STAT410 Project

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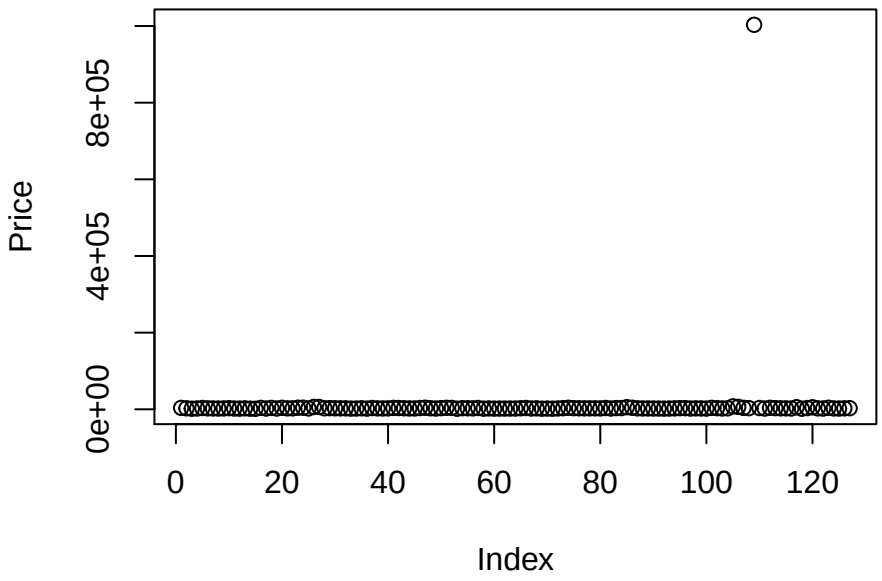
```
#data cleaning, Tokyo hostels as population
suppressPackageStartupMessages(library(tidyverse))
hostel=read.csv("japanHostel.csv")

Tokyo=filter(hostel,City=="Tokyo")
head(Tokyo)
```

```
##   X                hostel.name  City price.from                Distance
## 1 3      &And Hostel Akihabara Tokyo       3600  7.8km from city centre
## 2 4           &And Hostel Ueno Tokyo       2600  8.7km from city centre
## 3 5 &And Hostel-Asakusa North- Tokyo       1500 10.5km from city centre
## 4 6     1night1980hostel Tokyo Tokyo       2100  9.4km from city centre
## 5 7        328 Hostel & Lounge Tokyo       3300 16.5km from city centre
## 6 9   3Q House - Asakusa Smile Tokyo       2500 10.2km from city centre
##   summary.score rating.band atmosphere cleanliness facilities location.y
## 1           8.7    Fabulous        8.0         7.0        9.0        8.0
## 2           7.4   Very Good        8.0         7.5        7.5        7.5
## 3           9.4      Superb        9.5         9.5        9.0        9.0
## 4           7.0   Very Good        5.5         8.0        6.0        6.0
## 5           9.3      Superb        8.7         9.7        9.3        9.1
## 6            NA        <NA>         NA          NA         NA         NA
##   security staff valueformoney      lon      lat
## 1     10.0  10.0           9.0 139.7775 35.69745
## 2      7.0   8.0           6.5 139.7837 35.71272
## 3      9.5  10.0           9.5 139.7984 35.72790
## 4      8.5   8.5           6.5 139.7869 35.72438
## 5      9.3   9.7           8.9 139.7455 35.54804
## 6       NA    NA            NA       NA       NA
```

```
#Check NA
c( table(is.na(Tokyo$price.from)), table(is.na(Tokyo$Distance)) )
```

```
## FALSE FALSE
##   127   127
```

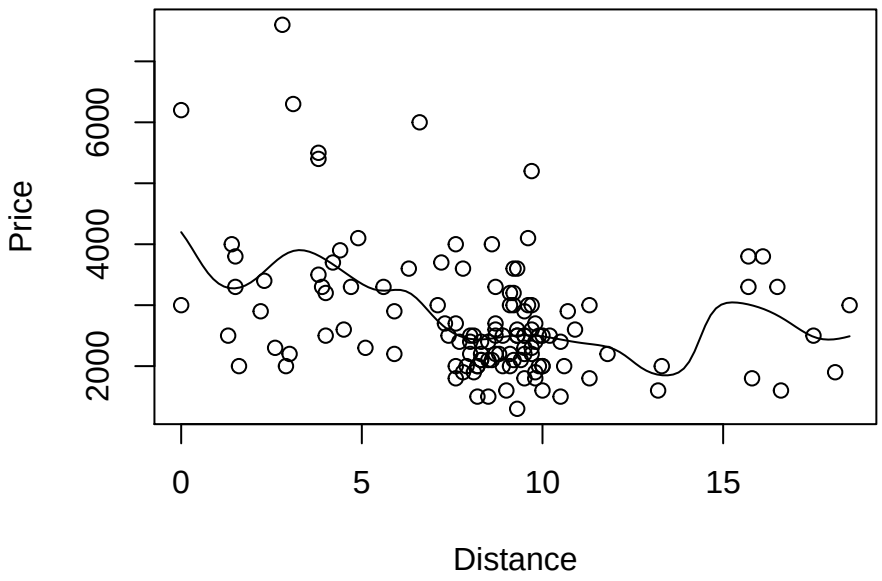
```
Tokyo$Distance=as.numeric(gsub("km.*","",Tokyo$Distance))
Tokyo=Tokyo%>%rename(Price=price.from)%>%select(Price, Distance)

#From the plot we see there is a obvious outlier
