Lecture 8.2
Structure from Motion

Thomas Opsahl

More-than-two-view geometry

Correspondences (matching)

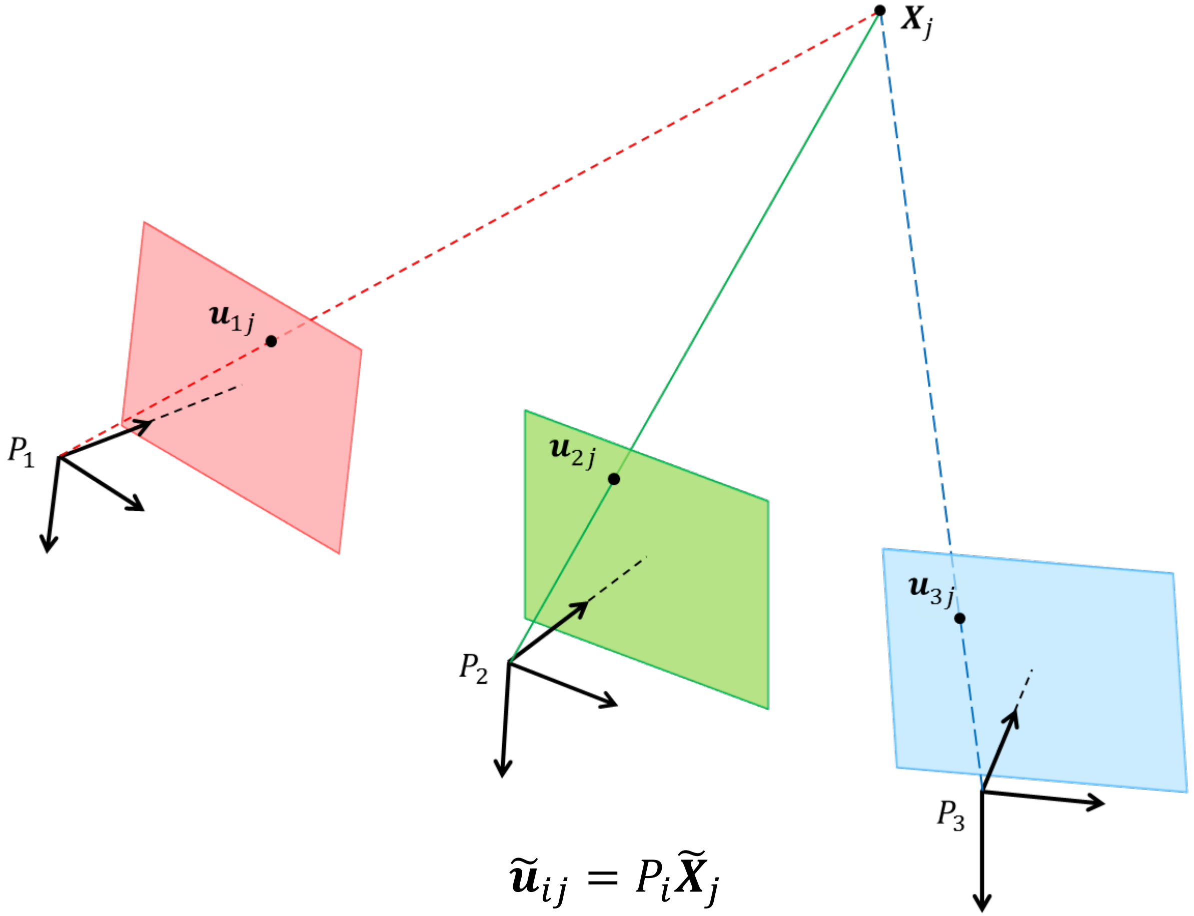
- More views enables us to reveal and remove more mismatches than we can do in the two-view case
- More views also enables us to predict correspondences that can be tested with or without the use of descriptors

Scene geometry (structure)

- Effect of more views on determining the 3D structure of the scene?

Camera geometry (motion)

- Effect of more views on determining camera poses?

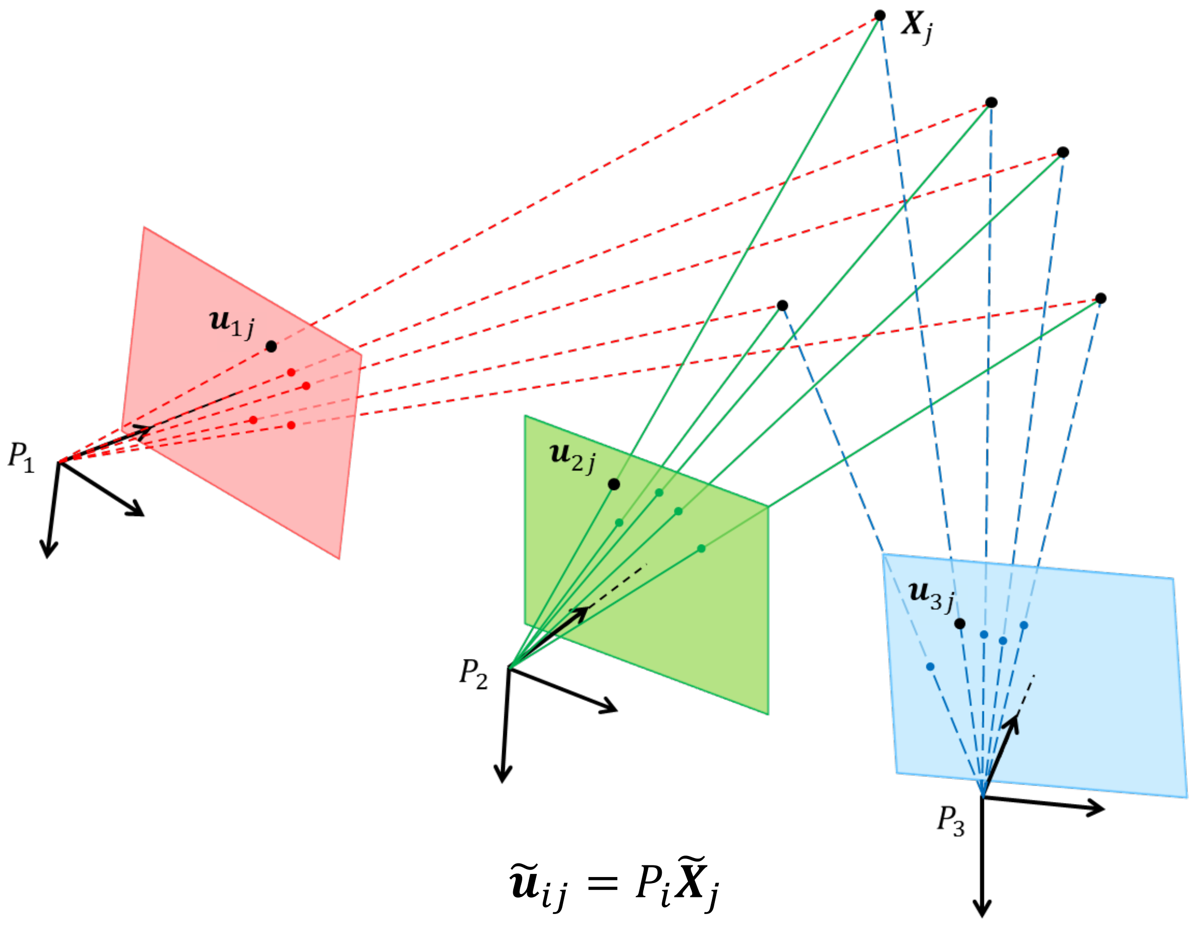


$$\tilde{\boldsymbol{u}}_{ij} = P_i \tilde{\boldsymbol{X}}_j$$

Structure from Motion

Problem

Given m images of n fixed 3D points, estimate the m projection matrices P_j and the n points $\boldsymbol{X}_j$ from the $m \cdot n$ correspondences $\boldsymbol{u}_{ij} \leftrightarrow \boldsymbol{u}_{kj}$



