STAT430 Assignment3

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Question 5.8

```
chemical_data <- data.frame(Temperature=factor(c(rep(150,6), rep(160,6),
                                                  rep(170,6))),
                            Pressure=factor(rep(c(200,200,215,215,230,230),3)),
                            Chem=c(90.4,90.2,90.7,90.6,90.2,90.4,90.1,90.3,
                                   90.5,90.6,89.9,90.1,90.5,90.7,90.8,90.9,
                                   90.4,90.1))
```

(a) First denote a effects model:

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk}$$

where τ_i is the effect of ith level of the row factor, β_j is the effect of jth level of the column factor, $(\tau\beta)_{ij}$ is the effect of the interaction between τ_i and β_j, and ϵ_{ijk} is a random error component. And k is the numbers of replicate in each factor.

Now, we are interested in testing hypotheses about the equality of row treatment effects by stating:

$H_0 : \tau_1 = \tau_2 = \tau_3$

$H_a :$ at least one τ_i is different than the other, where i $\in \{1, 2, 3\}$

Similarly, we can set our hypotheses for column treatment effects and interaction effects between row and column treatments.

```r
anova(lm(Chem~Temperature*Pressure,data=chemical_data))

## Analysis of Variance Table
##
## Response: Chem
##                      Df  Sum Sq Mean Sq F value    Pr(>F)
## Temperature           2 0.30111 0.15056  8.4687 0.0085392 **
## Pressure              2 0.76778 0.38389 21.5937 0.0003673 ***
## Temperature:Pressure  4 0.06889 0.01722  0.9687 0.4700058
## Residuals             9 0.16000 0.01778
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Based on the result, we can see that the main effects of temperature and pressure are significant since we have both p-values less than 0.05. However, we see that the interaction effect is not significant as we obtained a p-value of $0.47 > 0.05$. Since We know both treatments are significant, we can further perform a LSD test to see which level of treatments are different.

```r
library(agricolae)
# Fisher LSD: Compare the temperature regardless of pressure.
LSD.test(chemical_data$Chem, chemical_data$Temperature, DFerror=9,
         MSerror=0.01778, console=T)

##
## Study: chemical_data$Chem ~ chemical_data$Temperature
##
## LSD t Test for chemical_data$Chem
##
## Mean Square Error:  0.01778
##
```

```
## chemical_data$Temperature,  means and individual ( 95 %) CI
##
##     chemical_data.Chem       std r      LCL      UCL  Min  Max
## 150           90.41667 0.2041241 6 90.29352 90.53981 90.2 90.7
## 160           90.25000 0.2664583 6 90.12686 90.37314 89.9 90.6
## 170           90.56667 0.2943920 6 90.44352 90.68981 90.1 90.9
##
## Alpha: 0.05 ; DF Error: 9
## Critical Value of t: 2.262157
##
## least Significant Difference: 0.1741518
##
## Treatments with the same letter are not significantly different.
##
##     chemical_data$Chem groups
## 170           90.56667      a
## 150           90.41667     ab
## 160           90.25000      b
```

```
# Fisher LSD: Compare the pressure regardless of temperautre.
LSD.test(chemical_data$Chem, chemical_data$Pressure, DFerror=9,
         MSerror=0.01778, console=T)
```

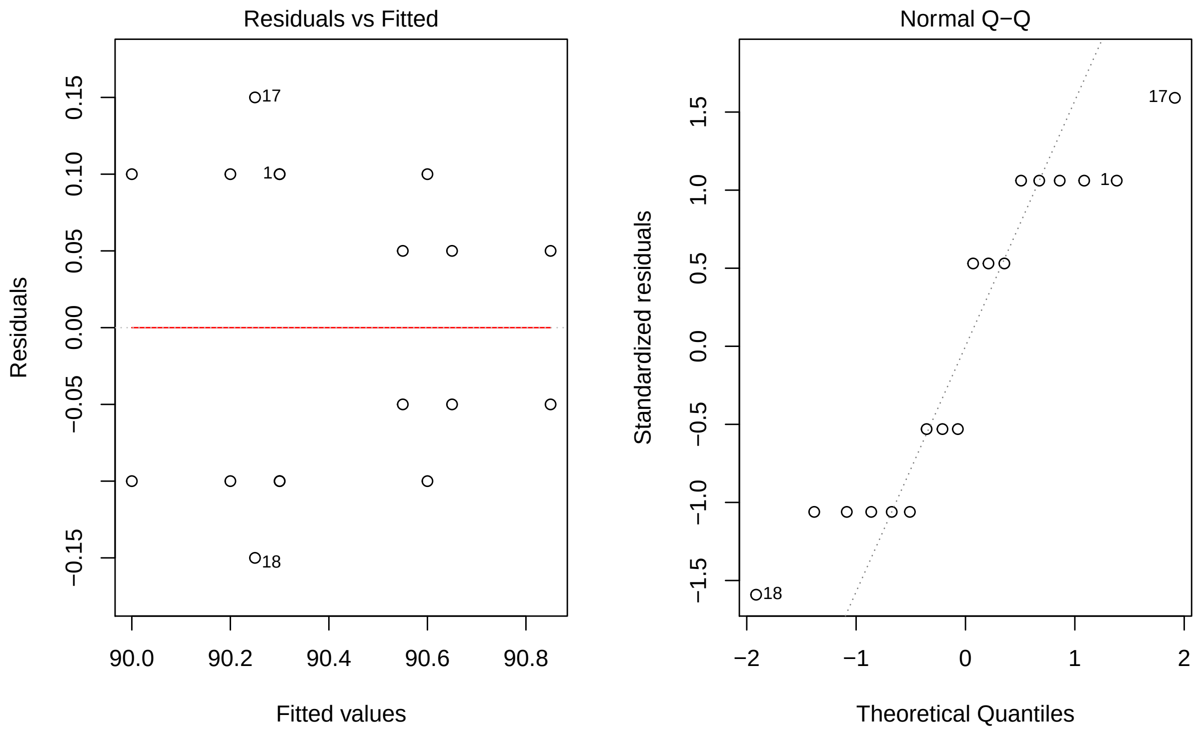
```
##
## Study: chemical_data$Chem ~ chemical_data$Pressure
##
## LSD t Test for chemical_data$Chem
##
## Mean Square Error:  0.01778
##
## chemical_data$Pressure,  means and individual ( 95 %) CI
```

