

## Abstract

- Empirical q-factor model: Market, size, investment, profitability
- 38/73 anomalies are insignificant (NYSE breakpoints, VW decile)
- Q model outperforms Fama–French and Carhart models for the rest
  - Average of  $|\alpha|$  from 10–1s:  $q=0.20\%$ , Carhart=0.33%, FF=0.55%
  - # of insignificant  $\alpha$ s:  $q=5/35$ , Carhart=19/35, FF=27/35
  - # of null-rejecting GRS stats:  $q=20/35$ , Carhart=24/35, FF=28/35

## Introduction

$$E[r^i] - r^f = \beta^i E[\text{MKT}] + \beta_{\text{ME}}^i E[r_{\text{ME}}] + \beta_{\text{I/A}}^i E[r_{\text{I/A}}] + \beta_{\text{ROE}}^i E[r_{\text{ROE}}]$$

- 2×3×3 by size, investment/assets, ROE
- Jan 1972–Dec 2012
- $E[r_{\text{ME}}]=0.31\%$ /month,  $E[r_{\text{I/A}}]=0.45\%$  (0.69  $\rho$  with HML),  $E[r_{\text{ROE}}]=0.58\%$  (0.50  $\rho$  with UMD)
- Q model explains FF ( $\alpha$ s are insignificant), while the converse is false
- Explains the momentum effect well
  - Earnings momentum  $\alpha$ s:  $q=0.16\%$ , Carhart=0.34%, FF=0.55%
  - Price momentum  $\alpha$ s:  $q=0.24\%$ , Carhart=0.06%, FF=1.12%
- Explains the 25 size-B/M portfolios well
  - MAPE:  $q=0.11\%$ , Carhart=0.11%, FF=0.10%
- Underperforms in explaining the operating accrual anomaly and the R&D-to-market anomaly
- Ceteris paribus, investment  $\uparrow \Rightarrow$  return  $\downarrow$ , and profitability  $\uparrow \Rightarrow$  return  $\uparrow$
- Investment- rather than consumption-based asset pricing model

## 1. Conceptual Framework

- Investment-based asset pricing model

$$\max_{I_{i0}} \underbrace{\frac{\Pi_{i0} A_{i0}}{\text{OCF at } 0}}_{\text{investment+adjustment}} - \underbrace{I_{i0} - \frac{a}{2} \left( \frac{I_{i0}}{A_{i0}} \right)^2 A_{i0}}_{\text{investment+adjustment}} + \underbrace{E_0[M_1 \Pi_{i1} A_{i1}]}_{\text{firm value}}$$

- First order condition

$$1 + a \frac{I_{i0}}{A_{i0}} = E_0[M_1 \Pi_{i1}]$$

- Return implication

$$r_{i1}^S = \frac{P_{i1} + D_{i1}}{P_{i0}} = \frac{\Pi_{i1} A_{i1}}{E_0[M_1 \Pi_{i1} A_{i1}]} = \frac{\Pi_{i1}}{E_0[M_1 \Pi_{i1}]} = \frac{\Pi_{i1}}{1 + a \frac{I_{i0}}{A_{i0}}} \Rightarrow E[r_{i1}^S] = \frac{E[\Pi_{i1}]}{1 + a \frac{I_{i0}}{A_{i0}}}$$

- $E[r]$  increases as  $I_{i0}$  decreases

- ◆ Less investing stock earns more return
- ◆ Given profitability, low discount rate implies high marginal  $q$  so high investment
- ◆ Value premium (Low  $q$  means high B/M so high return)

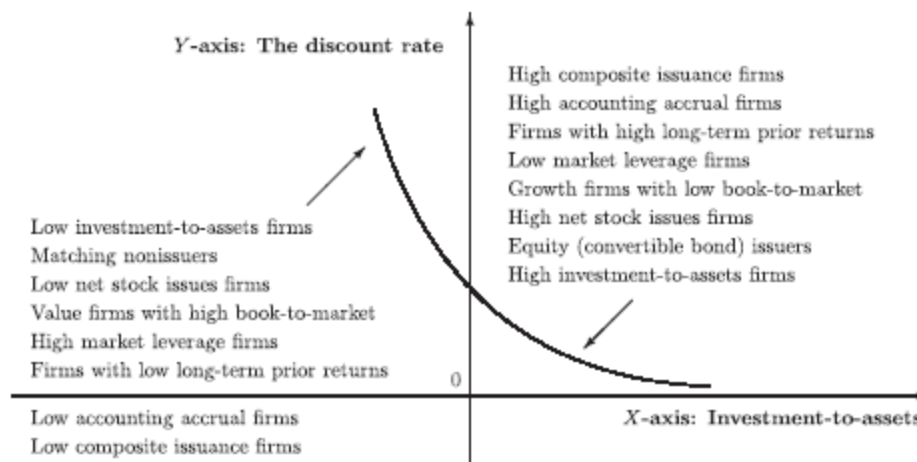


Figure 1. Stocks financing more tend to be right so low return

- $E[r]$  increases as  $E[\Pi]$  increases
  - ◆ More profitable stock earns more return
  - ◆ High profitability given investment implies high discount rate (otherwise additional investment)
  - ◆ Momentum premium (Momentum winner means high profitability so high return)
- Matching reduced-form factor model instead of structural model
  - Factor mimicking portfolio approach
  - High frequency return data are less subject to measurement error than accounting data
  - Structural model is more subject to specification error than factor model
  - Additional assumption is that stocks with similar investment/profitability comove together

## 2. Factors

- CRSP & annual/quarterly Compustat
  - Jan 1972–Dec 2012
  - Include stocks with quarterly earnings announcement dates & quarterly book equity data available only
  - Exclude financial firms and negative book equity firms
- $$I/A_t = \frac{\Delta \text{Assets}_t}{\text{Assets}_{t-1}} \text{ (annual)}, \quad \text{ROE}_t = \frac{\text{Income}_t}{\text{Book equity}_{t-1}} \text{ (quarterly)}$$
- 2×3×3 size-I/A-ROE portfolios
    - Separate 50-50 using NYSE ME at the end of each June
    - Separate 30-40-30 using NYSE I/A at the end of each June
      - ◆ I/A for the fiscal year ending in  $t-1$
    - Separate 30-40-30 using NYSE ROE at the beginning of each month
      - ◆ If the 4Q earnings in  $t-1$  become available on Mar 5 in  $t$ , then the earnings (divided by the 3Q book equity in  $t-1$ ) are employed to separate 30-40-30 at the beginning of April in  $t$
    - 18 VW portfolios are rebalanced monthly based on ROE, annually based on ME & I/A
      - ◆  $r_{\text{ME}}$ =average of 9 small–average of 9 big
      - ◆  $r_{\text{I/A}}$ =average of 6 low I/A–average of 6 high I/A
      - ◆  $r_{\text{ROE}}$ =average of 6 high ROE–average of 6 low ROE
      - ◆ MKT=VW market–one-month Treasury bill

