Idiosyncratic Risk and the Cross-section of Expected Stock Returns

Fangjian Fu (2009,JFE)

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Idiosyncratic risk and the cross-section of expected stock returns ☆

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A B S T R A C T

Theories such as Merton [1987. A simple model of capital market equilibrium with incomplete information. Journal of Finance 42, 483–510] predict a positive relation between idiosyncratic risk and expected return when investors do not diversify their portfolio. Ang, Hodrick, Xing, and Zhang [2006. The cross-section of volatility and expected returns. Journal of Finance 61, 259–299], however, find that monthly stock returns are negatively related to the one-month lagged idiosyncratic volatilities. I show that idiosyncratic volatilities are time-varying and thus, their findings should not be used to imply the relation between idiosyncratic risk and expected return. Using the exponential GARCH models to estimate expected idiosyncratic volatilities, I find a significantly positive relation between the estimated conditional idiosyncratic volatilities and expected returns. Further evidence suggests that Ang et al.'s findings are largely explained by the return reversal of a subset of small stocks with high idiosyncratic volatilities.

Abstract

1. Theoretically,
    - Idiosyncratic Risk ↑ ⇒ Expected Returns ↑
2. Empirically,
    - Idiosyncratic Risk ↑ ⇒ Expected Returns ↓
3. Using the EGARCH models,
    - E(Idiosyncratic Risk) ↑ ⇒ Expected Returns ↑
4. Ang et al's Finding?
    - Largely explained by return reversal

Contents

1. Introduction

2. Idiosyncratic Volatility and Its Time-series Property

3. Cross-section Return Tests

4. The Relation between Return and Lagged Idiosyncratic Volatility

5. Conclusion

High Risk High Return?

- Goetzman and Kumar (2008): # of stocks in reality?
  - Only 1 (25% ↑), No more than 3 (50% ↑), More than 10 (10% ↓)
- Campbell et al. (2001): # of stocks for diversification?
  - 50 stocks are needed to achieve diversification.
- What theory says is
  - Levy (1978), Merton (1987), Malkiel and Xu (2002)
  - Idiosyncratic Risk ↑ ⇒ Expected Return ↑
- What evidence says is
  - Ang et al. (2006), Kho and Kim (2014)
  - Idiosyncratic Risk ↑ ⇒ Expected Return ↓

High Risk High Return? (cont'd)

Reference	Conclusion
Pontiff (2006,JAE)	Idiosyncratic $\sigma \uparrow \Rightarrow$ Holding Cost $\uparrow \Rightarrow$ Arbitrage Profit $\downarrow$
Spiegel and Wang (2006)	EGARCH $\sigma \uparrow$, Liquidity $\downarrow \Rightarrow$ Return $\uparrow$; EGARCH σ is dominant.
Chua, Goh and Zhang (2007)	Expected Idiosyncratic σ by Using AR Model, Positive Relationship
Bali and Cakici (2008,JFQA)	Method-Sensitive σ-μ Relationship $\Rightarrow$ No Empirical Robustness
Ang et al. (2009,JFE)	$\exists$ Idiosyncratic Volatility Anomaly in G7 Countries
Jiang, Xu and Yao (2009,JFQA)	Lack of Information Content $\Rightarrow$ Idiosyncratic $\sigma \uparrow$, Return $\downarrow$
Boyer, Mitton and Vorkink (2010,RFS)	Harvey and Siddique (2000,JF); Idiosyncratic $\sigma \uparrow \Rightarrow$ E(Skewness) $\uparrow$
Huang et al. (2011,JIM)	Volatility anomaly can be explained by return reversal.
Eiling (2013,JF)	Exclude Human Capital $\Rightarrow$ Idiosyncratic σ is priced.
Brockman, Schutte and Yu (2013)	International Setting, EGARCH $\sigma \uparrow \Rightarrow$ Return $\uparrow$

Ang et al. (2006)

The Cross-Section of Volatility and Expected Returns

ANDREW ANG, ROBERT J. HODRICK, YUHANG XING, and XIAOYAN ZHANG*

ABSTRACT

We examine the pricing of aggregate volatility risk in the cross-section of stock returns. Consistent with theory, we find that stocks with high sensitivities to innovations in aggregate volatility have low average returns. Stocks with high idiosyncratic volatility relative to the Fama and French (1993, *Journal of Financial Economics* 25, 2349) model have abysmally low average returns. This phenomenon cannot be explained by exposure to aggregate volatility risk. Size, book-to-market, momentum, and liquidity effects cannot account for either the low average returns earned by stocks with high exposure to systematic volatility risk or for the low average returns of stocks with high idiosyncratic volatility.

Ang et al. (2006,JF)
Idiosyncratic $\text{Volatility}_{t\text{-}1} \uparrow \Rightarrow E(\text{Return}_t) \downarrow$

Low Volatility Puzzle
Round 1

Fu (2009,JF)
(1) Do the findings imply that the relationship between idiosyncratic risk and expected return is negative?
(2) If not necessary, what is the true empirical relation?
(3) If the true relation is not negative, how their findings are explained?

