# Breeden, Gibbons, and Litzenberger (1989)




## Abstract

- Compare CAPM and CCAPM, and demonstrate the matching performance of CCAPM against CAPM
- Adjust several measurement problems inherent in consumption data
  - The durables problem
  - The problem of measured consumption as an integral of spot consumption rates
  - The problem that consumption data are reported infrequently
  - The problem of pure sampling error in consumption measures
- Estimate CCAPM using (i) consumption and (ii) the portfolio maximally correlated to consumption
- The result displays $\gamma_0$ to be insignificant and $\gamma_1$ to be positive & significant


## I. A Synthesis of the CCAPM Theory

M risky assets and K individuals are assumed with the zero-beta z—the subscripts are suppressed for simplicity. Under some standard assumptions,

$$E_{t-1}\left[(R_t - R_t^z)\frac{U'(C_t)}{U'(C_{t-1})}\right] = 0.$$

Breeden and Litzenberger (1978) argue that (i) the consumption of an individual is an increasing function of the aggregate consumption, and (ii) the optimal marginal utility of an individual is a decreasing function of the aggregate consumption. So, with the representative agent assumption, the growth rate of the individual marginal utility $U'(\cdot)$ is equal to the growth rate of the aggregate marginal utility $f(\cdot)$, and

$$\frac{U'(C_t)}{U'(C_{t-1})} = \frac{f(C_t)}{f(C_{t-1})} \approx 1 - \underbrace{\frac{-C_{t-1}f'(C_{t-1})}{f(C_{t-1})}}_{b}\underbrace{\frac{C_t - C_{t-1}}{C_{t-1}}}_{c_t} = 1 - bc_t,$$

assuming CRRA. Further assume the M risky assets and the zero-beta to be

$$R_t = \alpha_c + \beta_c c_t + u_t, \qquad \beta_c = \frac{Cov[c_t, R_t]}{Var[c_t]}$$
$$R_t^z = \gamma_0 + u_t^z,$$

and manipulate the first order condition to derive the CCAPM as

$$E_{t-1}\left[(R_t - R_t^z)\frac{U'(C_t)}{U'(C_{t-1})}\right] \approx E_{t-1}[(R_t - R_t^z)(1 - bc_t)] = E[(R_t - R_t^z)(1 - bc_t)] = 0.$$
$$\Rightarrow E[R_t - R_t^z] = \frac{Cov[R_t - R_t^z, bc_t]}{E[1 - bc_t]} \Rightarrow \mu - \gamma_0 = \underbrace{\frac{bVar[c_t]}{1 - bE[c_t]}}_{\gamma_1}\beta_c.$$

Thus, the market price $\gamma_1$ of the consumption beta $\beta_c$ increases as (i) the consumption growth volatility $Var[c_t]$ increases, and (ii) the relative risk aversion b increases.

The tests in this paper are based on the unconditional moments of c and R. Though Cornell (1981) points out that the CCAPM of Breeden (1979) requires the conditional consumption beta to be a constant, Grossman and Shiller (1982) argue that the tests relying on unconditional moments do not ignore the concern of Cornell (1981). Tests relying on unconditional moments require no specification on the behavior of conditional moments.

## II. Econometric Problems Associated with Measured Consumption

Note that Expenditures≠Consumption, though the data relate them. The use of nondurables and services minimizes the measurement problem (Hall (1978)), but nondurables become more durable as the sampling interval decreases.

### *A. Description of the Consumption Data*

- The data are spliced
  - Nondurables and services consumption
  - For 1929–1939, consumption growth data (annual) are regressed on income growth data (monthly)
  - For 1939–1958, quarterly expenditures on nondurable goods and services are used
    - For 1939–1946, the data are deflated by the quarterly average of the monthly CPIs
    - For 1947–1958, real consumption data are available
  - For 1959–1982, matching quarterly data from their monthly counterparts are used

