Fama and MacBeth (1973)

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Abstract

This paper tests the relationship between average return and risk for New York Stock Exchange common stocks. The theoretical basis of the tests is the "two-parameter" portfolio model and models of market equilibrium derived from the two-parameter portfolio model. We cannot reject the hypothesis of these models that the pricing of common stocks reflects the attempts of risk-averse investors to hold portfolios that are "efficient" in terms of expected value and dispersion of return. Moreover, the observed "fair game" properties of the coefficients and residuals of the risk-return regressions are consistent with an "efficient capital market"—that is, a market where prices of securities fully reflect available information.

- β works!

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1. Theoretical Background

- MV model says that investors hold "efficient" portfolios under
  - Risk-averse price-taking individuals
  - Normal (or 2-parameter symmetric) asset returns
  - No transaction and information cost
- Portfolio risk is proportional to contributing asset covariances

$$\sigma\left(R_p\right) = \sum_{i=1}^{N} x_{ip} \left[\frac{\sum_{j=1}^{N} x_{jp} \sigma_{ij}}{\sigma\left(R_p\right)} \right] = \sum_{i=1}^{N} x_{ip} \frac{\operatorname{cov}\left(R_i, R_p\right)}{\sigma\left(R_p\right)}$$

- FOC relates covariance risks and expected returns of efficient portfolios and participating assets

$$E\left(R_i\right) - E\left(R_m\right) = S_m \left[\frac{\sum_{j=1}^{N} x_{jm} \sigma_{ij}}{\sigma\left(R_m\right)} - \sigma\left(R_m\right) \right] \qquad (1)$$

2. Testable Implications

- The FOC also relates expected returns and betas linearly

$$E(R_i) = \underbrace{E(R_0)}_{\text{zero-beta}} + \underbrace{[E(R_m) - E(R_0)]}_{\text{risk premium}} \beta_i \qquad (6)$$

- Three implications exist
  1. The relationship between $E(R_i)$ and β_i is linear
  2. β_i among other risks determines $E(R_i)$ exclusively
  3. $E(R_m) - E(R_0) > 0$ as investors are risk-averse
- Under homogenous expectations and short selling assumptions, the market portfolio is efficient in a market equilibrium (Black, 1972)

$$x_{im} \equiv \frac{\text{total market value of all units of asset } i}{\text{total market value of all assets}}$$

2. Testable Implications (cont.)

- The authors use the following regression

$$R_{it} = \gamma_{0t} + \gamma_{1t}\beta_i + \gamma_{2t}\beta_i^2 + \gamma_{3t}s_i + \eta_{it} \tag{7}$$

- They test five hypotheses
  1. $E\left(\gamma_{2t}\right) = 0$ as linear
  2. $E\left(\gamma_{3t}\right) = 0$ as exclusive
  3. $E\left(\gamma_{1t}\right) = E\left(R_{mt}\right) - E\left(R_{0t}\right) > 0$ as risk-averse
  4. $E\left(\gamma_{0t}\right) = R_{ft}$ as there exists a risk-free asset
  5. γ_{2t}, γ_{3t}, $\gamma_{1t} - \left[E\left(R_{mt}\right) - E\left(R_{0t}\right)\right]$, $\gamma_{0t} - E\left(R_{0t}\right)$ and η_{it} are fair games as the capital market is efficient
- s_i is is meant to be some measure of the risk of security i that is not deterministically related to β_i

3. Previous Work

- Douglas (1969): $\exists$priced risks other than β
- Miller and Scholes (1972): Criticize his techniques and data. Fama and MacBeth (1973) address these issues
- Friend and Blume (1970), Black, Jensen, and Scholes (1972): Show $E(\gamma_{0t}) > R_{ft}$ since 1940
- No paper tests Hypotheses 1 and 5

4. Methodology

- All NYSE common stocks, Jan 1926–Jun 1968, CRSP
- Measurement error causes endogeneity

$$\hat{\beta}_i = \frac{\widehat{\text{cov}}\left(R_i, R_m\right)}{\hat{\sigma}^2\left(R_m\right)} \neq \beta_i$$

- The authors employ portfolio betas rather than individual betas following Blume (1970)

$$\hat{\beta}_p = \frac{\widehat{\text{cov}}\left(R_p, R_m\right)}{\hat{\sigma}^2\left(R_m\right)} = \sum_{i=1}^{N} x_{ip} \frac{\widehat{\text{cov}}\left(R_i, R_m\right)}{\hat{\sigma}^2\left(R_m\right)} = \sum_{i=1}^{N} x_{ip} \hat{\beta}_i$$