Guo, Kassa and Ferguson (2014)

On the Relation between EGARCH Idiosyncratic Volatility and Expected Stock Returns

Hui Guo, Haimanot Kassa, and Michael F. Ferguson*

Abstract

A spurious positive relation between exponential generalized autoregressive conditional heteroskedasticity (EGARCH) estimates of expected month t idiosyncratic volatility and month t stock returns arises when the month t return is included in estimation of model parameters. We illustrate via simulations that this look-ahead bias is problematic for empirically observed degrees of stock return skewness and typical monthly return time series lengths. Moreover, the empirical idiosyncratic risk-return relation becomes negligible when expected month t idiosyncratic volatility is estimated using returns only up to month $t-1$.

Fu (2009,JFE)
E_{t-1}(Idiosyncratic Volatility$_t$) $\uparrow \Rightarrow$ E(Return$_t$) $\uparrow$

Low Volatility Puzzle

Round 2

Guo, Kassa and Ferguson (2014,JFQA)
(1) The positive idiosyncratic risk-return relation is driven by a look-ahead bias accidentally introduced by standard methods of estimating month t EGARCH idiosyncratic volatility.
(2) When month t EGARCH idiosyncratic volatility is forecast using returns only up through month $t-1$, there is no significant cross-sectional relation between EGARCH idiosyncratic volatility and returns.

1. Introduction

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3. Cross-section Return Tests

