Problem 1 - Pricing European option if Stock price follows a normal distribution

Suppose the stock price S obeys a normal (instead of a lognormal) distribution:

$$S_T \sim N(S_0 e^{rT}, \sigma\sqrt{T})$$

We'd like to calculate the price of an option which pays $\max(S_T - K, 0)$ at expiry T.

The way to do it is to calculate the following expectation value:

$$C = e^{-rT} E^Q[\max(S-K,0)] = e^{-rT} \int_{-\infty}^{\infty} \max(S_0 e^{rT} + \sigma\sqrt{T}x - K, 0) \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = e^{-rT} \int_{d}^{\infty} (S_0 e^{rT} + \sigma\sqrt{T}x - K) \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

Where $d = \dfrac{K - S_0 e^{rT}}{\sigma\sqrt{T}}$.

Your task is to find the option price by solving the above integral.

Hint: you can express your final answer with the following function: $N(z) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{\infty}^{z} e^{\frac{-x^2}{2}} dx$

Notice that $N(-z) = 1 - N(z) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{z}^{\infty} e^{\frac{-x^2}{2}} dx$

Problem 2 Monte Carlo Simulation

Use Monte Carlo Simulation to find the expected number of times for flipping a coin (which contains 2 faces: head and tail) until the first time you get two consecutive heads.

Hint: represent the sequence of coin tosses as a series of heads (H) and tails (T). Every time simulate a random variable X which is uniformly distributed between 0 and 1 (via np.random.rand() as done in the class), and take the experiment result as 'H' if X>0.5 and 'T' otherwise (so with probability 1/2 you get either H or T each time). Repeat the experiment many (e.g., 10000) times and count the average number of times it takes to get 2 consecutive heads (HH).