	Mean	$\alpha$	$\beta_{MKT}$	$\beta_{SMB}$	$\beta_{HML}$	$\beta_{UMD}$	$R^2$		$r_{1/A}$	$r_{ROE}$	MKT	SMB	HML	UMD
$r_{ME}$	0.31 (2.12)	0.23 (1.62)	0.17 (4.33)	0.04 (0.02)	0.17 (1.59)	0.03 (0.15)	0.06	$r_{ME}$	-0.11 (0.02)	-0.31 (0.00)	0.25 (0.00)	0.95 (0.00)	-0.07 (0.13)	0.01 (0.90)
		0.09 (1.09)	0.02 (1.59)	0.99 (57.37)	0.17 (7.05)	0.03 (0.15)	0.93	$r_{1/A}$		0.06 (0.20)	-0.36 (0.00)	-0.22 (0.00)	0.69 (0.00)	0.05 (0.31)
		0.01 (0.15)	0.02 (2.40)	0.99 (61.51)	0.19 (7.34)	0.03 (2.16)	0.94	$r_{ROE}$			-0.19 (0.00)	-0.38 (0.00)	-0.09 (0.06)	0.50 (0.00)
$r_{1/A}$	0.45 (4.95)	0.52 (5.93)	-0.15 (-5.58)	-0.06 (-3.66)	0.39 (-0.81)	0.05 (1.97)	0.13	MKT				0.28 (0.00)	-0.32 (0.00)	-0.15 (0.00)
		0.33 (4.85)	-0.06 (-3.66)	-0.02 (-0.81)	0.39 (11.98)	0.05 (1.97)	0.50	SMB					-0.23 (0.00)	-0.01 (0.79)
		0.28 (3.85)	-0.05 (-3.24)	-0.02 (-0.87)	0.41 (11.94)	0.05 (1.97)	0.52	HML						-0.15 (0.00)
$r_{ROE}$	0.58 (4.81)	0.63 (5.62)	-0.11 (-2.38)	-0.09 (-0.98)	-0.33 (-4.38)	-0.20 (-1.48)	0.04							
		0.77 (6.94)	-0.09 (-2.08)	-0.33 (-5.75)	-0.20 (-2.38)	-0.20 (-2.38)	0.20							
		0.50 (4.75)	-0.03 (-0.98)	-0.33 (-4.38)	-0.10 (-1.48)	0.28 (6.27)	0.40							

Table 1.  $r_{ME} \approx SMB$ ,  $r_{1/A} \approx HML$ ,  $r_{ROE} \approx UMD$ ; q model explains FF, while FF doesn't explain q model

### 3. Empirical Results

- 73 anomaly variables & Fama-French industries
  - McLean and Pontiff (2013) use 82
  - Green, Hand, and Zhang (2013) use 60 among 330 identified
  - Harvey, Liu, and Zhu (2013) identify 314, but many are macroeconomic factors
- NYSE breakpoints, VW decile portfolios
  - Microcaps: ME below the 20 NYSE percentile, 60% of #, 3% of ME in NYSE-Amex-NASDAQ
  - Annual sorting: Sort in June  $t$  using variables in  $t-1$
  - Monthly sorting with earnings: Use earnings immediately after the announcement dates
  - Monthly sorting with other accounting variables: Lag four months after the announcement dates

Panel A: Momentum				Panel D: Profitability			
SUE-1	Earnings surprise (1-month holding period), Foster, Olsen, and Shevlin (1984)	SUE-6	Earnings surprise (6-month holding period), Foster, Olsen, and Shevlin (1984)	ROE	Return on equity, Haugen and Baker (1996)	ROA	Return on assets, Balakrishnan, Bartov, and Faurel (2010)
Abr-1	Cumulative abnormal stock returns around earnings announcements (1-month holding period), Chan, Jegadeesh, and Lakonishok (1996)	Abr-6	Cumulative abnormal stock returns around earnings announcements (6-month holding period), Chan, Jegadeesh, and Lakonishok (1996)	RNA	Return on net operating assets, Soliman (2008)	PM	Profit margin, Soliman (2008)
RE-1	Revisions in analysts' earnings forecasts (1-month holding period), Chan, Jegadeesh, and Lakonishok (1996)	RE-6	Revisions in analysts' earnings forecasts (6-month holding period), Chan, Jegadeesh, and Lakonishok (1996)	ATO	Asset turnover, Soliman (2008)	CTO	Capital turnover, Haugen and Baker (1996)
R6-1	Price momentum (6-month prior returns, 1-month holding period), Jegadeesh and Titman (1993)	R6-6	Price momentum (6-month prior returns, 6-month holding period), Jegadeesh and Titman (1993)	GP/A	Gross profits-to-assets, Novy-Marx (2013)	F	F-score, Piotroski (2000)
R11-1	Price momentum (11-month prior returns, 1-month holding period), Fama and French (1996)	I-Mom	Industry momentum, Moskowitz and Grinblatt (1999)	TES	Tax expense surprise, Thomas and Zhang (2011)	TI/BI	Taxable income-to-book income, Green, Hand, and Zhang (2013)
				RS	Revenue surprise, Jegadeesh and Livnat (2006)	NEI	Number of consecutive quarters with earnings increases, Barth, Elliott, and Finn (1999)
				FP	Failure probability, Campbell, Hilscher, and Szilagyi (2008)	O	O-score, Dichev (1998)
Panel B: Value-versus-growth				Panel E: Intangibles			
B/M	Book-to-market equity, Rosenberg, Reid, and Lanstein (1985)	A/ME	Market leverage, Bhandari (1988)	OC/A	Organizational capital-to-assets, Eisfeldt and Papanikolaou (2013)	BC/A	Brand capital-to-assets, Belo, Lin, and Vitorino (2014)
Rev	Reversal, De Bondt and Thaler (1985)	E/P	Earnings-to-price, Basu (1983)	Ad/M	Advertising expense-to-market, Chan, Lakonishok, and Sougiannis (2001)	RD/S	R&D-to-sales, Chan, Lakonishok, and Sougiannis (2001)
EF/P	Analysts' earnings forecasts-to-price, Elgers, Lo, and Pfeiffer (2001)	CF/P	Cash flow-to-price, Lakonishok, Shleifer, and Vishny (1994)	RD/M	R&D-to-market, Chan, Lakonishok, and Sougiannis (2001)	RC/A	R&D capital-to-assets, Li (2011)
D/P	Dividend yield, Litzenger and Ramaswamy (1979)	O/P	Payout yield, Boudoukh et al. (2007)	H/N	Hiring rate, Belo, Lin, and Bazdresch (2014)	OL	Operating leverage, Novy-Marx (2011)
NO/P	Net payout yield, Boudoukh et al. (2007)	SG	Sales growth, Lakonishok, Shleifer, and Vishny (1994)	G	Corporate governance, Gompers, Ishii, and Metrick (2003)	AccQ	Accrual quality, Francis et al. (2005)
LTG	Long-term growth forecasts of analysts, La Porta (1996)	Dur	Equity duration, Dechow, Sloan, and Soliman (2004)	Ind	Industries, Fama and French (1997)		
Panel C: Investment				Panel F: Trading frictions			
ACI	Abnormal corporate investment, Titman, Wei, and Xie (2004)	I/A	Investment-to-assets, Cooper, Gulen, and Schill (2008)	ME	The market equity, Banz (1981)	Ivol	Idiosyncratic volatility, Ang et al. (2006)
NOA	Net operating assets, Hirshleifer et al. (2004)	$\Delta P/A$	Changes in property, plant, and equipment plus changes in inventory scaled by assets, Lyandres, Sun, and Zhang (2008)	Tvol	Total volatility, Ang et al. (2006)	Svol	Systematic volatility, Ang et al. (2006)
IG	Investment growth, Xing (2008)	NSI	Net stock issues, Pontiff and Woodgate (2008)	MDR	Maximum daily return, Bali, Cakici, and Whitelaw (2011)	$\beta$	Market beta, Frazzini and Pedersen (2014)
CEI	Composite issuance, Daniel and Titman (2006)	NXF	Net external financing, Bradshaw, Richardson, and Sloan (2006)	D- $\beta$	Dimson's beta, Dimson (1979)	S-Rev	Short-term reversal, Jegadeesh (1990)
IvG	Inventory growth, Belo and Lin (2011)	IvC	Inventory changes, Thomas and Zhang (2002)	Disp	Dispersion of analysts' earnings forecasts, Diether, Malloy, and Scherbina (2002)	Turn	Share turnover, Datar, Naik, and Radcliffe (1998)
OA	Operating accruals, Sloan (1996)	TA	Total accruals, Richardson et al. (2005)	I/P	1/share price, Miller and Scholes (1982)	Dvol	Dollar trading volume, Brennan, Chordia, and Subrahmanyam (1998)
POA	Percent operating accruals, Hafzalla, Lundholm, and Van Winkle (2011)	PTA	Percent total accruals, Hafzalla, Lundholm, and Van Winkle (2011)	Illiq	Illiquidity as absolute return-to-volume, Amihud (2002)		