plot(Tokyo$Price, ylab="Price")
```




```
#Remove obs 109 where the price is considered as an outlier
Tokyo=Tokyo[-109,]

#Check the extreme of distance
c( max(Tokyo$Distance), min(Tokyo$Distance) )
```

```
## [1] 18.5  0.0
```

```
#There is sort of relationship b/w dist and price, but not quite, fit lm model
plot(Tokyo$Distance, Tokyo$Price, xlab="Distance", ylab="Price")
lines(ksmooth(Tokyo$Distance, Tokyo$Price, kernel="normal", bandwidth=2))
```

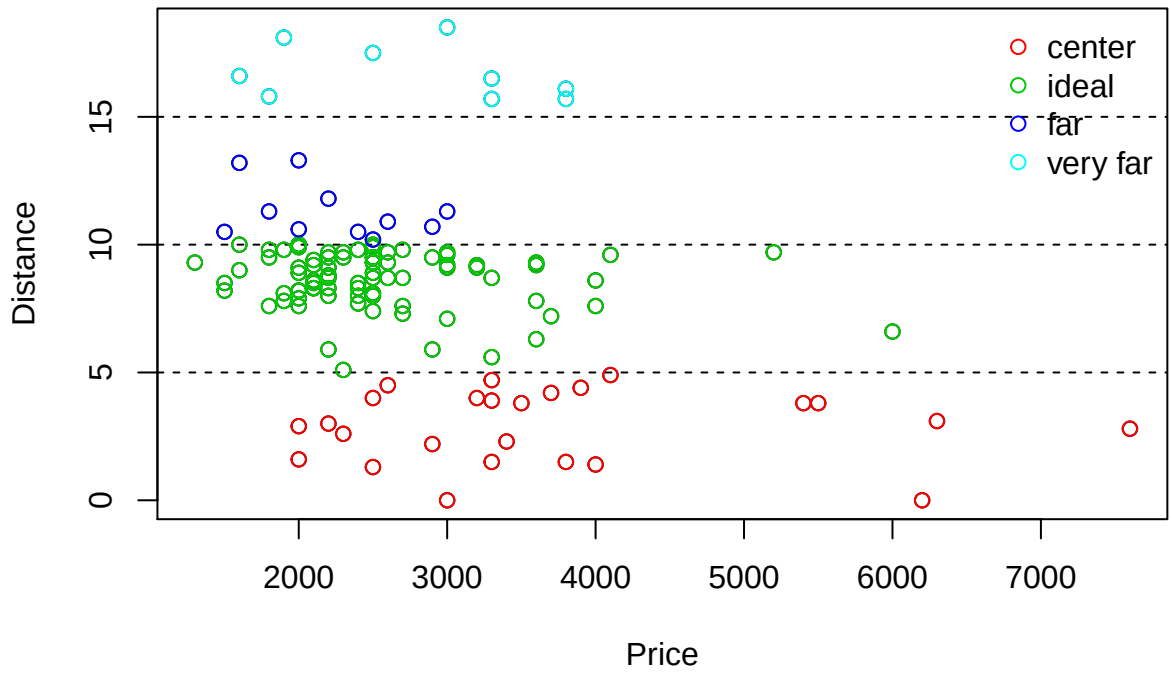



```
dis <- Tokyo$Distance
pric <- Tokyo$Price
lm1 <- lm(dis~pric)
summary(lm1)
```

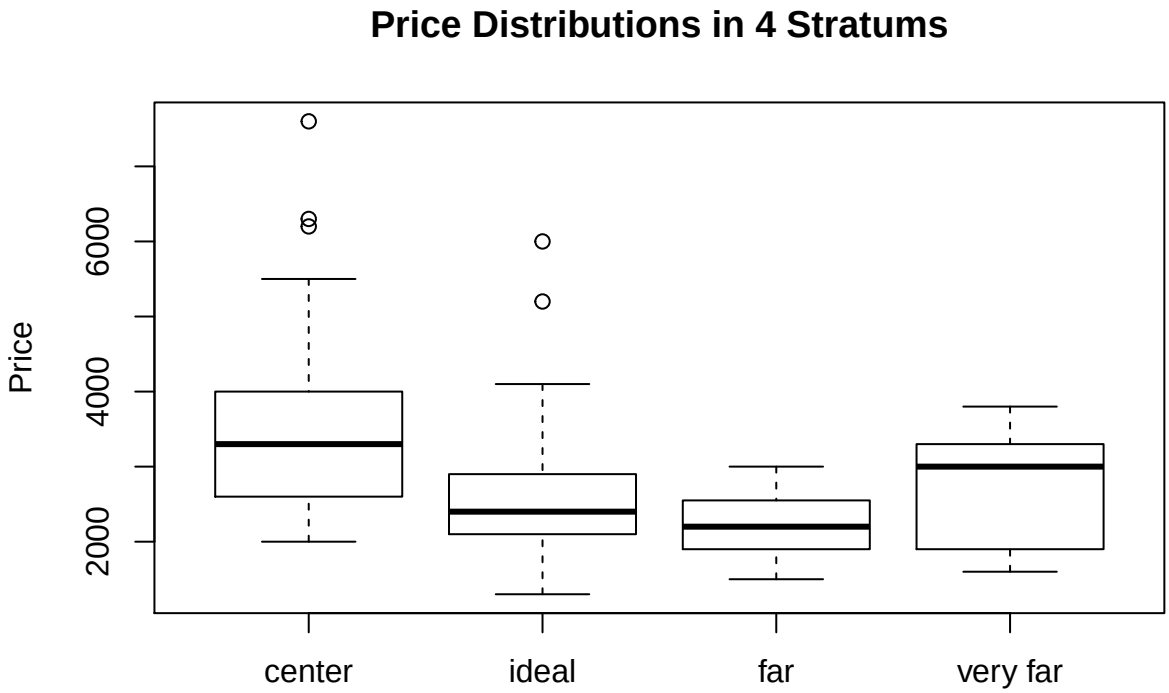
```
##
## Call:
## lm(formula = dis ~ pric)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.0170 -1.2773  0.0771  1.3136 10.4830
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 11.6851653  0.8644199  13.518  < 2e-16 ***
## pric        -0.0012227  0.0002922  -4.185 5.37e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.408 on 124 degrees of freedom
## Multiple R-squared:  0.1237, Adjusted R-squared:  0.1167
## F-statistic: 17.51 on 1 and 124 DF,  p-value: 5.365e-05
```

```
#Reverse the axis to visualize stratums
plot(y=Tokyo$Distance, x=Tokyo$Price, xlab="Price", ylab="Distance")
abline(h=c(5,10,15), lty=2)
points(x=Tokyo$Price[Tokyo[,"Distance"]<=5], y=Tokyo$Distance[Tokyo[,"Distance"]<=5], col=2)
points(x=Tokyo$Price[Tokyo[,"Distance"]>5&Tokyo[,"Distance"]<=10],
       y=Tokyo$Distance[Tokyo[,"Distance"]>5&Tokyo[,"Distance"]<=10], col=3)
```

```
points(x=Tokyo$Price[Tokyo[,"Distance"]>10&Tokyo[,"Distance"]<=15],
       y=Tokyo$Distance[Tokyo[,"Distance"]>10&Tokyo[,"Distance"]<=15], col=4)
points(x=Tokyo$Price[Tokyo[,"Distance"]>15], y=Tokyo$Distance[Tokyo[,"Distance"]>15], col=5)
legend("topright", col=c(2,3,4,5), pch=c(1,1,1,1),
       c("center", "ideal", "far", "very far"), bty="n")
```




```
y1=Tokyo$Price[Tokyo[,"Distance"]<=5]; y2=Tokyo$Price[Tokyo[,"Distance"]>5&Tokyo[,"Distance"]<=10]
y3=Tokyo$Price[Tokyo[,"Distance"]>10&Tokyo[,"Distance"]<=15]; y4=Tokyo$Price[Tokyo[,"Distance"]>15]

