Abstract

- ▶ For CCAPM to better hold, investors must revise their consumption decisions together
 - Fourth quarters
 - Contraction periods
- ▶ Consumption betas estimated using 4Q YOY consumption growth well explain the cross-section
- According to the past literature, low consumption beta stocks are considered as insurance, so high consumption beta stocks have high expected returns
 - ▶ Weak empirical support
 - One stream of papers has developed more general consumer preferences
 - ▶ Another stream says costly consumption adjustments
 - ▶ Lettau and Ludvigson (2005): CCAPM must hold under an economy with more general consumption-based asset pricing models
- Some papers argue that CCAPM better performs as the return frequency decreases
 - ▶ Consumption decisions may be made infrequently because costly
 - Rather consumers may revise their decisions at some specific periods or during contractions
 - Uncertainty about yearend bonuses/tax consequences of capital gains (losses)

I. Other Related Literature

- ► CCAPM relates consumption growth to explain the cross-section, but is disadvantageous as macroeconomic factors are observed infrequently or with measurement errors
- ➤ CAPM adopts some portfolio returns such as the market returns as proxies, but the true market portfolio may be unobservable
- ▶ The authors use Fama-French model as a benchmark in evaluating the performance of CCAPM

II. The Model

Nonlinear CCAPM uses

$$\begin{split} m_{t,t+j} &= \delta^{j} g_{c,t+j}^{-\gamma} \\ E\big[R_{i,t+j}\big] &= \lambda_{c\gamma j} \beta_{ic\gamma,j} \\ \beta_{ic\gamma,j} &= \frac{Cov \left[R_{i,t+j}, g_{c,t+j}^{-\gamma}\right]}{Var \left[g_{c,t+j}^{-\gamma}\right]}, \end{split}$$

with both betas and the lambda to be negative. The linearized version of Breeden et al. (1989) uses

$$\begin{split} E\big[R_{i,t+j}\big] &= \lambda_{cj}\beta_{icj} \\ \beta_{icj} &= \frac{Cov\big[R_{i,t+j},g_{c,t+j}\big]}{Var\big[g_{c,t+j}\big]}. \end{split}$$

with both betas and the lambda to be positive.

III. Data and Empirical Analysis

		Panel A:	Annual Cons	umption Grov	vth (%)	
	Q1-Q1	Q2-Q2	Q3-Q3	Q4-Q4	Annual-Annual	Dec-De
Mean	2.38	2.38	2.41	2.44	2.40	2.49
SD	1.38	1.31	1.29	1.38	1.21	1.43
Min	-0.36	-0.27	-0.49	-0.78	-0.07	-0.79
Max	5.72	5.40	4.83	5.70	4.52	5.17
		Panel B: 0	Quarterly Con	sumption Gro	owth (%)	
	Q4-Q1	Q1-Q2	Q2-Q3	Q3-Q4		
Mean	3.36	3.60	3.64	3.80		
SD	1.96	1.80	1.72	2.08		
Min	-2.68	-3.52	-0.88	-1.12		
Max	7.20	7.24	6.84	10.84		
	Panel C: Qu	arterly Cons	amption as Pe	rcentage of A	nnual Consumption (%))
			Q1	Q2	Q3	Q4
Not seas	onally	Mean	23.55	24.63	25.06	26.76
adjust	ed data	SD	0.26	0.12	0.16	0.31
Seasonal	lly	Mean	24.77	24.93	25.07	25.23
adjust	ed data	SD	0.13	0.07	0.06	0.14

Table I. Q4-Q4 exhibits the least \mbox{Min} and the greatest \mbox{Max}

	Low		Book-to-market		High
	Pan	el A: Average Ann	ual Excess Return	s (%)	
Small	6.19	12.47	12.24	15.75	17.19
	5.99	9.76	12.62	13.65	15.07
Size	6.93	10.14	10.43	13.23	13.94
	7.65	7.91	11.18	12.00	12.35
Big	7.08	7.19	8.52	8.75	9.50
		Panel B: Cons	umption Betas		
Small	3.46	5.51	4.26	4.75	5.94
	2.89	3.03	4.79	4.33	5.21
Size	2.88	4.10	4.35	4.79	5.71
	2.57	3.35	3.90	4.77	5.63
Big	3.39	2.34	2.83	4.07	4.41
		Panel C	: t-values		
Small	0.93	1.71	1.59	1.83	2.08
	0.98	1.27	2.02	1.83	2.10
Size	1.15	1.93	2.17	2.07	2.39
	1.14	1.75	1.90	2.26	2.39
Big	1.71	1.32	1.67	2.15	2.00

