On the Relation between EGARCH Idiosyncratic Volatility and Expected Stock Returns

Guo, Kassa and Ferguson (2014, JFQA)

Authors

Hui Guo
University of Cincinnati

Haimanot Kassa
Miami University

Michael F. Ferguson
University of Cincinnati

On the Relation between EGARCH Idiosyncratic Volatility and Expected Stock Returns

Hui Guo, Haimanot Kassa, and Michael F. Ferguson*

Abstract

A spurious positive relation between exponential generalized autoregressive conditional heteroskedasticity (EGARCH) estimates of expected month t idiosyncratic volatility and month t stock returns arises when the month t return is included in estimation of model parameters. We illustrate via simulations that this look-ahead bias is problematic for empirically observed degrees of stock return skewness and typical monthly return time series lengths. Moreover, the empirical idiosyncratic risk-return relation becomes negligible when expected month t idiosyncratic volatility is estimated using returns only up to month $t-1$.

Why Out-of-Sample Estimates?

$$(1) \quad R_{i,t} - r_{f,t} = \alpha_i + \beta_i(R_{m,t} - r_{f,t}) + s_i\mathrm{SMB}_t + h_i\mathrm{HML}_t + \boxed{\varepsilon_{i,t}},$$

$$(2) \quad \varepsilon_{i,t} \sim \boxed{N}(0, \boxed{\sigma^2_{i,t}}),$$

$$(3) \quad \ln \boxed{\sigma^2_{i,t}} = a_i + \sum_{l=1}^{p} b_{i,l} \ln \sigma^2_{i,t-l} + \sum_{k=1}^{q} c_{i,k} \left\{ \theta \left(\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}} \right) + \gamma \left[\left| \frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}} \right| - \left(\frac{2}{\pi} \right)^{½} \right] \right\}.$$

Assume the Distribution of Residual and Process of Conditional Variance

$$(4) \quad \boxed{L}(R_{i,t}) = -\frac{1}{2}\log(2\pi) - \frac{1}{2}\log\left(\sigma^2_{i,t}\right) - \frac{\boxed{\varepsilon^2_{i,t}}}{\boxed{2\sigma^2_{i,t}}}.$$

Since the Distribution is assumed, the likelihood function of residual can be derived.

$$\frac{\partial L(R_{i,t})}{\partial \varepsilon_t} \varepsilon_t = -\frac{\varepsilon_t^2}{\sigma_t^2} \boxed{< 0}, \quad \frac{\partial L(R_{i,t})}{\partial \sigma_t} \sigma_t = \frac{\varepsilon_t^2}{\sigma_t^2} \boxed{> 0}$$

$$(4) \quad L(R_{i,t}) = -\frac{1}{2}\log(2\pi) - \frac{1}{2}\log\left(\sigma_{i,t}^2\right) - \frac{\boxed{\varepsilon_{i,t}^2}}{2\sigma_{i,t}^2}.$$

The joint likelihood function is strictly governed by time t residual.

$$(5) \quad \sum_{\tau=1}^{\boxed{t}} L(R_{i,\tau}) = -\frac{t}{2}\log(2\pi) - \frac{1}{2}\sum_{\tau=1}^{t}\log(\sigma_{i,\tau}^2) - \sum_{\tau=1}^{\boxed{t}} \frac{\varepsilon_{i,\tau}^2}{2\sigma_{i,\tau}^2}.$$

$$(6) \quad \mathrm{E}(\mathrm{IVOL}_t) = \exp(\ln \sigma_{i,t}^2)$$
$$= \exp\left[\boxed{a_{i,t}} + \sum_{l=1}^{p} \boxed{b_{i,l,t}} \ln \sigma_{i,t-l}^2 + \sum_{k=1}^{q} \boxed{c_{i,k,t}} \left\{ \theta\left(\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}}\right) + \gamma\left[\left|\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}}\right| - \left(\frac{2}{\pi}\right)^{1/2}\right]\right\}\right].$$

Then, Estimators are also governed by time t residual as well.

⇒ As a result, $\mathrm{E}(\mathrm{IVOL}_t)$ is distortedly measured.

$$(7) \quad \sum_{\tau=1}^{\boxed{t-1}} L(R_{i,\tau}) = -\frac{t-1}{2}\log(2\pi) - \frac{1}{2}\sum_{\tau=1}^{t-1}\log(\sigma_{i,\tau}^2) - \sum_{\tau=1}^{\boxed{t-1}} \frac{\varepsilon_{i,\tau}^2}{2\sigma_{i,\tau}^2}.$$

$$(8) \quad \mathrm{E}(\mathrm{IVOL\_O}_t) = \exp(\ln \sigma_{i,t}^2)$$
$$= \exp\left[\boxed{a_{i,t-1}} + \sum_{l=1}^{p} \boxed{b_{i,l,t-1}} \ln \sigma_{i,t-l}^2 + \sum_{k=1}^{q} \boxed{c_{i,k,t-1}} \left\{ \theta\left(\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}}\right) + \gamma\left[\left|\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}}\right| - \left(\frac{2}{\pi}\right)^{1/2}\right]\right\}\right].$$

Instead, time t residual should be excluded from the joint likelihood function.

⇒ Distortion in $\mathrm{E}(\mathrm{IVOL}_t)$ will be eliminated.

TABLE 4

In-Sample EGARCH Idiosyncratic Volatility and Expected Stock Returns

	$E(IVOL_t)$	$E(IVOL_{t-1})$	$\Delta_1 E(IVOL_t)$	$\Delta_2 E(IVOL_t)$	$UE(IVOL_t)$	Adj. R^2
Panel A. Simple Returns						
1	0.138** (6.607)					0.030
2		0.000 (0.019)				0.019
3	0.211** (11.746)	–0.125** (–13.040)				0.037
4	0.086** (3.685)		0.125** (13.040)			0.037
5	0.065** (2.641)		0.097** (11.854)	0.078** (9.946)		0.040
6	0.074** (2.990)				0.228** (11.471)	0.039
Panel B. Log Returns						
7	0.019 (0.983)					0.026
8		–0.070** (–4.068)				0.021
9	0.094** (5.881)	–0.129** (–13.258)				0.033
10	–0.035 (–1.553)		0.129** (13.258)			0.033
11	–0.057* (–2.365)		0.100** (12.376)	0.083** (10.580)		0.037
12	–0.050* (–2.066)				0.238** (12.005)	0.035

Look-ahead Bias → Cause Positive Relation!

TABLE 5

Out-of-Sample EGARCH Idiosyncratic Volatility and Expected Stock Returns

	ln(ME)	ln(BE/ME)	RET(−2, −7)	ln(TURN)	ln(CVTURN)	E(IVOL_O$_t$)	Adj. R^2
Panel A. July 1963–Dec. 2006							
1						0.015 (0.995)	0.014
2	–0.089* (–2.296)	0.211** (3.714)				0.006 (0.475)	0.033
3	–0.145** (–3.693)	0.171** (3.224)	0.702** (3.987)	–0.059 (–0.794)	–0.453** (–6.077)	0.003 (0.460)	0.054
Panel B. Sept. 1931–June 1963							
4						0.028 (1.523)	0.015
5	–0.260** (–3.021)					0.002 (0.130)	0.035
6	–0.309** (–3.912)		0.745 (1.760)	–0.121 (–1.844)	–0.300** (–2.614)	0.004 (0.373)	0.070
Panel C. Sept. 1931–Dec. 2009							
7						0.024* (2.185)	0.015
8	–0.208** (–5.010)					0.002 (0.247)	0.030
9	–0.264** (–6.727)		0.700** (3.307)	–0.089 (–1.808)	–0.384** (–6.259)	0.004 (0.703)	0.056

Out-of-Sample Estimates → No Positive Relation

The Objective of Kim (2015)

- To Test Whether Low-volatility Anomaly Can Be Explained by Introducing Time-varying Volatility Model
  - Low-volatility Anomaly: Ang et al. (2006, 2009), Kho and Kim (2014)
  - Time-varying Volatility: Fu (2009), Chua, Goh and Zhang (2010), Brockman, Schutte and Yu (2012)
- To Investigate the Influence of Look-ahead Bias
  - Out-of-Sample Volatility Estimate: Fink, Fink and He (2012), Guo, Kassa and Ferguson (2014)
  - In-Sample: Fu (2009, 2010), Brockman, Schutte and Yu (2012)
- To Check Whether Reversal Can Explain Low-volatility Anomaly
  - Fu (2009), Huang et al. (2010, 2011), Fink, Fink and He (2012), Kang, Lee and Shim (2014)
- To Apply This Research Scheme into Korean Market

Fu (2009)

Table 5 Fama-MacBeth Regressions of Stock Returns on Idiosyncratic Volatility and Firm Characteristics

Model	*BETA*	*Ln(ME)*	*Ln(BE/ME)*	*Ret(−2,−7)*	*Ln(TURN)*	*Ln(CVTURN)*	$E(IVOL_t)$	$IVOL_{t-1}$	$IVOL_t$	$\overline{R^2}$ (%)
1	0.02	−0.12	0.23							3.82
	(0.08)	(−3.11)	(4.97)							
2	0.14	−0.17	0.19	0.64	−0.12	−0.44				5.73
	(0.93)	(−4.52)	(4.38)	(3.09)	(−2.05)	(−6.79)				
3							0.11			3.02
							(9.05)			
4		0.25	0.60				0.13			4.98
		(7.28)	(12.58)				(11.41)			
5		0.19	0.48	0.93	−0.48	−0.73	0.15			6.89
		(5.01)	(10.70)	(4.74)	(−7.34)	(−11.82)	(13.65)			
6		−0.21	0.18	0.67	−0.09	−0.39		−0.02		5.56
		(−5.76)	(4.04)	(3.36)	(−1.24)	(−6.48)		(−3.73)		
7		0.41	0.44	1.61	−0.55	−0.83			0.31	10.42
		(14.53)	(10.57)	(8.55)	(−8.54)	(−13.59)			(20.56)	

1. Size Effect + Book-to-Market Effect
2. Model 1 + Momentum Effect + Liquidity Effect
3. Only Conditional Idiosyncratic Volatility Effect (Hereafter Volatility Effect)
4. Model 1 (Except BETA) + Volatility Effect
5. Model 2 + Volatility Effect
6. Model 2 + Lagged Volatility Effect
7. Model 2 + Contemporaneous Volatility Effect