Table 2. 73 anomaly variables & Fama-French industries

	R6-1	A/ME	Rev	EF/P	D/P	O/P	SG	LTG	ACI	NXF
$m$	0.48	0.43	-0.39	0.45	0.27	0.35	-0.27	0.01	-0.27	-0.30
$t_m$	1.43	1.82	-1.57	1.73	0.94	1.53	-1.34	0.02	-1.70	-1.55
	TA	RNA	PM	ATO	CTO	F	TES	TI/BI	RS	O
$m$	-0.19	0.13	0.10	0.22	0.20	0.37	0.32	0.13	0.29	-0.08
$t_m$	-1.31	0.61	0.40	1.11	1.11	1.28	1.92	0.86	1.82	-0.37
	BC/A	RD/S	RC/A	H/N	G	AccQ	ME	Ivol	Tvol	MDR
$m$	0.18	0.01	0.32	-0.25	0.03	-0.18	-0.24	-0.54	-0.37	-0.31
$t_m$	0.73	0.06	1.27	-1.47	0.09	-0.79	-0.90	-1.56	-0.95	-0.94
	$\beta$	D- $\beta$	S-Rev	Disp	Turn	I/P	Dvol	Illiq		
$m$	-0.13	0.07	-0.31	-0.33	-0.12	-0.00	-0.26	0.27		
$t_m$	-0.36	0.30	-1.39	-1.24	-0.43	-0.01	-1.30	1.14		

Table 3. Insignificant anomalies

	SUE-1	SUE-6	Abr-1	Abr-6	RE-1	RE-6	R6-6	R11-1	I-Mom	B/M	E/P	CF/P	NO/P	Dur	I/A	NOA	$\Delta$ PI/A	IG
$m$	0.45	0.24	0.73	0.30	0.89	0.60	0.85	1.18	0.51	0.70	0.59	0.52	0.66	-0.54	-0.42	-0.38	-0.51	-0.41
$\alpha$	0.50	0.27	0.76	0.31	1.02	0.71	0.92	1.29	0.58	0.75	0.69	0.63	0.84	-0.62	-0.50	-0.38	-0.57	-0.45
$\alpha_{FF}$	0.55	0.39	0.84	0.38	1.20	0.94	1.12	1.52	0.68	0.01	0.05	0.01	0.52	-0.06	-0.15	-0.52	-0.41	-0.26
$\alpha_C$	0.34	0.18	0.62	0.19	0.56	0.37	0.06	0.09	-0.18	-0.01	0.01	-0.06	0.49	-0.08	-0.09	-0.41	-0.36	-0.20
$\alpha_q$	0.16	0.02	0.64	0.26	0.12	0.03	0.24	0.24	0.00	0.21	0.17	0.22	0.36	-0.27	0.14	-0.38	-0.26	0.05
$t_m$	3.59	2.17	5.50	3.11	3.43	2.58	3.17	3.52	2.33	2.88	2.63	2.44	3.23	-2.59	-2.45	-2.55	-3.43	-2.93
$t$	4.26	2.68	5.84	3.33	4.13	3.28	3.63	4.18	2.68	3.05	3.12	3.01	4.45	-2.98	-2.94	-2.52	-3.91	-3.16
$t_{FF}$	4.50	3.62	5.93	3.89	4.81	4.52	4.47	4.99	3.25	0.04	0.34	0.08	3.51	-0.44	-1.09	-3.30	-2.93	-1.99
$t_C$	2.62	1.69	4.37	2.06	2.56	2.15	0.51	0.67	-1.11	-0.06	0.03	-0.40	3.33	-0.56	-0.61	-2.69	-2.48	-1.51
$t_q$	1.12	0.18	4.07	2.18	0.43	0.14	0.71	0.54	0.01	1.15	0.76	1.04	2.38	-1.32	1.08	-1.90	-1.85	0.39
$ \alpha $	0.16	0.11	0.13	0.08	0.19	0.14	0.17	0.21	0.16	0.22	0.23	0.20	0.23	0.24	0.17	0.15	0.15	0.13
$ \alpha_{FF} $	0.17	0.13	0.16	0.11	0.27	0.23	0.19	0.26	0.15	0.07	0.10	0.08	0.17	0.11	0.12	0.17	0.13	0.13
$ \alpha_C $	0.11	0.09	0.12	0.08	0.11	0.09	0.10	0.13	0.06	0.06	0.09	0.07	0.15	0.08	0.10	0.14	0.12	0.11
$ \alpha_q $	0.05	0.07	0.13	0.07	0.10	0.11	0.08	0.13	0.13	0.08	0.10	0.14	0.12	0.08	0.09	0.12	0.14	0.09
$p$	0.00	0.00	0.00	0.01	0.04	0.21	0.00	0.00	0.09	0.04	0.01	0.05	0.00	0.00	0.00	0.00	0.00	0.00
$p_{FF}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.19	0.18	0.43	0.00	0.15	0.01	0.00	0.00	0.00
$p_C$	0.00	0.00	0.00	0.01	0.16	0.12	0.00	0.00	0.45	0.29	0.38	0.37	0.00	0.41	0.02	0.00	0.01	0.00
$p_q$	0.42	0.04	0.00	0.02	0.46	0.08	0.00	0.01	0.03	0.35	0.13	0.02	0.00	0.72	0.01	0.00	0.00	0.01

