

SDSC3001 Tutorial 2

Graph search

2024.09.19

Content

- 1. Examples of Graph Data, And their applications.
- 2. Some basic concepts
- 3. How to describe a Graph in Python?
- 4. Breadth-first/ Depth-first Search
- 5. Game: Painting a Picture.

Graph Data

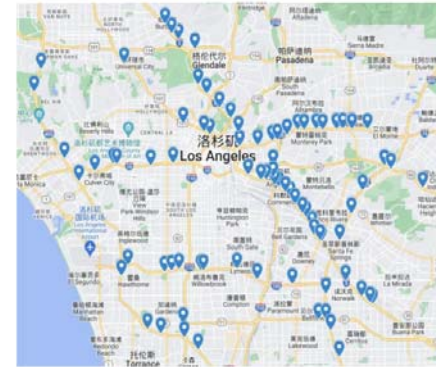
Graph: Nodes and edges



Social network

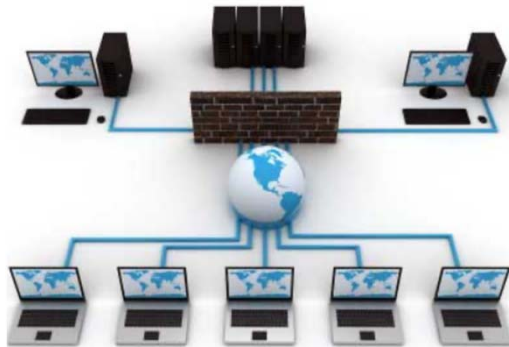


(a)

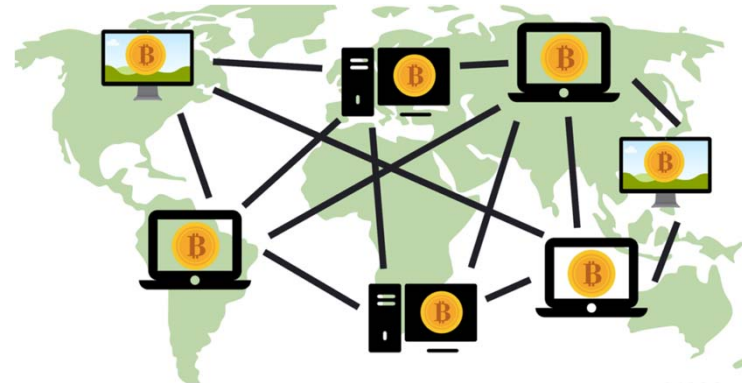


(b)

Traffic network



Computer network

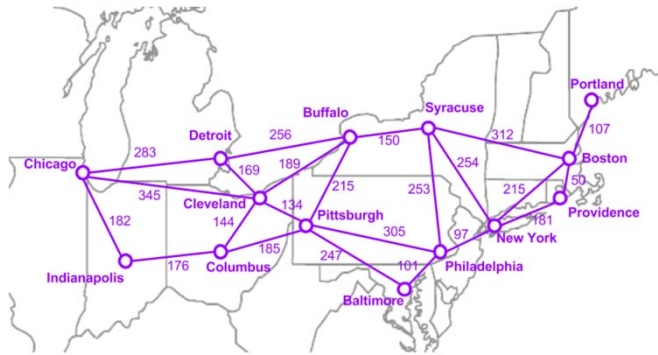


Bitcoin network

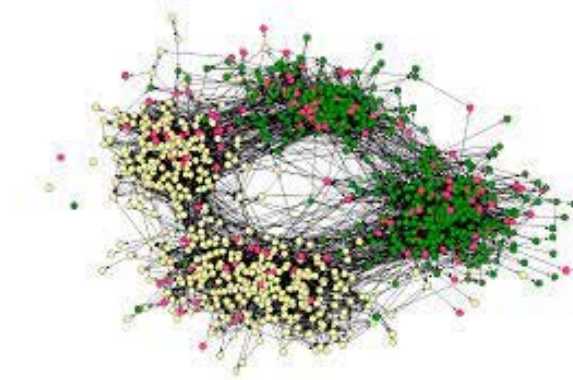
Graph Data Applications

Problems on Graph

- **Graph Search:** Given a source node and a destination node, find a path between them.
- **Dense Subgraph Mining:** Find dense subgraph in a graph.
- **Graph Clustering:** Find similar nodes in a group.
- **Graph Representation Learning:** Learn representations for node/edge/subgraph.



