Date Posted: November 26, 2014

Last Revised: October 21, 2017

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The Cross-section of Conditional Heteroskedasticity and Expected Return

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Abstract

This paper sheds light on the cross-sectional relation between idiosyncratic volatility and expected returns in Korean stock market. Consistently with Fu (2009), the result shows the existence of first-order autocorrelation in monthly idiosyncratic volatility. However, no positive relation between risk and return is observed when an out-of-sample EGARCH idiosyncratic volatility estimate is used. Instead, its full-sample counterpart exhibits a positive relation. In addition to Fink, Fink and He (2012) and Guo, Kassa and Ferguson (2014), this evidence suggests that look-ahead bias causes a spurious positive relation Fu argues and that the low-volatility anomaly of Ang et al. (2006) cannot be reversed by introducing time-varying volatility model. Short-term return reversal seems to partially cause the anomaly as argued by Huang et al. (2010), but the negative relation between risk and return is robust after controlling this reversal.

Keywords: Idiosyncratic Risk, GARCH, Cross-sectional Return, Low-volatility Anomaly, Return Reversal

JEL Classification: G12, G14

1 Introduction and Literature Review

Innumerable scholars have investigated the relationship between risk and return based on both theoretical and empirical perspectives. Capital Asset Pricing Model, hereafter CAPM, emphasizes the role of a systematic risk as a determinant of an expected return under an equilibrium (Sharpe, 1964; Black, Jensen and Scholes, 1972; Fama and MacBeth, 1973). An unsystematic risk of a stock is not considered in the model since it is diversifiable. In reality, however, the equilibrium is rarely achieved (Campbell et al., 2001) and the unsystematic risk is not washed away because of an under-diversification (Blume and Friend, 1975; Goetzmann and Kumar, 2008). Several researchers including Levy (1978), Merton (1987) and Malkiel and Xu (2006) hypothesize the positive relationship between the unsystematic risk and the expected return since investors require an additional compensation for bearing the unsystematic risk. Pastor and Veronesi (2009) emphasize the role of idiosyncratic risk in determining asset price as well. Their hypotheses are seemingly plausible in terms of risk-averse investors.

The empirical findings of nascent researches are contradictory to each other. While Fama and MacBeth report mixed results, Longstaff (1989) exhibits a negative but insignificant cross-sectional relation between the variance and the mean of portfolio returns. On the other hand, Lehman (1990) shows there exists a positive and significant risk premium for the residual risk in time-serial perspectives. Consistently, Goyal and Santa-Clara (2003) also find a positive and significant relation between the average idiosyncratic volatilities and the aggregate excess returns. Subsequent researches cast a doubt on the conclusion of Goyal and Santa-Clara and present the outcomes directly refute the relation (Bali et al., 2005; Wei and Zhang, 2005).

Among them, Ang et al. (2006) even document a negative cross-sectional relation between an idiosyncratic risk and an expected return. Reportedly, the relationship is both statistically and economically significant. Furthermore, follow-up researches consistently establish the negative cross-sectional relation between an idiosyncratic volatility and an expected return in worldwide markets including Korean market as well (Guo and Savickas, 2007; Ang et al., 2009; Kim and Byun, 2011; Yun, Ku and Eom, 2011; Kang, Lee and Sim, 2014; Kho and Kim, 2014). This negative relation is incomprehensible in terms of traditional theories.

Various researches to deny or justify the negative relation have been conducted. Bali and Cakici

(2008) rebut the relation by stating that the results drawn by Ang et al. (2006) are too sensitive to validate their negative relation. Pointedly, Bali and Cakici argue that diverse setups such as the data frequency, weighting schemes, utilized breakpoints and screening crucially influence the results. Huang et al. (2010, 2011) also point out that short-term return reversal and January effect govern the negative risk-return relation in the cross-section and the relation is reversed after their controlling for these variables. On the contrary, Pontiff (2006) insists that the negative risk-return relationship is reasonable because the idiosyncratic risk to which investors are exposed is a sort of holding costs, which limits an arbitrage opportunity from mispriced securities and hence lowers the expected return. Besides, information contents about future earnings, an expected idiosyncratic skewness, lottery-like characteristics, liquidity biases and a human capital have been mentioned as the variables of interest because of the economic meanings implied in the variables as well as their high correlations with the idiosyncratic volatility (Jiang, Xu and Yao, 2009; Boyer, Mitton and Vorkink, 2010; Bali, Cakici and Whitelaw, 2011; Han and Lesmond, 2011; Eiling, 2013).

Especially, some researches focus on the time-varying properties of estimated idiosyncratic volatilities. Fu (2009) contends that the negative relationship exhibited in many previous researches is because of the autocorrelation of idiosyncratic volatilities and that the positive risk-return relationship is discovered by introducing Exponential Generalized Autoregressive Conditional Heteroskedasticity model, hereafter EGARCH model, to accurately estimate time-varying volatilities. Previously, numerous papers adopt the GARCH models to empirically investigate the time-serial relationship between conditional volatilities and expected returns (French, Schwert and Stambaugh, 1987; Bollerslev, Engle and Wooldridge, 1988; Glosten, Jagannathan and Runkle, 1993; Cho, 1994; Kim and Whang, 1996; Lee and Chae, 1996), but Fu firstly introduces the EGARCH model into the cross-sectional study; although unpublished, Spiegel and Wang (2005) demonstrate similar results. Except Glosten, Jagganathan and Runkle, most findings show the positive time-serial relationship between conditional volatilities and expected returns. Choosing the best model among the GARCH models is an empirical issue, but according to Fu, considering the EGARCH model is reasonable because of both theoretical advantages and empirical performances established by past papers (Pagan and Schwert, 1990; Engle and Mustafa, 1992; Engle and Ng, 1993; Hentschel, 1995; Ryu, 2012). Besides, microstructure debate mentioned by Bali and Cakici is successfully avoided by using monthly data. Andersen and Bollerslev (1998) also substantiate the empirical performance of out-of-sample estimates measured by conditional heteroskedasticity model as well. Consistently, Chua, Goh and Zhang (2010) employ the time-series model, which is different with the model of Fu, to decompose an idiosyncratic volatility into expected and unexpected portions and show the positive relationship between expected volatilities and expected returns, which is similar to the relationship discovered by Fu.

Contrarily, Guo, Kassa and Ferguson (2014) dispute the argument of Fu by reporting the results of their Monte-Carlo simulation, which shows that the in-sample EGARCH volatility used by Fu and the full-sample EGARCH volatility have an overestimation bias when the distribution of returns are positive-skewed and the biased estimates, in turn, spuriously display the positive risk-return relationship in the cross-section; generalized lambda distribution developed by Ramberg et al. (1979) is applied to conduct the Monte-Carlo simulation. Guo, Kassa and Ferguson also state that the out-of-sample EGARCH volatility, which excludes a contemporaneous return observation and genuinely predicts the future volatility, doesn’t exhibit significant predictability for the future return and the negative risk-return relationship suggested by Ang et al. (2006) cannot be explained by introducing the out-of-sample EGARCH volatility.

Similarly, Fink, Fink and He (2012) also provide an empirical evidence that exhibits an influence of look-ahead bias. Fink, Fink and He introduce diverse volatility estimates including in-sample and out-of-sample EGARCH estimates to explain cross-sectional returns. By conducting both portfolio approach and cross-sectional analysis, their findings also suggest that the positive relation between risk and return documented by Fu is spuriously caused by look-ahead bias. The results of Fink, Fink and He are robust with various control variables as well. These consequences simultaneously undermine the positive risk-return relation Fu argues.

Fu (2010) refutes these counterarguments by suggesting that EGARCH volatility estimates are sensitive to settings adopted by the researchers and showing that the results generated from the settings of Guo, Kassa and Ferguson are vulnerable to noises since their settings limit the number of iteration when the parameters are numerically estimated. Fu (2010) contends that the positive relation between idiosyncratic risk and return is robust when the volatility estimates are computed in stable settings that allow enough iteration to estimate parameters.

Since the findings from past literatures are strikingly different with each other, this paper sheds light on this controversy by introducing more reliable settings than previous researches. The primary goal of this paper is to investigate whether the time-varying volatility scheme proposed by Fu (2009) can explain so-called low-volatility anomaly reported by numerous studies including Ang et al. (2006, 2009) and Kho and Kim. Then, this paper also examines the influence of look-ahead bias documented by both Guo, Kassa and Ferguson and Fink, Fink and He on both computing EGARCH volatility estimates and conducting cross-sectional regression Fama and MacBeth suggest.

The remainder of this paper continues as follows. In Section 2, the model whereby conditional volatility is measured is introduced. Specifically, the difference among EGARCH estimates, which have been used in preceding researches, is revisited: the in-sample, out-of sample and full-sample EGARCH idiosyncratic volatility estimates. Since Fu (2010) points out that the difference in numerical methods leads to the conflicting results, I also introduce the way I adopt to estimate the parameters in the model. In Section 3, the sample data used in this paper is described. By replicating both settings and methods adopted by foregoing references with the data from Korean stock market, the finding of this paper is able to be directly compared to the references. In Section 4, the empirical consequences are provided and interpreted. To investigate whether the interpretation is robust or not, I conduct additional analysis with different EGARCH estimates to juxtapose the outcomes from different treatments. Lastly, Section 5 contains the conclusion of this paper.

2 Economic Model

Conditional heteroskedasticity models, i.e. time-series volatility models, have been developed to accurately forecast the future variance, which exhibits a clustering characteristic (Engle, 1982; Bollerslev, 1986; Nelson, 1991; Glosten, Jagannathan and Runkle, 1993). Since ARCH model firstly proposed by Engle, the researchers have suggested diverse variations of the model with their own specifications to closely reflect the reality and precisely estimate the future risk.

As aforementioned, Fu (2009) exploits both the factor model of Fama and French (1993) for expected returns and the EGARCH model for conditional idiosyncratic volatilities. Specifically, this model can be written as the below equations.

$$R_{it} - r_t = \alpha_i + \beta_i(R_{mt} - r_t) + s_i SMB_t + h_i HML_t + \varepsilon_{it}, \text{where } \varepsilon_{it} \sim N(0, \sigma_{it}^2)$$

$$\ln \sigma_{it}^2 = a_i + \sum_{l=1}^{p} b_{i,l} \ln \sigma_{i,t-l}^2 + \sum_{k=1}^{q} c_{i,k} \left\{ \theta \left(\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}} \right) + \gamma \left[\left| \frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}} \right| - \left(\frac{2}{\pi} \right)^{1/2} \right] \right\}$$

Where R_i is the return for the corresponding firm i, r is the risk-free return, R_m is the market return and both SMB and HML are the factors proposed by Fama and French. The coefficient α, β, s and h for the first equation and a, b, c, θ and γ for the second equation are the parameters to be estimated. The parameters are able to be numerically estimated by maximizing the log-likelihood function under the normality assumption imposed on ε of the first equation. The above equations simultaneously yield an expected return and a conditional volatility, respectively.

Introducing the normality assumption gives below log-likelihood function L to be maximized. Since all ε are assumed to follow a joint normal distribution, the joint log-likelihood function has the summation form below. Hence, the maximum likelihood estimator $\widehat{\boldsymbol{\theta}}_{\mathrm{ML}}$ can be attained by directly maximizing the entire the joint function, i.e. the summation of L.

$$\sum_{\tau=1}^{t} L(R_{i,\tau}) = -\frac{t}{2}\log(2\pi) - \frac{1}{2}\sum_{\tau=1}^{t} \log(\sigma_{i,\tau}^2) - \sum_{\tau=1}^{t} \frac{\varepsilon_{i,\tau}^2}{2\sigma_{i,\tau}^2}$$

$$\widehat{\boldsymbol{\theta}}_{\mathrm{ML}} = \underset{\boldsymbol{\theta}\in\mathbb{R}^P}{\operatorname{argmax}} \sum_{\tau=1}^{t} L(\boldsymbol{\theta}; R_{i,\tau})$$

Where $\boldsymbol{\theta} = (\alpha \quad \beta \quad \cdots \quad \gamma)^{\mathsf{T}}$ is the vector of P parameters to be estimated. Unfortunately, L, the function to be maximized is non-linear in terms of the parameter $\boldsymbol{\theta}$. Therefore, numerical methods such as Newton's method must be utilized to consistently estimate $\widehat{\boldsymbol{\theta}}_{\mathrm{ML}}$. With the estimated parameters and given information up to t, the conditional volatility $E(IVOL)$ of the time t can be computed as below.

$$\begin{aligned} E(IVOL_t^{\mathrm{In}}) &= E_t(IVOL_t) \\ &= \exp(\ln \sigma_{it}^2) \\ &= \exp\left[a_{i,t} + \sum_{l=1}^{p} b_{i,l,t} \ln \sigma_{i,t-l}^2 + \sum_{k=1}^{q} c_{i,k,t} \left\{\theta\left(\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}}\right) + \gamma\left[\left|\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}}\right| - \left(\frac{2}{\pi}\right)^{1/2}\right]\right\}\right] \end{aligned}$$

Where E_t is an expectation operator conditioned by the given information in t. To clarify the meaning of the estimated variable, the notation $E(IVOL^{\mathrm{In}})$ is used and the in-sample EGARCH idiosyncratic volatility is calculated as a result.

However, according to Guo, Kassa and Ferguson (2014), $E(IVOL_t^{\mathrm{In}})$ is positively correlated with ε_t. Specifically, $L(R_t)$ is negatively correlated with ε_t by nature.

$$\frac{\partial L(R_t)}{\partial \varepsilon_t}\varepsilon_t = -\frac{\varepsilon_t^2}{\sigma_t^2} \in (-\infty, 0]$$

$$\frac{\partial L(R_t)}{\partial \sigma_t}\sigma_t = \frac{\varepsilon_t^2}{\sigma_t^2} - 1 \in \begin{cases} (0, +\infty), & \sigma_t^2 < \varepsilon_t^2 \\ (-\infty, 0], & \sigma_t^2 \geq \varepsilon_t^2 \end{cases}$$

Thus, the maximization process repeatedly tunes up the values of parameters to increase σ_t and compensate the damaged log-likelihood function as long as σ_t^2 is smaller than ε_t^2. As a result, $E(IVOL_t^{\mathrm{In}})$ is distortedly measured by the optimization procedure. Guo, Kassa and Ferguson designate this problem as look-ahead bias. Since the returns observed in realities are highly positive-skewed, the positive look-ahead bias tends to dominate the negative look-ahead bias.