```
##
##     chemical_data.Chem       std r      LCL      UCL  Min  Max
## 200           90.36667 0.2160247 6 90.24352 90.48981 90.1 90.7
## 215           90.68333 0.1471960 6 90.56019 90.80648 90.5 90.9
## 230           90.18333 0.1940790 6 90.06019 90.30648 89.9 90.4
##
## Alpha: 0.05 ; DF Error: 9
## Critical Value of t: 2.262157
##
## least Significant Difference: 0.1741518
##
## Treatments with the same letter are not significantly different.
##
##     chemical_data$Chem groups
## 215           90.68333      a
## 200           90.36667      b
## 230           90.18333      c
```

From the LSD test results, we can see that temperature level of 160 °C is different than temperature level of 170 °C. On the other hand, we observe that all the pressure levels are significantly different than the other.

(b)

```
fit <- aov(lm(Chem~Temperature*Pressure,data=chemical_data))
par(mfrow=c(1,2))
plot(fit, 1); plot(fit, 2)
```



There is nothing unusual with the residual vs fitted value plot as since we can observe from the above plot that there is a constant band throughout the graph. The normal QQplot shows that many points are off the theoretical line. However, in order to check whether the normality assumption fails, we can perform a Shapiro-Wilk's test.

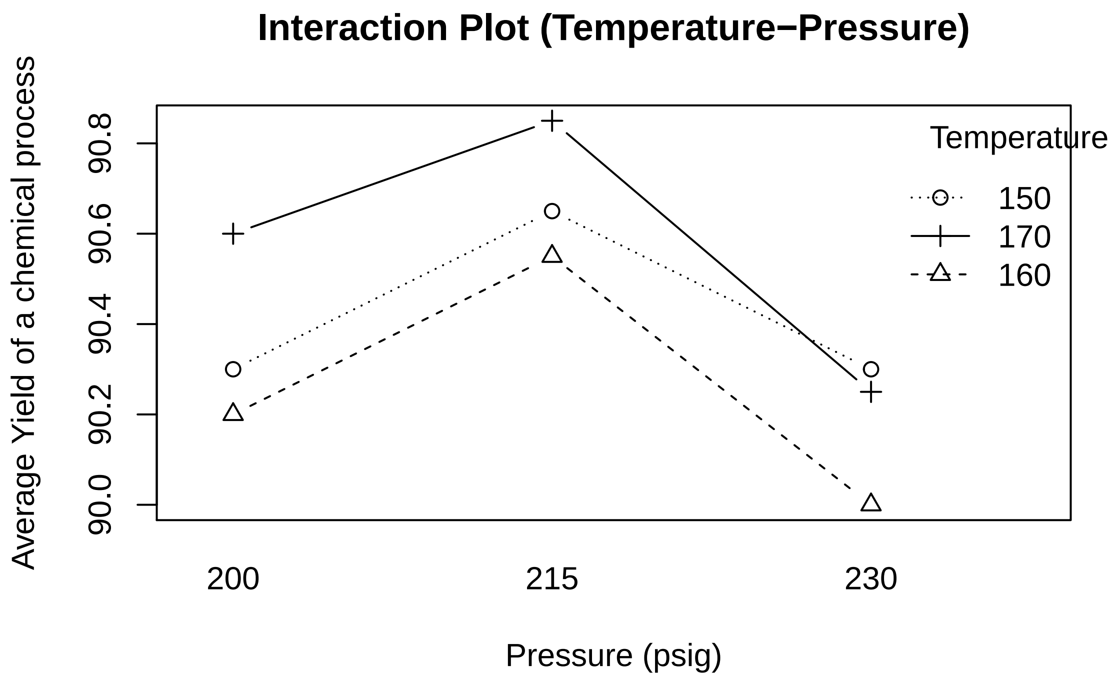
```
# H0: Data follows a normal distribution
shapiro.test(chemical_data$Chem)