### *B. Interval versus Spot Consumption (the Summation Bias)*

- The moments from available consumption data underestimate the true moments
  - Assuming the consumption rate C and the wealth P to be random walking and the interval T to be 0.25 (quarterly sampling under annual environment),

$$C_t = C_{t-1} + \sum_{i=1}^{n} \Delta_i^C \Rightarrow C_{Q1} = \frac{1}{T}\sum_{i=1}^{n} C_i \Delta t \Rightarrow C_{Q2} - C_{Q1} = \int_0^T \frac{t}{T}\Delta_t^C dt + \int_T^{2T} \frac{2T-t}{T}\Delta_t^C dt$$
$$\Rightarrow \mathrm{Var}\left[C_{Q2} - C_{Q1}\right] = \int_0^T \left(\frac{t}{T}\right)^2 \sigma_C^2 dt + \int_T^{2T} \left(\frac{2T-t}{T}\right)^2 \sigma_C^2 dt = \frac{2}{3}\sigma_C^2 T$$
$$\Rightarrow \mathrm{Cov}\left[C_{Q2} - C_{Q1}, P_{2T} - P_T\right] = \frac{1}{2}\sigma_{aC}T, \qquad \Rightarrow \beta_{aC}^{sum} = \frac{3}{4}\beta_{aC}^{spot}.$$

- In addition, the consumption data are subject to the first-order autocorrelation
  - The result in Table 1 displays the estimate of $\rho_1$ to be 0.29

$$\rho_1 = \frac{\mathrm{Cov}\left[C_{Q3} - C_{Q2}, C_{Q2} - C_{Q1}\right]}{\mathrm{Var}\left[C_{Q2} - C_{Q1}\right]} = \frac{\frac{1}{6}\sigma_C^2 T}{\frac{2}{3}\sigma_C^2 T} = \frac{1}{4}.$$

### *C. Infrequent Reporting of Consumption: The Maximum Correlation Portfolio*

- Introduce the maximally correlated portfolio (MCP) and restate the CCAPM using it
  - Find the minimum-variance portfolio that has a consumption beta of $\beta_{C,NB}$ (no borrowing)

$$\begin{aligned}
\boldsymbol{\mu} - \gamma_0\mathbf{1} &= \boldsymbol{\beta}_{MCP}(\mu_{MCP} - \gamma_0)\\
\boldsymbol{\beta}_{MCP} &= \frac{\mathbf{V}\mathbf{w}_{MCP}}{\mathbf{w}_{MCP}^T\mathbf{V}\mathbf{w}_{MCP}} = \frac{\boldsymbol{\beta}_C}{\beta_{C,NB}}\\
\mathbf{w}_{MCP} &= \theta\mathbf{V}^{-1}\mathbf{V}_{aC}\\
\theta &= \frac{\beta_{C,NB}}{\beta_C^T V^{-1}\beta_C \mathrm{Var}[c]} = \frac{\beta_{C,NB}}{w_C^T V w_C/\mathrm{Var}[c]} = \frac{\beta_{C,NB}}{R^2}.
\end{aligned}$$

- If there's a riskless asset, a unit beta MCP has weights that equal the regression's coefficients divided by the R-squared value of the regression (with any residual wealth in the riskless asset)
- Betas with respect to such a unit-beta MCP equal the asset's direct consumption betas

### D. *Sampling Error In Reported Consumption*

- The measurement error in the consumption growth c causes the price $\gamma_1$ of the measured beta to be biased

$$\mu_i - \gamma_0 = \gamma_1^{\text{True}} \beta_{ci}^{\text{True}} = \gamma_1^{\text{True}} \frac{\text{Var}\,[c_t + \varepsilon_t]}{\text{Var}[c_t]} \beta_{ci}^{\text{Measured}} = \gamma_1^{\text{Measured}} \beta_{ci}^{\text{Measured}}.$$

- If the measurement error exists, then $\gamma_1^{\text{Measured}} > \gamma_1^{\text{True}}$

