

American Monte Carlo

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MAIN POINTS

- What's the problem?
- Conditional expectation by regression
- An example

WHAT'S THE PROBLEM?

- It's difficult to price an option with early exercise using Monte Carlo (Why?)
- But some options require Monte Carlo to price
 - Path-dependent options
 - Options on baskets
 - Options priced using models with many factors

WHAT'S THE PROBLEM?

- Options with early exercise are called American if you can exercise at any time, or Bermudan if you can exercise only at certain times
- Hence a Monte Carlo scheme that allows you to price options with early exercise is called American Monte Carlo, or AMC for short

WHAT'S THE PROBLEM?

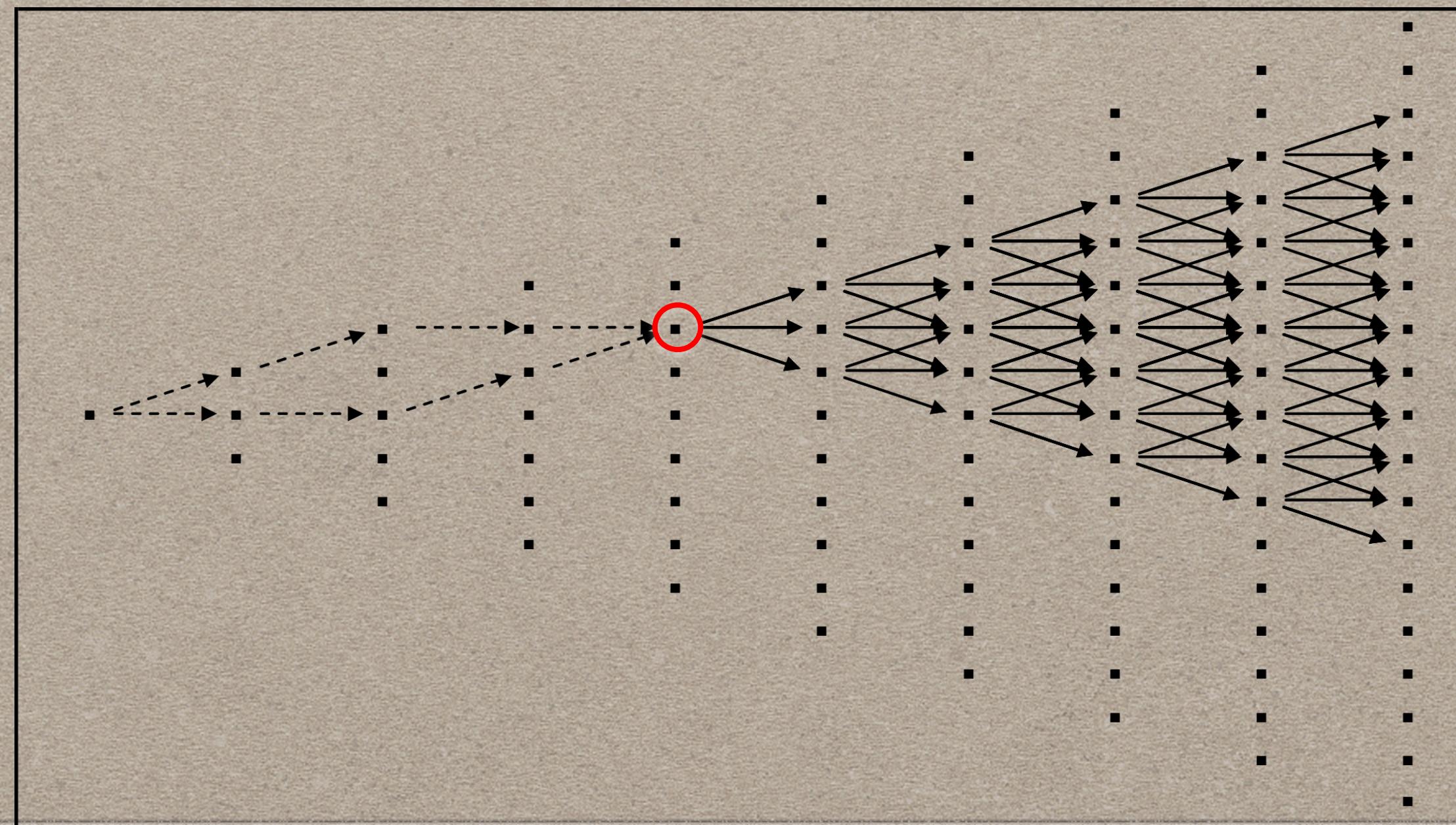
- But Monte Carlo doesn't use backward induction
- So the problem is: How can we compute the continuation value?

WHAT'S THE PROBLEM?

