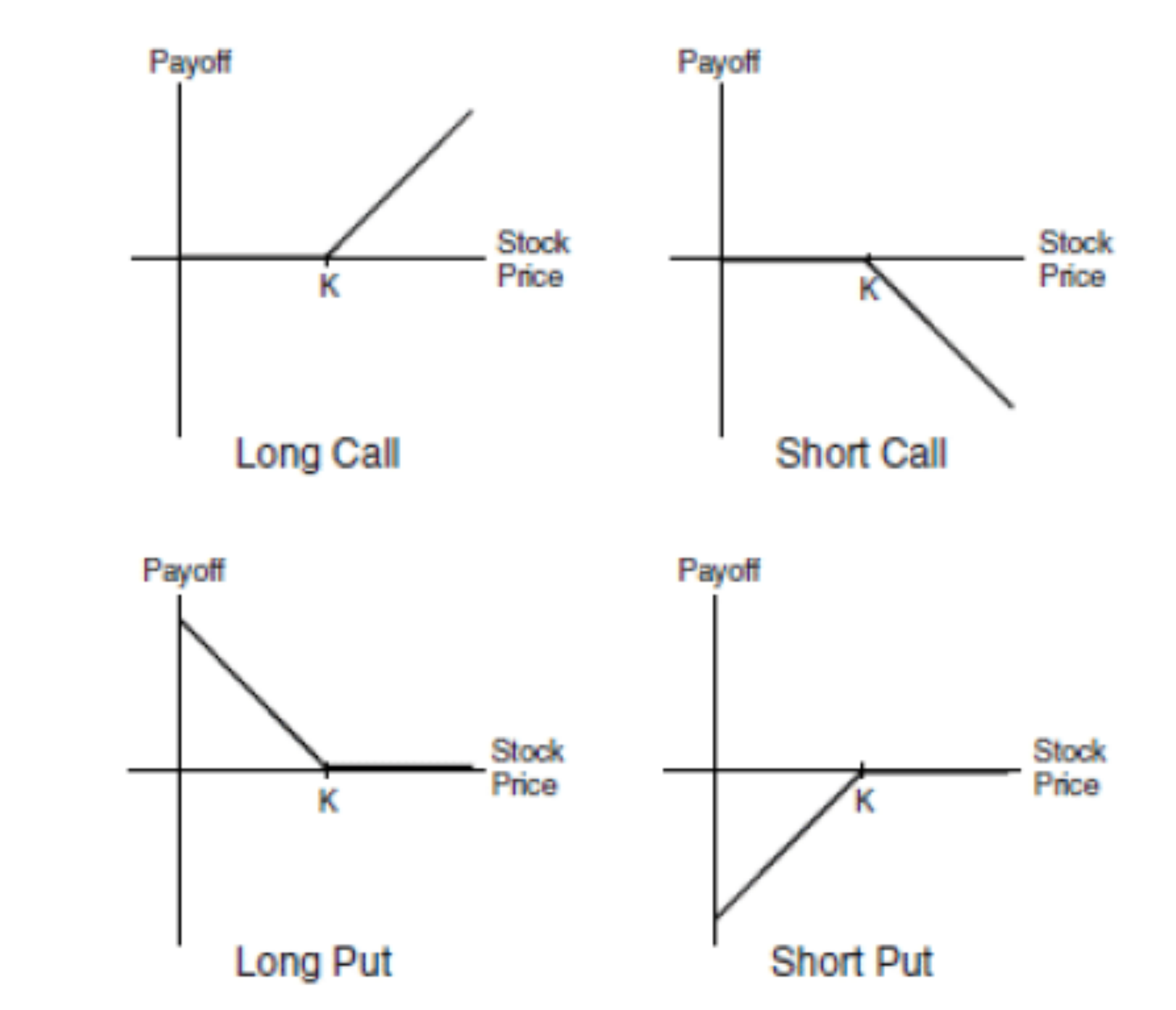
Introduction to Option Pricing

Steven Li

WHAT IS AN OPTION

- A FINANCIAL CONTRACT THAT ALLOWS YOU TO BUY/SELL AN UNDERLYING ASSET AT A FIXED PRICE (CALLED THE STRIKE PRICE)
  - Right to buy - Call
  - Right to sell - Put
- Based on the different type of underlying assets option can be classified into
  - Stock option
  - FX option
  - Interest Rate Option (Cap, Floor, Swaption)
  - Bond Option
  - CDS Option
- Based on when the option can be exercised:
  - European option - Plain Vanilla
  - American/Bermudan Option
  - Asian Option
  - Barrier Option (Exotic)

Option Pay Off (European)



Factors affecting Option Price

- Underlying Asset Price
- Strike Price
- Risk-Free Interest Rate
- Time to Maturity
- Dividend (if any)
- Volatility - the only unobservable quantity

Factor	**Call option price**	**Put option price**
Stock price	+	-
Execution price	-	+
Time to maturity	+	+
Volatility	+	+
Dividends	-	+
Interest rates	+	-

Option Pricing Method

- Black-Scholes Formula For European Options
- Assuming asset price is S(t), under the appropriate pricing measure (which we call the 'risk neutral measure' (every riskless payoff is discounted at risk free interest rate of r)

$$CallPrice(0) = e^{-rT}E^{RN}[(S_T - K)^+]$$

$$PutPrice(0) = e^{-rT}E^{RN}[(K - S_T)^+]$$

Normally We assume that S(T) follows LogNormal Distribution under risk neutral measure, that is

$\ln S(T) \sim N\left[\ln S(0) + (r - \frac{1}{2}\sigma^2)T, \sigma^2 T\right]$, in which case

$$CallPrice(0) = S_0 N(d_1) - Ke^{-rT}N(d_2)\ , \quad PutPrice(0) = Ke^{-rT}N(-d_2) - S_0 N(-d_1)$$

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, \qquad d_2 = d_1 - \sigma\sqrt{T}$$

σ is the (implied) volatility of the option. It can be constant, time dependent or even stochastic.