| Time Period | $T$ | $\bar{c}$ | $\widehat{SD}(c)$ | $\hat{\rho}_1$ | $\hat{\rho}_2$ | $\hat{\rho}_3$ | $\hat{\rho}_4$ | $\hat{\rho}_8$ | $SD^*$ $(\hat{\rho}_k)$ | $\widehat{SD}$ $(\hat{\rho}_1)$ | $\widehat{SD}$ $(\hat{\rho}_k)$ | $Q_{12}$ | $p$-Value |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel A: Quarterly Consumption Data | | | | | | | | | | | | | |
| 39Q2–82Q4 | 175 | 0.00543 | 0.00951 | 0.29 | 0.03 | −0.00 | 0.07 | 0.02 | 0.08 | 0.07 | 0.08 | 23.93 | 0.02 |
| 39Q2–52Q4 | 55 | 0.00665 | 0.01517 | 0.30 | 0.03 | −0.04 | 0.08 | 0.08 | 0.13 | 0.12 | 0.14 | 11.26 | 0.51 |
| 53Q1–67Q4 | 60 | 0.00463 | 0.00549 | 0.21 | 0.09 | 0.11 | −0.01 | −0.22 | 0.13 | 0.12 | 0.14 | 11.25 | 0.51 |
| 68Q1–82Q4 | 60 | 0.00511 | 0.00487 | 0.36 | 0.01 | 0.26 | 0.09 | −0.31 | 0.13 | 0.12 | 0.14 | 25.95 | 0.01 |
| Panel B: Monthly Consumption Data | | | | | | | | | | | | | |
| 1959–1982 | 287 | 0.00178 | 0.00447 | −0.28 | −0.02 | −0.14 | −0.12 | −0.19 | 0.06 | 0.05 | 0.06 | 43.09 | 0.00 |
| 1959–1970 | 143 | 0.00199 | 0.00467 | −0.31 | −0.11 | 0.18 | −0.08 | −0.17 | 0.08 | 0.08 | 0.09 | 33.49 | 0.00 |
| 1971–1982 | 144 | 0.00156 | 0.00427 | −0.24 | 0.07 | 0.09 | −0.16 | −0.16 | 0.08 | 0.08 | 0.09 | 20.56 | 0.06 |
| Panel C: Quarterly Sampling of Monthly Consumption Data | | | | | | | | | | | | | |
| 59Q2–82Q4 | 95 | 0.00521 | 0.00568 | 0.13 | −0.13 | 0.20 | 0.04 | −0.17 | 0.10 | 0.09 | 0.11 | 13.42 | 0.34 |
| 59Q2–70Q4 | 47 | 0.00576 | 0.00506 | 0.13 | −0.15 | 0.13 | −0.03 | −0.04 | 0.15 | 0.13 | 0.15 | 10.61 | 0.56 |
| 71Q1–82Q4 | 47 | 0.00468 | 0.00623 | 0.12 | −0.07 | 0.22 | −0.10 | −0.26 | 0.14 | 0.13 | 0.15 | 11.40 | 0.50 |

Table I. The consumption growth data are autocorrelated

## III. Empirical Characteristics of Consumption Betas and the MCP

- Construct 12 portfolios using the SIC codes
  - Follow Sharpe (1982) with one exception that the category of the consumer goods is separated to increase the variability of the consumption betas
- Use the aggregated portfolios rather than the individual securities not to introduce measurement errors
- Following Chen, Roll, and Ross (1986), employ the junk bond premium, which reflects the default probability evaluated by investors who relate the probability to determine how much to consume
  - The premium displays a strong relation with the real consumption growth
- Real simple returns (rather than nominal continuously-compounded ones)

| Asset (SIC Codes) | Number of Firms | | | Spliced Consumption, Quarterly 1929–1982 ($T = 215$) | | | Max.-Correlation Cons. Portfolio, Monthly 1926–1982 ($T = 684$) | | | CRSP Value-Weighted Index Monthly 1926–1982 ($T = 684$) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1/26 | 6/54 | 12/82 | $\hat{\beta}_c$ | $t(\hat{\beta})$ | $R^2$ | $\hat{\beta}_{MCP}$ | $t(\hat{\beta})$ | $R^2$ | $\hat{\beta}_{CRSP}$ | $t(\hat{\beta})$ | $R^2$ |
| U.S. Treasury bills | — | — | — | −0.11 | −1.27 | 0.01 | 0.03 | 3.86 | 0.02 | 0.01 | 2.04 | 0.01 |
| Long-term govt. bonds | NA | NA | NA | −0.01 | −0.02 | 0.00 | 0.07 | 2.53 | 0.01 | 0.07 | 4.93 | 0.03 |
| Long-term corp. bonds | NA | NA | NA | 0.24 | 0.91 | 0.00 | 0.07 | 2.52 | 0.01 | 0.08 | 6.62 | 0.06 |
| Junk bond premium | NA | NA | NA | 2.45 | 6.85 | 0.18 | 0.63 | 18.52 | 0.33 | 0.33 | 20.45 | 0.38 |
| Petroleum (13, 29) | 46 | 51 | 69 | 4.31 | 6.37 | 0.16 | 1.41 | 20.61 | 0.38 | 0.92 | 38.63 | 0.69 |
| Finance & real estate (60–69) | 16 | 43 | 234 | 5.85 | 6.30 | 0.16 | 1.50 | 18.81 | 0.34 | 1.19 | 75.95 | 0.89 |
| Consumer durables (25, 30, 36, 37, 50, 55, 57) | 69 | 157 | 180 | 6.86 | 6.80 | 0.18 | 1.79 | 22.03 | 0.42 | 1.29 | 80.79 | 0.91 |