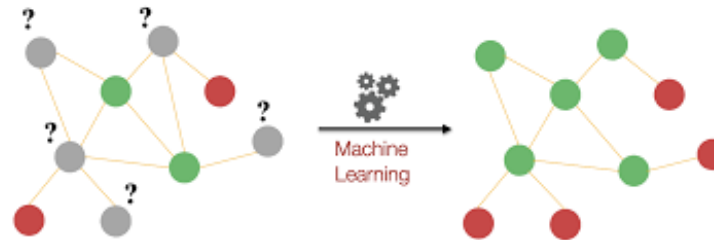
(a)



(b)



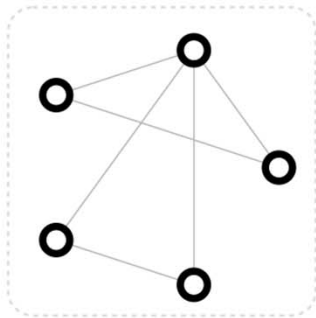
(c)



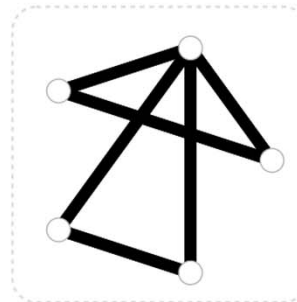
(d)

Some concepts in Graph

Definition: A graph represents the relations (*edges*) between a collection of entities (*nodes*).



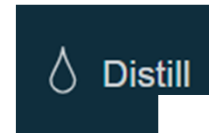
- V** Vertex (or node) attributes
e.g., node identity, number of neighbors
- E** Edge (or link) attributes and directions
e.g., edge identity, edge weight
- U** Global (or master node) attributes
e.g., number of nodes, longest path



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Same meaning:

- Vertex, Node, Entity
- Edge, Link



A Gentle Introduction to Graph Neural Networks

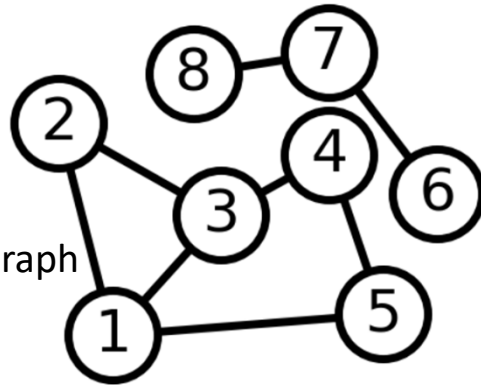
Neural networks have been adapted to leverage the structure and properties of graphs. We explore the components needed for building a graph neural network - and motivate the design choices behind them.

<https://distill.pub/2021/gnn-intro/>

How to define a Graph in practice?

1. Array (adjacency matrix)

Most of the elements are zero
Sparse matrix



Undirected Graph

symmetric matrix

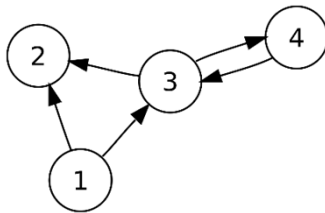
$$A_{8,8} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{matrix} & \left\{ \begin{array}{cccccccc} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right\} \end{matrix}$$

8 × 8 matrix

$a_{ij} = 1$, if node i, j are connected

$a_{ij} = 0$, otherwise

Directed Graph



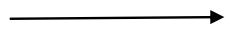
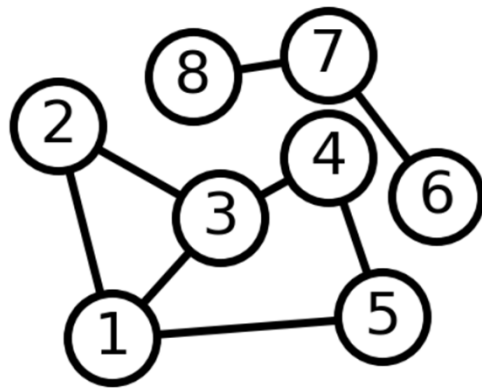
Q: What is $a_{ii} = 1$?