The false introduction of the information in t is able to be solved by manipulating above joint log-likelihood function as below.

$$\sum_{\tau=1}^{t-1} L(R_{i,\tau}) = -\frac{t-1}{2}\log(2\pi) - \frac{1}{2}\sum_{\tau=1}^{t-1}\log(\sigma_{i,\tau}^2) - \sum_{\tau=1}^{t-1}\frac{\varepsilon_{i,\tau}^2}{2\sigma_{i,\tau}^2}$$

Unlike the joint log-likelihood function with the information in t, this joint log-likelihood function is disengaged from the tricky distortion from the look-ahead bias. Hence, the estimated parameters don't cause any spurious positive bias between σ_t and ε_t as a result.

Based on these estimated parameters, another EGARCH idiosyncratic volatility is derived as below.

$$\begin{aligned} E\left(IVOL_t^{\text{Out}}\right) &= E_{t-1}(IVOL_t) \\ &= \exp(\ln \sigma_{it}^2) \\ &= \exp\left[a_{i,t-1} + \sum_{l=1}^{p} b_{i,l,t-1}\ln\sigma_{i,t-l}^2 + \sum_{k=1}^{q} c_{i,k,t-1}\left\{\theta\left(\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}}\right) + \gamma\left[\left|\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}}\right| - \left(\frac{2}{\pi}\right)^{1/2}\right]\right\}\right] \end{aligned}$$

Similarly, the notation $E(IVOL^{\text{Out}})$ is adopted to distinguish the above out-of-sample EGARCH idiosyncratic volatility from the in-sample EGARCH idiosyncratic volatility mentioned above. The one-step ahead conditional variance can be measured by simultaneously utilizing both the estimated parameters and the observations in $t-1$. This variance estimates don't exhibit any problematic bias by which faulty correlations can occur.

To avoid unnecessary waste of computing resources, the full-sample EGARCH idiosyncratic volatility is exploited instead of the in-sample EGARCH idiosyncratic volatility, which requires intensive iteration during the recursive estimating procedures, as in Guo, Kassa and Ferguson. Technically, the two EGARCH idiosyncratic volatilities are different in detail, but the two volatilities are similar to each other in terms of the look-ahead bias. Pointedly, the joint log-likelihood function to be maximized can be denoted as below.

$$\sum_{\tau=1}^{T} L(R_{i,\tau}) = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{\tau=1}^{T}\log(\sigma_{i,\tau}^2) - \sum_{\tau=1}^{T}\frac{\varepsilon_{i,\tau}^2}{2\sigma_{i,\tau}^2}$$

Where T is the end of the sample. The look-ahead bias naturally arises in the above joint log-likelihood function because the function also contains $L(R_t)$ since the full-sample is used.

Based on these estimated parameters, $E\left(IVOL^{\text{Full}}\right)$, i.e. the full-sample EGARCH idiosyncratic volatility is represented as below.

$$
\begin{aligned}
E\left(IVOL_t^{\text{Full}}\right) &= E_T(IVOL_t) \\
&= \exp(\ln \sigma_{it}^2) \\
&= \exp\left[a_i + \sum_{l=1}^{p} b_{i,l} \ln \sigma_{i,t-l}^2 + \sum_{k=1}^{q} c_{i,k} \left\{\theta\left(\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}}\right) + \gamma \left[\left|\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}}\right| - \left(\frac{2}{\pi}\right)^{1/2}\right]\right\}\right]
\end{aligned}
$$

Since these EGARCH models incorporate non-linear objective function to be optimized, maximum likelihood estimator cannot be obtained analytically. Instead, the function is numerically maximized by iterative computation. If the model includes only one parameter, then the maximum likelihood estimator θ_{ML} can be indirectly denoted as below.

$$
\theta_{\text{ML}} = \underset{\theta}{\operatorname{argmax}}\, \mathcal{L}(\theta)
$$

$$
\text{where } \mathcal{L}(\theta) \coloneqq \sum_{\tau=1}^{T} L(\theta; R_\tau)
$$

Where $\mathcal{L}$ is a likelihood function of joint normal distribution; hereafter, this notation is adopted for convenience. If $\mathcal{L}$ is maximized at θ_{ML}, then its first order condition is equal to zero as well. Since $\mathcal{L}$ is non-linear in terms of θ, just below equation is satisfied.

$$
\frac{d\mathcal{L}(\theta_{\text{ML}})}{d\theta} = 0
$$

Instead of finding closed-form solution, Newton's method attempts to construct a sequence $\{\theta_k\}$ that converges toward θ_{ML} such that above equation is satisfied. By optimizing second order Taylor series, below univariate sequence $\{\theta_k\}$ and corresponding multivariate sequence $\{\boldsymbol{\theta}_k\}$ is obtained.

$$\theta_{k+1} = \theta_k - \left[\frac{d^2\mathcal{L}(\theta_k)}{d\theta^2}\right]^{-1} \frac{d\mathcal{L}(\theta_k)}{d\theta}$$

$$\begin{aligned}\boldsymbol{\theta}_{k+1} &= \boldsymbol{\theta}_k - \left[\frac{\partial^2\mathcal{L}(\boldsymbol{\theta}_k)}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}^\top}\right]^{-1} \frac{\partial\mathcal{L}(\boldsymbol{\theta}_k)}{\partial\boldsymbol{\theta}} \\ &= \boldsymbol{\theta}_k - [\mathbf{H}(\boldsymbol{\theta}_k)]^{-1}\mathbf{g}(\boldsymbol{\theta}_k)\end{aligned}$$

If the hessian $\mathbf{H}$ of the objective function $\mathcal{L}$ is directly available, then $\boldsymbol{\theta}_{\mathrm{ML}}$ is attained by imposing enough convergence criteria and computing above sequence iteratively. For instance, Berndt–Hall–Hall–Hausman method, hereafter BHHH method, proposed by Berndt et al. (1970) suggest that $\mathbf{H}$ is able to be substituted by the expectation of the outer product of gradients matrix since the information matrix equality is viable.

$$\mathbf{H}_{k+1} = -\sum_{\tau=1}^{T} \frac{\partial\mathcal{L}(\boldsymbol{\theta}_k)}{\partial\boldsymbol{\theta}^\top}\frac{\partial\mathcal{L}(\boldsymbol{\theta}_k)}{\partial\boldsymbol{\theta}}$$

Instead of adopting BHHH method, other approximations can be utilized to compute the sequence $\{\boldsymbol{\theta}_k\}$ if there exists no straightforward formula for $\mathbf{H}$. Quasi-Newton method uses the hessian approximation and diverse approximation is proposed by past literatures as well. For example, Broyden–Fletcher–Goldfarb–Shanno method, hereafter BFGS method, incorporates below formula to efficiently approximate hessian. Other methods such as Davidon–Fletcher–Powell method can also be utilized, but SAS provides only BFGS method among them to conduct optimization by applying Quasi-Newton method.

$$\mathbf{H}_{k+1} = \mathbf{H}_k - \frac{(\mathbf{H}_k \Delta\boldsymbol{\theta}_{k+1})(\mathbf{H}_k \Delta\boldsymbol{\theta}_{k+1})^\top}{\Delta\boldsymbol{\theta}_{k+1}^\top \mathbf{H}_k \Delta\boldsymbol{\theta}_{k+1}} + \frac{\Delta\mathbf{g}_{k+1}\Delta\mathbf{g}_{k+1}^\top}{\Delta\mathbf{g}_{k+1}^\top \Delta\boldsymbol{\theta}_{k+1}}$$

$$\text{where } \mathbf{H}_k \coloneqq \mathbf{H}(\boldsymbol{\theta}_k)$$

$$\mathbf{g}_k \coloneqq \mathbf{g}(\boldsymbol{\theta}_k)$$

In addition to Quasi-Newton method, SAS also provides Trust Region method to estimate $\boldsymbol{\theta}_{\mathrm{ML}}$. In lieu of the formula for hessian approximation of Quasi-Newton method, Trust Region method utilizes finite difference to obtain $\mathbf{H}$ whereby the sequence $\{\boldsymbol{\theta}_k\}$ is computed repeatedly. Specification of Trust Region method is able to be denoted as below.

$$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k - \lambda_k[\mathbf{H}(\boldsymbol{\theta}_k)]^{-1}\mathbf{g}(\boldsymbol{\theta}_k)$$

$$\text{where } \mathbf{H}(\boldsymbol{\theta}_k) \coloneqq \begin{pmatrix} \frac{\partial^2\mathcal{L}}{\partial\theta_{1k}^2} & \frac{\partial^2\mathcal{L}}{\partial\theta_{1k}\partial\theta_{2k}} & \cdots & \frac{\partial^2\mathcal{L}}{\partial\theta_{1k}\partial\theta_{Pk}} \\ \frac{\partial^2\mathcal{L}}{\partial\theta_{2k}\partial\theta_{1k}} & \frac{\partial^2\mathcal{L}}{\partial\theta_{2k}^2} & \cdots & \frac{\partial^2\mathcal{L}}{\partial\theta_{2k}\partial\theta_{Pk}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2\mathcal{L}}{\partial\theta_{Pk}\partial\theta_{1k}} & \frac{\partial^2\mathcal{L}}{\partial\theta_{Pk}\partial\theta_{2k}} & \cdots & \frac{\partial^2\mathcal{L}}{\partial\theta_{Pk}^2} \end{pmatrix}$$

$$\frac{\partial^2\mathcal{L}}{\partial\theta_{1k}^2} \approx \frac{\mathcal{L}(\theta_{1k}+\Delta) - 2\mathcal{L}(\theta_{1k}) + \mathcal{L}(\theta_{1k}-\Delta)}{\Delta^2}$$

Moreover, Trust Region method incorporates the step size λ by which the sequence is converged more gradually than other aforementioned methods. In order to determine λ, SAS adopts the way introduced by Moré and Sorensen (1983), which considers reliability of hessian matrix obtained by finite difference. Since finite difference to acquire $\mathbf{H}$ requires myriad repetition and λ limits the extent of each optimization step, Trust Region method is numerically expensive and significantly sacrifices efficiency as well. However, stable convergence is an important matter in estimating $\boldsymbol{\theta}_{\mathrm{ML}}$ as mentioned in Fu (2010). In this aspect, Trust Region method guarantees the convergence of $\{\boldsymbol{\theta}_k\}$ that is relatively stable than the convergence other methods provide. Hence, I utilize Trust Region method with 10,000 iterations to estimate EGARCH parameters whereby EGARCH idiosyncratic volatility estimates are available. Based upon these estimates, the cross-sectional relationship between EGARCH idiosyncratic volatility and expected return is investigated by applying both the cross-sectional approach given by Fama and MacBeth (1973) and the time-serial approach appeared on Gibbons, Ross and Shanken (1989), Fama and French and so forth.

3 Data Description

The sample data this paper incorporates include all common stocks traded on KOSPI and KOSDAQ during January 1987 to August 2014. The sample acquired from DataGuide database is recorded with daily, monthly and yearly frequencies, respectively. Following Ang et al. (2006) and Fu (2009), I compute monthly idiosyncratic volatility estimates with daily data by applying the factors of Fama and French (1993) as explanatory variables. Specifically, the model is able to be denoted as below.

$$R_{i\tau} - r_\tau = \alpha_{it} + b_{it}(R_{m\tau} - r_\tau) + s_{it}SMB_\tau + h_{it}HML_\tau + \varepsilon_{i\tau}$$

$$IVOL_{it} = \sqrt{\frac{N(t)}{N(t)-4} \times \sum_{\tau=1}^{N(t)} \hat{\varepsilon}_{i\tau}^2}$$

Where $R_{i\tau}$ is the raw return of the firm i in the day $\tau \in t$. The risk-free rate of return, r_τ, is the one-month yield of Monetary Stabilization Bonds maturing 364 days as in Kho and Kim (2014); since the time-series of r_τ are only available after January 1987, the other data are also excluded as mentioned above. $R_{m\tau}$ is the return of Korean stock market and the return of KOSPI index is employed as a proxy. SMB_τ and HML_τ are the small-minus-big and high-minus-low factors. All the specifications used to construct these factors are same with Fama and French as well; six value-weighted portfolios, from SL to BH, are formed and rebalanced at the end of each June based upon both size and book-to-market ratio. Size is the market value of equity at the end of June and book-to-market ratio is the year-end book-value of equity divided by the year-end market value of equity, respectively. The regression parameters α_{it}, b_{it}, s_{it} and h_{it} are recursively estimated for each firm i in each month t by regressing the excess return $R_i - r$ on $R_m - r$, SMB and HML. Ordinary least squares method is applied to estimate the parameters.

$IVOL$, one of the main variables of this paper, is the sample standard deviation of regression residuals. The deviation is scaled by the square root of $N(t)$, the number of trading days in the month t with non-zero trading volume since $IVOL$ implies the extent of monthly idiosyncratic volatility by definition. If $N(t)$ is smaller than 15, then $IVOL$ of corresponding month is not computed as in Fu. The pooled sample contains 364,228 firm-month observations. The average $IVOL$ is 14.21% and the standard deviation of $IVOL$ is 8.56%, respectively.

