow\ var(p_t) = 0 \Rightarrow n_t = \frac{1}{1-\alpha}(y_t - a_t)$
2. In general, when prices are sticky, up to 1st order $d_t \simeq 0$
   $\Rightarrow n_t \simeq \frac{1}{1-\alpha}(y_t - a_t)$

2.3.1 Relationship between $\hat{MC}_t$ and y_t

An additional unit of output needs $\frac{1}{MPN_t}$ unit of labor. Firms pay this labour with W_t, thus, $\frac{W_t}{MPN_t}$ is the nominal marginal cost for a firm. To get real marginal cost, we have $\frac{W_t}{P_t}\frac{1}{MPN_t}$. This is how firm decides their labour demand. Therefore,

$$-\mu_t = mc_t = w_t - p_t - mpn_t$$

It can be shown that

$$mc_t = \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t - \frac{1+\varphi}{1-\alpha} a_t - log(1-\alpha)$$

New notations:

- $\tilde{mc}_t = mc_t - mc_t^n$
- $mc_t^n = -\mu_t^n = \frac{\varepsilon}{\varepsilon - 1} \equiv M$
- $y_t^n =$ natural level of y_t, ie. the level of y_t if price were flexible
- Thus, the term with $\sim$ means the difference between itself and perfect competitive market level

It can be shown that $\hat{MC}_t = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(y_t - y_t^n)$,

Interpretation: $\tilde{y}_t \uparrow \Rightarrow \hat{MC}_t \uparrow$ due to diminishing marginal return. Thus,

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa \tilde{y}_t$$

This is called the New Keynesian Phillips Curve.

Now, rewrite Euler Equation with natural terms, we can get

$$\tilde{y}_t = E(\tilde{y}_{t+1}) - \frac{1}{\sigma}(i_t - E_t(\pi_{t+1}) - r_t^n)$$

This is called the Dynamic IS Curve.
Interpretation:

- $E_t(\pi_{t+1}) \uparrow \Rightarrow P_t^* \uparrow$ because firms afraid next period they can't change the price.
- $\tilde{y}_t \uparrow \Rightarrow \pi_t \uparrow$ because $\hat{MC}_t \uparrow$, firms need to increase price to maintain at the same markup.

2.4 Solving NK Model

1. DIS:
$$\tilde{y}_t = E(\tilde{y}_{t+1}) - \frac{1}{\sigma}(i_t - E_t(\pi_{t+1}) - r_t^n)$$
   DIS goes into AD, since it is households' optimal basket
2. NKPC:
$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa \tilde{y}_t$$
   NKPC goes into AS, because it is determined by firms
3. MP:
$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t$$
   MP goes into AD because interest rate affects demand (if no borrowing)

To solve the model, we need to express endogenous variables in terms of exogenous variables. One method is use guess and verify. We assume some endogenous variables can be expressed by exogenous variables v_t with some coefficients (θ_π, θ_y). If in the end these 2 coefficients can be expressed by exogenous variables, we have found a solution.

$$\tilde{y}_t = \theta_y \cdot v_t \text{ , where } \theta_y < 0$$

$$\pi_t = \theta_\pi \cdot v_t \text{ , where } \theta_\pi < 0$$

However, this solution may not be unique. It has been shown that

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0 \Rightarrow \exists \text{ ! solution}$$

The Taylor Principle ($\phi_\pi > 1$) can partly fulfill this condition.

2.4.1 Scenario 1: if there is a shock i.e. $v_t \uparrow$

- $\tilde{y}_t \downarrow$, $\pi_t \downarrow$
  - $v_t \uparrow \Rightarrow i_t \uparrow \Rightarrow y_t \downarrow \rightarrow \tilde{y}_t \downarrow \Rightarrow c_t \downarrow \Rightarrow \pi_t \downarrow$
  - i_t is the indirect channel that affects $\tilde{y}_t$ and π_t from the shock of v_t

2.4.1.1 Differences between NK Model and Classical Model

1. $$v_t \uparrow \Rightarrow i_t \uparrow \text{(direct)}, \pi_t \downarrow \text{(indirect)}$$

   - In NK model, $v_t \uparrow \Rightarrow i_t \uparrow$ because direct effect dominates. This is called the liquidity effect.
   - In classical model, there is no liquidity effect because indirect effect dominates since prices are fully flexible, i.e. $v_t \uparrow \Rightarrow i_t \downarrow$

2. $$y_t = E_t(y_{t+1}) - \frac{1}{\sigma}(i_t - E_t(\pi_{t+1}) - \rho)$$

   - In NK model, $r_t = i_t \uparrow -E_t(\pi_{t+1}) \downarrow \Rightarrow r_t \uparrow \Rightarrow y_t \downarrow$
     - nominal term i_t affects real term y_t, hence this is the monetary non-neutrality
   - In classical model, $r_t = i_t \uparrow -E_t(\pi_{t+1}) \downarrow$
     - Both are affected. But since the price is fully flexible, $E_t(\pi_{t+1})$ can quickly fully adjust to offset the change in i_t such that r_t unchanged.
     - nominal term does not affect real term, this is called monetary neutrality.