- The continuation value is the value of the option if we continue to hold it
- As such it is the risk-neutral expectation of the payoff of the option
- So we need to be able to calculate conditional expectations in Monte Carlo

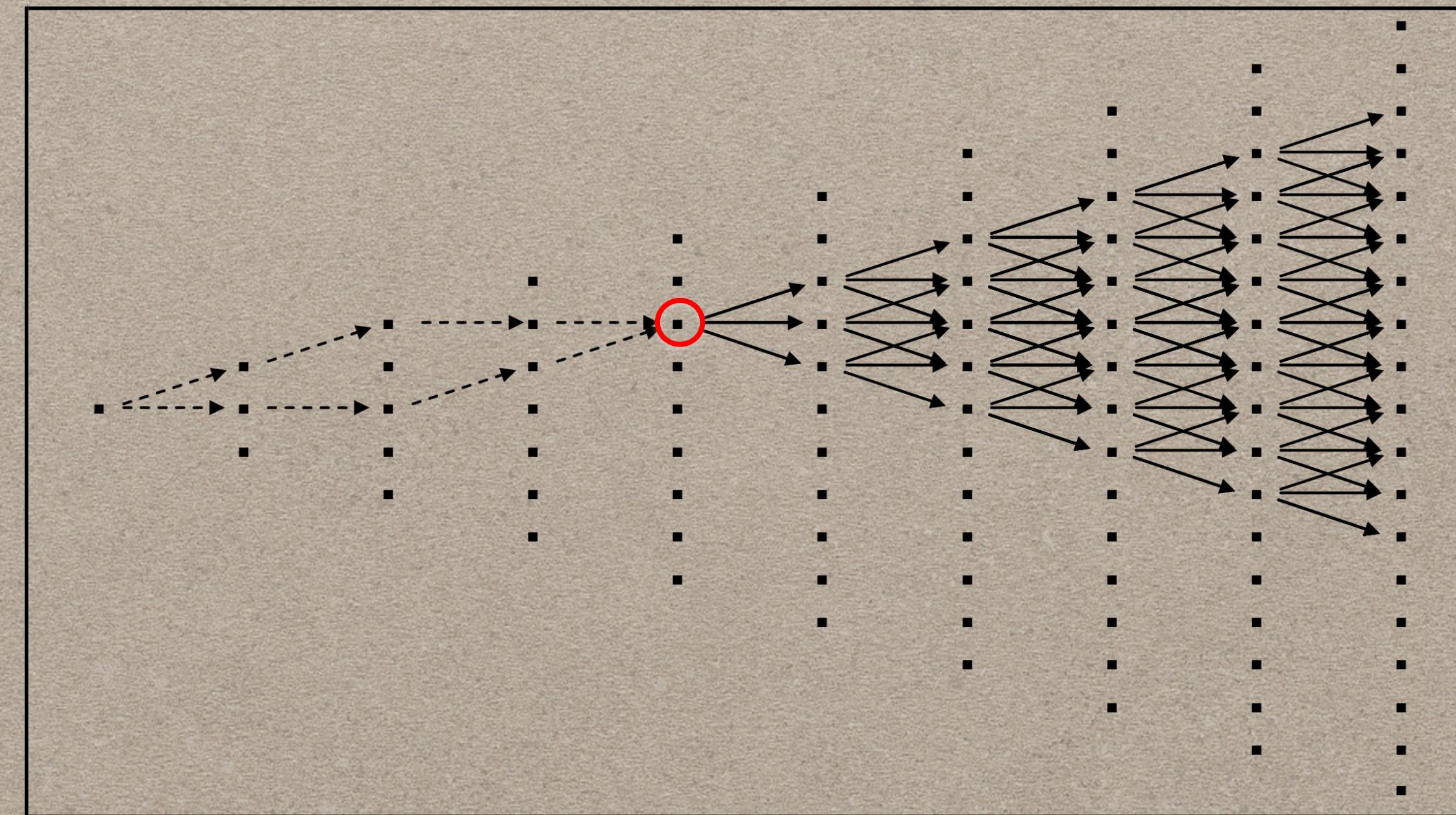
WHAT'S THE PROBLEM?

- In a tree the conditional expectation comes for free
- For example the conditional expectation at the circled point here is the value calculated on the subtree that starts from the point



