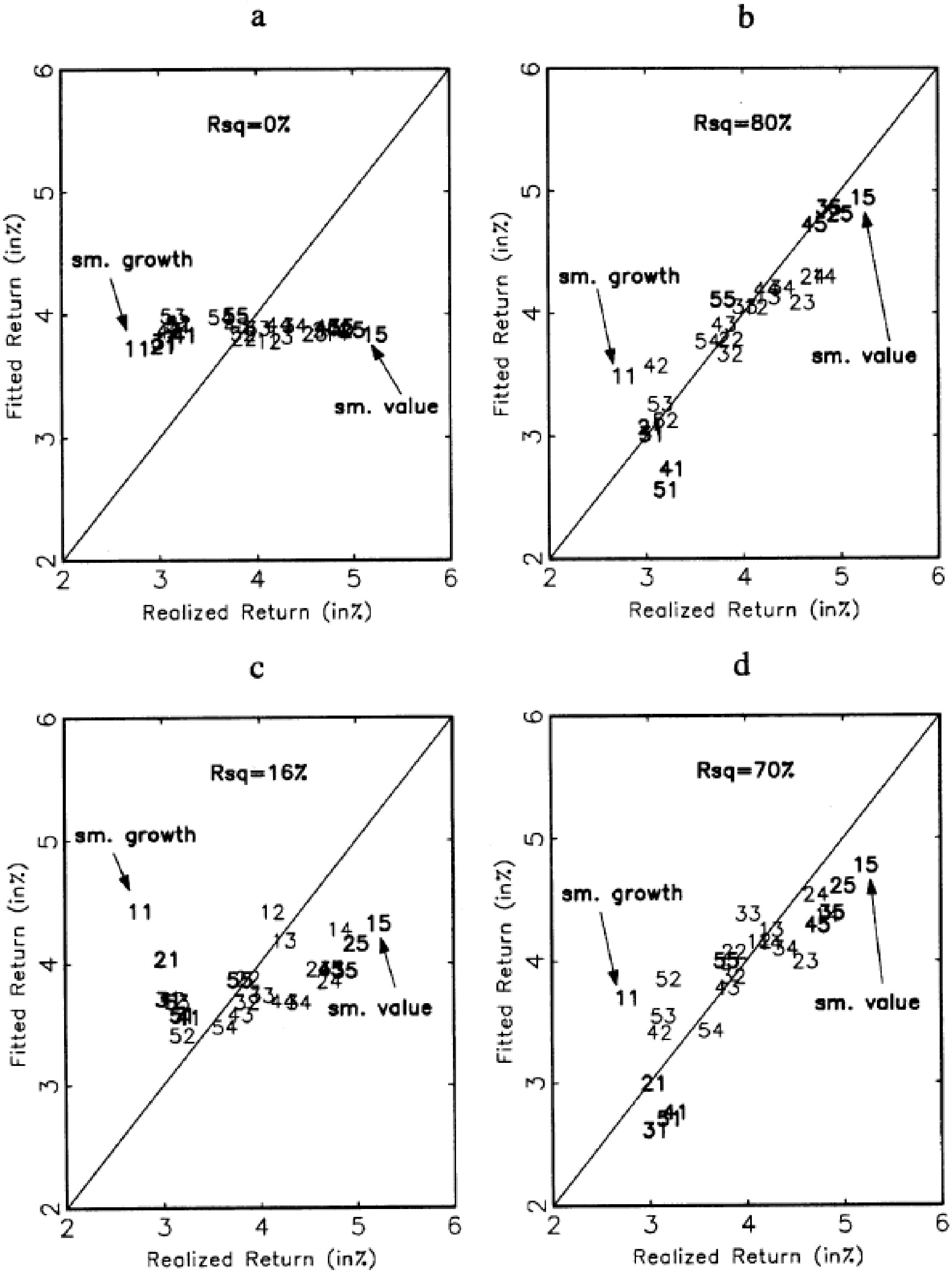
- Lettau and Ludvigson (2001, JPE), "이론적 모형 vs. 실증적 모형"
  - Lettau, Martin, and Sydney Ludvigson, 2001, "Resurrecting the (C)CAPM: A Cross-Sectional Test When Risk Premia Are Time-Varying," *Journal of Political Economy*, vol. 109, no. 6, pp. 1238-1287