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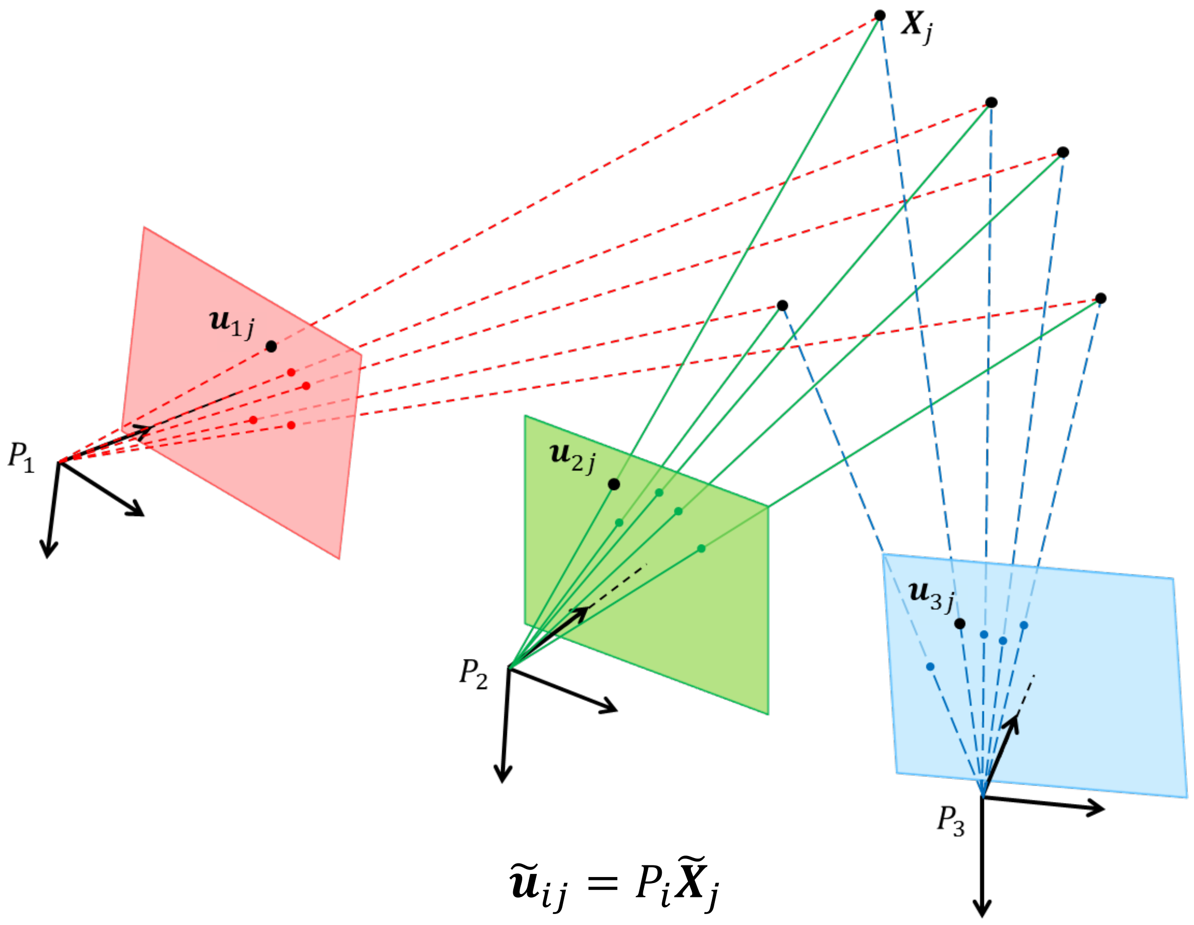
- We can solve for structure and motion when
  $$2mn \geq 11m + 3n - 15$$
- In the general/uncalibrated case, cameras and points can only be recovered up to a projective ambiguity ($\tilde{\boldsymbol{u}}_{ij} = P_i Q^{-1} Q \tilde{\boldsymbol{X}}_j$)
- In the calibrated case, they can be recovered up to a similarity (scale)
  - Known as Euclidean/metric reconstruction



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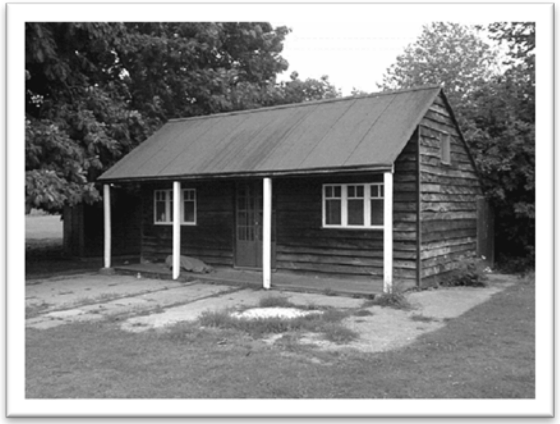
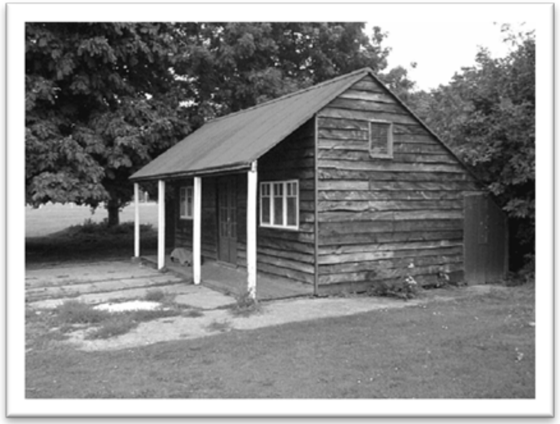
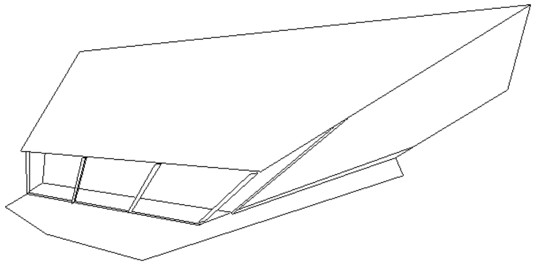
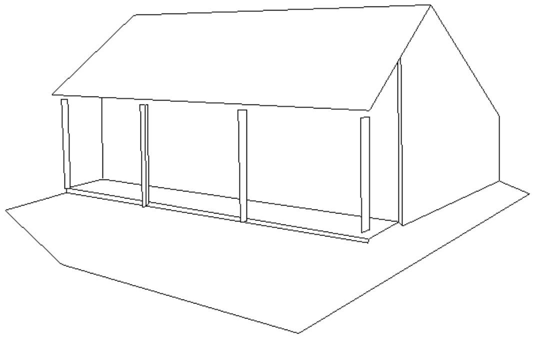






Projective reconstruction



Metric reconstruction

Images courtesy of Hartley & Zisserman http://www.robots.ox.ac.uk/~vgg/hzbook/

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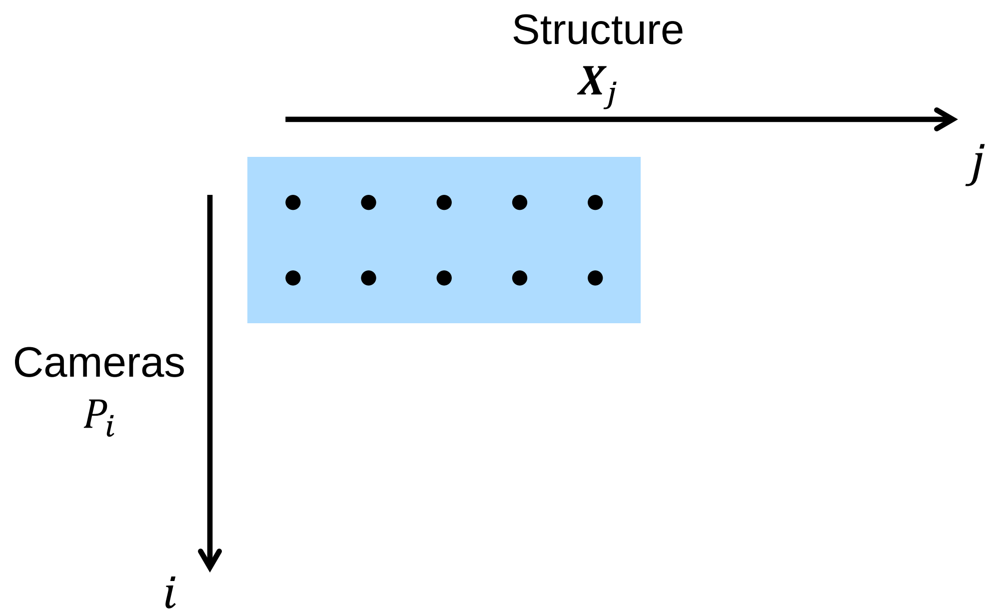
- This problem has been studied extensively and several different approaches have been suggested

- We will take a look at a couple of these
  - Sequential structure from motion
  - Bundle adjustment

Sequential structure from motion

- Initialize motion from two images
  - $F \rightarrow (P_1, P_2)$
  - $E \rightarrow (P_1, P_2) = (K_1[I \quad \mathbf{0}], K_2[{}^1R_2 \quad {}^1\boldsymbol{t}_2])$

- Initialize the 3D structure by triangulation



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- For each additional view
  - Determine the projection matrix P_i, e.g. from 2D-3D correspondences $\boldsymbol{u}_{ij} \leftrightarrow \boldsymbol{X}_j$
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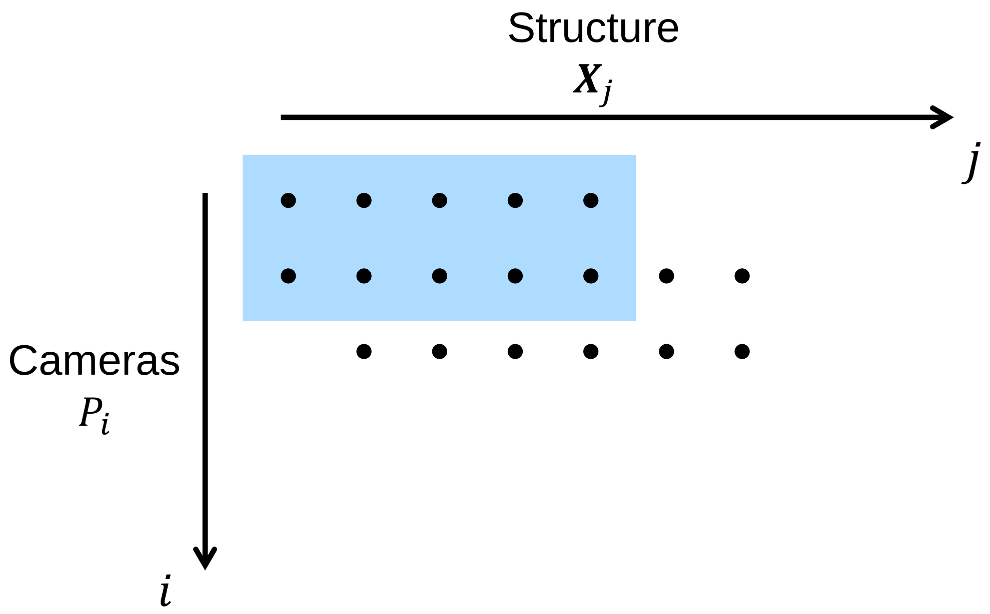