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Idiosyncratic Volatility

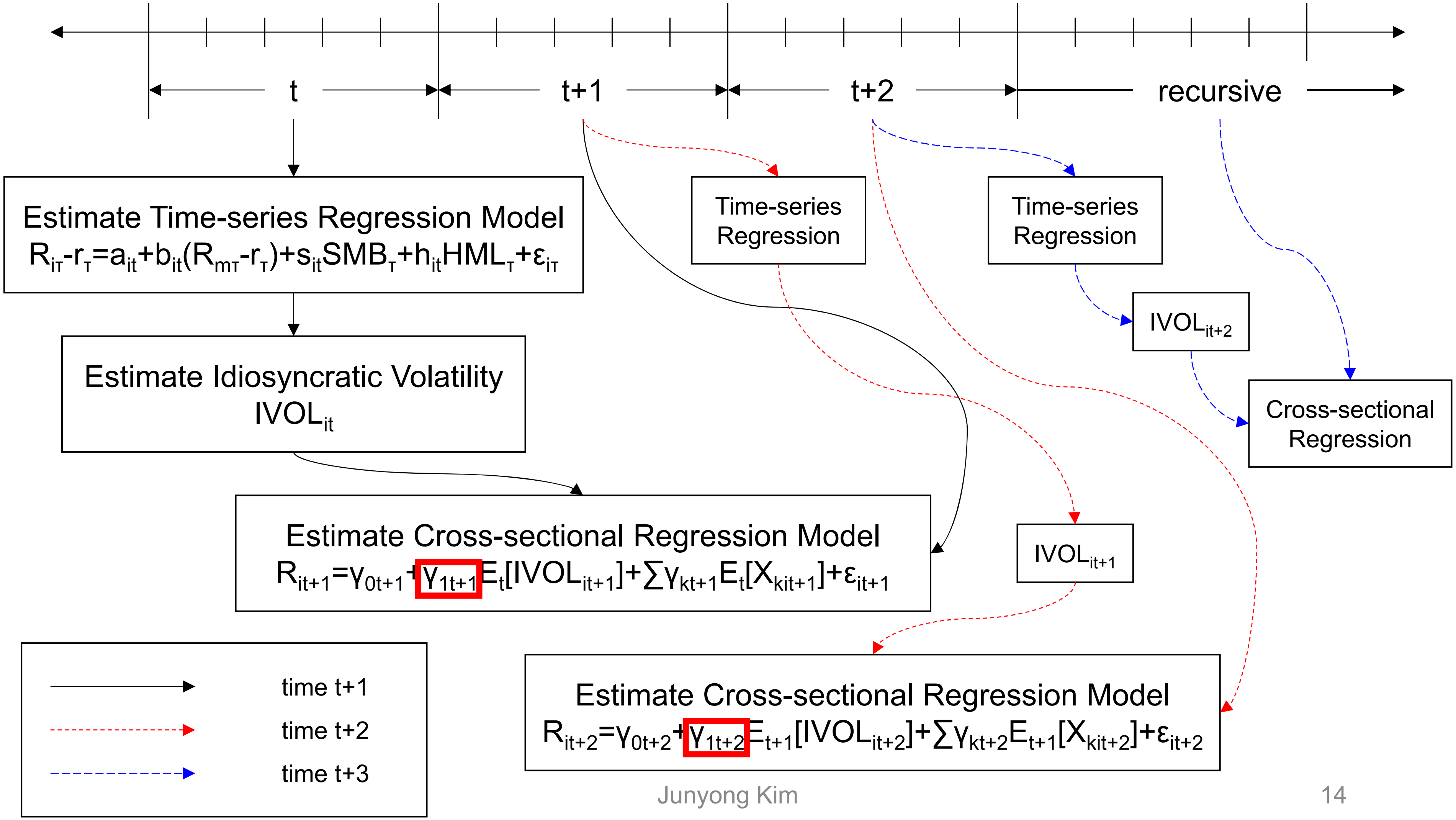

Table 1 Time-series Properties of Idiosyncratic Volatility

	N	Mean	S.D.	C.V.	Skew	Autocorrelation at lags								
						1	2	3	4	5	6	11	12	13
$IVOL$	26,189	16.87	9.94	0.55	1.65	0.33	0.27	0.24	0.20	0.19	0.18	0.12	0.14	0.11
$Ln\left(\frac{IVOL_t}{IVOL_{t-1}}\right)$	26,068	−0.004	0.54	366.24	−0.03	−0.42	−0.04	0.01	−0.02	−0.01	0.01	−0.02	0.03	−0.02

- $\exists$ Compensation for Bearing Idiosyncratic Risk?
  - $R_{it} = \gamma_{0t} + \gamma_{1t}E_{t-1}[IVOL_{it}] + \sum \gamma_{kt}E_{t-1}[X_{kit}] + \varepsilon_{it}$
  - $H_0: \gamma_{1t} = 0, H_1: \gamma_{1t} \neq 0$ (Merton (1987) predict $\gamma_{1t} > 0$)
- $R_{i\tau} - r_\tau = \alpha_{it} + b_{it}(R_{m\tau} - r_\tau) + s_{it}SMB_\tau + h_{it}HML_\tau + \varepsilon_{i\tau}$
  - $\tau \in t$ for the day, t for the month, b_i, s_i, h_i for the factor loadings
- $IVOL_{it} = \sqrt{\frac{\sum \hat{\varepsilon}_{i\tau}^2}{N(t)-4} \times N(t)}$
  - $N(\cdot)$ for the number of trading days
  - Standard deviation of the regression residuals
  - CRSP, NYSE+Amex+Nasdaq, from July 1963 to December 2006
  - Minimum 15 trading days, non-zero trading volume

Table 1 Time-series Properties of Idiosyncratic Volatility (Replication)

	N	Mean	S.D.	C.V.	Skew	Autocorrelation at lags 1	2	3	4	5	6	11	12	13
$IVOL$	2,623	15.05	8.41	0.55	1.66	0.45	0.35	0.31	0.28	0.26	0.24	0.16	0.15	0.13
$Ln\left(\frac{IVOL_t}{IVOL_{t-1}}\right)$	2,585	-0.01	0.47	382.26	0.18	-0.40	-0.05	-0.01	-0.01	-0.00	0.00	0.00	0.01	-0.02

- $R_{i\tau} - r_\tau = \alpha_{it} + b_{it}(R_{m\tau} - r_\tau) + s_{it}SMB_\tau + h_{it}HML_\tau + \varepsilon_{i\tau}$
  - $\tau \in t$ for the day, t for the month, b_i, s_i, h_i for the factor loadings
- $IVOL_{it} = \sqrt{\frac{\sum \hat{\varepsilon}_{i\tau}^2}{N(t)-4} \times N(t)}$
  - $N(\cdot)$ for the number of trading days
  - Standard deviation of the regression residuals
  - Dataguide, KOSPI+KOSDAQ, from January 1987 to August 2014
  - Minimum 15 trading days, non-zero trading volume
- Ang et al.'s Assumption: $E_t(IVOL_{it+1})=IVOL_{it}$, i.e. Random Walk
- Not Appropriate for a Typical Stock's Idiosyncratic Volatility Process

Table 2 Do Monthly Idiosyncratic Volatilities Follow a Random Walk Process

Variables	N	Mean	Median	Q1	Q3	RW rejected (%)
Model: $IVOL_{i,t+1} - IVOL_{i,t} = \gamma_{0i} + \gamma_{1i} IVOL_{i,t} + \eta_i,\quad i = 1, 2, \ldots, N,\ \ t = 1, 2, \ldots, T_i$						
γ_1	20,979	−0.61	−0.60	−0.76	−0.45	89.97
$t(\gamma_1)$	20,979	−6.81	−6.40	−8.43	−4.85	
Model: $LnIVOL_{i,t+1} - LnIVOL_{i,t} = \gamma_{0i} + \gamma_{1i} LnIVOL_{i,t} + \eta_i,\quad i = 1, 2, \ldots, N,\ \ t = 1, 2, \ldots, T_i$						
γ_1	20,979	−0.56	−0.55	−0.70	−0.41	87.81
$t(\gamma_1)$	20,979	−6.38	−5.99	−7.86	−4.51	

*Dickey-Fuller critical t-statistics (from Fuller, 1996)

Sample size	Critical t-statistics (1%)
25	−3.75
50	−3.59
100	−3.50
250	−3.45
500	−3.44

- Dickey-Fuller Test: $E_t(IVOL_{it+1}) = IVOL_{it} \Rightarrow \gamma_{1i} = 0$ (Random Walk)
    - At least 30 months of consecutive observations
- This results suggest that it is not appropriate to describe a typical stock's idiosyncratic volatility process as a random walk.