Table II. Smaller firms and more growth firms have higher expected returns and Q4-Q4 betas

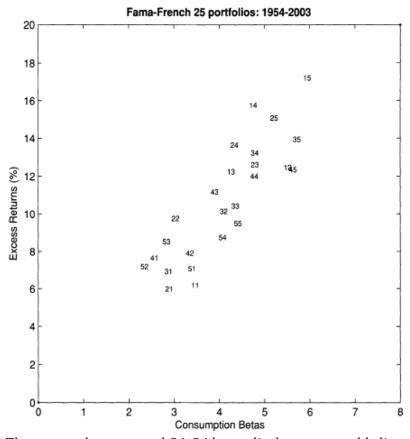


Figure 1. The expected returns and Q4-Q4 betas display a reasonably linear relation

	Constant	Δc	R_m	SMB	HML	log(ME)	log(B/M)	$R^2(\operatorname{adj} R^2)$
Estimate	0.14	2.56						0.73
t-value	0.05	3.89						0.71
Shanken-t	0.02	1.98						
Estimate	11.31		-0.56					0.00
t-value	2.05		-0.09					-0.04
Shanken-t	2.05		-0.08					
Estimate	10.43		-3.26	3.12	5.83			0.80
t-value	2.66		-0.70	1.62	3.11			0.77
Shanken-t	2.37		-0.57	1.03	2.12			
Estimate	11.75	1.58	-3.76	3.00	5.75			0.87
t-value	2.98	3.64	-0.81	1.56	3.07			0.84
Shanken-t	1.95	2.26	-0.50	0.83	1.71			
Estimate	16.20					-0.87	3.46	0.84
t-value	2.95					-1.43	3.00	0.83
Estimate	12.19	0.71				-0.71	2.66	0.86
t-value	2.41	1.62				-1.23	2.12	0.84
Estimate	22.22		-3.80	-0.67	0.96	-1.07	3.04	0.87
t-value	3.50		-0.88	-0.23	0.37	-1.51	2.87	0.84

Table III. Q4-Q4 betas survive solely and after Fama-French betas, but weaken after characteristics

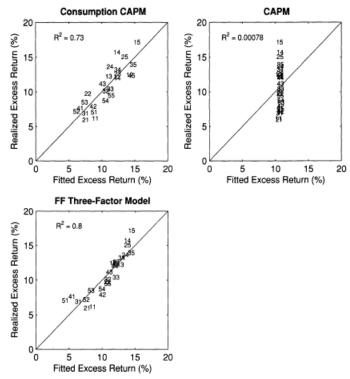


Figure 2. In terms of linearity, CCAPM and Fama-French model are matching

			P	anel A: Pr	icing Erro	rs			
		CCAPM o	·				t-value		
-1.59	1.00	2.67	5.10	5.27	-0.27	0.21	0.69	1.42	1.33
-1.21	1.46	2.12	2.71	3.30	-0.26	0.42	0.67	0.90	1.05
0.05	1.25	1.03	2.20	2.04	0.01	0.43	0.40	0.75	0.69
2.01	0.57	1.79	1.58	0.16	0.57	0.21	0.67	0.60	0.06
-0.83	0.99	3.34	-0.98	-2.37	-0.30	0.39	1.29	-0.44	-0.98
		$CAPM_{\alpha}$					t-value		
-5.14	1.84	3.84	7.65	8.08	-1.38	0.61	1.45	2.90	2.82
-3.99	1.51	4.58	5.87	6.79	-1.62	0.79	2.12	2.64	2.90
-2.23	2.45	3.36	5.33	6.31	-1.34	1.52	1.97	2.56	2.55
-0.55	0.90	3.68	4.65	3.98	-0.36	0.66	2.41	2.48	1.96
-0.46	0.40	2.22	1.85	1.88	-0.38	0.44	1.99	1.26	0.98
		FF3 α					t-value		
-3.98	-0.83	0.36	2.81	2.23	-2.18	-0.72	0.38	2.76	2.49
-2.80	-0.71	1.09	0.42	0.70	-2.41	-0.71	0.96	0.42	0.74
-0.31	-0.13	-0.79	-0.06	-0.22	-0.39	-0.13	-0.82	-0.05	-0.19
2.19	-1.54	-0.17	0.03	-1.13	2.09	-1.28	-0.16	0.02	-0.82
1.93	-0.22	1.06	-1.70	-2.97	2.01	-0.24	0.96	-1.93	-2.50
]	Panel B: G	ibbons, Ro	ss, and Sh	anken Tes	t		
			CCAPI	M.		CAPM	ſ		FF3
GRS			1.30			2.07			1.65
p-value			0.26			0.04			0.12

