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Check out the source code on GitHub

Correlation Ratio for Unsupervised Learning of Multi-modal Deformable Registration

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INTRODUCTION

Background:

- Deep learning (DL) has shown promise in multi-modal image registration, where mutual information (MI) or modality-independent descriptors (MIND) are often used as the similarity measure.
- **Correlation ratio** (CR) [1], though historically effective, is underexplored in DL due to non-differentiability.

Objective:

- Investigating the CR as an alternative to MI for multi-modal deformable registration.

Key Challenge:

- The original implementation of CR involves discretely iterating through intensity ranges, making it non-differentiable for DL training.

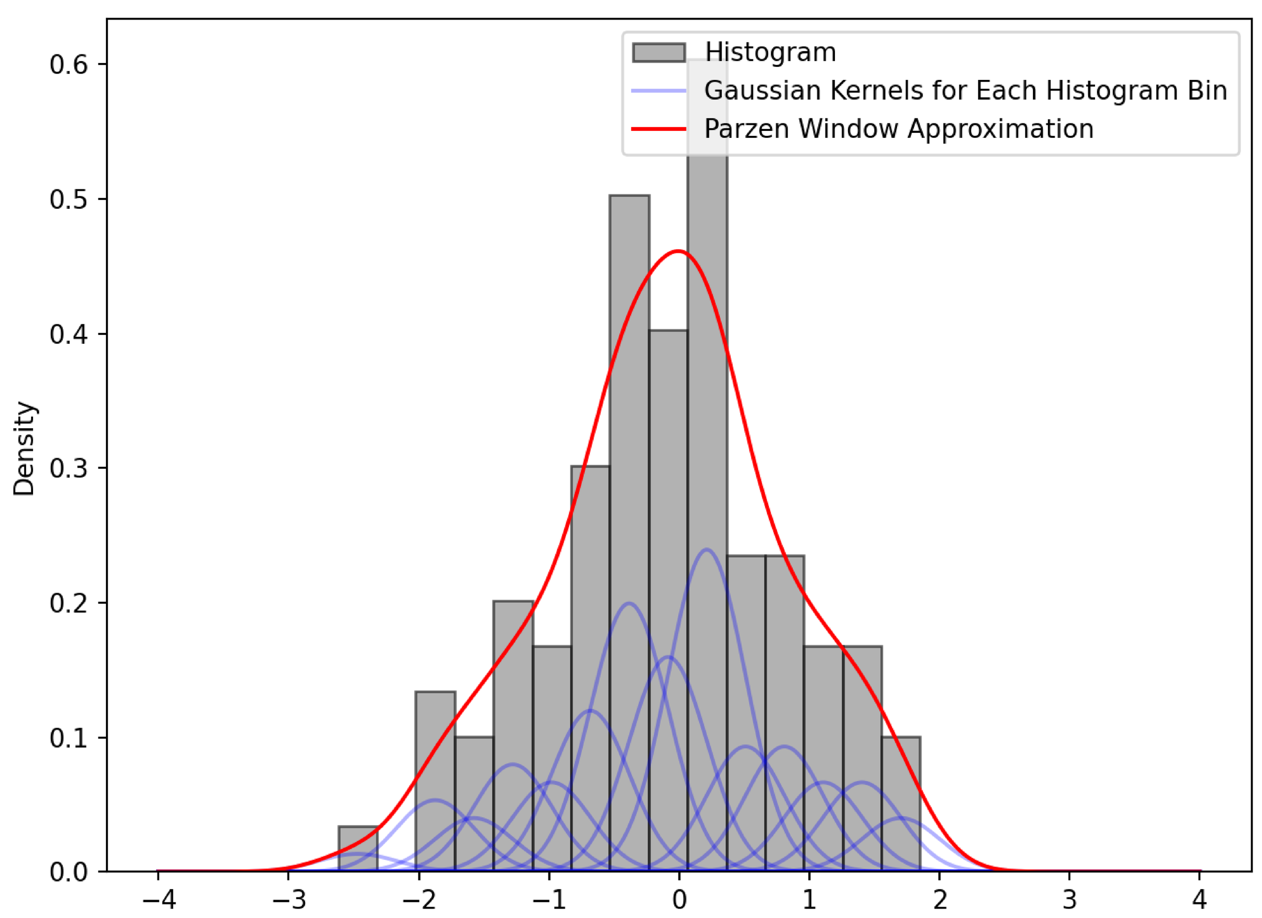

Fig. 1 *Illustration of Parzen-window approximation used to estimate a discrete histogram.*

Our Contribution:

- A **differentiable** CR using Parzen windowing for DL-based registration.
- Extensive experiments to find the **optimal regularization hyperparameter** when using CR.