The first row of Table 1 contains the statistical properties of $IVOL$. As in Fu, the statistics displayed in this table are firstly computed with the time-series of each firm and the cross-sectional average of these statistics is subsequently recomputed. The cross-sectional average of 2,623 time-serial means is 15.05% and the cross-sectional average of time-serial standard deviations are 8.41%. The average coefficient of variation across firms is 0.55 and this exhibits that the standard deviation of $IVOL$ time-series is roughly 55% of the average of $IVOL$ time-series. This result is quite close to the result Fu reports; the cross-sectional averages for mean, standard deviation and coefficient of variation are 16.87%, 9.94% and 0.55, respectively. Although the number of firms analyzed in this paper is smaller than the number of firms covered by Fu, the similarity of the results reasonably justifies the comparison. The cross-sectional average of time-serial skewness is 1.66. Guo, Kassa and Ferguson (2014) argue that the positive-skewed return distribution is the source of look-ahead bias by which the spurious positive relation between return and idiosyncratic risk arises. The remainder of the first row shows sample autocorrelations with different orders. The average first order autocorrelation is 0.45 and gradually decays. Similarly, the second row of Table 1 also reports the statistics for the first order difference of log $IVOL$. The average of the first order autocorrelation is -0.40 in this case and the averages for the higher order autocorrelations are close to zero. As mentioned by Fink, Fink and He (2012), one-month lagged $IVOL$ is able to be used as an unbiased forecastor if $IVOL$ follows a random walk. For instance, the analysis of Ang et al. implicitly assumes this. The evidence of Table 1, however, exhibits that this assumption is weak and it is highly doubtful that the process of $IVOL$ follows a random walk, as documented by Fu. Besides, the approaches that incorporate the autoregressive property of $IVOL$ are justified by this evidence (Chua, Goh and Zhang, 2010; Brockman, Schutte and Yu, 2012).

Since the first difference of log $IVOL$ does not imply that it is not a white noise, further researches are required to judge whether $IVOL$ follows a random walk process or not. As in Fu, I conduct Dickey–Fuller tests proposed by Dickey and Fuller (1979) to determine whether a random walk hypothesis for $IVOL$ is accurate. Specifically, the model is able to be illustrated as below.

$$\Delta IVOL_{i,t+1} = \gamma_{0i} + \gamma_{1i} IVOL_{i,t} + \eta_{i,t+1}$$

The null hypothesis of Dickey–Fuller test is that the coefficient γ_1 is equal to zero; i.e. $IVOL$ follows a random walk without a unit root. If γ_1 is significantly different from zero, then the null hypothesis is rejected; the alternative hypothesis of this test is that the process has a unit-root in it. However, a traditional t-test is invalid since the test statistic does not follow a standard t-distribution. Instead of applying t-test to conduct hypothesis test, using Dickey–Fuller distribution is more reasonable. So-called Dickey–Fuller tables constructed by conducting Monte-Carlo simulations have been documented by various textbooks and literatures. I use the probability function of Dickey–Fuller distribution provided by SAS to conduct the hypothesis test reported in this paper.

The first two rows of Table 2 report the results of Dickey–Fuller tests for all stocks. The third column of these rows exhibits the cross-sectional average of γ_1 estimates and the cross-sectional average of t-statistics for corresponding estimates, respectively. The median, the value of the first quartile and the third quartile are also reported. In the last column, I report the percentage of stocks for which the null hypothesis is rejected at 1% significance level. To keep the consistency with the analysis of Fu, I excluded the stocks without at least 30 months of continuous observations as well. This condition reduces the number of stocks from 2,623 to 2,386. The cross-sectional average of γ_1 among these stocks is -0.53 and the average of corresponding t-statistics is -7.53. Consistently with Fu, the null hypothesis of Dickey–Fuller test is rejected in approximately 93% of stocks. Investigations on log $IVOL$ reported in the rest two rows of Table 2 also yield similar consequencies. As discussed above, these results strongly suggest that assuming that $IVOL$ follows a random walk is riskier and that reflecting autoregressive characteristics of $IVOL$ in the model is plausible enough. Fu points out that the findings of Ang et al. can be suffer from severe measurement errors since one-month lagged $IVOL$ is used as a proxy. In this respect, the result of Table 2 substantiates the argument of Fu as well.

All above things considered, estimating idiosyncratic risk with a different method that considers the time-serial tendency of $IVOL$ seems to be necessary to accurately determine whether the relation between expected idiosyncratic risk and expected return is positive or negative. Based upon the reasons introduced in foregoing sections, I adopted EGARCH model of Nelson (1991) to estimate expected $IVOL$, i.e. conditional heteroskedasticity, instead of one-month lagged $IVOL$. As mentioned in Section 2, he specification of estimated EGARCH models is able to be denoted as below.

$$R_{it} - r_t = \alpha_i + \beta_i(R_{mt} - r_t) + s_i SMB_t + h_i HML_t + \varepsilon_{it}, \text{where } \varepsilon_{it} \sim N(0, \sigma_{it}^2)$$

$$\ln \sigma_{it}^2 = a_i + \sum_{l=1}^{p} b_{i,l} \ln \sigma_{i,t-l}^2 + \sum_{k=1}^{q} c_{i,k} \left\{ \theta \left(\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}} \right) + \gamma \left[\left| \frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}} \right| - \left(\frac{2}{\pi} \right)^{1/2} \right] \right\}$$

Since the out-of-sample EGARCH idiosyncratic volatility estimate does not exhibit any look-ahead bias documented by both Guo, Kassa and Ferguson and Fink, Fink and He, I firstly conduct the analysis same with Fu with $E(IVOL^{\text{Out}})$. R_{it}, the monthly raw return, is utilized instead of the daily raw return, which is employed to estimated $IVOL$. As other ARCH models do, EGARCH model requires the determination of specific lag length as well. To consider the comparability with other literatures, I also choose the best model among EGARCH (p, q) models with $p, q \in \{1,2,3\}$. Specifically, all nine separate EGARCH model combinations are firstly estimated and the best model in terms of Akaike Information criterion, hereafter AIC, is ultimately determined. The formula to compute AIC can be written as below.

$$AIC(\boldsymbol{\theta}_{p,q}) = -2 \ln \mathcal{L}^*(\boldsymbol{\theta}_{p,q}) + 2k$$

Where $\mathcal{L}^*$ is a value of maximized log-likelihood function. Then, the estimate $\boldsymbol{\theta}^*$ of the best EGARCH model among the estimates $\{\boldsymbol{\theta}_{p,q}\}$ of all EGARCH combinations is ultimately chosen as below.

$$\boldsymbol{\theta}^* = \underset{\boldsymbol{\theta} \in \{\boldsymbol{\theta}_{p,q}\}}{\operatorname{argmin}} AIC(\boldsymbol{\theta})$$

As aforementioned, Fu (2010) points out that the convergence status matters to the results of successive researches. To avoid potential measurement error, I exclude some estimates from the set of possible estimates $\{\boldsymbol{\theta}_{p,q}\}$ if the objective function whereby the estimate is determined is not properly converged. Thus, if an idiosyncratic volatility model of a stock as of month t satisfies corresponding convergence status under all different identifications, the set of potential candidates $\{\boldsymbol{\theta}_{p,q}\}$ also consists of nine different estimates and hence, nine different $E(IVOL^{\text{Out}})$ as well. Additionally, at least 30 historical observations are required to be qualified for estimation in order to keep the comparability with literatures.

Consistently with Fu (2009), an expanding window of sample data is used to estimate EGARCH model parameters with the above minimum requirements, i.e. past 30 monthly returns. Since the risk-free rate of return r, the one-month yield of Monetary Stabilization Bonds maturing 364 days, is only available after January 1987, $E(IVOL^{\text{Out}})$ is estimated from July 1989 to August 2014 as well.

The computed expected idiosyncratic EGARCH volatility $E(IVOL^{\text{Out}})$ is employed in succeeding examinations including both Fama–MacBeth regression test and portfolio analysis. The pooled average of $E(IVOL^{\text{Out}})$ is 16.39% and its standard deviation is 13.84%, respectively. The sample correlation between $E(IVOL^{\text{Out}})$ and $IVOL$ is 0.23 and significant at the 99% confidence level as well. Among 303,583 observations of $E(IVOL^{\text{Out}})$, EGARCH (3, 1) model provides 48,397 observations, which are 15.94% of total observations. This number is close to 16.58% documented by Fu (2009) as well; Fink, Fink and He also report that EGARCH (3, 1) model, which generates 16.47% of their observations, is the best in terms of AIC. However, the observations generated from EGARCH (1, q) models are 105,460 (34.74%), which exceed the observations generated from EGARCH (3, q) models, 103,721 (34.17%). EGARCH (1, 1) model generates 34,302 observations (11.30%). This number is bigger than 7.41% reported by Fu (2009).

Table 3 displays the descriptive statistics for the variables used in the pooled sample. I investigate all common stocks listed in Korean stock market including both KOSPI and KOSDAQ during the time from January 1989 to August 2014, i.e. 308 months. All the sample data are acquired from DataGuide database. The average of RET, the monthly raw return recorded in percentage terms, is 1.20% and corresponding standard deviation is 22.09%. As in Fu (2009), the sample excludes the observations, which have RET greater than 300% so as to prevent any unintended problem such as the distortion caused by some extremely high RETs. The mean and standard deviation of $XRET$, the monthly excess return that is computed by subtracting one-month riskless yield from monthly raw return, are 0.75 and 22.10%, separately. The average of monthly idiosyncratic volatility and monthly expected EGARCH idiosyncratic volatility are 14.21% and 16.39%, individually.

The systematic risk $BETA$ is measured as in Fama and French (1992). Firstly, the estimates of betas using former 60 months observations are computed across firms. Then, each stock is assigned to the 7-by-7 portfolios based on its size and beta; differently with previous literatures, I reduced the number of portfolios from 10-by-10 to 7-by-7 as the number of stocks traded on KOSPI and KOSDAQ is smaller than the number of stocks traded on NYSE, AMEX and NASDAQ. The portfolios are recursively formed each month and the equal-weighted returns of all portfolios are computed. The returns of these portfolios constructed by sizes and betas are regressed on both contemporary and one-month lagged market return together. The $BETA$s of the portfolios are measured as the sum of these two posterior coefficients. These ex-post portfolio $BETA$s are reassigned to each stock contained into size-beta portfolios to avoid an errors-in-variables problem. The average of $BETA$ is 1.02 and its median is 1.00, respectively.

According to previous studies, the firm-specific variables such as the size, book-to-market ratio, liquidity and past performance help explain cross-sectional expected return. The sample data include these control variables for subsequent cross-sectional tests as well. The descriptive statistics for these variables are also exhibited in Table 3. The market value of equity ME is the total market value of common equity recorded in the end of June. As in Fama and French (1992), BE/ME is the book value of common equity recorded in the end of fiscal year divided by the market value of equity observed in the end of December. $RET(-2,-7)$ is the cumulative return during the month $t-7$ to $t-2$; as in Jegadeesh (1990), the $t-1$ return is excluded to avoid potential errors driven by bid-ask spreads. Many studies have shown that there exists a momentum effect in the stock market of the United States, but the empirical evidences observed in Korean stock market are contrary to each other. (Kho, 1997; Park and Jee, 2006; Kim, 2012) As in Chordia, Subrahmanyam and Anshuman (2001), $TURN$ and $CVTURN$ are the mean turnover and the coefficient of variation of turnover, respectively; the sample requires past 36 monthly data to compute these variables. The variables ME, BE/ME, $IVOL$, $RET(-2,-7)$, $TURN$ and $CVTURN$ are winsorized at 0.5% and 99.5% level each month to reduce the distortive impact of outliers; $E(IVOL^{\mathrm{Out}})$ is independently winsorized at 2.5% and 97.5% level since it is maximum likelihood estimator, which is not computed with closed-form solution.

In order to capture potential cross-sectional patterns among these variables, their sample correlations are estimated. Table 4 displays the results. The numbers shown in the table are the time-serial averages of cross-sectional sample correlations calculated with monthly data. In order to adjust positive-skewed distributions observed in the variables such as ME, BE/ME, $TURN$ and $CVTURN$, they are substituted by their log transformations. First of all, the correlation coefficient between RET and contemporaneous $IVOL$ is 0.15 and statistically significant at 1% level. This is consistent with the results reported by Fu (2009) and Fink, Fink and He; the correlation coefficients reported in their results are 0.14

and 0.1283, respectively. Though excluded in the table, the t-statistic for the correlation coefficient between RET and $IVOL$ is 12.40. However, the correlation coefficient between RET and one-month lagged $IVOL$ is −0.05 and corresponding t-statistic is −6.21, respectively; these numbers are excluded in the table as well. This result seems to be the source of low-volatility anomaly and is also consistent with the precedents. Fu (2009) reports that the estimated correlation between the monthly return and the one-month lagged idiosyncratic volatility is −0.016 and its corresponding t-statistic is −2.79, respectively. Fink, Fink and He also present that the same correlation obtained by their sample is 0.0156, which is noticeably lower than 0.1283, the correlation between the return and the contemporaneous idiosyncratic volatility.

These consequences imply that the low-volatility anomaly shown in previous literatures is reasonably suspected. However, the estimated correlation between RET and $E(IVOL^{\mathrm{Out}})$ is −0.04, which is different with Fu (2009); the correlation displayed on his table is 0.09. Fu (2009) concludes that EGARCH idiosyncratic volatility, unlike conventional idiosyncratic volatility, retrieves the positive relation between idiosyncratic risk and return theorized by such as Merton (1987) and Malkiel and Xu (2006). On the other hand, the correlation between $M\_EGARCH(t-1)$ and $RET(t)$ shown by Fink, Fink and He is 0.0089, which is far from 0.1394, the sample correlation between $M\_EGARCH(t)$ and $RET(t)$. Fink, Fink and He bring up a look-ahead bias to account for this discrepancy. The estimated correlation exhibited in this paper is close to the counterpart appeared on Fink, Fink and He. The t-statistic for the correlation coefficient is -7.30.