- They obtain portfolio betas based on ranked values of individual betas to reduce the loss of information
  - But positively correlated measurement errors exaggerate portfolio betas ($\hat{\beta}_p$ over/underestimates high/low β_ps)
  - They minimize these errors by forming portfolios with ranked values and then obtaining portfolio betas with subsequent data

4. Methodology (cont.)

	Periods					Periods			
	1	2	3	4	5	6	7	8	9
Portfolio formation period ...	1926–29	1927–33	1931–37	1935–41	1939–45	1943–49	1947–53	1951–57	1955–61
Initial estimation period	1930–34	1934–38	1938–42	1942–46	1946–50	1950–54	1954–58	1958–62	1962–66
Testing period	1935–38	1939–42	1943–46	1947–50	1951–54	1955–58	1959–62	1963–66	1967–68
No. of securities available	710	779	804	908	1,011	1,053	1,065	1,162	1,261
No. of securities meeting data requirement	435	576	607	704	751	802	856	858	845

- For each period, the authors form twenty portfolios using ranked values of individual betas (estimated once) from a portfolio formation period, estimate portfolio betas using individual betas (updated yearly) from an initial estimation period, and estimate monthly regressions using portfolio returns from a testing period

$$R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it} \tag{8}$$

$$\sigma^2(R_i) = \beta_i^2 \sigma^2(R_m) + \sigma^2(\epsilon_i) + 2\beta_i \mathrm{cov}(R_m, \epsilon_i) \tag{9}$$

$$R_{pt} = \hat{\gamma}_{0t} + \hat{\gamma}_{1t}\hat{\beta}_{pt-1} + \hat{\gamma}_{2t}\hat{\beta}^2_{pt-1} + \hat{\gamma}_{3t}\bar{s}_{pt-1}(\hat{\epsilon}_i) + \hat{\eta}_{pt} \tag{10}$$

- $\hat{\beta}^2_{pt-1}$ is an average of $\hat{\beta}_i$s (not a squared average) and $\bar{s}_{pt-1}$ is an average of $s(\hat{\epsilon}_i)$ (not $s^2(\hat{\epsilon}_i)$)

