电路理论

——二阶电路的暂态分析

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电气与电子工程学院

本章学习内容

9.1 概述

9.2 零输入响应（自然响应）

9.3 直流电源激励下的响应

讲授学时：2

本章学习目标与难点

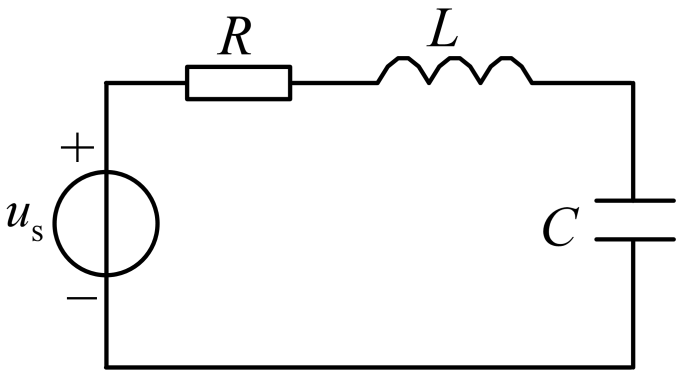
目标

1. 掌握二阶电路零输入响应的变化规律；
2. 掌握直流电源激励的二阶电路响应计算方法；
3. 掌握自由分量与强制分量、暂态分量与稳态分量、阶跃响应与冲激响应等概念。

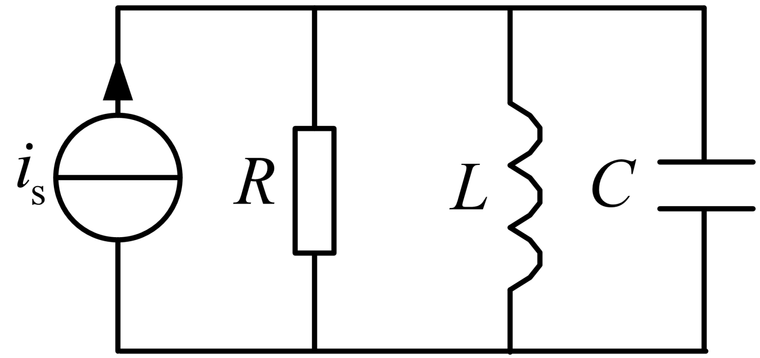
难点

理解自然响应的变化规律，列写一般二阶电路的微分方程。

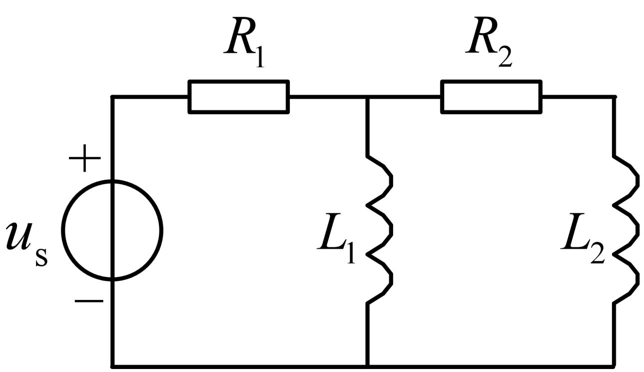
9.1 概述



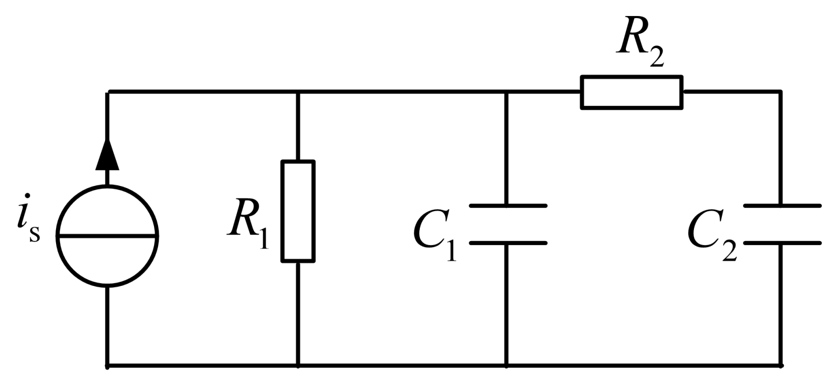
*RLC*串联电路



*RLC*并联电路

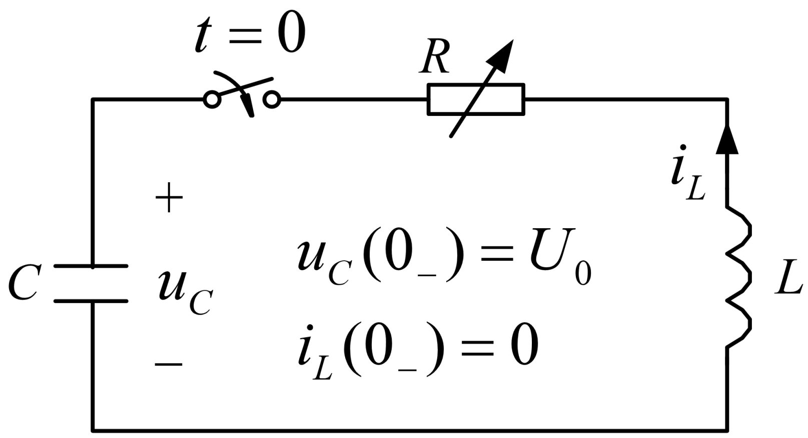


一般二阶*RLL*电路



一般二阶*RCC*电路

9.2 零输入响应（自然响应）



$$\begin{cases} LC\dfrac{\mathrm{d}^2u_C}{\mathrm{d}t^2}+RC\dfrac{\mathrm{d}u_C}{\mathrm{d}t}+u_C=0 \\ u_C(0_+)=u_C(0_-)=U_0 \\ \left.\dfrac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{0_+}=\dfrac{i_C(0_+)}{C}=\dfrac{i_L(0_+)}{C}=0 \end{cases}$$

特征方程： $LCs^2+RCs+1=0$

特征根： $$s_{1,2}=\frac{-RC\pm\sqrt{(RC)^2-4LC}}{2LC}=-\frac{R}{2L}\pm\sqrt{(\frac{R}{2L})^2-\frac{1}{LC}}$$

令$\alpha=\dfrac{R}{2L}$ (衰减系数)，$\omega_0=\sqrt{\dfrac{1}{LC}}$ (谐振角频率) $=-\alpha\pm\sqrt{\alpha^2-\omega_0^2}$

9.2 零输入响应（自然响应）

特征根：

$$s_{1,2}=\frac{-RC\pm\sqrt{(RC)^2-4LC}}{2LC}=-\frac{R}{2L}\pm\sqrt{(\frac{R}{2L})^2-\frac{1}{LC}}=-\alpha\pm\sqrt{\alpha^2-\omega_0^2}$$

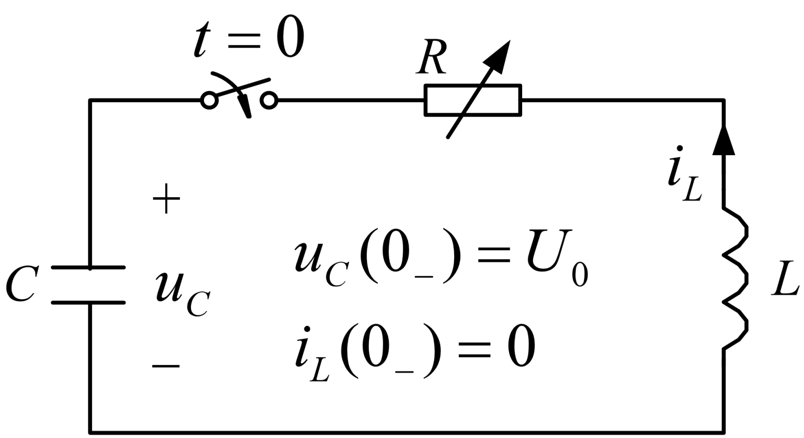
零状态响应的三种情况

（1） $\alpha>\omega_0 \quad \rightarrow R>2\sqrt{\frac{L}{C}}$ 两个不相等负实根 过阻尼

（2） $\alpha<\omega_0 \quad \rightarrow R<2\sqrt{\frac{L}{C}}$ 两个共轭复根 欠阻尼

（3） $\alpha=\omega_0 \quad \rightarrow R=2\sqrt{\frac{L}{C}}$ 两个相等负实根 临界阻尼

9.2 零输入响应（自然响应）



Overdamped——过阻尼

(1) $\alpha > \omega_0 \quad \rightarrow R > 2\sqrt{\dfrac{L}{C}}$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$u_C = k_1 e^{s_1 t} + k_2 e^{s_2 t}$$

