1. (20 points) For each of the following question, there is only one right answer, write your answer in the following table. (2 points each)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)

(1) Among the following input-output equations, which one represents a time-invariant system? Here $y(t)$ denotes the system output and $x(t)$ the system input.

(a) $y(t) = x(1-t)$ **(b)** $y(t) = x(t/3)$ **(c)** $y(t) = \begin{cases} 0, & t < 0 \\ x(t-2), & t \geq 0 \end{cases}$ **(d)** $y(t) = \begin{cases} 0, & x(t) < 0 \\ x(t), & x(t) \geq 0 \end{cases}$

(2) Among the following signals, which has a period（周期） of 4?

(a) $\sum_{n=-\infty}^{+\infty} e^{-\left(\frac{t}{4}-n\right)} u\left(\frac{t}{4}-n\right)$ **(b)** $e^{j(\pi t -1)}$ **(c)** $\sin\left(\frac{n}{5}-\pi\right)$ **(d)** $2\sin\left(\frac{\pi}{8}n\right) - \cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right)$

(3) The convolution（卷积） sum of $\{u[n+1] - u[n-2]\} * \{u[n+1] - u[n-2]\}$ is _____ .

(a) $\delta[n+2] + \delta[n+1] + \delta[n] + \delta[n-1] + \delta[n-2]$ **(b)** $\delta[n] + 2\delta[n-1] + 3\delta[n-2] + 2\delta[n-3] + \delta[n-4]$

(c) $\delta[n+2] + 2\delta[n+1] + 3\delta[n] + 2\delta[n-1] + \delta[n-2]$ **(d)** $\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4]$

(4) Which one of the following signals has the widest frequency band（频带）?

(a) $e^{-t}u(t)$ **(b)** $\int_0^t e^{-\tau}d\tau$ **(c)** $u(t)$ **(d)** $\delta(t)$

(5) Given that $x(t)$ has the Fourier transform $X(j\omega)$, express the Fourier transform of $y(t) = x(1-t) + x(-1-t)$ in terms of $X(j\omega)$ as _____.

(a) $2\cos\omega X(j\omega)$ **(b)** $2\cos\omega X(-j\omega)$ **(c)** $2j\sin\omega X(j\omega)$ **(d)** $-2j\sin\omega X(-j\omega)$

(6) Given that $X(e^{j\omega}) = j\cos 2\omega \sin 3\omega$, the inverse Fourier transform of it is a _____ function.

(a) real and even **(b) real and odd** **(c) purely imaginary** **(d) neither even nor odd（非奇非偶）**

(7) Let $x(t)$ be a signal with Nyquist（奈奎斯特） rate ω_0, then the Nyquist rate of $\dfrac{dx(t)}{dt}$ is __.

(a) ω_0 **(b)** $2\omega_0$ **(c)** $\omega_0/2$ **(d) cannot be determined**

(8) If signal $x(t)$ is real and even, and the Laplace transform $X(s)$ of $x(t)$ has a pole at $s = -2+j$, then $X(s)$ must have some other poles as following except for _____.

(a) $-2-j$ **(b)** $2+j$ **(c)** $2-j$ **(d)** 0

(9) A stable LTI system has system function $H(s) = \dfrac{4e^{-s}}{s^2 - 2s - 3}$, its impulse response $h(t)$ is _____.

(a) $(e^{3t-3} - e^{-t})u\,t(-\;$ **(b)** $-e^{3t-3}u(1-t) + e^{-t}u\,t(-$

(c) $(e^{3t-3} - e^{1-t})u(1-t)$ **(d)** $e^{3t-3}u(1-t) - e^{1-t}u(t-1)$

(10) Discrete time signal $x[n]$ has z-transform $X(z) = \dfrac{1}{z+2}$, which statement below is true?

(a) If $3^{-n}x[n]$ is not absolutely summable（绝对可和）, the ROC of $X(z)$ is $|z| > 2$.

(b) If $x[n] * 3^{-n}u[n]$ is absolutely summable, the ROC of $X(z)$ is $|z| > 2$.

(c) If $3^{n}x[n]$ is absolutely summable, the ROC of $X(z)$ is $|z| < 2$.

(d) If $x[n] * 3^{n}u[n]$ is not absolutely summable, the ROC of $X(z)$ is $|z| < 2$.