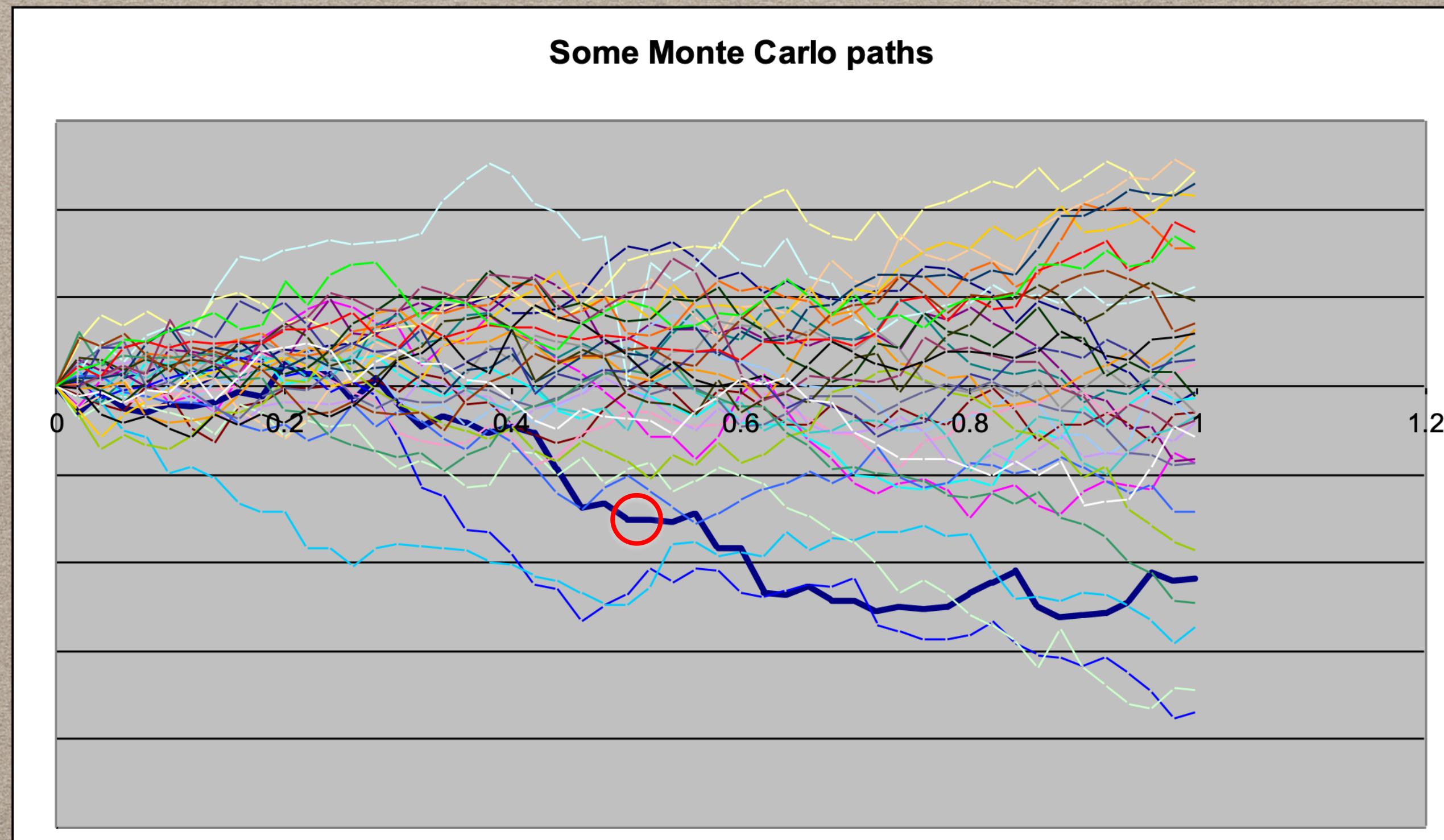
WHAT'S THE PROBLEM?

- BTW, that's why it is difficult to price path-dependent options with trees: this conditional value does not depend on how we get there (e.g. on which one of the dashed lines)