Table IV. In terms of absorbing alphas, CCAPM and Fama–French model are matching

A. Implied Coefficient of Relative Risk Aversion

	Δc	R_m	SMB	HML	HJ distance	<i>p</i> -value
Estimate t-value	33.01 25.45				0.29	0.69
Estimate t-value		2.10 6.44			0.74	0.08
Estimate t-value		1.90 4.12	0.56 0.85	$\frac{2.61}{5.02}$	0.63	0.10

Table V. In terms of Hansen–Jagannathan distance, CCAPM outperforms the Fama–French model

		Panel A:	Annual	Excess Re	turns and (Consumpt	ion Betas		
	Q1-Q1 H	Excess Ret	ırns (%)			Q1-Q1 (Consumption	n Betas	
3.88	9.80	10.75	13.93	14.69	5.10	6.02	4.30	4.83	5.80
4.34	8.62	11.29	12.21	13.14	2.64	3.02	3.99	3.23	4.60
5.90	9.04	9.55	11.64	12.22	2.03	2.52	3.17	3.74	4.25
7.12	6.93	10.24	10.51	10.78	2.39	1.68	2.44	3.77	5.23
6.63	6.59	7.83	8.01	8.29	3.11	1.84	2.15	3.60	4.55
	Q2-Q2 H	Excess Retu	ırns (%)			n Betas			
4.61	10.95	11.54	14.83	15.67	5.31	4.81	4.28	4.38	5.14
5.58	9.55	12.08	12.78	13.90	2.03	2.46	3.23	2.64	3.60
6.85	10.06	10.32	12.23	12.82	1.93	1.70	2.83	2.51	2.95
7.66	7.91	10.94	11.16	11.38	1.90	0.60	1.24	2.81	3.10
7.18	7.00	8.44	8.60	8.79	3.03	0.15	0.89	1.88	2.73
	Q3-Q3 Excess Returns (%) Q3-Q3 Consumption						n Betas		
5.52	11.81	12.05	15.51	16.56	3.30	2.76	2.62	2.98	3.63
6.01	9.64	12.62	13.25	14.44	-0.02	0.54	1.84	1.11	2.52
7.35	10.64	10.45	13.03	13.33	0.01	0.34	1.41	0.66	2.80
8.51	8.26	11.37	11.99	11.81	0.19	0.11	0.10	1.95	2.09
7.64	7.47	8.67	8.75	9.10	1.41	-0.13	1.04	1.34	1.55
			Panel	B: Cross-S	ectional Re	gression			
		Constant		Q1-Q1	Q2-0	Q2	Q3-Q3	R^2	$(\operatorname{adj} R^2)$
Estima	ate	4.98		1.17					0.27
t-value	,	2.03		2.38					0.24
Estima	ate	7.52			0.8	7			0.18
t-value	,	3.08			1.6	7			0.14
Estima	ate	8.61					1.36		0.30
t-value	•	3.10					2.69		0.27
			Panel C:	Bootstrap	Simulation	n p-values			
			Q1-Q1		Q2-Q2	(Q3-Q3	Q	4-Q4
CSR ir	ntercept		4.98		7.52		8.61		0.14
p-value			0.046		0.136		0.177	(0.0003
CSR sl			1.17		0.87		1.36	9	2.56
p-valu	e		0.196		0.254		0.137	(0.023
CSR a	$\operatorname{dj} R^2$		0.24		0.14		0.27	(0.71
p-value			0.279		0.394		0.209		0.010
HJ-dis	tance		0.31		0.33		0.31	(0.29
p-valu	e		0.815		0.620		0.682	(0.886
4	Sharpe ratio)	0.45		-0.44	-	-0.64		0.70
p-valu			0.135		0.977		0.998	(0.012
GRS			2.76		1.96		1.92		1.30
p-valu	e		0.892		0.859		0.966	(0.981