Table II. 12 portfolios are formed using the SIC codes and their consumption betas are reported

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Basic industries (10, 12, 14, 24, 26, 28, 33) | 94 | 207 | 194 | 5.45 | 6.95 | 0.18 | 1.48 | 21.98 | 0.41 | 1.09 | 100.80 | 0.94 |
| Food & tobacco (1, 20, 21, 54) | 64 | 103 | 81 | 3.25 | 5.69 | 0.13 | 0.99 | 18.62 | 0.34 | 0.76 | 58.15 | 0.83 |
| Construction (15–17, 32, 52) | 5 | 28 | 53 | 7.36 | 7.06 | 0.19 | 1.57 | 19.16 | 0.35 | 1.20 | 61.22 | 0.85 |
| Capital goods (34, 35, 38) | 39 | 120 | 191 | 5.31 | 6.74 | 0.18 | 1.45 | 21.10 | 0.39 | 1.08 | 85.90 | 0.92 |
| Transportation (40–42, 44, 45, 47) | 78 | 85 | 46 | 5.15 | 4.97 | 0.10 | 1.27 | 13.52 | 0.21 | 1.19 | 49.04 | 0.78 |
| Utilities (46, 48, 49) | 24 | 102 | 176 | 3.73 | 6.10 | 0.15 | 1.04 | 19.40 | 0.35 | 0.75 | 46.34 | 0.76 |
| Textiles & trade (22, 23, 31, 51, 53, 56, 59) | 46 | 101 | 119 | 5.63 | 7.84 | 0.22 | 1.66 | 30.49 | 0.58 | 0.95 | 48.73 | 0.78 |
| Services (72, 73, 75, 80, 82, 89) | 3 | 4 | 57 | 4.21 | 4.18 | 0.08 | 1.65 | 12.97 | 0.20 | 0.80 | 12.82 | 0.19 |
| Leisure (27, 58, 70, 78, 79) | 12 | 31 | 59 | 7.35 | 6.95 | 0.18 | 1.85 | 23.03 | 0.44 | 1.22 | 49.82 | 0.78 |
| CRSP value-weighted | NA | NA | NA | 4.92 | 7.06 | 0.19 | 1.37 | 23.73 | 0.45 | 1.00 | — | — |

Table II. (cont.)

- Table II exhibits that the consumption betas, the MCP betas, and the market betas are correlated significantly
  - Corr[market beta,consumption beta]=0.96
  - Corr[market beta,MCP beta]=0.94
  - Corr[consumption beta,MCP beta]=0.98
  - Goods with high income elasticities of demand display high consumption betas
    - Consumer durables, construction, and recreation & leisure
  - Goods with lower income elasticities of demand demonstrate low consumption betas
    - Utilities, Petroleum, food & agriculture, and transportation
- Table III displays the composition of the MCP
  - The MCP mostly consists of long-term government bonds, the junk bond premium, and the CRSP value-weighted index
  - Though the weight on the CRSP index is negative (−0.51), the MCP and the CRSP index exhibit a positive correlation (0.67)

| Asset | Weight | $t$-Statistic |
|---|---|---|
| U.S. Treasury bills | 0.01 | 0.02 |
| Long-term government bonds | 0.54 | 1.05 |
| Long-term corporate bonds | −0.31 | −0.64 |
| Junk bond premium | 0.59 | 2.71 |
| Petroleum | 0.27 | 1.13 |
| Banking, finance and real estate | −0.17 | 0.38 |
| Consumer durables | 0.10 | 0.44 |
| Basic industries | 0.33 | 0.90 |
| Agriculture, food, and tobacco | −0.35 | −1.45 |
| Construction | −0.11 | −0.80 |
| Capital goods | 0.03 | 0.11 |
| Transportation | −0.29 | −2.25 |
| Utilities | 0.18 | 0.72 |
| Textiles, retail stores, and wholesalers | 0.49 | 2.69 |
| Services | 0.08 | 1.39 |
| Recreation and leisure | 0.13 | 1.17 |
| CRSP value-weighted index | −0.51 | −0.38 |
| | 1.00 | |

