University of Waterloo

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MTE 203 Project 2

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Table of Contents

List of Figures

List of Tables

Question 1

The shape of the linear aerospike engine (LAE)'s faceplate can be modelled mathematically, given the diagram in the project description. Because it involves probalas, Cartesian coordinates are used over cylindrical and spherical coordinates. The front faceplate is put onto the x-y plane, with the origin located at the top-left corner of the faceplate. The z-plane is out of page, as shown below.

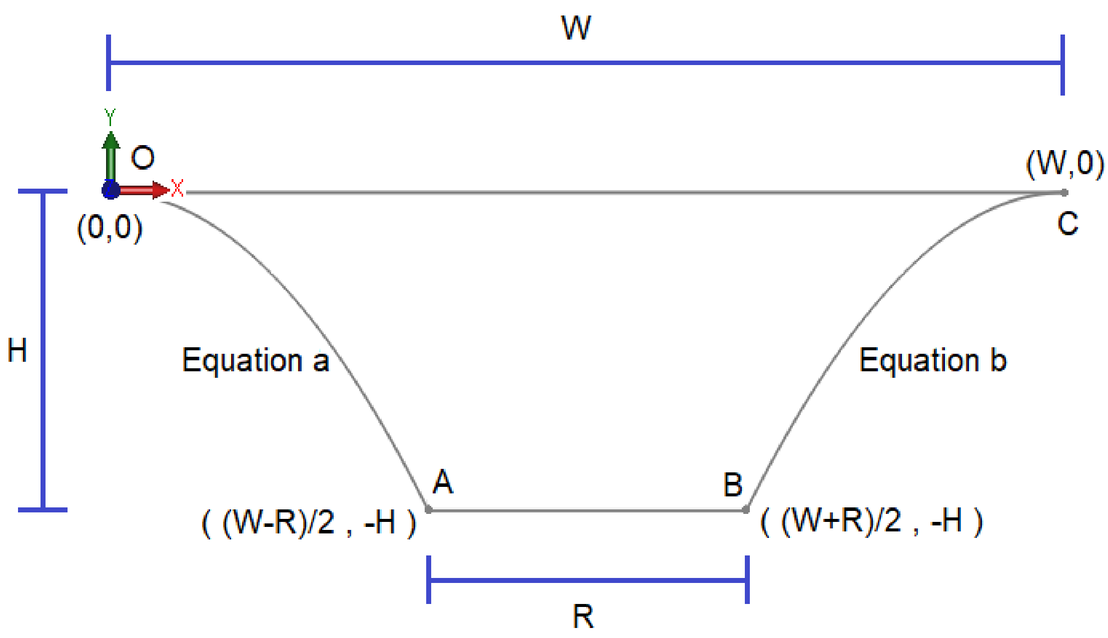

Figure 1: Diagram of the front faceplate of the LAE

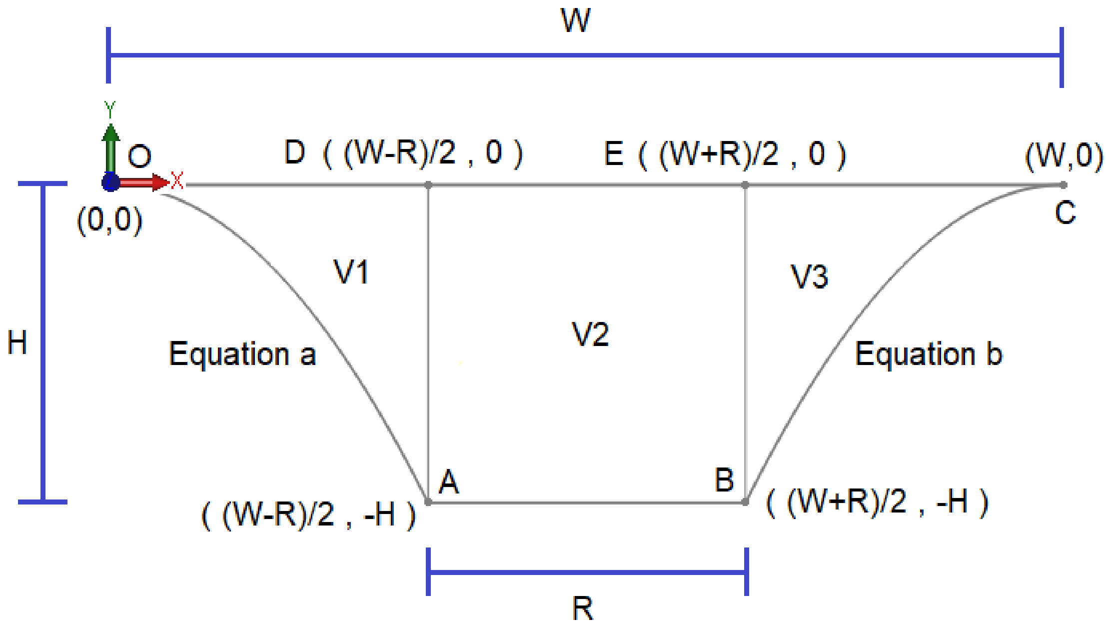

Figure 2: Diagram of the front faceplate of the LAE with coordinates

The faceplate is broken down into 3 components, V1, V2 and V3; the boundary is drawn using vertical lines that start from the two lower corners. Please note that the physical faceplate is not touched - the division is merely to allow each region to be bounded by only one equation, and thus make the calculation easier.

According to the structure of the LAE and the diagram above, one resonable assumption I made is that the faceplate is symmetric about the vertical line that connects the midpoint of W and R. Therefore, V3 has the exact same area as V1, and furthermore, the exact same volumn if the height on the z-axis is uniform (which it is, represented by the constant D).

I can now start defining each coordinate. A is at half of R left of half of W, with a height of -H, so A = ((W-R)/2 , -H). Likewise, B = ((W+R)/2 , -H). C, D and E are simply (W,0), ((W+R)/2 , 0) and ((W-R)/2 , 0), respectively.

Knowing the vertex of the parabola is at O = (0,0), I can write Equation a with

$$(y - 0) = \alpha(x - 0)^2$$

Given the coordinate of A