Table VI. Findings from the rest quarters are noisier with low R^{2} and significant $\boldsymbol{\gamma}$

B. Alternative Empirical Specifications

		Pa	nel A: Con	sumption	Growth	l.				
	Cor	Monthly Consumption Data			Quarte sumptio	-	Annual Consumption Date			
Monthly growth Quarterly growth	Dec	Month-Month Dec-Mar, Mar-Jun Jun-Sep, Sep-Dec Dec-Dec		Qu	Quarter-Quarter					
Annual growth	nnual growth Dec-Dec)ec	Q4-Q4			Annual-Annual			
	Pa	anel B: (Cross-Sect	ional Reg	ression l	Results				
	Cons	Monthl; umption			Quarterly Consumption Data			Annual Consumption Data		
	λ_0	λ_1	R^2	λ_0	λ ₁	R^2	λ_0	λ ₁	R^2	
Monthly return	7.70	0.02	0.00							
t-value	2.61	0.17	-0.04							
Quarterly return	8.34	0.03	0.00	4.52	0.33	0.22				
t-value	2.80	0.15	-0.04	1.83	1.59	0.18				
Annual return	-1.83	2.01	0.41	-1.19	2.68	0.69	10.12	1.32	0.21	
t-value	-0.51	2.33	0.38	-0.37	3.49	0.68	3.70	1.61	0.18	

Table VII. Results deteriorate as the return frequency increases

	Constant	Δc	R_m	SMB	$_{ m HML}$	$R^2(\operatorname{adj} R^2)$
Estimate	-1.10	2.81				0.89
t-value	-0.33	3.86				0.86
Shanken-t	-0.16	1.84				
Estimate	9.07		-1.46	2.64	5.76	0.87
t-value	1.94		-0.27	1.39	3.11	0.68
Shanken-t	1.75		-0.23	0.88	2.12	

Table VIII. The cross-sectional regression with 2×3 Fama–French portfolios—CCAPM outperforms

C. Other Portfolios

	C	CAPM		Far	na-French '	Three-Fact	or Model	
	Constant	Δc	R^2	Constant	R_m	SMB	HML	R^2
			18 8	Size Portfolios				
Estimate	-0.44	2.60	0.81	9.09	-1.01	3.36	-0.05	0.99
t-value	-0.09	1.68	0.80	0.78	-0.09	1.43	-0.01	0.99
Shanken-t	-0.04	0.85		0.75	-0.08	1.05	-0.01	
			18 E	3/M Portfolios				
Estimate	2.62	1.79	0.80	-0.58	8.53	0.27	4.62	0.95
t-value	0.97	2.94	0.79	-0.10	1.37	0.05	1.80	0.94
Shanken-t	0.63	1.87		-0.09	1.08	0.04	1.29	
			19 I	E/P Portfolios				
Estimate	1.94	2.09	0.53	-1.96	10.05	-0.02	6.44	0.96
t-value	0.93	3.85	0.50	-0.36	1.67	0.00	2.75	0.95
Shanken-t	0.55	2.22		-0.27	1.21	0.00	1.81	
			19 C	F/P Portfolios				
Estimate	2.81	1.72	0.59	-1.33	9.41	1.64	6.09	0.90
t-value	1.19	3.46	0.56	-0.27	1.69	0.40	2.61	0.88
Shanken-t	0.79	2.22		-0.21	1.25	0.29	1.75	

Table IX. CCAPM and Fama-French model are comparable in explaining some other portfolios