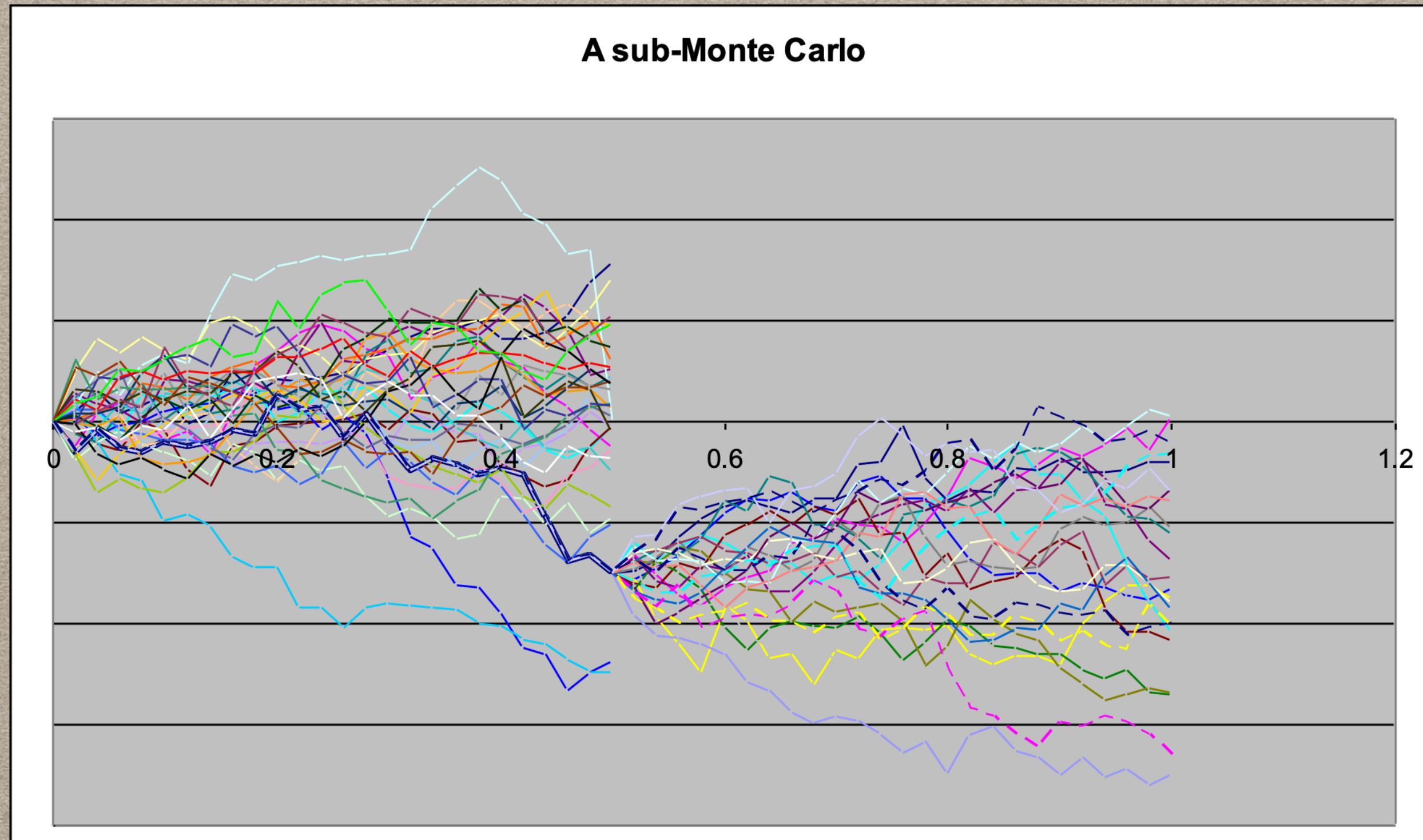
WHAT'S THE PROBLEM?

- In a Monte Carlo the situation is different
- There is no sub-Monte Carlo starting from the circled point



WHAT'S THE PROBLEM?

- To calculate the continuation value, we could start a sub-Monte Carlo there



WHAT'S THE PROBLEM?

- And that was how people did this for a while (this scheme is called the Broadie-Glasserman scheme)
- But this is very expensive computationally
- We need to do a sub-Monte Carlo for each path
- To reduce from the computational burden we can start a sub-Monte Carlo at a discrete number of points and interpolate elsewhere

MAIN POINTS

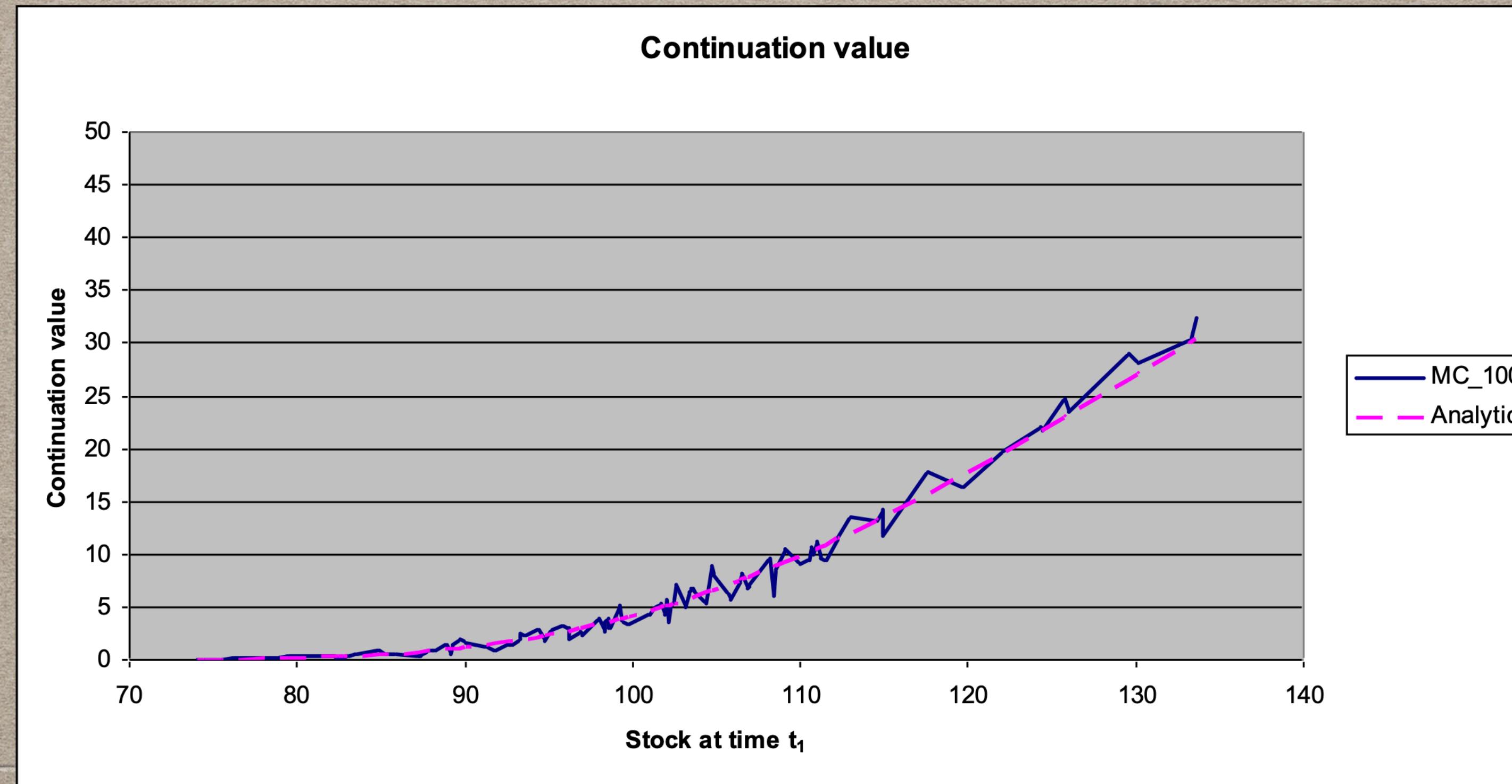
- What's the problem?
- *Conditional expectation by regression*
- An example