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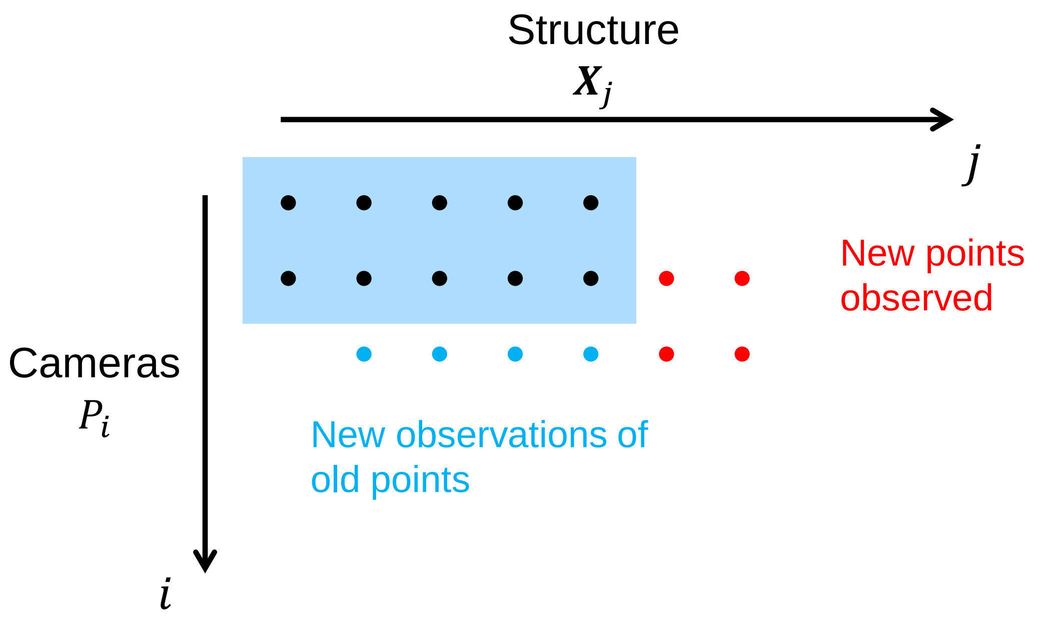

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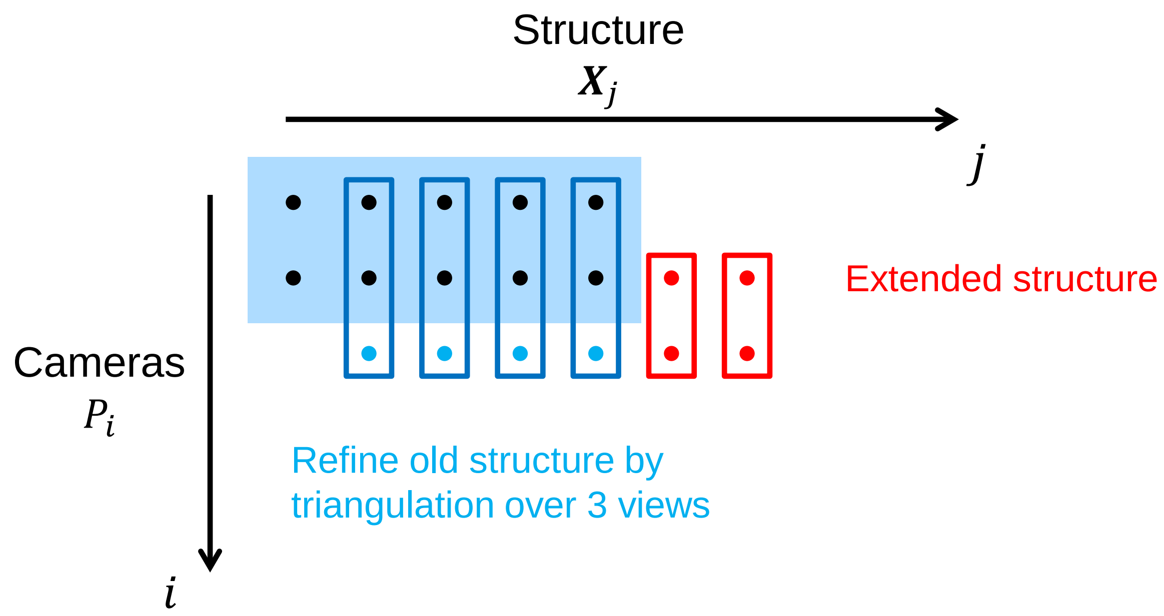

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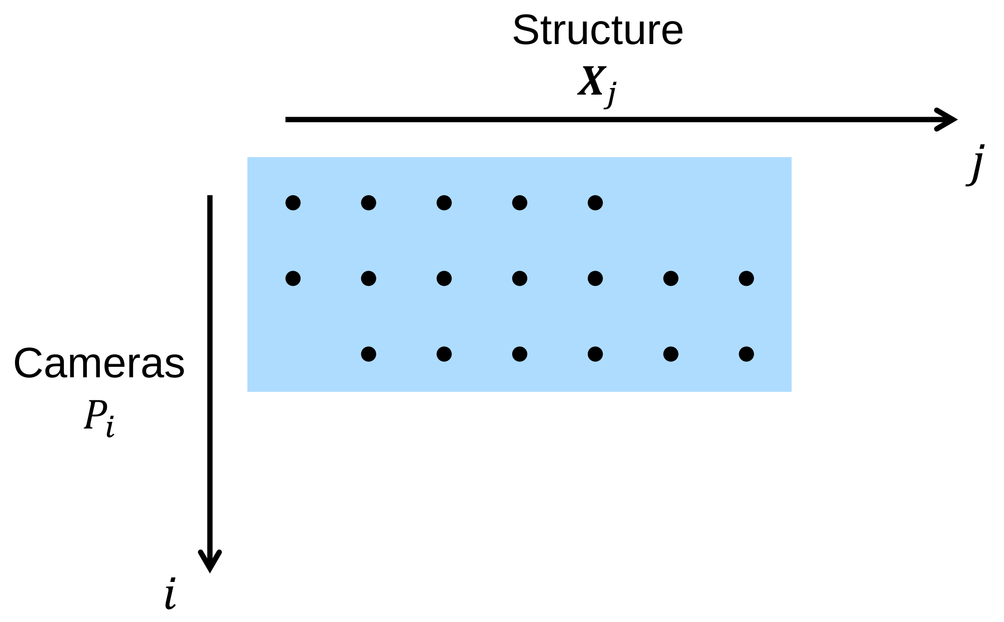

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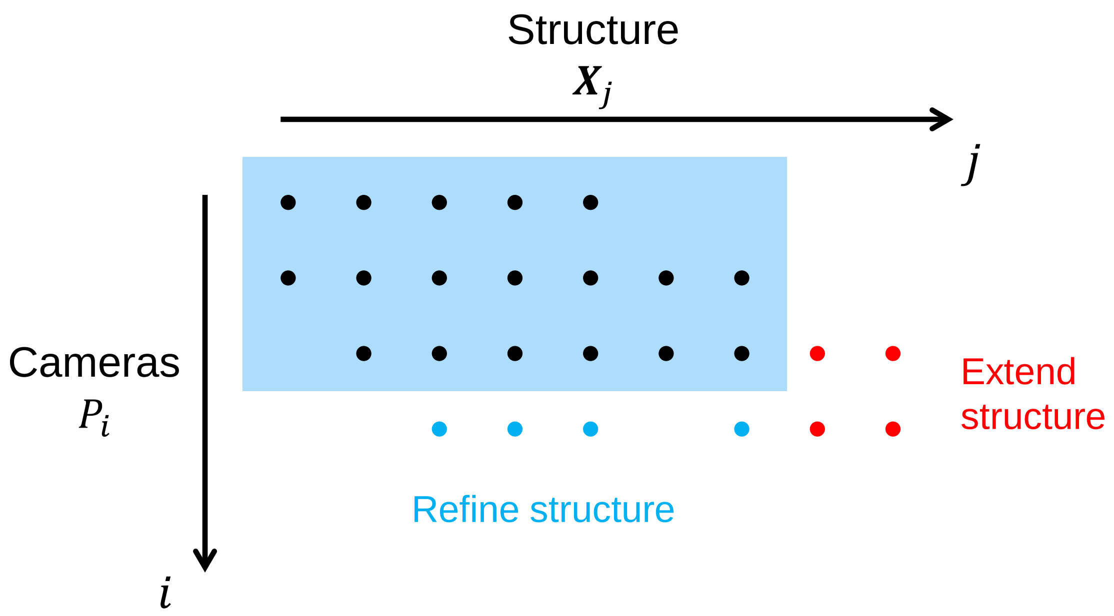

