

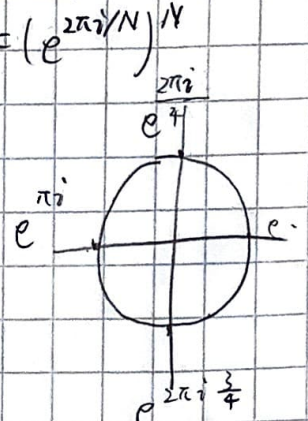
1. (Kynä & paperi) Laske käsin lukujonon $x(n) = (5, 1, -1, 0)^T$ diskreetti Fourier-muunnos

$$X(k) = \sum_{k=0}^{N-1} x(k) W_N^{-kn}$$

$$N=4$$

$$X(n) = \begin{bmatrix} 5 \\ 1 \\ -1 \\ 0 \end{bmatrix}$$

$$W_N = e^{j2\pi/N}$$



$$\begin{bmatrix} W_N^0 & W_N^{-0} & W_N^{-0} & W_N^{-0} \\ W_N^0 & W_N^{-1} & W_N^{-2} & W_N^{-3} \\ W_N^0 & W_N^{-2} & W_N^{-4} & W_N^{-6} \\ W_N^0 & W_N^{-3} & W_N^{-6} & W_N^{-9} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$X(n) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 5 \\ 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 6-j \\ 5 \\ 6+j \end{bmatrix}$$

2. (Kynä & paperi) Laske nopean Fourier-muunnoksen algoritmia jäljitellen jonon $x(n) = (-1, 3, 1, 0)$ diskreetti Fourier-muunnos. Voit käyttää hyväksi tietoa, että lukujonon $(-1, 1)$ DFT on $(0, -2)$ ja lukujonon $(3, 0)$ DFT on $(3, 3)$.

$$x(n) = (-1, 3, 1, 0)$$

$$X_0(n) = (-1, 1)^T \quad X_1(n) = (3, 0)^T$$

$$X_{00}(n) = -1 \quad X_{01}(n) = 1 \quad X_{10}(n) = 3 \quad X_{11}(n) = 0$$

$$X_{00}(0) = -1 \quad X_{01}(0) = 1 \quad X_{10}(0) = 3 \quad X_{11}(0) = 0$$

$$X_0(0) = X_{00}(0) + W_2^{-1} X_{01}(0) = -1 + 1 = 0$$

$$X_0(1) = X_{00}(1 - \frac{2}{2}) + W_2^{-1} X_{01}(1 - \frac{2}{2}) = -1 + (-1) = -2$$

$$X_1(0) = X_{10}(0) + W_2^{-1} X_{11}(0) = 3 + 0 = 3$$

$$X_1(1) = X_{10}(1 - \frac{2}{2}) + W_2^{-1} X_{11}(1 - \frac{2}{2}) = 3 - 0 = 3$$

$$X(0) = X_0(0) + W_4^{-1} X_1(0) = 0 + 3 = 3$$

$$X(1) = X_0(1) + W_4^{-1} X_1(1) = -2 + 3(-j) = -3j - 2$$

$$X(2) = X_0(2 - \frac{4}{2}) + W_4^{-2} X_1(2 - \frac{4}{2}) = 0 + (-1)3 = -3$$

$$X(3) = X_0(3 - \frac{4}{2}) + W_4^{-3} X_1(3 - \frac{4}{2}) = -2 + (j)3 = 3j - 2$$

$$X_n = [3, -3j - 2, -3, 3j - 2]$$