Intuition: When $v_t > 0$,

- $r_t \uparrow = i_t \uparrow\uparrow -E_t(\pi_{t+1}) \downarrow$ due to price rigidity
- From DIS, $r_t \uparrow \Rightarrow \tilde{y}_t \downarrow$
- $r_t \downarrow \Rightarrow$ return to savings $\uparrow \Rightarrow$ consumption $\downarrow = y_t \downarrow \Rightarrow$ output gap 0 $\Rightarrow$ inflation 0 (From NKPC)
- $\pi_t \downarrow$ can be inferred from NKPC
- $y_t \downarrow \Rightarrow n_t \downarrow = y_t \downarrow -a_t \Rightarrow$ real wage $\downarrow \Rightarrow (w_t - p_t) - mpn_t = mc \downarrow \Rightarrow P_t^* \downarrow$ (to get same markup that maximizes profit) $\Rightarrow \pi_t \downarrow$

2.4.2 Scenario 2: if there is a technological shock i.e. $a_t \uparrow$

1. $y_t^n = \psi_{ya} a_t \uparrow$ automatically

2. $\tilde{y}_t = y_t \uparrow - y_t^n \uparrow\uparrow \Rightarrow \tilde{y}_t \downarrow$

   - $\uparrow y_t < \uparrow\uparrow y_t^n$ because of price rigidity
   - $a_t \uparrow \Rightarrow MC_t \downarrow \Rightarrow P_t \downarrow$ so that firms can achieve optimal markup at this price level. But price not entirely change to the new price.

3. $y_t = a_t \uparrow + n_t \Rightarrow y_t \uparrow$

4. $\pi_t \downarrow$ can be seen from NKPC since $\tilde{y}_t < 0$

   - Intuition: $a_t \uparrow \Rightarrow mc \downarrow \Rightarrow P_t^* \downarrow \Rightarrow \pi_t \downarrow$

5. $n_t = y_t \uparrow - a_t \uparrow$ but $n_t \downarrow$

   - $\frac{\partial n_t}{\partial a_t} < 0$ if $\sigma > 1$
   - i.e. the wealth effect dominates

6. $i_t = \rho + \phi_\pi \pi_t (< 0) + \phi_y \tilde{y}_t (< 0) \Rightarrow i_t \downarrow$

Question: $a_t \uparrow$, CB reacts by $i_t \downarrow$, Is this a desirable property?
Answer: Yes. Because the cuts in interest rate helps stimulates AD and therefore helps to close output gap and deflation

Question: Is there room for improvement?
Answer: Yes. CB can cut i_t more by $\uparrow \phi_\pi$ and $\uparrow \phi_y$, so that the gap can be closed faster.

2.5 Summary

- When there is price rigidity, the competitiveness of the market has negative correlation with firms' markup

- Current aggregate price level is weighted by adjusters' and non-adjusters' average price

- Markup is inverse Marginal Cost (If prof called you at 3am, you had to answer this!!)

- Marginal Cost $\uparrow \Rightarrow$ Markup $\downarrow \Rightarrow$ Price $\uparrow \Rightarrow$ inflation $\uparrow$

- NKPC: $\pi_t = \beta E_t(\pi_{t+1}) + \lambda \hat{MC}_t = \beta E_t(\pi_{t+1}) + \kappa \tilde{y}_t$. Note the relationship between inflation, marginal cost, and output gap.

- DIS: $\tilde{y}_t = E_t(\tilde{y}_{t+1}) - \frac{1}{\sigma}(i_t - E_t(\pi_{t+1}) - r_t^n)$

- MP rule: $i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t$

- When there's an MP shock ($v_t \uparrow$), $i_t \uparrow$ due to liquidity effect, while in classical model, the shock causes $i_t \downarrow$
- Because of liquidity effect and price rigidity, $r_t = i_t \uparrow\uparrow - E_t(\pi_{t+1}) \downarrow \Rightarrow r_t \uparrow$. This shows the **Monetary non-neutrality**.

3 Monetary Policy Design

3.1 The Optimal Allocation

In a centralized economy, there's a social planner who can choose the optimal allocation, (π_t, y_t), that maximize the output and HH's utility.

1st problem of social planner: maximize aggregate consumption/output

$$\max \ C_t \equiv \left[\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

FOC implies $C_t(i) = C_t(j),\ N_t(i) = N_t(j),\ Y_t(i) = Y_t(j)\ \forall i,\ j \longrightarrow (*)$

2nd problem of social planner: maximize HH's utility

$$\max_{C_t, N_t} \ U(C_t, N_t)$$

$$\text{s.t.} \quad C_t(i) = A_t N_t(i)^{1-\alpha}$$

FOC implies $-\frac{U_{n,t}}{U_{c,t}} = MPN_t \longrightarrow (**)$

If $(*)$, $(**)$ are satisfied, then the economy achieves the optimal allocation.