#Check the distributions of all 4 stratums
boxplot(y1,y2,y3,y4, names=c("center", "ideal", "far", "very far"), ylab="Price",
        main="Price Distributions in 4 Stratums")
```




```
Tokyo=Tokyo%>%mutate(strata = cut(Distance,breaks=c(-0.1,5,10,15,20)))
Tokyo=mutate(Tokyo, strata = recode_factor(strata,
                                           "(-0.1,5]" = "center",
                                           "(5,10]" = "ideal",
                                           "(10,15]" = "far",
                                           "(15,20]" = "very far"))
head(Tokyo)
```

```
##   Price Distance   strata
## 1  3600      7.8    ideal
## 2  2600      8.7    ideal
## 3  1500     10.5      far
## 4  2100      9.4    ideal
## 5  3300     16.5 very far
## 6  2500     10.2      far
```

```
mu=mean(Tokyo$Price)
```

SRS

```
y=Tokyo$Price
N=length(y)
n=50
ybar=NULL;sv=NULL
```

```r
for(i in 1:10000){
  s=sample(1:N,n,r=F) #without replacement
  ybar[i]=mean(y[s])
  sv[i]=var(y[s])
}

low=ybar-qt(0.975,d=n-1)*sqrt((1-n/N)*sv/n)
up=ybar+qt(0.975,d=n-1)*sqrt((1-n/N)*sv/n)

# compute the coverage probability
cover_prob=sum( (low<=mu)*(up>=mu) )/10000
cover_prob
```

```
## [1] 0.9374
```

SRS with Replacement

```r
ybar_rT=NA
for(i in 1:10000){
  s_r=sample(1:N,n,r=T) #with replacement
  ybar_rT[i]=mean(y[s_r])
}
```

```r
#MSE(SRS vs SRS_replacement)
c(SRS=mean((ybar-mu)^2), SRS_replace=mean((ybar_rT-mu)^2))
```

```
##         SRS SRS_replace
##    13059.21    21373.41
```

Regression Estimation

```r
n=50
reg_mu=NA;reg_var=NA
mu_x=mean(Tokyo$Distance)

for(i in 1:10000){
  s=sample(1:N,n,r=F) #without replacement
  xi=Tokyo$Distance[s]
  yi=Tokyo$Price[s]
  x_bar=mean(xi)
  y_bar=mean(yi)
  b=sum((xi-x_bar)*(yi-y_bar))/sum((xi-x_bar)^2)
  a=y_bar-(b*x_bar)
  reg_mu[i]=a+(b*mu_x)
  reg_var[i]=( (N-n)/(N*n*(n-2)) )*sum((yi-a-b*xi)^2)
}

low_reg=reg_mu-qt(0.975,d=n-1)*sqrt(reg_var)
up_reg=reg_mu+qt(0.975,d=n-1)*sqrt(reg_var)

# compute the coverage probability
```

```r
cover_prob_reg=sum( (low_reg<=mu)*(up_reg>=mu) )/10000
cover_prob_reg
```

```