Table 5. Fama-MacBeth Regressions

Kim (2015)

Model	$BETA$	$\ln(ME)$	$\ln(BE/ME)$	$RET(-2,-7)$	$\ln(TURN)$	$\ln(CVTURN)$	$E(IVOL_t^{Out})$	$IVOL_{t-1}$	$IVOL_t$	$\bar{R}^2$ (%)
1	0.38 (1.06)	-0.25 (-1.73)	0.94 (4.23)							5.92
2	0.75 (2.44)	-0.46 (-3.24)	0.62 (3.69)	-0.68 (-1.48)	-0.76 (-6.17)	-0.33 (-1.54)				9.64
3							-0.03 (-4.62)			0.80
4		-0.30 (-2.10)	0.86 (3.84)				-0.03 (-5.79)			5.95
5		-0.49 (-3.40)	0.58 (3.41)	-0.65 (-1.39)	-0.71 (-5.75)	-0.31 (-1.44)	-0.03 (-5.51)			9.76
6		-0.65 (-4.59)	0.50 (2.89)	-0.64 (-1.37)	-0.62 (-4.87)	-0.18 (-0.81)		-0.14 (-7.26)		10.48
7		0.56 (4.65)	1.34 (7.68)	-0.71 (-1.83)	-1.15 (-9.80)	-0.89 (-3.52)			0.65 (11.98)	17.17

Model	$BETA$	$Ln(ME)$	$Ln(BE/ME)$	$Ret(-2,-7)$	$Ln(TURN)$	$Ln(CVTURN)$	$E(IVOL_t)$	$IVOL_{t-1}$	$IVOL_t$	$\overline{R^2}$ (%)
1	0.02 (0.08)	−0.12 (−3.11)	0.23 (4.97)							3.82
2	0.14 (0.93)	−0.17 (−4.52)	0.19 (4.38)	0.64 (3.09)	−0.12 (−2.05)	−0.44 (−6.79)				5.73
3							0.11 (9.05)			3.02
4		0.25 (7.28)	0.60 (12.58)				0.13 (11.41)			4.98
5		0.19 (5.01)	0.48 (10.70)	0.93 (4.74)	−0.48 (−7.34)	−0.73 (−11.82)	0.15 (13.65)			6.89
6		−0.21 (−5.76)	0.18 (4.04)	0.67 (3.36)	−0.09 (−1.24)	−0.39 (−6.48)		−0.02 (−3.73)		5.56
7		0.41 (14.53)	0.44 (10.57)	1.61 (8.55)	−0.55 (−8.54)	−0.83 (−13.59)			0.31 (20.56)	10.42

Table V. Equal-Weighted Fama-MacBeth Regressions with Fu (2009)'s Specification

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$M\_EGARCH(T)$ (×10)	2.402*** (14.049)						
$M\_EGARCH(t)$ (×10)		3.181*** (18.936)					
$D\_SQRET(t)$ (×10)			3.193*** (10.041)				
$M\_EGARCH(t–1)$ (×10)				0.009 (0.230)			
$M\_SQRET(t–1)$ (×10)					−0.153 (−1.196)		
$D\_SQRET(t–1)$ (×10)						−0.254*** (−3.654)	
$D\_SQRET\_ARMA(t–1)$ (×10)							−0.159 (−1.195)
LNME	0.223***	0.310***	0.407***	−0.161***	−0.148***	−0.179***	−0.174***
	(5.251)	(6.592)	(10.630)	(−3.616)	(−4.125)	(−4.239)	(−4.795)
LNBEME	0.462***	0.565***	0.387***	0.183***	0.176***	0.170***	0.164***
	(8.277)	(9.920)	(6.886)	(3.288)	(3.349)	(3.069)	(3.090)
CRETURN	0.007***	0.006***	0.013***	0.006***	0.006***	0.006***	0.006***
	(4.410)	(3.466)	(7.140)	(3.263)	(3.249)	(3.538)	(3.733)
LNTURN	−0.581***	−0.707***	−0.552***	−0.172**	−0.162**	−0.137*	−0.141**
	(−9.017)	(−10.547)	(−8.344)	(−2.284)	(−2.518)	(−1.861)	(−2.0526)
LNCVTURN	−0.773***	−0.859***	−0.791***	−0.421***	−0.402***	−0.364***	−0.391***
	(−8.101)	(−8.862)	(−8.537)	(−5.551)	(−6.007)	(−4.929)	(−5.686)
R^2	0.071	0.087	0.102	0.052	0.051	0.056	0.058

Look-ahead Bias → Positive Relation

No Look-ahead Bias → Negative or No Relation

Fink, Fink and He (2012)

Fu (2009)

Table 6 Summary Statistics for Portfolios Formed on Conditional Idiosyncratic Volatility

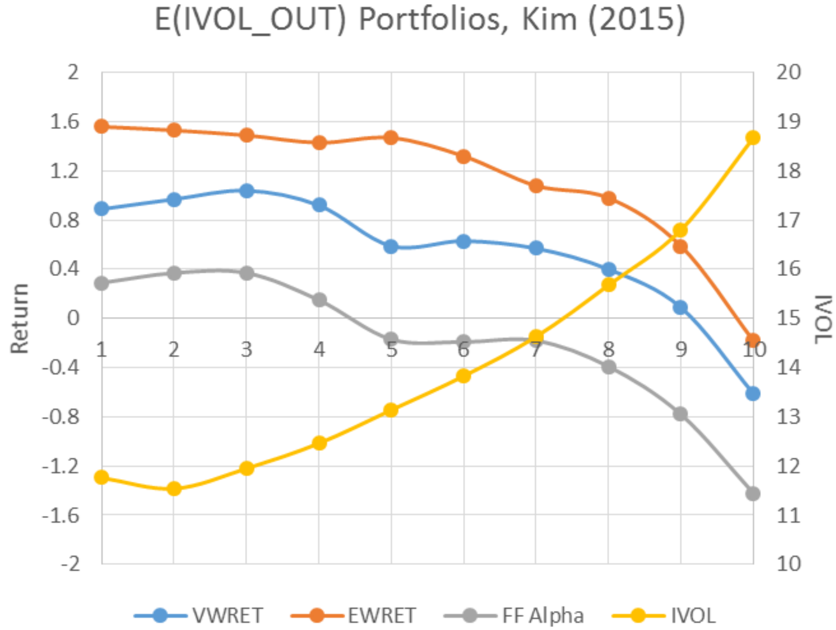
Variables	Portfolios formed on E(*IVOL*)									
	Low	2	3	4	5	6	7	8	9	High
Port. VWRET	0.90	0.96	0.97	0.98	1.00	1.02	1.17	1.18	1.28	2.65
Port. EWRET	0.54	0.77	0.79	0.80	0.78	0.82	0.85	0.91	1.41	5.33
E(*IVOL*)	3.19	5.17	6.52	7.80	9.19	10.78	12.73	15.34	19.58	36.35
IVOL	6.74	7.80	8.98	10.29	11.80	13.50	15.46	17.72	20.81	27.29
BETA	0.90	1.00	1.08	1.16	1.23	1.29	1.36	1.40	1.44	1.46
ME (*$mil, med*)	113.03	177.16	161.38	119.04	85.80	63.04	45.68	33.83	23.72	14.19
BE/ME (*med*)	0.90	0.78	0.75	0.74	0.73	0.71	0.68	0.64	0.59	0.52
FF Alphas	0.03	0.01	−0.02	−0.02	−0.05	−0.06	0.04	0.01	0.13	1.45

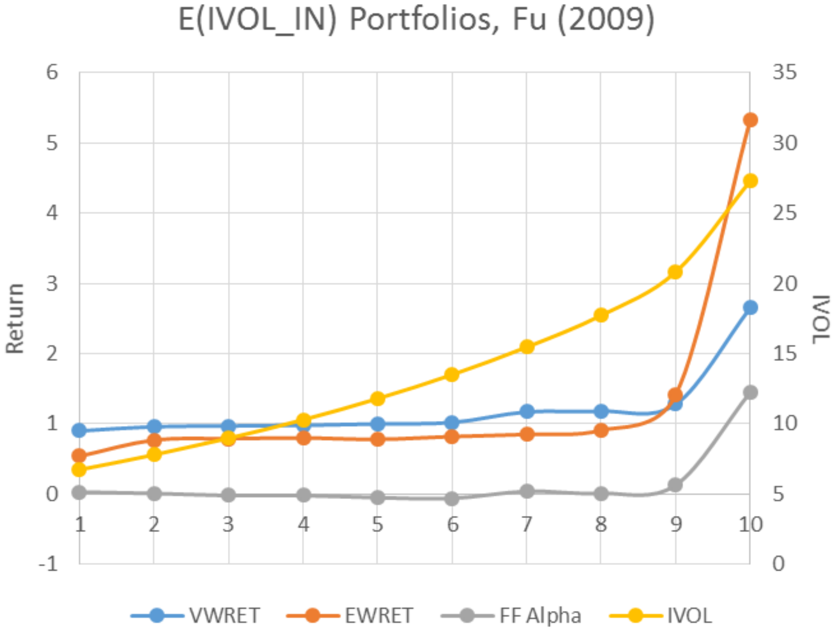
- $E(IVOL_{10})$-$E(IVOL_1)$=33.16%
- $IVOL_{10}$-$IVOL_1$=20.55%
- Value-Weighted Long-Short Return
  - 2.65%-0.90%=1.75%/Month
- Equal-Weighted Long-Short Return
  - 5.33%-0.54%=4.79%/Month
- Fama-French Alpha for Value-Weighted Long-Short Excess Return
  - 1.45%-0.03%=1.42%/Month
- Gibbons-Ross-Shanken Statistic=5.92 (reject H_0)
- This result contrasts sharply with the findings of Ang et al. (2006).