- **Abstract**
  - about CAPM+CCAPM=(C)CAPM
  - cross-section of average stock returns
  - conditioning variable: log consumption-wealth ratio
  - a. conditional specification$\gtrsim$unconditional specification (in explanation, existing)
  - b. Fama-French model$\approx$conditional specification (in explanation)
  - possible to explain "value premium"

1 Introduction

- CAPM$\Rightarrow$failed in empirical
  - $^{\exists}$cross-sectional explanation?
    - ✓ Fama and French (1992, JF), Fama and French (1993, JFE): $^{\nexists}$explanation
    - ✓ failure of CAPM: figure 1a (CAPM) $\Leftrightarrow$ figure 1b (Fama-French)
  - FF (1993), Fama and French (1995, JF)
    - ✓ Fama-French model: SMB, HML $\rightarrow$ proxies for unobeserved common risk
    - ✓ interpretation $\rightarrow$ controversial
- Why did CAPM fail?
  - $^{\nexists}$effects of time-varying investment opportunities in the model?
    - ✓ intertemporal CAPM $\rightarrow$ Breeden (1979, JFE)
    - ✓ unfortunately, dissapointing empirically
  - research $\rightarrow$ portfolio-based models substituted CAPM
    - ✓ $^{\nexists}$additional performance than CAPM
- strong points of consumption-based framework?
  - ✓ Merton (1973, Econometrica): $^{\nexists}$hedging component of asset demand
  - ✓ Roll (1977): $^{\nexists}$adequate proxy for market returns?
- paper's main point
  - conditional CCAPM
    - ✓ stochastic discount factor=conditional (scaled) factor model



- Figure 1a: the relationship measured by CAPM
- Figure 1b: measured by Fama-French
- Figure 1c: measured by Consumption CAPM
- Figure 1d: measured by Scaled CCAPM (main topic of the paper)

2 Linear Factor Models with Time-varying Coefficients

- $\nexists$arbitrage $\Rightarrow$ $\exists$SDF M_{t+1} s.t.

$$\begin{aligned} 1 &= E_t[M_{t+1}(1+R_{i,t+1})] \\ M_{t+1} &= a_t+b_tR_{e,t+1} && \text{conditional linear factor model} \\ M_{t+1} &= a+bR_{e,t+1} && \text{unconditional model} \end{aligned}$$

  - conditional moment $E_t(\cdot)$→time-varying $\Rightarrow$ parameter b_t will not be constant.
  - scaling for a_t and b_t?
    - ✓ $a_t=\gamma_0+\gamma_1 z_t$
    - ✓ $b_t=\eta_0+\eta_1 z_t$
    - ✓ $\Rightarrow$ $M_{t+1}=(\gamma_0+\gamma_1 z_t)+(\eta_0+\eta_1 z_t)R_{e,t+1}=\gamma_0+\gamma_1 z_t+\eta_0 R_{e,t+1}+\eta_1(z_tR_{e,t+1})$

$$1 = E_t\{[\gamma_0+\gamma_1 z_t+\eta_0 R_{e,t+1}+\eta_1(z_tR_{e,t+1})](1+R_{i,t+1})\}$$

A. Application of Conditional Factor Pricing to the (C)CAPM

1. Consumption CAPM: $M_{t+1}\approx a_t+b_t\Delta c_{t+1}$
2. CAPM and Human Capital CAPM: $M_{t+1}=a_t+b_{vwt}R_{vw,t+1}+b_{\Delta yt}\Delta y_{t+1}$

B. The Conditioning Variable

$$\begin{aligned} c_t-w_t &\approx E_t\sum_{i=1}^{\infty}\rho_w^i(r_{m,t+i}-\Delta c_{t+i}) && \text{log consumption-wealth ratio} \\ \Rightarrow cay_t &:= c_t-\omega a_t-(1-\omega)y_t && \text{observable proxy} \\ &\approx E_t\sum_{i=1}^{\infty}\rho_w^i(r_{m,t+i}-\Delta c_{t+i})-(1-\omega)\nu_t \end{aligned}$$

- ✓ justification of *cay*: Lettau and Ludvigson (2001, JF)

3 Econometric Specification and Tests

- from 1963:3 to 1998:3 → 141 observations & 25 portfolios

4 Empirical Results

TABLE 1
FAMA-MACBETH REGRESSIONS USING 25 FAMA-FRENCH PORTFOLIOS: λ_j COEFFICIENT ESTIMATES ON BETAS IN CROSS-SECTIONAL REGRESSION