3.1.1 1st Distortion

$(**)$ can be violated due to price rigidity/monopolistic competition:

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{MPN_t}{M} < MPN_t$$

Intuition 1: When prices are flexible, HH giving up 1 unit of consumption increase utility by MPN_t, which is the optimal wage offered by the firm. Now firms earn markup due because of the monopolistic competition nature of the market. Part of this markup is funded from labors' wage, so workers do not earn what they should have earn.

Intuition 2: Market power $\Rightarrow$ firms charge $M > 1$ $\Rightarrow$ over charge prices compared to perfect competition $\Rightarrow$ demand for goods $\downarrow \Rightarrow$ under production $\Rightarrow$ under hire $\Rightarrow$ unemployment

One can subsidize labor (τ), with $\tau = \frac{1}{\varepsilon}$, to fix the 1st distortion.

3.1.2 2nd Distortion

$(*)$ can be violated due to price rigidity/monopolistic competition:

$$
\begin{aligned}
P_t(i) &\neq P_t(j) \\
C_t(i) &\neq C_t(j) \\
N_t(i) &\neq N_t(j) \\
Y_t(i) &\neq Y_t(j)
\end{aligned}
$$

These distortions enter $d_t = log\left[\int_0^1 (\frac{P_t(i)}{P_t})^{-\frac{\varepsilon}{\varepsilon-1}}\right] di$. CB's objective is to fix these distortions.

Assumptions:

1. τ^* is implemented $\Rightarrow y_t^n = y_t^e$
2. No inherited relative price distortions, $P_{t-1}(i) = P_{t-1}(j) \quad \forall i, j$. i.e. no price distortion in the past.

One solution is to set $\phi_\pi \to \infty$, if firms change $P_t(i) \Rightarrow \pi_t \uparrow,\ i_t \to \infty \Rightarrow$ profit loss of all firms $\Rightarrow$ no one want to change the price $\Rightarrow \pi_t = 0 \Rightarrow \tilde{y}_t = 0 \Rightarrow y_t = y_t^n = y_t^e \ \because \tau^*$

Divine coincidence: $(\pi_t = 0,\ \tilde{y}_t = 0)$ is possible

Example 1: When CB sets $i_t = r_t^n$, the divine coincidence achieved!!
But the solution is not unique as $\kappa(\phi_\pi - 1) + (1-\beta)\phi_y = -\kappa < 0$!!

Example 2: $i_t = r_t^n + \phi_\pi \pi_t + \phi_y \tilde{y}_t$, with $\phi_\pi,\ \phi_y$ are sufficiently large
This satisfy $\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$

However, $(r_t^n,\ \tilde{y}_t)$ are not observable, one can use a simple rule:

$$i_t = f(\text{Observables})$$

$$\Leftrightarrow i_t = \phi_\pi \pi_t + \phi \hat{y}_t$$

3.2 Optimal MP in the presence of cost-push shocks

Recall, NKPC: $\pi_t = \beta E_t(\pi_{t+1}) + \kappa(y_t - y_t^n)$. y_t^n is a good indicator when we think about welfare, because $y_t^n = y_t^e$ (due to τ^*).

Note that:
y_t^n: output when price is flexible, monopolistic competition market can have this output level
y_t^e: output when there is no welfare loss, monopolistic competition market cannot achieve this level unless they have τ^*

However, there are reasons that $y_t^n \neq y_t^e$, ie. even if price is flexible in monopolistic market, there is still welfare loss because:

1. **no** τ^*: at a given output level, firms need more labor because of the absence of τ^*, HH are reluctant to work at the $MRS = MPN_t$ level.
2. desired price markup changes overtime, we need τ^* changing overtime
3. wage markup change overtime: firms need to give higher wage to workers (labor union).

Define welfare relevant output gap (efficient output gap)

$$x_t = y_t - y_t^e$$

Rewriting NKPC,

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa x_t + u_t$$
$$u_t = \kappa(y_t^e - y_t^n) \neq 0$$

- u_t is so called the cost-push/markup shocks
- divine coincidence ($\pi_t = 0, x_t = 0$) no longer possible due to u_t
- there is a tradeoff between (π_t, x_t) facing by CB
- CB now need to find out the optimal values of (π_t, x_t)

3.2.1 Welfare Loss Function

Welfare loss is given by

$$-E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{U_t - U}{U_c \cdot C} \right)$$
$$\approx \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) y_t^2 + \frac{\varepsilon}{\lambda} \pi_t^2 \right]$$

Take one period from the above infinite sum, we have period welfare loss:

$$\mathscr{L} = \frac{1}{2} \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \mathrm{var}(y_t) + \frac{\varepsilon}{\lambda} \mathrm{var}(\pi_t) \right]$$
$$\Leftrightarrow \mathscr{L} = \pi_t^2 + v \cdot \tilde{y}_t^2$$
$$\Leftrightarrow \mathscr{L} = \pi_t^2 + v \cdot x_t^2$$