## [1] 0.9293
```

Unequal Probability Random Sampling with Replacement

```r
#Assign prob using the 1/x form because we expect closer dist has higher prob
x=Tokyo$Distance

for(i in seq_along(x)){
  if(x[i]==0){
    #Set this as second min, because 1/0=Inf
    x[i]=min(x[x!=min(x)])
  }
}

p=(1/x)/sum(1/x)

ybar_unprobb=NA; mu_HH=NA; mu_HT=NA; mu_GUPE=NA
pi=1-(1-p)^n
for(i in 1:10000){
  ss=sample(1:N, n, r=T, prob=p)
  mu_HH[i]=mean(y[ss]/p[ss])/N
  ssu=unique(ss)
  mu_HT[i]=sum(y[ssu]/pi[ssu])/N
  mu_GUPE[i]=sum(y[ss]/pi[ss])/sum(1/pi[ss])
}

#Mean
c( True_mean=mu, HH_estimate=mean(mu_HH), HT_estimate=mean(mu_HT),
   GUPE_estimate=mean(mu_GUPE))
```

```
##     True_mean   HH_estimate   HT_estimate GUPE_estimate
##      2769.841      2769.836      2769.302      2838.021
```

```r
#MSE
c( SRS=mean((ybar-mu)^2), HH=mean((mu_HH-mu)^2), HT=mean((mu_HT-mu)^2),
   GUPE=mean((mu_GUPE-mu)^2))
```

```
##      SRS       HH       HT     GUPE
## 13059.21 60680.79 83226.80 27444.25
```

Stratified Random Sampling, Proportional Allocation

```r
c( Popn_mean=mean(Tokyo$Price), Popn_var=var(Tokyo$Price) )
```

```
##   Popn_mean    Popn_var
##    2769.841 1088363.175
```

```r
(strata_table=table(Tokyo$strata))
```

```
##
##   center    ideal      far very far
##       25       81       11        9
```

```
N1=as.numeric(strata_table[1]); N2=as.numeric(strata_table[2])
N3=as.numeric(strata_table[3]); N4=as.numeric(strata_table[4])
n=50

#Proportional
(prop_table=round((table(Tokyo$strata)/nrow(Tokyo)*n), 0))
```

```
##
##   center    ideal      far very far
##       10       32        4        4
```

```
n_p1=as.numeric(prop_table[1]); n_p2=as.numeric(prop_table[2])
n_p3=as.numeric(prop_table[3]); n_p4=as.numeric(prop_table[4])


#Stratified Random Sampling(Prop Allocation)
ybar_strat=NA;var_strat=NA

for(i in 1:10000){
  s1=sample(1:N1, n_p1)
  s2=sample(1:N2, n_p2)
  s3=sample(1:N3, n_p3)
  s4=sample(1:N4, n_p4)
  # sum(N_h*ybar_h)/N
  ybar_strat[i]=( (mean(y1[s1])*N1)+(mean(y2[s2])*N2)+(mean(y3[s3])*N3)+(mean(y4[s4])*N4) )/N