			Factors$_{t+1}$				$\widehat{cay}_t \cdot$ Factors$_{t+1}$		R^2
ROW	CONSTANT	$\widehat{cay}_t$	R_{vw}	Δy	SMB	HML	R_{vw}	Δy	$(\bar{R}^2)$
1	4.18		−.32						.01
	(4.47)		(−.27)						−.03
	(4.45)		(−.27)						
2	3.21		−1.41	1.26					.58
	(3.37)		(−1.20)	(3.42)					.54
	(1.87)		(−.67)	(1.90)					
3	1.87		1.33		.47	1.46			.80
	(1.31)		(.83)		(.94)	(3.24)			.77
	(1.21)		(.76)		(.86)	(2.98)			
4	3.70	−.52	−.06				1.14		.31
	(3.88)	(−.22)	(−.05)				(3.59)		.21
	(2.61)	(−.15)	(−.03)				(2.41)		
5	3.70		−.08				1.16		.31
	(3.86)		(−.07)				(3.58)		.25
	(2.60)		(−.44)				(2.41)		
6	5.18	−.44	−1.99	.56			.34	−.17	.77
	(5.59)	(−1.60)	(−1.73)	(2.12)			(1.67)	(−2.40)	.71
	(3.32)	(−.95)	(−1.02)	(1.26)			(.99)	(−1.42)	
7	3.81		−2.22	.59			.63	−.08	.75
	(4.02)		(−1.88)	(2.20)			(2.79)	(−2.52)	.70
	(2.80)		(−1.31)	(1.53)			(1.94)	(−1.75)	

- Row 1: CAPM
- Row 2: Human Capital CAPM (hereafter HC-CAPM)
- Row 3: Fama-French Model (hearafter FFM)
- Row 4, Row 5: Scaled CAPM (Hereafter S-CAPM)
- Row 6, Row 7: Scaled HC-CAPM (Hereafter S-HC-CAPM)

$$
\begin{aligned}
E(R_{i,t+1}) &= 5.18-0.44\beta_{cay,i,t}-1.99\beta_{vw,i,t}+0.34\beta_{cay,i,t}\beta_{vw,i,t}+0.56\beta_{\Delta y,i,t}-0.17\beta_{cay,i,t}\beta_{\Delta y,i,t} \\
&\approx (5.18-0.44\beta_{cay,i,t})+(-1.99+0.34\beta_{cay,i,t})\beta_{vw,i,t}+(0.56-0.17\beta_{cay,i,t})\beta_{\Delta y,i,t} \\
&\approx 3.81+(-2.22+0.63\beta_{cay,i,t})\beta_{vw,i,t}+(0.59-0.08\beta_{cay,i,t})\beta_{\Delta y,i,t}
\end{aligned}
$$

- ✓ counter-intuitive results for Row 1: i.e. high beta, low expected returns, but not significant
- ✓ Row 3: powerful FFM, intuitive and value premium capture
- ✓ Row 6, Row 7: S-HC-CAPM, *cay* factors can make additional explanation for expected stock returns, almost same as FFM's R-squared

TABLE 2

FAMA-MACBETH REGRESSIONS USING 25 FAMA-FRENCH PORTFOLIOS: TESTS FOR JOINT SIGNIFICANCE

Row	ALL	Factors_{t+1}	$\widehat{cay}_t \cdot \text{Factors}_{t+1}$	f_{t+1} AND $\widehat{cay}_t \cdot f_{t+1}$ FOR EACH FACTOR f	
				R_{vw}	Δy
1	.798				
	.798				
2	.000				
	.022				
3	.000				
	.002				
4	.000	.963	.000		
	.000	.975	.016		
5	.000	.948	.000		
	.003	.965	.016		
6	.000	.001	.000	.008	.000
	.000	.092	.021	.079	.040
7	.000	.001	.000	.001	.000
	.001	.032	.002	.012	.004