- Loss $\uparrow$ if volatility of output gap $\uparrow$
- Loss $\uparrow$ if inflation is too volatile

3.2.2 CB's Problem

$$\min \quad E_0 \sum_{t=0}^{\infty} [\pi_t^2 + v \cdot x_t^2]$$
$$s.t. \quad \pi_t = \beta E_t(\pi_{t+1}) + \kappa x_t + u_t, \quad \forall t$$

3.2.2.1 Discretionary Policy

- Minimize welfare loss period by period, $\mathscr{L} = \pi_t^2 + v \cdot x_t^2$
- can only decide/influence on today's variables π_t, x_t, not future variables

$$\min \quad \mathscr{L} = \pi_t^2 + v \cdot x_t^2$$
$$\text{s.t.} \quad \pi_t = \kappa x_t + z_t$$
$$\text{where } z_t = u_t + \beta E_t(\pi_{t+1})$$

FOC yields:

$$x_t = -\frac{\kappa}{v}\pi_t$$

Interpretation: Given a cost-push shock (u_t), the first best allocation ($\pi_t = 0, x_t = 0$) is impossible. Given this tradeoff (π_t, x_t), the CB is willing to decrease x_t by $-\frac{\kappa}{v}$ for each percentage point increase in inflation

Discussion: v

1. $v = 0 \Rightarrow \mathscr{L} = \pi_t^2$
   the optimal MP is $\pi_t = 0 \ \ \forall \ t$
   according to NKPC, u can back x_t
2. $v = \infty \Rightarrow \mathscr{L} = \pi_t^2 + \infty \cdot x_t^2$
   the optimal MP is $x_t = 0 \ \ \forall \ t$
   by plugging $x_t = 0$ into NKPC, we can backup the implied π_t
3. In general, the smaller is v, then the stabilization of π_t is more important thant the stabilization of output gap x_t

Rewriting DIS:

$$x_t = E_t(x_{t+1}) - \frac{1}{\sigma}(r_t - r_t^e)$$
$$\text{where} \quad r_t^e = \rho + \sigma E_t(y_{t+1}^e - y_t^e)$$

Continue to solve for optimal MP

$$\begin{cases} x_t = -\frac{\kappa}{v}\pi_t \\ \pi_t = \kappa x_t + \beta E_t(\pi_{t+1}) + u_t \end{cases}$$

We will have:

$$\pi_t = \frac{v}{(1-\beta\rho_u)v + \kappa^2} \cdot u_t$$
$$x_t = -\frac{\kappa}{(1-\beta\rho_u)v + \kappa^2} \cdot u_t$$

Interpretation: when there is a cost-push shock ($u_t \uparrow$), $\pi_t \uparrow$ and $x_t \downarrow$, meaning that generating a recession is optimal. This illustrate the tradeoff.

3.2.2.2 Commitment Policy

$$\min_{\{x_t\}_{t=0}^{\infty},\{\pi_t\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + v \cdot x_t^2]$$
$$\text{s.t.} \quad \pi_t = \beta E_t(\pi_{t+1}) + \kappa x_t + u_t, \quad \forall t$$

By solving forward, we get

$$\pi_t = \kappa x_t + \kappa \sum_{j=0}^{\infty} \beta^j E_t(x_{t+j}) + \frac{1}{1-\beta\rho_u} \cdot u_t$$

Interpretation: CB can balance tradeoff between t and any $t+j$ period.

Commitment MP always dominates discretionary MP in terms of welfare because commitment MP is minimizing entire past and future of welfare loss.

3.3 Summary

- In a centralized economy, social planner can achieve optimal allocation (π_t, y_t) that maximize HH's utility and output level
- In decentralized/price-rigid economy, some actions are needed to fix 2 distortions due to price rigidity, in order to reach the divine coincidence
- In practice, there exists cost-push shock that prevent the economy to reach the divine coincidence
- The cost-push shocks bring tradeoffs between inflation and efficient output gap
- CB has to choose (π_t, x_t) such that the welfare loss is minimized
- CB can implement either discretionary MP or commitment MP to address cost-push shocks

4 Criticism and Defenses of the NK Model

Three equations of NK model:

$$\tilde{y} = E_t(\tilde{y}_{t+1}) - \frac{1}{\sigma}(i_t - E_t(\pi_{t+1}) - r_t^n)$$

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa \tilde{y}_t$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t$$

4.1 The Missing Disinflation

According to NKPC,

$y \downarrow \Rightarrow mc \downarrow$ (mc is a function of y) $\Rightarrow M \uparrow \Rightarrow p \downarrow$ (to maintain at the optimal value of M) $\Rightarrow \pi \downarrow$

However, after the 2008 financial crisis, there is no disinflation when an economy is in recession, from the data. The slope of the NKPC seems to be zero.