  var_strat[i]=( ((N1/N)^2)*( ((N1-n_p1)/N1)*(var(y1[s1]))/n_p1)+
                 ((N2/N)^2)*( ((N2-n_p2)/N2)*(var(y2[s2]))/n_p2)+
                 ((N3/N)^2)*( ((N3-n_p3)/N3)*(var(y3[s3]))/n_p3)+
                 ((N4/N)^2)*( ((N4-n_p4)/N4)*(var(y4[s4]))/n_p4))
}

low_strat=ybar_strat-qt(0.975,d=n-1)*sqrt(var_strat)
up_strat=ybar_strat+qt(0.975,d=n-1)*sqrt(var_strat)

# compute the coverage probability
cover_prob_strat=sum( (low_strat<=mu)*(up_strat>=mu) )/10000
cover_prob_strat
```

```
## [1] 0.9385
```

```
#Mean comparison
c( True_Popn=mu, SRS=mean(ybar), Stratified_PA=mean(ybar_strat) )
```

```
##     True_Popn           SRS Stratified_PA
##      2769.841      2769.091      2769.042
```

```
#MSE comparison
c( SRS=mean((ybar-mu)^2), Stratified_PA=mean((ybar_strat-mu)^2) )
```

```
##           SRS Stratified_PA
##      13059.21      10755.63
```

Stratified Random Sampling, Optimum Allocation

```
#Optimum
#var of within each stratum
sigma_sq=tapply(Tokyo$Price, Tokyo$strata, var)
sigma_sq_1=as.numeric(sigma_sq[1]); sigma_sq_2=as.numeric(sigma_sq[2])
sigma_sq_3=as.numeric(sigma_sq[3]); sigma_sq_4=as.numeric(sigma_sq[4])
sigma1=sqrt(sigma_sq_1); sigma2=sqrt(sigma_sq_2)
sigma3=sqrt(sigma_sq_3); sigma4=sqrt(sigma_sq_4)
#standard deviation within each stratum
#c(sigma1,sigma2,sigma3,sigma4)

n_o1=round( (n*N1*sigma1)/sum((N1*sigma1)+(N2*sigma2)+(N3*sigma3)+(N4*sigma4)), 0)
n_o2=round( (n*N2*sigma2)/sum((N1*sigma1)+(N2*sigma2)+(N3*sigma3)+(N4*sigma4)), 0)
n_o3=round( (n*N3*sigma3)/sum((N1*sigma1)+(N2*sigma2)+(N3*sigma3)+(N4*sigma4)), 0)
n_o4=round( (n*N4*sigma4)/sum((N1*sigma1)+(N2*sigma2)+(N3*sigma3)+(N4*sigma4)), 0)

ybar_strat_Opti=NA;var_strat_opti=NA
#Stratified Random Sampling(Optimum Allocation)
for(i in 1:10000){
    ss1=sample(1:N1, n_o1)
    ss2=sample(1:N2, n_o2)
    ss3=sample(1:N3, n_o3)
    ss4=sample(1:N4, n_o4)
    ybar_strat_Opti[i]=
      ( (mean(y1[ss1])*N1)+(mean(y2[ss2])*N2)+(mean(y3[ss3])*N3)+(mean(y4[ss4])*N4) )/N

    var_strat_opti[i]=( ((N1/N)^2)*( ((N1-n_o1)/N1)*(var(y1[ss1]))/n_o1)+
                  ((N2/N)^2)*( ((N2-n_o2)/N2)*(var(y2[ss2]))/n_o2)+