Table 2

Statistic	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	Portfolios for Estimation Period 1934–38										Portfolios for Estimation Period 1934–38									
$\hat{\beta}_{p,t-1}$	.322	.508	.651	.674	.695	.792	.921	.942	.970	1.005	1.046	1.122	1.181	1.192	1.196	1.295	1.335	1.396	1.445	1.458
$s(\hat{\beta}_{p,t-1})$	.027	.027	.025	.023	.028	.026	.032	.029	.034	.027	.028	.031	.035	.028	.029	.032	.032	.053	.039	.053
$r(R_p, R_m)^2$	.709	.861	.921	.936	.912	.941	.932	.946	.933	.958	.959	.956	.951	.969	.966	.966	.967	.922	.958	.927
$s(R_p)$	.040	.058	.072	.074	.077	.087	.101	.103	.106	.109	.113	.122	.128	.128	.129	.140	.144	.154	.156	.160
$s(\hat{\epsilon}_p)$	.022	.022	.020	.019	.023	.021	.026	.024	.028	.022	.023	.026	.029	.023	.024	.026	.026	.043	.032	.043
$\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.085	.075	.083	.078	.090	.095	.109	.106	.111	.097	.094	.124	.120	.122	.132	.125	.129	.158	.145	.170
$s(\hat{\epsilon}_p)/\bar{s}_{p,t-1}(\hat{\epsilon}_i)$ ..	.259	.293	.241	.244	.256	.221	.238	.226	.252	.227	.245	.210	.242	.188	.182	.208	.202	.272	.221	.253
	Portfolios for Estimation Period 1942–46										Portfolios for Estimation Period 1942–46									
$\hat{\beta}_{p,t-1}$	.467	.537	.593	.628	.707	.721	.770	.792	.805	.894	.949	.952	1.010	1.038	1.254	1.312	1.316	1.473	1.631	1.661
$s(\hat{\beta}_{p,t-1})$	.045	.041	.044	.037	.027	.032	.035	.035	.028	.040	.031	.036	.040	.030	.034	.039	.041	.084	.083	.077
$r(R_p, R_m)^2$	.645	.745	.753	.829	.919	.898	.889	.898	.934	.896	.942	.923	.917	.954	.958	.951	.945	.839	.867	.887
$s(R_p)$	.035	.037	.041	.041	.044	.046	.049	.050	.050	.057	.059	.060	.063	.064	.077	.081	.081	.097	.105	.106
$s(\hat{\epsilon}_p)$	.021	.019	.020	.017	.013	.015	.016	.016	.013	.018	.014	.016	.018	.014	.016	.018	.019	.039	.038	.036
$\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.055	.055	.063	.058	.058	.063	.064	.064	.062	.069	.073	.074	.085	.077	.096	.083	.086	.134	.117	.122
$s(\hat{\epsilon}_p)/\bar{s}_{p,t-1}(\hat{\epsilon}_i)$ ..	.382	.345	.317	.293	.224	.238	.250	.250	.210	.261	.192	.216	.212	.182	.167	.217	.221	.291	.325	.295
	Portfolios for Estimation Period 1950–54										Portfolios for Estimation Period 1950–54									
$\hat{\beta}_{p,t-1}$	.418	.590	.694	.751	.777	.784	.929	.950	.996	1.014	1.117	1.123	1.131	1.134	1.186	1.235	1.295	1.324	1.478	1.527
$s(\hat{\beta}_{p,t-1})$	.042	.047	.045	.037	.038	.035	.050	.038	.035	.029	.039	.027	.044	.033	.037	.049	.045	.046	.058	.086
$r(R_p, R_m)^2$	.629	.723	.798	.872	.878	.895	.856	.913	.933	.954	.934	.968	.919	.952	.944	.915	.933	.934	.917	.841
$s(R_p)$	.019	.025	.028	.029	.030	.030	.036	.036	.037	.038	.042	.041	.043	.042	.044	.047	.049	.050	.056	.060
$s(\hat{\epsilon}_p)$	.012	.013	.013	.010	.010	.010	.014	.011	.010	.008	.011	.007	.012	.009	.010	.014	.013	.013	.016	.024
$\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.040	.044	.046	.048	.051	.051	.052	.053	.054	.057	.066	.057	.066	.060	.064	.064	.065	.068	.076	.088
$s(\hat{\epsilon}_p)/\bar{s}_{p,t-1}(\hat{\epsilon}_i)$ ..	.300	.295	.283	.208	.196	.196	.269	.208	.185	.140	.167	.123	.182	.150	.156	.219	.200	.192	.210	.273
	Portfolios for Estimation Period 1958–62										Portfolios for Estimation Period 1958–62									
$\hat{\beta}_{p,t-1}$	.626	.635	.719	.801	.817	.860	.920	.950	.975	.995	1.013	1.019	1.037	1.048	1.069	1.081	1.092	1.098	1.269	1.388
$s(\hat{\beta}_{p,t-1})$	.043	.048	.039	.046	.047	.033	.037	.038	.032	.037	.038	.031	.036	.033	.036	.038	.045	.045	.048	.065
$r(R_p, R_m)^2$	.783	.745	.851	.835	.838	.920	.913	.915	.939	.925	.922	.948	.934	.945	.936	.931	.907	.910	.922	.886
$s(R_p)$	.030	.031	.033	.037	.038	.038	.041	.042	.043	.044	.045	.045	.046	.046	.047	.048	.049	.049	.056	.063
$s(\hat{\epsilon}_p)$	.014	.016	.013	.015	.015	.011	.012	.012	.011	.012	.013	.010	.012	.011	.012	.013	.015	.015	.016	.021
$\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.049	.052	.056	.059	.064	.061	.070	.069	.068	.064	.069	.066	.067	.062	.070	.072	.076	.068	.070	.078
$s(\hat{\epsilon}_p)/\bar{s}_{p,t-1}(\hat{\epsilon}_i)$ ..	.286	.308	.232	.254	.234	.180	.171	.174	.162	.188	.188	.152	.179	.177	.171	.180	.197	.220	.228	.269

4.C. Some Observations on the Approach

- $\hat{\beta}_{pt-1}$ increases from left to right, so the portfolios are formed as intended
- $s(\hat{\epsilon}_p)$ is about 18–29% of $\bar{s}_{pt-1}(\hat{\epsilon}_i)$, so forming the portfolios addresses the measurement error issue as intended
  - That is, portfolio betas are more precise with lower standard errors than individual betas
  - Extreme/moderate portfolios have high/low ratios, so more/less subject to the measurement error issue

