Fixed Income Review

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January 1, 2019

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1 Forward Measure

1.1 Intuitive Definition

To start with, recall in the HJM framework we have the discount value of a bond price is a martingale in risk neutral measure. This is true because any tradeable assets must be a martingale in risk neutral measure otherwise you have arbitrage oppurtunity

$$d(D(t)B(t,T)) = -\sigma^* D(t)B(t,T)d\widetilde{W}(t) \tag{1}$$

Then recall the definition of the fundamental theorem that there exists a measure between two tradeable assets that those two can form a martingale.

If we further assume the measure is at time T, which is the same expiration time of the zero-coupon bonds, then we have:

$$\frac{V(t)}{B(0,T)} = E^T \frac{V(T)}{B(T,T)} = E^T V(T) \tag{2}$$

$$V(t) = B(0,T)E^T V(T) \tag{3}$$

The idea behind this is that, we cannot find a way using bond to hedge our payoff then get money at any condition. Recall the similar risk neutral pricing formula for any contract. But first, think of risk neutral measure as a measure that you can find a martingale such that it prevents you from being using bank account to make arbitrage

$$V(t) = E^B(V(T)D(T)) \tag{4}$$

Since T, B measures are both defined on same event space. The we can naturally get the transformation of the probability on same event to be

$$\widetilde{P}^T(A) = \frac{1}{B(0,T)} \int_A D(T)d\widetilde{P}^B(A) \tag{5}$$

1.2 Link To Risk Neutral Measure

1.2.1 Non-Model Based

We often hear someone say *forward price is martingale under T measure*. Firstly, without proof, this statement can be think of as:

- First forward price is the number of bond you hold to hedge the price at time T. Aka the ratio between the expected payoff of contract vs the bond
- Under T measure, this value should equal to $\frac{V(t)}{B(t,T)}$ at time t
- In fact, the ratio value mentioned in first item should equal to $\frac{V(0)}{B(0,T)}$ at time 0

- According to the Tower Theorem, this second and third items are chained up to force the evolution of f from time 0 to time t, the f is a martingale

More intuitively, you can see using T measure just include/offset the factor of $B(0,T)$ and $D(T)$ in the risk neutral measure.

$$f(0) = \frac{E^B V(T)D(T)}{E^B D(T)} = B(0,T)E^B V(T)D(T) = E^T f(T) \tag{6}$$

1.2.2 Model Based

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