CONDITIONAL EXPECTATION BY REGRESSION

- Let's think a bit more about the interpolation idea
- Here's an example where we run a simulation with 100 paths until a time t_1 , then for each path calculate the continuation value using another MC with 100 paths
- We graph the continuation value as calculated vs the true value

CONDITIONAL EXPECTATION BY REGRESSION

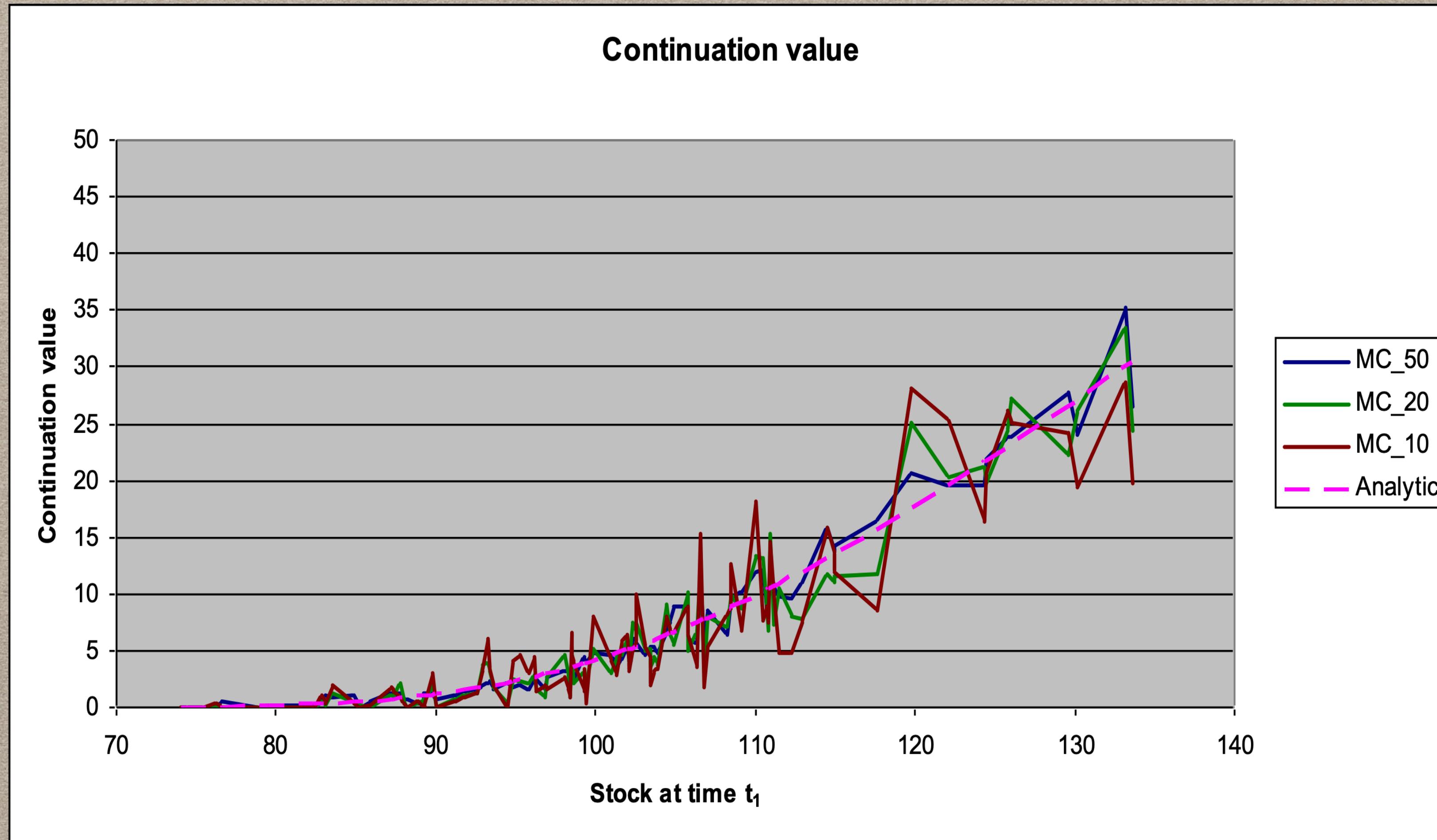
- The result is very noisy
- So instead of using interpolation, we should use a “best fit” curve