Table 6. Summary Statistics for E(IVOL^{Out}) Portfolios

Kim (2015)

Variable	Portfolios Formed on $E(IVOL^{Out})$									
	Low	2	3	4	5	6	7	8	9	High
Port. $VWRET$	0.89	0.97	1.04	0.92	0.59	0.63	0.57	0.40	0.09	-0.61
Port. $EWRET$	1.56	1.53	1.49	1.43	1.47	1.32	1.08	0.98	0.59	-0.18
$E(IVOL^{Out})$	4.92	7.21	8.77	10.23	11.81	13.67	16.04	19.47	25.52	46.35
$IVOL$	11.76	11.54	11.95	12.46	13.14	13.83	14.63	15.68	16.80	18.67
$BETA$	1.01	1.02	1.02	1.03	1.03	1.03	1.03	1.03	1.03	1.02
ME (1B, med.)	51.67	59.47	61.51	56.53	52.70	47.22	44.05	40.05	36.47	32.76
BE/ME (med.)	1.14	1.16	1.10	1.06	1.02	0.98	0.92	0.86	0.80	0.76
FF Alphas	0.29	0.37	0.37	0.15	-0.17	-0.19	-0.18	-0.39	-0.78	-1.42
	(1.29)	(1.57)	(1.49)	(0.57)	(-0.80)	(-0.76)	(-0.69)	(-1.18)	(-2.65)	(-4.43)





Low-vol is Consistent! Is Fu Wrong?

Table IV. Abnormal Returns of Portfolios Sorted on Idiosyncratic Volatility

Panel A

Variable	Ranking on *M_EGARCH(t)*					
	1 Low	2	3	4	5 High	5–1
Exchange Traded Stocks	0.040	0.020	−0.053	0.012	0.603***	0.563**
	(0.959)	(0.456)	(−0.899)	(0.131)	(2.865)	(2.359)
Controlling for Size	−0.671***	−0.708***	−0.660***	−0.308***	2.354***	3.025***
	(−11.456)	(−12.434)	(−10.872)	(−4.252)	(11.170)	(12.883)
Controlling for Lag Return	0.012	−0.022	−0.087	0.048	0.903***	0.891***
	(0.314)	(−0.510)	(−1.558)	(0.531)	(4.846)	(4.307)
Controlling for Liquidity	0.038	−0.019	−0.107*	0.017	0.766***	0.728***
	(0.903)	(−0.453)	(−1.939)	(0.172)	(4.449)	(3.677)

Variable	Ranking on *M_EGARCH(t−1)*					
	1 Low	2	3	4	5 High	5–1
Exchange Traded Stocks	0.033	0.015	0.0829	−0.010	−0.226*	−0.260*
	(0.707)	(0.345)	(1.5350)	(−0.113)	(−1.924)	(−1.773)
Controlling for Size	−0.007	0.023	−0.0188	−0.023	−0.224**	−0.218*
	(−0.102)	(0.384)	(−0.3045)	(−0.321)	(−2.098)	(−1.711)
Controlling for Lag Return	0.034	0.049	0.0545	−0.031	−0.158	−0.192
	(0.724)	(1.163)	(1.1392)	(−0.415)	(−1.615)	(−1.589)
Controlling for Liquidity	0.071	0.033	0.0217	−0.007	−0.120	−0.192
	(1.501)	(0.758)	(0.4242)	(−0.106)	(−1.109)	(−1.368)

M_EGARCH(t) = In-Sample EGARCH Volatility: ∃ Look-ahead Bias → Positive Relation
M_EGARCH(t-1) = Out-of-Sample EGARCH Volatility: No Look-ahead Bias → Negative Relation
M_EGARCH(T) = Full-Sample EGARCH Volatility: ∃ Look-ahead Bias → Positive Relation
D_SQRET(t) = Contemporaneous Idiosyncratic Volatility: ∃ Look-ahead Bias → Positive Relation
D_SQRET(t-1) = Lagged Idiosyncratic Volatility: No Look-ahead Bias → Negative Relation

Fink, Fink and He (2012)

Table IV. Abnormal Returns of Portfolios Sorted on Idiosyncratic Volatility

Panel B

Variable	1 Low	2	3	4	5 High	5–1
Ranking on *M_EGARCH(T)*						
Exchange Traded Stocks	0.007	−0.002	0.032	−0.000	0.477**	0.470*
	(0.156)	(−0.045)	(0.536)	(−0.005)	(2.262)	(1.923)
Controlling for Size	−0.481***	−0.479***	−0.475***	−0.227***	1.466***	1.946***
	(−8.145)	(−7.536)	(−7.60)	(−3.143)	(7.550)	(8.854)
Controlling for Lag Return	−0.058	0.007	0.036	−0.101	0.639***	0.698***
	(−1.431)	(0.176)	(0.570)	(−1.188)	(3.561)	(3.403)
Controlling for Liquidity	0.054	−0.044	0.036	0.010	0.409**	0.355*
	(1.198)	(−0.998)	(0.510)	(0.110)	(2.250)	(1.684)
Ranking on *D_SQRET(t)*						
Exchange Traded Stocks	0.014	0.064	0.147**	−0.085	0.559	0.573
	(−0.244)	(1.492)	(1.991)	(−0.593)	(1.397)	(1.298)
Controlling for Size	−0.696***	−0.997***	−0.757***	−0.077	2.268***	2.964***
	(−10.137)	(−12.461)	(−10.434)	(−0.860)	(8.643)	(9.780)
Controlling for Lag Return	−0.025	0.065	0.004	0.068	0.438	0.462
	(−0.419)	(1.340)	(0.063)	(0.541)	(1.150)	(1.083)
Controlling for Liquidity	−0.012	0.066	0.032	−0.016	1.090***	1.101***
	(−0.204)	(1.245)	(0.376)	(−0.104)	(3.021)	(2.766)
Ranking on *D_SQRET(t–1)*						
Exchange Traded Stocks	0.084*	0.046	0.032	−0.336***	−1.166***	−1.250***
	(1.844)	(1.170)	(0.492)	(−3.271)	(−6.955)	(−6.319)
Controlling for Size	0.081	0.165**	0.087	−0.151*	−0.924***	−1.005***
	(1.012)	(2.241)	(1.217)	(−1.880)	(−7.132)	(−5.994)
Controlling for Lag Return	0.101**	0.075*	−0.094	−0.239***	−1.125***	−1.226***
	(2.433)	(1.930)	(−1.548)	(−3.085)	(−8.582)	(−8.253)
Controlling for Liquidity	0.102**	0.083*	−0.033	−0.204**	−0.857***	−0.959***
	(1.938)	(1.899)	(−0.526)	(−2.226)	(−6.494)	(−5.957)

Fink, Fink and He (2012)

Table 7. Miscellanies for E(IVOL^{Full})

Kim (2015)

Panel A. Descriptive Statistics

Variable	Mean	Std. dev.	Median	Q1	Q3	Skew.	N
$E(IVOL^{Full})$	16.84	11.51	13.44	9.65	19.98	2.69	372,388

Panel B. Cross-sectional Correlations

Variable	RET	$\ln(1+RET)$	$IVOL$	$BETA$	$\ln(ME)$	$\ln(BE/ME)$	$RET(-2,-7)$	$\ln(TURN)$	$\ln(CVTURN)$
$E(IVOL^{Full})$	0.05*	-0.02*	0.37*	0.01*	-0.27*	-0.13*	0.02	0.24*	0.11*

Panel C. Fama-MacBeth Regressions of Stock Returns

Model	$\ln(ME)$	$\ln(BE/ME)$	$RET(-2,-7)$	$\ln(TURN)$	$\ln(CVTURN)$	$E(IVOL_t^{Full})$	$\bar{R}^2$ (%)
1						0.07 (4.01)	2.18
2	0.04 (0.33)	1.26 (6.29)				0.09 (5.81)	6.50
3	-0.17 (-1.34)	0.88 (5.56)	-0.26 (-0.62)	-0.89 (-8.21)	-0.52 (-2.19)	0.11 (7.39)	10.10

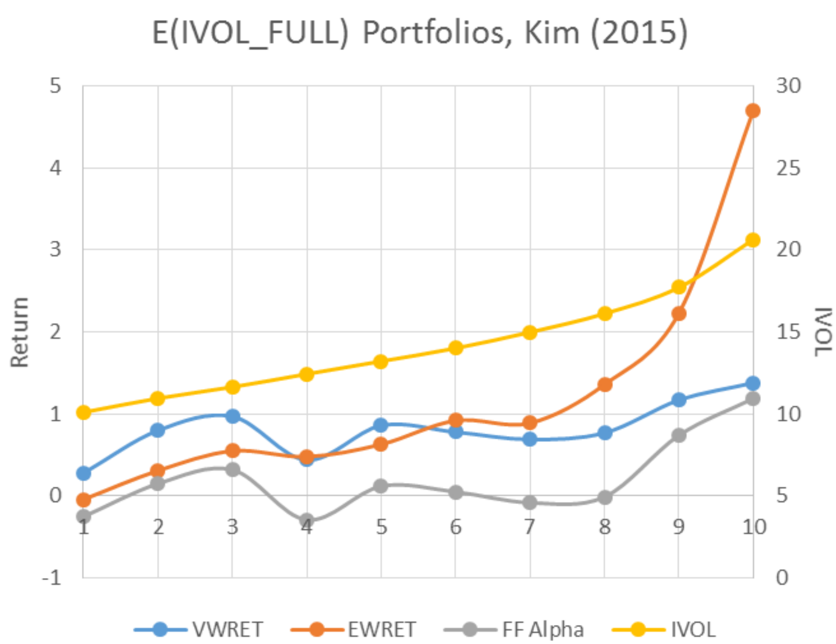
Model	$BETA$	$\ln(ME)$	$\ln(BE/ME)$	$RET(-2,-7)$	$\ln(TURN)$	$\ln(CVTURN)$	$E(IVOL_t^{Out})$	$IVOL_{t-1}$	$IVOL_t$	$\bar{R}^2$ (%)
1	0.38 (1.06)	-0.25 (-1.73)	0.94 (4.23)							5.92
2	0.75 (2.44)	-0.46 (-3.24)	0.62 (3.69)	-0.68 (-1.48)	-0.76 (-6.17)	-0.33 (-1.54)				9.64
3							-0.03 (-4.62)			0.80
4		-0.30 (-2.10)	0.86 (3.84)				-0.03 (-5.79)			5.95
5		-0.49 (-3.40)	0.58 (3.41)	-0.65 (-1.39)	-0.71 (-5.75)	-0.31 (-1.44)	-0.03 (-5.51)			9.76
6		-0.65 (-4.59)	0.50 (2.89)	-0.64 (-1.37)	-0.62 (-4.87)	-0.18 (-0.81)		-0.14 (-7.26)		10.48
7		0.56 (4.65)	1.34 (7.68)	-0.71 (-1.83)	-1.15 (-9.80)	-0.89 (-3.52)			0.65 (11.98)	17.17