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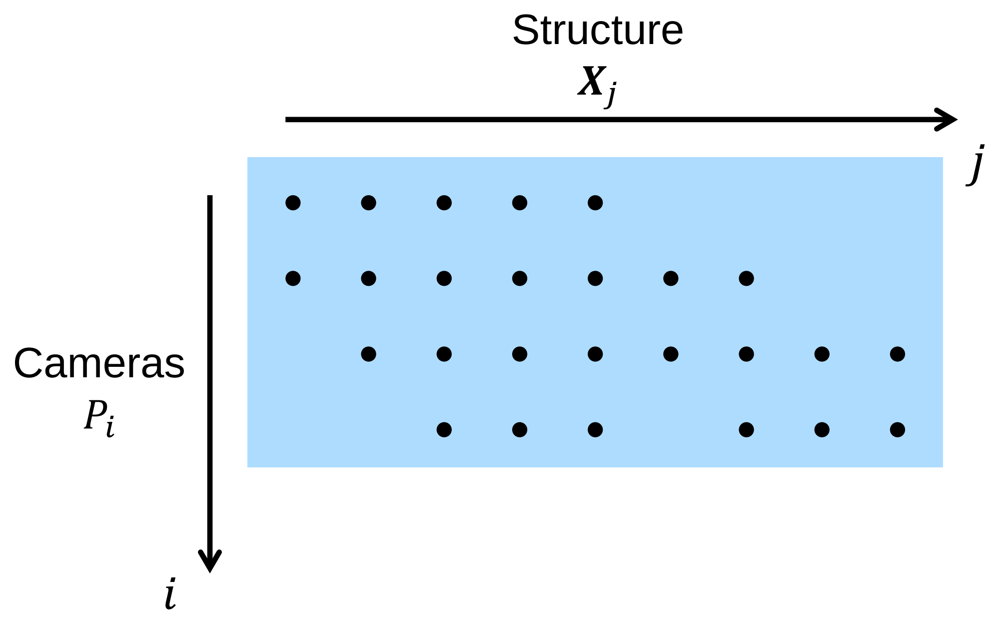

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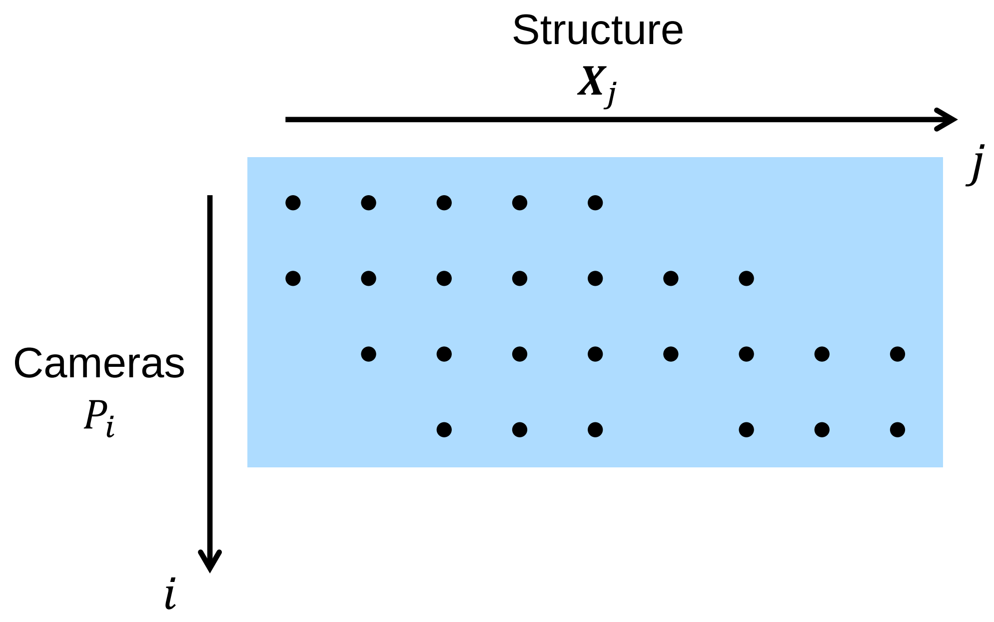
- The resulting structure and motion can be refined in a process known as bundle adjustment



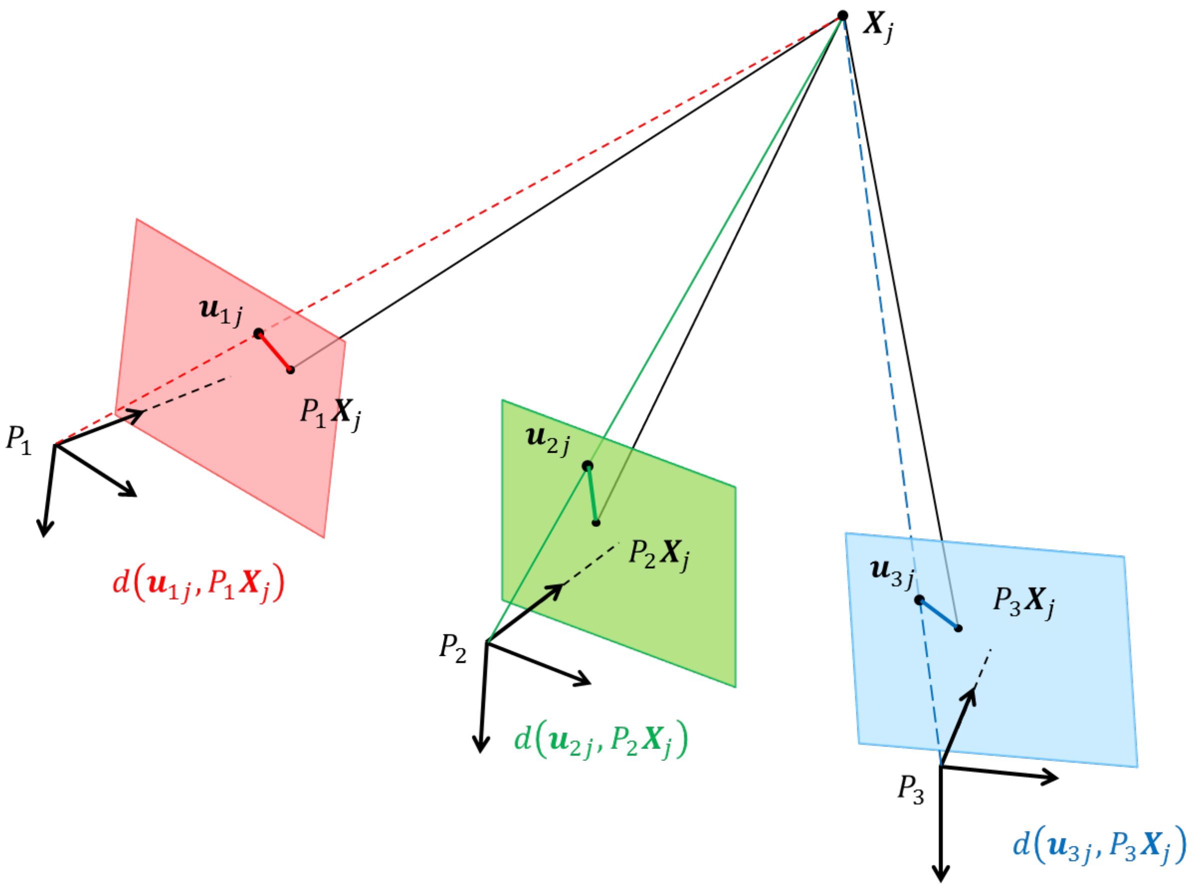
Bundle adjustment

- Non-linear method that refines structure and motion by minimizing the sum of squared reprojection errors

$$\epsilon = \sum_{i=1}^{m} \sum_{j=1}^{n} d\left(\tilde{\boldsymbol{u}}_{ij}, P_i \tilde{\boldsymbol{X}}_j\right)^2$$

- Camera calibration can be solved as part of bundle adjustment by including intrinsic parameters and skew parameters in the cost function

- Need initial estimates for all parameters!
  - 3 per 3D point
  - ~12 per camera depending on parameterization
  - Some intrinsic parameters, like the focal length, can be initialized from image EXIF data