CONDITIONAL EXPECTATION BY REGRESSION

- Such a best fit curve would probably be pretty close to the true value
- But we still have the problem that we have to run 10000 paths
- We can try to run fewer paths for the sub-Monte Carlo scheme used to calculate the continuation value
- The next graphs shows the result of 50 paths, 20 paths and 10 paths

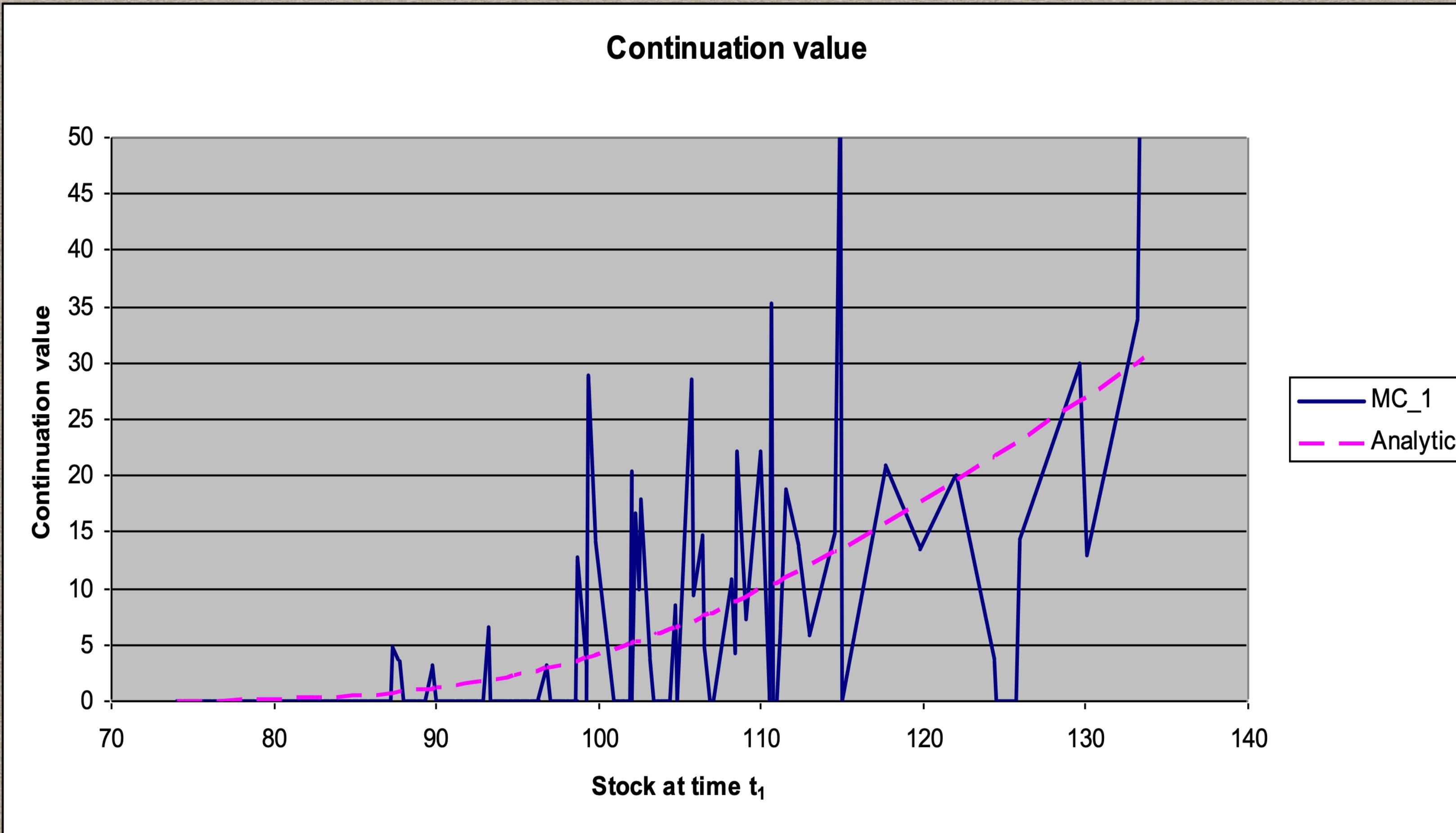
CONDITIONAL EXPECTATION BY REGRESSION



CONDITIONAL EXPECTATION BY REGRESSION

- It's clear that fewer paths means more noise
- But it still makes sense to do the best fit curve
- Taken to the extreme, we can try to calculate the continuation value with a Monte-Carlo scheme using only 1 path