The rest of the numbers in the first row of Table 4 is consistent with the results demonstrated by past literatures as well. Consistently with Fama and French (1992), the correlation between $BETA$ and RET is −0.00. Furthermore, the average monthly cross-sectional correlation between $\ln(ME)$ and RET is −0.02 and the correlation between $\ln(BE/ME)$ and RET is 0.05, respectively. $RET(-2,-7)$, which accounts for momentum effects of return, is negatively correlated to RET. Since the momentum effects in Korean stock market is evidently ambiguous according to past references, the negative correlation is also reasonable (Kho, 1997; Kim, 2012). The last two columns in the first row of Table 4 display that return is negatively related to $\ln(TURN)$, the liquidity variable. However, the correlation between RET and $\ln(CVTURN)$ is positive. According to the result of Fu (2009), the sign of the correlation is negative, but the estimates are statistically insignificant at 1% level in both cases.

The correlation between $IVOL$ and $E(IVOL^{\text{Out}})$ is 0.23 and statistically significant at 1% level as well. The correlation Fu (2009) exhibits is 0.46, but Fink, Fink and He report that the correlation between $D\_SQRET(t)$ and $M\_EGARCH(t-1)$ is 0.2522, which is close to 0.23 of this paper. This consequence imply that a look-ahead bias can spuriously introduce a strong correlation between $IVOL$ and $E(IVOL)$. The rest of the third row is consistent with the numbers of Fu (2009) except the correlation between $BETA$ and $E(IVOL^{\text{Out}})$. The idiosyncratic volatility is relatively higher for small firms, growth firms and liquid firms. Similarly, the EGARCH idiosyncratic volatility exhibits same signs for these correlations.

4 Main Result

To test whether idiosyncratic volatility determines expected return, Fama and MacBeth (1972) cross-sectional regressions are conducted. As in Fu (2009) and Guo, Kassa and Ferguson (2014), these regressions include several control variables of Fama and French (1992) to distinguish the effects of idiosyncratic volatility. Specifications of the variables are aforementioned in the previous section. Sample data used in this paper include all common stocks listed on KOSPI and KOSDAQ during January 1989 to August 2014; the size of these data is relatively bigger than the samples used in preceding researches. The sample of Yun, Ku and Eom (2011) contains non-financial stocks traded on KOSPI during January 1991 to December 2008. Kang, Lee and Sim (2014) use the data from February 2002 to March 2009. The data of Kho and Kim (2014) include the sample period from January 1990 to December 2012. The cross-sectional regressions can be denoted as below.

$$R_{it} = \boldsymbol{\gamma}_t^{\top}\mathbf{x}_{it} + \varepsilon_{it}, \text{where } \ i = 1,2,\cdots,N_t, t = 1,2,\cdots,T$$

Specifically, R in the left-hand side is the observed return and $\mathbf{x} = (1 \quad BETA \quad \ln(ME) \quad \cdots \quad IVOL)^{\top}$ in the right-hand side is the vector of independent variables; the composition of explanatory variables are different for each model. ε in the right-hand side is a cross-sectional regression residual. As mentioned by Petersen (2009), the standard error of Fama–MacBeth estimator exhibits no bias when the dependent variable does not have a firm effect. This justifies the use of Fama–MacBeth regressions with these data. Fama–MacBeth estimator can be written as below.

$$\hat{\boldsymbol{\gamma}} = \frac{1}{T}\sum_{t=1}^{T}\hat{\boldsymbol{\gamma}}_t$$

For each month t, the above cross-sectional regressions are conducted and the ordinary least squares estimators are computed, respectively. Then, the sample average of 308 monthly estimates is reported on Table 5. To avoid potential bias occurred by heteroskedasticity or autocorrelation, the adjusted standard error estimator proposed by Newey and West (1987) is used to compute corresponding t-statistic for each estimate. Conventional t-test is also conducted, but only the t-statistic with Newey–West standard error is reported in this paper since the estimator is consistent with heteroskedasticity and autocorrelation.

As in preceding literatures such as Fama and French, the market value of common equity in the end of June is utilized to account for subsequent 12 monthly returns. On the other hand, the year-end book value of common equity and the year-end market value of common equity is employed to compute the book-to-market ratio. The ratio is assigned to the next 12 months from the end of June in the subsequent years. The results of Fama–MacBeth regressions are recorded in Table 5. The first model of the table includes three variables that have been regarded as major determinants of expected return by past researches: $BETA$, $\ln(ME)$ and $\ln(BE/ME)$. As in Fama and French and Fu, the cross-sectional returns are seemingly uncorrelated with $BETA$, the proxy for systematic risk. The average cross-sectional coefficient for $BETA$ is 0.38 and statistically insignificant. Contrarily, the coefficient for $\ln(ME)$ is -0.25 and its corresponding t-statistic is -1.73. This result exhibits that the size of each stock is negatively related to its return. The third column of the first row shows the positive relation between expected return and book-to-market ratio. The coefficient for $\ln(BE/ME)$ is 0.94 and statistically significant at 1% level as well; corresponding t-statistic is 4.23. The cross-sectional R-squared for Model 1 is 5.92% on average.

The second model in Table 5 includes additional variables to consider both momentum effect and liquidity effect. The first three columns of the second row demonstrate that the former results are consistent with additional control variables. However, the coefficient for $BETA$ is 0.75 and statistically significant with its t-statistic estimated as 2.44. This result is interesting and even theoretically desirable, but it is inconsistent with the existing empirical findings reported by foregone references. Xu and Zhao (2014) argue that the estimate of systematic risk is able to account for a cross-sectional distribution of return when the estimate is accurately tuned up by idiosyncratic volatility. According to the result of Fu (2009), the t-statistic for the coefficient of $BETA$ is improved with additional control variables; 0.08 for the first model is increased to 0.93 for the second model. However, the argument on systematic risk is far from the major concern of this paper, and hence the result is not closely reviewed. The signs of the rest two coefficients are parallel to the ones appeared on the previous model.

According to the momentum studies in Korean stock market, the momentum effect in Korean market is randomly vary with time and its existence is doubtful (Kho, 1997; Park and Jee, 2006; Kim, 2012). The coefficient for $RET(-2,-7)$ and its t-statistic are -0.68 and -1.48, respectively. Both Fu and Guo, Kassa and Ferguson report that the momentum effect is positive and statistically significant. The second model in Table 5 of Fu shows that the corresponding coefficient is 0.64 and its t-statistic is 3.09. Similarly, the third model in Table 5 of Guo, Kassa and Ferguson exhibits that the coefficient is 0.702 and its t-statistic is 3.987. This discrepancy seems to come from different samples reflecting local characteristics of Korean market.

The last two columns of the second row report the coefficients for liquidity variables. According to liquidity studies such as Chordia, Subrahmanyam and Anshuman (2001), investors prefer liquid stocks to illiquid ones and which is why illiquid stocks should be compensated by their expected returns. Liquidity effects are controlled by adding associated variables as well. Identically to Chordia, Subrahmanyam and Anshuman, $TURN$ is the time-serial mean turnover in the previous 36 months. Similarly, $CVTURN$ is corresponding coefficient of variation as well. As anticipated, the result shows that two liquidity variables seem to be negatively related to cross-sectional expected return. The coefficient for $\ln(TURN)$ is -0.76 and corresponding t-statistic is -6.17. Also, the coefficient for $\ln(CVTURN)$ is -0.33 and corresponding t-statistic is -1.54. The average R-squared for Model 2 is 9.64%.

$E(IVOL^{\mathrm{Out}})$, the main variable of interest, is used as an independent variable with the above-mentioned control variables in the model 3, 4 and 5 of Table 5. Fu exploits an in-sample EGARCH idiosyncratic volatility as an explanatory variable and demonstrates the positive cross-sectional relation between average return and expected idiosyncratic volatility. As already mentioned, however, this in-sample estimate naturally contains a look-ahead bias. Considering the counterarguments of Guo, Kassa and Ferguson and Fink, Fink and He (2012), this paper alters the estimate with an out-of-sample EGARCH idiosyncratic volatility and conducts the regression analyses same with Fu. First of all, $E(IVOL^{\mathrm{Out}})$ is solely regressed on realized return. The third model displays the resulting coefficient. Consistently with the counterarguments, the estimate does not have a positive sign Fu argues. Instead, the average regression coefficient for $E(IVOL^{\mathrm{Out}})$ is -0.03 and its corresponding t-statistic is -4.62, respectively. The time-serial mean of R-squared for Model 3 is 0.80%. This negative sign is consistent with the control variables introduced above as well. When $\ln(ME)$ and $\ln(BE/ME)$ are joined together with $E(IVOL^{\mathrm{Out}})$, the t-statistic for the coefficient is changed to -5.79 and the coefficient remains unchanged. Likewise, the coefficient is not changed when the momentum and liquidity variables are combined to the regression specification. The t-statistic of the coefficient of $E(IVOL^{\mathrm{Out}})$ increases from −5.79 to −5.51. The R-squared estimates for Model 4 and 5 are 5.95% and 9.76%, respectively; the additional contribution made by $E(IVOL^{\mathrm{Out}})$ seems to be marginal.

Instead of $E(IVOL^{\mathrm{Out}})$, Model 6 of Table 5 includes one-month lagged idiosyncratic volatility introduced by Ang et al. (2006) as an independent variable. Based upon the findings of Ang et al., $\hat{\gamma}$ for the one-month lagged $IVOL$ is anticipated to have a negative sign. Consistent with this anticipation, the coefficient for $IVOL_{t-1}$ is -0.14 and its t-statistic is -7.26, respectively. The mean R-squared for Model 5 is 10.48%. This result reconfirms the existence of the low-volatility anomaly that has been verified by earlier studies (Yun, Ku and Eom, 2011; Kang, Lee and Sim, 2014; Kho and Kim, 2014). Fink, Fink and He document equivalent results in Table 5 as well; the regression coefficient for $D\_SQRET(t-1)$ is −0.0254 and statistically significant at 1% level. Similarly, Table 5 of Fu shows a negative sign for the coefficient of $IVOL_{t-1}$.

Lastly, Model 7 of Table 5 combines contemporaneous idiosyncratic volatility as an explanatory variable instead of the lagged volatility. According to the theoretical aspects raised by Levy (1978), Merton (1987) and Malkiel and Xu (2006), a return is positively related to the contemporaneous idiosyncratic volatility when investors are risk-averse and portfolio diversification is imperfect; i.e. investors require an additional compensation for bearing undiversifiable risks. The last row of Table 5 exhibits resulting regression coefficients. The estimated coefficient for $IVOL_t$ is 0.65 and its t-statistic is 11.98. As mentioned by Fu, the genuine relation between expected return and expected idiosyncratic volatility is unobservable since the correlation between unexpected shocks in realized return and realized idiosyncratic volatility is unknown. In spite of this problem, the estimated coefficient can be employed as a benchmark. Interestingly, the coefficient for $\ln(ME)$ in Model 7 increases to 0.56 and statistically significant at 1% level. This result is desirable according to Merton and it is even consistent with the result of Fu as well. Simultaneously, however, this positive coefficient is inconsistent in terms of Fama and French; the correlation between $\ln(ME)$ and $IVOL_t$, which can also be a source of multicollinearity, is negative and statistically significant at 1% level. The average R-squared for the last model is 17.17%, which is definitely superior to the other models in Table 5.

Unlike the results Fu reports, the regression coefficients for $E(IVOL^{\mathrm{Out}})$ and $IVOL_t$ show substantial differences. The coefficient for $E(IVOL)$ documented in Table 5 of Fu is 0.15 with other five control variables and statistically significant at 1% level. Likewise, the coefficient for $IVOL_t$ is 0.31 with its t-statistic estimated as 20.56. On the other hand, the values of the coefficients for $E(IVOL^{\mathrm{Out}})$ and $IVOL_t$ estimated in this paper is −0.03 and 0.65, respectively. This evidence implies that the positive relation between EGARCH idiosyncratic volatility and expected return is invalid as an in-sample EGARCH estimate contains look-ahead bias whereby the results are distorted. Contrarily, this consequence vindicates the counterarguments raised by Guo, Kassa and Ferguson and Fink, Fink and He.

These results suggest that the low-volatility anomaly raised by Ang et al. cannot be reversed by introducing time-varying volatility proposed by Fu when the look-ahead bias is properly excluded. With the Fama–MacBeth cross-sectional regressions, time-serial approaches are also made to conduct Gibbons–Ross–Shanken test, hereafter GRS test, suggested by Gibbons, Ross and Shanken (1989). GRS test statistic can be written as below.

$$\frac{T-N-K}{N}\frac{\widehat{\boldsymbol{\alpha}}^{\top}\widehat{\boldsymbol{\Sigma}}^{-1}\widehat{\boldsymbol{\alpha}}}{1+\widehat{\boldsymbol{\mu}}^{\top}\widehat{\boldsymbol{\Omega}}^{-1}\widehat{\boldsymbol{\mu}}}\sim F_{N,T-N-K}$$

Where T is the length of used time-series, N is the number of portfolios to be tested and K is the number of factors used as independent variables. Since this paper constructs decile portfolios based on $E(IVOL^{\mathrm{Out}})$, N is equal to 10. Next K is equal to 3 as the test uses Fama–French three factors of Fama and French (1993) to compute the alphas of the decile portfolios. $\widehat{\boldsymbol{\alpha}}=(\hat{\alpha}_1 \quad \hat{\alpha}_2 \quad \cdots \quad \hat{\alpha}_N)^{\top}$ is the vector of Fama–French alphas. $\widehat{\boldsymbol{\mu}}=(\hat{\mu}_1 \quad \hat{\mu}_2 \quad \cdots \quad \hat{\mu}_K)^{\top}$ is the vector of sample averages of the Fama–French factors. $\widehat{\boldsymbol{\Sigma}}$ is an $N\times N$ sample covariance matrix obtained from regression residuals and $\widehat{\boldsymbol{\Omega}}$ is a $K\times K$ sample covariance matrix obtained from Fama–French factors. The null hypothesis to be tested with GRS test statistic is $\boldsymbol{\alpha}=\mathbf{0}$. Under this hypothesis and conventional assumptions, GRS statistic follows F-distribution. If the portfolios constructed based on $E(IVOL^{\mathrm{Out}})$ are all efficient in terms of Fama–French three factors, then any additional compensations for the portfolios will be equal to zero on average as well. The findings demonstrated by past references have shown that $\boldsymbol{\alpha}\neq\mathbf{0}$ when the decile portfolios are constructed based on $IVOL_{t-1}$.