In [graph theory](#), a **loop** (also called a **self-loop** or a *buckle*) is an [edge](#) that connects a [vertex](#) to itself.



Represent Graph in Python

2. Neighbor Dictionary (adjacency table)

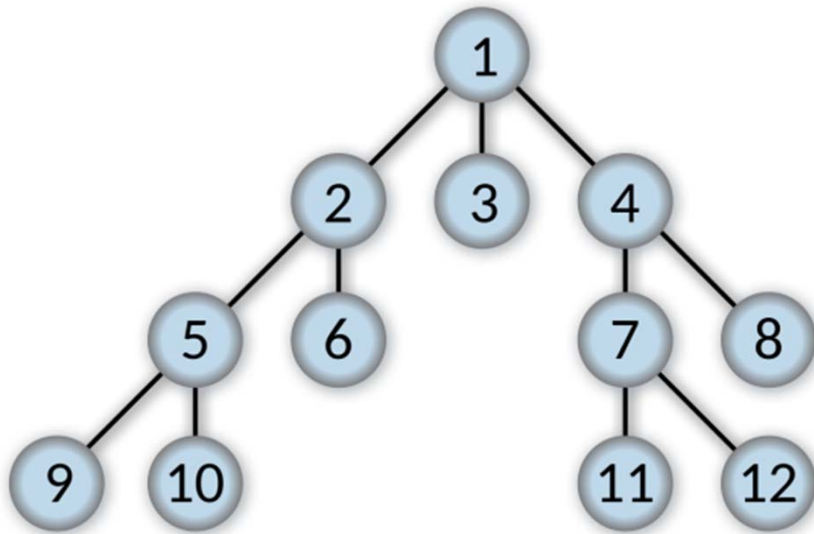


Graph=

```
{1: [2, 3, 5],  
 2: [1, 3],  
 3: [1, 4],  
 4: [3, 5],  
 5: [1, 4],  
 6: [7],  
 7: [6, 8],  
 8: [7]}
```

BFS: Breadth-First Search (See Jupyter)

Running example



Initial	
Waiting List	[1]
Visited List	[]

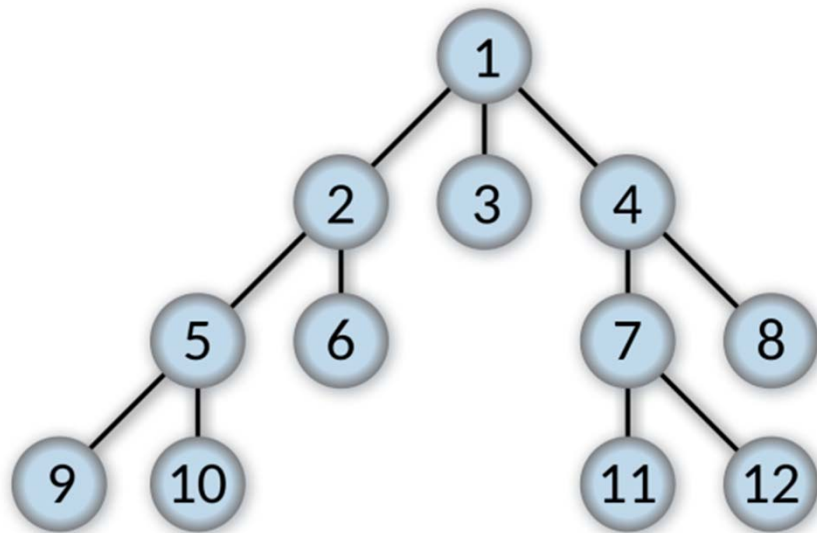
Deal with 1	
Waiting List	[2 3 4]
Visited List	[1]

Deal with 2	
Waiting List	[3 4 5 6]
Visited List	[1 2]

Deal with 3	
Waiting List	[4 5 6]
Visited List	[1 2 3]