CONDITIONAL EXPECTATION BY REGRESSION



CONDITIONAL EXPECTATION BY REGRESSION

- The “best fit” is the informal name for regression
- So in order to find the continuation value we need to perform regression
- There are lots of types of regression
- Basically the field of statistics is nothing else than “regressology”
- The simplest and most widespread regression is the linear regression

CONDITIONAL EXPECTATION BY REGRESSION

- Other types of regression
 - Kernel-based, or non-parametric regression
 - Non-linear regression
 - Generalized linear model (GLM)
 - Generalized additive model (GAM)
 - Tension splines

CONDITIONAL EXPECTATION BY REGRESSION

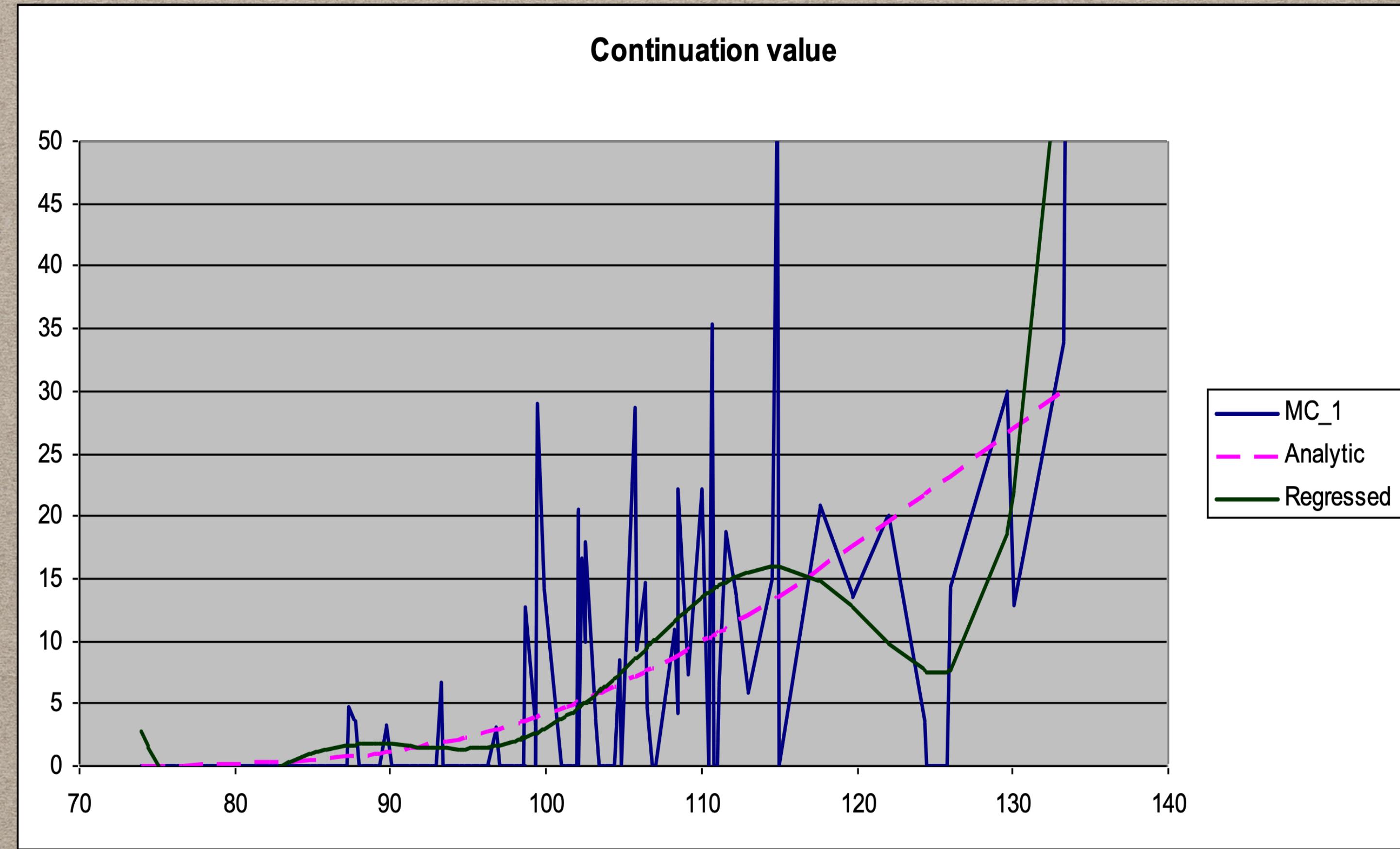
- The first American Monte Carlo algorithm was devised by Longstaff-Schwartz
- It used linear regression