Table 6 displays the resulting specifications for these decile portfolios formed based on $E(IVOL^{\mathrm{Out}})$, which is recursively estimated month by month. All the portfolios equally contain 10% of stocks in the market. For instance, Portfolio 1 in Table 6 includes 10% of stocks with the lowest $E(IVOL^{\text{Out}})$s. The returns of that portfolios are computed with both equal-weight scheme and value-weight one. The third row of Table 6 describes the average $E(IVOL^{\text{Out}})$ for these decile portfolios. The level of expected idiosyncratic volatility monotonically increases from 4.92% for the first portfolio to 46.35% for the last portfolio. Similarly, the level of ex-post idiosyncratic volatility monotonically increases from 11.76% to 18.67%. On the other hand, the level of average return for value-weighted portfolio decreases from 0.89% to −0.61% and the decreasing pattern is not monotonic. Likewise, the level of average return for equal-weighted portfolio decreases from 4.56% to −0.18% and the pattern is not monotonic as well. This pattern of returns is also consistent after controlling the factors. The level of Fama–French alpha for the portfolio decreases from 0.29% for Portfolio 1 to −1.42% for Portfolio 10.

Consistently with the results of Table 5, these evidence shows that the returns of stocks are not positively related to their corresponding idiosyncratic volatilities as argued by Fu. Table 6 of Fu shows that the returns of value-weighted decile portfolios increase from 0.90% for the first portfolio to 2.65% for the last one along with the increase of expected idiosyncratic volatility from 3.19% to 36.35%. On the contrary, Table 4 of Fink, Fink and He demonstrates that the returns of value-weighted quintile portfolios sorted on $M\_EGARCH(t-1)$ decrease from 0.33% for the first portfolio to −0.226% for the last portfolio; the volatility for each portfolio is not reported. GRS test statistic for these decile portfolios is estimated as 2.79 and corresponding p-value is 0.26%. The null hypothesis for this GRS test is rejected at 1% significance level. All above things considered, the positive relation between return and idiosyncratic volatility documented by Fu is spurious and the consequences appeared on Table 6 contradicts the findings of Fu, which are based upon the in-sample EGARCH idiosyncratic volatility.

If the counterarguments made by Guo, Kassa and Ferguson and Fink, Fink and He are reasonable, then the results reported in the previous tables will be changed with the use of EGARCH idiosyncratic volatility with look-ahead bias. To check the robustness of these approaches, the full-sample EGARCH idiosyncratic volatility, i.e. $E\left(IVOL^{\text{Full}}\right)$ is exploited. According to Guo, Kassa and Ferguson, the in-sample EGARCH idiosyncratic volatility and the full-sample one equivalently have a look-ahead bias as they incorporate unavailable observations to estimate $\boldsymbol{\theta}_{\text{ML}}$ whereby $E(IVOL)$s are predicted. The panels in Table 7 replicate the forms adopted by previous tables to describe miscellaneous changes occurred by replacing $E(IVOL^{\text{Out}})$ with $E\left(IVOL^{\text{Full}}\right)$.

Panel A displays the descriptive statistics for $E\left(IVOL^{\text{Full}}\right)$. The pooled average of $E\left(IVOL^{\text{Full}}\right)$ is 16.84%, which is close to the average of $E(IVOL^{\text{Out}})$ estimated as 16.39%. Panel B contains sample cross-sectional correlations; the methods to compute these estimates are already introduced in the previous tables. Unlike $E(IVOL^{\text{Out}})$, $E\left(IVOL^{\text{Full}}\right)$ is positively correlated with RET. The monthly average of sample cross-sectional correlations between $E\left(IVOL^{\text{Full}}\right)$ and RET is 0.05 and statistically significant at 1% level. This result is close to the result Fu reports; according to Table 4 of Fu, the average correlation between $E(IVOL)$ and RET is 0.09 and statistically significant at 1% level as well. This consequence also suggests that the positive relation between these two variables is spurious.

Panel C reports three results of Fama–MacBeth regressions. The first model in Panel C of Table 7 exhibits the resulting coefficient when $E\left(IVOL^{\text{Full}}\right)$ is solely included as an independent variable. As anticipated, the estimate has a positive sign. The average regression coefficient for $E\left(IVOL^{\text{Full}}\right)$ is 0.07 and its corresponding t-statistic is 4.01, respectively. The time-serial average of R-squared for Model 1 is 2.18%. These consequences are closer to the findings of Table 5 in Fu than the consequences of Table 5 in this paper. This positive sign is even consistent with several control variables. When $\ln(ME)$ and $\ln(BE/ME)$ are combined with $E\left(IVOL^{\text{Full}}\right)$, the coefficient increases from 0.07 to 0.09 and its t-statistic is also changed to 5.81. Similarly, the coefficient increases from 0.09 to 0.11 when the momentum and liquidity variables are joined further. The corresponding t-statistic is also changed to 7.39. The R-squared estimates for Model 2 and 3 are 6.50% and 10.10%, respectively. These findings are parallel to the results in Table 5 of Fu and the ones in Table 5 of Fink, Fink and He, simultaneously.

Since $E\left(IVOL^{\text{Full}}\right)$ and $E(IVOL^{\text{In}})$ involve unobtainable observations to estimate parameters, the trading scheme that incorporates these variables are inapplicable. However, this paper conduct portfolio analyses to compare the consequences with the ones introduced in the previous tables. Table 8 displays the resulting statistics for the decile portfolios formed based on $E\left(IVOL^{\text{Full}}\right)$. Entire portfolios equally have 10% of stocks traded on the market. The returns of both value-weighted portfolios and equal-weighted portfolios are computed. The third row of Table 8 exhibits the average $E\left(IVOL^{\text{Full}}\right)$ for these decile portfolios. The level of $E\left(IVOL^{\text{Full}}\right)$ monotonically increases from 6.33% for the first portfolio to 41.44% for the last portfolio. Likewise, the level of ex-post idiosyncratic volatility monotonically increases from 10.11% for Portfolio 1 to 20.64% for Portfolio 10.

Along with these increases, the level of average return for value-weighted portfolio also increases from 0.28% to 1.38%. Correspondingly, the level of average return for equal-weighted portfolio increases from −0.04% for the first portfolio to 4.70% for the last portfolio as well. This pattern of returns is also persistent after considering Fama–French factors. The level of Fama–French alpha for the portfolio increases from −0.25% for Portfolio 1 to 1.19% for Portfolio 10. According to these results, investors can construct zero-investment portfolio by buying Portfolio 10 and selling Portfolio 1. The anticipated excessive return for this portfolio in terms of Fama–French factors is 1.44% per each month. GRS test statistic for these decile portfolios is estimated as 1.50 and corresponding p-value is 13.77%. The null hypothesis for this GRS test is not rejected even at 10% significance level. According to these results, $E\left(IVOL^{\mathrm{Full}}\right)$ is not a decent criterion that is able to generate distinct differences in portfolio returns.

From Table 5 to Table 8, cross-sectional and time-series regressions are conducted by using both $E(IVOL^{\mathrm{Out}})$ and $E\left(IVOL^{\mathrm{Full}}\right)$ to test hypotheses based upon Fama–MacBeth coefficients and Gibbons–Ross–Shanken statistics. The findings of Table 1 and Table 2 imply that an idiosyncratic volatility does not follow a random walk process. Unfortunately, however, subsequent consequences suggest that the introduce of an EGARCH idiosyncratic volatility is unable to account for the low-volatility anomaly. Specifically, Table 7 and Table 8 apparently indicate the defect of using $E\left(IVOL^{\mathrm{Full}}\right)$ or $E(IVOL^{\mathrm{In}})$ mentioned by both Guo, Kassa and Ferguson and Fink, Fink and He: a look-ahead bias. Even these outcomes substantially attenuate the contention of Fu because the EGARCH idiosyncratic volatilities used in this paper are computed under the consideration of optimization issues raised by Fu (2010). Fu (2010) refutes the assertions of Guo, Kassa and Ferguson by stating the effects of limiting maximum iterations in estimating parameters. This paper adopts the settings more reliable than the ones used in previous references; maximum number of iterations are extended and Trust Region method is applied instead of Quasi-Newton method. The findings confirm that the low-volatility anomaly is still consistent after controlling a look-ahead bias of an EGARCH idiosyncratic volatility.

Lastly, this paper revisits the effects of return reversal. According to the literatures such as Bali and Cakici (2008) and Huang et al. (2010, 2011), the low-volatility anomaly demonstrated by Ang et al. is inconsistent after controlling the effect of short-term return reversal. As shown in Table 5, an idiosyncratic volatility of individual stock is positively related to contemporaneous realized return. Therefore, the existence of reversal effect can cause the volatility anomaly as well; a stock with both high idiosyncratic volatility and high return in this month tends to exhibit low return driven by the reversal effect in the next month. Fu (2009) conducts portfolio analyses to confirm the effects of short-term return reversal into the low-volatility anomaly. Similarly to Fu (2009), this paper firstly constructs quintile portfolios based upon one-month lagged idiosyncratic volatility. According to relative Korean studies, the expected return for zero-investment portfolio constructed along with lagged idiosyncratic volatility seems to be about 1.5% per month; the average excess return of the zero-investment portfolio that Kang, Lee and Sim document is 1.27% per month and the average return of the portfolio Kho and Kim report is 1.57% per month after excluding potential trading costs.

Table 9 contains the performance of quintile portfolios constructed based upon one-month lagged $IVOL$. As anticipated, the outcomes are consistent with many previous literatures. The value-weighted excess returns of these quintile portfolios decrease from 0.63% for the first portfolio to -1.13% for the last portfolio along with the increase in lagged idiosyncratic volatility from 6.59% to 25.45%. The average excess return of the zero-investment portfolio is 1.76% per each month. Since the portfolio returns reported by Kho and Kim are computed with excluding 0.35% per each transaction, these two results are very similar to each other. Moreover, Fama–French alpha for this portfolio is estimated as 2.00% per each month. Table 9 displays lagged returns for these quintile portfolios to capture the relation between idiosyncratic volatility and contemporaneous return as well. Contrary to subsequent returns, the one-month lagged value-weighted excess returns for the portfolios monotonically increase from 0.43% for Portfolio 1 to 12.62% for Portfolio 5. On the other hand, subsequent idiosyncratic volatility does not exhibit a reversing behavior. For reference, Xu and Zhao argue that the systematic risk reverses over time and that this reversal attenuates the relation between return and beta.

GRS statistic for these quintile portfolios is 5.42 and the null hypothesis is rejected even at 1% significant level. However, only the Fama–French alpha for the last portfolio is statistically significant at 1% level. According to this consequence, the bonus return for the zero-investment portfolio comes not from the superior performance of buying low-volatile stocks, but from the inferior performance of selling high-volatile ones. This phenomenon is commonly observed in previous studies as well (Ang et al., 2006; Fu, 2009; Fink, Fink and He, 2012; Kho and Kim, 2014). These seemingly inefficient stocks account for just 3.6% of the market in terms of their market capitalizations. Meanwhile, the other 96.4% do not exhibit any anomalous phenomenon at a glance. Therefore, the low-volatility anomaly is able to be explained by interpreting the unusual outcomes observed in high-volatile stocks.

To examine this phenomenon, further researches are done by scrutinizing the sample data. As in Fu (2009), the stocks assigned to the last two quintile portfolios are specifically investigated. These stocks are re-sorted along with their one-month lagged returns to control the effect of short-term return reversal and another set of quintile portfolios are reconstructed based upon this classification. Table 10 describes the performances of these quintile portfolios. As anticipated, RET_ts for the quintile portfolios monotonically decrease from 1.53% for Portfolio 1 to −0.69% for Portfolio 5, while RET_{t-1}s for the portfolios monotonically increase from −21.70% to 36.82%; since these portfolios are highly volatile in terms of $IVOL_{t-1}$, the dispersion of their one-month lagged returns is remarkably larger than the same dispersion displayed on Table 9. Similarly, $XRET^{EW}$s and $XRET^{VW}$s decrease across portfolios with the increase in RET_{t-1}s. Especially, the last two columns of Table 10 confirm that these volatile stocks are generally inefficient in terms of Fama–French three factors. These consequences imply the existence of reversal effect in the group of high-volatile stocks and the extent of reversal is considerable enough.

Meanwhile, this scheme involving reversal effects provides only partial explanation to interpret the low-volatility anomaly. According to the findings of Ang et al., the anomaly is consistent with lots of extant anomalies except the reversal effect; refer to Table 7 of Ang et al. in order to find more details. Huang et al. (2010) contend that the winner-minus-loser factor that reflects the reversal effect is able to account for the abnormal return observed in the zero-investment portfolios, but additional portfolio approaches are not done in the study. Similarly with Ang et al., the stocks in the sample of this paper are sorted along with their past one-month returns and then re-sorted based upon their idiosyncratic volatilities to control the effects of short term return reversal. The resulting performances observed from each of 5-by-5 portfolios are displayed on Figure 1 to Figure 4, respectively. According to Figure 1 and Figure 3, the high returns for the low-volatile portfolios are consistently observed.

Table 11 exhibits the performances of quintile portfolios sorted by idiosyncratic volatilities after controlling past one-month returns. Similar to the findings of Table 9, the returns for each of quintile portfolios are monotonically decreases along with the increase in one-month lagged idiosyncratic volatilities; $XRET^{VW}$s and $FF\alpha$ illustrate similar results as well. These consequences imply that the reversal effect partially explains the abnormal pattern of returns captured in idiosyncratic volatility portfolios, but the reversal is not enough to fully account for the low-volatility anomaly.