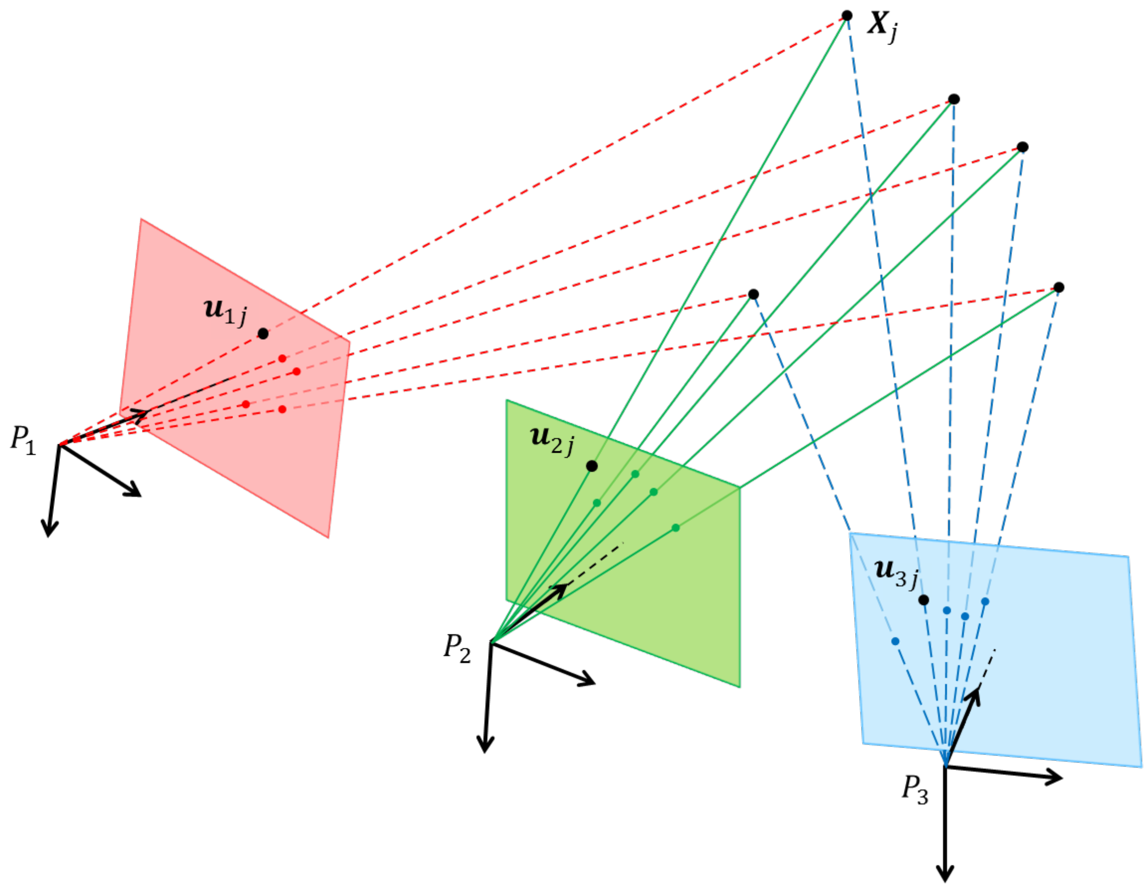


Bundle adjustment

- There are several strategies that deals with the potentially extreme number of parameters

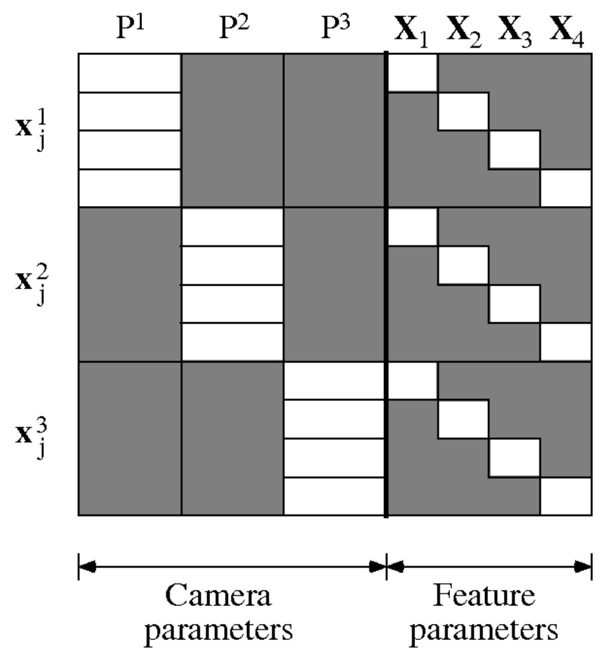
- Reduce the number of parameters by not including all the views and/or all the points
  - Perform bundle adjustment only on a subset and compute missing views/points based on the result
  - Divide views/points into several subsets which are bundle adjusted independently and merge the results

- Interleaved bundle adjustment
  - Alternate minimizing the reprojection error by varying only the cameras or only the points
  - This is viable since each point is estimated independently given fixed cameras, and similarly each camera is estimated independently from fixed points



Bundle adjustment

- Sparse bundle adjustment
  - For each iteration, iterative minimization methods need to determine a vector $\boldsymbol{\Delta}$ of changes to be made in the parameter vector
  - In Levenberg-Marquardt each such step is determined from the equation
    $$(J^T J + \lambda I)\boldsymbol{\Delta} = -J^T \boldsymbol{\epsilon}$$
    where J is the Jacobian matrix of the cost function and $\boldsymbol{\epsilon}$ is the vector of errors
  - For the bundle adjustment problem the Jacobian matrix has a sparse structure that can be exploited in computations



The sparse structure of the Jacobian matrix for a bundle adjustment problem with 3 cameras and 4 3D points

Figure courtesy of Hartley & Zisserman http://www.robots.ox.ac.uk/~vgg/hzbook/

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- Combined with parallel processing the before mentioned strategies has made it possible to solve extremely large SfM problems

- S. Agarwal et al, *Building Rome in a Day,* 2011
  - Cluster of 62-computers
  - 150 000 unorganized images from Rome
  - ~37 000 image registered
  - Total processing time ~21 hours
  - SfM time ~7 hours

- J. Heinly et al, *Reconstructing the World in Six Days*, 2015
  - 1 dual processor PC with 5 GPU's (CUDA)
  - ~96 000 000 unordered images spanning the globe
  - ~1.5 000 000 images registered
  - Total processing time ~5 days
  - SfM time ~17 hours

Bundle adjustment

- SBA – Sparse Bundle Adjustment
  - A generic sparse bundle adjustment C/C++ package based on the Levenberg-Marquardt algorithm
  - Code (C and Matlab mex) available at http://www.ics.forth.gr/~lourakis/sba/
  - CVSBA is an OpenCV wrapper for SBA www.uco.es/investiga/grupos/ava/node/39/

- Ceres
  - By Google (used in production since 2010)
  - A C++ library for modeling and solving large, complicated optimization problems like SfM
  - Homepage: www.ceres-solver.org
  - Code available on GitHub https://github.com/ceres-solver/ceres-solver

- GTSAM – Georgia Tech Smoothing and Mapping
  - A C++ library based on factor graphs that is well suited for SfM ++
  - Code (C++ library and Matlab toolbox) available at https://borg.cc.gatech.edu/borg/download

- g^2o – General Graph Optimization
  - Open source C++ framework for optimizing graph-based nonlinear error functions
  - Homepage: https://openslam.org/g2o.html
  - Code available on GitHub https://github.com/RainerKuemmerle/g2o

Bundle adjustment

- Bundler
  - A structure from motion system for unordered image collections written in C and C++
  - SfM based on a modified version SBA (default) or Ceres
  - Homepage: http://www.cs.cornell.edu/~snavely/bundler/
  - Code available on GitHub https://github.com/snavely/bundler_sfm