##
##  Shapiro-Wilk normality test
##
## data:  chemical_data$Chem
## W = 0.97363, p-value = 0.8625
```

Based on the result, we can conclude that the assumption of normality is satisified.

(c)

```r
with(chemical_data, interaction.plot(x.factor=Pressure,
            trace.factor=Temperature, response=Chem, fun=mean,
            type="b", legend=T, pch=c(1,2,3), xlab="Pressure (psig)",
            ylab="Average Yield of a chemical process",
            main="Interaction Plot (Temperature-Pressure)"))
```



From the interaction graph, we can see that under the condition temperature of 170 °C and pressure of 215 psig, we will have the highest chemical yield.

Question 5.14

We can construct a effects model same as the previous question. For drill speed effect:

$H_0 : \tau_1 = \tau_2$

$H_a : \tau_1$ is different than τ_2

Similarly, we can set our hypotheses for column treatment effects and interaction effects between row and column treatments.

```
mechanic_data <- data.frame(Drill.Speed=factor(c(rep(125,8), rep(200,8))),
                            Feed.Rate=factor(rep(c(0.015,0.015,0.030,0.030,
                                                   0.045,0.045,0.060,0.060),2)),
                            Thrust.Force=c(2.70,2.78,2.45,2.49,2.60,2.72,2.75,
                                           2.86,2.83,2.86,2.85,2.80,2.86,2.87,
                                           2.94,2.88))
anova(lm(Thrust.Force~Drill.Speed*Feed.Rate, data=mechanic_data))

## Analysis of Variance Table
##
## Response: Thrust.Force
##                       Df   Sum Sq  Mean Sq F value    Pr(>F)
## Drill.Speed            1 0.148225 0.148225 57.0096 6.605e-05 ***
## Feed.Rate              3 0.092500 0.030833 11.8590  0.002582 **
## Drill.Speed:Feed.Rate  3 0.041875 0.013958  5.3686  0.025567 *
## Residuals              8 0.020800 0.002600
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Based on the result, we can conclude that both treatments drill speed, feed rate and their interaction are significant since we have p-values less than 0.05. Therefore, we can perform LSD tests on both treatments and interaction.

```r
# Fisher LSD: Compare the drill speed regardless of feed rate.
LSD.test(mechanic_data$Thrust.Force, mechanic_data$Drill.Speed, DFerror=8,
         MSerror=0.0026, console=T)

##
## Study: mechanic_data$Thrust.Force ~ mechanic_data$Drill.Speed
##
## LSD t Test for mechanic_data$Thrust.Force
##
## Mean Square Error:  0.0026
##
## mechanic_data$Drill.Speed,  means and individual ( 95 %) CI
##
##     mechanic_data.Thrust.Force        std r      LCL      UCL  Min  Max
## 125                    2.66875 0.14327172 8 2.627178 2.710322 2.45 2.86
## 200                    2.86125 0.04051014 8 2.819678 2.902822 2.80 2.94
##
## Alpha: 0.05 ; DF Error: 8
## Critical Value of t: 2.306004
##
## least Significant Difference: 0.0587918
##
## Treatments with the same letter are not significantly different.
##
##     mechanic_data$Thrust.Force groups
## 200                    2.86125      a
## 125                    2.66875      b

# Fisher LSD: Compare the feed rate regardless of drill speed.
LSD.test(mechanic_data$Thrust.Force, mechanic_data$Feed.Rate, DFerror=8,
         MSerror=0.0026, console=T)
```

```
##
## Study: mechanic_data$Thrust.Force ~ mechanic_data$Feed.Rate
##
## LSD t Test for mechanic_data$Thrust.Force
##
## Mean Square Error:  0.0026
##
## mechanic_data$Feed.Rate,  means and individual ( 95 %) CI
##
##       mechanic_data.Thrust.Force        std r      LCL      UCL  Min  Max
## 0.015                     2.7925 0.06994045 4 2.733708 2.851292 2.70 2.86
## 0.03                      2.6475 0.20661962 4 2.588708 2.706292 2.45 2.85
## 0.045                     2.7625 0.12816006 4 2.703708 2.821292 2.60 2.87
## 0.06                      2.8575 0.07932003 4 2.798708 2.916292 2.75 2.94
##
## Alpha: 0.05 ; DF Error: 8
## Critical Value of t: 2.306004
##
## least Significant Difference: 0.08314416
##
## Treatments with the same letter are not significantly different.
##
##       mechanic_data$Thrust.Force groups
## 0.06                      2.8575      a
## 0.015                     2.7925     ab
## 0.045                     2.7625      b
## 0.03                      2.6475      c
```

