Analysis of the White Elephant Gift Exchange Problem

Junye Li & Matt Kretchmar
Department of Mathematics and Computer Science
Denison University
Granville, OH 43023
Email: li_j9@denison.edu, kretchmar@denison.edu

***Abstract*—something abstract goes here**

I. Definition of the Game

The *White Elephant Gift Exchange* (WEGE) is a western party game played during the traditional Christmas season. A number of people come to a party, each bearing a wrapped but unlabelled gift. The gifts are all placed under a tree and the participants all draw random numbers to determine the order of play. The game starts by Player 1 selecting a random gift from under the tree[1], opening it, and allowing the party members to view the gift. During the second round, Player 2 has a choice: they may either steal Player 1's gift or they may select another random wrapped gift from under the tree, open and show it. If Player 1's gift is stolen, then they must now select and unwrap another random gift. Play continues in this way where, during each turn, a player may steal an opened gift from someone else or may open a random wrapped gift. A round ends when someone selects and opens a wrapped gift from under the tree. The game ends when all gifts have been finally opened. To make sure there are no infinite gift-stealing cycles, a gift may only be stolen once per round and then it remains frozen until the next round.

In this paper we make a number of modifications and simplifications to the standard game. We assume the game has n participants with n wrapped gifts. We establish the *value* of the gifts to be 1, 2, 3, ..., n. Note that in the original game, the values are opinions independently determined by the participants (two people may value those "slippers" quite differently depending on how much they want slippers). The simplified game also differs in that the players know *a priori* the value of the gifts but still do not know which wrapped gift has which value until it is opened during the game. The simplified version does not have the people bring the gifts, but instead the gifts are all wrapped by some third-party.

1) n gifts are wrapped and placed under the tree.
2) The gifts have values $1, 2, ..., n$ to all players.
3) The gifts are identically wrapped so that no player can determine the value before opening a gift.
4) n people randomly select numbers $1...n$ to determine order of play.
5) The game features n rounds of play.
6) Round k begins with Player k.
   a) Person k may either steal another already-opened gift from someone else (Players $1...k-1$ all have gifts now) or they may open a random wrapped gift.
   b) If, during a round, Person j has their gift stolen, they may either steal someone else's opened gift or opt to open a new random gift.
   c) During a round, a gift cannot be stolen more than once.
   d) A round ends where someone chooses to open a wrapped gift.
7) The game ends after n rounds when all the gifts have been opened and all players have a gift.

The goal of this paper is to determine optimal play for the game. During each round, participants have choices about which gifts to steal or when to opt instead to open a randomly selected gift. Since opening a gift is a random process, optimal play will involve stochastic choices and expected values. The goal, of course, is for each person to maximize their expected present value at the end of the game. We seek to determine *strategies* for each player about how to maximize the expected value of their gift. Given optimal play, we also seek to determine the *expected value* for each person at the end of the game.

II. State and Action Spaces

The WEGE game progresses through a series of states and actions. A state of the game is a (partial) assignment of gifts to people. We denote a state of the n-player game with a vector S:

$$S = (v_1, v_2, v_3, ..., v_n)$$

where v_i is the value of the gift held by person i at some point in the game. If a player does not hold a gift yet, then the value $v = 0$ is used. For example, the state

$$S = (3, 1, 4, 0, 0)$$

is the state of a five-player game at the end of Round 3. The first three players all have gifts (Player 1 has gift value

[1] It is considered poor form to select your own gift. Presumably you can identify your own wrapped gift.

3, Player 2 has gift 1, and Player 3 has gift value 4). The last two players do not yet have gifts. Note that by looking for the location of the zeros, we can determine from the state we can determine the current round.

We must also keep track of which gifts have been stolen so far during a round. We augment the state vector with an asterisk (v_i^*) when Player i's gift has already been stolen this round. It is now "frozen" and cannot be stolen again to the next round. Thus the example:

$$S = (3, 0, 4^*, 0, 0)$$

shows a state where Player 1 has gift value 3 (which has not yet been stolen), Player 3 has gift value 4 (which has been stolen and thus may not be stolen again) and Player 2 has no gift because their gift has just been stolen (in this case, by Player 3). So it is Player 2's turn and they may either steal the value 3 gift from Player 1 or they may select a wrapped gift (gift values 1, 2 and 5 still remain wrapped). Player 2 may not steal back gift value 4 this round because this gift has already been stolen once in this round; they may, however, steal it back during a subsequent round.

Acknowledgments

Thank people here.

References

[1] Aristotle, *Nicomachean Ethics*. New York, NY: Macmillan Publishing Company, 1962.