4.1.1 Explanation 1: The slope of NKPC is becoming flatter

Note that the slope of NKPC is given by

$$\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$$

- β: Time preference
- σ: Intertemporal elasticity of substitution
- φ: Labor supply elasticity

We do not have good reasons why the above parameters have changed over time. What we can possibly explain are:

- $1-\alpha$: Labor share, has been declining by 13% between 2000 and 2013 $\Rightarrow$ derease κ by around 15%
- ε: measures firms's profit $\left(\frac{1}{1-\varepsilon}\right)$, has risen from 6% to 9% after 2005 $\Rightarrow$ 20-30% reduction in $\varepsilon \Rightarrow$ 20-30% increase in κ
- θ: a measure of frequency of price change $\left(\frac{1}{1-\theta}\right)$, has been somewhat decreasing overtime $\Rightarrow \kappa$ falls by around 50%

4.1.2 Explanation 2: The right expected inflation

By running the following regression:

$$\pi_t = \beta_1 E^{MSC}(\pi_{t+1}) + \beta_2 E^{SPF}(\pi_{t+1}) + \kappa x_t + e_t$$

We can find that β_1 is significantly positive but not β_2. This indicates that NKPC is alive when we use consumer expectation rather than professionals' expectation.

Question: NKPC comes from price rigidity, which is a firms' problem, why consumers' expectations bother?
Possible answer: Consumers are closer to firms compared to professional bankers and investors. Those who are running firms are also consumers at the same time.

4.2 The intermediate NKPC is alive

Recall, the NKPC is derived in 2 steps:

1. Step 1:
$$\pi_t = \beta E_t(\pi_{t+1}) + \lambda \hat{mc}_t$$
Assumptions:
   - staggered price setting a la Calvo (a probability that firms can reset price)
   - optimal price setting by monopolistic competitive firms
   - constant frictionless desired markup μ

2. Step2:
$$\lambda \hat{mc}_t = \kappa \tilde{y}_t$$
$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa \tilde{y}_t$$
Assumptions (strong):
   - all output is consumed
   - perfect competition in labor market
     - HH believes that they can sell as much labor as they want

By using lagged economic indicators as IV (independent of the error term because of rational expectation), we can estimate the slope of the intermediate NKPC from the below regression specification.

$$\pi_t = \beta \pi_{t+1} + \lambda \hat{mc}_t - \underbrace{\beta(\pi_{t+1} - E_t(\pi_{t+1}))}_{e_{t+1}}$$

It turns out that when estimating the intermediate NKPC, λ is positive, which aligns with NKPC. However, the author of this paper (Gali and Gertler (1999)) is subject to weak instrument problem.

4.3 The professionals don't know ECON101

The inflations and output gaps observed are the equilibrium of AS and AD curve, which moves dynamically. In order to examine the slope of NKPC (AS), we need to fix AS and use the movement of AD to draw out the AS curve.

How can we achieve that? there are 2 equations for the problem

$$\text{MP: } \pi_t = -\frac{\lambda}{\kappa} x_t + \varepsilon_t^m$$

$$\text{NKPC: } \pi_t = \kappa x_t + u_t$$

We can obtain the following, which the solutions are a linear combination between supply shock and demand shock:

$$\pi_t = \theta_1 u_t + \theta_2 \varepsilon_t^m$$

$$x_t = \theta_3 u_t + \theta_4 \varepsilon_t^m$$

"To examine the slope of NKPC (AS), we need to fix AS and use the movement of AD to draw out the AS curve".
We can use MP shocks (demand shocks ε_t^m) as IV, then we can eliminate supply shock and obtain the NKPC.

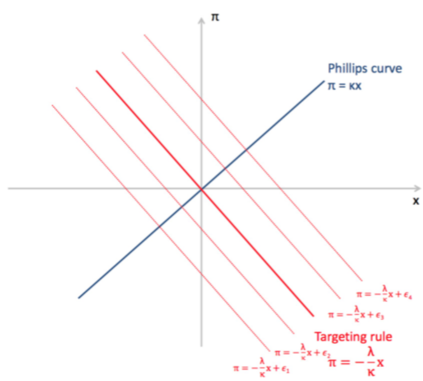


Figure 1: Drawing NKPC by moving AD

4.4 Summary

- The slope of NKPC is becoming flatter
- The right expectation (consumers' expectation) need to be used
- The intermediate NKPC is alive because the final NKPC needs strong assumptions
- What we observe is the results of AD AS, not solely NKPC.

5 AD & AS

Recall:

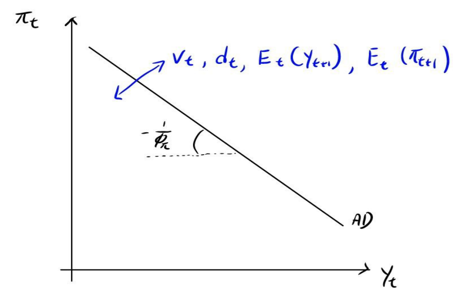
$$y_t = E_t(y_{t+1}) - (i_t + E_t(\pi_{t+1}) - \rho) + d_t$$

$$i_t = \rho + \phi_\pi \pi_t + v_t$$

Combining both equations, we get

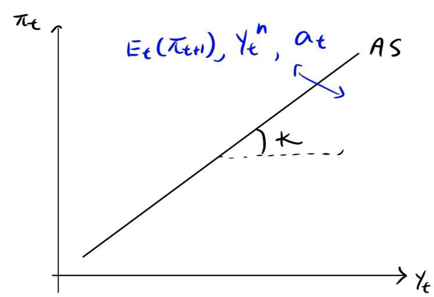
$$AD: \; y_t = E_t(y_{t+1}) + E_t(\pi_{t+1}) - \phi_\pi \pi_t - v_t + d_t$$

where d_t and v_t are demand shock and MP shock respectively.