Table III. The recipe of the MCP—Corr[consumption,MCP]=0.57

| Date | Number of Observations | Mean of T-bills | t-Statistic for Mean of T-bills | Standard Deviation |
|---|---|---|---|---|
| 1926–1982 | 684 | 0.0013 | 0.48 | 0.0204 |
| 1926–1945 | 240 | 0.0100 | 1.77 | 0.0253 |
| 1946–1965 | 240 | −0.0082 | −1.74 | 0.0211 |
| 1966–1982 | 204 | 0.0023 | 0.89 | 0.0106 |

| Date | Number of Observations | Mean of CRSP Return | t-Statistic for Mean of CRSP Return | Standard Deviation |
|---|---|---|---|---|
| 1926–1982 | 684 | 0.0767 | 2.88 | 0.2013 |
| 1926–1945 | 240 | 0.1002 | 1.61 | 0.2782 |
| 1946–1965 | 240 | 0.1039 | 3.70 | 0.1257 |
| 1966–1982 | 204 | 0.0172 | 0.44 | 0.1615 |

| Date | Number of Observations | Mean of MCP Return | t-Statistic for Mean of MCP Return | Standard Deviation |
|---|---|---|---|---|
| 1926–1982 | 684 | 0.0370 | 2.83 | 0.0987 |
| 1926–1945 | 240 | 0.0598 | 1.98 | 0.1351 |
| 1946–1965 | 240 | 0.0382 | 2.62 | 0.0651 |
| 1966–1982 | 204 | 0.0086 | 0.46 | 0.0786 |

Table IV. The properties of the market portfolio and the MCP—Corr[market,MCP]=0.67

- In Table IV, the mean and the standard deviation of the MCP are about half of the mean and the standard deviation of the CRSP index
  - Though the MCP contains the CRSP by −51%, these portfolios are positively correlated