$$u_C(0_+) = U_0 \rightarrow k_1 + k_2 = U_0$$

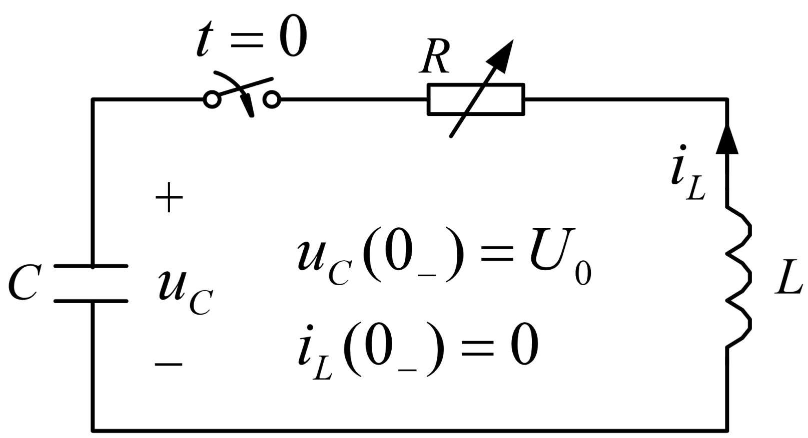
$$\left.\frac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{(0_+)} \rightarrow s_1 k_1 + s_2 k_2 = 0$$

$$\begin{cases} k_1 = \dfrac{s_2}{s_2 - s_1} U_0 \\ k_2 = \dfrac{-s_1}{s_2 - s_1} U_0 \end{cases}$$

$$\boxed{u_C = \frac{U_0}{s_2 - s_1}(s_2 e^{s_1 t} - s_1 e^{s_2 t})}$$

$$\boxed{i_L = C\frac{\mathrm{d}u_c}{\mathrm{d}t} = \frac{U_0}{L(s_2 - s_1)}(e^{s_1 t} - e^{s_2 t})}$$

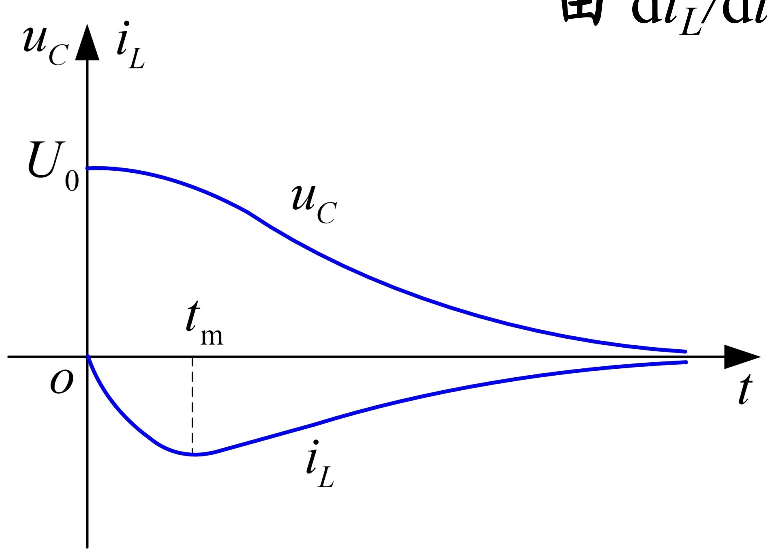
9.2 零输入响应（自然响应）



$$u_C=\frac{U_0}{s_2-s_1}(s_2e^{s_1t}-s_1e^{s_2t})$$

$$i_L=C\frac{\mathrm{d}u_c}{\mathrm{d}t}=\frac{U_0}{L(s_2-s_1)}(e^{s_1t}-e^{s_2t})$$

由 $\mathrm{d}i_L/\mathrm{d}t$ 可确定 i_L 为极小时的 t_m：

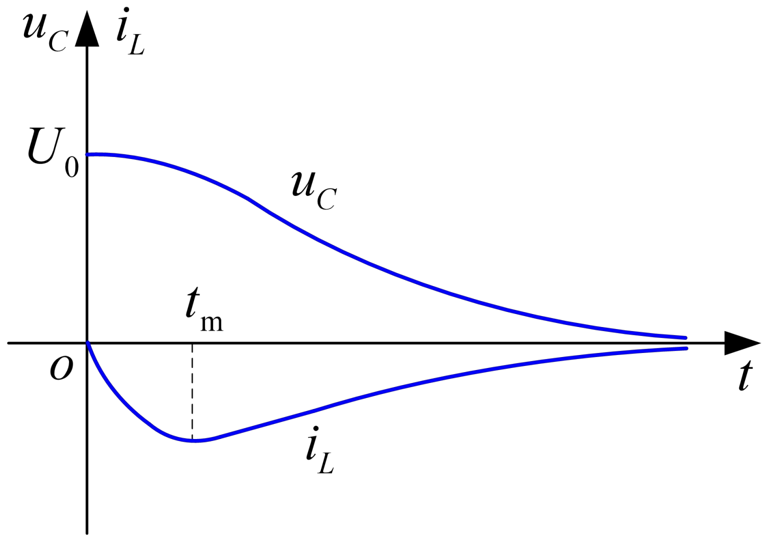


$$s_1e^{s_1t_m}-s_2e^{s_2t_m}=0$$

$$t_m=\frac{\ell n\dfrac{s_2}{s_1}}{s_1-s_2}$$

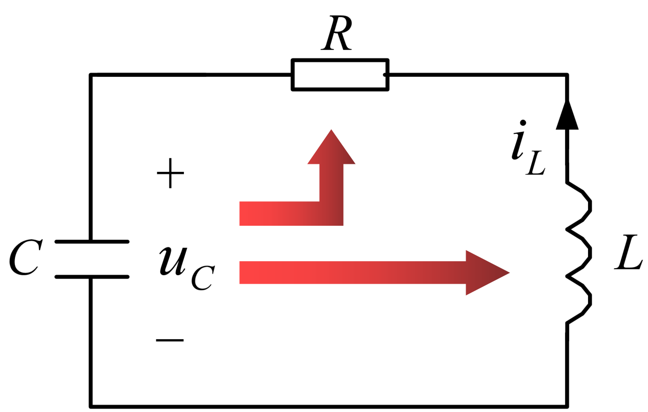
9.2 零输入响应（自然响应）

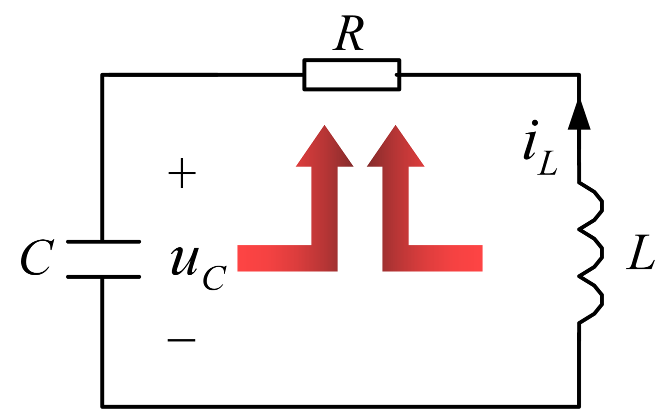
能量转换关系



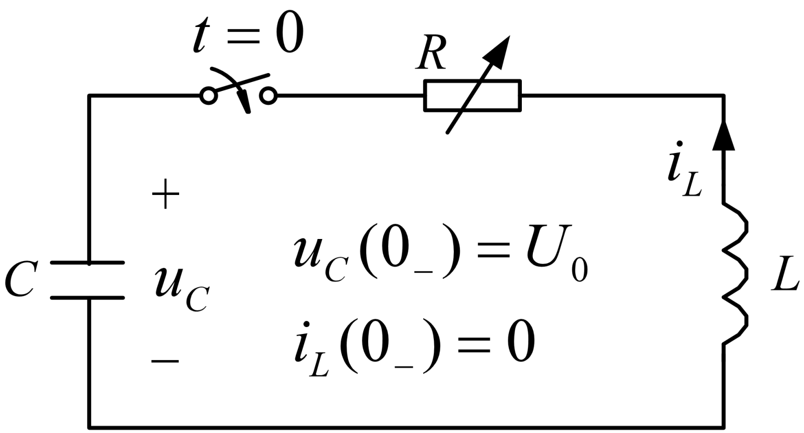
$0 < t < t_m$ $u_C \downarrow$ $i \uparrow$