Recall NKPC,

$$AS: \; \pi_t = \beta E_t(\pi_{t+1}) + \kappa(y_t - y_t^n) + u_t$$



5.1 Lesson 1: Demand Shock

There are 2 demand shocks component in AD, v_t and d_t. Let's talk about v_t here.

If $v_t < 0$ or a negative demand/MP shock, $i_t \downarrow$. The mechanism is

$$v_t < 0 \Rightarrow i_t \downarrow\downarrow \Rightarrow r_t \downarrow \Rightarrow AD \uparrow \Rightarrow y_t \uparrow$$

How r_t affects y_t is through mechanism below:

$$r_t \downarrow \Rightarrow c_t \uparrow \Rightarrow mc_t \uparrow \Rightarrow \mu_t \downarrow \Rightarrow p_t \uparrow \Rightarrow \pi_t \uparrow$$

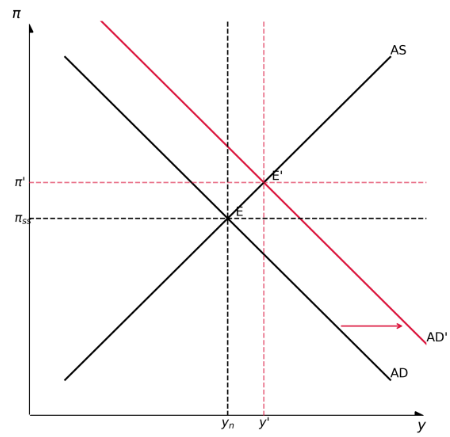


Figure 2: AD-AS Curve given $v_t < 0$

Figure 2: AD-AS Curve given $v_t < 0$

The above mechanism causes AD to shift right.

In classical model, $v_t < 0 \Rightarrow \bar{y}_t$ because prices are fully flexible, $E_t(\pi_{t+1})$ will fully offset changes in i_t s.t. $\bar{r}_t$.

5.2 Lesson 2: Productivity/Supply Shock

$$a_t \uparrow \Rightarrow mc_t \downarrow \Rightarrow \pi_t \downarrow \Rightarrow i_t \downarrow\downarrow \Rightarrow r_t \downarrow \Rightarrow y_t \uparrow$$

Note the roles of firms, CB, and consumers in above mechanism.

From the graph below, $\tilde{y}_1 = y_1 - y_n < 0$. This is because in response to $a_t \uparrow$, p_t needs to go down but due to rigidity, $p_t \downarrow$ insufficiently. The output should have been larger and the disinflation should have been lower.

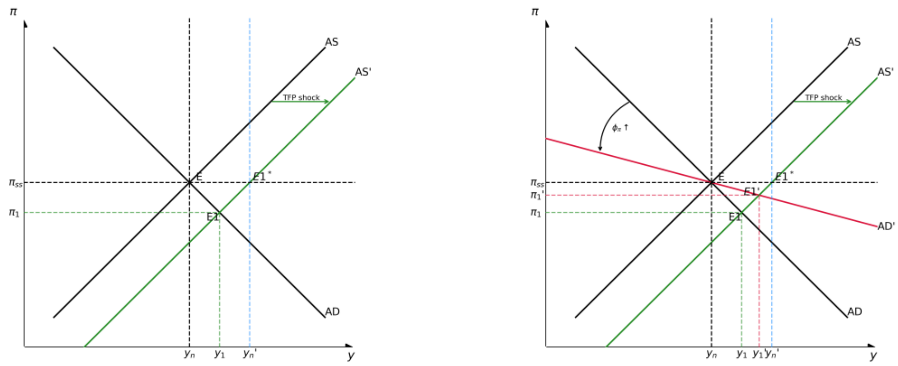
Is there a way to make the output level be at y_n'?

Yes! Since v_t is exogenous, we cannot shift the AD curve, but we can rotate it. By setting a larger ϕ_π, the slope of AD is becoming flatter. Therefore, y_1 will become larger (y_1') and disinflation become lower (π_1').

If $\phi_\pi \to \infty$, then the slope of AD will become completely flat.