2. **(10 points) For each of the following statements, determine whether it is true or false and mark with "T" or "F" in the bracket. (2 points / question)**

① **The system with input-output relation $y(t) = t\,x(t) + 2$ is invertible（可逆的）. ( )**

② **A causal LTI system must have both its impulse response $h(t)$ and step response $s(t)$ causal.**
( )

③ **Unit step function $u(t)$ has finite energy. ( )**

④ **Signal $x(t)$ with its FS representation $x(t) = \displaystyle\sum_{k=-100}^{100} 2^{k} e^{jk\frac{2\pi}{50}t}$ is not real. ( )**

⑤ **If an ideal low-pass filter with cut-off frequency $\omega_{Lc} = \dfrac{\pi}{4}$ is parallel connected（并联） with**

an ideal high-pass filter with cut-off frequency $\omega_{Hc} = \dfrac{3\pi}{4}$, we can get a band-stop filter. ( )

3. **(30 points) Short answer questions.**

(1) (6 points) Figure 1 shows signal $x(t)$, draw the graphs of $\dfrac{d\,x(t)}{dt}$ and its odd-part（奇部）
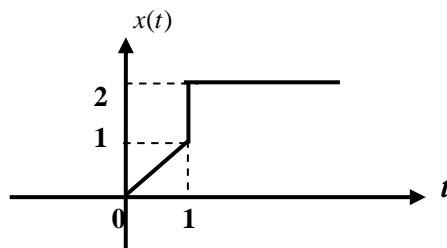
$Od\{x(t)\}$.

Figure 1

(2) (8 points) Consider a continuous-time causal LTI system with system function $H(s) = \dfrac{1}{s+2}$.

If input $x(t) = \sin 2t$, determine the corresponding output $y(t)$.

(3) (8 points) Determine the inverse z-transform $x[n]$ of $X(z) = \dfrac{z^3 + 2z^2 - 3.5z}{(z+1)(z-0.5)(z-2)}, 1 < |z| < 2$.

(4) (8 points) A discrete-time causal LTI system is described by the input-output equation

$$y[n] + 5y[n-1] + 3y[n-2] = v[n-1] + v[n-2]$$

Where $v[n]$ is the input, $y[n]$ is the output. Draw a direct-form block diagram（直接型框图）with least number（最少数量） of unit delayer（单位延时器） and construct state model based on this diagram.

4. (10 points) Consider an ideal low-pass filter with frequency response $H(j\omega) = \begin{cases} e^{-j\omega} & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$,

where ω_c is the cut-off frequency（截止频率）. Try to find the value of ω_c if we want to obtain an output signal $y(t)$ with its total energy 50% of that of the input $x(t) = 2e^{-t}u(t)$. （要求输出信号的总能量为输入信号总能量的 50%）

5. (15 points) A second-order causal LTI system is composed by two parallel connected causal systems S_1 and S_2. S_1 and S_2 are described by following equations respectively:

$$S_1: \quad \frac{d\,y_1(t)}{d\,t} + y_1(t) = x_1(t) \quad S_2: \quad \frac{d\,y_2(t)}{d\,t} + 3y_2(t) = x_2(t)$$

Suppose that the initial states of this second-order system are not zero, answer following questions:

(a) Determine the overall $H(s)$ and specify its ROC;

(b) Is the second-order system stable?

(c) It is known that when input $x(t) = u(t)$ the complete response $y(t) = \dfrac{4}{3}\left[1 - e^{-3t}\right]u(t)$, try to find zero-state response $y_{zs}(t)$, zero-input response $y_{zi}(t)$, and initial values（初始值）of $y(0^-)$, $y'(0^-)$.

(d) Specify natural response（自然响应） $y_h(t)$ and forced response（受迫响应） $y_p(t)$ in above

complete response $y(t) = \dfrac{4}{3}\left[1 - e^{-3t}\right]u(t)$.

6. (15 points) A discrete-time causal and stable LTI system S is represented by the following equation

$$y[n] + \frac{k}{4} y[n-1] = x[n] - \frac{k}{3} x[n-1]$$

Here k is a real number.

(a) Find expression of $H(z)$, specify its ROC and determine the unit sample response $h[n]$ in terms of k（用 k 表示）；

(b) Determine the range of value of k（确定 k 的取值范围）；

(c) If $k = 2$, find the response $y[n]$ for the input $x[n] = 1$；

(d) Suppose that S is invertible and its inverse system（逆系统） S_1 is also both causal and stable, find the expression of system function $H_1(z)$ of S_1 in terms of k. And determine the range of value of k by analyzing the locations（位置） of pole and zero of $H(z)$.