$t > t_m$ $u_C \downarrow$ $i \downarrow$





9.2 零输入响应（自然响应）



$u_C(0_-) = U_0$

$i_L(0_-) = 0$

underdamped——欠阻尼

（2） $\alpha < \omega_0 \quad \rightarrow R < 2\sqrt{\dfrac{L}{C}}$

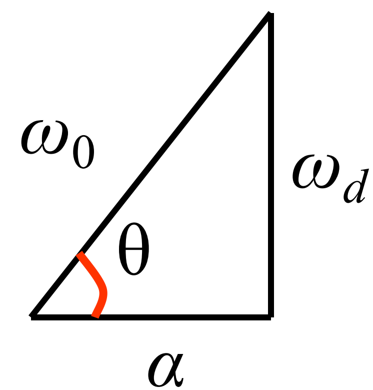
$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} \quad \text{固有振荡频率}$$

$$s_{1,2} = -\alpha \pm \mathrm{j}\sqrt{\omega_0^2 - \alpha^2} = -\alpha \pm \mathrm{j}\omega_\mathrm{d}$$

$$u_C = k_1 \mathrm{e}^{s_1 t} + k_2 \mathrm{e}^{s_2 t} = k\mathrm{e}^{-\alpha t}\sin(\omega_\mathrm{d} t + \theta)$$

初始条件：

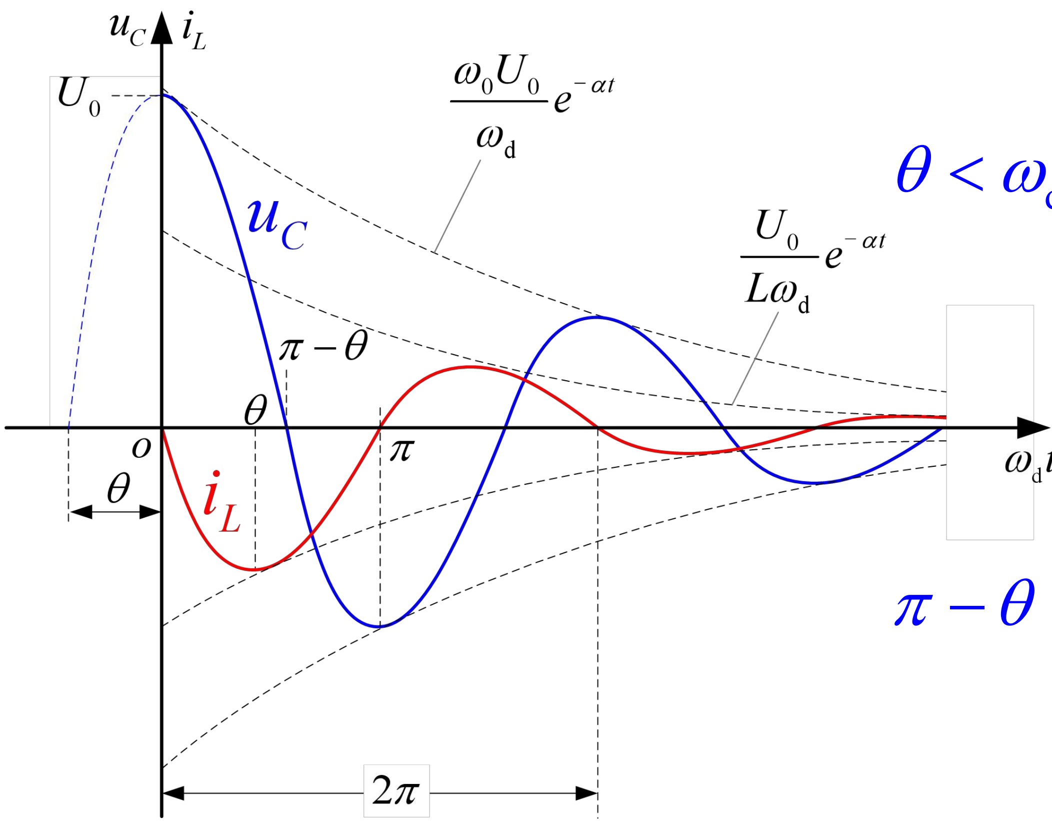
$$\begin{cases} u_C(0^+) = U_0 \rightarrow k\sin\theta = U_0 \\ \dfrac{du_C}{dt}(0^+) = 0 \rightarrow k(-\alpha)\sin\theta + k\omega_d\cos\theta = 0 \end{cases}$$



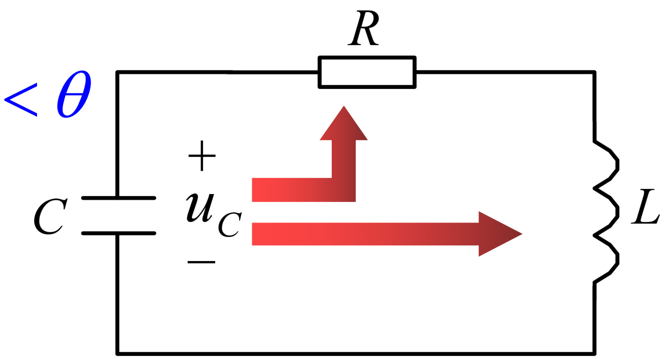
$$k = \frac{U_0}{\sin\theta}, \quad \theta = arctg\frac{\omega_d}{\alpha} \qquad \sin\theta = \frac{\omega_d}{\omega_0} \qquad k = \frac{\omega_0}{\omega_d}U_0$$

9.2 零输入响应（自然响应）

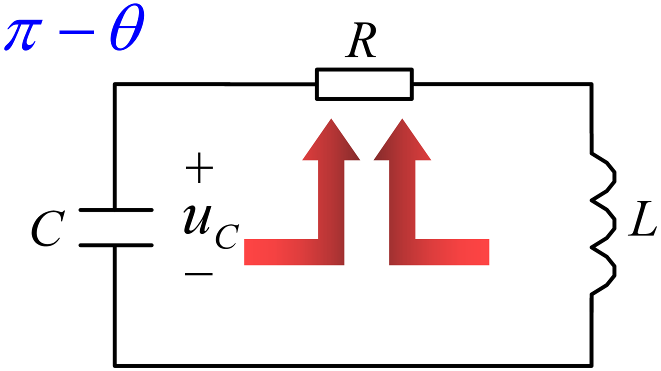
$$u_C = \frac{\omega_0}{\omega_d} U_0 e^{-\alpha t} \sin(\omega_d t + \theta)$$