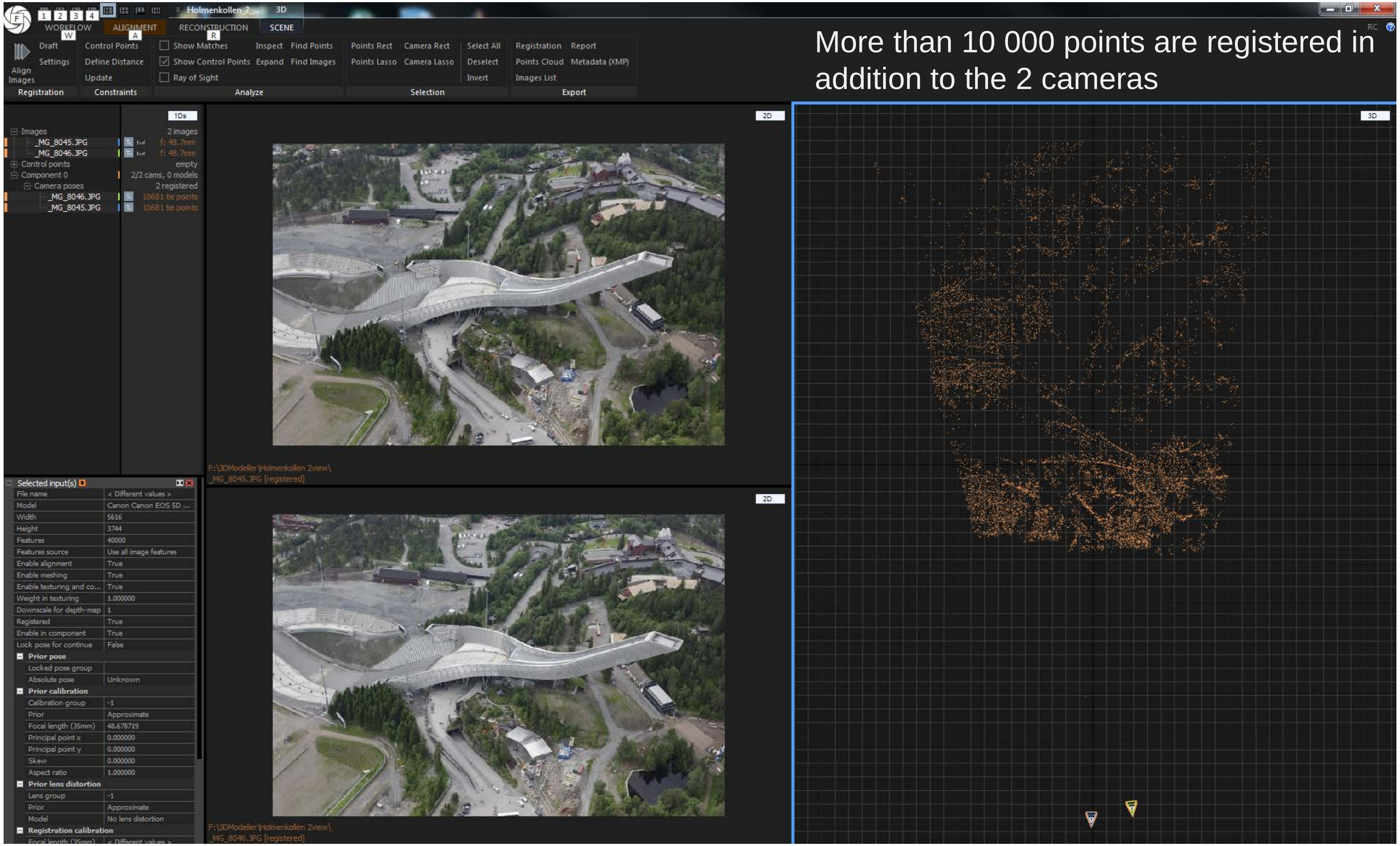
- VisualSfM
  - A GUI application for 3D reconstruction using structure from motion
  - Output works with other tools that performs dense 3D reconstruction
  - Homepage: http://ccwu.me/vsfm/

- RealityCapture
  - A state-of-the-art photogrammetry software that automatically extracts accurate 3D models from images, laser-scans and other input
  - Homepage: https://www.capturingreality.com/

Example

Holmenkollen 2-view SfM

More than 10 000 points are registered in addition to the 2 cameras



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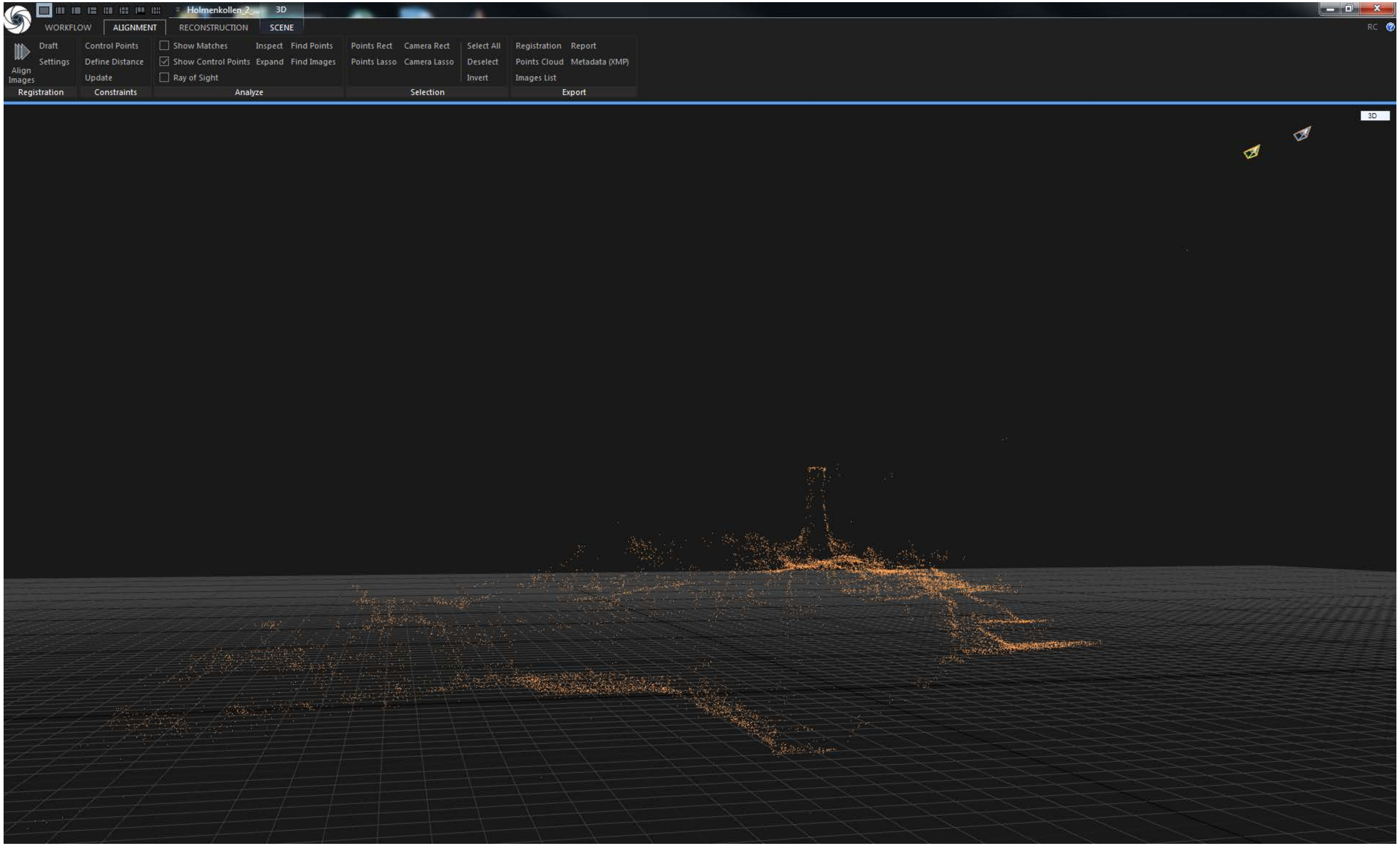