Put-Call Parity

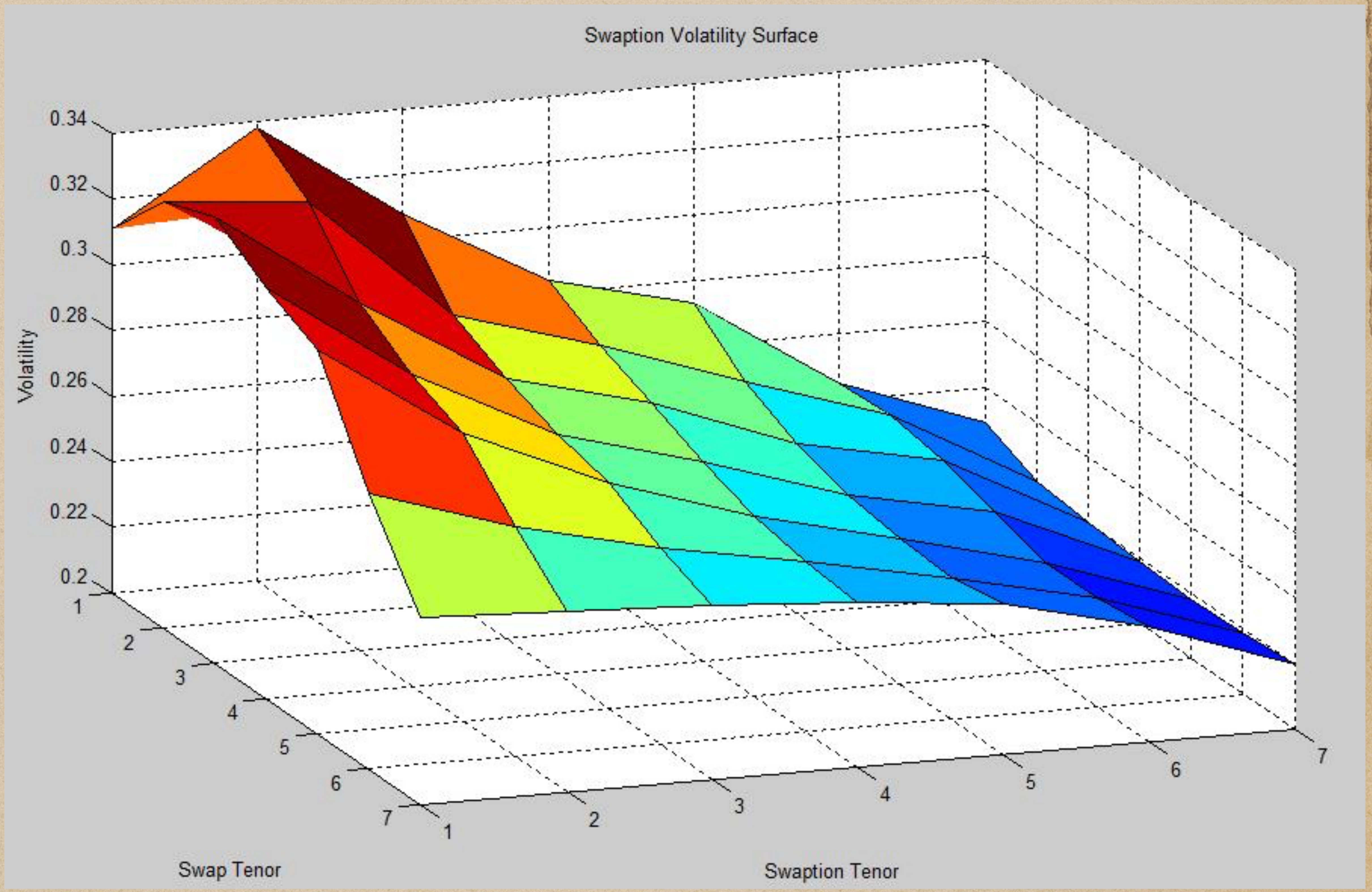
$$CallPrice(t) - PutPrice(t) = S_t - Ke^{-r(T-t)}$$

This is a general relationship which is model agnostic.

Exercise: Derive the option pricing formula if the underlying follows a normal instead of lognormal distribution

Implied Volatility and Option price

- Option volatility is not a constant but can vary according to strike price as well as underlying maturity
- Starting from vanilla option prices we can back up the implied volatility corresponding to different strike prices and expiries, this leads to the implied volatility surface.



Pricing American Option

- Closed form formula does not exist, need to resort to numerical methods
- Common methods
  - Tree Method (binomial/trinomial method)
  - PDE (Explicit, Implicit, Crank-Nicolsen)
  - American Monte-Carlo
  - Neural network/Deep learning