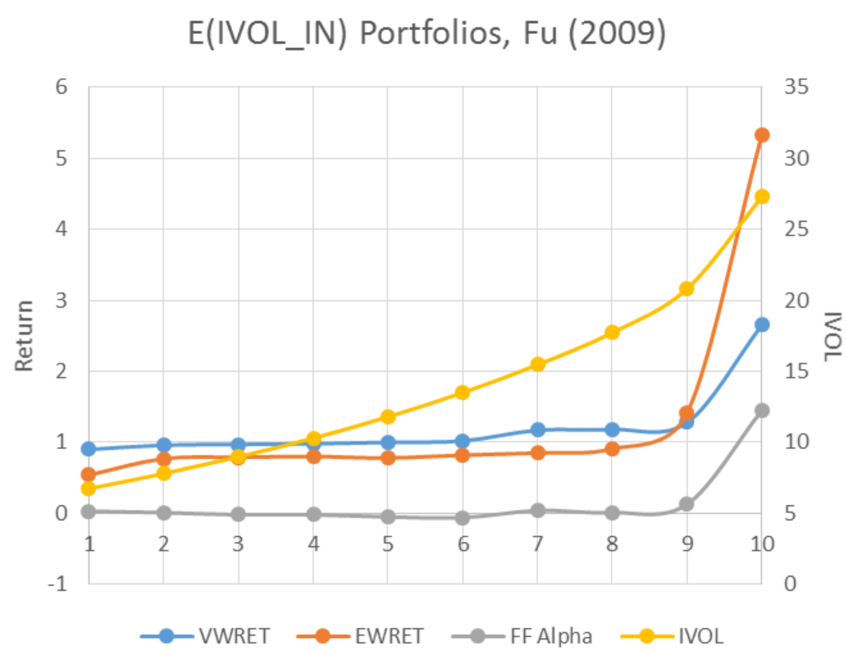
Relation is reversed.

Table 8. Summary Statistics for $E(IVOL^{Full})$ Portfolios

Kim (2015)

Variable	Portfolios Formed on $E(IVOL^{Full})$									
	Low	2	3	4	5	6	7	8	9	High
Port. $VWRET$	0.28	0.80	0.97	0.44	0.86	0.78	0.69	0.77	1.17	1.38
Port. $EWRET$	-0.04	0.31	0.55	0.48	0.63	0.92	0.89	1.36	2.23	4.70
$E(IVOL^{Full})$	6.33	8.43	9.95	11.38	12.93	14.75	17.04	20.27	25.71	41.44
$IVOL$	10.11	10.96	11.65	12.44	13.22	14.01	14.99	16.12	17.72	20.64
$BETA$	1.00	1.02	1.03	1.02	1.03	1.03	1.03	1.02	1.03	1.03
ME (1B, med.)	63.11	64.00	61.07	53.04	47.81	42.23	37.88	34.59	29.93	26.93
BE/ME (med.)	1.11	1.07	1.02	0.98	0.94	0.90	0.86	0.81	0.77	0.74
FF Alphas	-0.25 (-1.65)	0.15 (0.92)	0.32 (1.70)	-0.29 (-1.32)	0.12 (0.48)	0.05 (0.16)	-0.08 (-0.22)	-0.01 (-0.03)	0.74 (1.65)	1.19 (1.93)





Positive relation is spurious!

Table 8 Return Dispersion of Portfolios Sorted by Idiosyncratic Volatility

IVOL portfolio	*N*	IVOL (*t*−1)	ME(*t*−1) ($mil)	MKT share (%)	RET (*t*)	VWXRET (*t*)	FF-3F alpha (*t*)	RET (*t*−1)	VWXRET (*t*−1)	FF-3F alpha (*t*−1)
1 (Low)	574,915	4.30	1885.04	43.00	1.10	0.52	0.074	0.44	0.39	−0.04
							(1.75)			(−0.94)
2	574,293	7.58	1451.81	33.08	1.34	0.57	0.034	0.55	0.63	0.05
							(0.76)			(1.07)
3	574,694	11.06	653.93	14.91	1.37	0.64	0.058	0.61	0.76	−0.02
							(0.83)			(−0.13)
4	574,707	16.17	294.70	6.72	1.19	0.29	−0.353	0.77	0.79	0.26
							(−3.60)			(2.65)
5 (High)	574,915	32.32	100.44	2.29	1.08	−0.40	−1.146	4.11	1.66	0.85
							(−7.00)			(3.14)
5-1							−1.220			0.89
							(−6.45)			(3.26)

- Replicate Ang et al.'s results by following their methods and offer an explanation
- High Idiosyncratic σ_t → High R_t → Tend to reverse in the following month

1. Only 2 out of 5 alphas are statistically significant.
   - The other 3 portfolios of stocks with low volatilities do not realize abnormal returns.
2. These 40% of stocks in the high-IVOL portfolios tend to be small firms.
   - Their total market capitalization is only 9% of the whole market.
3. The patterns for the metrics are not monotonically increasing or decreasing.

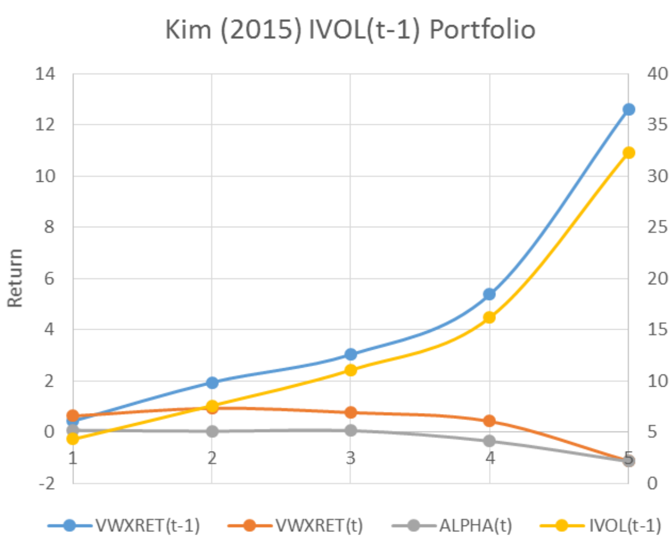
- Why These Stocks Earn Low Returns in the Subsequent Month?

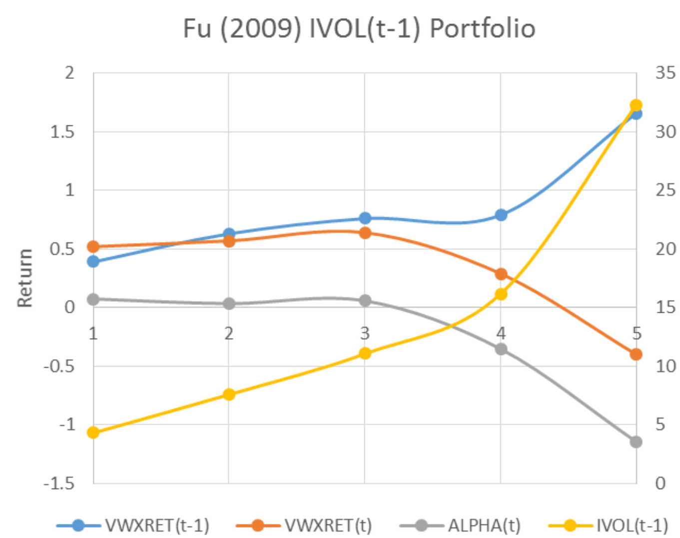
Fu (2009)

Table 9. Portfolios Sorted by Idiosyncratic Volatility

Kim (2015)

$IVOL$ portfolio	N	$IVOL_{t-1}$	$IVOL_t$	ME_{t-1}	Mkt. Share	RET_t	$XRET_t^{VW}$	$FF\alpha_t$	RET_{t-1}	$XRET_{t-1}^{VW}$	$FF\alpha_{t-1}$
1 (Low)	73,061	6.59	9.64	929.00	45.82	1.57	0.63	0.14 (1.17)	-1.94	0.43	-0.23 (-0.48)
2	73,261	9.69	11.69	503.44	24.83	1.65	0.93	0.28 (1.65)	-1.28	1.94	1.18 (2.14)
3	73,259	12.52	13.32	341.90	16.86	1.67	0.76	0.04 (0.17)	0.10	3.03	2.22 (3.71)
4	73,226	16.46	15.44	180.14	8.89	1.28	0.42	-0.24 (-0.91)	2.40	5.37	4.60 (5.95)
5 (High)	72,849	25.45	20.09	72.96	3.60	0.18	-1.13	-1.86 (-4.97)	7.07	12.62	11.61 (9.71)
5-1								-2.00 (-4.73)			11.84 (10.73)





∃ Low-Volatility Anomaly in Korean Market

Fu (2009)

Table 9 Return Dispersion of High-IVOL Stocks Sorted by the One-month Lagged Return

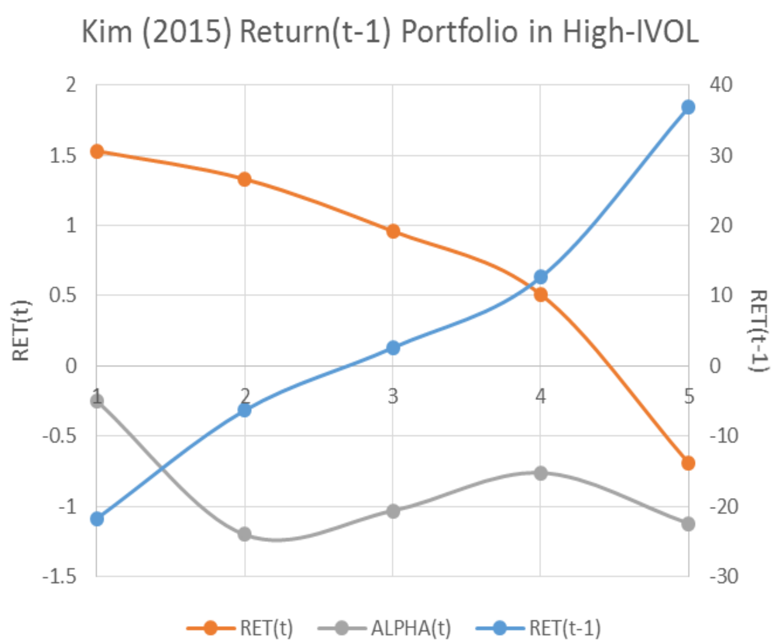
Portfolio sorted by RET(t−1)	N	RET(t−1)	RET(t)	EWXRET(t)	VWXRET(t)	IVOL(t)	ME(t−1) ($mil)	FF-3F alpha (EWXRET(t))	FF-3 alpha (VWXRET(t))
1 (Low)	232,405	−22.67	3.35	2.84	0.56	28.10	155.23	1.77	−0.33
								(6.55)	(−1.52)
2	223,492	−8.39	1.17	0.93	0.35	21.63	193.63	−0.07	−0.41
								(−0.46)	(−2.67)
3	228,808	0.00	0.90	0.67	0.09	20.32	193.69	−0.48	−0.57
								(−3.62)	(−4.54)
4	233,511	9.15	0.45	0.02	−0.06	19.35	228.15	−0.83	−0.70
								(−7.23)	(−6.02)
5 (High)	231,406	33.78	−0.21	−0.62	−0.10	21.15	216.79	−1.40	−0.69
								(−9.49)	(−5.66)