	NSI	CEI	IvG	IvC	OA	POA	PTA	ROE	ROA	GP/A	NEI	FP	OC/A	Ad/M	RD/M	OL	Svol
$m$	-0.68	-0.57	-0.41	-0.45	-0.30	-0.46	-0.40	0.80	0.62	0.34	0.39	-0.67	0.56	0.79	0.63	0.39	-0.60
$\alpha$	-0.78	-0.79	-0.47	-0.51	-0.33	-0.53	-0.50	0.96	0.78	0.32	0.40	-1.06	0.65	0.82	0.47	0.44	-0.72
$\alpha_{FF}$	-0.64	-0.50	-0.29	-0.38	-0.37	-0.32	-0.29	1.17	1.00	0.50	0.63	-1.44	0.61	0.15	0.22	0.37	-0.66
$\alpha_C$	-0.54	-0.40	-0.19	-0.30	-0.33	-0.25	-0.27	0.85	0.67	0.45	0.43	-0.67	0.40	0.32	0.31	0.33	-0.62
$\alpha_q$	-0.26	-0.22	-0.03	-0.28	-0.56	-0.12	-0.10	0.05	0.09	0.11	0.18	-0.17	0.09	0.11	0.60	-0.05	-0.37
$t_m$	-4.13	-2.96	-2.77	-3.05	-2.32	-3.02	-2.57	3.11	2.70	2.18	3.31	-1.98	4.07	2.96	2.31	2.06	-2.57
$t$	-4.86	-4.79	-3.29	-3.35	-2.47	-3.64	-3.50	4.02	3.67	2.02	3.45	-3.80	4.69	3.08	1.81	2.22	-3.12
$t_{FF}$	-4.28	-3.72	-2.10	-2.61	-2.84	-2.42	-2.06	5.43	5.40	3.25	6.03	-6.44	4.52	0.79	0.93	1.91	-2.88
$t_C$	-3.58	-2.93	-1.34	-1.97	-2.32	-1.88	-1.82	4.03	3.59	2.85	3.73	-3.79	2.97	1.37	1.40	1.76	-2.59
$t_q$	-1.75	-1.50	-0.20	-1.84	-3.90	-0.87	-0.67	0.37	0.72	0.71	1.68	-0.57	0.66	0.39	2.40	-0.27	-1.42
$ \alpha $	0.18	0.19	0.14	0.16	0.15	0.12	0.12	0.18	0.15	0.06	0.19	0.16	0.14	0.23	0.13	0.11	0.18
$ \alpha_{FF} $	0.18	0.15	0.11	0.12	0.13	0.11	0.11	0.24	0.23	0.14	0.23	0.23	0.15	0.13	0.17	0.11	0.19
$ \alpha_C $	0.15	0.15	0.10	0.10	0.12	0.11	0.10	0.15	0.14	0.14	0.15	0.12	0.13	0.18	0.21	0.12	0.16
$ \alpha_q $	0.11	0.12	0.11	0.08	0.15	0.12	0.08	0.09	0.07	0.11	0.09	0.13	0.11	0.11	0.27	0.12	0.11
$p$	0.00	0.00	0.01	0.00	0.00	0.01	0.01	0.01	0.07	0.25	0.00	0.00	0.00	0.04	0.24	0.54	0.01
$p_{FF}$	0.00	0.00	0.03	0.01	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.18	0.02	0.07	0.01
$p_C$	0.00	0.00	0.11	0.04	0.00	0.01	0.02	0.00	0.04	0.01	0.00	0.00	0.00	0.07	0.01	0.06	0.06
$p_q$	0.02	0.01	0.08	0.56	0.00	0.00	0.11	0.05	0.75	0.38	0.05	0.00	0.02	0.07	0.00	0.09	0.20

Table 4. Significant anomalies; q model is superior

	SUE-1	SUE-6	Abr-1	Abr-6	RE-1	RE-6	R6-6	R11-1	I-Mom	B/M	E/P	CF/P	NO/P	Dur	I/A	NOA	$\Delta$ PI/A	IG
$\alpha_3^q$	0.21	0.07	0.67	0.30	0.05	−0.05	0.37	0.42	0.15	0.42	0.29	0.30	0.21	−0.38	0.09	−0.35	−0.28	0.00
$t_3^q$	1.43	0.42	3.95	2.48	0.18	−0.22	0.96	0.85	0.48	2.13	1.21	1.42	1.26	−1.72	0.68	−1.71	−2.00	0.00
$ \alpha_3^q $	0.05	0.07	0.12	0.08	0.14	0.15	0.10	0.16	0.09	0.11	0.12	0.15	0.12	0.11	0.09	0.12	0.11	0.13
$p_3^q$	0.62	0.13	0.00	0.00	0.20	0.02	0.00	0.00	0.10	0.15	0.04	0.01	0.01	0.31	0.01	0.01	0.03	0.00
	NSI	CEI	IvG	IvC	OA	POA	PTA	ROE	ROA	GP/A	NEI	FP	OC/A	Ad/M	RD/M	OL	Svol	
$\alpha_3^q$	−0.18	−0.10	0.03	−0.28	−0.44	−0.06	−0.01	−0.14	−0.09	0.12	0.14	0.03	0.21	0.37	0.89	0.07	−0.25	
$t_3^q$	−1.18	−0.65	0.23	−1.86	−2.83	−0.38	−0.05	−0.84	−0.60	0.79	1.26	0.09	1.31	1.24	2.86	0.37	−0.91	
$ \alpha_3^q $	0.10	0.12	0.15	0.11	0.13	0.09	0.11	0.10	0.09	0.12	0.10	0.17	0.13	0.12	0.30	0.12	0.11	
$p_3^q$	0.04	0.01	0.01	0.26	0.02	0.07	0.04	0.03	0.55	0.36	0.05	0.00	0.01	0.06	0.00	0.11	0.12	

Table 5. Without  $r_{ME}$ , the size factor; q model is still superior

- Q model outperforms Fama-French and Carhart models for the rest
  - Average of  $|\alpha|$  from 10-1: q=0.20% (0.23% without  $r_{ME}$ ), Carhart=0.33%, FF=0.55%
  - Average of MAPE: q=0.11% (0.12% without  $r_{ME}$ ), Carhart=0.12%, FF=0.16%
  - # of insignificant  $\alpha$ : q=5/35, Carhart=19/35, FF=27/35
  - # of null-rejecting GRS: q=20/35, Carhart=24/35, FF=28/35



- In most cases except the value-versus-growth category, the operating accrual anomaly, and the R&D-to-market anomaly, q model shows the minimum MAPE and the maximum GRS P-value; both are computed using decile portfolios