- For specification

$$
\begin{aligned}
E(R_{i,t+1}) &= E(R_{0,t})+\beta_{vwi}\lambda_{vw} && \text{static CAPM} \\
&= E(R_{0,t})+\beta_{vwi}\lambda_{vw}+\beta_{SMBi}\lambda_{SMB}+\beta_{HMLi}\lambda_{HML} && \text{FFM} \\
&= E(R_{0,t})+\beta_{zi}\lambda_{z}+\beta_{vwi}\lambda_{vw}+\beta_{vwzi}\lambda_{vwz} && \text{S-CAPM} \\
&= E(R_{0,t})+\beta_{zi}\lambda_{z}+\beta_{vwi}\lambda_{vw}+\beta_{vwzi}\lambda_{vwz}+\beta_{\Delta y}\lambda_{\Delta y}+\beta_{vyzi}\lambda_{\Delta yz} && \text{S-HC-CAPM} \\
&= E(R_{0,t})+\beta_{zi}\lambda_{z}+\beta_{\Delta ci}\lambda_{\Delta c}+\beta_{\Delta czi}\lambda_{\Delta cz} && \text{S-CAPM}
\end{aligned}
$$

TABLE 3

CONSUMPTION CAPM: FAMA-MACBETH REGRESSIONS USING 25 FAMA-FRENCH PORTFOLIOS

A. λ_j COEFFICIENT ESTIMATES OF BETAS IN CROSS-SECTIONAL REGRESSIONS

Row	Constant	$\widehat{cay}_t$	Δc_{t+1}	$\widehat{cay}_t \cdot \Delta c_{t+1}$	R^2 ($\bar{R}^2$)
1	3.24		.22		.16
	(4.93)		(1.27)		.13
	(4.46)		(1.15)		
2	4.28	−.13	.02	.06	.70
	(6.10)	(−.43)	(.20)	(3.12)	.66
	(4.24)	(−.30)	(.14)	(2.17)	
3	4.10		−.02	.07	.69
	(6.82)		(−.14)	(3.20)	.66
	(5.14)		(−.10)	(2.41)	

B. TESTS FOR JOINT SIGNIFICANCE

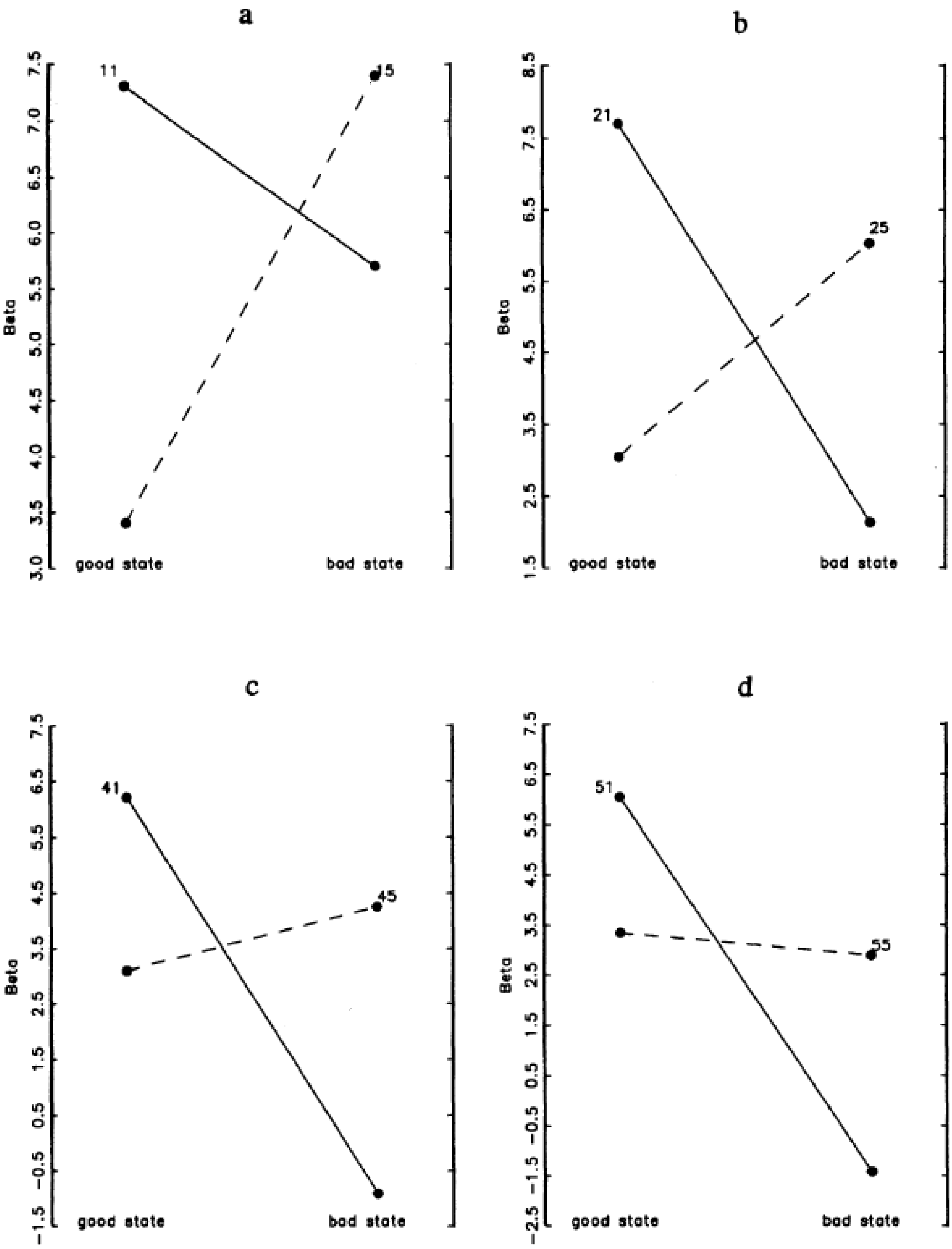
Row	All	Δc_{t+1}	$\widehat{cay}_t \cdot \Delta c_{t+1}$	Δc_{t+1} and $\widehat{cay}_t \cdot \Delta c_{t+1}$
1	.205			
	.249			
2	.000	.840	.002	.000
	.001	.888	.030	.009
3	.000	.893	.001	.000
	.001	.919	.016	.001