Table 2 Do Monthly Idiosyncratic Volatilities Follow a Random Walk Process (Replication)

Variables	N	Mean	Median	Q1	Q3	RW rejected (%)
Model: $IVOL_{i,t+1} - IVOL_{i,t} = \gamma_{0i} + \gamma_{1i} IVOL_{i,t} + \eta_{i,t}$						
γ_1	2,386	$\boxed{-0.53}$	-0.53	-0.64	-0.42	$\boxed{93.08}$
$t(\gamma_1)$	2,386	$\boxed{-7.53}$	-7.06	-8.48	-5.51	
Model: $Ln\,IVOL_{i,t+1} - Ln\,IVOL_{i,t} = \gamma_{0i} + \gamma_{1i}\,Ln\,IVOL_{i,t} + \eta_{i,t}$						
γ_1	2,386	$\boxed{-0.52}$	-0.51	-0.63	-0.41	$\boxed{94.13}$
$t(\gamma_1)$	2,386	$\boxed{-6.83}$	-6.93	-8.25	-5.45	

- Dickey-Fuller Test
  - Dickey and Fuller (1979,JASA)
  - Random Walk: $Y_t = \varphi Y_{t-1} + \varepsilon_t$ (H_0: $\varphi=1$) $\Rightarrow$ $\Delta Y_t = \gamma Y_{t-1} + \varepsilon_t$ (H_0: $\gamma=0$)
  - $Y_t = \varphi Y_{t-1} + \varepsilon_t$ ($\varepsilon_t \sim^{i.i.d.} (0,\sigma^2)$) $\Rightarrow$ $\sqrt{T}(\varphi^* - \varphi) \to^d N(0, 1-\varphi^2)$
  - If $\varphi=1$, then $\sqrt{T}(\varphi^* - \varphi) = \sqrt{T}(\varphi^* - 1) \to^p 0$ $\therefore$ Hypothesis Test is Impossible!
- In more than 90% of the stocks, H_0 is rejected at 1% significance level.
- $\exists$ Non-Random-Walk Time-series Structure in Idiosyncratic Volatility

EGARCH Model

$$R_{it} - r_t = \alpha_i + \beta_i(R_{mt} - r_t) + s_i SMB_t + h_i HML_t + \varepsilon_{it}$$

$$\varepsilon_{it} \sim N\left(0, \sigma_{it}^2\right)$$

$$\ln \sigma_{it}^2 = a_i + \sum_{l=1}^{p} b_{i,l} \ln \sigma_{i,t-l}^2 + \sum_{k=1}^{q} c_{i,k} \left\{ \theta \left(\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}} \right) + \gamma \left[\left| \frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}} \right| - (2/\pi)^{1/2} \right] \right\}$$

- Minimum 30 observations
- 1≤p≤3, 1≤q≤3 lag selection through Akaike Information Criteria
- Use SAS to construct the 1-month-ahead out-of-sample forecast of month t EGARCH idiosyncratic volatility
  - Guo, Kassa and Fergusson (2014,JFQA)

EGARCH Model (cont’d)

- Theoretical Literature
  - Engle (1982,EMA): Autoregressive CH
  - Bollerslev (1986,JEM): Generalized ARCH
  - Nelson (1991,EMA): Exponential GARCH
- Empirical Literature
  - French, Schwert and Stambaugh (1987,JFE): GARCH
  - Bollerslev, Engle and Wooldridge (1988,JPE): multivariate
  - Pagan and Schwert (1990,JEM): comparision approach
  - Engle and Mustafa (1992,JEM): based on price of options
  - Engle and Ng (1993,JMCB): Lagrange Multiplier Test