	SUE-1	SUE-6	Abr-1	Abr-6	RE-1	RE-6	R6-6	R11-1	I-Mom	B/M	E/P	CF/P	NO/P	Dur	I/A	NOA	$\Delta$ PI/A	IG
$\beta_{MKT}$	-0.08	-0.06	-0.06	-0.03	-0.05	-0.07	-0.09	-0.14	-0.11	-0.03	-0.12	-0.15	-0.18	0.11	0.02	-0.02	0.05	-0.02
$\beta_{ME}$	0.10	0.09	0.07	0.09	-0.15	-0.19	0.27	0.40	0.31	0.46	0.25	0.19	-0.32	-0.23	-0.11	0.06	-0.05	-0.11
$\beta_{I/A}$	0.02	-0.11	-0.13	-0.16	0.04	-0.12	-0.07	0.04	-0.03	1.45	0.99	1.01	1.03	-0.85	-1.37	-0.01	-0.77	-0.82
$\beta_{ROE}$	0.48	0.45	0.28	0.18	1.33	1.12	1.02	1.48	0.82	-0.51	-0.09	-0.24	0.02	0.24	0.15	-0.01	0.16	-0.07
$t_{\beta_{MKT}}$	-1.82	-1.53	-1.31	-1.20	-0.76	-1.24	-1.17	-1.43	-1.72	-0.59	-2.02	-2.41	-3.86	1.67	0.62	-0.55	1.33	-0.71
$t_{\beta_{ME}}$	1.94	1.27	0.67	1.82	-1.42	-1.98	1.43	1.74	1.86	5.37	1.90	1.66	-4.40	-1.61	-1.81	0.54	-0.94	-1.95
$t_{\beta_{I/A}}$	0.18	-0.97	-1.25	-2.24	0.25	-0.82	-0.27	0.12	-0.13	12.74	5.76	6.79	10.25	-5.69	-15.50	-0.04	-6.98	-10.91
$t_{\beta_{ROE}}$	5.75	5.95	3.26	2.94	10.09	9.96	5.31	5.67	4.90	-5.98	-0.66	-1.78	0.19	1.87	2.29	-0.12	1.93	-1.06
ME	0.69	0.75	-0.01	0.03	0.77	0.87	0.40	0.52	0.62	-2.46	-0.73	-0.89	1.23	0.41	0.88	0.07	0.63	0.22
I/A	-1.46	-0.96	-1.37	-1.13	-0.80	0.72	-4.07	-3.83	-1.18	-9.70	-1.11	-5.63	-14.43	3.95	83.89	55.72	61.16	34.03
ROE	5.80	3.38	1.59	1.49	6.58	6.47	4.14	5.34	1.61	-5.68	0.21	-0.72	1.18	0.49	1.63	-1.24	0.84	0.50
$t_{ME}$	4.91	5.38	-0.29	1.31	8.75	9.65	4.92	4.95	3.67	-10.31	-4.09	-4.57	7.74	5.23	7.75	1.71	7.56	6.16
$t_{I/A}$	-3.30	-2.57	-2.36	-2.58	-1.22	1.13	-5.54	-4.66	-1.79	-17.06	-1.20	-5.47	-13.81	2.73	32.74	18.28	30.77	22.38
$t_{ROE}$	16.46	19.07	13.38	15.47	29.77	27.86	16.00	17.06	10.24	-29.57	1.30	-4.83	7.27	2.22	10.00	-7.89	5.10	3.24
	NSI	CEI	IvG	IvC	OA	POA	PTA	ROE	ROA	GP/A	NEI	FP	OC/A	Ad/M	RD/M	OL	Svol	
$\beta_{MKT}$	0.04	0.24	-0.03	0.04	0.03	-0.01	0.06	-0.10	-0.14	0.05	0.02	0.44	-0.13	0.04	0.16	-0.06	0.04	
$\beta_{ME}$	0.17	0.26	0.12	0.00	0.28	0.15	0.21	-0.41	-0.38	0.03	-0.10	0.43	0.25	0.50	0.66	0.26	0.31	
$\beta_{I/A}$	-0.68	-1.06	-0.96	-0.65	-0.02	-0.90	-0.90	0.10	-0.10	-0.24	-0.30	0.17	0.35	1.42	0.21	0.16	-0.21	
$\beta_{ROE}$	-0.32	-0.12	0.05	0.18	0.29	0.05	0.04	1.50	1.31	0.52	0.63	-1.61	0.51	-0.27	-0.58	0.54	-0.43	
$t_{\beta_{MKT}}$	0.99	6.27	-0.77	1.01	0.80	-0.19	1.50	-2.57	-4.48	1.20	0.88	6.46	-3.74	0.50	2.51	-1.22	0.53	
$t_{\beta_{ME}}$	2.24	3.79	2.85	-0.07	4.41	3.20	3.28	-6.56	-6.41	0.51	-2.53	2.45	5.69	2.85	6.75	2.63	2.30	
$t_{\beta_{I/A}}$	-6.14	-13.11	-11.81	-5.49	-0.21	-9.61	-8.72	1.05	-1.23	-2.35	-3.78	0.63	3.52	6.03	1.21	1.34	-1.30	
$t_{\beta_{ROE}}$	-4.07	-1.42	0.56	1.95	4.59	1.04	0.55	20.71	16.86	7.08	10.83	-8.79	7.12	-1.37	-4.10	4.85	-3.54	
ME	-1.37	-1.79	0.26	0.19	-0.24	-0.36	-0.36	2.81	2.66	0.39	2.34	-3.09	-1.31	-1.34	-4.39	-1.31	-0.19	
I/A	27.04	14.80	37.85	44.80	10.15	11.12	16.14	3.56	5.32	-1.29	5.35	-3.91	-13.77	-10.71	-3.22	-5.71	0.66	
ROE	-1.71	-1.41	0.42	1.07	0.88	1.02	0.36	16.95	14.71	3.94	4.32	-8.74	1.52	-3.33	-2.80	1.86	-0.64	
$t_{ME}$	-6.44	-7.77	4.27	4.86	-5.13	-9.39	-6.76	10.56	10.50	10.84	11.58	-10.76	-9.44	-9.83	-9.47	-8.43	-3.95	
$t_{I/A}$	13.85	14.37	23.42	34.89	5.19	7.90	12.93	4.16	6.88	-2.05	11.45	-4.18	-11.38	-12.17	-2.54	-4.70	1.37	
$t_{ROE}$	-11.87	-9.07	3.31	8.13	5.06	7.42	2.56	29.02	27.97	23.88	27.36	-25.56	7.97	-12.60	-9.21	11.42	-4.03	

Table 6. Loadings and characteristics across categories

- $r_{ROE}$  explains the momentum category and the profitability category
- $r_{I/A}$  explains the value-versus-growth category and the investment category (except OA)
- OA is problematic (High OA means high ROE so high  $\beta_{ROE}$ , while low m)
  - On the other hand, POA is not (Earnings in the denominator unrelate them)
- RD/M is problematic (High R&D means low ROE so low  $\beta_{ROE}$ , while high m)