Table 3

Period	Statistic																			
	$\bar{\hat{\gamma}}_0$	$\bar{\hat{\gamma}}_1$	$\bar{\hat{\gamma}}_2$	$\bar{\hat{\gamma}}_3$	$\overline{\hat{\gamma}_0 - R_f}$	$s(\hat{\gamma}_0)$	$s(\hat{\gamma}_1)$	$s(\hat{\gamma}_2)$	$s(\hat{\gamma}_3)$	$\rho_0(\hat{\gamma}_0 - R_f)$	$\rho_M(\hat{\gamma}_1)$	$\rho_0(\hat{\gamma}_2)$	$\rho_0(\hat{\gamma}_3)$	$t(\bar{\hat{\gamma}}_0)$	$t(\bar{\hat{\gamma}}_1)$	$t(\bar{\hat{\gamma}}_2)$	$t(\bar{\hat{\gamma}}_3)$	$t(\overline{\hat{\gamma}_0 - R_f})$	$\bar{r}^2$	$s(r^2)$
Panel A:																				
1935–6/68 ..	.0061	.0085	. . .	. . .	.0048	.038	.066	. . .	. . .	$\boxed{.15}$	$\boxed{.02}$	. . .	. . .	3.24	$\boxed{2.57}$	. . .	. . .	$\boxed{2.55}$	.29	.30
1935–45	.0039	.0163	. . .	. . .	.0037	.052	.098	. . .	. . .	.10	—.03	. . .	. . .	.86	1.92	. . .	. . .	.82	.29	.29
1946–55	.0087	.0027	. . .	. . .	.0078	.026	.041	. . .	. . .	.18	.07	. . .	. . .	3.71	.70	. . .	. . .	3.31	.31	.32
1956–6/68 ..	.0060	.0062	. . .	. . .	.0034	.030	.044	. . .	. . .	.27	.15	. . .	. . .	2.45	1.73	. . .	. . .	1.39	.28	.29
1935–40	.0024	.0109	. . .	. . .	.0023	.064	.116	. . .	. . .	.07	—.09	. . .	. . .	.32	.79	. . .	. . .	.31	.23	.30
1941–45	.0056	.0229	. . .	. . .	.0054	.034	.069	. . .	. . .	.23	.15	. . .	. . .	1.27	2.55	. . .	. . .	1.22	.37	.28
1946–50	.0050	.0029	. . .	. . .	.0044	.031	.047	. . .	. . .	.20	.04	. . .	. . .	1.27	.48	. . .	. . .	1.10	.39	.33
1951–55	.0123	.0024	. . .	. . .	.0111	.019	.035	. . .	. . .	.20	.08	. . .	. . .	5.06	.53	. . .	. . .	4.56	.24	.29
1956–60	.0148	—.0059	. . .	. . .	.0128	.020	.034	. . .	. . .	.37	.18	. . .	. . .	5.68	—1.37	. . .	. . .	4.89	.22	.31
1961–6/68 ..	.0001	.0143	. . .	. . .	—.0029	.034	.048	. . .	. . .	.22	.09	. . .	. . .	.03	2.81	. . .	. . .	—.80	.32	.27
Panel B:																				
1935–6/68 ..	.0049	.0105	—.0008	. . .	.0036	.052	.118	.056	. . .	$\boxed{.03}$	$\boxed{—.11}$	$\boxed{—.11}$	. . .	1.92	$\boxed{1.79}$	$\boxed{—.29}$	. . .	$\boxed{1.42}$	.32	.31
1935–45	.0074	.0079	.0040	. . .	.0073	.061	.139	.074	. . .	—.10	—.31	—.21	. . .	1.39	.65	.61	. . .	1.36	.32	.30
1946–55	—.0002	.0217	—.0087	. . .	—.0012	.036	.095	.034	. . .	.04	.00	.00	. . .	—.07	2.51	—2.83	. . .	—.38	.36	.32
1956–6/68 ..	.0069	.0040	.0013	. . .	.0043	.054	.116	.053	. . .	.17	.07	.03	. . .	1.56	.42	.29	. . .	.97	.30	.30
1935–40	.0013	.0141	—.0017	. . .	.0012	.069	.160	.075	. . .	—.13	—.36	—.35	. . .	.16	.75	—.19	. . .	.14	.24	.30
1941–45	.0148	.0004	.0108	. . .	.0146	.050	.111	.073	. . .	—.04	—.19	—.04	. . .	2.28	.03	1.15	. . .	2.24	.39	.29
1946–50	—.0008	.0152	—.0051	. . .	—.0015	.037	.104	.032	. . .	.14	.04	.00	. . .	—.18	1.14	—1.24	. . .	—.32	.44	.32
1951–55	.0004	.0281	—.0122	. . .	—.0008	.030	.085	.035	. . .	—.17	—.14	—.01	. . .	.10	2.55	—2.72	. . .	—.20	.28	.29