## IV. Testing the CCAPM and the CAPM

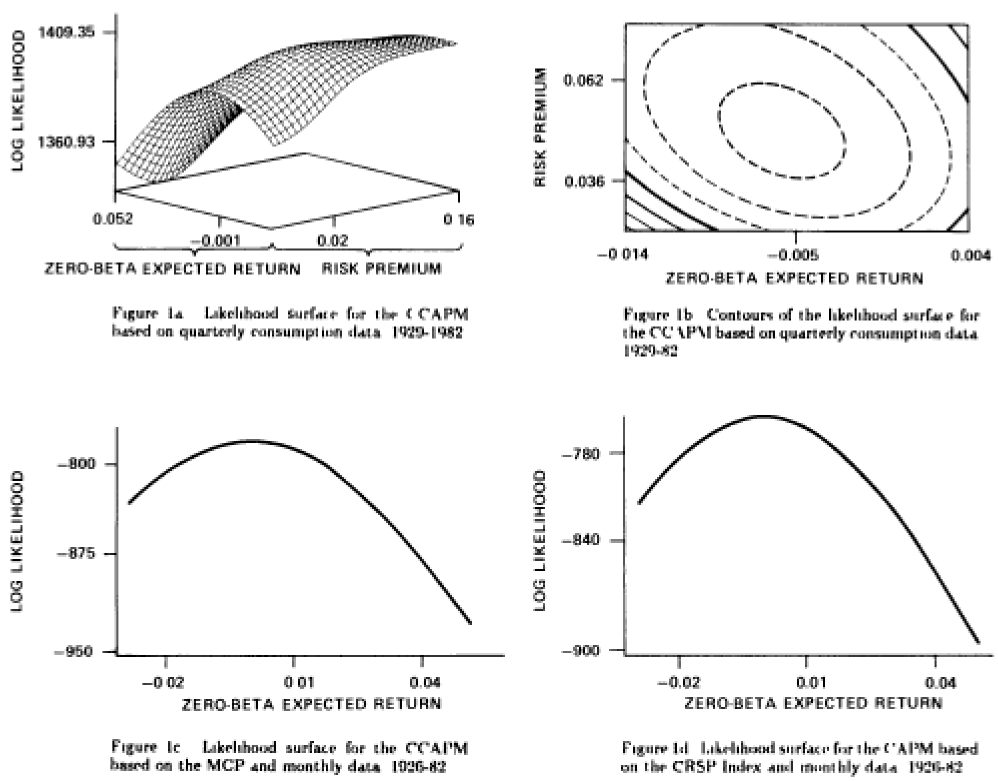


Figure 1. ML estimation using consumption, MCP, and CRSP data

- Both the time-series regressions and the cross-sectional regressions are estimated based on the concentrated log-likelihood functions following Kandel (1984) and Shanken (1985)

$$R_{it} = \alpha_{ci} + \beta_{ci}c_t + u_{it}$$
$$\mu_i = \gamma_0 + \gamma_1\beta_{ci}$$
$$L(\gamma_0, \gamma_1) = \frac{\mathbf{e}^\top\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{e}}{1 + \frac{\gamma_1^2}{s_c^2}}.$$

| Date | Number of Observations | $F$-test: Betas Equal ($p$-Value) | $F$-test: Betas = Zero ($p$-Value) | $\hat{\gamma}_0$ (SE($\hat{\gamma}_0$)) | $\hat{\gamma}_1$ (SE($\hat{\gamma}_1$)) | LR Test of $H_0$ ($p$-Value) |
|---|---|---|---|---|---|---|
| Panel A: Spliced Quarterly Consumption Data, Adjusted for Summation Bias, 1929–1982 | | | | | | |
| 1929Q2– | 215 | 3.874 | 3.912 | −0.0061 | 0.0478 | 28.03 |
| 1982Q4 | | (<0.001) | (<0.001) | (0.0044) | (0.0133) | (0.021) |
| 1929Q2– | 40 | 4.319 | 4.241 | 0.0484 | 0.0329 | 26.45 |
| 1939Q1 | | (0.001) | (0.001) | (0.0091) | (0.0189) | (0.034) |
| 1939Q2– | 32 | 0.502 | 1.410 | −0.2558 | 0.5850 | 6.84 |
| 1947Q1 | | (0.908) | (0.261) | (0.0859) | (0.2507) | (0.962) |
| 1947Q2– | 48 | 1.006 | 1.182 | −0.0699 | 0.2928 | 14.82 |
| 1959Q1 | | (0.476) | (0.334) | (0.0469) | (0.1865) | (0.464) |
| 1959Q2– | 95 | 2.257 | 2.277 | 0.0015 | 0.0187 | 20.95 |
| 1982Q4 | | (0.009) | (0.008) | (0.0028) | (0.0062) | (0.138) |
| Panel B: Unspliced Quarterly Consumption Data, Adjusted for Summation Bias, 1947–1982 | | | | | | |
| 1947Q2– | 144 | 1.342 | 1.695 | −0.0325 | 0.2136 | 19.87 |
| 1982Q4 | | (0.182) | (0.052) | (0.0256) | (0.1430) | (0.177) |
| 1959Q2– | 96 | 1.398 | 1.450 | −0.0007 | 0.0528 | 16.29 |
| 1982Q4 | | (0.165) | (0.137) | (0.0040) | (0.0179) | (0.363) |
| Panel C: Unspliced Monthly Consumption Data, Adjusted for Summation Bias, 1959–1982 | | | | | | |
| 1959 Feb– | 287 | 1.316 | 1.581 | −0.0008 | 0.0804 | 10.47 |
| 1982 Dec | | (0.186) | (0.069) | (0.0034) | (0.0263) | (0.789) |

Table V. Insignificant $\gamma_0$s and positive & significant $\gamma_1$s

- By and large, Table V demonstrates that
  - The zero-beta rate is insignificant
  - The market price of the consumption betas is positive and significant
  - The linearity hypothesis is rejected, mainly due to the first subperiod
  - The estimated betas are not dispersed enough
    - The resulting cross-sectional regressions may lose power
  - Except for the subperiod 1929–1939, Table V provides positive support for the CCAPM
- Table VI displays that
  - The zero-beta rates from the MCP and the CRSP are both insignificant
  - Both the MCP and the CRSP are ex post inefficient