	Panel A: SUE-1										Panel B: R6-6									
	Low	2	3	4	5	6	7	8	9	High	Low	2	3	4	5	6	7	8	9	High
$m$	0.36	0.34	0.35	0.28	0.44	0.43	0.64	0.64	0.64	0.80	0.02	0.28	0.45	0.52	0.46	0.47	0.50	0.55	0.66	0.87
$\alpha$	-0.13	-0.15	-0.15	-0.20	-0.01	-0.03	0.20	0.18	0.19	0.38	-0.61	-0.24	-0.02	0.07	0.03	0.04	0.07	0.11	0.18	0.31
$\alpha_{FF}$	-0.12	-0.15	-0.15	-0.18	-0.01	-0.02	0.21	0.24	0.21	0.43	-0.68	-0.29	-0.07	0.03	-0.01	0.01	0.05	0.10	0.21	0.44
$\alpha_C$	0.00	-0.06	-0.07	-0.10	0.01	0.02	0.21	0.16	0.13	0.34	-0.03	0.16	0.25	0.24	0.11	0.03	-0.02	-0.05	-0.04	0.03
$t_m$	1.47	1.41	1.36	1.17	2.00	1.86	3.01	3.06	2.96	3.89	0.06	1.02	1.89	2.38	2.24	2.37	2.49	2.58	2.85	2.83
$t$	-1.39	-1.92	-1.88	-2.38	-0.18	-0.35	2.58	2.54	2.64	5.14	-3.60	-2.07	-0.28	1.00	0.47	0.84	1.61	1.78	2.40	2.13
$t_{FF}$	-1.29	-1.73	-1.75	-2.30	-0.08	-0.28	2.81	3.43	2.87	5.86	-4.34	-2.49	-0.89	0.40	-0.23	0.18	0.92	1.64	2.76	3.38
$t_C$	0.01	-0.69	-0.80	-1.18	0.07	0.29	2.63	2.13	1.69	4.55	-0.29	2.18	4.24	4.27	1.77	0.55	-0.27	-0.88	-0.63	0.34
	The $q$ -factor model regressions										The $q$ -factor model regressions									
$\alpha_q$	0.05	0.00	0.04	0.05	0.00	-0.03	0.09	0.02	0.04	0.21	0.00	0.04	0.11	0.11	-0.01	-0.07	-0.10	-0.11	-0.04	0.24
$\beta_{MKT}$	1.03	1.00	1.02	0.94	0.96	0.98	0.98	1.01	0.97	0.95	1.19	1.05	1.00	0.96	0.94	0.94	0.95	0.97	1.03	1.10
$\beta_{ME}$	-0.16	0.04	0.00	0.02	0.00	-0.04	-0.05	-0.03	0.01	-0.05	0.16	-0.04	-0.08	-0.08	-0.06	-0.05	-0.02	0.04	0.13	0.43
$\beta_{I/A}$	0.00	-0.16	-0.12	-0.25	0.06	0.03	0.06	0.07	0.06	0.02	-0.34	-0.08	0.02	0.07	0.09	0.14	0.16	0.12	0.02	-0.40
$\beta_{ROE}$	-0.22	-0.12	-0.20	-0.19	-0.06	-0.01	0.13	0.21	0.20	0.26	-0.74	-0.36	-0.20	-0.09	0.00	0.09	0.15	0.23	0.28	0.28
$t_q$	0.42	-0.04	0.37	0.53	-0.04	-0.32	1.21	0.31	0.44	2.63	-0.02	0.24	0.92	1.21	-0.09	-1.32	-1.73	-1.71	-0.45	1.34
$t_{\beta_{MKT}}$	32.75	39.64	39.24	34.70	43.64	41.42	51.90	55.30	42.68	37.45	24.15	25.44	35.35	40.62	42.64	49.23	55.21	55.07	38.75	27.49
$t_{\beta_{ME}}$	-3.61	0.97	0.09	0.55	0.09	-0.85	-1.37	-1.22	0.18	-1.50	1.42	-0.48	-1.23	-1.76	-1.22	-1.25	-0.53	1.50	3.25	4.71
$t_{\beta_{I/A}}$	-0.01	-2.39	-1.99	-3.30	1.04	0.48	1.12	1.23	0.99	0.36	-2.39	-0.69	0.21	0.95	1.65	3.22	3.93	2.80	0.32	-3.27
$t_{\beta_{ROE}}$	-3.19	-2.56	-3.62	-3.48	-1.53	-0.20	2.81	5.77	4.15	7.30	-5.87	-3.93	-2.86	-1.53	0.05	2.50	4.42	6.62	5.26	3.26
	Characteristics in the $q$ -factor model										Characteristics in the $q$ -factor model									
ME	1.51	1.53	1.38	1.41	1.39	1.64	1.93	1.85	1.77	2.20	0.46	1.15	1.61	1.90	2.07	2.18	2.16	2.07	1.73	0.86
I/A	11.60	11.42	10.26	8.54	6.59	7.33	7.96	8.26	9.00	10.14	12.62	10.46	9.48	8.83	8.79	8.62	8.71	8.63	8.44	8.56
ROE	-0.80	1.94	2.05	2.34	2.72	3.24	3.59	3.75	3.95	5.00	-0.71	1.73	2.45	2.74	2.94	3.11	3.24	3.34	3.43	3.43

Table 7.  $r_{ROE}$  explains both the earnings momentum effect and the price momentum effect

	Panel A: Earnings momentum			Panel B: Price momentum		
	SUE-12	SUE-13-36	SUE-37-60	R6-12	R6-13-36	R6-37-60
$m$	0.16	-0.11	0.13	0.57	-0.24	-0.03
$\alpha$	0.19	-0.08	0.16	0.60	-0.25	-0.05
$\alpha_{FF}$	0.31	-0.04	0.14	0.86	-0.04	0.07
$\alpha_C$	0.10	0.01	0.14	0.09	-0.02	0.06
$\alpha_q$	-0.01	0.04	0.07	0.17	-0.09	0.08
$t_m$	1.57	-1.58	1.69	2.67	-1.84	-0.29
$t$	2.09	-1.09	2.09	2.86	-1.91	-0.56
$t_{FF}$	3.31	-0.56	1.90	4.41	-0.37	0.85
$t_C$	1.17	0.11	1.83	0.83	-0.18	0.76
$t_q$	-0.06	0.51	0.80	0.71	-0.70	0.86
$ \alpha $	0.09	0.07	0.07	0.13	0.11	0.08
$ \alpha_{FF} $	0.11	0.09	0.09	0.17	0.09	0.08
$ \alpha_C $	0.08	0.09	0.09	0.07	0.09	0.09
$ \alpha_q $	0.07	0.07	0.07	0.06	0.08	0.08
$p$	0.01	0.10	0.10	0.01	0.00	0.08
$p_{FF}$	0.00	0.01	0.01	0.00	0.00	0.05
$p_C$	0.01	0.02	0.05	0.04	0.04	0.02
$p_q$	0.03	0.00	0.03	0.08	0.00	0.00

Table 8. Q model explains the momentum effect, while the reversal effect is insignificant