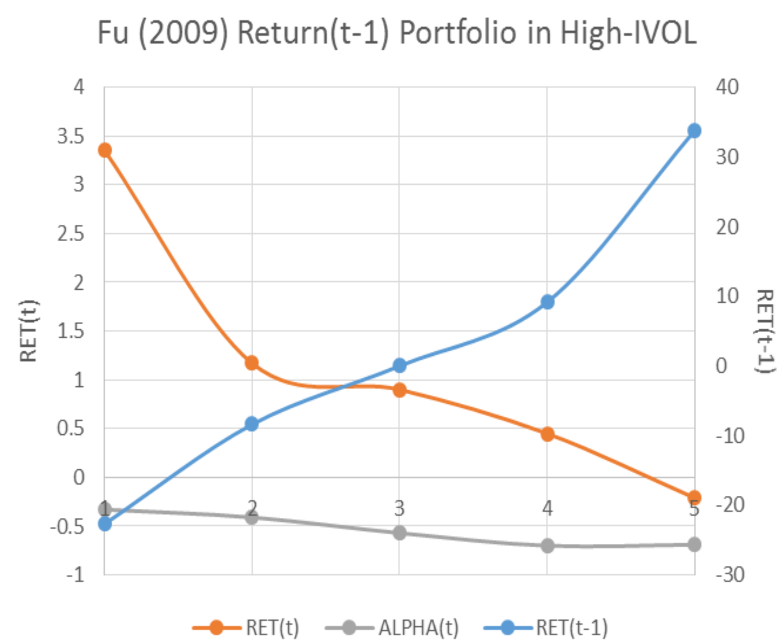
- Focus on the 40% of “trouble-making” firms (i.e. stocks in Portfolios 4 and 5)
- RET(t-1): increases from -22.67% for the lowest to 33.78% for the highest
- RET(t): decreases from 3.35% to -0.21% monotonically across these portfolios
- The alphas: the negative abnormal returns concentrate in high-past-return firms
- $IVOL_{t-1} \uparrow \Rightarrow RET_{t-1} \uparrow \Rightarrow RET_t \downarrow \because \exists$ Return Reversal
    - Jegadeesh (1990,JF)
    - Huang et al. (2011,JIM)
    - Ang et al. (2006,JF) vs. Bali and Cakici (2008,JFQA)

Table 10. High-IVOL Portfolios Sorted by Lagged Return

Kim (2015)

RET_{t-1} portfolio	N	RET_{t-1}	RET_t	$XRET_t^{EW}$	$XRET_t^{VW}$	$IVOL_{t-1}$	$IVOL_t$	ME_{t-1}	$FF\alpha_t^{EW}$	$FF\alpha_t^{VW}$
1 (Low)	28,827	-21.70	1.53	1.04	-0.13	21.53	19.38	98.34	0.77 (1.66)	-0.25 (-0.54)
2	29,366	-6.31	1.33	0.84	-0.99	20.27	17.12	107.19	0.44 (1.39)	-1.20 (-3.01)
3	29,391	2.59	0.96	0.47	-0.66	20.22	16.38	119.44	-0.06 (-0.21)	-1.03 (-2.84)
4	29,360	12.61	0.51	0.02	-0.50	20.60	16.77	138.03	-0.44 (-1.70)	-0.76 (-2.50)
5 (High)	29,131	36.82	-0.69	-1.18	-1.04	22.11	19.09	170.24	-1.49 (-4.63)	-1.12 (-2.52)





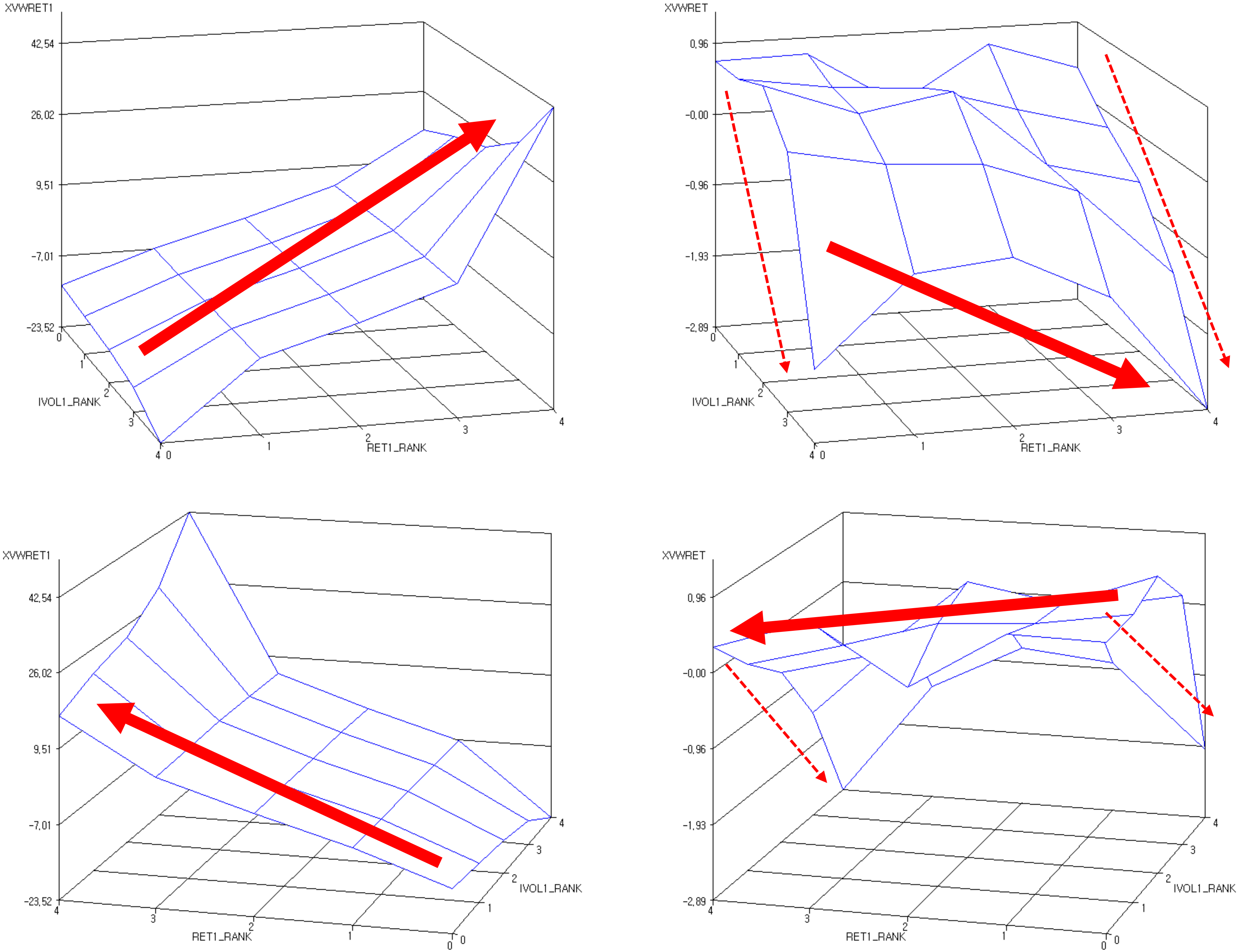
∃ Return Reversal in High-IVOL Stocks

Ang et al. (2006)

Table VII
Alphas of Portfolios Sorted on Idiosyncratic Volatility

		Ranking on Idiosyncratic Volatility					
		1 Low	2	3	4	5 High	5-1
NYSE Stocks Only		0.06	0.04	0.02	−0.04	−0.60	−0.66
		[1.20]	[0.75]	[0.30]	[−0.40]	[−5.14]	[−4.85]
Size Quintiles	Small 1	0.11	0.26	0.31	0.06	−0.43	−0.55
		[0.72]	[1.56]	[1.76]	[0.29]	[−1.54]	[−1.84]
	2	0.19	0.20	−0.07	−0.65	−1.73	−1.91
		[1.49]	[1.74]	[−0.67]	[−5.19]	[−8.14]	[−7.69]
	3	0.12	0.21	0.03	−0.27	−1.49	−1.61
		[1.23]	[2.40]	[0.38]	[−3.36]	[−10.1]	[−7.65]
	4	0.03	0.22	0.17	−0.03	−0.82	−0.86
		[0.37]	[2.57]	[2.47]	[−0.45]	[−6.61]	[−4.63]
	Large 5	0.09	0.04	0.03	0.14	−0.17	−0.26
		[1.62]	[0.72]	[0.51]	[1.84]	[−1.40]	[−1.74]
Controlling for Size		0.11	0.18	0.09	−0.15	−0.93	−1.04
		[1.30]	[2.49]	[1.35]	[−1.99]	[−6.81]	[−5.69]
Controlling for Book-to-Market		0.61	0.69	0.71	0.50	−0.19	−0.80
		[3.02]	[2.80]	[2.49]	[1.47]	[−0.48]	[−2.90]
Controlling for Leverage		0.11	0.11	0.08	−0.24	−1.12	−1.23
		[2.48]	[2.20]	[1.19]	[−2.45]	[−7.81]	[−7.61]
Controlling for Liquidity		0.08	0.09	−0.01	−0.16	−1.01	−1.08
		[1.71]	[1.53]	[−0.09]	[−1.62]	[−8.61]	[−7.98]
Controlling for Volume		−0.03	0.02	−0.01	−0.39	−1.25	−1.22
		[−0.49]	[0.39]	[−0.32]	[−7.11]	[−10.9]	[−8.04]
Controlling for Turnover		0.11	0.03	−0.11	−0.49	−1.34	−1.46
		[2.49]	[0.58]	[−1.79]	[−6.27]	[−11.0]	[−10.7]
Controlling for Bid–Ask Spreads		−0.07	−0.01	−0.09	−0.49	−1.26	−1.19
		[−1.21]	[−0.18]	[−1.14]	[−5.36]	[−9.13]	[−6.95]
Controlling for Coskewness		−0.02	−0.00	0.01	−0.37	−1.40	−1.38
		[−0.32]	[−0.02]	[0.08]	[−2.30]	[−6.07]	[−5.02]
Controlling for Dispersion in Analysts' Forecasts		0.12	−0.07	0.11	0.01	−0.27	−0.39
		[1.57]	[−0.76]	[1.12]	[0.09]	[−1.76]	[−2.09]