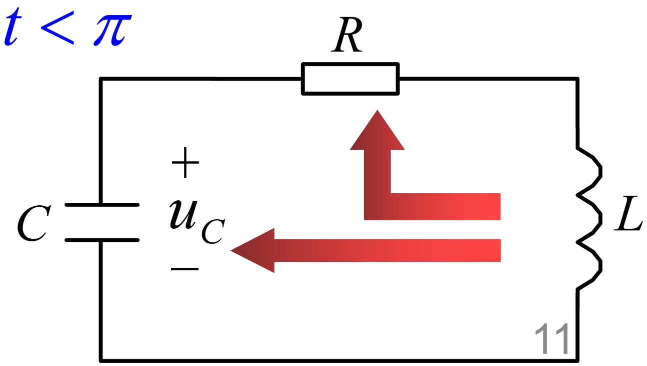
$0 < \omega_d t < \theta$



$\theta < \omega_d t < \pi - \theta$



$\pi - \theta < \omega_d t < \pi$



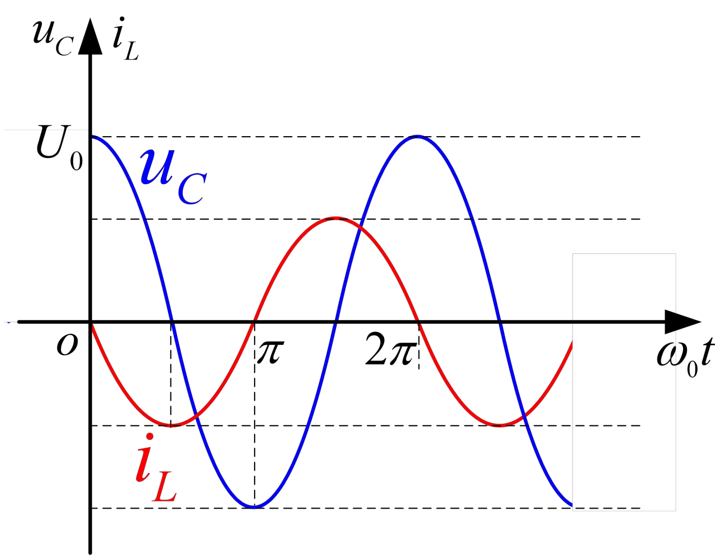
9.2 零输入响应（自然响应）

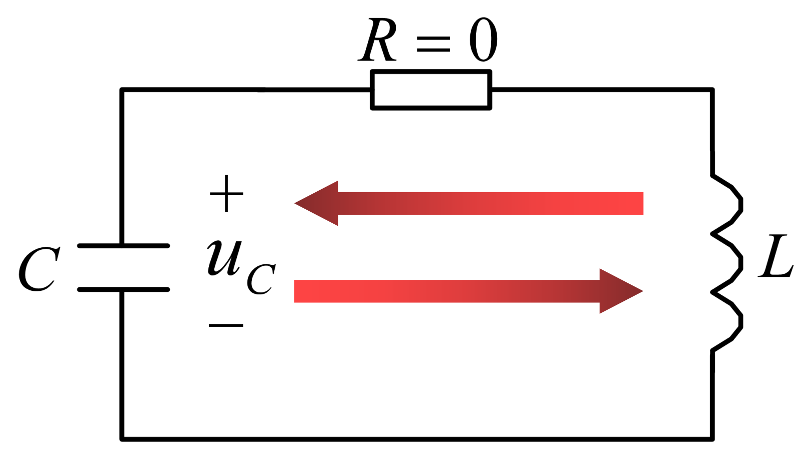
特例：R=0 时　$\alpha = 0$，$\omega_d = \omega_0 = \dfrac{1}{\sqrt{LC}}$，$\theta = \dfrac{\pi}{2}$

$$u_C = U_0 \sin(\omega_0 t + 90°) = u_L$$

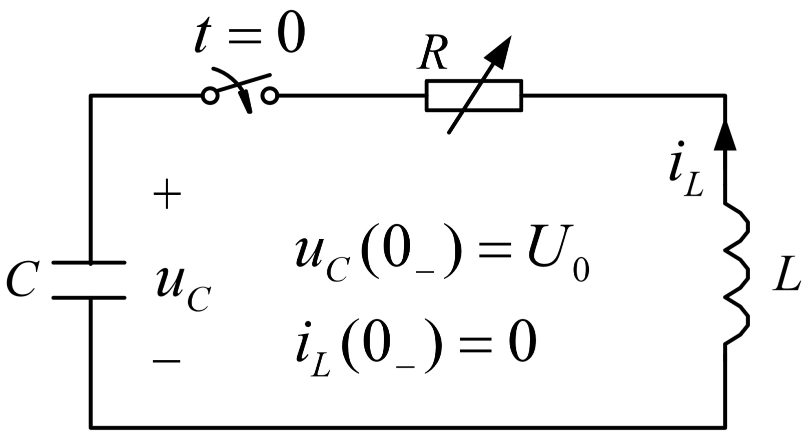
$$i = -\frac{U_0}{\omega_0 L}\sin \omega_0 t$$

→ 等幅振荡





9.2 零输入响应（自然响应）



Critically damped——临界阻尼

(3) $\alpha=\omega_0 \quad \rightarrow R=2\sqrt{\dfrac{L}{C}}$

$$s_{1,2}=-\alpha$$

$$u_C=(k_1+k_2t)\mathrm{e}^{-\alpha t}$$

初始条件：

$$\begin{cases} u_c(0^+)=U_0 \rightarrow k_1=U_0 \\ \dfrac{\mathrm{d}u_c}{\mathrm{d}t}(0^+)=0 \rightarrow k_1(-\alpha)+k_2=0 \end{cases}$$

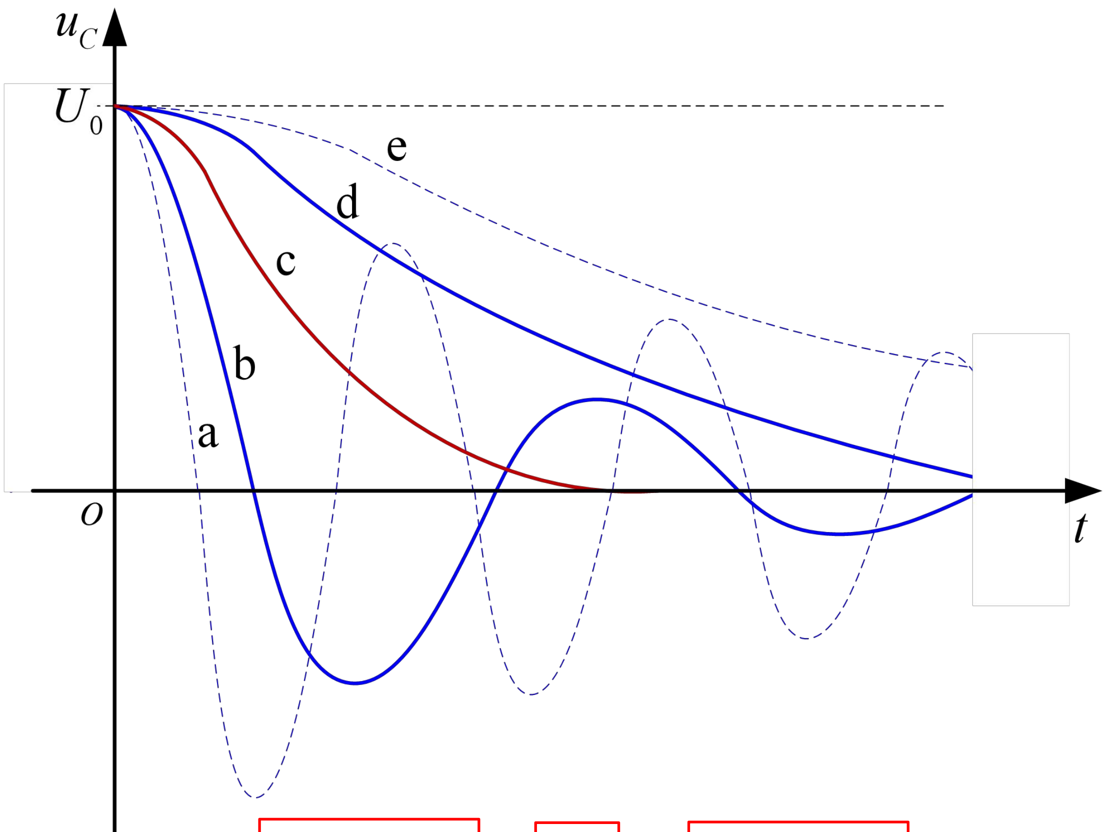
$$\begin{cases} k_1=U_0 \\ k_2=U_0\alpha \end{cases}$$

$$u_C=U_0e^{-\alpha t}(1+\alpha t)$$

$$i_C=C\frac{\mathrm{d}u_C}{\mathrm{d}t}=-\frac{U_0}{L}te^{-\alpha t}$$

非振荡电路

9.2 零输入响应（自然响应）



$$R_e > R_d > R_c > R_b > R_a$$

过阻尼　　临界阻尼　　欠阻尼

$R > 2\sqrt{\dfrac{L}{C}}$ 过阻尼，非振荡放电

$$u_C = k_1 e^{s_1 t} + k_2 e^{s_2 t}$$

$R = 2\sqrt{\dfrac{L}{C}}$ 临界阻尼，非振荡放电

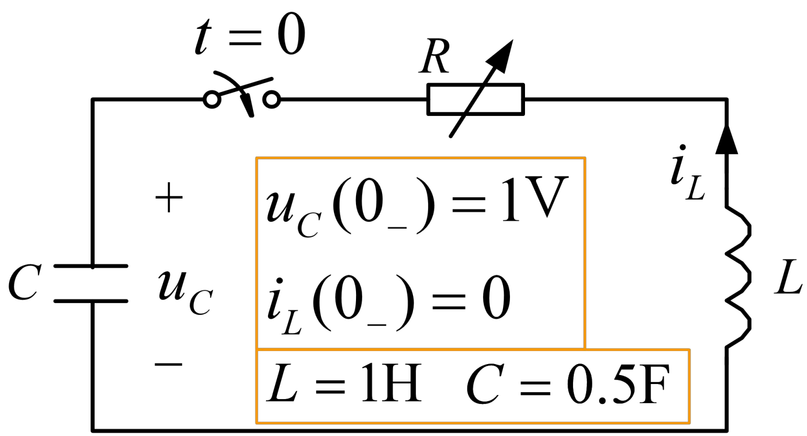
$$u_C = k_1 e^{-\alpha t} + k_2 t e^{-\alpha t}$$