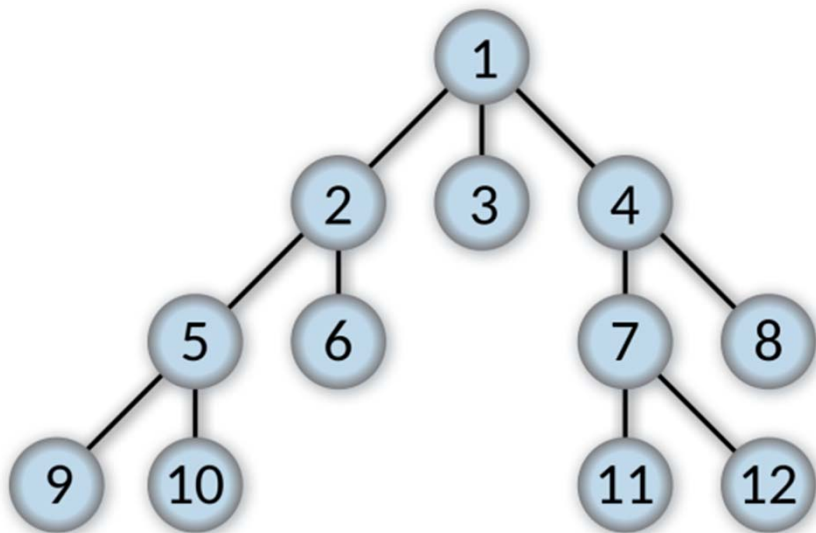
BFS: Breadth-First Search

Running example



BFS: Breadth-First Search

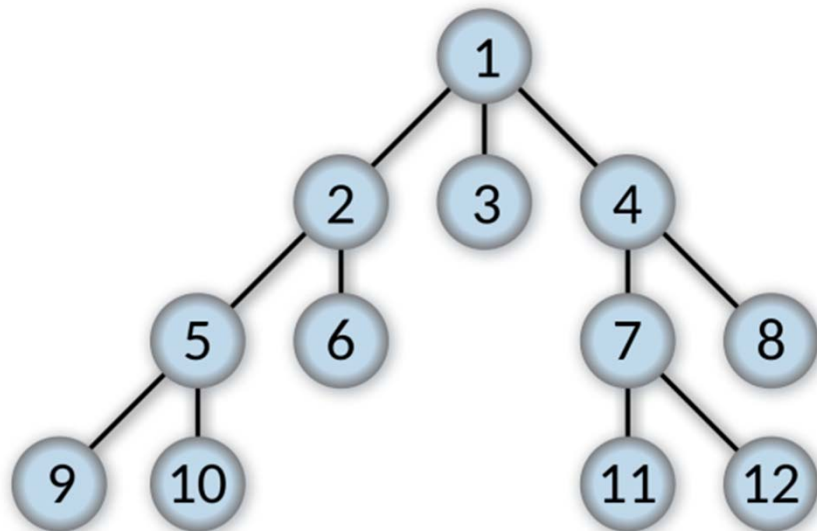
Running example



BFS: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]

DFS: Depth-First Search

Running example



Initial	
Waiting List	[1]
Visited List	[]

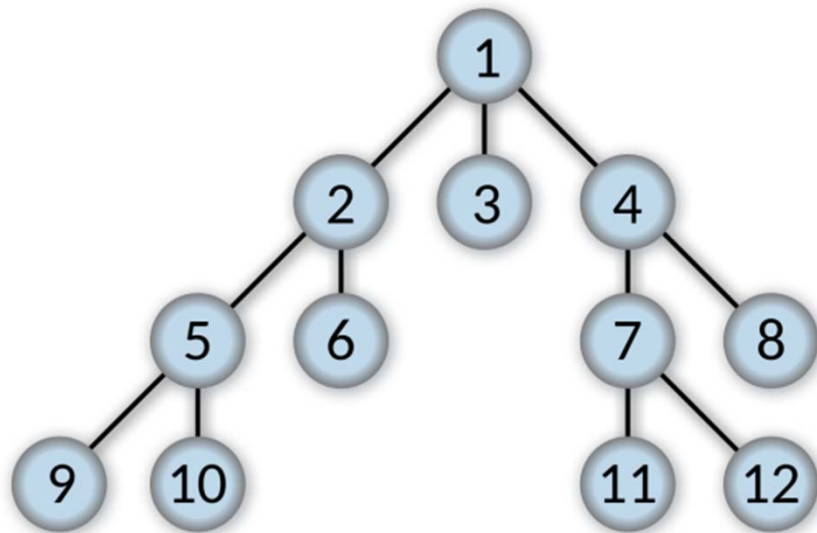
Deal with 1	
Waiting List	[2 3 4]
Visited List	[1]