Since in AD, we have

$$y_t = E_t(y_{t+1}) - \phi_\pi \pi_t + E_t(\pi_{t+1})$$



(a) TFP shock (b) Policy Intervention

Figure 3: AD-AS Curve given $y^n \uparrow$

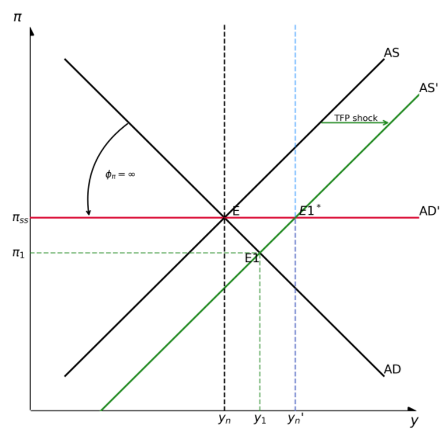


Figure 4: The Extreme Policy

$$\phi_\pi \uparrow\uparrow \Rightarrow i_t \downarrow\downarrow\downarrow \Rightarrow y_t \uparrow\uparrow\uparrow \Rightarrow \tilde{y}_t \downarrow\downarrow$$

5.3 Lesson 3: Cost-push Shock

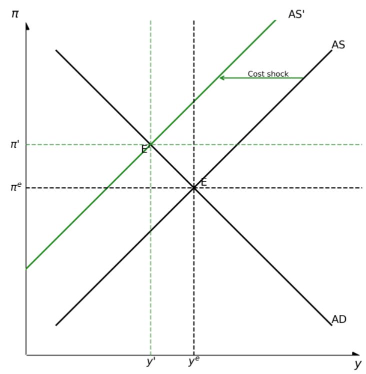
Recall:

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa(y_t - y_t^e) + u_t$$

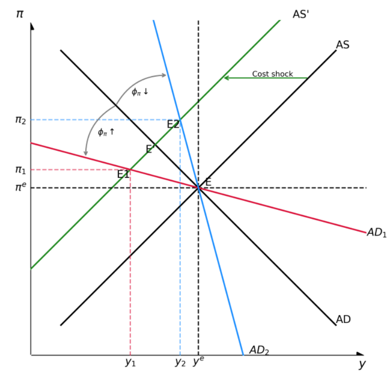
Where u_t is the cost-push shock. If there is a positive cost-push shock,

$$u_t \uparrow \Rightarrow \pi_t \uparrow \Rightarrow i_t \uparrow\uparrow \Rightarrow r_t \uparrow \Rightarrow c_t \downarrow \Rightarrow y_t \downarrow \text{ (recession)}$$

Alternative Policy scenarios:



(a) Cost-push shock



(b) TFP shock

Figure 5: AD-AS Curve given $y^n \uparrow$

- $\phi_\pi \uparrow \Rightarrow$ flatter AD $\Rightarrow y_1 < y'$(worse recession, bad), lower inflation (good)
- $\phi_\pi \downarrow \Rightarrow$ steeper AD $\Rightarrow y_2 > y'$(smaller recession, good), higher inflation (bad)

Ultimately, the the region between $\phi_\pi \to \infty$ and $\phi_\pi = 0$ can achieved by FOMC depending on $\mathscr{L}$

Recall that CB's problem is

$$\min \ \mathscr{L} = \pi_t^2 + \lambda \tilde{y}_t^2$$

1. Case 1: $\lambda = 0 \Rightarrow \mathscr{L} = \pi_t^2 \Rightarrow$ FOMC pick $\pi_t = 0$, i.e. $\phi_\pi = \infty$
2. Case 2: $\lambda = \infty \Rightarrow \mathscr{L} = \pi_t^2 + \infty \cdot \tilde{y}_t^2 \Rightarrow \mathscr{L} = \tilde{y}_t^2 \Rightarrow$ FOMC pick $\tilde{y}_t = 0$, i.e. strict output gap target

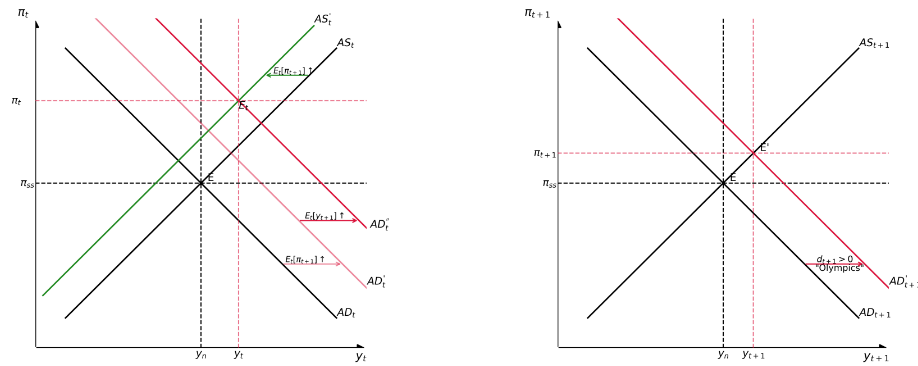
5.4 Lesson 4: The Role of Expectation

$$AD: \ y_t = E_t(y_{t+1}) + E_t(\pi_{t+1}) - \phi_\pi \pi_t - v_t + d_t$$
$$AS: \ \pi_t = \beta E_t(\pi_{t+1}) + \kappa(y_t - y_t^n) + u_t$$