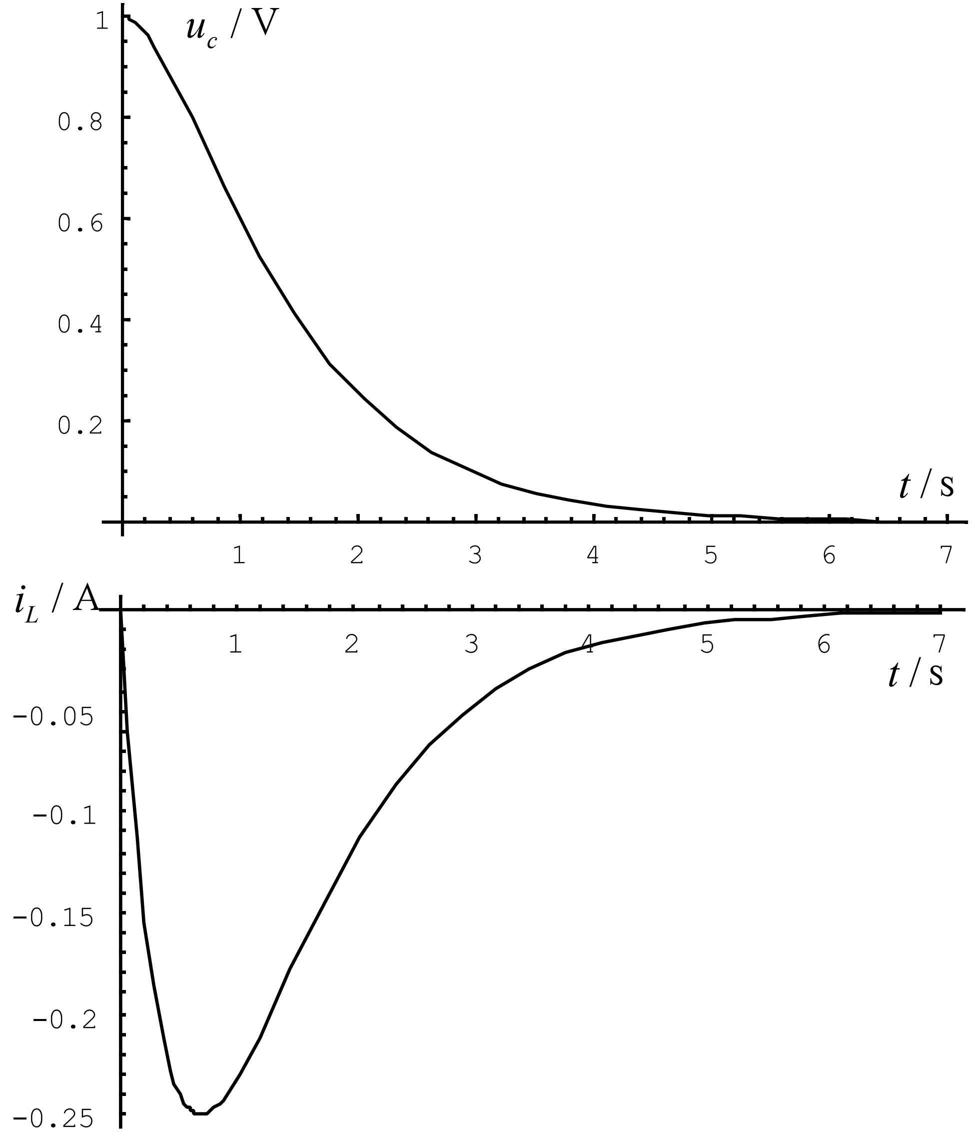
$R < 2\sqrt{\dfrac{L}{C}}$ 欠阻尼，振荡放电

$$u_C = k e^{-\alpha t}\sin(\omega_d t + \theta)$$

9.2 零输入响应（自然响应）

Multisim仿真

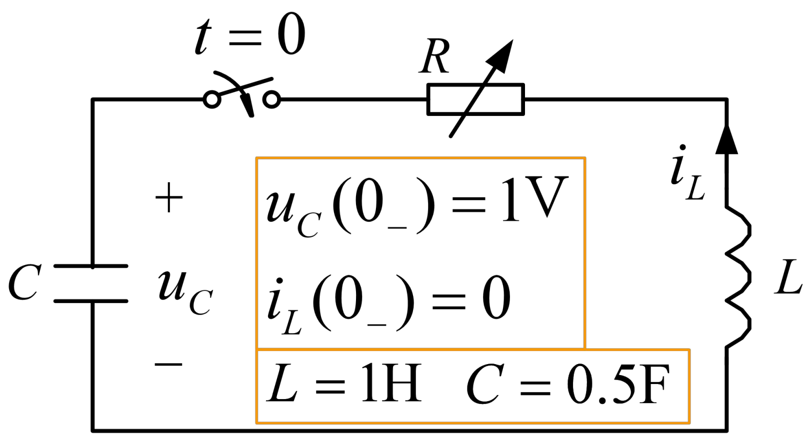


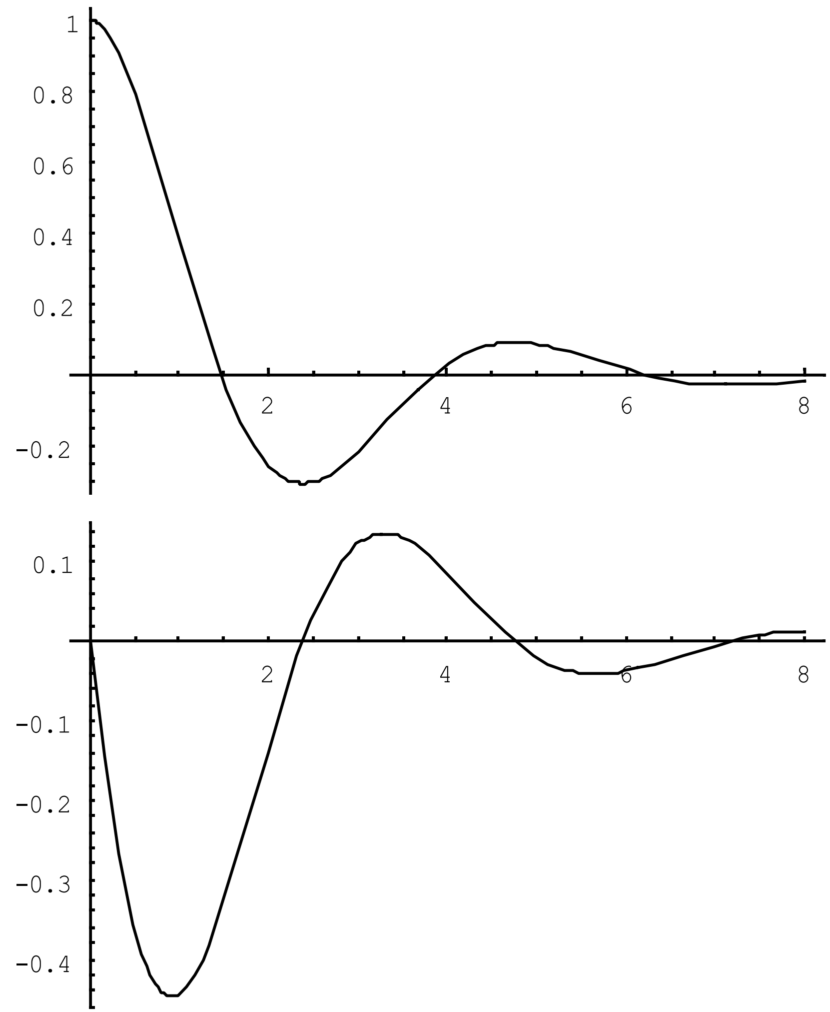


$R=3\Omega$ **过阻尼**

$$\begin{bmatrix} u_c \\ i_L \end{bmatrix} = \begin{bmatrix} (2e^{-t}-e^{-2t})\text{V} \\ (-e^{-t}+e^{-2t})\text{A} \end{bmatrix}$$