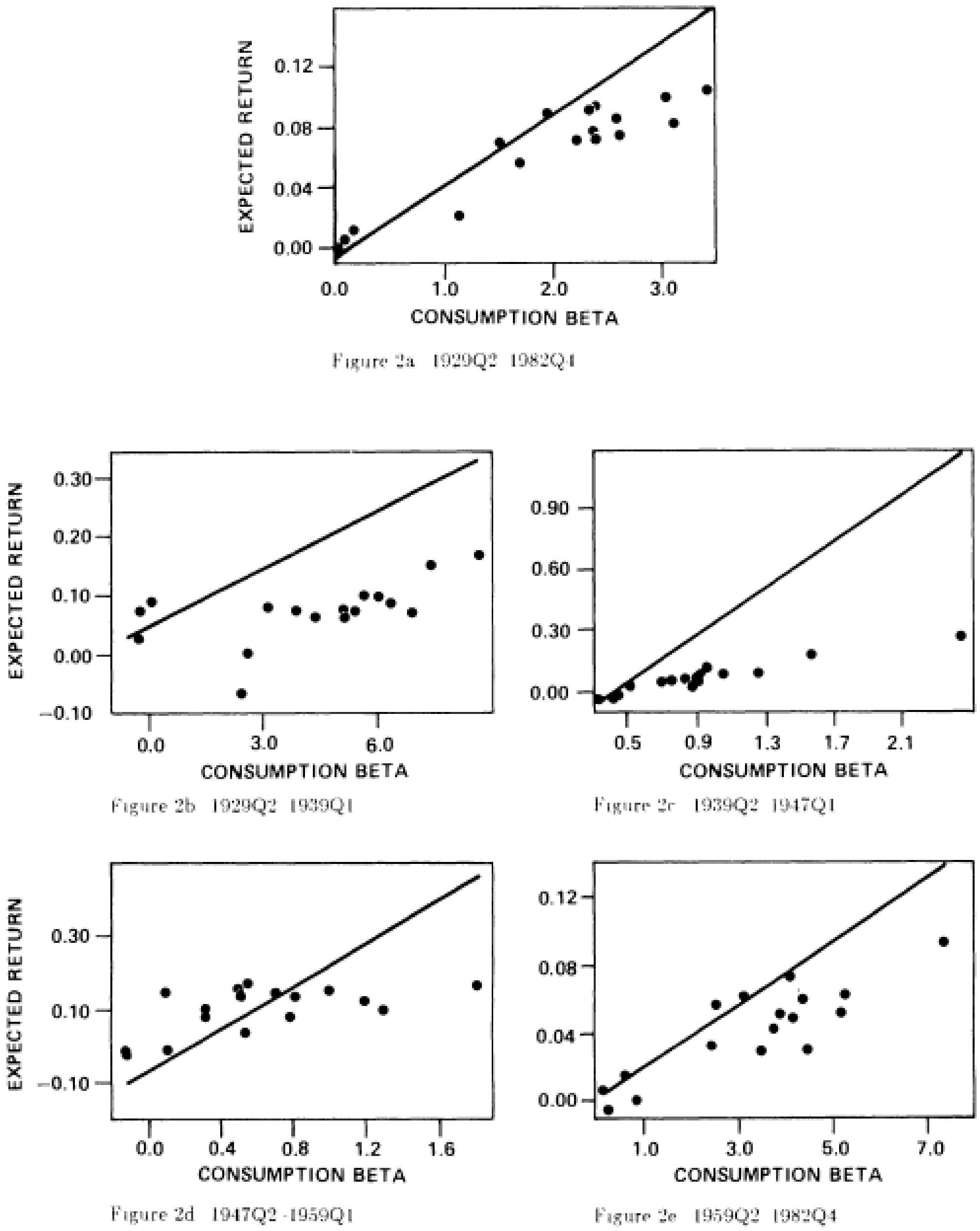


Figure 2. The linearity assumption at a glance—the authors say they're reasonably linear