	Panel A: OA										Panel B: POA									
	Low	2	3	4	5	6	7	8	9	High	Low	2	3	4	5	6	7	8	9	High
$m$	0.50	0.56	0.66	0.60	0.57	0.60	0.52	0.40	0.40	0.20	0.64	0.56	0.60	0.61	0.46	0.52	0.55	0.43	0.39	0.18
$\alpha$	-0.06	0.08	0.22	0.18	0.16	0.20	0.07	-0.03	-0.08	-0.39	0.12	0.07	0.16	0.14	0.00	0.09	0.09	-0.03	-0.10	-0.41
$\alpha_{FF}$	0.10	0.14	0.22	0.13	0.11	0.16	0.10	-0.04	-0.05	-0.27	-0.01	0.03	0.13	0.12	0.01	0.11	0.22	0.08	-0.06	-0.32
$\alpha_C$	0.11	0.19	0.20	0.08	0.13	0.13	0.10	0.00	-0.02	-0.22	-0.03	0.05	0.07	0.15	0.00	0.12	0.22	0.12	-0.02	-0.28
$t_m$	1.71	2.38	3.15	2.97	2.77	3.07	2.40	1.76	1.67	0.65	2.39	2.35	2.82	2.60	2.04	2.49	2.53	1.87	1.62	0.58
$t$	-0.50	0.81	2.96	2.38	2.01	2.51	1.10	-0.39	-0.96	-3.65	1.04	0.77	1.92	1.75	0.05	1.32	1.27	-0.28	-1.09	-3.26
$t_{FF}$	0.86	1.47	3.11	1.77	1.40	2.05	1.53	-0.47	-0.65	-3.22	-0.05	0.33	1.48	1.40	0.19	1.53	3.13	0.86	-0.73	-3.32
$t_C$	0.98	1.74	2.58	1.08	1.54	1.56	1.46	0.02	-0.21	-2.49	-0.30	0.50	0.75	1.68	0.06	1.85	3.20	1.15	-0.22	-2.95
The $q$ -factor model regressions											The $q$ -factor model regressions									
$\alpha_q$	0.39	0.22	0.24	-0.06	0.06	-0.02	0.00	-0.12	-0.21	-0.17	-0.04	0.07	0.12	0.26	0.06	-0.04	0.19	0.10	-0.20	-0.16
$\beta_{MKT}$	1.08	1.02	0.96	0.95	0.93	0.91	0.98	0.93	1.04	1.11	1.11	1.05	0.96	1.00	0.98	0.95	0.97	0.93	1.03	1.10
$\beta_{ME}$	0.02	-0.09	-0.12	-0.03	-0.09	-0.04	-0.09	0.05	0.03	0.30	0.23	-0.04	-0.06	-0.10	-0.07	-0.04	-0.12	-0.03	0.09	0.37
$\beta_{I/A}$	-0.56	-0.09	0.06	0.27	0.22	0.22	0.01	0.01	-0.03	-0.58	0.35	0.26	0.14	0.05	-0.05	0.06	-0.25	-0.30	-0.11	-0.55
$\beta_{ROE}$	-0.26	-0.12	-0.05	0.16	0.00	0.17	0.14	0.11	0.22	0.03	-0.12	-0.20	-0.04	-0.19	-0.02	0.17	0.09	0.07	0.22	-0.06
$t_q$	3.13	1.62	3.02	-0.78	0.66	-0.19	0.00	-1.23	-2.23	-1.86	-0.40	0.75	1.41	2.89	0.71	-0.55	2.54	0.95	-1.91	-1.34
$t_{\beta_{MKT}}$	32.16	35.50	56.68	48.78	38.87	42.85	44.43	41.06	35.50	39.68	43.24	37.37	30.61	43.73	47.87	40.75	41.32	35.95	50.05	34.83
$t_{\beta_{ME}}$	0.46	-2.09	-4.28	-1.17	-2.16	-1.19	-2.55	1.19	0.82	6.79	6.54	-0.91	-1.34	-3.12	-2.22	-1.54	-3.55	-0.85	1.94	7.96
$t_{\beta_{I/A}}$	-6.04	-0.80	1.05	5.83	2.75	3.14	0.12	0.08	-0.42	-9.86	6.29	3.76	2.36	0.95	-0.67	1.13	-4.84	-5.17	-1.25	-6.41
$t_{\beta_{ROE}}$	-3.55	-1.79	-1.08	4.17	0.05	4.06	3.21	2.13	3.83	0.69	-2.42	-2.70	-0.70	-4.44	-0.46	4.34	2.02	1.23	3.57	-1.24
Characteristics in the $q$ -factor model											Characteristics in the $q$ -factor model									
ME	0.81	1.48	1.93	2.21	2.37	2.14	2.37	1.78	1.46	0.58	0.90	1.10	1.41	1.69	1.73	1.97	2.47	2.35	1.49	0.54
1/A	13.87	9.06	7.72	7.37	7.45	7.78	8.63	7.13	11.49	24.02	5.16	6.97	6.48	8.85	8.52	8.38	10.81	11.41	14.22	16.28
ROE	1.88	2.61	2.90	2.91	2.86	2.90	3.09	2.94	2.89	2.75	1.22	1.82	2.24	2.47	2.69	3.21	3.75	3.96	3.51	2.24

Table 9. OA is problematic (high profitability so high  $\beta_{ROE}$ , while low  $m$ ), while POA is not