$$(-H - 0) = \alpha((W - R)/2 - 0)^2$$

Simplify, and

$$\alpha = \frac{-4H}{(W - R)^2}$$

So Equation a becomes

$$y = \frac{-4H}{(W - R)^2}\, x^2$$

Question 2

Part a

To calculate the volume of V1, a triple integral is needed, with f(x,y,z) = 1. However, since the height on the z-axis is D, a constant, which is independent of x and y, the triple integral can be easily converted into a double integral.

$$\iiint_V 1\, dz\, dy\, dx = \iint_{A1} \int_0^D dz\, dy\, dx = \iint_{A1} D\, dy\, dx = D \iint_{A1} dy\, dx$$

Now I need to find the boundary condition for A1. I am using y as a function of x, so it makes sense to have the order *dydx*. The upper bound for y is the x-axis, which is 0. The lower bound for y is the equation above, y as a function of x. For x, the bounds need to be numbers, so in this case, 0 to (W-R)/2. So the integral is set up

$$V1 = D \int_0^{\frac{W-R}{2}} \int_{\frac{-4H}{(W-R)^2} x^2}^{0} dy\, dx$$

$$V1 = D \int_0^{\frac{W-R}{2}} \frac{4H}{(W-R)^2}\, x^2\, dx$$

$$V1 = D\, \frac{4H}{(W-R)^2} [\frac{x^3}{3}]_0^{\frac{W-R}{2}}$$

$$V1 = \frac{4DH(W-R)^3}{(W-R)^2 3 \times 8}$$

$$V1 = \frac{DH(W-R)}{6} = V3$$

To calculate the volume of V2, a triple integral is still needed, with f(x,y,z) = 1. However, since the height on the z-axis is D, a constant, which is independent of x and y, and y is independent of x, all bounds are going to be constants, so the integral is simplified to

$$V2 = \int_{(W-R)/2}^{(W+R)/2} \int_{-H}^{0} \int_{0}^{D} 1\, dz\, dy\, dx$$

$$V2 = DHR$$

So combine V1, V2 and V3 to calculate the final volume V

$$V = V1 + V2 + V3 = 2 \times \frac{DH(W-R)}{6} + DHR$$

$$V = \frac{1}{3}DHW + \frac{2}{3}DHR$$

Given W=3, H=1, R=1, D=3 for the outer LAE, in meters

$$V_{outer} = \frac{1}{3}DHW + \frac{2}{3}DHR = \frac{1}{3}3 \times 1 \times 3 + \frac{2}{3}3 \times 1 \times 1 = 3 + 2 = 5m^3$$