## V. Conclusion

- CCAPM versus CAPM
- Address two econometric issues in consumption data
  - The summation bias issue—the betas are fine-tuned correspondingly
  - The data frequency issue—the MCP is introduced
- Unlike the existing evidence, insignificant $\gamma_0$s and positive & significant $\gamma_1$s are reported
- The risk–return relation implied by CCAPM is reasonably linear, given the poor quality of the consumption data
- Both the market portfolio and the MCP are mean-variance inefficient

| Date | Number of Observations | $\hat{\gamma}$ ($SE(\hat{\gamma})$) | LR Test (*p*-Value) |
|---|---|---|---|
| **Panel A: Mean-Variance Efficiency Tests on the MCP** | | | |
| 1926–1982 | 684 | −0.0009 | 26.86 |
| | | (0.0027) | (0.029) |
| 1926–1945 | 240 | 0.0064 | 49.21 |
| | | (0.0054) | (<0.001) |
| 1946–1965 | 240 | −0.0151 | 40.96 |
| | | (0.0049) | (<0.001) |
| 1966–1982 | 204 | 0.0016 | 19.25 |
| | | (0.0024) | (0.203) |
| **Panel B: Mean-Variance Efficiency Tests on CRSP Index** | | | |
| 1926–1982 | 684 | 0.0000 | 26.77 |
| | | (0.0027) | (0.031) |
| 1926–1945 | 240 | 0.0076 | 49.98 |
| | | (0.0053) | (<0.001) |
| 1946–1965 | 240 | −0.0125 | 36.62 |
| | | (0.0047) | (0.001) |
| 1966–1982 | 204 | 0.0016 | 19.23 |
| | | (0.0024) | (0.204) |

Table VI. The zero-beta rates are insignificant with both the MCP and the CRSP—the MCP is never inferior

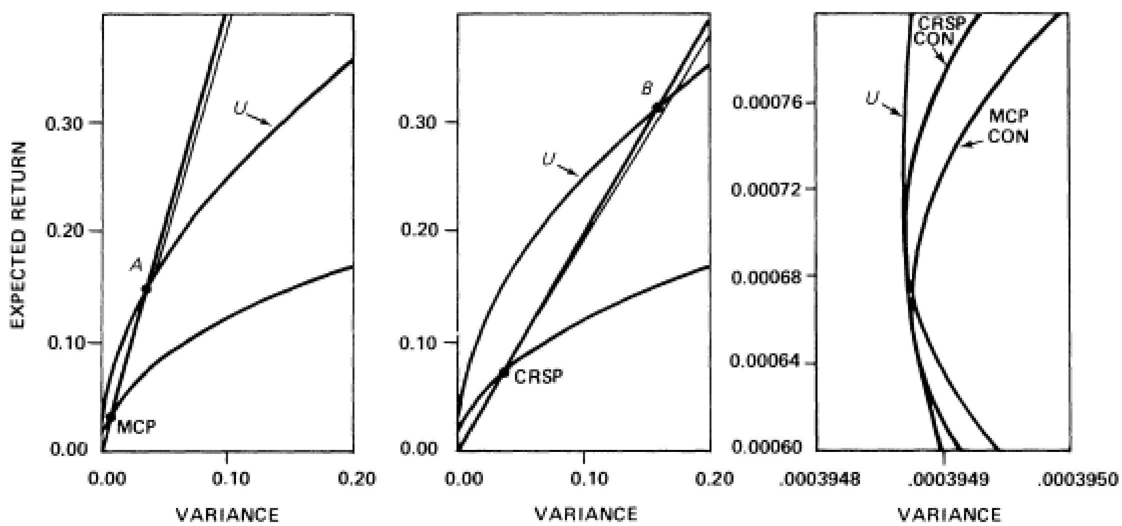


Figure 3a. Unconstrained *ex post* efficient frontier ($U$) versus frontier constrained so that MCP is efficient.

Figure 3b. Unconstrained *ex post* efficient frontier ($U$) versus frontier constrained so that CRSP Index is efficient.

Figure 3c. Unconstrained *ex post* efficient frontier ($U$) versus frontier constrained so that CRSP Index is efficient (CRSP CON.) versus frontier constrained so that MCP is efficient (MCP CON.). Note the grid is very fine.

Figure 3. The MCP versus the CRSP index—both are about equally inefficient