- Row 1, Row 2 in Panel A: CCAPM and S-CCAPM
- Row 1, Row 2 in Panel B: CCAPM and S-CCAPM

TABLE 4
PRICING ERRORS

Portfolio	CAPM	CAPM Scaled	CCAPM	CCAPM Scaled	HC-CAPM Scaled	Fama-French
				A. Individual Portfolios		
S1B1	−1.0378	−1.1857	−1.7303	−1.0083	−.6451	−.8611
S1B2	.2601	.0859	−.3686	−.1109	−.0435	.0055
S1B3	.3494	.1532	−.0105	−.0809	−.1852	−.0276
S1B4	.9048	1.0021	.4774	.3803	.5386	.3722
S1B5	1.3108	.7976	.8278	.3704	.3504	.1555
S2B1	−.7860	−.7558	−1.0749	−.0362	−.1149	−.1416
S2B2	−.0189	.3796	−.0847	−.2823	−.2661	.0478
S2B3	.6786	.8639	.5696	.5310	.7516	.5134
S2B4	.7561	.6316	.7800	.0964	.0753	.3244
S2B5	1.0436	.4944	.7576	.3070	−.0309	.0757
S3B1	−.8062	−.2985	−.7324	.3645	−.0300	−.0310
S3B2	−.0659	.1049	.1019	−.0868	.1712	.2053
S3B3	.0486	.2463	.1996	−.4457	−.0357	−.0291
S3B4	.4098	.0956	.6398	.2218	−.1837	.1066
S3B5	.9036	1.1403	.8631	.4109	.7183	.0225
S4B1	−.6513	−.1532	−.3754	.4252	.1218	.4184
S4B2	−.8487	−.2545	−.6870	−.3924	−.4210	−.5041
S4B3	−.1903	−.2714	.1383	−.0626	−.2180	−.1749
S4B4	.2339	−.3943	.4651	.0130	−.3444	−.0347
S4B5	.7553	−.2413	.6952	.3610	.0678	−.0073
S5B1	−.7959	−.9014	−.4572	.4159	.6455	.5342
S5B2	−.8186	−.7850	−.2918	−.7343	−.2759	.0035
S5B3	−.9287	−.3369	−.6240	−.4895	−.1888	−.1980
S5B4	−.4252	−.2523	.0760	.1226	−.2116	−.2715
S5B5	−.2811	−.1650	−.1547	−.2901	−.2457	−.5043
				B. Pricing Errors of Aggregated Portfolios		
S1	.872	.784	.899	.513	.416	.421
S2	.740	.649	.731	.305	.362	.257
S3	.573	.542	.590	.334	.341	.094
S4	.601	.274	.516	.306	.269	.333
S5	.697	.572	.378	.458	.356	.336
B1	.825	.762	1.004	.549	.415	.484
B2	.541	.411	.377	.398	.267	.253
B3	.545	.451	.393	.382	.370	.239
B4	.599	.571	.542	.209	.314	.276
B5	.924	.673	.709	.351	.375	.211
Average	.705	.589	.648	.393	.352	.308
χ^2	63.67*	27.24	53.03*	33.88	27.54	45.33*