5 Conclusion

Fu (2009) contends that the negative relation between one-month lagged idiosyncratic risk and expected return reported by Ang et al. (2006) ignores the time-varying property of idiosyncratic volatility and that the genuine positive relation between the variables is able to be restored by introducing EGARCH models to measure expected idiosyncratic risk, as hypothesized by such as Levy (1978), Merton (1987) and Malkiel and Xu (2006). However, the recent arguments insisted by Guo, Kassa and Ferguson (2014) and Fink, Fink and He (2012) point out that look-ahead bias severely distorts the sign of the relation between risk and return. This paper adopts more reliable settings to empirically investigate whether EGARCH idiosyncratic volatility estimates are able to capture cross-sectional expected returns. The finding of this paper suggests following points.

First of all, the behavior of idiosyncratic risk apparently exhibits the autoregressive characteristics as maintained by Fu. In this aspect, past literatures reflecting this feature can be vindicated. However, the positive relation between idiosyncratic risk and expected return is inconsistent with out-of-sample EGARCH estimates. Successive findings show that look-ahead bias of in-sample EGARCH estimates creates a spurious positive relation between risk and return mentioned by Fu. Besides, this result seems to be unaffected by the convergence status that Fu (2010) suspects. By employing more stable settings to estimate parameters, the evidence supports that the arguments of Guo, Kassa and Ferguson and Fink, Fink and He are still valid with the convergence issues as well.

Furthermore, subsequent consequences suggest that the anomalous result reported by Ang et al. (2006) can be partially explained by short-term return reversal in the stocks with high idiosyncratic risk. This finding is consistent with recent references such as Fu, Huang et al. (2010) and Kang, Lee and Sim (2014). However, the abnormality observed in quintile portfolios formed by idiosyncratic volatility still exists after fully controlling one-month reversal effect. The dynamic relation between systematic risk and idiosyncratic risk suggested by Xu and Zhao (2014) and the exploitation of the information implied in options prices proposed by Buss and Vilkov (2012) seems to be promising in further expansion of this study.

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Appendix

Table 1. Autocorrelations of Idiosyncratic Volatilities

This table exhibits the elementary statistics of idiosyncratic volatilities of individual stocks. Firstly, the time-serial means and other statistics for individual stocks are computed and then the cross-sectional means of the corresponding statistics are recomputed. The sample from DataGuide contains all common stocks traded on KOSPI and KOSDAQ during January 1987 to August 2014. $IVOL$, i.e. idiosyncratic volatility, is computed as below.

$$R_{i\tau} - r_\tau = \alpha_{it} + b_{it}(R_{m\tau} - r_\tau) + s_{it}SMB_\tau + h_{it}HML_\tau + \varepsilon_{i\tau}$$

$$IVOL_{it} = \sqrt{\frac{N(t)}{N(t)-4} \times \sum_{\tau=1}^{N(t)} \hat{\varepsilon}_{i\tau}^2}$$

Where $\tau \in t$ for the day, t for the month, i for the firm, m for the market and N for the number of trading days. Daily excess returns are regressed on corresponding factors proposed by Fama and French (1993). Each $IVOL$ requires at least 15 trading days without zero trading volume.

Variable	N	Mean	S.D.	C.V.	Skew.	Autocorrelation at Lags								
						1	2	3	4	5	6	11	12	13
$IVOL$	2,623	15.05	8.41	0.55	1.66	0.45	0.35	0.31	0.28	0.26	0.24	0.16	0.15	0.13
$\ln\left(\frac{IVOL_t}{IVOL_{t-1}}\right)$	2,585	−0.01	0.47	382.26	0.18	−0.40	−0.05	−0.01	−0.01	−0.00	0.00	0.00	0.01	−0.02

Table 2. Dickey–Fuller Tests for Idiosyncratic Volatilities

This table displays the results of Dickey–Fuller test suggested by Dickey and Fuller (1979) to determine whether the time-series of monthly idiosyncratic volatilities follow a random walk process or not. Firstly, γ_1 for each model and corresponding t value are computed for each firm and then the computed statistics across stocks are summarized below. The sample from DataGuide contains all common stocks traded on KOSPI and KOSDAQ during January 1987 to August 2014. The last column of the table introduces the percentage of cases for the rejection of the null hypothesis at 1% significance level; the null hypothesis of Dickey–Fuller test is that the time-series follows a random walk process. Each model requires at least 30 months of observations.

Parameter	N	Mean	Median	Q1	Q3	RW Rejected (%)
Model: $IVOL_{i,t+1} - IVOL_{i,t} = \gamma_{0i} + \gamma_{1i} IVOL_{i,t} + \eta_{i,t}$, where $i = 1,2,\cdots,N$ and $t = 1,2,\cdots,T_i$						
γ_1	2,386	−0.53	−0.53	−0.64	−0.42	93.08
$t(\gamma_1)$	2,386	−7.53	−7.06	−8.48	−5.51	
Model: $\ln IVOL_{i,t+1} - \ln IVOL_{i,t} = \gamma_{0i} + \gamma_{1i} \ln IVOL_{i,t} + \eta_{i,t}$, where $i = 1,2,\cdots,N$ and $t = 1,2,\cdots,T_i$						
γ_1	2,386	−0.52	−0.51	−0.63	−0.41	94.13
$t(\gamma_1)$	2,386	−6.83	−6.93	−8.25	−5.45	

Table 3. Variable Descriptive Statistics

This table shows the major descriptive statistics of panel sample. The sample from DataGuide includes entire common stocks traded on KOSPI and KOSDAQ during January 1989 to August 2014. RET is the monthly raw return recorded in percentages. $XRET$ is the monthly excess return defined as the raw return minus the one-month risk-free return; as in Kho and Kim (2014), the one-month yield of Monetary Stabilization Bonds maturing 364 days is used as the risk-free return. $BETA$, ME and BE/ME are measured as in Fama and French (1992). $BETA$ is the beta measured from the 7-by-7 portfolios constructed by the size and corresponding pre-ranking beta. ME, i.e. the market value of equity, is the total market value recorded in the end of June. BE/ME is the book value of common equity recorded in the end of fiscal year divided by the market value of equity recorded in the end of December. $IVOL$ is the monthly sample variance of regression residuals estimated by daily returns regressed on three factors of Fama and French (1993): $R_m - r$, SMB and HML. $E(IVOL^{\mathrm{Out}})$ is the one-month-ahead out-of-sample conditional variance measured by EGARCH model. $RET(-2,-7)$ is the cumulative return during the month $t-7$ to $t-2$; as in Jegadeesh (1990), the $t-1$ return is excluded to avoid potential errors driven by bid-ask spreads. As in Chordia, Subrahmanyam and Anshuman (2001), $TURN$ and $CVTURN$ are the mean turnover and the coefficient of variation of turnover, respectively; the sample includes past 36 months data. Winsorization is conducted for each month to prevent possible biases caused by extreme observations or recording errors; the smallest and largest 0.5% of the observations for ME, BE/ME, $IVOL$, $RET(-2,-7)$, $TURN$ and $CVTURN$, and the smallest and largest 2.5% of the observations for $E(IVOL^{\mathrm{Out}})$ are altered. As in Fu (2009), observations containing monthly returns greater than 300% are excluded.

Variable	Mean	Std. dev.	Median	Q1	Q3	Skew.	N
RET (%)	1.20	22.09	−0.64	−8.96	8.15	2.70	383,386
$XRET$ (%)	0.75	22.10	−1.09	−9.42	7.73	2.68	383,386
$\ln(1+RET)$ (%)	−1.09	22.93	−0.64	−9.39	7.83	−4.00	383,386
$IVOL$	14.21	8.56	11.78	8.14	18.21	1.77	364,288
$E(IVOL^{\mathrm{Out}})$	16.39	13.84	12.44	8.47	19.23	3.69	303,583
$BETA$	1.02	0.19	1.00	0.92	1.11	2.50	324,462
$\ln(ME)$	10.89	1.56	10.69	9.90	11.67	0.69	369,928
$\ln(BE/ME)$	−0.10	0.88	−0.08	−0.60	0.44	−0.23	351,025
$RET(-2,-7)$	0.09	0.65	−0.02	−0.22	0.23	6.70	368,318
$\ln(TURN)$ (%)	−0.09	1.28	−0.05	−0.89	0.78	−0.42	331,406
$\ln(CVTURN)$	0.31	0.41	0.30	0.05	0.57	0.09	330,553

Table 4. Cross-sectional Correlations

This table illustrates the time-serial averages of cross-sectional correlation coefficients among the variables. Firstly, sample correlation coefficients across stocks are computed for each month and then the averages for the time-series of coefficients are recomputed. The sample from DataGuide includes entire common stocks traded on KOSPI and KOSDAQ during January 1989 to August 2014. The previous table includes all specifications for the variables appeared in this table. The correlations marked with an asterisk represent that the coefficient is significant at 1% level.

Variable	$\ln(1+RET)$	$IVOL$	$E(IVOL^{\text{Out}})$	$BETA$	$\ln(ME)$	$\ln(BE/ME)$	$RET(-2,-7)$	$\ln(TURN)$	$\ln(CVTURN)$
RET	0.94*	0.15*	−0.04*	−0.00	−0.02	0.05*	−0.01	−0.05*	0.01
$\ln(1+RET)$		0.06*	−0.06*	−0.00	0.00	0.06*	−0.00	−0.07*	−0.01
$IVOL$			0.23*	−0.00	−0.35*	−0.08*	−0.00	0.19*	0.18*
$E(IVOL^{\text{Out}})$				0.01	−0.16*	−0.14*	0.03*	0.18*	0.09*
$BETA$					0.05*	0.02*	−0.02*	0.14*	−0.15*
$\ln(ME)$						−0.24*	0.01	−0.19*	−0.41*
$\ln(BE/ME)$							−0.01	−0.24*	0.11*
$RET(-2,-7)$								−0.06*	0.06*
$\ln(TURN)$									−0.12*

Table 5. Fama–MacBeth Regressions of Returns

This table reports the time-serial averages of cross-sectional regression coefficients estimated based on the methodology suggested by Fama and MacBeth (1973). The figures in parentheses denote corresponding t-statistics for the estimates stated above. The sample from DataGuide includes entire common stocks traded on KOSPI and KOSDAQ during January 1989 to August 2014. Definitions of variables contained in this table and winsorizing criteria associated with the variables are already explained in previous tables. To prevent possible underestimation caused by heteroskedasticity or autocorrelation, the standard error of Newey and West (1987) is utilized to keep the consistency of t-statistic. The last column exhibits the time-serial mean of R-squares for respective cross-sectional regression models.

Model	$BETA$	$\ln(ME)$	$\ln(BE/ME)$	$RET(-2,-7)$	$\ln(TURN)$	$\ln(CVTURN)$	$E(IVOL_t^{\mathrm{Out}})$	$IVOL_{t-1}$	$IVOL_t$	$\bar{R}^2$ (%)
1	0.38 (1.06)	−0.25 (−1.73)	0.94 (4.23)							5.92
2	0.75 (2.44)	−0.46 (−3.24)	0.62 (3.69)	−0.68 (−1.48)	−0.76 (−6.17)	−0.33 (−1.54)				9.64
3							−0.03 (−4.62)			0.80
4		−0.30 (−2.10)	0.86 (3.84)				−0.03 (−5.79)			5.95
5		−0.49 (−3.40)	0.58 (3.41)	−0.65 (−1.39)	−0.71 (−5.75)	−0.31 (−1.44)	−0.03 (−5.51)			9.76
6		−0.65 (−4.59)	0.50 (2.89)	−0.64 (−1.37)	−0.62 (−4.87)	−0.18 (−0.81)		−0.14 (−7.26)		10.48
7		0.56 (4.65)	1.34 (7.68)	−0.71 (−1.83)	−1.15 (−9.80)	−0.89 (−3.52)			0.65 (11.98)	17.17

Table 6. Summary Statistics for Portfolios Formed on $E(IVOL^{\mathrm{Out}})$

This table demonstrates the performance of each of decile portfolios formed on $E(IVOL^{\mathrm{Out}})$. $E(IVOL^{\mathrm{Out}})$ is an EGARCH one-month-ahead out-of-sample conditional variance. The variance is recursively estimated and then the decile portfolios are constructed month by month. The sample from DataGuide includes entire common stocks traded on KOSPI and KOSDAQ during January 1989 to August 2014. The first portfolio contains the most stable stocks in terms of $E(IVOL^{\mathrm{Out}})$ and vice versa for the last portfolio. The first row and the second row consist of the decile portfolios' value-weighted and equal-weighted monthly returns, respectively. The third row and the fourth row exhibit the extent of monthly volatility for each portfolio: $E(IVOL^{\mathrm{Out}})$ for EGARCH volatility and $IVOL$ for realized daily volatility converted into monthly volatility. As in Fama and French (1992), $BETA$ is the ex-post beta measured from the seven-by-seven portfolios constructed by the size and the pre-ranking beta. ME, the market value of equity, and BE/ME, the book-to-market ratio, display their pooled medians, respectively. The last row reports the intercept and its t-statistic of three-factor regression model suggested by Fama and French (1993). The Gibbons–Ross–Shanken statistic of Gibbons, Ross and Shanken (1989) for the estimated intercepts is 2.79 and associated p-value is 0.26%, respectively.