METHODS

Overall Loss Function:

Regularization hyperparameter

$$\mathcal{L}(I_f, I_m \circ \phi) = \mathcal{L}_{CR}(I_f, I_m \circ \phi) + \lambda \mathcal{L}_{Reg}(\phi)$$

Correlation Ratio (CR): $\eta(Y|X) = \frac{Var(E(Y|X))}{Var(X)}$

- $E(Y|X)$: Conditional expectation of Y given X
- $Var(\cdot)$: Variance

Approximating the Probability Density Function of X:

The PDF is approximated using Parzen-window (*Gaussian kernels)* applied to the histogram of X:

- $\omega_{ik}(X; h, bin) = \frac{1}{h\sqrt{2\pi}} \exp(-\frac{(x_i - \mathrm{bin}_k)^2}{2h^2})$
- ω_{ik}: Contribution to the k-th intensity bin from the i-th voxel
- h: Standard deviation of the Gaussian kernel

Conditional expectation: $E(Y|X) = \bar{y}_k = \frac{\sum_i \omega_{ik} y_i}{\sum_i \omega_{ik}}$

Variance Components:

- *Conditional variance:*
  - $Var(E(Y|X)) = \sum_k n_k (\bar{y}_k - \bar{y})^2$
- *Overall Variance:*
  - $Var(X) = \frac{1}{|\Omega|} \sum_i (x_i - \bar{x})^2$

where n_k: normalization factor; $\bar{x}, \bar{y}$: the mean intensity of X and Y, respectively

CR loss for DL-based registration:

- $\mathcal{L}_{CR}(I_f, I_m \circ \phi) = -\frac{1}{2}(\eta(I_f | I_m \circ \phi) + \eta(I_m \circ \phi | I_f))$

Task: T1w to T2w registration using IXI dataset

Comparative models:

- **VoxelMorph** [2] and **TransMorph** [3] trained using MI and CR.
- **SyN** [4] using MI.

Hyperparameter Optimization:

- The hyperparameter λ was optimized over 20 trials for each model using the Optuna package.

RESULTS

Fig. 2 *Quantitative and qualitative results comparing different deformable registration methods.* ***(a)*** *Hyperparameter optimization results for λ.* ***(b)*** *and* ***(c)*** *Objective value versus translation (b) and rotation (c) between the images shown on top.* ***(d)*** *Example MR coronal slices extracted from input pairs (I_f, I_m), and the resulting ($I_m \circ \phi$) for different methods with optimal λ.*