Multisim仿真



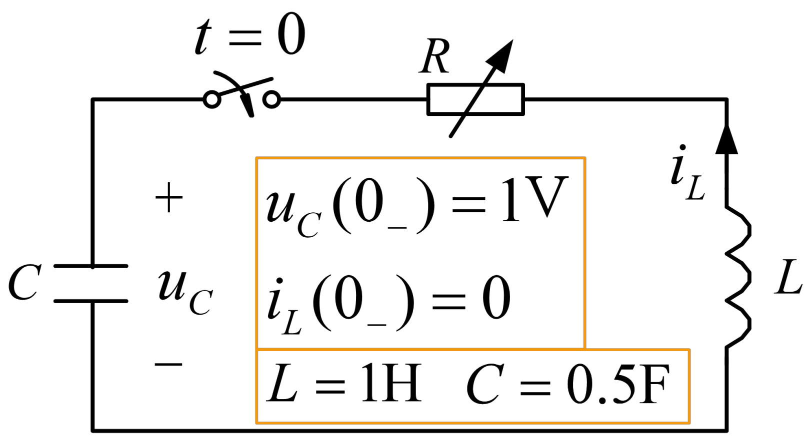


$\boxed{R=1\Omega}$ **欠阻尼**

$$\begin{bmatrix} u_C \\ i_L \end{bmatrix} = \begin{bmatrix} e^{-0.5t}(\cos\frac{\sqrt{7}}{2}t+\frac{1}{\sqrt{7}}\sin\frac{\sqrt{7}}{2}t)\text{V} \\ (-\frac{2}{\sqrt{7}}e^{-0.5t}\sin\frac{\sqrt{7}}{2}t)\text{A} \end{bmatrix}$$

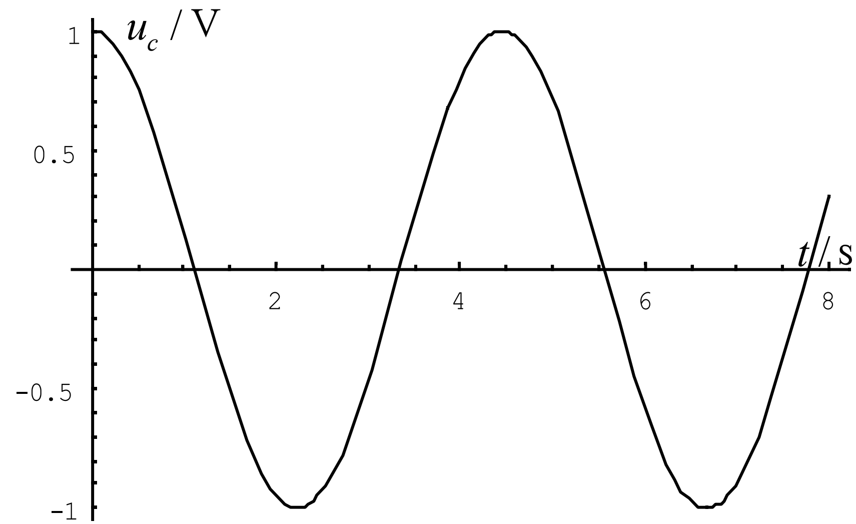
9.2 零输入响应（自然响应）

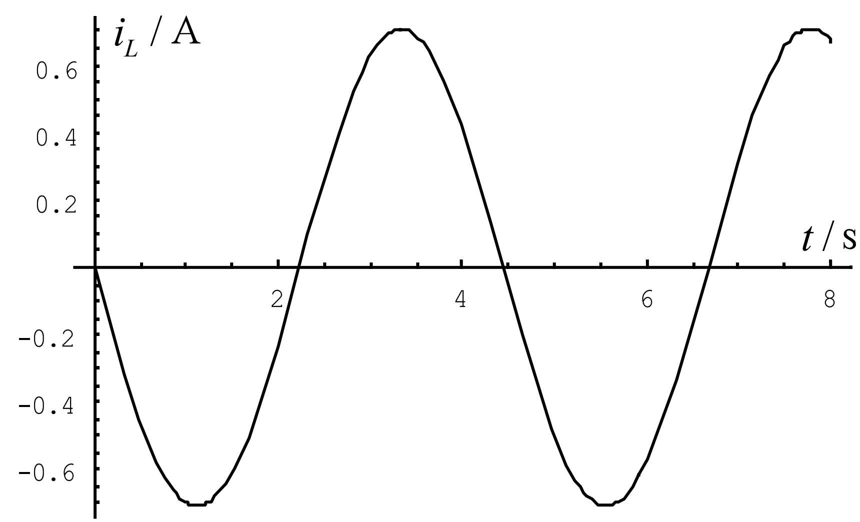
Multisim仿真



$\boxed{R=0}$ 无阻尼

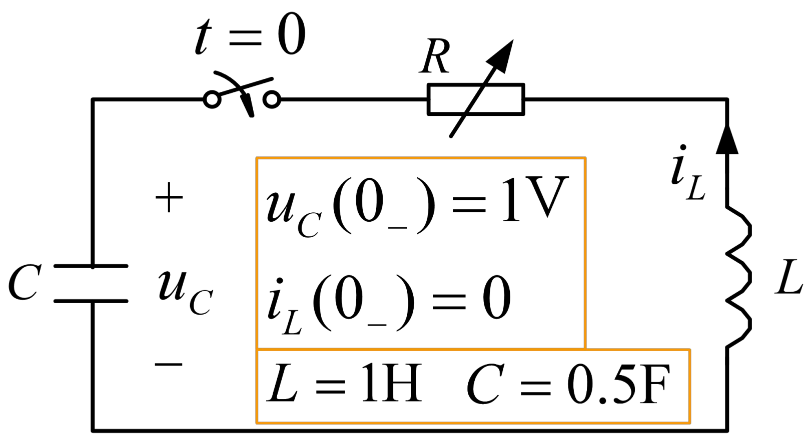
$$\begin{bmatrix} u_C \\ i_L \end{bmatrix} = \begin{bmatrix} (\cos\sqrt{2}t)\text{V} \\ (-\dfrac{1}{\sqrt{2}}\sin\sqrt{2}t)\text{A} \end{bmatrix}$$