Deal with 2	
Waiting List	[2 3 7 8]
Visited List	[1 4]

Deal with 3	
Waiting List	[2 3 7]
Visited List	[1 4 8]

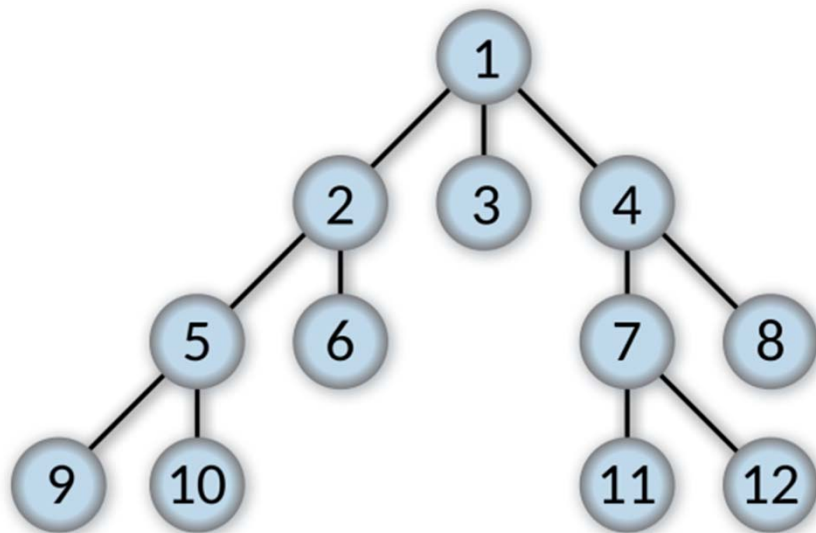
DFS: Depth-First Search

Running example



DFS: Depth-First Search

Running example



DFS: [1, 4, 8, 7, 12, 11, 3, 2, 6, 5, 10, 9]

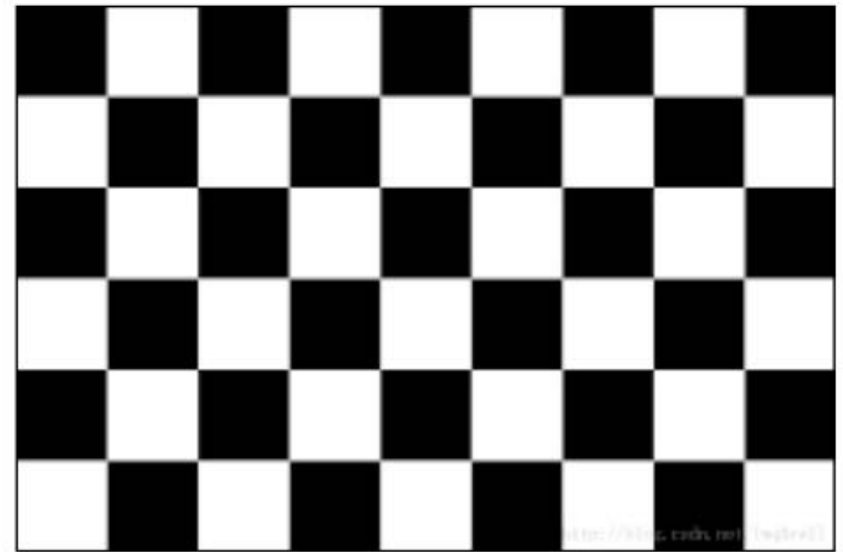
Now, let's do some arts!



Draw a picture by graph search

Basic logic

- A pixel is a node.
- The neighbors of a pixel is the node near it.
- We can color a node and then spread the color to its neighbor nodes.
- It just like a flood!
- <https://observablehq.com/@mbostock/randomized-flood-fill>



Thank you!