From the LSD test results, we can see that the two drill speed levels are significant different. And for feed rate, we observe that feed rate of 0.06 and rate of 0.045, rate of 0.06

and 0.03, and rate of 0.045 and 0.03 are significant different.

```
mechanic_data$Interact <- with(mechanic_data,
                               interaction(Drill.Speed, Feed.Rate))
int_fit <- aov(Thrust.Force~Interact, data=mechanic_data)
LSD.test(int_fit, "Interact", group=FALSE, console = TRUE)
```

```
##
## Study: int_fit ~ "Interact"
##
## LSD t Test for Thrust.Force
##
## Mean Square Error:  0.0026
##
## Interact,  means and individual ( 95 %) CI
##
##           Thrust.Force         std r      LCL      UCL  Min  Max
## 125.0.015        2.740 0.056568542 2 2.656856 2.823144 2.70 2.78
## 125.0.03         2.470 0.028284271 2 2.386856 2.553144 2.45 2.49
## 125.0.045        2.660 0.084852814 2 2.576856 2.743144 2.60 2.72
## 125.0.06         2.805 0.077781746 2 2.721856 2.888144 2.75 2.86
## 200.0.015        2.845 0.021213203 2 2.761856 2.928144 2.83 2.86
## 200.0.03         2.825 0.035355339 2 2.741856 2.908144 2.80 2.85
## 200.0.045        2.865 0.007071068 2 2.781856 2.948144 2.86 2.87
## 200.0.06         2.910 0.042426407 2 2.826856 2.993144 2.88 2.94
##
## Alpha: 0.05 ; DF Error: 8
## Critical Value of t: 2.306004
##
## Comparison between treatments means
##
```

```
##                          difference pvalue signif.        LCL          UCL
## 125.0.015 - 125.0.03          0.270 0.0007     ***  0.1524164  0.387583601
## 125.0.015 - 125.0.045         0.080 0.1553         -0.0375836  0.197583601
## 125.0.015 - 125.0.06         -0.065 0.2382         -0.1825836  0.052583601
## 125.0.015 - 200.0.015        -0.105 0.0734       . -0.2225836  0.012583601
## 125.0.015 - 200.0.03         -0.085 0.1341         -0.2025836  0.032583601
## 125.0.015 - 200.0.045        -0.125 0.0398       * -0.2425836 -0.007416399
## 125.0.015 - 200.0.06         -0.170 0.0103       * -0.2875836 -0.052416399
## 125.0.03 - 125.0.045         -0.190 0.0058      ** -0.3075836 -0.072416399
## 125.0.03 - 125.0.06          -0.335 0.0002     *** -0.4525836 -0.217416399
## 125.0.03 - 200.0.015         -0.375 0.0001     *** -0.4925836 -0.257416399
## 125.0.03 - 200.0.03          -0.355 0.0001     *** -0.4725836 -0.237416399
## 125.0.03 - 200.0.045         -0.395 0.0001     *** -0.5125836 -0.277416399
## 125.0.03 - 200.0.06          -0.440 0.0000     *** -0.5575836 -0.322416399
## 125.0.045 - 125.0.06         -0.145 0.0217       * -0.2625836 -0.027416399
## 125.0.045 - 200.0.015        -0.185 0.0067      ** -0.3025836 -0.067416399
## 125.0.045 - 200.0.03         -0.165 0.0120       * -0.2825836 -0.047416399
## 125.0.045 - 200.0.045        -0.205 0.0038      ** -0.3225836 -0.087416399
## 125.0.045 - 200.0.06         -0.250 0.0012      ** -0.3675836 -0.132416399
## 125.0.06 - 200.0.015         -0.040 0.4554         -0.1575836  0.077583601
## 125.0.06 - 200.0.03          -0.020 0.7051         -0.1375836  0.097583601
## 125.0.06 - 200.0.045         -0.060 0.2731         -0.1775836  0.057583601
## 125.0.06 - 200.0.06          -0.105 0.0734       . -0.2225836  0.012583601
## 200.0.015 - 200.0.03          0.020 0.7051         -0.0975836  0.137583601
## 200.0.015 - 200.0.045        -0.020 0.7051         -0.1375836  0.097583601
## 200.0.015 - 200.0.06         -0.065 0.2382         -0.1825836  0.052583601
## 200.0.03 - 200.0.045         -0.040 0.4554         -0.1575836  0.077583601
## 200.0.03 - 200.0.06          -0.085 0.1341         -0.2025836  0.032583601
## 200.0.045 - 200.0.06         -0.045 0.4032         -0.1625836  0.072583601
```

And by performing a LSD test on the treatments interaction, we can observe some significant difference between interactions by looking at the p-values result above. More importantly, we can see from the result that when the drill speed is 200 and feed rate is 0.06, the **thrust force is maximized with an average value of 2.91**. This concludes that we shall operate the process under this condition.