                  ((N3/N)^2)*( ((N3-n_o3)/N3)*(var(y3[ss3]))/n_o3)+
                  ((N4/N)^2)*( ((N4-n_o4)/N4)*(var(y4[ss4]))/n_o4))
}

low_strat_opti=ybar_strat_Opti-qt(0.975,d=n-1)*sqrt(var_strat_opti)
up_strat_opti=ybar_strat_Opti+qt(0.975,d=n-1)*sqrt(var_strat_opti)

# compute the coverage probability
cover_opti_strat=sum( (low_strat_opti<=mu)*(up_strat_opti>=mu) )/10000
cover_opti_strat
```

```
## [1] 0.941
```

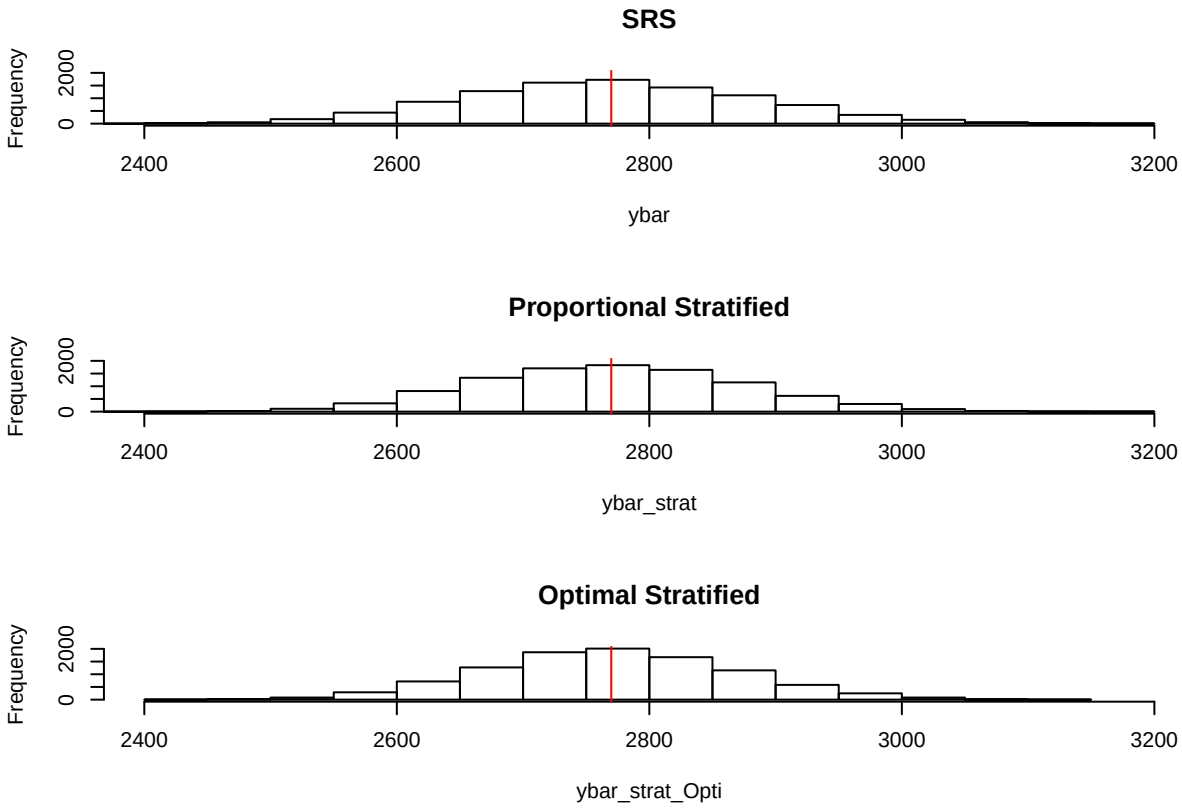
```
#Mean comparison
c( True_Popn=mu, SRS=mean(ybar), Stratified_PA=mean(ybar_strat),
   Stratified_Opti = mean(ybar_strat_Opti))
```

```
##       True_Popn             SRS   Stratified_PA Stratified_Opti
##        2769.841        2769.091        2769.042        2769.828
```

```
#MSE comparison
c( SRS=mean((ybar-mu)^2), Stratified_Prop=mean((ybar_strat-mu)^2),
   Stratified_Opti=mean((ybar_strat_Opti-mu)^2) )
```

```
##             SRS Stratified_Prop Stratified_Opti
##       13059.210       10755.629        9446.465
```

```r
par(mfrow=c(3,1))
hist(ybar, xlim=c(2400,3200), ylim=c(0,2000), main="SRS"); abline(v=mu, col=2)
hist(ybar_strat, xlim=c(2400,3200), ylim=c(0,2000),
     main="Proportional Stratified"); abline(v=mu, col=2)
hist(ybar_strat_Opti, xlim=c(2400,3200), ylim=c(0,2000),
     main="Optimal Stratified"); abline(v=mu, col=2)
```



Stratified Sampling With Unequal Probability

```r
# Within each stratum, the distances have unequal prob to be selected
x1=Tokyo$Distance[Tokyo[,"Distance"]<=5]
x2=Tokyo$Distance[Tokyo[,"Distance"]>5 & Tokyo[,"Distance"]<=10]
x3=Tokyo$Distance[Tokyo[,"Distance"]>10 & Tokyo[,"Distance"]<=15]
x4=Tokyo$Distance[Tokyo[,"Distance"]>15]

for(i in seq_along(x1)){
  if(x1[i]==0){
    #Set this as second min, because 1/0=Inf
    x1[i]=min(x1[x1!=min(x1)])
  }
}

x1_prob=(1/x1)/sum(1/x1); x2_prob=(1/x2)/sum(1/x2)
x3_prob=(1/x3)/sum(1/x3); x4_prob=(1/x4)/sum(1/x4)
```

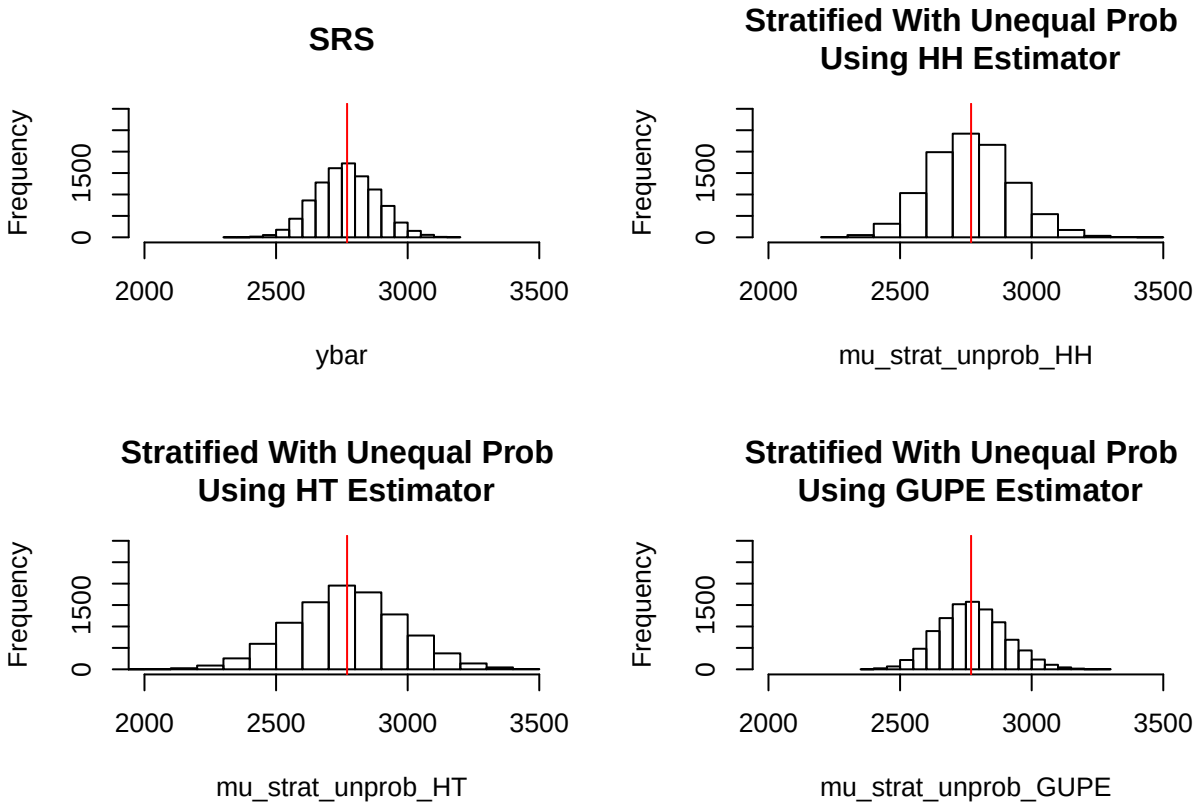
```r
#HH, HT estimator
pi_1=1-((1-x1_prob)^n_o1); pi_2=1-((1-x2_prob)^n_o2);
pi_3=1-((1-x3_prob)^n_o3); pi_4=1-((1-x4_prob)^n_o4)