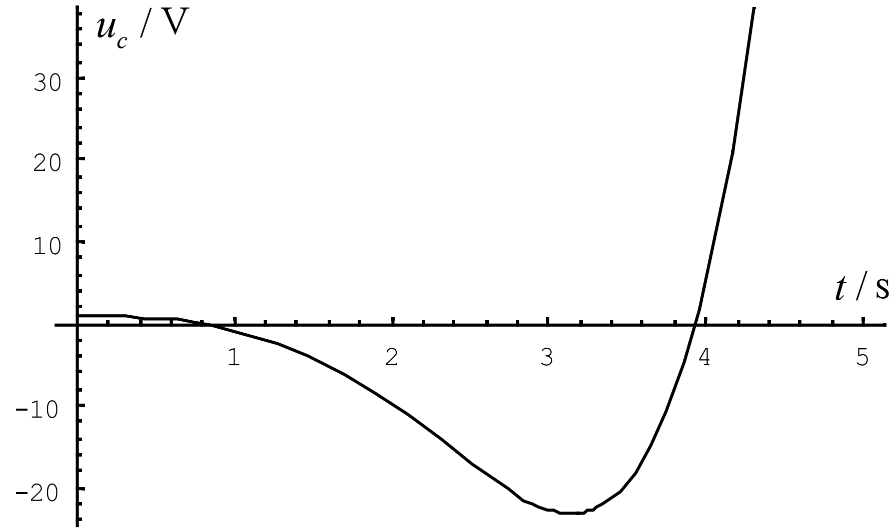




9.2 零输入响应（自然响应）

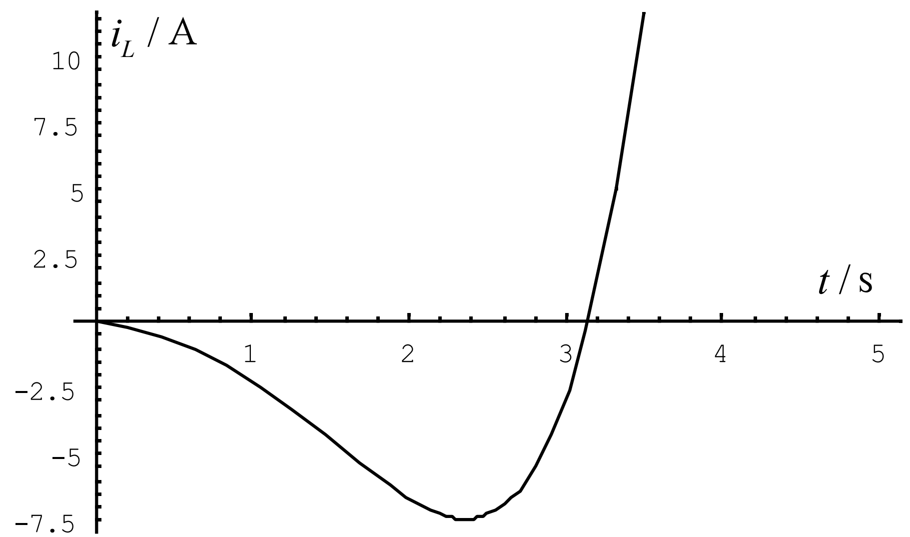
Multisim仿真





$R=-2\Omega$ **负阻尼**

$$\begin{bmatrix} u_C \\ i_L \end{bmatrix} = \begin{bmatrix} e^t(\cos t - \sin t)\text{V} \\ (-e^t \sin t)\text{A} \end{bmatrix}$$



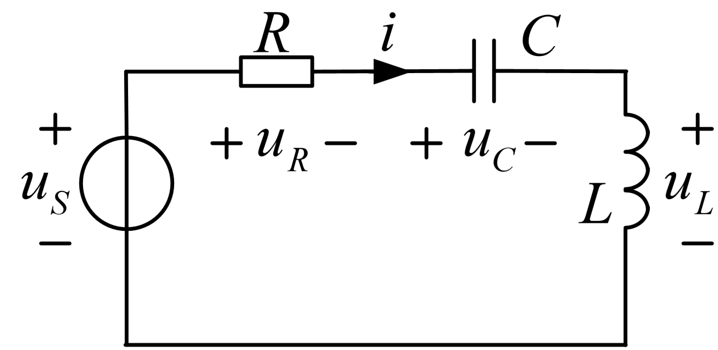
9.3 直流电源激励下的响应

以阶跃响应为例来分析二阶*RLC*电路的零状态响应。

1. *RLC*串联电路的阶跃响应

根据KVL和支路电压–电流关系,可得

$$LC\frac{d^2u_C}{dt^2}+RC\frac{du_C}{dt}+u_C=U_0$$



$$u_S=U_0\varepsilon(t)$$

二阶常系数线性非齐次微分方程

初始条件为: $u_C(0_+)=u_C(0_-)=0$

$$i_L(0_+)=i_L(0_-)=0$$

9.3 直流电源激励下的响应

方程的解为 $$u_C = u_{Ch} + u_{Cp}$$

齐次解为 $$u_{Ch} = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

特征方程 $$LCs^2 + RCs + 1 = 0$$

特征根(固有频率) $$\begin{cases} s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0} \\ s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0} \end{cases}$$

与RLC串联电路零输入响应一样，RLC串联电路的固有频率s_1和s_2也可以是两个不相等的负实数，两个相等的负实数，一对共轭复数和一对共轭虚数。

9.3 直流电源激励下的响应

阶跃激励下的稳态分量u_{Cp}=U$_0$

$$u_C = u_{Ch} + u_{Cp} = K_1 e^{s_1 t} + K_2 e^{s_2 t} + U_0$$

根据初始条件,有
$$\begin{cases} u_C(0_+) = K_1 + K_2 + U_0 = 0 \\ \left.\dfrac{du_C}{dt}\right|_{t=0_+} = K_1 s_1 + K_2 s_2 = 0 \end{cases}$$

$$\Rightarrow K_1 = \frac{s_2}{s_1 - s_2}\mathrm{U}_0,\quad K_2 = \frac{s_1}{s_2 - s_1}\mathrm{U}_0$$

电容电压为

$$u_C = \left[\frac{1}{s_1 - s_2}(s_2 e^{s_1 t} - s_1 e^{s_2 t}) + 1\right] U_0 \varepsilon(t)$$

9.3 直流电源激励下的响应

RLC串联充电电路也可以区分为:

1.过阻尼 $\alpha > \omega_0$ 电路参数满足 $R > 2\sqrt{L/C}$

2.临界阻尼 $\alpha = \omega_0$ $R = 2\sqrt{L/C}$

3.欠阻尼 $\alpha < \omega_0$ $R < 2\sqrt{L/C}$

4.无阻尼 $\alpha=0$ (即$R=0$)

下面仅讨论过阻尼和欠阻尼两种不同情况的阶跃响应。

9.3 直流电源激励下的响应

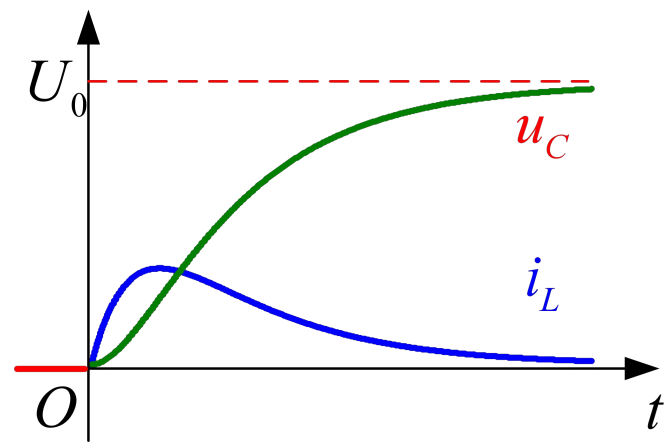
1.过阻尼

$$u_C = \left[\frac{1}{s_1 - s_2}(s_2 e^{s_1 t} - s_1 e^{s_2 t}) + 1\right] U_0 \varepsilon(t)$$

$$i_L = i = C\frac{du_C}{dt} = \frac{s_1 s_2}{L(s_1 - s_2)}(e^{s_1 t} - e^{s_2 t}) U_0 \varepsilon(t)$$

由于$s_1<0$、$s_2<0$及$|s_2|>|s_1|$ $\Rightarrow$ $e^{s_1 t} - e^{s_2 t} > 0$

使电容电压u_C和电感电流i_L永远不改变方向。电容元件在全部时间内一直在充电。



9.3 直流电源激励下的响应

2.欠阻尼

$$\begin{cases} s_1 = -\alpha + \sqrt{-(\omega_0^2 - \alpha^2)} = -\alpha + j\omega_d \\ s_2 = -\alpha - \sqrt{-(\omega_0^2 - \alpha^2)} = -\alpha - j\omega_d \end{cases}$$

或表示成极坐标形式 $\begin{cases} s_1 = \omega_0 e^{j(90^\circ + \theta)} \\ s_1 = \omega_0 e^{-j(90^\circ + \theta)} \end{cases}$

其中 $\theta = \arctan(\alpha / \omega d)$

电容电压 $u_C = \left\{1 + \dfrac{1}{2j\omega_d}\omega_0 e^{-\alpha t}\left[e^{j(j\omega_d t - 90^\circ - \theta)} - e^{-j(j\omega_d t - 90^\circ - \theta)}\right]\right\}U_0\varepsilon(t)$