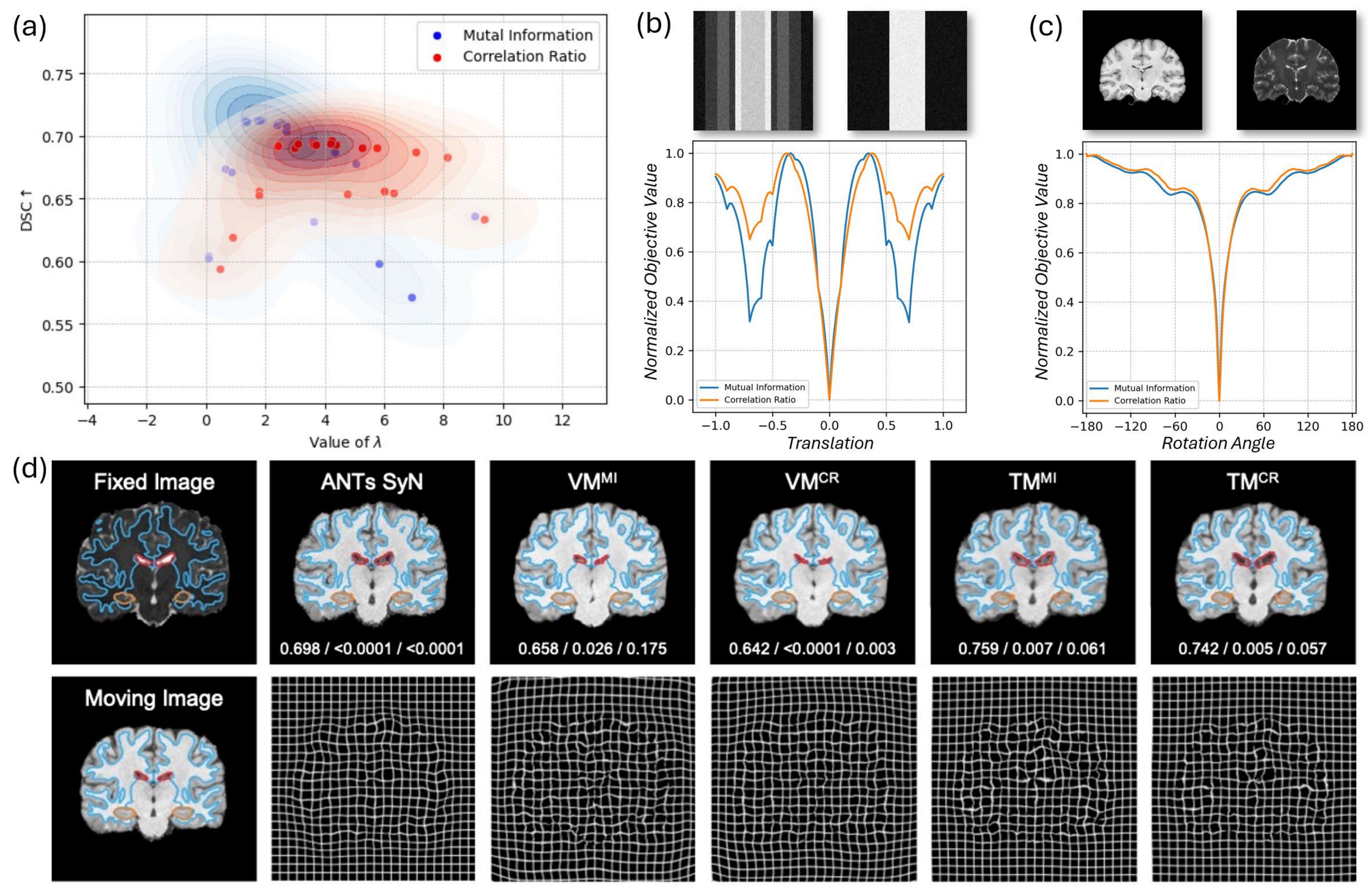

Table. 1 *Quantitative results on the brain registration task.*

Method	λ	DSC ↑	%NDV ↓	% of $\lvert J(\phi)\rvert < 0$ ↓
ANTs SyN (MI)	–	0.636 ± 0.048	< 0.0001	< 0.0001
VoxelMorph (MI)	4.5	0.594 ± 0.071	0.004 ± 0.004	0.063 ± 0.051
TransMorph (MI)	1.7	0.713 ± 0.028	0.009 ± 0.005	0.108 ± 0.063
VoxelMorph (CR)	7.7	0.594 ± 0.063	0.002 ± 0.004	0.006 ± 0.009
TransMorph (CR)	4.2	0.691 ± 0.031	0.014 ± 0.009	0.119 ± 0.056

Metrics: DSC; the percentage of NDV [5]; the percentage of voxels with $|J(\phi)| < 0$.

CONCLUSIONS

Accuracy: **Comparable DSC** between CR and MI (VM: 0.59 vs. 0.59; TM: 0.71 vs. 0.69)

Speed: CR is **~6x faster** than MI to compute (good for instance optimization)

Stability: CR shows **smoother optimization landscapes**

Additional Read: See [6], where **CR outperforms MI** for DL-based affine registration

REFERENCES

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