Table 3 Variable Descriptive Statistics for the Pooled Sample

Variables	Mean	Std dev.	Median	Q1	Q3	Skew	*N*
RET (%)	1.18	16.86	0.15	−6.52	6.78	2.35	2,947,826
XRET(%)	0.71	16.87	−0.27	−6.99	6.34	2.35	2,947,826
Ln(1+*RET*) (%)	−0.12	16.09	0.15	−6.74	6.56	−0.54	2,947,826
IVOL	14.17	13.91	10.41	6.33	17.41	6.94	2,946,521
E(*IVOL*)	12.67	10.91	10.29	6.46	15.18	2.38	2,867,821
BETA	1.22	0.36	1.17	0.94	1.46	0.31	1,721,356
Ln(*ME*)	4.29	2.03	4.16	2.82	5.64	0.34	2,804,878
Ln(*BE*/*ME*)	−0.39	1.09	−0.35	−0.97	0.20	0.27	2,145,253
RET(−2,−7)	1.07	0.39	1.03	0.85	1.22	2.90	2,758,743
Ln(*TURN*) (%)	1.39	1.09	1.39	0.67	2.13	−0.08	2,041,658
Ln(*CVTURN*)	4.15	0.44	4.14	3.92	4.48	0.14	2,038,647

Variable	Description	Reference
RET	Monthly Raw Return	
XRET	Monthly Excess Return	
IVOL	Monthly Fama-French Idiosyncratic Volatility (Daily Return)	Ang et al. (2006)
E(IVOL)	Monthly Fama-French Idiosyncratic Volatility (EGARCH)	Fu (2009)
BETA	Portfolio Beta (10×10)	
ME	Market Value of Equity	Fama and French (1992)
BE/ME	Book-to-Market Equity	
RET(-2,-7)	Compound Gross Return from Month t-7 to t-2	Jegadeesh (1990)
TURN	Average Turnover Ratio of the Previous 36 Months	Chordia, Subrahmanyam and Anshuman (2001)
CVTURN	Coefficient of Variation of these 36 Monthly Turnovers	

Table 3 Variable Descriptive Statistics for the Pooled Sample (Replication)

Variables	Mean	Std dev.	Median	Q1	Q3	Skew	N
RET (%)	1.20	22.09	-0.64	-8.96	8.15	2.70	383,386
$XRET$ (%)	0.75	22.10	-1.09	-9.42	7.73	2.68	383,386
$Ln(1 + RET)$ (%)	-1.09	22.93	-0.64	-9.39	7.83	-4.00	383,386
$IVOL$	14.21	8.56	11.78	8.14	18.21	1.77	364,288
$E(IVOL)$	16.39	13.84	12.44	8.47	19.23	3.69	303,583
$BETA$	1.02	0.19	1.00	0.92	1.11	2.50	324,462
$Ln(ME)$	10.89	1.56	10.69	9.90	11.67	0.69	369,928
$Ln(BE/ME)$	-0.10	0.88	-0.08	-0.60	0.44	-0.23	351,025
$RET(-2, -7)$	0.09	0.65	-0.02	-0.22	0.23	6.70	368,318
$Ln(TURN)$ (%)	-0.09	1.28	-0.05	-0.89	0.78	-0.42	331,406
$Ln(CVTURN)$	0.31	0.41	0.30	0.05	0.57	0.09	330,553

- Dataguide, KOSPI+KOSDAQ, from July 1989 to August 2014
  - 2,673 Firms × 302 Months, Unbalanced Panel
  - 522 Months for Fu’s Data
- Data Size: 2,947,826 Returns for Fu’s Data vs. 383,386 Returns for Kim’s Data
- Skewness ≥ 3.00 ⇒ Logarithm
- Winsorization: ME, BE/ME, IVOL, RET(-2,-7), TURN, CVTURN 1%, E(IVOL) 5%
  - To avoid giving extreme observations heavy weight & potential data recording errors

1. Introduction

2. Idiosyncratic Volatility and Its Time-series Property

3. Cross-section Return Tests

4. The Relation between Return and Lagged Idiosyncratic Volatility

5. Conclusion

Table 4 Cross-sectional Simple Correlations

	Ln(1+*RET*)	*IVOL*	E(*IVOL*)	*BETA*	*Ln*(*ME*)	*Ln*(*BE*/*ME*)	*RET*(−2,−7)	*Ln*(*TURN*)	*Ln*(*CVTURN*)
RET	0.98*	0.14*	0.09*	−0.01	−0.01*	0.03*	0.02*	−0.02*	−0.00
Ln(1+*RET*)		0.05*	0.03*	−0.03*	0.02*	0.04*	0.04*	−0.03*	−0.02*
IVOL			0.46*	0.34*	−0.39*	−0.05*	−0.12*	0.16*	0.31*
E(*IVOL*)				0.35*	−0.34*	−0.11*	−0.04*	0.20*	0.30*
BETA					−0.34*	−0.04*	−0.03*	0.41*	0.23*
Ln(*ME*)						−0.21*	−0.03*	0.04*	−0.57*
Ln(*BE*/*ME*)							0.06*	−0.12*	0.16*
RET(−2,−7)								0.00	0.06*
Ln(*TURN*)									0.02*
Ln(*CVTURN*)									

- Correlation between Risk and Return
  - Idiosyncratic Volatility_t ↑ ⇒ Return_t ↑
  - Idiosyncratic Volatility_{t-1} ↑ ⇒ Return_t ↓
  - E_{t-1}(Idiosyncratic Volatility_t) ↑ ⇒ Return_t ↑
- Consistency with the Findings in the Literature
  - Size ↓, Liquidity ↓, BE/ME ↑, Past Return ↑ ⇒ Return ↑
  - The relation between return and BETA is flat. (Fama and French (1992))
- Correlation between Risk and Characteristic
  - Size ↓, BE/ME ↓, BETA ↑, Liquidity ↑ ⇒ E(Idiosyncratic Volatility) ↑
- The correlation between IVOL and E(IVOL) is a significant 0.46.