mu_strat_unprob_HH=NA; mu_strat_unprob_HT=NA
mu_strat_unprob_GUPE=NA
for(i in 1:10000){
  ss_1=sample(1:N1, n_o1, r=T, prob=x1_prob)
  ss_2=sample(1:N2, n_o2, r=T, prob=x2_prob)
  ss_3=sample(1:N3, n_o3, r=T, prob=x3_prob)
  ss_4=sample(1:N4, n_o4, r=T, prob=x4_prob)
  #tau_hat/N where tau_hat=sum(tau_hat_h)
  mu_strat_unprob_HH[i]=( mean(y1[ss_1]/x1_prob[ss_1])+mean(y2[ss_2]/x2_prob[ss_2])+
    mean(y3[ss_3]/x3_prob[ss_3])+mean(y4[ss_4]/x4_prob[ss_4]) )/N

  su1=unique(ss_1); su2=unique(ss_2); su3=unique(ss_3); su4=unique(ss_4)
  mu_strat_unprob_HT[i]=( sum(y1[su1]/pi_1[su1])+sum(y2[su2]/pi_2[su2])+
    sum(y3[su3]/pi_3[su3])+sum(y4[su4]/pi_4[su4]) )/N

  #tau_hat_h = mu_hat_g * Nh
  mu_strat_unprob_GUPE[i]=( (sum(y1[ss_1]/pi_1[ss_1])/sum(1/pi_1[ss_1])*N1)+
    (sum(y2[ss_2]/pi_2[ss_2])/sum(1/pi_2[ss_2])*N2)+
    (sum(y3[ss_3]/pi_3[ss_3])/sum(1/pi_3[ss_3])*N3)+
    (sum(y4[ss_4]/pi_4[ss_4])/sum(1/pi_4[ss_4])*N4) )/N

}

#Mean comparison
c( True_Popn=mu, SRS=mean(ybar), HH=mean(mu_strat_unprob_HH),
   HT=mean(mu_strat_unprob_HT), GUPE=mean(mu_strat_unprob_GUPE))
```

```
## True_Popn       SRS        HH        HT      GUPE
##  2769.841  2769.091  2769.753  2770.644  2772.695
```

```r
#MSE Comparison
c( SRS=mean((ybar-mu)^2), HH=mean((mu_strat_unprob_HH-mu)^2),
   HT=mean((mu_strat_unprob_HT-mu)^2), GUPE=mean((mu_strat_unprob_GUPE-mu)^2) )
```

```
##      SRS       HH       HT     GUPE
## 13059.21 24500.02 42766.13 15606.86
```

```r
par(mfrow=c(2,2))
hist(ybar, main="SRS", xlim=c(2000,3500), ylim=c(0,3000)); abline(v=mu, col=2)
hist(mu_strat_unprob_HH, main="Stratified With Unequal Prob \n Using HH Estimator",
     xlim=c(2000,3500), ylim=c(0,3000)); abline(v=mu, col=2)
hist(mu_strat_unprob_HT, main="Stratified With Unequal Prob \n Using HT Estimator",
     xlim=c(2000,3500), ylim=c(0,3000)); abline(v=mu, col=2)
hist(mu_strat_unprob_GUPE, main="Stratified With Unequal Prob \n Using GUPE Estimator",
     xlim=c(2000,3500), ylim=c(0,3000)); abline(v=mu, col=2)
```



Results

```
#Mean comparison