Example

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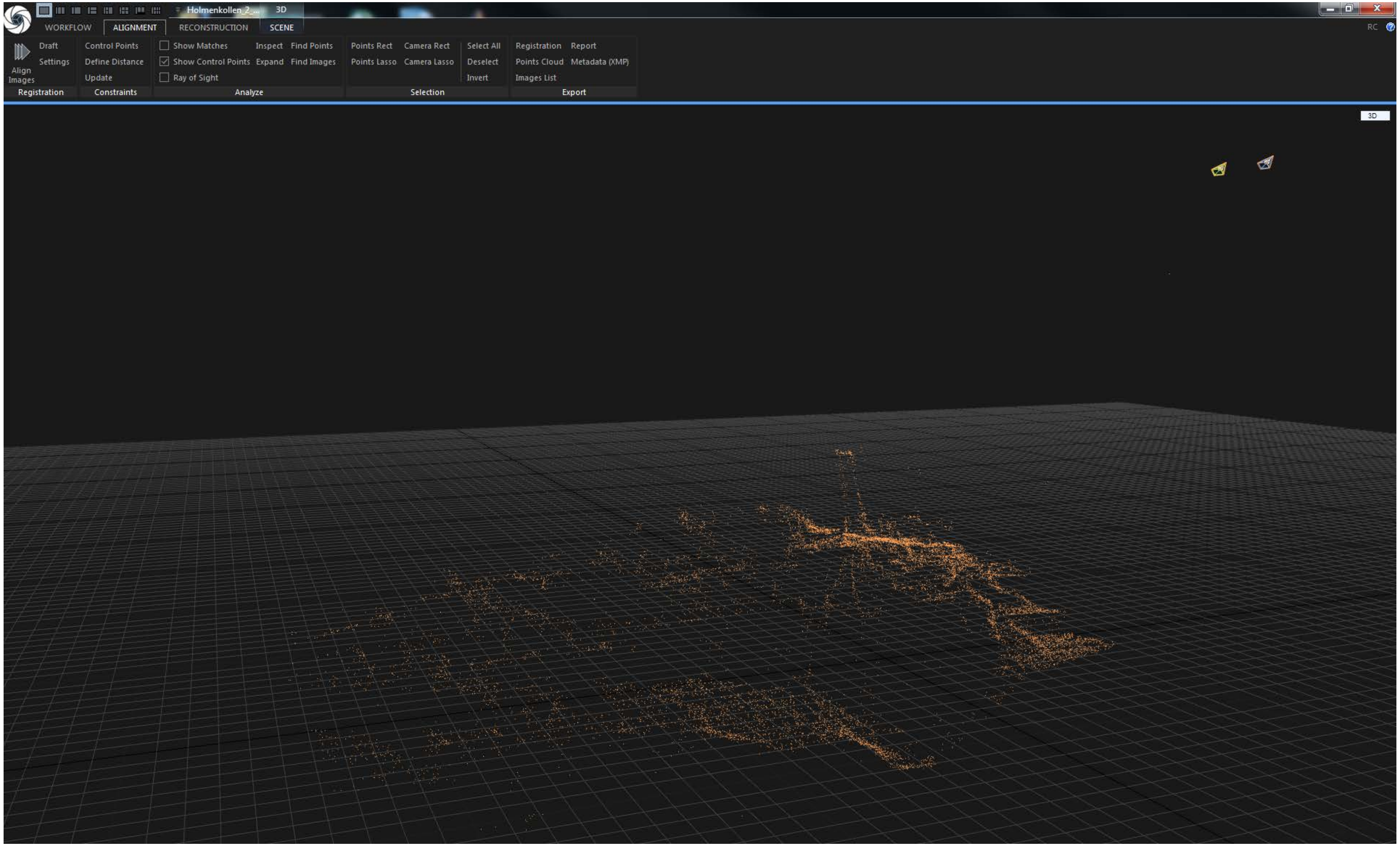
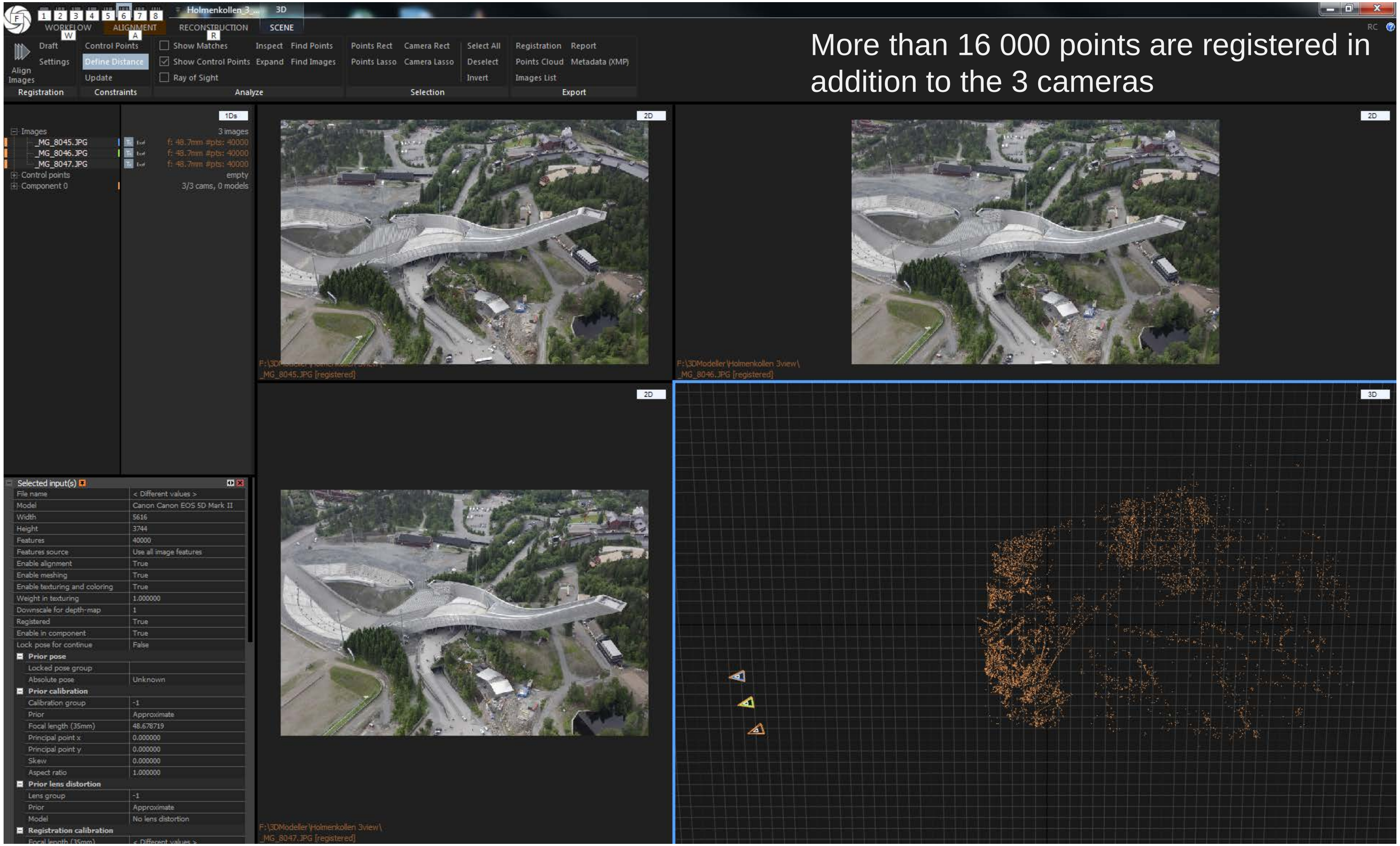