D. Contraction Beta and Expansion Beta

	Panel A: Tir	ne-Series Regression ar	nd GRS Test	
	CCAP	M (CMP)	Fama-Fr	ench Model
	α	t-value	α	t-value
1	2.45	0.99	3.26	1.54
2	3.08	0.78	-0.96	-0.30
3	5.28	1.61	1.58	0.69
4	-3.42	-0.91	-4.08	-1.70
5	1.05	0.31	0.06	0.02
6	-0.79	-0.29	-0.98	-0.59
7	4.38	1.47	6.24	2.73
8	-0.38	-0.12	-1.18	-0.91
9	-2.83	-0.74	-4.59	-1.71
10	0.39	0.14	-0.98	-0.64
11	1.40	0.37	1.68	0.96
12	-4.51	-1.39	-4.79	-2.06
13	-1.13	-0.36	-2.70	-1.51
14	0.54	0.21	-1.59	-0.91
15	-1.83	-0.61	0.59	0.25
16	0.19	0.07	-1.25	-0.74
17	0.98	0.33	1.46	1.15
GRS	1	.51	2	.93
<i>p</i> -value	C	.15	0.	.00

Panel B: Cross-Sectional Regression Constant CMPSMB HML log(ME) log(B/M) \mathbb{R}^2 R_m Estimate 6.90 3.61 0.09 t-value 2.83 0.540.06 Shanken-t 2.810.44Estimate 6.01 -0.680.122.60-1.24t-value 1.530.53-0.48-0.30-0.08Shanken-t 1.51 0.47-0.37-0.23Estimate 5.750.00 0.66 -0.33t-value 1.83 1.06 0.26 -0.52

Table X. 17 industry portfolios—the consumption mimicking portfolio as the factor

$$R_{i,t+4} = \alpha_{i,cont}I_t + \alpha_{i,exp}(1 - I_t) + \beta_{i,cont}\Delta c_{t+4}I_t + \beta_{i,exp}\Delta c_{t+4}(1 - I_t) + \epsilon_{i,t+4}.$$
 (13)

	Intercept	Contraction	Expansion	$R^2(\operatorname{adj} R^2)$
Estimate	0.86	0.98	0.23	0.65
t-value	0.50	6.11	0.67	0.62
Estimate	0.84	1.06		0.65
t-value	0.50	7.51		0.62
Estimate	6.10		1.40	0.33
t-value	4.71		4.78	0.26

Table XI. Betas estimated during contraction explain the cross-section more precisely

E. Further Comparison of CCAPM and the Fama and French Three-Factor Model

	$\Delta \mathbf{c}$	R_m	SMB	$_{ m HML}$	$\log(\mathrm{ME})$	log(B/M)	$R^2(\operatorname{adj} R^2)$
Estimate	2.59						0.73
t-value	3.72						0.73
Shanken-t	1.88						
Estimate		9.71					-0.26
t-value		3.49					-0.26
Shanken-t		2.42					
Estimate		7.09	3.03	6.24			0.73
t-value		2.79	1.58	3.31			0.71
Shanken-t		1.79	0.95	2.13			
Estimate	1.67	7.78	2.92	6.21			0.79
t-value	3.84	3.06	1.52	3.30			0.76
Shanken-t	2.39	1.70	0.81	1.84			
Estimate					1.88	3.20	0.81
t-value					9.67	2.03	0.76
Estimate	2.75				0.01	0.29	0.74
t-value	3.09				0.03	0.18	0.72
Estimate		-1.13	7.27	3.04	1.29	2.39	0.77
t-value		-0.29	3.26	1.17	3.28	2.06	0.72