1956–60	.0128	—.0015	—.0020	. . .	.0108	.030	.072	.029	. . .	.35	.11	.26	. . .	3.38	—.16	—.54	. . .	2.84	.25	.31
1961–6/68 ..	.0029	.0077	.0034	. . .	—.0000	.066	.138	.064	. . .	.14	.06	—.01	. . .	.42	.53	.51	. . .	—.01	.34	.29
Panel C:																				
1935–6/68 ..	.0054	.0072	. . .	.0198	.0041	.052	.065	. . .	.868	$\boxed{.04}$	$\boxed{—.12}$	. . .	$\boxed{—.04}$	2.10	$\boxed{2.20}$	. . .	$\boxed{.46}$	$\boxed{1.59}$	.32	.31
1935–45	.0017	.0104	. . .	.0841	.0015	.073	.083	. . .	.921	—.00	—.26	. . .	—.08	.26	1.41	. . .	1.05	.24	.32	.31
1946–55	.0110	.0075	. . .	—.1052	.0100	.032	.056	. . .	.609	.08	.02	. . .	—.20	3.78	1.47	. . .	—1.89	3.46	.34	.32
1956–6/68 ..	.0042	.0041	. . .	.0633	.0016	.040	.052	. . .	.984	.12	.08	. . .	.03	1.28	.96	. . .	.79	.50	.30	.29
1935–40	.0036	.0119	. . .	—.0170	.0035	.082	.105	. . .	.744	—.03	—.26	. . .	—.18	.37	.97	. . .	—.19	.36	.25	.30
1941–45	—.0006	.0085	. . .	.2053	—.0009	.061	.052	. . .	1.091	.07	—.29	. . .	—.02	—.08	1.25	. . .	1.46	—.11	.41	.30
1946–50	.0069	.0081	. . .	—.0920	.0062	.034	.066	. . .	.504	.14	.06	. . .	—.02	1.56	.95	. . .	—1.41	1.40	.42	.33
1951–55	.0150	.0069	. . .	—.1185	.0138	.029	.043	. . .	.702	.06	—.18	. . .	—.32	4.05	1.24	. . .	—1.31	3.72	.27	.29
1956–60	.0127	—.0081	. . .	.0728	.0107	.037	.045	. . .	1.164	.15	.15	. . .	.21	2.68	—1.40	. . .	.48	2.26	.26	.30
1961–6/68 ..	—.0014	.0122	. . .	.0570	—.0044	.042	.055	. . .	.850	.10	.00	. . .	—.19	—.32	2.12	. . .	.64	—.98	.33	.27
Panel D:																				
1935–6/68 ..	.0020	.0114	—.0026	.0516	.0008	.075	.123	.060	.929	$\boxed{—.09}$	$\boxed{—.09}$	$\boxed{—.12}$	$\boxed{—.10}$	.55	$\boxed{1.85}$	$\boxed{—.86}$	$\boxed{1.11}$	$\boxed{.20}$	.34	.31
1935–45	.0011	.0118	—.0009	.0817	.0010	.103	.146	.079	1.003	—.20	—.23	—.24	—.15	.13	.94	—.14	.94	.11	.34	.31
1946–55	.0017	.0209	—.0076	—.0378	.0008	.042	.096	.038	.619	—.10	—.00	—.01	—.20	.44	2.39	—2.16	—.67	.20	.36	.32
1956–6/68 ..	.0031	.0034	—.0000	.0966	.0005	.065	.122	.055	1.061	.12	.03	.01	—.05	.59	.34	—.00	1.11	.10	.32	.29
1935–40	.0009	.0156	—.0029	.0025	.0008	.112	.171	.085	.826	—.16	—.23	—.26	—.12	.07	.78	—.29	.03	.06	.26	.30
1941–45	.0015	.0073	.0014	.1767	.0012	.092	.109	.072	1.181	—.28	—.21	—.22	—.18	.12	.52	.15	1.16	.10	.43	.31
1946–50	.0011	.0141	—.0040	—.0313	.0004	.047	.106	.042	.590	—.10	.03	—.01	—.12	.18	1.03	—.73	—.41	.07	.44	.33
1951–55	.0023	.0277	—.0112	—.0443	.0011	.037	.085	.034	.651	—.11	—.13	—.01	—.28	.48	2.53	—2.54	—.53	.23	.29	.30
1956–60	.0103	—.0047	—.0020	.0979	.0083	.049	.078	.032	1.286	—.16	.19	—.01	.02	1.63	—.47	—.49	.59	1.31	.28	.30
1961–6/68 ..	—.0017	.0088	.0013	.0957	—.0046	.073	.144	.066	.887	.20	.00	.01	—.15	—.21	.58	.19	1.02	—.60	.35	.29