Question 5.24

We can construct a effects model same as the previous question. For cycle time effect:

$H_0 : \tau_1 = \tau_2 = \tau_3$

H_a : at least one τ_i is different than the other, where i $\in \{1, 2, 3\}$

Similarly, we can set our hypotheses for the other two treatment effects, two-way interaction between each of the two treatments and three-way interaction.

```r
#Three-factor factorial design
cloth_data <- data.frame(Cycle.Time=factor(c(rep(40,18),rep(50,18),rep(60,18))),
                         Temperature=factor(rep(c(300,350),each=9,times=3)),
                         Operator=factor(rep(c(1,2,3),each=3,times=6)),
                         Cloth=c(23,24,25,27,28,26,31,32,29,24,23,28,38,36,35,
                                 34,36,39,36,35,36,34,38,39,33,34,35,37,39,35,
                                 34,38,36,34,36,31,28,24,27,35,35,34,26,27,25,
                                 26,29,25,36,37,34,28,26,24))
anova(lm(Cloth~Cycle.Time*Temperature*Operator, data=cloth_data))
```

```
## Analysis of Variance Table
##
## Response: Cloth
##                                 Df Sum Sq Mean Sq F value    Pr(>F)
## Cycle.Time                       2 436.00 218.000 66.5085 8.141e-13 ***
## Temperature                      1  50.07  50.074 15.2768 0.0003934 ***
## Operator                         2 261.33 130.667 39.8644 7.439e-10 ***
## Cycle.Time:Temperature           2  78.81  39.407 12.0226 0.0001002 ***
## Cycle.Time:Operator              4 355.67  88.917 27.1271 1.982e-10 ***
## Temperature:Operator             2  11.26   5.630  1.7175 0.1938948
## Cycle.Time:Temperature:Operator  4  46.19  11.546  3.5226 0.0158701 *
## Residuals                       36 118.00   3.278
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Based on the result, we can see that only the interaction between temperature and operator is not significant since we have a p-value of $0.19 > 0.05$. Other than that, every other terms are significant. Therefore, we can perform LSD tests on three treatments.

```
# Fisher LSD: Compare the cycle time regardless of temperature, operator.
LSD.test(cloth_data$Cloth, cloth_data$Cycle.Time, DFerror=36,
         MSerror=3.278, console=T)

##
## Study: cloth_data$Cloth ~ cloth_data$Cycle.Time
##
## LSD t Test for cloth_data$Cloth
##
## Mean Square Error:  3.278
##
## cloth_data$Cycle.Time,  means and individual ( 95 %) CI
##
##    cloth_data.Cloth      std  r      LCL      UCL Min Max
## 40         29.88889 5.378485 18 29.02341 30.75437  23  39
## 50         35.55556 2.120550 18 34.69008 36.42103  31  39
## 60         29.22222 4.557548 18 28.35674 30.08770  24  37
##
## Alpha: 0.05 ; DF Error: 36
## Critical Value of t: 2.028094
##
## least Significant Difference: 1.223971
##
## Treatments with the same letter are not significantly different.
##
```

```
##    cloth_data$Cloth groups
## 50         35.55556      a
## 40         29.88889      b
## 60         29.22222      b
```

```r
# Fisher LSD: Compare the temperature regardless of cycle time, operator.
LSD.test(cloth_data$Cloth, cloth_data$Temperature, DFerror=36,
         MSerror=3.278, console=T)
```

```
##
## Study: cloth_data$Cloth ~ cloth_data$Temperature
##
## LSD t Test for cloth_data$Cloth
##
## Mean Square Error:  3.278
##
## cloth_data$Temperature,  means and individual ( 95 %) CI
##
##     cloth_data.Cloth      std  r      LCL      UCL Min Max
## 300         30.59259 4.837826 27 29.88593 31.29925  23  39
## 350         32.51852 5.184076 27 31.81186 33.22518  23  39
##
## Alpha: 0.05 ; DF Error: 36
## Critical Value of t: 2.028094
##
## least Significant Difference: 0.9993685
##
## Treatments with the same letter are not significantly different.
##
##     cloth_data$Cloth groups
## 350         32.51852      a
```

```
## 300         30.59259      b

# Fisher LSD: Compare the operator regardless of cycle time, temperature.
LSD.test(cloth_data$Cloth, cloth_data$Operator, DFerror=36,
         MSerror=3.278, console=T)