	Low	2	3	4	High	H-L	Low	2	3	4	High	H-L	Low	2	3	4	High	H-L
	$m$						$\alpha \ (\bar{\alpha}=0.29)$						$\alpha_{FF} \ (\bar{\alpha}_{FF}=0.10)$					
Small	0.08	0.72	0.84	0.95	1.11	1.02	-0.60	0.13	0.30	0.45	0.59	1.19	-0.54	0.02	0.13	0.18	0.16	0.70
2	0.32	0.69	0.86	0.87	0.99	0.67	-0.34	0.12	0.33	0.39	0.48	0.82	-0.21	0.00	0.09	0.07	0.04	0.25
3	0.38	0.71	0.77	0.77	1.02	0.65	-0.25	0.16	0.28	0.32	0.56	0.81	-0.09	0.04	0.03	0.00	0.13	0.22
4	0.52	0.59	0.73	0.74	0.84	0.32	-0.07	0.08	0.24	0.29	0.40	0.47	0.14	-0.02	0.03	0.00	0.02	-0.12
Big	0.40	0.54	0.54	0.61	0.56	0.16	-0.07	0.10	0.14	0.25	0.19	0.26	0.16	0.11	0.09	0.00	-0.16	-0.32
	$t_m$						$t \ (p=0.00)$						$t_{FF} \ (p_{FF}=0.00)$					
Small	0.20	2.02	2.57	3.07	3.30	4.59	-2.55	0.64	1.66	2.54	2.98	5.62	-4.84	0.23	1.58	2.53	1.97	5.66
2	0.90	2.20	3.03	3.29	3.31	2.93	-1.94	0.77	2.34	2.77	2.60	3.70	-2.59	0.01	1.36	0.92	0.45	2.19
3	1.15	2.49	3.03	3.05	3.93	2.76	-1.77	1.40	2.27	2.52	3.37	3.63	-1.21	0.52	0.35	0.02	1.28	1.70
4	1.73	2.35	2.91	3.08	3.24	1.43	-0.57	0.76	2.11	2.33	2.53	2.09	1.74	-0.23	0.27	0.03	0.17	-0.87
Big	1.70	2.53	2.71	3.04	2.43	0.79	-0.79	1.27	1.42	2.10	1.17	1.24	2.66	1.29	0.92	-0.02	-1.31	-2.32
	$\alpha_C \ (\bar{\alpha}_C=0.11)$						$\alpha_q \ (\bar{\alpha}_q=0.11)$						$\beta_{MKT}$					
Small	-0.48	0.03	0.12	0.18	0.22	0.70	-0.25	0.27	0.31	0.30	0.32	0.57	1.11	0.96	0.92	0.88	0.96	-0.15
2	-0.18	0.03	0.09	0.10	0.04	0.22	-0.14	0.02	0.03	0.07	0.10	0.24	1.14	1.02	1.01	0.94	1.01	-0.13
3	-0.04	0.04	0.09	0.03	0.16	0.20	-0.01	-0.03	-0.04	-0.01	0.14	0.15	1.10	1.05	1.01	0.95	1.01	-0.09
4	0.15	-0.01	0.07	0.03	0.09	-0.06	0.18	-0.14	-0.01	0.02	0.06	-0.12	1.09	1.07	1.06	0.98	1.02	-0.08
Big	0.17	0.07	0.07	-0.03	-0.13	-0.31	0.10	-0.04	0.06	-0.01	-0.04	-0.13	0.98	0.98	0.93	0.86	0.90	-0.09
	$t_C \ (p_C=0.00)$						$t_q \ (p_q=0.00)$						$t_{\beta_{MKT}}$					
Small	-4.00	0.36	1.58	2.59	2.53	5.72	-1.48	2.24	3.09	3.68	2.72	2.91	25.50	29.26	36.30	39.11	27.04	-2.63
2	-2.28	0.37	1.34	1.40	0.44	1.88	-1.21	0.29	0.37	0.67	0.89	1.25	33.73	51.06	53.11	42.18	34.08	-2.43
3	-0.50	0.53	0.93	0.28	1.40	1.43	-0.09	-0.30	-0.37	-0.05	1.16	0.92	39.22	53.02	28.69	37.92	26.87	-1.60
4	1.87	-0.16	0.74	0.24	0.75	-0.42	1.50	-1.58	-0.06	0.21	0.44	-0.61	34.34	42.24	31.78	31.00	25.98	-1.30
Big	2.89	0.91	0.70	-0.36	-1.02	-2.12	1.32	-0.49	0.65	-0.06	-0.23	-0.70	52.33	40.01	30.63	30.86	25.72	-2.09
	Low	2	3	4	High	H-L	Low	2	3	4	High	H-L	Low	2	3	4	High	H-L
	$\beta_{ME}$						$\beta_{I/A}$						$\beta_{ROE}$					
Small	1.14	1.18	1.08	1.01	0.99	-0.15	-0.66	-0.32	-0.11	0.17	0.53	1.19	-0.40	-0.39	-0.31	-0.26	-0.35	0.05
2	0.93	0.94	0.83	0.72	0.85	-0.07	-0.72	-0.13	0.25	0.42	0.59	1.31	-0.05	-0.08	-0.02	-0.09	-0.20	-0.14
3	0.72	0.63	0.49	0.43	0.47	-0.24	-0.77	0.03	0.37	0.54	0.78	1.56	0.00	0.05	0.04	-0.09	-0.16	-0.16
4	0.40	0.33	0.24	0.20	0.18	-0.21	-0.70	0.18	0.39	0.60	0.77	1.47	0.04	0.09	-0.01	-0.14	-0.16	-0.21
Big	-0.22	-0.09	-0.21	-0.10	-0.14	0.08	-0.39	0.12	0.30	0.61	0.80	1.20	0.14	0.16	-0.05	-0.06	-0.25	-0.39
	$t_{\beta_{ME}}$						$t_{\beta_{I/A}}$						$t_{\beta_{ROE}}$					
Small	16.49	18.22	23.29	33.68	14.79	-1.31	-5.09	-3.64	-1.46	3.06	5.90	8.59	-3.34	-4.73	-5.43	-7.53	-6.82	0.40
2	16.68	32.58	19.26	12.82	13.78	-0.70	-9.48	-2.32	3.93	6.26	8.97	10.46	-0.68	-1.84	-0.56	-1.76	-3.24	-1.25
3	14.60	16.16	5.21	7.05	4.85	-1.80	-11.40	0.43	3.35	6.24	9.13	12.41	0.02	0.93	0.54	-1.23	-2.08	-1.50
4	6.26	6.97	3.22	3.63	2.14	-1.54	-8.47	2.51	4.05	5.50	7.28	9.57	0.68	1.62	-0.19	-1.82	-1.98	-1.65
Big	-7.59	-2.67	-5.22	-1.63	-2.15	1.06	-9.47	2.19	4.98	4.87	6.25	8.00	4.37	3.89	-0.89	-0.68	-2.93	-3.99
	$ME$						$I/A$						$ROE$					
Small	0.07	0.07	0.07	0.06	0.05	-0.03	10.90	12.67	9.00	7.02	1.77	-9.13	-1.38	0.65	0.79	0.56	-0.91	0.46
2	0.33	0.33	0.33	0.33	0.32	-0.01	17.87	14.63	11.22	7.57	3.92	-13.94	2.10	2.34	2.01	1.47	0.05	-2.05
3	0.76	0.76	0.76	0.76	0.76	0.00	18.30	13.64	9.95	8.14	4.47	-13.82	3.35	2.86	2.20	1.63	0.65	-2.70
4	1.86	1.83	1.78	1.77	1.82	-0.04	15.28	11.31	8.86	7.06	5.18	-10.09	4.24	3.14	2.28	1.58	0.74	-3.50
Big	15.98	13.55	11.19	9.94	8.45	-7.53	12.59	11.36	7.49	8.22	6.44	-6.14	5.44	3.71	2.64	2.24	1.05	-4.38

Table 10. Q model versus Fama–French and Carhart models in explaining Fama–French 25 size-B/M portfolios

Panel A: Sharpe ratios								Panel B: Maximum Sharpe ratios										
MKT	SMB	HML	UMD	$r_{ME}$	$r_{I/A}$	$r_{ROE}$		CAPM	FF	Carhart	$q$							
0.10	0.06	0.13	0.16	0.10	0.24	0.22		0.10	0.21	0.30	0.43							
Panel C: Anomaly portfolios																		
	SUE-1	SUE-6	Abr-1	Abr-6	RE-1	RE-6	R6-6	R11-1	I-Mom	B/M	E/P	CF/P	NO/P	Dur	I/A	NOA	$\Delta PI/A$	IG
$S_{H-L}$ $S_m$	0.14	0.09	0.23	0.15	0.16	0.13	0.15	0.16	0.10	0.14	0.12	0.11	0.16	0.12	0.11	0.12	0.17	0.14
	0.27	0.26	0.28	0.23	0.25	0.21	0.31	0.28	0.20	0.21	0.24	0.21	0.30	0.24	0.24	0.29	0.25	0.26
	NSI	CEI	IvG	IvC	OA	POA	PTA	ROE	ROA	GP/A	NEI	FP	OC/A	Ad/M	RD/M	OL	Svol	All
$S_{H-L}$ $S_m$	0.21	0.14	0.13	0.14	0.10	0.15	0.12	0.15	0.13	0.10	0.14	0.10	0.18	0.14	0.12	0.10	0.14	0.48
	0.30	0.32	0.23	0.25	0.25	0.23	0.23	0.24	0.21	0.18	0.26	0.28	0.26	0.23	0.21	0.17	0.29	1.60

Table 11.  $r_{I/A}$  and  $r_{ROE}$  show highest Sharpe ratios so are most efficient

#### 4. Conclusion

- Empirical q-factor model outperforms FF model
  - Especially superior in explaining the earnings momentum effect and the price momentum effect
- Introduce a rational investment-based asset pricing model behind
- Quiet about the rational versus irrational debate