5. Results

- Table 3 picks $\hat{\beta}_p$ in Panel A, $\hat{\beta}_p$ and $\hat{\beta}_p^2$ in Panel B, $\hat{\beta}_p$ and $\bar{s}_p\left(\hat{\epsilon}_i\right)$ in Panel C, and all explanatory variables in Panel D
- The authors estimate the regression parameters from the monthly regressions and compute the following t-statistics from the monthly estimates to test the hypotheses above

$$t\left(\bar{\hat{\gamma}}_j\right) = \frac{\bar{\hat{\gamma}}_j}{\frac{s\left(\hat{\gamma}_j\right)}{\sqrt{n}}}$$

- In short, they insist that
  - The relationship is linear as $\bar{\hat{\gamma}}_2$ is insignificant
  - Idiosyncratic volatilities are not priced as $\bar{\hat{\gamma}}_3$ is insignificant
  - Betas are positively priced as $\bar{\hat{\gamma}}_1$ is positively significant
  - Whether $E\left(\gamma_{0t}\right) = R_{ft}$ is ambiguous as $\overline{\hat{\gamma}_0 - R_f}$ is insignificant in Panels B–D, but significant in Panel A (most efficient)
  - The capital market is efficient (i.e. the parameters are fair games) as the serial correlations are insignificant

Table 4

Period	$\overline{R_m}$	$\overline{R_m - R_f}$	$\bar{\hat{\gamma}}_1$	$\bar{\hat{\gamma}}_0$	$\overline{R_f}$	$\frac{\overline{R_m - R_f}}{s(R_m)}$	$\frac{\bar{\hat{\gamma}}_1}{s(R_m)}$	$s(R_m)$	$s(R_m)$
	Statistic*								
1935–6/68	.0143	$\boxed{.0130}$	$\boxed{.0085}$	$\boxed{.0061}$	$\boxed{.0013}$	.2136	.1388	.061	.066
1935–45	.0197	.0195	.0163	.0039	.0002	.2207	.1844	.089	.098
1946–55	.0112	.0103	.0027	.0087	.0009	.2378	.0614	.043	.041
1956–6/68	.0121	.0095	.0062	.0060	.0026	.2387	.1560	.040	.044
1935–40	.0132	.0132	.0109	.0024	.0001	.1221	.1009	.108	.116
1941–45	.0274	.0272	.0229	.0056	.0002	.4715	.3963	.058	.069
1946–50	.0077	.0070	.0029	.0050	.0007	.1351	.0564	.052	.047
1951–55	.0148	.0136	.0024	.0123	.0012	.4174	.0735	.033	.035
1956–60	.0090	.0070	—.0059	.0148	.0020	.2080	—.1755	.034	.034
1961–6/68	.0141	.0111	.0143	.0001	.0030	.2567	.3294	.043	.048

Period	$s(\hat{\gamma}_0)$	$s(R_f)$	$t(\bar{R}_m)$	$t(\overline{R_m - R_f})$	$t(\bar{\hat{\gamma}}_1)$	$t(\bar{\hat{\gamma}}_0)$	$\rho_M(R_m)$	$\rho_M(R_m - R_f)$	$\rho_M(\hat{\gamma}_1)$	$\rho_M(\hat{\gamma}_0)$	$\rho_M(R_f)$
	Statistic*										
1935–6/68	.038	.0012	4.71	$\boxed{4.28}$	$\boxed{2.57}$	$\boxed{3.24}$	—.01	—.01	.02	.14	.98
1935–45	.052	.0001	2.56	2.54	1.92	.86	—.07	—.07	—.03	.10	.88
1946–55	.026	.0004	2.84	2.60	.70	3.71	.09	.09	.07	.10	.94
1956–6/68	.030	.0009	3.72	2.92	1.73	2.45	.14	.14	.15	.25	.92
1935–40	.064	.0001	1.04	1.04	.79	.32	—.13	—.13	—.09	.07	.72
1941–45	.034	.0001	3.68	3.65	2.55	1.27	.14	.14	.15	.21	.83
1946–50	.031	.0003	1.15	1.05	.48	1.27	.09	.09	.04	.18	.97
1951–55	.019	.0004	3.51	3.22	.53	5.06	—.02	—.01	.08	—.07	.89
1956–60	.020	.0007	2.07	1.60	—1.37	5.68	.12	.13	.18	.13	.80
1961–6/68	.034	.0008	3.08	2.44	2.81	.03	.13	.13	.09	.21	.93

