COMP.SGN.100 Signaalinkäsittelyn perusteet,
Harjoitus 3, 22.-23.3.2021

1. (*Kynä & paperi*) Laske funktion

$$x(t) = \begin{cases} 1, & \text{kun } -\frac{\pi}{4} \le t \le \frac{\pi}{4}, \\ 0, & \text{muissa välin } [-\pi, \pi] \text{ pisteissä,} \end{cases}$$

Fourier-sarja. Välin $[-\pi, \pi]$ ulkopuolella x on periodinen: $x(t) = x(t + 2\pi)$.

Fourier-sarja $X(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} x(t) e^{-int} dt = \frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x(t) e^{-int} dt$

$= \frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{-int} dt = \frac{1}{2\pi} \Big/_{-\frac{\pi}{4}}^{\frac{\pi}{4}} -\frac{1}{in} e^{-int} dt = \frac{1}{2\pi} \Big/_{-\frac{\pi}{4}}^{\frac{\pi}{4}} -\frac{1}{in} e^{-int}$

$= \frac{1}{2\pi} \frac{e^{in\frac{\pi}{4}} + e^{-in\frac{\pi}{4}})}{+in} = \frac{1}{2\pi} \frac{i2\sin(n\frac{\pi}{4})}{-in}$

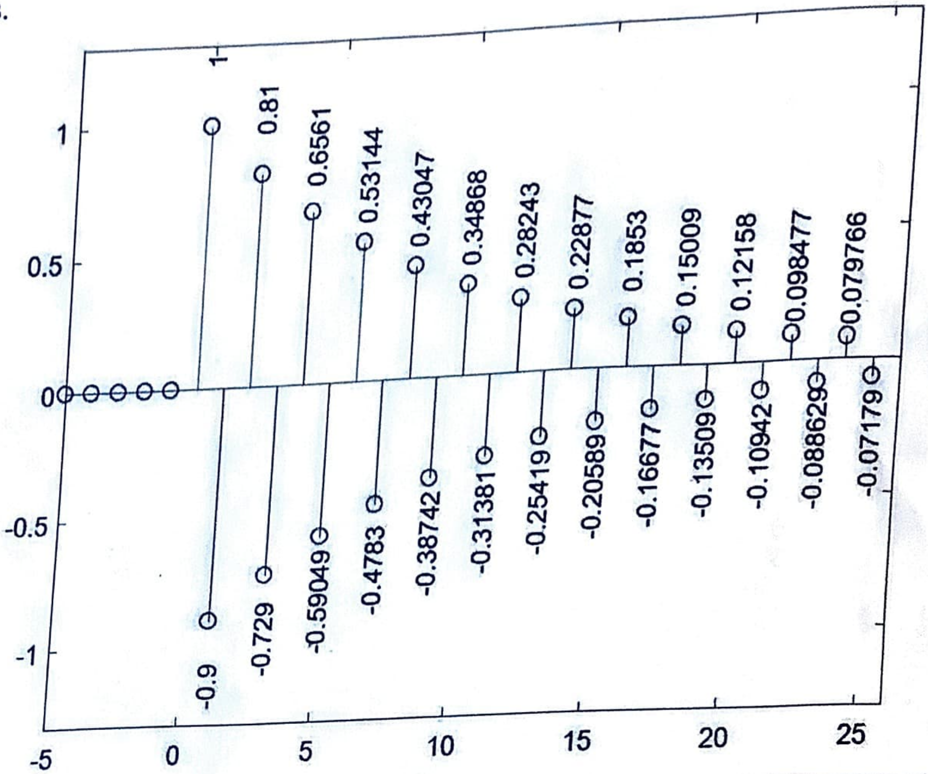
$= -\frac{\sin(n\frac{\pi}{4})}{\pi n}$

$=$

$$X(n) = \begin{cases} \frac{1}{4} \frac{\sin(\frac{n\pi}{4})}{\frac{n\pi}{4}}, & \text{kun } n \neq 0 \\ \frac{1}{4}. & \text{kun } n = 0 \end{cases}$$

$X(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} x(t)\, dt = \frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 1\, dt = \frac{1}{2\pi}\left(\frac{\pi}{4} + \frac{\pi}{4}\right) = \frac{\frac{\pi}{2}}{2\pi} = \frac{1}{4}.$

2. (*Kynä & paperi*) Laske alla olevan kuvan äärettömän pitkän signaalin diskreettiaikainen Fourier-muunnos.



$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-jwn}$$

$$= \sum_{n=0}^{\infty} x(n)\, e^{-jwn}$$

huomataan kerroin on $-0,9$

$$x(n) = 1 \cdot (-0,9^n) = (-0,9)^n$$

$$= \sum_{n=0}^{\infty} (-0,9)^n e^{-jwn}$$

$$= \sum_{n=0}^{\infty} (-0,9\, e^{-jw})^n = \frac{1}{1-(-0,9\, e^{-jw})}$$