c( True_Popn=mu, SRS=mean(ybar), reg_est = mean(reg_mu),
   Stratified_PA=mean(ybar_strat),
   Stratified_Opti = mean(ybar_strat_Opti),
   Stratified_GUPE=mean(mu_strat_unprob_GUPE))
```

```
##       True_Popn             SRS         reg_est   Stratified_PA
##        2769.841        2769.091        2762.894        2769.042
## Stratified_Opti Stratified_GUPE
##        2769.828        2772.695
```

```
#MSE Comparison
c( SRS=mean((ybar-mu)^2), reg_est=mean((reg_mu-mu)^2),
   Stratified_Prop=mean((ybar_strat-mu)^2),
   Stratified_Opti=mean((ybar_strat_Opti-mu)^2),
   Stratified_GUPE=mean((mu_strat_unprob_GUPE-mu)^2) )
```

```
##             SRS         reg_est Stratified_Prop Stratified_Opti
##       13059.210       12161.453       10755.629        9446.465
## Stratified_GUPE
##       15606.855
```

```
#Approximate Bias
c( SRS=mean(ybar)-mu, reg_est=mean(reg_mu)-mu,
```

```
   Stratified_Prop=mean(ybar_strat)-mu,
   Stratified_Opti=mean(ybar_strat_Opti)-mu,
   Stratified_GUPE=mean(mu_strat_unprob_GUPE)-mu )
```

```
##             SRS         reg_est Stratified_Prop Stratified_Opti
##     -0.75006984     -6.94758049     -0.79913690     -0.01289895
## Stratified_GUPE
##      2.85377939
```

```
#Coverage of CI comparison
c( SRS=cover_prob, reg_est=cover_prob_reg,
   Stratified_Prop=cover_prob_strat,
   Stratified_Opti=cover_opti_strat )
```

```
##             SRS         reg_est Stratified_Prop Stratified_Opti
##          0.9374          0.9293          0.9385          0.9410
```

```
par(mfrow=c(3,2))
hist(ybar, xlim=c(2400,3200), ylim=c(0, 2000), main="SRS"); abline(v=mu, col=2)
hist(reg_mu, xlim=c(2400,3200), ylim=c(0, 2000),
     main="Regression Estimate"); abline(v=mu, col=2)
hist(ybar_strat, xlim=c(2400,3200), ylim=c(0, 2000),
     main="Proportional Stratified"); abline(v=mu, col=2)
hist(ybar_strat_Opti, xlim=c(2400,3200), ylim=c(0, 2000),
     main="Optimal Stratified"); abline(v=mu, col=2)
hist(mu_strat_unprob_GUPE, xlim=c(2400,3200), ylim=c(0, 2000),
     main="Unequal Probability Stratified \n with GUPE Estimator"); abline(v=mu, col=2)
```