Variable	Portfolios Formed on $E(IVOL^{\mathrm{Out}})$									
	Low	2	3	4	5	6	7	8	9	High
Port. $VWRET$	0.89	0.97	1.04	0.92	0.59	0.63	0.57	0.40	0.09	−0.61
Port. $EWRET$	1.56	1.53	1.49	1.43	1.47	1.32	1.08	0.98	0.59	−0.18
$E(IVOL^{\mathrm{Out}})$	4.92	7.21	8.77	10.23	11.81	13.67	16.04	19.47	25.52	46.35
$IVOL$	11.76	11.54	11.95	12.46	13.14	13.83	14.63	15.68	16.80	18.67
$BETA$	1.01	1.02	1.02	1.03	1.03	1.03	1.03	1.03	1.03	1.02
ME (1B, med.)	51.67	59.47	61.51	56.53	52.70	47.22	44.05	40.05	36.47	32.76
BE/ME (med.)	1.14	1.16	1.10	1.06	1.02	0.98	0.92	0.86	0.80	0.76
FF Alphas	0.29	0.37	0.37	0.15	−0.17	−0.19	−0.18	−0.39	−0.78	−1.42
	(1.29)	(1.57)	(1.49)	(0.57)	(−0.80)	(−0.76)	(−0.69)	(−1.18)	(−2.65)	(−4.43)

Table 7. Miscellaneous Statistics for $E(IVOL^{\mathrm{Full}})$

This table consists of three panels containing elementary informations about $E(IVOL^{\mathrm{Full}})$. Panel A shows the major descriptive statistics for $E(IVOL^{\mathrm{Full}})$. Panel B illustrates the time-serial averages of cross-sectional correlation coefficients between $E(IVOL^{\mathrm{Full}})$ and other variables. Panel C reports the time-serial averages of cross-sectional regression coefficients estimated based on the methodology suggested by Fama and MacBeth (1973). The forms of each table reflects the forms of corresponding tables previously suggested; Table 3 for Panel A, Table 4 for Panel B and Table 5 for Panel C, respectively. The sample from DataGuide includes entire common stocks traded on KOSPI and KOSDAQ during January 1989 to August 2014. Previous tables include all specifications for the variables appeared in this table. The correlations of Panel B marked with an asterisk represent that the coefficient is significant at 1% level. The figures of Panel C in parentheses denote corresponding t-statistics for the estimates stated above; the consistent standard error estimator proposed by Newey and West (1987) is applied to compute t-statistics.

Panel A. Descriptive Statistics

Variable	Mean	Std. dev.	Median	Q1	Q3	Skew.	N
$E(IVOL^{\mathrm{Full}})$	16.84	11.51	13.44	9.65	19.98	2.69	372,388

Panel B. Cross-sectional Correlations

Variable	RET	$\ln(1+RET)$	$IVOL$	$BETA$	$\ln(ME)$	$\ln(BE/ME)$	$RET(-2,-7)$	$\ln(TURN)$	$\ln(CVTURN)$
$E(IVOL^{\mathrm{Full}})$	0.05*	−0.02*	0.37*	0.01*	−0.27*	−0.13*	0.02	0.24*	0.11*

Panel C. Fama–MacBeth Regressions of Stock Returns

Model	$\ln(ME)$	$\ln(BE/ME)$	$RET(-2,-7)$	$\ln(TURN)$	$\ln(CVTURN)$	$E(IVOL_t^{\mathrm{Full}})$	$\bar{R}^2$ (%)
1						0.07 (4.01)	2.18
2	0.04 (0.33)	1.26 (6.29)				0.09 (5.81)	6.50
3	−0.17 (−1.34)	0.88 (5.56)	−0.26 (−0.62)	−0.89 (−8.21)	−0.52 (−2.19)	0.11 (7.39)	10.10

Table 8. Summary Statistics for Portfolios Formed on $E(IVOL^{\text{Full}})$

This table demonstrates the performance of each of decile portfolios formed on $E(IVOL^{\text{Full}})$. $E(IVOL^{\text{Full}})$ is an EGARCH one-month-ahead out-of-sample conditional variance. The variance is estimated at once and then the decile portfolios are constructed month by month. The sample from DataGuide includes entire common stocks traded on KOSPI and KOSDAQ during January 1989 to August 2014. The first portfolio contains the most stable stocks in terms of $E(IVOL^{\text{Full}})$ and vice versa for the last portfolio. Previous tables include all specifications for the variables appeared in this table. The last row reports the intercept and its t-statistic of three-factor regression model suggested by Fama and French (1993). The Gibbons–Ross–Shanken statistic for the estimated intercepts is 1.50 and associated p-value is 13.77%, respectively.

Variable	Portfolios Formed on $E(IVOL^{\text{Full}})$									
	Low	2	3	4	5	6	7	8	9	High
Port. $VWRET$	0.28	0.80	0.97	0.44	0.86	0.78	0.69	0.77	1.17	1.38
Port. $EWRET$	−0.04	0.31	0.55	0.48	0.63	0.92	0.89	1.36	2.23	4.70
$E(IVOL^{\text{Full}})$	6.33	8.43	9.95	11.38	12.93	14.75	17.04	20.27	25.71	41.44
$IVOL$	10.11	10.96	11.65	12.44	13.22	14.01	14.99	16.12	17.72	20.64
$BETA$	1.00	1.02	1.03	1.02	1.03	1.03	1.03	1.02	1.03	1.03
ME (1B, med.)	63.11	64.00	61.07	53.04	47.81	42.23	37.88	34.59	29.93	26.93
BE/ME (med.)	1.11	1.07	1.02	0.98	0.94	0.90	0.86	0.81	0.77	0.74
FF Alphas	−0.25 (−1.65)	0.15 (0.92)	0.32 (1.70)	−0.29 (−1.32)	0.12 (0.48)	0.05 (0.16)	−0.08 (−0.22)	−0.01 (−0.03)	0.74 (1.65)	1.19 (1.93)

Table 9. Portfolios Sorted by $IVOL$s

This table exhibits the performance of each of quintile portfolios formed on $IVOL_{t-1}$. $IVOL$ is the monthly sample variance of regression residuals estimated by daily returns regressed on three factors of Fama and French (1993). The variance is recursively estimated and then the quintile portfolios are constructed month by month. The sample from DataGuide includes entire common stocks traded on KOSPI and KOSDAQ during January 1989 to August 2014. The first portfolio contains the most stable stocks in terms of $IVOL$ and vice versa for the last portfolio. N is the number of observations for the pooled sample. RET is the monthly raw return recorded in percentages. $XRET^{\mathrm{VW}}$ is the excess return of value-weighted portfolio. $FF\alpha$ is the intercept estimated by the monthly return of value-weighted portfolio regressed on three factors of Fama and French. Previous tables include all specifications for other variables appeared in this table. The Gibbons–Ross–Shanken statistic for the estimated intercepts is 5.42 and corresponding p-value is 0.01%, respectively.

$IVOL$ portfolio	N	$IVOL_{t-1}$	$IVOL_t$	ME_{t-1}	Mkt. Share	RET_t	$XRET_t^{\mathrm{VW}}$	$FF\alpha_t$	RET_{t-1}	$XRET_{t-1}^{\mathrm{VW}}$	$FF\alpha_{t-1}$
1 (Low)	73,061	6.59	9.64	929.00	45.82	1.57	0.63	0.14 (1.17)	−1.94	0.43	−0.23 (−0.48)
2	73,261	9.69	11.69	503.44	24.83	1.65	0.93	0.28 (1.65)	−1.28	1.94	1.18 (2.14)
3	73,259	12.52	13.32	341.90	16.86	1.67	0.76	0.04 (0.17)	0.10	3.03	2.22 (3.71)
4	73,226	16.46	15.44	180.14	8.89	1.28	0.42	−0.24 (−0.91)	2.40	5.37	4.60 (5.95)
5 (High)	72,849	25.45	20.09	72.96	3.60	0.18	−1.13	−1.86 (−4.97)	7.07	12.62	11.61 (9.71)
5−1								−2.00 (−4.73)			11.84 (10.73)

Table 10. Portfolios of High-$IVOL$ Stocks Sorted by Lagged Returns

This table displays the performance of each of quintile portfolios formed on RET_{t-1}. $IVOL$ is the monthly sample variance of regression residuals estimated by daily returns regressed on three factors of Fama and French (1993). The variance is recursively estimated and then the quintile portfolios are constructed month by month. The sample from DataGuide includes entire common stocks traded on KOSPI and KOSDAQ during January 1989 to August 2014. The first portfolio contains the stocks recorded high returns among others in the previous month and vice versa for the last portfolio. *N* is the number of observations for the pooled sample. RET is the monthly raw return recorded in percentages. $XRET^{\mathrm{EW}}$ and $XRET^{\mathrm{VW}}$ are the excess return of the equal-weighted portfolio and the value-weighted portfolio, respectively. $FF\alpha^{\mathrm{EW}}$ and $FF\alpha^{\mathrm{VW}}$ are the intercepts estimated by the monthly returns of value-weighted and equal-weighted regressed on three factors of Fama and French, respectively. Previous tables include all specifications for other variables appeared in this table. The Gibbons–Ross–Shanken statistic for the estimated intercepts of the equal-weighted portfolio is 5.30 and corresponding p-value is 0.01%, respectively. Also, the Gibbons–Ross–Shanken statistic for the estimated intercepts of the value-weighted portfolio is 4.29 and corresponding p-value is 0.09%, respectively.

RET_{t-1} portfolio	N	RET_{t-1}	RET_t	$XRET_t^{\mathrm{EW}}$	$XRET_t^{\mathrm{VW}}$	$IVOL_{t-1}$	$IVOL_t$	ME_{t-1}	$FF\alpha_t^{\mathrm{EW}}$	$FF\alpha_t^{\mathrm{VW}}$
1 (Low)	28,827	−21.70	1.53	1.04	−0.13	21.53	19.38	98.34	0.77 (1.66)	−0.25 (−0.54)
2	29,366	−6.31	1.33	0.84	−0.99	20.27	17.12	107.19	0.44 (1.39)	−1.20 (−3.01)
3	29,391	2.59	0.96	0.47	−0.66	20.22	16.38	119.44	−0.06 (−0.21)	−1.03 (−2.84)
4	29,360	12.61	0.51	0.02	−0.50	20.60	16.77	138.03	−0.44 (−1.70)	−0.76 (−2.50)
5 (High)	29,131	36.82	−0.69	−1.18	−1.04	22.11	19.09	170.24	−1.49 (−4.63)	−1.12 (−2.52)

Table 11. Portfolios Controlled by Lagged Returns and Sorted by $IVOL$s

This table exhibits the performance of each of quintile portfolios firstly controlled by RET_{t-1} and then formed on $IVOL_{t-1}$. RET is the monthly raw return recorded in percentages. $IVOL$ is the monthly sample variance of regression residuals estimated by daily returns regressed on three factors of Fama and French (1993). The variance is recursively estimated and then the quintile portfolios are constructed month by month. The sample from DataGuide includes entire common stocks traded on KOSPI and KOSDAQ during January 1989 to August 2014. The first portfolio contains the most stable stocks in terms of $IVOL$ and vice versa for the last portfolio, but lagged returns are firstly controlled. N is the number of observations for the pooled sample. $XRET^{\mathrm{VW}}$ is the excess return of value-weighted portfolio. $FF\alpha$ is the intercept estimated by the monthly return of value-weighted portfolio regressed on three factors of Fama and French. Previous tables include all specifications for other variables appeared in this table. The Gibbons–Ross–Shanken statistic for the estimated intercepts is 7.06 and corresponding p-value is 0.003%, respectively.

$IVOL$ portfolio	N	$IVOL_{t-1}$	$IVOL_t$	ME_{t-1}	Mkt. Share	RET_t	$XRET_t^{\mathrm{VW}}$	$FF\alpha_t$	RET_{t-1}	$XRET_{t-1}^{\mathrm{VW}}$	$FF\alpha_{t-1}$
1 (Low)	72,572	7.18	10.19	929.30	45.77	1.72	0.30	0.33 (2.66)	0.36	1.74	1.53 (2.91)
2	73,531	10.15	12.01	485.85	23.93	1.70	0.10	−0.09 (−0.51)	0.72	1.39	1.16 (2.26)
3	73,476	12.82	13.53	327.08	16.11	1.46	0.38	0.12 (0.62)	1.15	1.00	0.74 (1.36)
4	73,493	16.36	15.38	203.15	10.00	1.22	−0.20	−0.45 (−1.77)	1.66	1.52	1.18 (1.97)
5 (High)	72,584	24.18	19.03	85.14	4.19	0.24	−1.62	−1.86 (−5.56)	2.42	4.49	4.11 (4.74)
5−1								−2.19 (−5.59)			2.58 (3.80)

Figure 1. $XRET_t^{VW}$s of Portfolios Sorted by RET_{t-1} and $IVOL$s

This figure displays the present nominal performance of each of 5-by-5 portfolios constructed based upon past one-month returns and one-month lagged idiosyncratic volatilities. Firstly, the stocks are sorted and grouped along with RET_{t-1}s and then each group is reclassified into five subgroups depend on $IVOL_{t-1}$s. RET is the monthly raw return recorded in percentages. $IVOL$ is the monthly sample variance of regression residuals estimated by daily returns regressed on three factors of Fama and French (1993): $R_m - r$, SMB and HML. $IVOL$s are iteratively computed and then the 5-by-5 portfolios are formed and updated month by month. The sample from DataGuide includes entire common stocks traded on KOSPI and KOSDAQ during January 1989 to August 2014: 308 months. The vertical axis on the figure exhibits the average present excess returns for each of value-weighted portfolios; as in Kho and Kim (2014), the one-month yield of Monetary Stabilization Bonds maturing 364 days is used as the risk-free return.