Table 5 Fama-MacBeth Regressions of Stock Returns on Idiosyncratic Volatility and Firm Characteristics

Model	*BETA*	*Ln(ME)*	*Ln(BE/ME)*	*Ret(−2,−7)*	*Ln(TURN)*	*Ln(CVTURN)*	$E(IVOL_t)$	$IVOL_{t-1}$	$IVOL_t$	$\overline{R^2}$ (%)
1	0.02	−0.12	0.23							3.82
	(0.08)	(−3.11)	(4.97)							
2	0.14	−0.17	0.19	0.64	−0.12	−0.44				5.73
	(0.93)	(−4.52)	(4.38)	(3.09)	(−2.05)	(−6.79)				
3							0.11			3.02
							(9.05)			
4		0.25	0.60				0.13			4.98
		(7.28)	(12.58)				(11.41)			
5		0.19	0.48	0.93	−0.48	−0.73	0.15			6.89
		(5.01)	(10.70)	(4.74)	(−7.34)	(−11.82)	(13.65)			
6		−0.21	0.18	0.67	−0.09	−0.39		−0.02		5.56
		(−5.76)	(4.04)	(3.36)	(−1.24)	(−6.48)		(−3.73)		
7		0.41	0.44	1.61	−0.55	−0.83			0.31	10.42
		(14.53)	(10.57)	(8.55)	(−8.54)	(−13.59)			(20.56)	

1. Size Effect + Book-to-Market Effect
2. Model 1 + Momentum Effect + Liquidity Effect
3. Only Conditional Idiosyncratic Volatility Effect (Hereafter Volatility Effect)
4. Model 1 (Except BETA) + Volatility Effect
5. Model 2 + Volatility Effect
6. Model 2 + Lagged Volatility Effect
7. Model 2 + Contemporaneous Volatility Effect

Guo, Kassa and Ferguson (2014)

TABLE 5

Out-of-Sample EGARCH Idiosyncratic Volatility and Expected Stock Returns

	ln(ME)	ln(BE/ME)	RET(−2, −7)	ln(TURN)	ln(CVTURN)	E(IVOL_O$_t$)	Adj. R^2
Panel A. July 1963–Dec. 2006							
1						0.015 (0.995)	0.014
2	–0.089* (–2.296)	0.211** (3.714)				0.006 (0.475)	0.033
3	–0.145** (–3.693)	0.171** (3.224)	0.702** (3.987)	–0.059 (–0.794)	–0.453** (–6.077)	0.003 (0.460)	0.054
Panel B. Sept. 1931–June 1963							
4						0.028 (1.523)	0.015
5	–0.260** (–3.021)					0.002 (0.130)	0.035
6	–0.309** (–3.912)		0.745 (1.760)	–0.121 (–1.844)	–0.300** (–2.614)	0.004 (0.373)	0.070
Panel C. Sept. 1931–Dec. 2009							
7						0.024* (2.185)	0.015
8	–0.208** (–5.010)					0.002 (0.247)	0.030
9	–0.264** (–6.727)		0.700** (3.307)	–0.089 (–1.808)	–0.384** (–6.259)	0.004 (0.703)	0.056

Table 6 Summary Statistics for Portfolios Formed on Conditional Idiosyncratic Volatility

Variables	Portfolios formed on E(*IVOL*)									
	Low	2	3	4	5	6	7	8	9	High
Port. VWRET	0.90	0.96	0.97	0.98	1.00	1.02	1.17	1.18	1.28	2.65
Port. EWRET	0.54	0.77	0.79	0.80	0.78	0.82	0.85	0.91	1.41	5.33
E(*IVOL*)	3.19	5.17	6.52	7.80	9.19	10.78	12.73	15.34	19.58	36.35
IVOL	6.74	7.80	8.98	10.29	11.80	13.50	15.46	17.72	20.81	27.29
BETA	0.90	1.00	1.08	1.16	1.23	1.29	1.36	1.40	1.44	1.46
ME ($*mil, med*)	113.03	177.16	161.38	119.04	85.80	63.04	45.68	33.83	23.72	14.19
BE/*ME* (*med*)	0.90	0.78	0.75	0.74	0.73	0.71	0.68	0.64	0.59	0.52
FF Alphas	0.03	0.01	−0.02	−0.02	−0.05	−0.06	0.04	0.01	0.13	1.45

- $E(IVOL_{10})-E(IVOL_1)=33.16\%$
- $IVOL_{10}-IVOL_1=20.55\%$
- Value-Weighted Long-Short Return
  - 2.65%-0.90%=1.75%/Month
- Equal-Weighted Long-Short Return
  - 5.33%-0.54%=4.79%/Month
- Fama-French Alpha for Value-Weighted Long-Short Excess Return
  - 1.45%-0.03%=1.42%/Month
- Gibbons-Ross-Shanken Statistic=5.92 (reject H_0)
- This result contrasts sharply with the findings of Ang et al. (2006).