CONDITIONAL EXPECTATION BY REGRESSION

- In linear regression the best fit is a linear combination of some functions called “basis functions”
- In Excel use “LINEST” or “TREND”, In Python use `sklearn.linear_model.LinearRegression` or `scipy.stats.linregress`
- Let's do the linear regression for our 100 points in the graph on slide 17
- We use as basis functions the value of the stock at time t_1 and its powers up to power 6

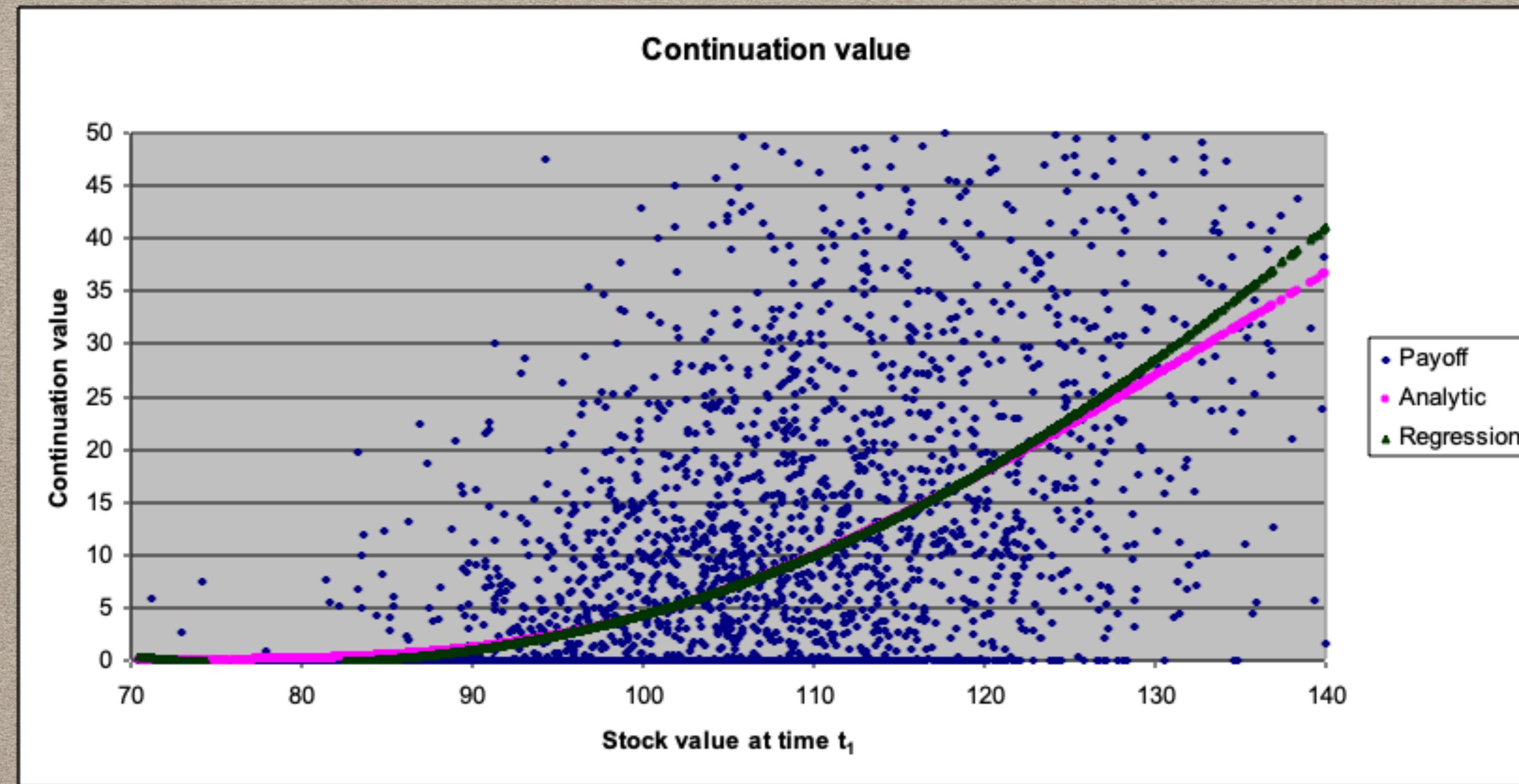
CONDITIONAL EXPECTATION BY REGRESSION

- The result is pretty far from the true value



CONDITIONAL EXPECTATION BY REGRESSION

- But we used only 100 paths
- Here's a graph where we use 4000 paths



MAIN POINTS

- What's the problem?
- Conditional expectation by regression
- *An example*

AN EXAMPLE

- Price of the Bermudan put = \$4.69
- Price of European put = \$3.59
- So the price of the early exercise feature is \$1.10
- Price given by AMC with 4000 paths and 6 basis functions is between \$4.5 and \$4.9

Exercise - Now it is your turn to give it a try! Let's play around with Jupiter Notebook