Given W=2, H=0.9, R=0.8, D=2.9 for the inner LAE, in meters

$$V_{inner} = \frac{1}{3}DHW + \frac{2}{3}DHR = \frac{1}{3}2.9 \times 0.9 \times 2 + \frac{2}{3}2.9 \times 0.9 \times 0.8 = 1.74 + 1.392$$
$$= 3.132m^3$$

So subtract V_{outer} by V_{inner} to get the final shell volume

$$V_{shell} = V_{outer} - V_{inner} = 5 - 3.132 = 1.868m^3$$

Part b

To determine the mass of the shell, the equation is

$$m = \rho V$$

Where m is mass, ρ is density, and V is volume. The density of pure Titanium is 4506kg/m^3 [1], so

$$m = \rho V = 4506kg/m^3 \times 1.868m^3 = 8417.208kg$$

To get a visual representation and to confirm the calculated result, a CAD file has been modeled in Solidworks.

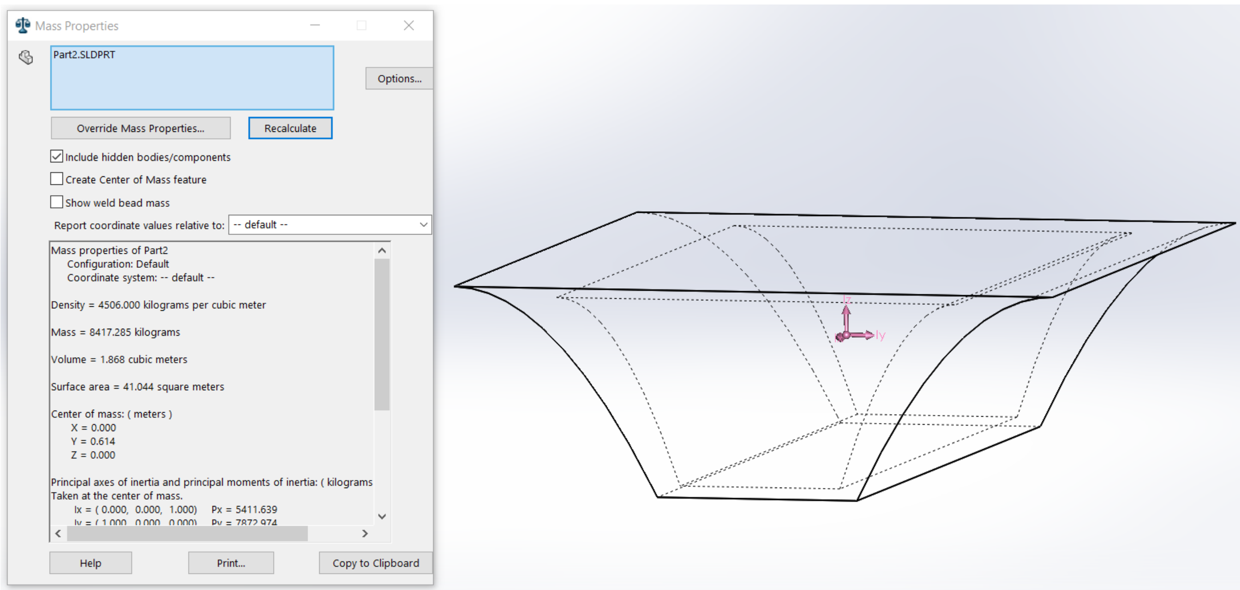

Figure 3: Solidworks model of the LAE

As shown above, the inner LAE (extruded cut) is located exactly at the centre of the outer LAE, meaning, the new LAE shell has faceplates on all sides, including top and bottom. For top and bottom, they in fact have the same thickness, (3 - 2.9)/2 = 0.05m to be specific.