- The difference between $\overline{R_m - R_f}$ and $\bar{\hat{\gamma}}_1$ and that between $\bar{R}_f$ and $\bar{\hat{\gamma}}_0$ are noticeable
  - Sharpe (1964) and Lintner (1965) derive the beta representation using both risky and risk-free assets
  - Black (1972) derives the representation using risky assets only
  - SL version predicts $E(\gamma_{0t}) = E(R_{0t}) = R_{ft}$ and $E(\gamma_{1t}) = E(R_{mt}) - E(R_{0t}) = E(R_{mt}) - R_{ft}$ in equilibrium

Table 5

Period	$s^2(\tilde{\gamma}_0)$	$s^2(\hat{\gamma}_0)$	$\overline{s^2(\tilde{\phi}_0)}$	F	$s^2(\tilde{\gamma}_1)$	$s^2(\hat{\gamma}_1)$	$\overline{s^2(\tilde{\phi}_1)}$	F	$s^2(\tilde{\gamma}_2)$	$s^2(\hat{\gamma}_2)$	$\overline{s^2(\tilde{\phi}_2)}$	F	$s^2(\tilde{\gamma}_3)$	$s^2(\hat{\gamma}_3)$	$\overline{s^2(\tilde{\phi}_3)}$	F
Panel A:																
1935–6/68 ...	.00105	.00142	.00037	3.84	.00401	.00436	.00035	12.46	...	...	...	...	...	...	...	...
1935–45	.00182	.00273	.00091	3.00	.00863	.00950	.00087	10.92	...	...	...	...	...	...	...	...
1946–55	.00057	.00066	.00009	7.33	.00163	.00171	.00008	21.38	...	...	...	...	...	...	...	...
1956–6/68 ...	.00077	.00090	.00013	6.92	.00181	.00193	.00012	16.08	...	...	...	...	...	...	...	...
1935–40	.00265	.00404	.00139	2.91	.01212	.01347	.00135	9.98	...	...	...	...	...	...	...	...
1941–45	.00086	.00118	.00032	3.69	.00452	.00481	.00029	16.59	...	...	...	...	...	...	...	...
1946–50	.00086	.00094	.00008	11.75	.00216	.00224	.00008	28.00	...	...	...	...	...	...	...	...
1951–55	.00027	.00036	.00009	4.00	.00113	.00121	.00008	15.12	...	...	...	...	...	...	...	...
1956–60	.00032	.00041	.00009	4.56	.00104	.00112	.00008	21.50	...	...	...	...	...	...	...	...
1961–6/68 ...	.00100	.00114	.00014	8.14	.00217	.00231	.00014	16.50	...	...	...	...	...	...	...	...
Panel B:																
1935–6/68 ...	.00092	.00267	.00175	1.52	.00564	.01403	.00839	1.67	.00121	.00318	.00197	1.61	...	...	...	...
1935–45	.00057	.00377	.00320	1.18	.00372	.01941	.01569	1.24	.00171	.00548	.00377	1.45	...	...	...	...
1946–55	.00053	.00112	.00059	1.90	.00651	.00897	.00245	3.66	.00063	.00112	.00049	2.29	...	...	...	...
1956–6/68 ...	.00155	.00294	.00139	2.12	.00667	.01338	.00671	1.99	.00122	.00278	.00156	1.78	...	...	...	...
1935–40	.00018	.00476	.00458	1.04	.00374	.02555	.02181	1.17	.00041	.00566	.00524	1.08	...	...	...	...
1941–45	.00101	.00254	.00153	1.66	.00389	.01225	.00836	1.46	.00327	.00527	.00201	2.62	...	...	...	...
1946–50	.00084	.00136	.00052	2.62	.00862	.01071	.00209	5.12	.00066	.00103	.00037	2.78	...	...	...	...
1951–55	.00024	.00090	.00066	1.36	.00447	.00729	.00282	2.58	.00058	.00120	.00062	1.94	...	...	...	...
1956–60	.00037	.00087	.00050	1.74	.00289	.00517	.00228	2.27	.00033	.00083	.00050	1.66	...	...	...	...
1961–6/68 ...	.00232	.00431	.00199	2.16	.00928	.01894	.00966	1.96	.00182	.00410	.00227	1.81	...	...	...	...
Panel C:																
1935–6/68 ...	.00192	.00266	.00075	3.55	.00285	.00428	.00142	3.01	...	...	...	...	.341	.753	.412	1.83
1935–45	.00394	.00533	.00139	3.83	.00433	.00717	.00283	2.52	...	...	...	...	.535	.847	.313	2.71
1946–55	.00083	.00101	.00018	5.61	.00261	.00310	.00050	6.20	...	...	...	...	.165	.370	.206	1.80
1956–6/68 ...	.00100	.00164	.00063	2.60	.00178	.00270	.00092	2.93	...	...	...	...	.304	.968	.664	1.46