##
## Study: cloth_data$Cloth ~ cloth_data$Operator
##
## LSD t Test for cloth_data$Cloth
##
## Mean Square Error:  3.278
##
## cloth_data$Operator,  means and individual ( 95 %) CI
##
##   cloth_data.Cloth      std  r      LCL      UCL Min Max
## 1         29.11111 5.571871 18 28.24563 29.97659  23  39
## 2         34.44444 3.776432 18 33.57897 35.30992  26  39
## 3         31.11111 4.377602 18 30.24563 31.97659  24  39
##
## Alpha: 0.05 ; DF Error: 36
## Critical Value of t: 2.028094
##
## least Significant Difference: 1.223971
##
## Treatments with the same letter are not significantly different.
##
##   cloth_data$Cloth groups
## 2         34.44444      a
## 3         31.11111      b
## 1         29.11111      c
```

From the LSD test results, we can first see that cycle time level of 50 is significant different than levels of 40 and 60. Then, two temperature levels are significant different than each other. Moreover, three operator levels are also significant different than each other.

Then, we can also perform LSD tests on the two significant interactions.

```
cloth_data$CycleTemp <- with(cloth_data,
                               interaction(Cycle.Time, Temperature))
CycleTemp_fit <- aov(Cloth~CycleTemp, data=cloth_data)
LSD.test(CycleTemp_fit, "CycleTemp", group=FALSE, console = TRUE)

##
## Study: CycleTemp_fit ~ "CycleTemp"
##
## LSD t Test for Cloth
##
## Mean Square Error:  16.50926
##
## CycleTemp,  means and individual ( 95 %) CI
##
##           Cloth      std r      LCL      UCL Min Max
## 40.300 27.22222 3.073181 9 24.49905 29.94540  23  32
## 40.350 32.55556 6.002314 9 29.83238 35.27873  23  39
## 50.300 35.55556 1.943651 9 32.83238 38.27873  33  39
## 50.350 35.55556 2.403701 9 32.83238 38.27873  31  39
## 60.300 29.00000 4.415880 9 26.27682 31.72318  24  35
## 60.350 29.44444 4.952553 9 26.72127 32.16762  24  37
##
## Alpha: 0.05 ; DF Error: 48
## Critical Value of t: 2.010635
##
## Comparison between treatments means
```

```
##
##                  difference pvalue signif.         LCL        UCL
## 40.300 - 40.350 -5.3333333 0.0076      **  -9.1844859 -1.4821808
## 40.300 - 50.300 -8.3333333 0.0001     *** -12.1844859 -4.4821808
## 40.300 - 50.350 -8.3333333 0.0001     *** -12.1844859 -4.4821808
## 40.300 - 60.300 -1.7777778 0.3580          -5.6289303  2.0733748
## 40.300 - 60.350 -2.2222222 0.2517          -6.0733748  1.6289303
## 40.350 - 50.300 -3.0000000 0.1239          -6.8511526  0.8511526
## 40.350 - 50.350 -3.0000000 0.1239          -6.8511526  0.8511526
## 40.350 - 60.300  3.5555556 0.0696       .  -0.2955970  7.4067081
## 40.350 - 60.350  3.1111111 0.1109          -0.7400414  6.9622637
## 50.300 - 50.350  0.0000000 1.0000          -3.8511526  3.8511526
## 50.300 - 60.300  6.5555556 0.0013      **   2.7044030 10.4067081
## 50.300 - 60.350  6.1111111 0.0025      **   2.2599586  9.9622637
## 50.350 - 60.300  6.5555556 0.0013      **   2.7044030 10.4067081
## 50.350 - 60.350  6.1111111 0.0025      **   2.2599586  9.9622637
## 60.300 - 60.350 -0.4444444 0.8175          -4.2955970  3.4067081
```

From this result, we can observe from the result that when the cycle time is 50 and temperatures of 300 and 350, the **cloth scores are maximized with both average values of 35.556**.

```
cloth_data$CycleOperator <- with(cloth_data,
                               interaction(Cycle.Time, Operator))
CycleOperator_fit <- aov(Cloth~CycleOperator, data=cloth_data)
LSD.test(CycleOperator_fit, "CycleOperator", group=FALSE, console = TRUE)
```

```
##
## Study: CycleOperator_fit ~ "CycleOperator"
##
## LSD t Test for Cloth
```

