94.867: Decision Analytics for Business and Policy
Spring 2019
Assignment 5: Stochastic simulation and optimization

Assigned: April 03; Due: May 01, 3 pm.
Please post the assignment in pdf format with file name "Lastname_94867_HW5.pdf".

Introduction

This problem will address a common problem known as *revenue management*. Consider a business that can sell a finite set of goods before a deadline. Prices typically evolve over time: they can go up (e.g., airfares, hotel rates) or down (e.g., retail promotions). A core objective in revenue management is to determine the number of items to sell in each period, given demand characteristics and a menu of prices over time. We address this problem in the context of hotel pricing. Here is the specific setting considered in this problem.

Super Bowl LIV will take place in February, 2020 in Miami, FL. A hotel in the Miami area is obviously excited about this prospect. We assume that we are in February 2019 and that customers will make bookings throughout the year for the night of the Super Bowl. Unfortunately, since this is an extraordinary event, the hotel does not have perfect demand information, and can only approximate it probabilistically.

The goal of this problem is to determine the number of rooms for the night of the Super Bowl to sell in each month from February 2019 to January 2020 to maximize total expected revenues. This is referred as the hotel's *booking policy*. We assume that:

- Demand D_t is modeled as a Poisson variable with mean $\lambda = 9$ per month.
- The hotel has 100 rooms available, and none of them is booked initially.
- All the demand involves one-night stays only.
- The price is \$100/room in month 1 (February 2019), and increases by \$10/month, up to \$210 in month 12 (January 2020).

Stages 1 and 3 are based on the materials on Monte Carlo simulation; Stage 2 is based on the materials on Markov decision processes. For each computational question, please show your work by providing screenshots of your code. A starting point for all each computation is provided in Revenue_Management.m and Revenue_Management.R. You do not have to follow the proposed structure exactly, as there are multiple correct approaches for each question.

Stage 1: Simulation of "Accept All" Booking Policy [30 pts]

We first simulate the evolution of the system over the 12-month horizon under a simple policy that accepts all bookings as long as there are sufficient rooms available in the hotel. For example, if 12 customers place a booking request in the first month, the hotel will book 12 rooms; if 10 customers then place a booking request in the second month, the hotel will book 10 rooms; the process is repeated until all rooms are booked.

We will sample realizations of demand for every single month, and characterize the resulting performance of the system. Please answer each question in three settings: (i) with 100 samples of the 12-month horizon, (ii) with 1,000 samples of the 12-month horizon, and (iii) with 10,000 samples of the 12-month horizon.

a. Write a simulation program that draws N realizations of the system and that samples the demand for every month from its probability distribution. For each of the N trials, store the inventory at the end of each month (i.e., the number of remaining rooms available at the end of each month) and the total revenue across the 12 months. Please show your work. **[12 pts]**

b. Provide the sample average of the total revenue across the 12-month horizon, its sample standard deviation and its confidence interval. Plot its frequency distribution. Comment briefly. **[10 pts]**

c. Provide the sample estimate of the probability that the hotel will not be full at the end of the 12-month horizon and its confidence interval. Comment briefly. **[8 pts]**

Stage 2: Optimization of Booking Policy [40 pts]

We now want to determine the optimal booking policy, i.e., the policy that maximizes the expected total revenues of the hotel for the Super Bowl night.

We index the 12 months by $t = 1, ..., 12$. We denote the price of each room sold during month t by p_t (i.e., $p_1 = \$100, \cdots p_{12} = \$210$) and the demand in month t by D_t. At the beginning of each month t, the hotel determines a booking limit, i.e., the maximum number of rooms to book in month t at price p_t. This is denoted by u_t. Specifically, if the demand is less than the booking limit in month t, the hotel will sell D_t rooms; if the demand is larger than the booking limit in month t, the hotel will sell u_t rooms.

The optimization of the booking policy is formulated as a Markov decision process. We consider a finite horizon with 12 periods, indexed by $t = 1, \cdots, 12$. The state in period t is defined as the number of rooms that remain available at the beginning of month t. It is denoted by x_t, and we have $x_1 = 100$. The state variable x_t can take any integer value between 0 and 100. The decision in period t is defined by the booking limit for month t, that is, u_t. The decision variable u_t can take any integer value between 0 and x_t.

a. Complete the formulation of the Markov decision process by specifying the transition function, the objective function and the Bellman equation. **[10 pts]**

b. Implement the Markov decision process computationally. Please show your work. [Note: The full model can take up to 2-3 minutes to solve.] **[15 pts]**

   Hint: To simplify the computations, we will assume that there will never be more than 30 booking requests in any month.

The optimal policy is given in the booking_limits.csv file, so you can continue the problem even if you did not get the optimal booking policy in the previous question. Each row represents the number of remaining rooms; each column represents the number of remaining months before the Super Bowl.

c. Characterize the optimal booking policy by answering the following questions. Please comment briefly on each answer. **[15 pts]**
   - What is your booking limit for the first month?
   - For a given number of rooms available, how does the booking limit vary over time?
   - For a given month, how does the booking limit vary as a function of the number of rooms available?

Stage 3: Simulation of Optimal Booking Policy [30 pts]

We now want to test the optimal policy found in Stage 2 in the simulation environment developed in Stage 1. Recall that the booking policy captures the maximal number of rooms to sell in each month, as a function of the number of rooms available.

a. Modify your simulation to capture the new policy. Please describe what changed, and what did not. **[12 pts]**

b. Under this new policy, compute the average total revenue, its standard deviation, and its confidence interval. Compare your answer with your results in Stage 1. Please comment briefly on your results, and the intuition behind them. **[10 pts]**

c. Under this new policy, compute the estimate of the probability that the hotel will not be full at the end of the 12-month horizon and its confidence interval. Compare your answer with your results in Stage 1. Please comment briefly on your results, and the intuition behind them. **[8 pts]**