Ang et al. (2006)

Table VI
Portfolio Sorted by Volatility

Rank	Mean	Std. Dev.	% Mkt Share	Size	B/M	CAPM Alpha	FF-3 Alpha
Panel B: Portfolios Sorted by Idiosyncratic Volatility Relative to FF-3							
1	1.04	3.83	53.5%	4.86	0.85	0.11	0.04
						[1.57]	[0.99]
2	1.16	4.74	27.4%	4.72	0.80	0.11	0.09
						[1.98]	[1.51]
3	1.20	5.85	11.9%	4.07	0.82	0.04	0.08
						[0.37]	[1.04]
4	0.87	7.13	5.2%	3.42	0.87	−0.38	−0.32
						[−2.32]	[−3.15]
5	−0.02	8.16	1.9%	2.52	1.10	−1.27	−1.27
						[−5.09]	[−7.68]
5-1	−1.06					−1.38	−1.31
	[−3.10]					[−4.56]	[−7.00]

High Past Volatility ⇒ Low Future Return (Severe)

$$\sqrt{\text{var}(\varepsilon_t^i)}$$

$$r_t^i = \alpha^i + \beta_{MKT}^i MKT_t + \beta_{SMB}^i SMB_t + \beta_{HML}^i HML_t + \boxed{\varepsilon_t^i}$$

Kho and Kim (2014)

〈표 2〉 변동성 포트폴리오의 수익률과 위험조정 수익률

Panel B: 고유변동성(IVol) 포트폴리오의 월 수익률(%)

	일별 고유변동성 포트폴리오						월별 고유변동성 포트폴리오					
	평균 주식수	평균	표준 편차	최대값	최소값	FF3 alpha	평균 주식수	평균	표준 편차	최대값	최소값	FF3 alpha
1. 전체 기간(1990. 1~2012. 12)에 대한 결과												
P1(Lowest)	135	0.77	8.95	56.47	-28.28	0.20	121	0.83	8.43	46.97	-19.96	0.19
P2	138	0.78	9.13	45.63	-23.60	0.02	121	1.13	10.51	69.72	-30.35	0.41
P3	138	1.14	10.58	52.14	-29.75	0.31	121	0.78	10.28	53.64	-30.90	-0.03
P4	138	0.59	11.51	51.68	-32.70	-0.20	121	0.28	10.80	47.56	-34.46	-0.63
P5(Highest)	135	-0.80	12.81	76.62	-42.13	-1.90	118	-0.39	10.83	42.55	-37.63	-1.07
P1-P5		1.57				1.44		1.22				0.59
		(2.37)				(3.06)		(2.47)				(1.34)
2. 하위기간 1(1990. 1~1999. 12)에 대한 결과												
P1-P5		0.42				0.33		0.64				0.03
		(0.35)				(0.52)		(0.75)				(0.04)
3. 하위기간 2(2000. 1~2012. 12)에 대한 결과												
P1-P5		2.45				2.50		1.67				1.44
		(3.36)				(3.88)		(2.88)				(2.81)

High Past Volatility ⇒ Low Future Return (Korean Market)

Table 7 Cross-sectional Relations between the Implied Cost of Capital and Idiosyncratic Volatility

	ICC	IVOL	E(IVOL)
Panel A: Simple correlations			
RET	0.06*	0.14*	0.07*
ICC		0.20*	0.17*
IVOL			0.54*
E(IVOL)			

MODEL	$Ln(ME)$	$Ln(BE/ME)$	$Ret(-2,-7)$	$Ln(TURN)$	$Ln(CVTURN)$	$E(IVOL_t)$	$IVOl_t$	$\overline{R^2}(\%)$
Panel B: Fama-MacBeth regressions (dependent variable: ICC)								
1	−0.05	0.03	−0.18	−0.01	−0.05	0.08		10.09
	(−31.94)	(6.52)	(−24.62)	(−1.60)	(−7.81)	(10.26)		
2	−0.06	0.03	−0.17	−0.02	−0.06		0.09	11.18
	(−31.42)	(6.14)	(−23.94)	(−3.92)	(−7.52)		(11.45)	