- Aggregate level of pricing error is significantly eliminated in S-HC-CAPM
- compare with FFM



- direct line: growth firm's beta comparison
- dot line: value firm's beta comparison
- $^{\exists}2$ states=$\{$good, bad$\}$
  - ✓ Figure 2a. small-growth vs. small-value
  - ✓ Figure 2b. semismall-growth vs. semismall-value
  - ✓ Figure 2c. semilarge-growth vs. semilarge-value
  - ✓ Figure 2d. large-growth vs. large-value

- Meaning of figure 2 is straightforward that all stocks have their own consumption beta that frequently changes over time by time: good-state means that investors' expectation is positive so consumption is grown up and vice versa.
- Value stock can give the investors more profitability in serious condition: that is investors' overlook for the overall economy is bad.
- Such a relationship can be justified in "Conditional linear factor model" framework, because this means that the "factor loading" of the portfolio can changes over time in intertemporal manner.

TABLE 6
FAMA-MACBETH REGRESSIONS INCLUDING CHARACTERISTICS
A. λ_j ESTIMATES ON BETAS IN CROSS-SECTIONAL REGRESSIONS INCLUDING SIZE

		Factors$_{t+1}$			$\widehat{cay}_t \cdot$ Factors$_{t+1}$				R^2
ROW	CONSTANT	R_{vw}	Δy	Δc	R_{vw}	Δy	Δc	SIZE	($\bar{R}^2$)
1	14.18	−3.60						−.57	.70
	(4.77)	(−2.78)						(−3.46)	.67
	(4.35)	(−2.54)						(−3.15)	
2	13.10	−3.05			.82			−.49	.75
	(4.71)	(−2.49)			(3.14)			(−3.24)	.73
	(3.79)	(−2.01)			(2.52)			(−2.61)	
3	12.03	−3.00	.51					−.41	.74
	(4.56)	(−2.52)	(2.00)					(−2.81)	.70
	(3.73)	(−2.06)	(1.63)					(−2.30)	
4	10.33	−2.68	.33		.59	−.02		−.33	.80
	(3.78)	(−2.33)	(1.36)		(2.63)	(−.59)		(−1.93)	.76
	(2.97)	(−1.84)	(1.07)		(2.07)	(−.46)		(−1.52)	
5	5.59			.04				−.18	.22
	(2.04)			(.35)				(−1.11)	.15
	(2.03)			(.35)				(−1.10)	
6	6.09			−.16			.08	−.15	.72
	(2.21)			(−1.45)			(3.23)	(−.87)	.68
	(1.66)			(−1.09)			(2.42)	(−.65)	

- # 1: CAPM
- # 2: S-CAPM
- # 3: HC-CAPM
- # 4: S-HC-CAPM
- # 5: CCAPM
- # 6: S-CCAPM
  - ✓ Higher explanations than other models are consistently observed in both of scaled model S-HC-CAPM and S-CCAPM

5 Alternative Estimation Methodologies

6 Conclusion

- Instead of assuming that the parameters of this function are fixed over time, this paper models the parameters as time-varying by scaling them with a proxy for the log consumption-wealth ratio.
- These scaled multifactor versions of the CCAM can explain a substantial fraction of the cross-sectional variation in average returns of size & B/M sorted pfo.
- (scaled) Consumption factor is important.
- High B/M pfo. → high correlated with scaled consumptions factors
- Eliminates residual size and B/M effects that remain in the CAPM
- The data suggest that the Fama-French factors are mimicking portfolios for risk factors associated with time-variation in risk remia. Once the (C)CAPM is modified to account for such time variation, it performs about as well as the Fama-French model in explaining the cross-sectional variation in average returns.
- The success of the (C)CAM model tested here rest with its relative accuracy rather than its ability to furnish a flawless description of reality.
  - ✓ key component: conditioning information
  - ✓ The conditional linear factor models ≠ unconditional models: i.e. investors' discount factor will not merely depend unconditionally on consumption growth or the market return, but instead will be a function of these factors conditional on information about future returns.
- justification for requiring more than one factor to explain
  - ✓ we can mitigate a common criticism of multifactor models that multiple factors are chosen without regard to economic theory.