```
##
## Mean Square Error:  6.762963
##
## CycleOperator,  means and individual ( 95 %) CI
##
##         Cloth      std r      LCL      UCL Min Max
## 40.1 24.50000 1.870829 6 22.36167 26.63833  23  28
## 40.2 31.66667 5.240865 6 29.52834 33.80500  26  38
## 40.3 33.50000 3.619392 6 31.36167 35.63833  29  39
## 50.1 36.33333 1.505545 6 34.19500 38.47166  35  39
## 50.2 36.50000 2.167948 6 34.36167 38.63833  34  39
## 50.3 33.83333 1.722401 6 31.69500 35.97166  31  36
## 60.1 26.50000 1.870829 6 24.36167 28.63833  24  29
## 60.2 35.16667 1.169045 6 33.02834 37.30500  34  37
## 60.3 26.00000 1.414214 6 23.86167 28.13833  24  28
##
## Alpha: 0.05 ; DF Error: 45
## Critical Value of t: 2.014103
##
## Comparison between treatments means
##
##              difference pvalue signif.         LCL         UCL
## 40.1 - 40.2  -7.1666667 0.0000     *** -10.1907213 -4.14261200
## 40.1 - 40.3  -9.0000000 0.0000     *** -12.0240547 -5.97594534
## 40.1 - 50.1 -11.8333333 0.0000     *** -14.8573880 -8.80927867
## 40.1 - 50.2 -12.0000000 0.0000     *** -15.0240547 -8.97594534
## 40.1 - 50.3  -9.3333333 0.0000     *** -12.3573880 -6.30927867
## 40.1 - 60.1  -2.0000000 0.1895          -5.0240547  1.02405466
## 40.1 - 60.2 -10.6666667 0.0000     *** -13.6907213 -7.64261200
## 40.1 - 60.3  -1.5000000 0.3231          -4.5240547  1.52405466
```

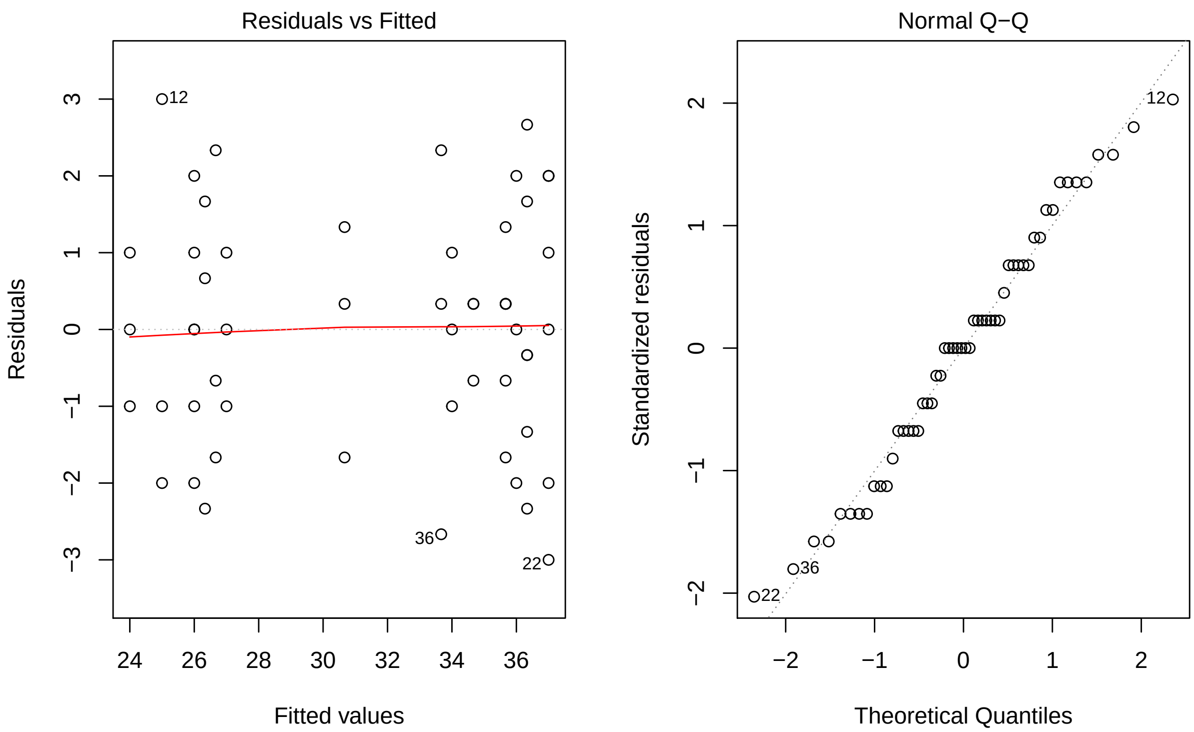
```
## 40.2 - 40.3  -1.8333333 0.2284         -4.8573880  1.19072133
## 40.2 - 50.1  -4.6666667 0.0033      **  -7.6907213 -1.64261200
## 40.2 - 50.2  -4.8333333 0.0024      **  -7.8573880 -1.80927867
## 40.2 - 50.3  -2.1666667 0.1559          -5.1907213  0.85738800
## 40.2 - 60.1   5.1666667 0.0013      **   2.1426120  8.19072133
## 40.2 - 60.2  -3.5000000 0.0243       *  -6.5240547 -0.47594534
## 40.2 - 60.3   5.6666667 0.0005     ***   2.6426120  8.69072133
## 40.3 - 50.1  -2.8333333 0.0656       .  -5.8573880  0.19072133
## 40.3 - 50.2  -3.0000000 0.0518       .  -6.0240547  0.02405466
## 40.3 - 50.3  -0.3333333 0.8253          -3.3573880  2.69072133
## 40.3 - 60.1   7.0000000 0.0000     ***   3.9759453 10.02405466
## 40.3 - 60.2  -1.6666667 0.2729          -4.6907213  1.35738800
## 40.3 - 60.3   7.5000000 0.0000     ***   4.4759453 10.52405466
## 50.1 - 50.2  -0.1666667 0.9121          -3.1907213  2.85738800
## 50.1 - 50.3   2.5000000 0.1028          -0.5240547  5.52405466
## 50.1 - 60.1   9.8333333 0.0000     ***   6.8092787 12.85738800
## 50.1 - 60.2   1.1666667 0.4412          -1.8573880  4.19072133
## 50.1 - 60.3  10.3333333 0.0000     ***   7.3092787 13.35738800
## 50.2 - 50.3   2.6666667 0.0825       .  -0.3573880  5.69072133
## 50.2 - 60.1  10.0000000 0.0000     ***   6.9759453 13.02405466
## 50.2 - 60.2   1.3333333 0.3792          -1.6907213  4.35738800
## 50.2 - 60.3  10.5000000 0.0000     ***   7.4759453 13.52405466
## 50.3 - 60.1   7.3333333 0.0000     ***   4.3092787 10.35738800
## 50.3 - 60.2  -1.3333333 0.3792          -4.3573880  1.69072133
## 50.3 - 60.3   7.8333333 0.0000     ***   4.8092787 10.85738800
## 60.1 - 60.2  -8.6666667 0.0000     *** -11.6907213 -5.64261200
## 60.1 - 60.3   0.5000000 0.7407          -2.5240547  3.52405466
## 60.2 - 60.3   9.1666667 0.0000     ***   6.1426120 12.19072133
```

From this result, we can observe from the result that when the cycle time is 50 and operator is number 2, the **cloth score is maximized with an average value of 36.5**.

Therefore, we shall consider operating the process by the conditions we concluded above.

Lastly, we need to check the model's adequacy(constant variance and normality assumptions).