1935–40	.00473	.00669	.00196	3.41	.00732	.01094	.00362	3.02	...	...	...	...	.270	.553	.282	1.96
1941–45	.00307	.00377	.00070	5.38	.00085	.00274	.00189	1.45	...	...	...	...	.840	1.189	.349	3.41
1946–50	.00103	.00117	.00014	8.36	.00386	.00439	.00053	8.28	...	...	...	...	.118	.254	.136	1.87
1951–55	.00061	.00083	.00022	3.77	.00140	.00188	.00047	4.00	...	...	...	...	.217	.493	.276	1.79
1956–60	.00079	.00134	.00055	2.44	.00106	.00204	.00098	2.08	...	...	...	...	.622	1.355	.734	1.85
1961–6/68 ...	.00109	.00177	.00068	2.60	.00212	.00300	.00088	3.41	...	...	...	...	.105	.722	.617	1.17
Panel D:																
1935–6/68 ...	.00150	.00566	.00406	1.39	.00608	.01521	.00913	1.66	.00061	.00362	.00301	1.21	.276	.864	.588	1.47
1935–45	.00233	.01065	.00832	1.28	.00402	.02118	.01716	1.23	...	.00624	.00644	.97	.392	1.001	.613	1.63
1946–55	.00013	.00176	.00163	1.08	.00647	.00916	.00269	3.41	.00061	.00148	.00087	1.70	.028	.383	.355	1.08
1956–6/68 ...	.00194	.00420	.00226	1.86	.00763	.01485	.00722	2.06	.00134	.00304	.00169	1.80	.374	1.125	.751	1.50
1935–40	.00157	.01263	.01106	1.14	.00457	.02910	.02453	1.19	...	.00723	.00886	.82	.120	.682	.562	1.21
1941–45	.00340	.00843	.00503	1.68	.00365	.01196	.00832	1.44	.00162	.00515	.00353	1.46	.720	1.395	.675	2.07
1946–50	.00023	.00220	.00197	1.12	.00858	.01119	.00261	4.29	.00083	.00180	.00096	1.87	.023	.348	.325	1.07
1951–55	.00006	.00136	.00130	1.05	.00442	.00719	.00277	2.60	.00039	.00116	.00077	1.51	.038	.424	.386	1.10
1956–60	.00092	.00239	.00147	1.62	.00328	.00602	.00274	2.20	.00037	.00103	.00066	1.56	.712	1.654	.941	1.76
1961–6/68 ...	.00260	.00539	.00279	1.93	.01060	.02081	.01021	2.04	.00202	.00440	.00238	1.85	.163	.787	.624	1.26

- Though some measurement errors exist, their main findings are still meaningful (large F)

VI. Conclusions

In sum our results support the important testable implications of the two-parameter model. Given that the market portfolio is efficient—or, more specifically, given that our proxy for the market portfolio is at least approximately efficient—we cannot reject the hypothesis that average returns on New York Stock Exchange common stocks reflect the attempts of risk-averse investors to hold efficient portfolios. Specifically, on average there seems to be a positive tradeoff between return and risk, with risk measured from the portfolio viewpoint. In addition, although there are "stochastic nonlinearities" from period to period, we cannot reject the hypothesis that on average their effects are zero and unpredictably different from zero from one period to the next. Thus, we cannot reject the hypothesis that in making a portfolio decision, an investor should assume that the relationship between a security's portfolio risk and its expected return is linear, as implied by the two-parameter model. We also cannot reject the hypothesis of the two-parameter model that no measure of risk, in addition to portfolio risk, systematically affects average returns. Finally, the observed fair game properties of the coefficients and residuals of the risk-return regressions are consistent with an efficient capital market—that is, a market where prices of securities fully reflect available information.