Suppose that there's an announcement that the Olympic games will be held in this economy at $t+1$, then

$$AD_{t+1} \Rightarrow y_{t+1} \uparrow, \ \pi_{t+1} \uparrow \Rightarrow E_t(y_{t+1}) \uparrow, \ E_t(\pi_{t+1}) \uparrow$$

$$AD_t \uparrow, \ AS_t \uparrow \Rightarrow y_t \uparrow, \ \pi_t \uparrow \ \text{(demand shock)}$$



(a) Equilibrium at t

(b) Equilibrium at $t+1$

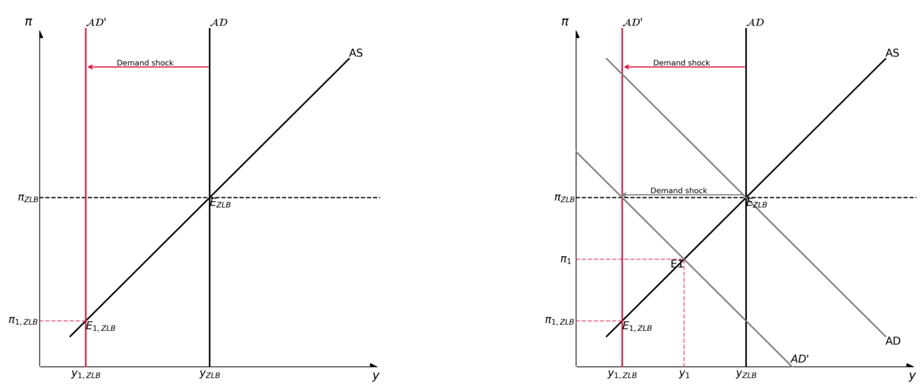
Figure 6: The Effect of Expectation

5.5 Lesson 5: Zero Lower Bound

Zero lower bound means CB is setting the nominal interest rate to 0, i.e. $i_t = 0$. Then, CB is not reacting to any inflation in the economy. Hence, the **AD is a vertical line**. AD becomes:

$$y_t = E_t(y_{t+1}) + E_t(\pi_{t+1}) + \rho + d_t$$

Then if $d_t < 0$ (negative demand shock), $y_{1,ZLB} < y_1$ (bigger recession than normal time) and $\pi_{1,ZLB} < \pi_1$ (bigger deflation than normal time).



(a) Zero Lower Bound

(b) Comparing to normal case

Figure 7: Demand Shock under ZLB

Intuition: In normal time, the conventional MP (Taylor rule) is active, it marks as counter-cyclical policy.

To mitigate the risk of hitting ZLB, the economy should raise i^{ss} because

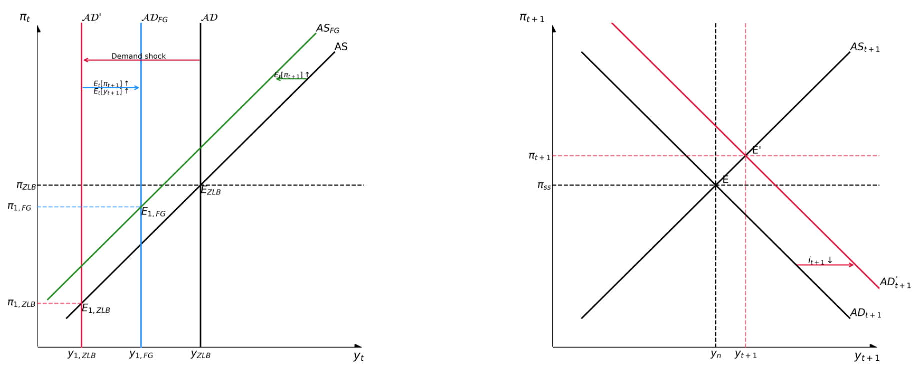
$$i^{ss} = r^{ss} + \pi^{ss}$$

where CB can only raise π^{ss} as no one knows about the real rate.

5.6 Lesson 6: Forward Guidance (Commitment MP)

Consider when there is ZLB in the economy and there is a negative demand shock $d_t < 0$, FG can mitigate the recession today.

$AD_{t+1} \uparrow \Rightarrow y_{t+1} \uparrow$, $\pi_{t+1} \uparrow \Rightarrow E_t(y_{t+1}) \uparrow$, $E_t(\pi_{t+1}) \uparrow \Rightarrow AD_t$ shifts right, AS_t shifts up



(a) Zero Lower Bound

(b) Forward guidance

Figure 8: Forward Guidance under ZLB

Hence, $y_{1,FG} > y_{1,ZLB}$ (smaller recession) and $\pi_{1,FG} > \pi_{1,ZLB}$ (smaller deflation).

For forward guidance to work, the CB need to have credibility.