Example

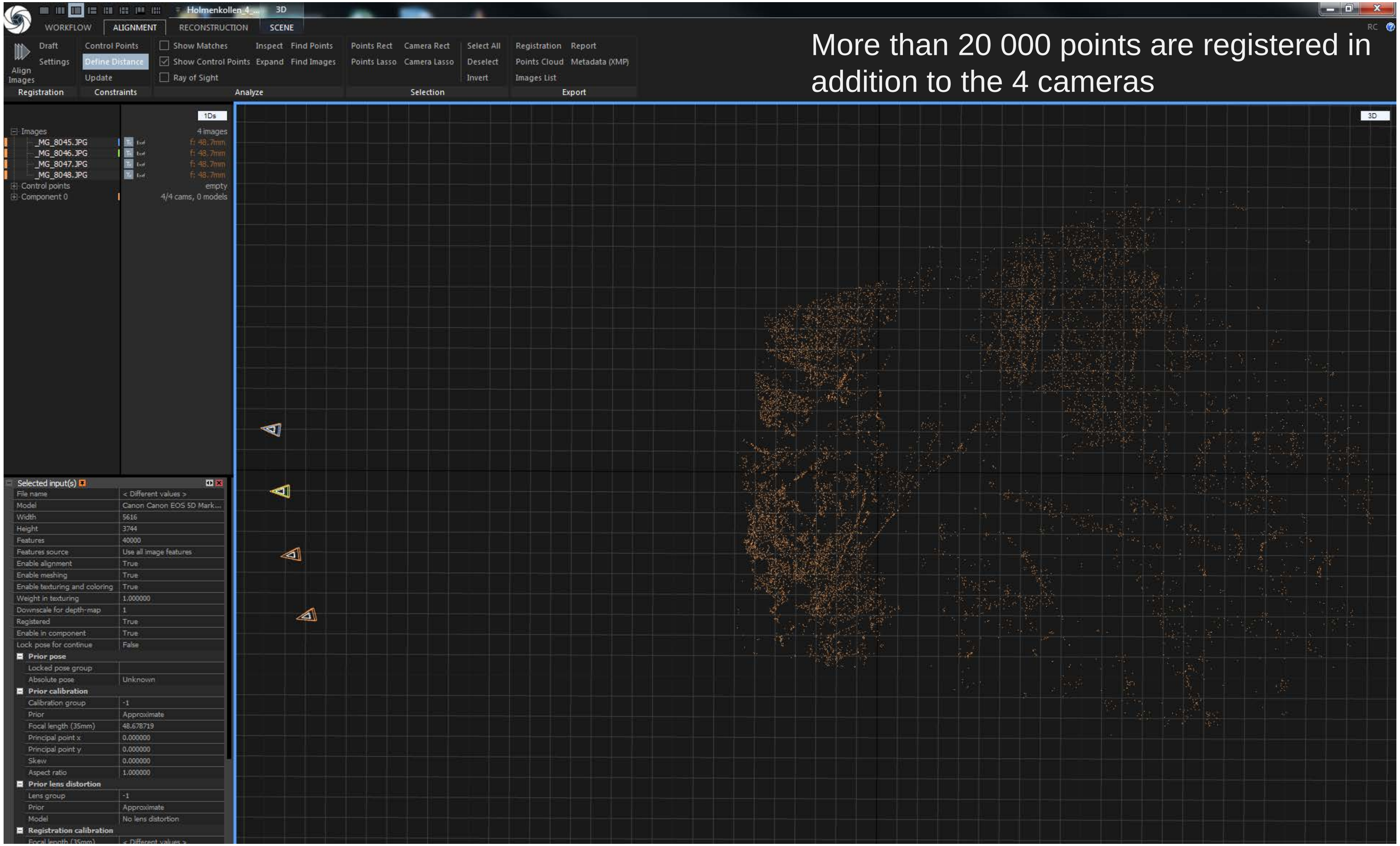
Holmenkollen 3-view SfM

More than 16 000 points are registered in addition to the 3 cameras



Example

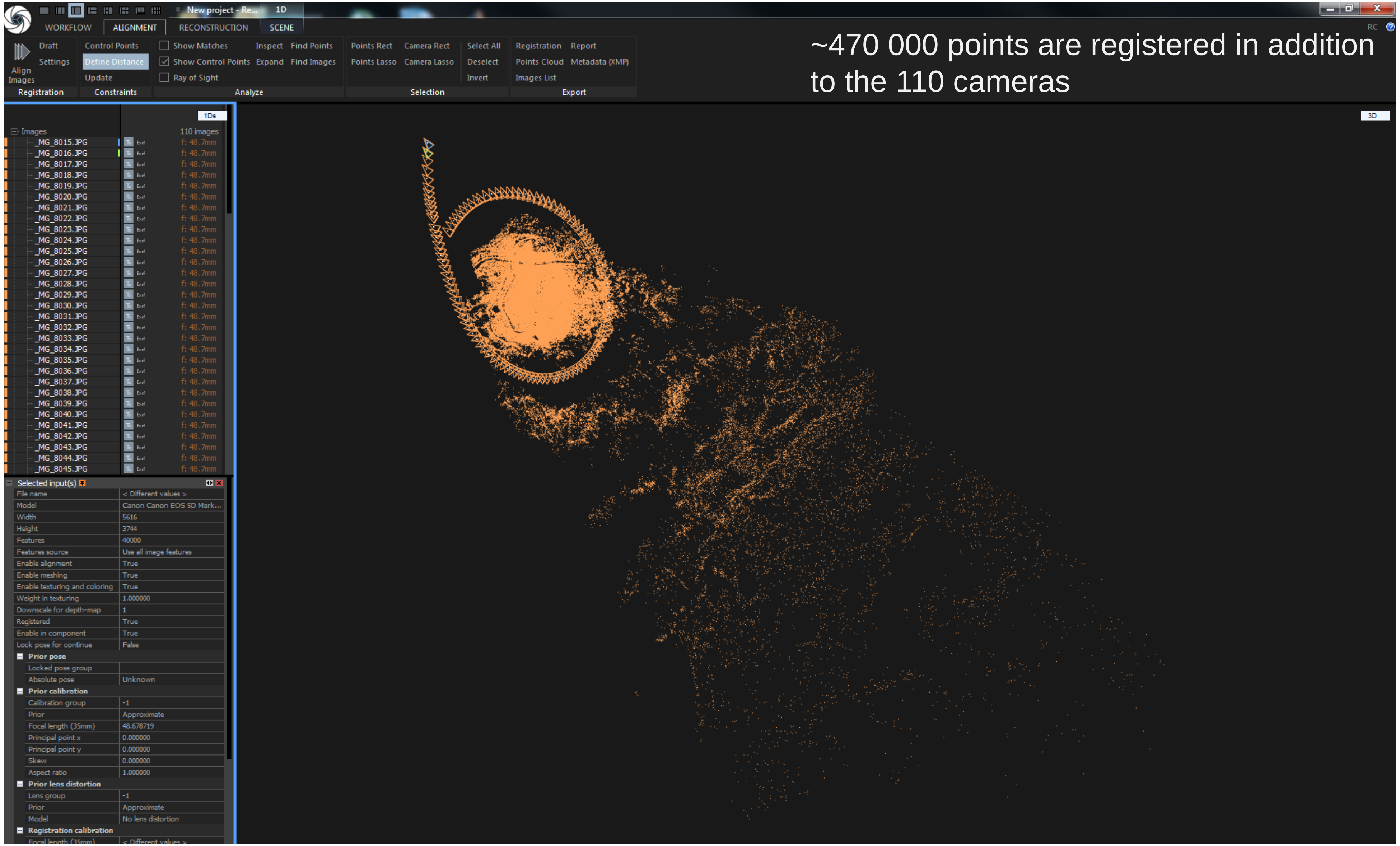
Holmenkollen 4-view SfM



More than 20 000 points are registered in addition to the 4 cameras

Example

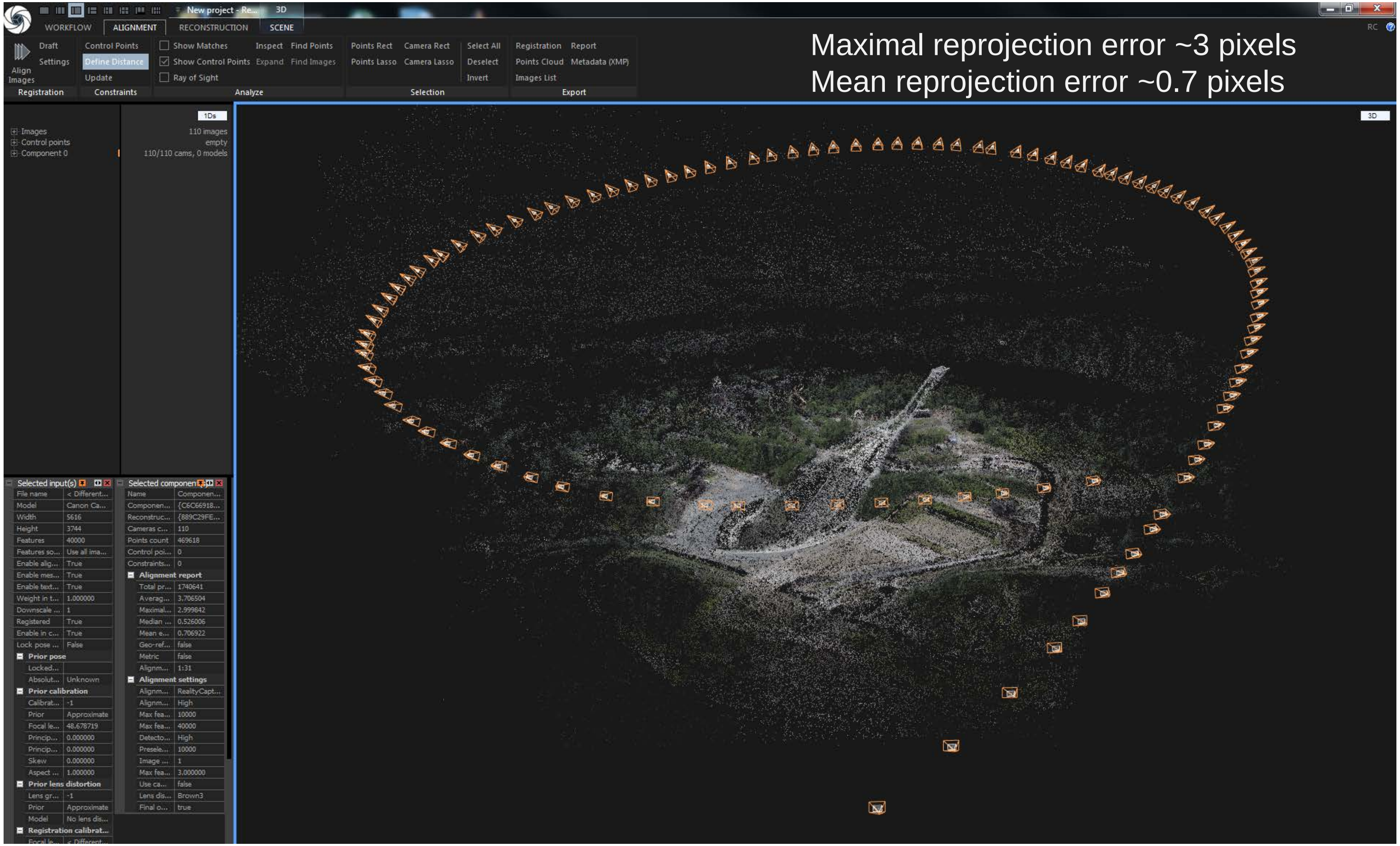
Holmenkollen 110-view SfM



~470 000 points are registered in addition to the 110 cameras

Example

Holmenkollen 110-view SfM

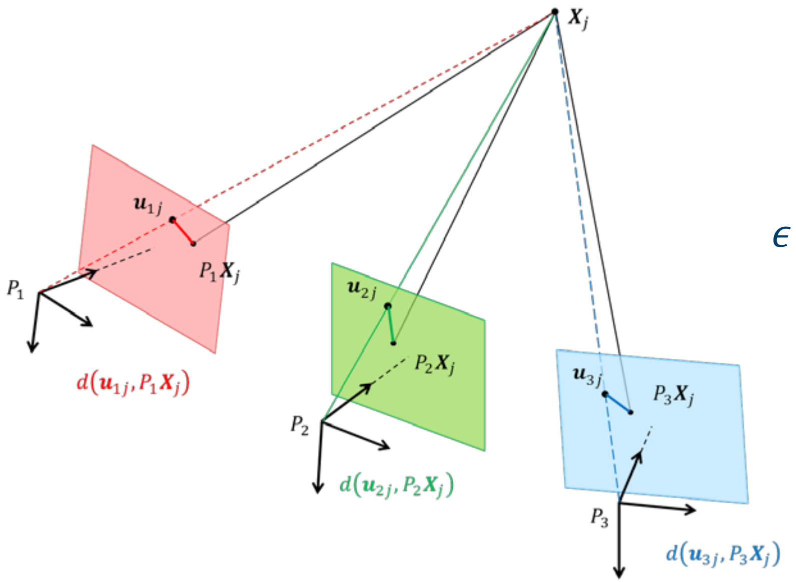


Summary

- Structure from motion
  - Sequential SfM
  - Bundle adjustment

- Additional reading:
  - Szeliski: 7.3-7.5

- Optional reading:
  - Snavely N. Seitz S. M., Szeliski R., *Modeling the World from Internet Photo Collections*, 2007
  - S. Agarwal et al, *Building Rome in a Day,* 2011
  - J. Heinly et al, *Reconstructing the World in Six Days*, 2015



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