The volume of the LAE shell computed by Solidworks is 1.868m^3, which consolidates with the calculated value. The mass of 8417kg is also confirmed by Solidworks after defining the density to be 4506kg/m^3.

Question 3

Part a

Considering the material used in the LAE is some type of metal, Titanium for example, traditional 3D printing techniques such as Fused Deposition Modeling (FDM) and Stereolithography (SLA) are inapplicable. The best method to 3D print a metal part is Selective Laser Sintering (SLS). Other methods including Direct Metal Laser Sintering (DMLS) and Selective Laser Melting (SLM) are very similar techniques, except they use a different way to melt the metal powder, which is irrelevant to the requirement of this project, as it does not affect the LAE's geometry.

SLS uses laser as the power source to sinter powdered material, aiming the laser automatically at points in space defined by a 3D model, binding the material together to create a solid structure [2]. Because it sinters each layer in a giant bucket of metal powder, the powder will remain inside the hollow part. And, it is impossible to remove the excess powder, unless to break the structure apart. Therefore, I need to print two structures, and finally weld them together to get the LAE shell.

Given the shape of the LAE, there are 3 main ways to 3D print it: along the x-axis, y-axis and z-axis. When printing along the x-axis, there is not a single flat surface of the LAE that touches the ground. Therefore, a lot of support material will be used to make the entire structure stand. Support material is undesirable because a) it requires manual cleanup and sanding, and b) they are expensive. Additionally, the location of the welded plates contains the critical region, which cuts the bottom faceplate of the LAE. From Figure 2 in the project description, it is clear that the nozzles are located at both sides of the LAE pointing down, and exhaust gases generated are pushed out of the nozzle. Therefore, the critical region that experiences the most mechanical stress and heat are the sides and bottom of the LAE. Therefore, it is better for the sides and the bottom to remain as one piece. In order to do this, printing along the z-axis would not work, as the front or back faceplate needs to be welded. The only option left is to print the LAE along the y-axis, shown in Figure 4 below. There may be support material involved, but overall it does not require much, and more importantly, the welded plates are on the top, outside the critical region.