```
fit <- aov(lm(Cloth~Cycle.Time*Temperature*Operator, data=cloth_data))
par(mfrow=c(1,2))
plot(fit, 1); plot(fit, 2)
```



The residuals vs fitted values plot and QQplot do not seem to have any unusual patterns. We can observe a constant band throughout the residuals vs fitted values plot and most of the points follow the theoretical line in the normal QQplot.

Question 5.46

(a)

$$SS_A = 118.667 - (96.333 + 12.167 + 10.000) = 0.167$$

(b)

$$DF_A = \frac{0.167}{0.0833} = 2$$

(c)

$$DF_B = \frac{96.333}{96.3333} = 1$$

(d)

$$MS_E = \frac{10.000}{6} = 1.667$$

(e)

```
pf(3.65, df1=2, df2=6, lower.tail=F)

## [1] 0.09181187
```

The p-value for the interaction test statistic is 0.09.

(f) We know the degree freedom for factor A is 2. Therefore, there are 2 + 1 = 3 levels.

(g) Similarly, we know there are 1 + 1 = 2 levels in factor B.

(h) We know the $DF_{error} = ab(n-1)$, and we know a = 3, b = 2, and $DF_{error} = 6$. Therefore, n = 2 which suggests that there are 2 replicates in this experiment.

(i) No, since we don't find the interaction between factor B and factor A is significant where we obtained a p-value greater than 0.05.

(j)

$$\hat{\sigma} = \sqrt{\frac{10}{6}} = 1.29$$

Question 5.52

(a) True.

$$SS_{error} = 185 - (50 + 80 + 30 + 10) = 15$$

$$DF_{error} = 11 - (1 + 2 + 2 + 1) = 5$$

$$\hat{\sigma} = \sqrt{\frac{15}{5}} = 1.73$$

(b)

$$DF_{error} = 11 - (1 + 2 + 2) = 6$$

(c)

$$SS_{error} = 185 - (50 + 80 + 30) = 25$$

$$MS_{error} = \frac{25}{6} = 4.167$$

(d) False, since $F_0 = 3.6$ less than the critical F value. Not significant.

```r
#F critical
qf(0.95, 2, 6)

## [1] 5.143253
```

$$F_0 = \frac{MS_{AB}}{MS_{error}} = \frac{15}{4.167} = 3.6$$