- Realized Return (RET) → Implied Cost of Capital (ICC): Consistent
  - Pastor, Sinha and Swaminathan (2008)
- Simple Return (R_t) → Log Return ($\ln(1+R_t)$): Consistent
- OLS Method → GLS Method: Consistent
  - Litzenberger and Ramaswamy (1979)
- Different Methods to Estimate Conditional Idiosyncratic Volatilities: Consistent
  - Chua, Goh and Zhang (2006) and Diavatopoulos, Doran and Peterson (2008,JFM)

1. Introduction

2. Idiosyncratic Volatility and Its Time-series Property

3. Cross-section Return Tests

4. The Relation between Return and Lagged Idiosyncratic Volatility

5. Conclusion

Table 8 Return Dispersion of Portfolios Sorted by Idiosyncratic Volatility

IVOL portfolio	*N*	IVOL (*t*−1)	ME(*t*−1) ($mil)	MKT share (%)	RET (*t*)	VWXRET (*t*)	FF-3F alpha (*t*)	RET (*t*−1)	VWXRET (*t*−1)	FF-3F alpha (*t*−1)
1 (Low)	574,915	4.30	1885.04	43.00	1.10	0.52	0.074 (1.75)	0.44	0.39	−0.04 (−0.94)
2	574,293	7.58	1451.81	33.08	1.34	0.57	0.034 (0.76)	0.55	0.63	0.05 (1.07)
3	574,694	11.06	653.93	14.91	1.37	0.64	0.058 (0.83)	0.61	0.76	−0.02 (−0.13)
4	574,707	16.17	294.70	6.72	1.19	0.29	−0.353 (−3.60)	0.77	0.79	0.26 (2.65)
5 (High)	574,915	32.32	100.44	2.29	1.08	−0.40	−1.146 (−7.00)	4.11	1.66	0.85 (3.14)
5-1							−1.220 (−6.45)			0.89 (3.26)

- Replicate Ang et al.'s results by following their methods and offer an explanation
- High Idiosyncratic σ_t → High R_t → Tend to reverse in the following month

1. Only 2 out of 5 alphas are statistically significant.
   - The other 3 portfolios of stocks with low volatilities do not realize abnormal returns.
2. These 40% of stocks in the high-IVOL portfolios tend to be small firms.
   - Their total market capitalization is only 9% of the whole market.
3. The patterns for the metrics are not monotonically increasing or decreasing.

- Why These Stocks Earn Low Returns in the Subsequent Month?

Table 9 Return Dispersion of High-IVOL Stocks Sorted by the One-month Lagged Return

Portfolio sorted by RET($t-1$)	N	RET($t-1$)	RET(t)	EWXRET(t)	VWXRET(t)	IVOL(t)	ME($t-1$) ($mil)	FF-3F alpha (EWXRET(t))	FF-3 alpha (VWXRET(t))
1 (Low)	232,405	−22.67	3.35	2.84	0.56	28.10	155.23	1.77	−0.33
								(6.55)	(−1.52)
2	223,492	−8.39	1.17	0.93	0.35	21.63	193.63	−0.07	−0.41
								(−0.46)	(−2.67)
3	228,808	0.00	0.90	0.67	0.09	20.32	193.69	−0.48	−0.57
								(−3.62)	(−4.54)
4	233,511	9.15	0.45	0.02	−0.06	19.35	228.15	−0.83	−0.70
								(−7.23)	(−6.02)
5 (High)	231,406	33.78	−0.21	−0.62	−0.10	21.15	216.79	−1.40	−0.69
								(−9.49)	(−5.66)

- Focus on the 40% of “trouble-making” firms (i.e. stocks in Portfolios 4 and 5)
- RET(t-1): increases from -22.67% for the lowest to 33.78% for the highest
- RET(t): decreases from 3.35% to -0.21% monotonically across these portfolios
- The alphas: the negative abnormal returns concentrate in high-past-return firms
- $IVOL_{t-1} \uparrow \Rightarrow RET_{t-1} \uparrow \Rightarrow RET_t \downarrow \because \exists$ Return Reversal
  - Jegadeesh (1990,JF)
  - Huang et al. (2011,JIM)
  - Ang et al. (2006,JF) vs. Bali and Cakici (2008,JFQA)

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Conclusion

- Levy (1978), Merton (1987) says
  - Under-diversification ⇒ High Idiosyncratic Risk, High Return
- Ang et al. (2006) says
  - Empirically, High Idiosyncratic Risk, Low Return
  - One-month Lagged Idiosyncratic Volatility ↓ ⇒ Stock Return ↑
- Fu (2009) says
  - Idiosyncratic Volatility = Time-varying
  - ∴ One-month Lagged Idiosyncratic Volatility ≠ Good Proxy
  - EGARCH Expected Idiosyncratic Volatility = Good Proxy
  - ⇒ High Idiosyncratic Risk, High Return is Observed!
  - Ang et al.'s Finding? Explained by Return Reversal

Q&A Session

Thanks for Listening