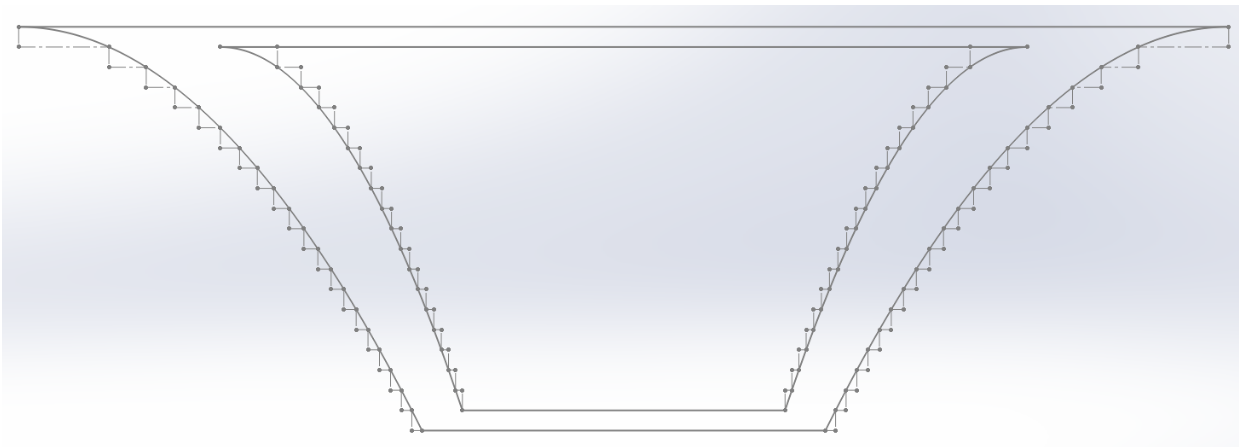

Figure 4: 3D printing the LAE along the y-axis

Because the uniform thickness of each layer, Z_L = 1cm, makes many rectangles in 2D and rectangular prisms in 3D, the Riemann sum is used to calculate the volume of the LAE shell. For 3D print of the physical structure, upper sum is the most appropriate method for the outer LAE and lower sum is the best method for the inner LAE, which is subtracted from the outer. Trapezoidal would be the most accurate representation of the structure and estimate of the volume, but it omits a small portion of the wall. In aerospace engineering, a little shortage of material may result in a small crack, and the small crack may be catastrophic. For this reason, it is better to use the upper sum to print the outer LAE, leaving the blank space for the lower sum of the inner LAE, and then clean up and sand off the undesirable part left over from the "staircase effect", at both the inner and outer wall, before welding the top faceplate. The mass of the 3D printed shell is 8632.9kg, calculated in MATLAB. The more detailed explanation is in Part b below.

Part b

The largest Z_L that could be used to limit the relative error within 0.2% is 0.5mm, and it gives a relative error of 0.13%. Any Z_L greater than 0.5mm will yield a minimum relative error greater than 0.25%, as shown in Table 1 below.

Mass (kg)	ZL (mm)	Relative Error
8427.912	0.50	0.13%
8438.635	1.00	0.25%
8456.134	1.50	0.46%
8460.119	2.00	0.51%
8470.875	2.50	0.64%
8508.688	3.00	1.09%
8511.165	3.50	1.12%
8503.188	4.00	1.02%
8608.352	4.50	2.27%
8577.032	5.00	1.90%
8585.665	5.50	2.00%
8573.449	6.00	1.86%
8602.037	6.50	2.20%
8623.299	7.00	2.45%
8646.467	7.50	2.72%
8631.359	8.00	2.54%
8719.475	8.50	3.59%
8813.550	9.00	4.71%
8789.945	9.50	4.43%
8632.937	10.00	2.56%

Table 1: Calculated result from MATLAB

The biggest challenge in the MATLAB algorithm is that it must take into consideration of the “unmatched” layer thickness, when the thickness of the structure is not divisible by the thickness of each layer. The upper faceplate for example, has a thickness of 0.05m = 50mm. If the thickness of each layer is 3mm, there will either be a total of 48mm printed (short of 2mm), or 51mm (has to sand off the 1mm). The solution to this challenge is that MATLAB always takes the upper limit for the outer LAE, and lower limit for in inner LAE, which is subtracted from the outer. For this reason, there is always going to be excess material used if the thickness is not divisible, and a technician will have to sand off the extra part.

Part c

One practical issue with the MATLAB code that I designed is that it does not take into consideration of the support material used. In 3D printing, support material plays a very important role, as each new layer must have something underneath to allow the layout. In SLS printing, the metal powder not only serves as material used, but also as the support material. However, as the slope becomes too large, the soft powder can on longer support the heavy weight of the structure. For this reason, as the print job get closer to the top faceplate of the LAE, the slope is too much. Therefore, some support material is still needed.

One method I would use to enhance the algorithm in the future is to add the volume needed for support material. A certain threshold of the difference in layer length can be defined, and if the average difference in layer length across any segment on the curve exceeds this value, a cylindrical support will be added. So, support cylinders will be sparsely distributed if the slope is moderately large, and densely distributed if the slope of the curve is extremely large.

Bonus

The simplification I had to make is that the rocket nozzles are attached onto the faceplate of the LAE, and this in reality can be troublesome. Welded parts may even be stronger than the original material, however, it is inconsistent on curves. It is ideal to be able to have uniform nozzles so it is much easier to model, simulate and troubleshoot. One of the potential ways is to take into account of the nozzles. For example, intentionally print the staircase effect so the surfaces are smooth, even though not perfect. So, the welding process may become easier.

Reference

[1] “Titanium - Element information, properties and uses,” *Royal Society of Chemistry - Advancing excellence in the chemical sciences.* [Online]. Available: http://www.rsc.org/periodic-table/element/22/titanium. [Accessed: 19-Jul-2017].

[2] “Laser Sintering,” *Stratasys Direct Manufacturing.* [Online]. Available: https://www.stratasysdirect.com/solutions/laser-sintering/. [Accessed: 19-Jul-2017].