Table XII. Cross-sectional regressions without $\boldsymbol{\gamma}$

CCAPM: â					CCAPM: $\tilde{\alpha}$					
-2.82	-1.77	1.20	3.45	1.85	-2.78	-1.80	1.21	3.44	1.80	
-1.55	1.87	0.23	2.41	1.59	-1.50	1.91	0.22	2.42	1.57	
-0.58	-0.48	-0.85	0.85	-0.81	-0.53	-0.47	-0.85	0.83	-0.86	
0.95	-0.79	1.07	-0.35	-2.18	1.01	-0.76	1.08	-0.37	-2.23	
-1.74	1.06	1.14	-1.81	-1.93	-1.71	1.12	1.20	-1.80	-1.93	
Three-Factor model: $\hat{\alpha}$					Three-Factor model: $\tilde{\alpha}$					
-2.36	0.87	-0.55	1.92	2.73	-3.30	-0.45	0.55	2.90	2.29	
-1.74	-1.03	0.52	0.13	1.20	-2.18	-0.42	1.27	0.46	0.72	
0.52	-0.71	-1.68	0.25	-0.49	0.33	0.11	-0.70	-0.01	-0.27	
2.23	-2.14	0.08	0.06	0.32	2.85	-1.32	-0.03	0.11	-1.03	
2.65	-0.40	0.20	-1.22	-1.37	2.54	0.13	1.34	-1.56	-2.88	
Four-Factor model: $\hat{\alpha}$					Four-Factor model: $\tilde{\alpha}$					
-1.64	-0.01	-0.54	1.73	1.94	-2.77	-1.36	0.68	2.84	1.57	
-0.82	0.48	-0.46	1.07	1.45	-1.43	0.95	0.50	1.31	0.88	
0.58	-1.20	-2.06	0.60	-1.38	0.36	-0.22	-0.92	0.26	-1.02	
1.66	-1.72	0.86	-0.37	-0.42	2.42	-0.86	0.64	-0.26	-1.82	
0.73	0.71	0.36	-1.13	-0.44	0.86	1.15	1.60	-1.52	-2.24	

Table XIII. Pricing errors from cross-sectional regressions— $\widehat{\alpha}~$ with γ and $~\widetilde{\alpha}~$ without γ

	Intercept	R_m	SMB	HML	$R^2(\operatorname{adj} R^2)$
		Fitted I	Beta		
Estimate	5.01	5.52	5.04	9.35	0.57
t-value	1.73	2.24	0.95	2.91	0.51
		Residual	Beta		
Estimate	10.71	-9.93	1.57	2.33	0.14
t-value	3.59	-1.96	0.80	1.14	0.02

Table XIV. Regress Fama-French betas on consumption betas

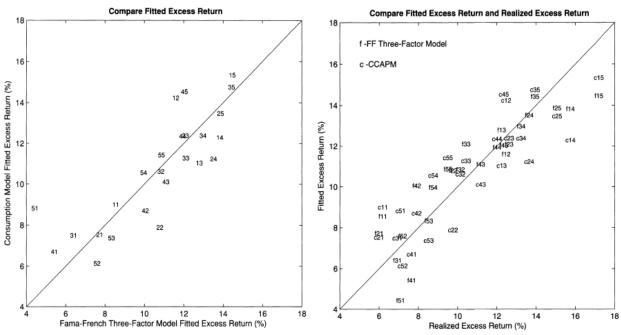


Figure 3. CCAPM E[R]×Fama-French E[R]

Figure 4. R×E[R] from both models

IV. Conclusion

- ▶ Betas estimated (i) with 4Q-4Q consumption growth and (ii) during contraction periods are better in explaining the cross-section—consumption decisions might be made mainly at that time
- There are two interesting findings—(i) the estimated consumption risk premium is high, and (ii) the significance of the fourth quarter betas disappears when the book-to-market ratio is added