Consistent with Controlling Suspected Variables


XVWRET1
42,54
26,02
9,51
-7,01
-23,52
IVOL1_RANK
RET1_RANK
XVWRET
0,96
-0,00
-0,96
-1,93
-2,89
IVOL1_RANK
RET1_RANK
XVWRET1
42,54
26,02
9,51
-7,01
-23,52
RET1_RANK
IVOL1_RANK
XVWRET
0,96
-0,00
-0,96
-1,93
-2,89
RET1_RANK
IVOL1_RANK

The Conclusion of Kim (2015)

1. <u>IVOL is autocorrelated.</u> → Fu is right.
   - Chua, Goh and Zhang are also right.
   - Maybe observations are not enough to estimate entire parameters.
2. However, <u>Low-volatility anomaly cannot be explained by introducing EGARCH model.</u> → Fu might be wrong.
   - Look-ahead bias matters. → GKS (2014) are right.
   - Fu (2010) rebuts this counterargument, but…
3. <u>Low-volatility anomaly is partially explained by return reversal.</u> → Fu is right, but just for high-IVOL group.
   - Extreme reversal is observed in high-IVOL group. (KLS, 2014)
   - But, after controlling reversal, Low-volatility anomaly still exists.
4. In Korea, <u>Anomaly is reality.</u> → Others except Time-Varying σ?
   - Consistent with Ang et al. (2006), Kho and Kim (2014)

On the Robustness of the Positive Relation between Expected Idiosyncratic Volatility and Expected Return

Fangjian Fu
Singapore Management University - Lee Kong Chian School of Business

December 20, 2010

Abstract:
My 2009 JFE paper ["Idiosyncratic Risk and the Cross-Section of Expected Stock Returns', Journal of Financial Economics, Vol. 91, pp. 24-37] documents a positive and statistically significant cross-sectional relation between expected idiosyncratic volatility (E(IVOL)) and expected stock return. A recent working paper titled "On the Relation between EGARCH Idiosyncratic Volatility and Expected Stock Returns" by Guo, Ferguson, and Kassa of University of Cincinnati suggests that the positive relation is driven by an in-sample approach to estimate E(IVOL). They fail to find a significant relation between return and their E(IVOL) estimated out of sample. I find that two estimation settings in their SAS code, one of which limits the maximum number of iterations and the other accepts estimates with a questionable convergence status, lead to potentially unreliable estimates and ultimately, the failure to find the positive relation between return and E(IVOL). Using more reliable settings, I re-estimate E(IVOL) strictly out of sample, and confirm a robust and significantly positive relation between return and E(IVOL), just as reported in my JFE paper.

Number of Pages in PDF File: 7

Keywords: Idiosyncratic Volatility, Expected Idiosyncratic Volatility, EGARCH

working papers series

Date posted: January 18, 2011

Download This Paper

An exploration on the settings of EGARCH estimation

Microsoft (254 months)	Number of final estimates generated by EGARCH(*p*,*q*)									Number (%) of months with failed convergence	Fraction of estimates different by > 1%	Fraction of status = "0 Converged"
	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)	(3,1)	(3,2)	(3,3)			
Maxiter=100	78	0	5	23	11	3	99	17	18	248 (97.6%)	66.5%	85.4%
Maxiter=500	6	1	1	27	8	18	66	48	79	1 (0.4%)	Benchmark	59.8%
Maxiter=1000	6	1	1	27	8	18	66	48	79	0	0.0%	59.8%

IBM (959 months)	Number of final estimates generated by EGARCH(*p*,*q*)									Number (%) of months with failed convergence	Fraction of estimates different by > 1%	Fraction of status = "0 Converged"
	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)	(3,1)	(3,2)	(3,3)			
Maxiter=100	50	57	25	50	140	10	156	236	234	725 (75.6%)	44.9%	89.5%
Maxiter=500	33	51	6	8	94	21	92	340	314	0	Benchmark	78.5%
Maxiter=1000	33	51	6	8	94	21	92	340	314	0	0.0%	78.5%

Eastman Kodak (959 months)	Number of final estimates generated by EGARCH(*p*,*q*)									Number (%) of months with failed convergence	Fraction of estimates different by > 1%	Fraction of status = "0 Converged"
	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)	(3,1)	(3,2)	(3,3)			
Maxiter=100	42	5	10	175	18	57	251	264	137	587 (61.2%)	32.1%	97.0%
Maxiter=500	8	8	5	100	57	79	235	247	220	0	Benchmark	87.3%
Maxiter=1000	8	8	5	100	57	79	235	247	220	0	0.0%	87.3%

Q&A Session

Thanks for Listening

Ang et al. (2006)

The Cross-Section of Volatility and Expected Returns

ANDREW ANG, ROBERT J. HODRICK, YUHANG XING, and XIAOYAN ZHANG*

ABSTRACT

We examine the pricing of aggregate volatility risk in the cross-section of stock returns. Consistent with theory, we find that stocks with high sensitivities to innovations in aggregate volatility have low average returns. Stocks with high idiosyncratic volatility relative to the Fama and French (1993, *Journal of Financial Economics* 25, 2349) model have abysmally low average returns. This phenomenon cannot be explained by exposure to aggregate volatility risk. Size, book-to-market, momentum, and liquidity effects cannot account for either the low average returns earned by stocks with high exposure to systematic volatility risk or for the low average returns of stocks with high idiosyncratic volatility.

Ang et al. (2006,JF)
Idiosyncratic $\text{Volatility}_{t-1} \uparrow \Rightarrow E(\text{Return}_t) \downarrow$

Low Volatility Puzzle

Round 1

Fu (2009,JF)
(1) Do the findings imply that the relationship between idiosyncratic risk and expected return is negative?
(2) If not necessary, what is the true empirical relation?
(3) If the true relation is not negative, how their findings are explained?

Fu (2009)

Idiosyncratic risk and the cross-section of expected stock returns ☆

Fangjian Fu *

ABSTRACT

Theories such as Merton [1987. A simple model of capital market equilibrium with incomplete information. Journal of Finance 42, 483–510] predict a positive relation between idiosyncratic risk and expected return when investors do not diversify their portfolio. Ang, Hodrick, Xing, and Zhang [2006. The cross-section of volatility and expected returns. Journal of Finance 61, 259–299], however, find that monthly stock returns are negatively related to the one-month lagged idiosyncratic volatilities. I show that idiosyncratic volatilities are time-varying and thus, their findings should not be used to imply the relation between idiosyncratic risk and expected return. Using the exponential GARCH models to estimate expected idiosyncratic volatilities, I find a significantly positive relation between the estimated conditional idiosyncratic volatilities and expected returns. Further evidence suggests that Ang et al.'s findings are largely explained by the return reversal of a subset of small stocks with high idiosyncratic volatilities.

Fu (2009,JFE)
$E_{t-1}(\text{Idiosyncratic Volatility}_t) \uparrow \Rightarrow E(\text{Return}_t) \uparrow$

Low Volatility Puzzle

Round 2

Guo, Kassa and Ferguson (2014,JFQA)
(1) The positive idiosyncratic risk-return relation is driven by a look-ahead bias accidentally introduced by standard methods of estimating month t EGARCH idiosyncratic volatility.
(2) When month t EGARCH idiosyncratic volatility is forecast using returns only up through month $t-1$, there is no significant cross-sectional relation between EGARCH idiosyncratic volatility and returns.

Fink, Fink and He (2012)

Expected Idiosyncratic Volatility Measures and Expected Returns

Jason D. Fink, Kristin E. Fink, and Hui He*

We find that idiosyncratic volatility forecasts using information available to traders at the time of the forecast are not related to expected returns. The positive relation documented in a number of other papers only exists when forward-looking information is incorporated into the volatility estimate. That positive relation is driven by the realized idiosyncratic volatility component that cannot be forecasted by investors. Our findings are robust to several different empirical tests, volatility forecasting models and time periods.

Look-ahead Bias Matters!