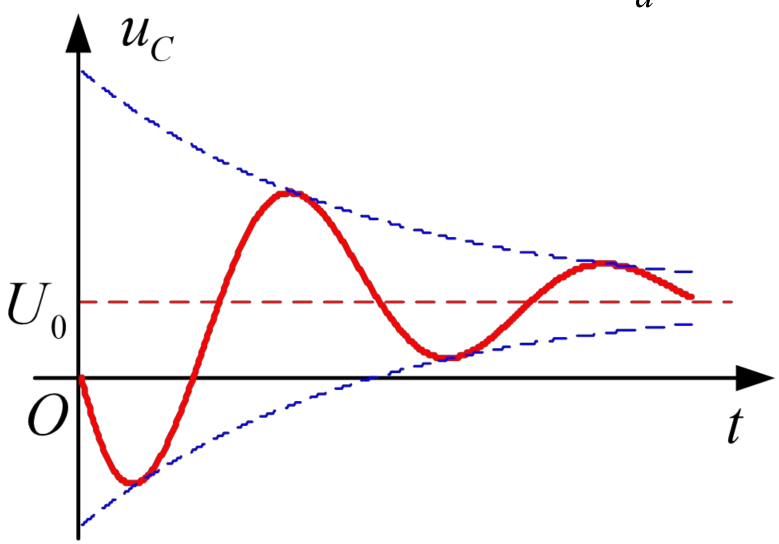
$$= \left[1 + \frac{\omega_0}{\omega_d}e^{-\alpha t}\sin(\omega_d t - 90^\circ - \theta)\right]U_0\varepsilon(t)$$

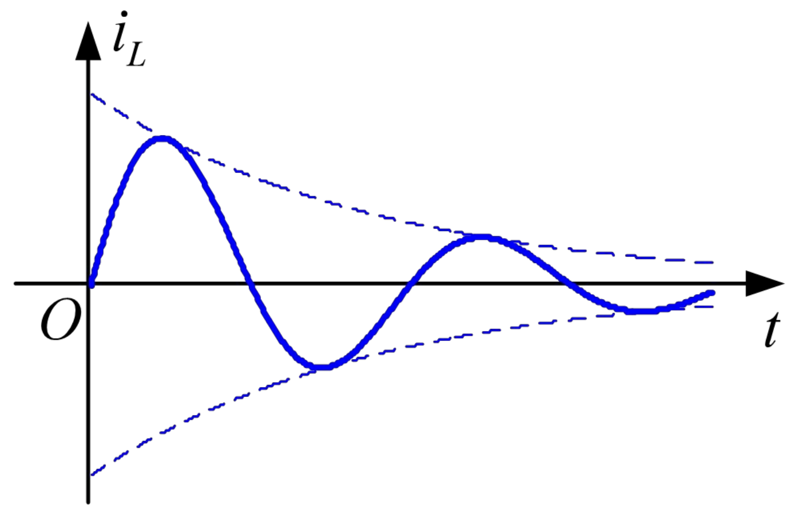
9.3 直流电源激励下的响应

$$= \left[1 - \frac{\omega_0}{\omega_d} e^{-\alpha t} \cos(\omega_d t - \theta)\right] U_0 \varepsilon(t)$$

根据电容的电压–电流关系　$i = C du_C / dt$

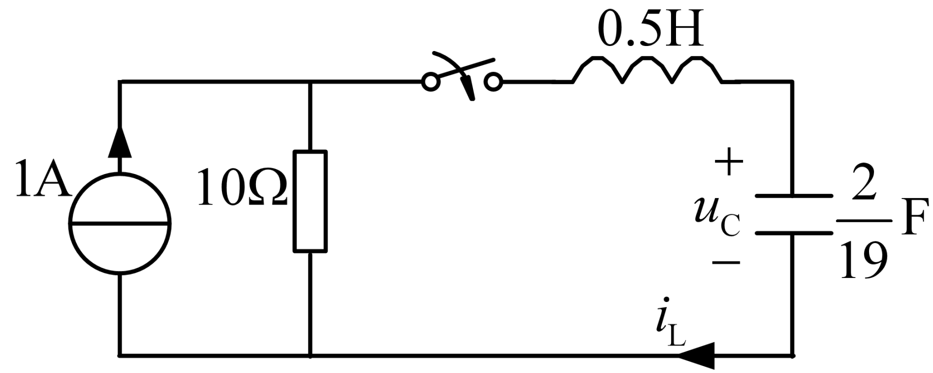
$$i_L = i = \left(\frac{1}{\omega_d L} e^{-\alpha t} \sin \omega_d t\right) U_0 \varepsilon(t)$$





9.3 直流电源激励下的响应

例9-1 求$i_L(t>0)$



$u_C(0_-)=15\text{V},\ i_L(0_-)=0$

解 初始条件：

$$u_C(0_+)=u_C(0_-)=15\text{V}$$

$$i_L(0_+)=i_L(0_-)=0$$

$$\left.\frac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{0_+}=\frac{i_C(0_+)}{C}=\frac{i_L(0_+)}{C}=0$$

微分方程： $$0.5\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{2}{19}\frac{\mathrm{d}u_C}{\mathrm{d}t}\right)+10\left(\frac{2}{19}\frac{\mathrm{d}u_C}{\mathrm{d}t}-1\right)+u_C=0$$

$$\Rightarrow \frac{\mathrm{d}^2u_C}{\mathrm{d}t^2}+20\frac{\mathrm{d}u_C}{\mathrm{d}t}+19u_C=190$$

9.3 直流电源激励下的响应

$$\frac{\mathrm{d}^2 u_C}{\mathrm{d}t^2}+20\frac{\mathrm{d}u_C}{\mathrm{d}t}+19u_C=190$$

$$u_C(0_+)=u_C(0_-)=15\mathrm{V}$$

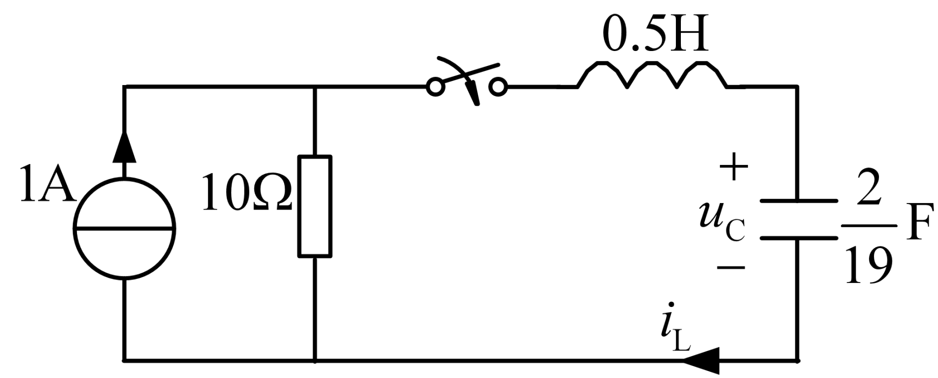
$$i_L(0_+)=i_L(0_-)=0$$

$$\left.\frac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{0_+}=\frac{i_C(0_+)}{C}=\frac{i_L(0_+)}{C}=0$$

$$p_{1,2}=-10\pm\sqrt{100-19}=\begin{cases}-1\\-19\end{cases}$$

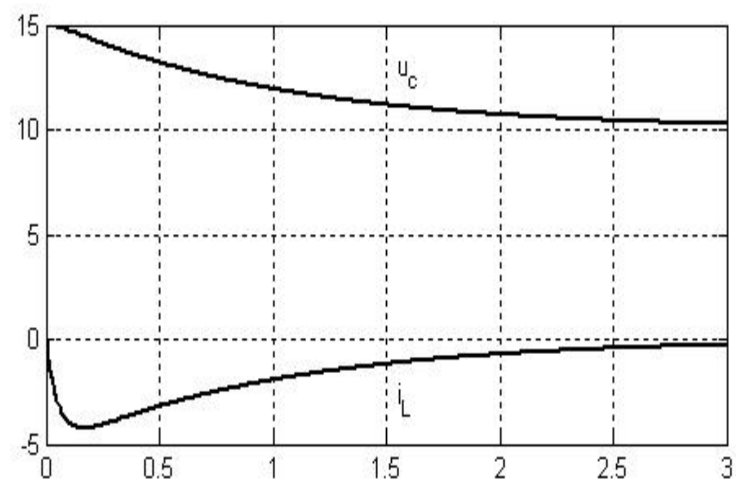
$$u_C=k_1\mathrm{e}^{-t}+k_1\mathrm{e}^{-19t}+10$$

$$k_1=\frac{95}{18}\quad k_2=-\frac{5}{18}$$



$$u_C(0_-)=15\mathrm{V},\ \ i_L(0_-)=0$$

$$i_L=C\frac{\mathrm{d}u_C}{\mathrm{d}t}=-\frac{95}{18}e^{-t}+\frac{95}{18}e^{-19t}$$



课后作业

- 9.2节：9-5，9-7，9-9
- 9.3节：9-13，9-15

谢谢聆听！！

刘旭 2023-4-17