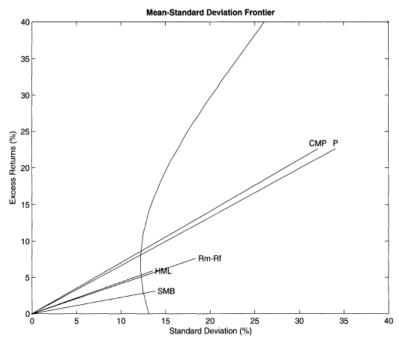


Figure 5. CMP as the most efficient portfolio versus Fama–French factors

			Panel A: T	ime-S	eries Regres	ssion R^2 's				
CCAPM					Three-Factor Model					
0.10	0.27	0.27	0.35	0.36	0.91	0.96	0.96	0.95	0.9	
0.13	0.26	0.40	0.45	0.46	0.94	0.94	0.92	0.94	0.98	
0.16	0.37	0.45	0.47	0.50	0.96	0.93	0.92	0.93	0.93	
0.14	0.31	0.43	0.49	0.53	0.92	0.86	0.91	0.87	0.89	
0.33	0.27	0.20	0.54	0.60	0.92	0.91	0.85	0.93	0.90	
		HML			CAPM					
0.08	0.00	0.00	0.02	0.03	0.56	0.63	0.58	0.56	0.58	
0.09	0.00	0.01	0.07	0.08	0.69	0.72	0.66	0.63	0.63	
0.12	0.00	0.05	0.08	0.12	0.81	0.76	0.70	0.66	0.5	
0.17	0.01	0.04	0.07	0.07	0.80	0.79	0.77	0.68	0.70	
0.11	0.00	0.00	0.07	0.11	0.84	0.88	0.81	0.75	0.68	
			Panel B:	Cross-	Sectional Re	gression				
		Intercept	CMP		R_m	SMB	HML	R^2	2 (adj R^2	
Estimate		-0.40	26.86						0.65	
t-value		-0.12	3.66						0.64	
Shanken-t		-0.09	2.53							
Estimate		10.43			-3.26	3.12	5.83		0.80	
t-value		2.66			-0.70	1.62	3.11		0.77	
Shanken-t		2.37			-0.57	1.03	2.12			
Estimate		10.24					5.23		0.53	
t-value		3.41					2.70		0.51	
Shanken-t		3.14					1.90			
Estimate		11.31			-0.56		(0.00	
t-value		2.05			-0.09				-0.04	
Shanken-t		2.05			-0.08					

Table XV. CMP moderately performs in both time-series and cross-sectional regressions

Appendix A: Linear Consumption Factor Model

The important point is

$$\frac{u'\big(c_{t+j}\big)}{u'(c_{t})} \approx 1 - \left(-\frac{u''(c_{t})}{u'(c_{t})}c_{t}\right) \frac{c_{t+j} - c_{t}}{c_{t}} = 1 - \gamma \big(g_{c,t+j} - 1\big),$$

by a Taylor approximation. Therefore,

$$E\big[R_{i,t+j}\big] = \frac{\gamma Var\big[g_{c,t+j}\big]}{1-\gamma E\big[g_{c,t+j}-1\big]} \frac{Cov\big[g_{c,t+j},R_{i,t+j}\big]}{Var\big[g_{c,t+j}\big]} = \lambda_{cj}\beta_{icj}.$$

Appendix B: A Model with Infrequent Adjustment of Consumption and Investment Plans

One group of utility maximizers will derive

$$E\left[R_{i,t+j}\left(1-\gamma(g_{c,t+j}^{1}-1)\right)\right]=0, \tag{B1}$$

while another group will not maximize, so

$$E\left[R_{i,t+j}\left(1-\gamma(g_{c,t+j}^2-1)\right)\right] = \epsilon_{it}. \tag{B2}$$

Mix them proportionally, so

$$\begin{split} g_{c,t+j}^{A} &= w_{t}g_{c,t+j}^{1} + (1-w_{t})g_{c,t+j}^{2}. \\ \Rightarrow E\left[R_{i,t+j}\left(1-\gamma(g_{c,t+j}^{A}-1)\right)\right] &= \gamma(1-w_{t})E\left[\left(g_{c,t+j}^{1}-g_{c,t+j}^{2}\right)R_{i,t+j}\right]. \\ \Rightarrow E\left[R_{i,t+j}\right] &= \frac{\gamma(1-w_{t})E\left[\left(g_{c,t+j}^{1}-g_{c,t+j}^{2}\right)R_{i,t+j}\right]}{1-\gamma E\left[g_{c,t+j}^{A}-1\right]} + \frac{\gamma Cov\left[\left(g_{c,t+j}^{A}-1\right),R_{i,t+j}\right]}{1-\gamma E\left[g_{c,t+j}^{A}-1\right]} \\ &= \frac{\gamma(1-w_{t})\varepsilon_{it}}{1-\gamma E\left[g_{c,t+j}^{A}-1\right]} + \frac{\gamma Cov\left[\left(g_{c,t+j}^{A}-1\right),R_{i,t+j}\right]}{1-\gamma E\left[g_{c,t+j}^{A}-1\right]} \\ &= \varepsilon_{it} + \lambda_{t} \frac{Cov\left[g_{c,t+j}^{A},R_{i,t+j}\right]}{Var\left[g_{c,t+j}^{A}\right]}, \end{split} \tag{B6}$$

since (B1) – (B2) = ϵ_{it} . Note that $\frac{\partial |\epsilon_{it}|}{\partial w_t} < 0$.