TABLE 1

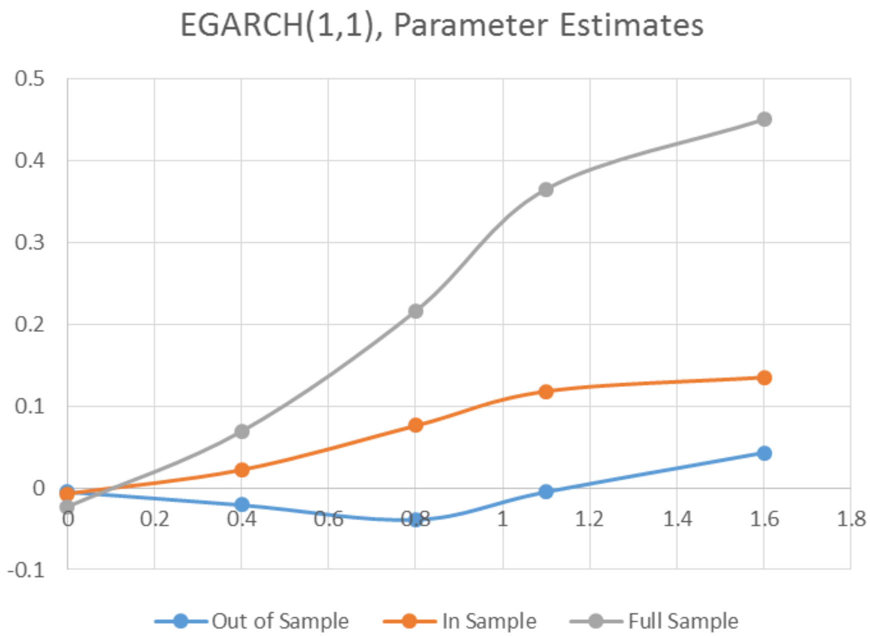
The Impact of Skewness on the Look-Ahead Bias:
Monte Carlo Simulation Using EGARCH(1, 1) Estimation

	Out of Sample	In Sample	Full Sample
Panel A. SKEW = 0.0			
Intercept	–0.000	0.000	0.001
	(–0.099)	(0.007)	(0.665)
E(IVOL)	–0.005	–0.007	–0.023
	(–0.134)	(–0.223)	(–0.732)
Adj. R^2	0.003	0.004	0.006
Panel B. SKEW = 0.4			
Intercept	0.001	–0.001	–0.004*
	(0.670)	(–0.510)	(–2.109)
E(IVOL)	–0.021	0.022	0.069*
	(–0.580)	(0.617)	(2.133)
Adj. R^2	0.004	0.003	0.004
Panel C. SKEW = 0.8			
Intercept	0.002	–0.005*	–0.012**
	(1.217)	(–2.581)	(–6.454)
E(IVOL)	–0.039	0.076*	0.216**
	(–1.418)	(2.362)	(6.203)
Adj. R^2	0.001	0.003	0.007
Panel D. SKEW = 1.1			
Intercept	–0.000	–0.007**	–0.021**
	(–0.012)	(–3.774)	(–11.715)
E(IVOL)	–0.005	0.118**	0.365**
	(–0.167)	(3.609)	(10.722)
Adj. R^2	0.002	0.003	0.007
Panel E. SKEW = 1.6			
Intercept	–0.002	–0.007**	–0.024**
	(–1.464)	(–4.331)	(–14.869)
E(IVOL)	0.043	0.135**	0.450**
	(1.452)	(4.294)	(14.057)
Adj. R^2	–0.000	0.001	0.009

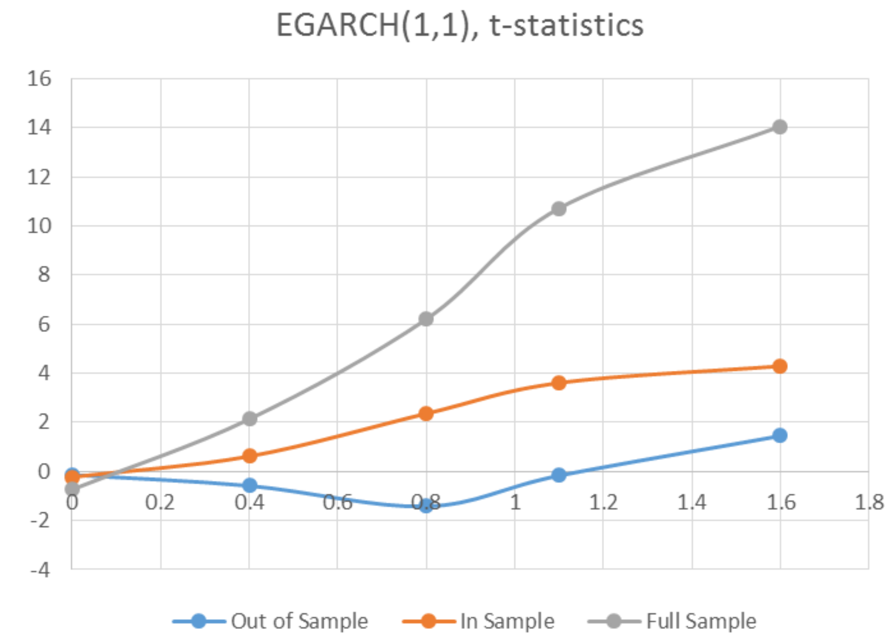
TABLE 2

The Impact of Skewness on the Look-Ahead Bias:
Monte Carlo Simulation Using Nine EGARCH Combinations

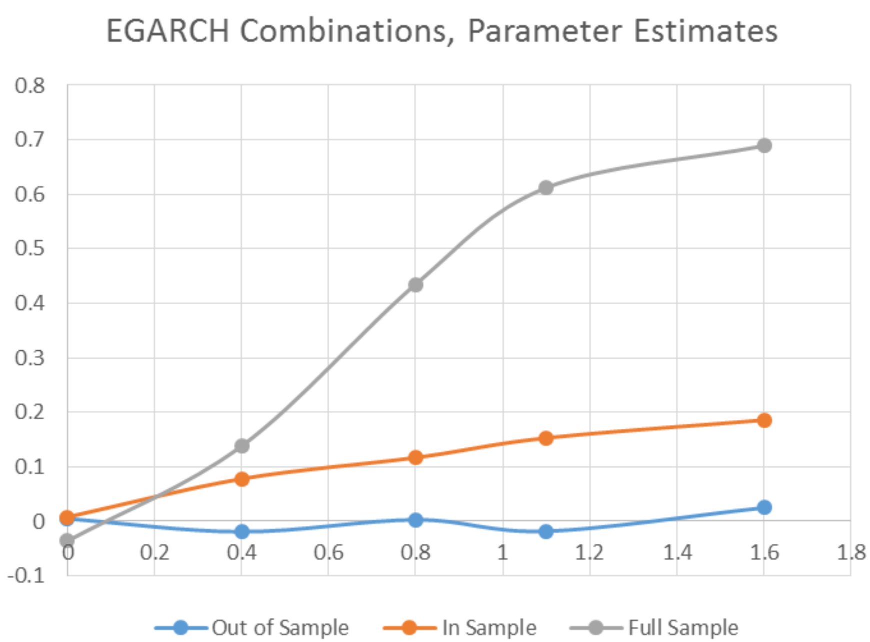
	Out of Sample	In Sample	Full Sample
Panel A. SKEW = 0.0			
Intercept	–0.000	–0.000	0.002
	(–0.152)	(–0.344)	(1.695)
E(IVOL)	0.005	0.008	–0.036
	(0.272)	(0.495)	(–1.356)
Adj. R^2	0.005	0.006	0.009
Panel B. SKEW = 0.4			
Intercept	0.001	–0.004*	–0.008**
	(0.877)	(–2.593)	(–5.684)
E(IVOL)	–0.019	0.078*	0.138**
	(–0.734)	(2.568)	(5.574)
Adj. R^2	0.002	0.005	0.006
Panel C. SKEW = 0.8			
Intercept	–0.001	–0.007**	–0.025**
	(–0.476)	(–6.653)	(–18.642)
E(IVOL)	0.003	0.117**	0.435**
	(0.138)	(6.693)	(16.679)
Adj. R^2	–0.000	0.003	0.020
Panel D. SKEW = 1.1			
Intercept	0.001	–0.009**	–0.034**
	(0.623)	(–7.008)	(–28.440)
E(IVOL)	–0.018	0.153**	0.612**
	(–0.871)	(6.611)	(26.195)
Adj. R^2	0.001	0.007	0.037
Panel E. SKEW = 1.6			
Intercept	–0.001	–0.009**	–0.036**
	(–1.621)	(–6.312)	(–25.895)
E(IVOL)	0.025	0.186**	0.689**
	(1.496)	(6.440)	(24.401)
Adj. R^2	–0.001	0.010	0.050



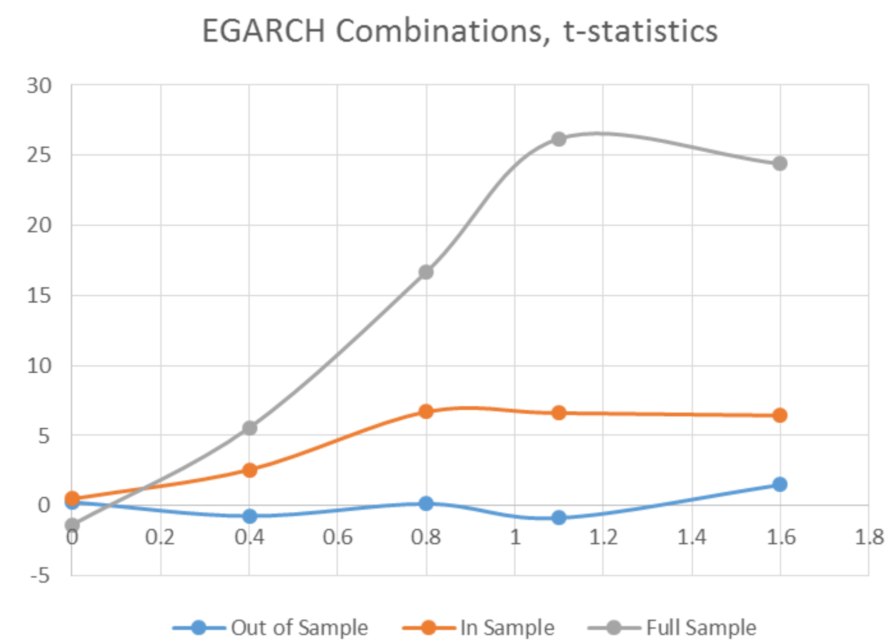
EGARCH(1,1), Parameter Estimates
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Out of Sample
In Sample
Full Sample



EGARCH(1,1), t-statistics
16
14
12
10
8
6
4
2
0
-2
-4
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Out of Sample
In Sample
Full Sample



EGARCH Combinations, Parameter Estimates
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Out of Sample
In Sample
Full Sample