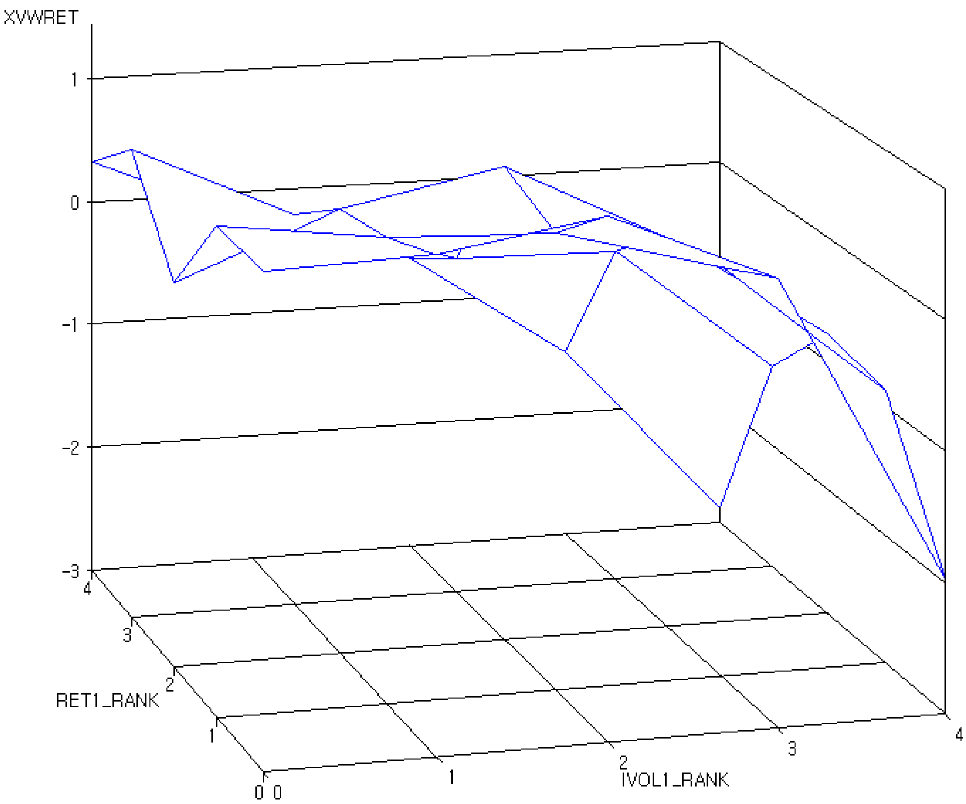


Figure 2. $XRET_{t-1}^{VW}$s of Portfolios Sorted by RET_{t-1} and $IVOL$s

This figure displays the past nominal performance of each of 25 portfolios constructed based upon past one-month returns and one-month lagged idiosyncratic volatilities. Firstly, the stocks are sorted and grouped along with RET_{t-1}s and then each group is reclassified into five subgroups depend on $IVOL_{t-1}$s. RET is the monthly raw return recorded in percentages. $IVOL$ is the monthly sample variance of regression residuals estimated by daily returns regressed on Fama–French three factors. $IVOL$s are iteratively computed and then the 25 portfolios are formed and updated month by month. The sample from DataGuide includes entire common stocks traded on KOSPI and KOSDAQ during January 1989 to August 2014: 308 months. The vertical axis on the figure exhibits the average one-month lagged excess returns for each of value-weighted portfolios; the one-month yield of Monetary Stabilization Bonds maturing 364 days is used as the risk-free return.

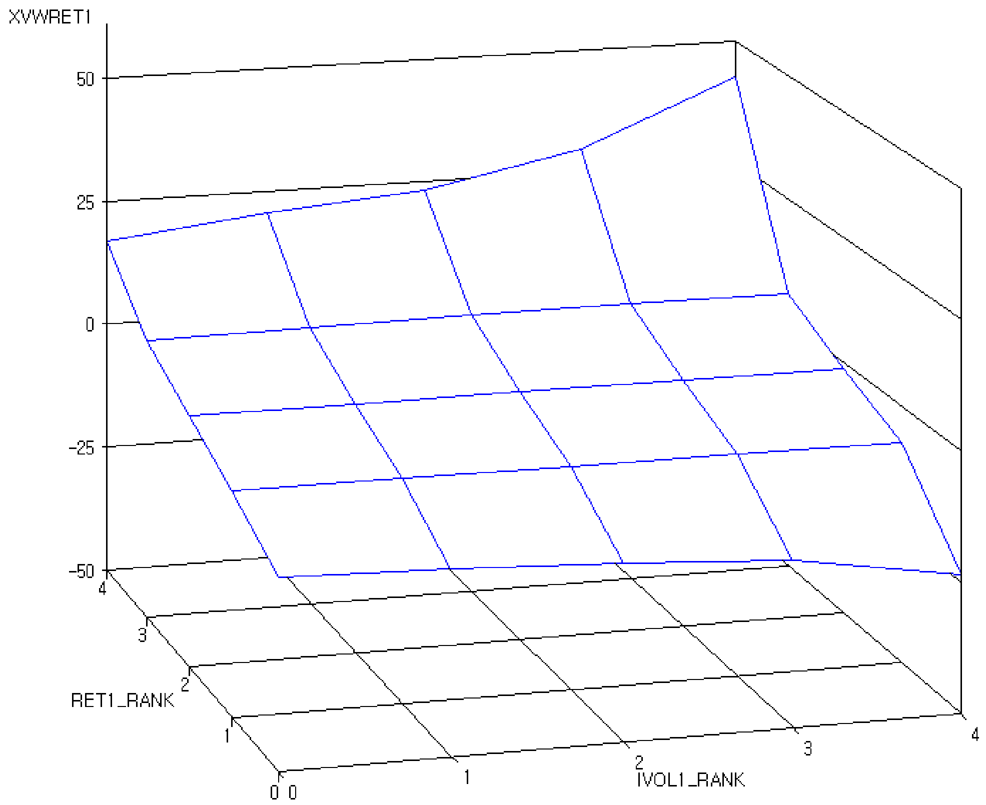


Figure 3. $FF\alpha_t$s of Portfolios Sorted by RET_{t-1} and $IVOL$s

This figure displays the present excess performance of each of 5-by-5 portfolios constructed based upon past one-month returns and one-month idiosyncratic volatilities. Firstly, the stocks are grouped along with RET_{t-1}s and then each group is reclassified into five subgroups depend on $IVOL_{t-1}$s. RET is the monthly raw return recorded in percentages. $IVOL$ is the monthly sample variance of regression residuals estimated by daily returns regressed on Fama–French three factors. $IVOL$s are iteratively computed and then the 5-by-5 portfolios are formed and updated month by month. The sample from DataGuide includes entire common stocks traded on KOSPI and KOSDAQ during January 1989 to August 2014: 308 months. The vertical axis on the figure exhibits the present Fama–French alphas for each of value-weighted portfolios; monthly excess returns of 25 value-weighted portfolios are regressed on Fama–French three factors, $R_m - r$, SMB and HML, and then their OLS intercepts are recorded. The Gibbons–Ross–Shanken statistic for the estimated alphas is 3.04 and corresponding p-value is 0.004%, respectively.

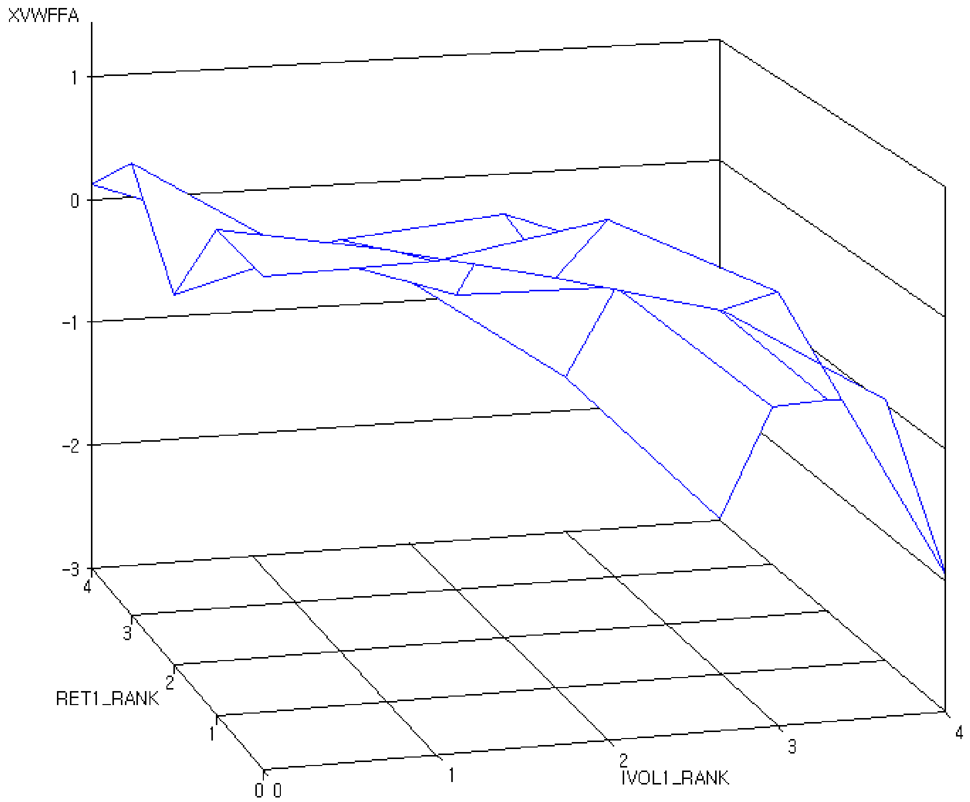
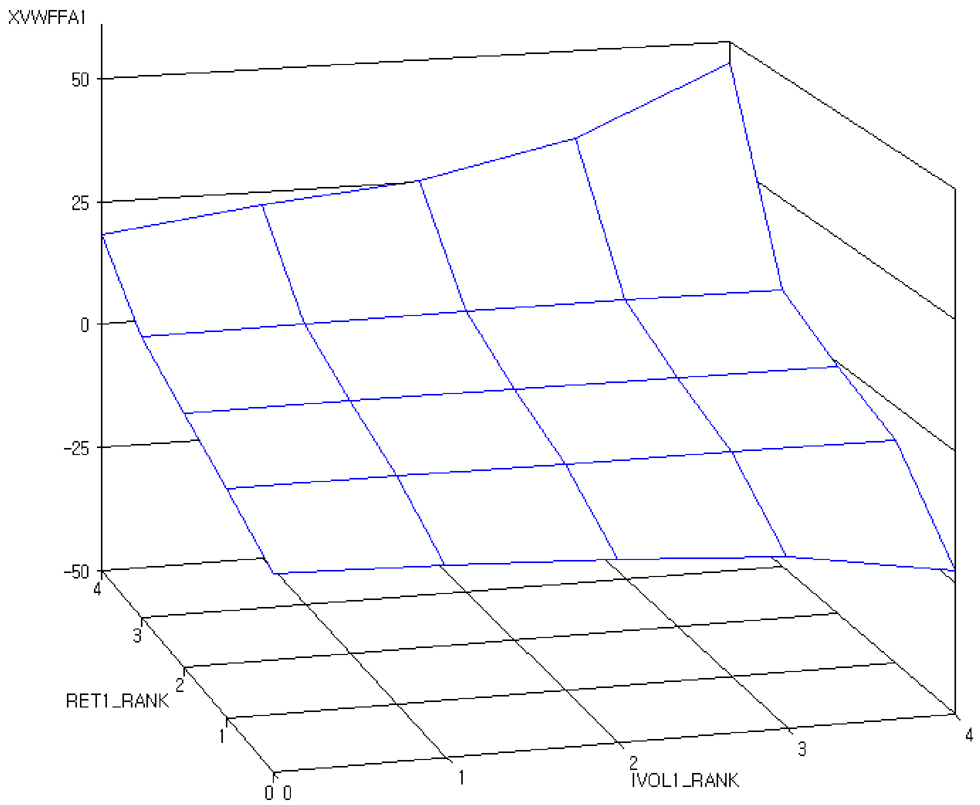


Figure 4. $FF\alpha_{t-1}$s of Portfolios Sorted by RET_{t-1} and $IVOL$s

This figure displays the past excess performance of each of 25 portfolios constructed based upon past one-month returns and one-month idiosyncratic volatilities. Firstly, the stocks are grouped along with RET_{t-1}s and then each group is reclassified into five subgroups depend on $IVOL_{t-1}$s. RET is the monthly raw return recorded in percentages. $IVOL$ is the monthly sample variance of regression residuals estimated by daily returns regressed on Fama–French three factors. $IVOL$s are iteratively computed and then the 25 portfolios are formed and updated month by month. The sample from DataGuide includes entire common stocks traded on KOSPI and KOSDAQ during January 1989 to August 2014: 308 months. The vertical axis on the figure exhibits the one-month lagged Fama–French alphas for each of value-weighted portfolios; monthly excess returns of 25 value-weighted portfolios are regressed on Fama–French three factors and then their OLS intercepts are recorded.



국문초록

조건부 이분산과 기대 수익률의 횡단면

본 논문은 한국 증권 시장에서 고유 변동성과 기대 수익률 간의 횡단면 관계를 규명한다. Fu (2009)의 결과와 마찬가지로, 본 논문의 분석 결과는 월별 고유 변동성에 1차 자기상관이 존재함을 시사한다. 그러나 외 표본을 사용한 EGARCH 고유 변동성 추정치를 사용하는 경우, 위험과 수익률 간의 양의 관계가 나타나지 않음을 보인다. 또한 전 표본을 사용한 변동성 추정치를 대신 사용하면 위험과 수익률 사이의 양의 관계가 관찰됨을 확인한다. 이는 예견 편의가 Fu의 결과와 같은 위험과 수익률 간의 가성적 양의 관계를 유발한다는 Fink, Fink and He (2012)와 Guo, Kassa and Ferguson (2014)의 설명과 일관되며, 시간 가변 변동성 모형의 도입을 통해 Ang et al. (2006)의 저변동성 이례현상을 설명할 수 없음을 시사한다. Huang et al. (2010)이 주장하는 단기 수익률 역전이 이러한 이례현상을 부분적으로 야기하는 듯하지만, 역전을 통제하더라도 위험과 수익률 간 음의 관계는 확고하게 나타난다.

핵심 단어: 고유 변동성, GARCH, 횡단면 수익률, 저변동성 이례현상, 수익률 역전

JEL 분류기호: G12, G14