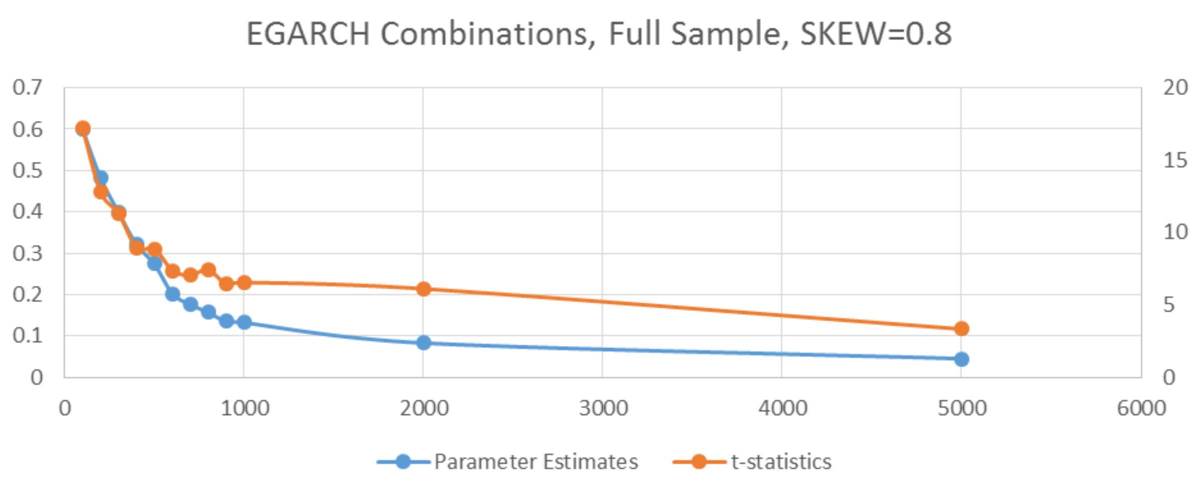


EGARCH Combinations, t-statistics
30
25
20
15
10
5
0
-5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Out of Sample
In Sample
Full Sample

TABLE 3

The Impact of Estimation Period Length on the Look-Ahead Bias: Monte Carlo Simulation Using Nine EGARCH Combinations over the Full Sample with SKEW = 0.8

	$T = 100$	$T = 200$	$T = 300$	$T = 400$
Intercept	–0.032**	–0.027**	–0.023**	–0.018**
	(–17.860)	(–13.489)	(–11.898)	(–9.436)
E(IVOL)	0.599**	0.482**	0.399**	0.321**
	(17.191)	(12.805)	(11.327)	(8.953)
Adj. R^2	0.053	0.022	0.014	0.011
	$T = 500$	$T = 600$	$T = 700$	$T = 800$
Intercept	–0.016**	–0.012**	–0.010**	–0.009**
	(–9.381)	(–7.801)	(–7.522)	(–7.994)
E(IVOL)	0.274**	0.202**	0.176**	0.157**
	(8.873)	(7.383)	(7.052)	(7.441)
Adj. R^2	0.008	0.006	0.005	0.004
	$T = 900$	$T = 1,000$	$T = 2,000$	$T = 5,000$
Intercept	–0.008**	–0.008**	–0.005**	–0.002**
	(–6.788)	(–6.842)	(–6.250)	(–3.208)
E(IVOL)	0.135**	0.132**	0.083**	0.045**
	(6.444)	(6.556)	(6.119)	(3.337)
Adj. R^2	0.004	0.004	0.003	0.006



Numerical Optimization

- Newton's method

$$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k - \left[\frac{\partial^2 \mathcal{L}(\boldsymbol{\theta}_k)}{\partial \boldsymbol{\theta}^2}\right]^{-1} \frac{\partial \mathcal{L}(\boldsymbol{\theta}_k)}{\partial \boldsymbol{\theta}} = \boldsymbol{\theta}_k - [\mathbf{H}(\boldsymbol{\theta}_k)]^{-1}\mathbf{g}(\boldsymbol{\theta}_k)$$

- If Hessian is known, then apply Neton's method directly.
  - Berndt-Hall-Hall-Hausman method (Berndt et al., 1970)

$$\mathbf{H}_{k+1} = -\sum_{\tau=1}^{T} \frac{\partial L(\boldsymbol{\theta}_k)}{\partial \boldsymbol{\theta}} \frac{\partial L(\boldsymbol{\theta}_k)}{\partial \boldsymbol{\theta}'}$$

- If Hessian is unknown, then apply Quasi-Newton method.
  - Broyden-Fletcher-Goldfarb-Shanno method, Efficient

$$\mathbf{H}_{k+1} = \mathbf{H}_k - \frac{(\mathbf{H}_k \Delta\boldsymbol{\theta}_{k+1})(\mathbf{H}_k \Delta\boldsymbol{\theta}_{k+1})'}{\Delta\boldsymbol{\theta}'_{k+1}\mathbf{H}_k \Delta\boldsymbol{\theta}_{k+1}} + \frac{\Delta\mathbf{g}_{k+1}\Delta\mathbf{g}'_{k+1}}{\Delta\mathbf{g}'_{k+1}\Delta\boldsymbol{\theta}_{k+1}}$$

- If Hessian is unknown, then apply Trust Region method.
  - Step Size (Moré and Sorensen, 1983), Stable

$$\mathbf{H}(\boldsymbol{\theta}_k) = \begin{pmatrix} \frac{\partial^2 \mathcal{L}}{\partial \theta_{1k}^2} & \cdots & \frac{\partial^2 \mathcal{L}}{\partial \theta_{1k} \partial \theta_{Pk}} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 \mathcal{L}}{\partial \theta_{Pk} \partial \theta_{1k}} & \cdots & \frac{\partial^2 \mathcal{L}}{\partial \theta_{Pk}^2} \end{pmatrix}$$

TABLE 6

Size, In-Sample EGARCH Idiosyncratic Volatility, and the Cross Section of Stock Returns

	BETA	ln(ME)	ln(BE/ME)	RET(−2, −7)	ln(TURN)	ln(CVTURN)	E(IVOL$_t$)	Δ_1E(IVOL$_t$)	Δ_2E(IVOL$_t$)	Adj. R^2
Panel A. Simple Returns										
1		0.204** (5.504)	0.444** (8.379)				0.164** (9.002)			0.045
2		0.049 (1.437)	0.308** (6.191)				0.070** (3.205)	0.095** (12.422)	0.076** (10.942)	0.053
3		0.127** (3.453)	0.392** (8.151)	0.910** (5.503)	–0.360** (–5.506)	–0.730** (–9.040)	0.184** (11.782)			0.065
4		–0.003 (–0.096)	0.281** (6.011)	0.891** (5.530)	–0.218** (–3.489)	–0.579** (–7.845)	0.088** (4.917)	0.089** (12.583)	0.072** (11.922)	0.070
5	–0.087 (–0.534)	–0.011 (–0.351)	0.275** (6.102)	0.907** (6.828)	–0.208** (–4.053)	–0.585** (–8.120)	0.090** (5.455)	0.088** (12.787)	0.071** (12.168)	0.076
Panel B. Log Returns										
6		0.223** (5.976)	0.476** (8.685)				0.049** (2.983)			0.043
7		0.065 (1.923)	0.337** (6.650)				–0.047* (–2.221)	0.096** (12.832)	0.079** (11.548)	0.051
8		0.146** (3.930)	0.412** (8.502)	1.044** (6.497)	–0.406** (–5.760)	–0.698** (–8.370)	0.074** (5.468)			0.063
9		0.017 (0.499)	0.301** (6.446)	1.025** (6.673)	–0.264** (–3.975)	–0.547** (–7.244)	–0.023 (–1.378)	0.088** (13.023)	0.073** (12.766)	0.068
10	–0.112 (–0.673)	0.007 (0.223)	0.295** (6.549)	1.037** (6.983)	–0.248** (–4.581)	–0.551** (–7.483)	–0.020 (1.310)	0.087** (13.320)	0.071** (13.203)	0.073

Control Look-ahead Bias → No Positive Size Effect

TABLE 7

The Cross Section of Expected Idiosyncratic Volatility

	ln(ME)	ln(BE/ME)	RET(−2, −7)	ln(TURN)	ln(CVTURN)	E(IVOL$_t$)	E(IVOL_O$_t$)	E(IVOL$_{t-1}$)	IVOL$_{t-1}$	Adj. R^2
1						0.631** (45.683)				0.277
2							0.507** (40.069)			0.163
3								0.579** (41.139)		0.234
4									0.678** (85.633)	0.454
5							0.172** (46.470)		0.618** (65.690)	0.482
6						0.294** (57.239)			0.550** (56.495)	0.502
7						0.445** (48.480)		0.322** (39.679)		0.325
8								0.211** (33.346)	0.582** (57.931)	0.480
9						0.250** (62.876)		0.104** (20.691)	0.522** (48.539)	0.508
10	–0.823** (–20.357)	–0.343** (–11.912)	–1.241** (–9.745)	0.376** (10.452)	0.214** (4.005)	0.130** (25.895)			0.482** (42.507)	0.517
11	–0.702** (–19.536)	–0.216** (–8.604)	–1.259** (–10.627)	0.212** (6.341)	0.041 (0.849)	0.204** (54.381)		0.063** (14.096)	0.448** (39.265)	0.535

IVOL$_{t-1}$ has the biggest information!

TABLE 8

Out-of-Sample EGARCH Idiosyncratic Volatility Estimated Using Daily Return Data

	ln(ME)	ln(BE/ME)	RET(−2, −7)	ln(TURN)	ln(CVTURN)	E(IVOL_D1)	E(IVOL_D2)	Adj. R^2
Panel A. July 1964–Dec. 2006								
1						0.017 (1.230)		0.022
2	–0.153** (–3.980)	0.157* (2.530)				–0.003 (–0.300)		0.037
3	–0.232** (–5.840)	0.117* (2.090)	0.822** (5.140)	–0.077 (–1.030)	–0.005** (–6.550)	–0.001 (–0.080)		0.055
4							0.019 (1.130)	0.027
5	–0.165** (–4.810)	0.144* (2.420)					–0.009 (–0.600)	0.040
6	–0.238** (–6.600)	0.112* (2.050)	0.811** (5.110)	–0.070 (–0.970)	–0.005** (–6.890)		–0.004 (–0.340)	0.057
Panel B. July 1964–Dec. 2009								
7						0.016 (1.170)		0.021
8	–0.169** (–3.600)	0.135* (2.220)				–0.005 (–0.480)		0.036
9	–0.237** (–5.180)	0.092 (1.640)	0.652** (2.730)	–0.056 (–0.710)	–0.004** (–4.120)	–0.003 (–0.410)		0.054
10							0.014 (0.860)	0.027
11	–0.191** (–4.120)	0.123* (2.110)					–0.014 (–1.010)	0.039
12	–0.255** (–5.640)	0.087 (1.600)	0.610* (2.380)	–0.043 (–0.560)	–0.004** (–4.090)		–0.012 (–0.950)	0.056

Using Daily Data → Consistent (No Positive Relation)