1. **(20 points) For each of the following question, there is only one right answer, write your answer in the following table. (2 points each)**

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
D	A	C	D	B	B	A	D	B	C

(1) Among the following input-output equations, which one represents a time-invariant system? Here *y*(*t*) denotes the system output and *x*(*t*) the system input.

(a) $y(t)=x(1-t)$ **(b)** $y(t)=x(t/3)$ **(c)** $y(t)=\begin{cases}0, & t<0\\ x(t-2), & t\ge 0\end{cases}$ **(d)** $y(t)=\begin{cases}0, & x(t)<0\\ x(t), & x(t)\ge 0\end{cases}$

(2) Among the following signals, which has a period（周期） of 4?

(a) $\sum_{n=-\infty}^{+\infty} e^{-\left(\frac{t}{4}-n\right)}u\left(\frac{t}{4}-n\right)$ **(b)** $e^{j(\pi t-1)}$ **(c)** $\sin\left(\frac{n}{5}-\pi\right)$ **(d)** $2\sin\left(\frac{\pi}{8}n\right)-\cos\left(\frac{\pi}{2}n+\frac{\pi}{6}\right)$

(3) The convolution（卷积） sum of $\{u[n+1]-u[n-2]\}*\{u[n+1]-u[n-2]\}$ **is ____ .**

(a) $\delta[n+2]+\delta[n+1]+\delta[n]+\delta[n-1]+\delta[n-2]$ **(b)** $\delta[n]+2\delta[n-1]+3\delta[n-2]+2\delta[n-3]+\delta[n-4]$

(c) $\delta[n+2]+2\delta[n+1]+3\delta[n]+2\delta[n-1]+\delta[n-2]$ **(d)** $\delta[n]+\delta[n-1]+\delta[n-2]+\delta[n-3]+\delta[n-4]$

(4) Which one of the following signals has the widest frequency band（频带）?

(a) $e^{-t}u(t)$ **(b)** $\int_0^t e^{-\tau}d\tau$ **(c)** $u(t)$ **(d)** $\delta(t)$

(5) Given that *x*(*t*) has the Fourier transform *X*(*jω*), express the Fourier transform of $y(t)=x(1-t)+x(-1-t)$ **in terms of *X*(*jω*) as _____.**

(a) $2\cos\omega X(j\omega)$ **(b)** $2\cos\omega X(-j\omega)$ **(c)** $2j\sin\omega X(j\omega)$ **(d)** $-2j\sin\omega X(-j\omega)$

(6) Given that $X(e^{j\omega})=j\cos 2\omega\sin 3\omega$**, the inverse Fourier transform of it is a ____ function.**

(a) real and even (b) real and odd (c) purely imaginary (d) neither even nor odd（非奇非偶）

(7) Let *x*(*t*) be a signal with Nyquist（奈奎斯特） rate ω_0**, then the Nyquist rate of** $\frac{dx(t)}{dt}$ **is __.**

(a) ω_0 **(b)** $2\omega_0$ **(c)** $\omega_0/2$ **(d) cannot be determined**

(8) If signal *x*(*t*) is real and even, and the Laplace transform *X*(*s*) of *x*(*t*) has a pole at $s=-2+j$ **, then *X*(*s*) must have some other poles as following except for ____.**

(a) $-2-j$ **(b)** $2+j$ **(c)** $2-j$ **(d) 0**

(9) A stable LTI system has system function $H(s)=\frac{4e^{-s}}{s^2-2s-3}$**, its impulse response *h*(*t*) is ____.**

(a) $(e^{3t-3}-e^{1-t})u(t-1)$ **(b)** $-e^{3t-3}u(1-t)+e^{1-t}u(t-1)$

(c) $(e^{3t-3}-e^{1-t})u(1-t)$ **(d)** $e^{3t-3}u(1-t)-e^{1-t}u(t-1)$

(10) Discrete time signal *x*[*n*] has z-transform $X(z)=\dfrac{1}{z+2}$**, which statement below is true?**

(a) If $3^{-n}x[n]$ **is not absolutely summable（绝对可和）, the ROC of** $X(z)$ **is** $|z|>2$**.**

(b) If $x[n]*3^{-n}u[n]$ **is absolutely summable, the ROC of** $X(z)$ **is** $|z|>2$**.**

(c) If $3^{n}x[n]$ **is absolutely summable, the ROC of** $X(z)$ **is** $|z|<2$**.**

(d) If $x[n]*3^{n}u[n]$ **is not absolutely summable, the ROC of** $X(z)$ **is** $|z|<2$**.**

2. (10 points) For each of the following statements, determine whether it is true or false and mark with "T" or "F" in the bracket. (2 points / question)

① **The system with input-output relation** $y(t)=t\,x(t)+2$ **is invertible（可逆的）. (F)**

② **A causal LTI system must have both its impulse response *h*(*t*) and step response *s*(*t*) causal. (T)**

③ **Unit step function *u*(*t*) has finite energy. (F)**

④ **Signal *x*(*t*) with its FS representation** $x(t)=\sum_{k=-100}^{100}2^{k}e^{jk\frac{2\pi}{50}t}$ **is not real. (T)**

⑤ **If an ideal low-pass filter with cut-off frequency** $\omega_{Lc}=\dfrac{\pi}{4}$ **is parallel connected（并联） with an ideal high-pass filter with cut-off frequency** $\omega_{Hc}=\dfrac{3\pi}{4}$**, we can get a band-stop filter. (T)**

3. (30 points) Short answer questions.

(1) (6 points) Figure 1 shows signal *x*(*t*), draw the graphs of $\dfrac{dx(t)}{dt}$ **and its odd-part** $Od\{x(t)\}$**.**

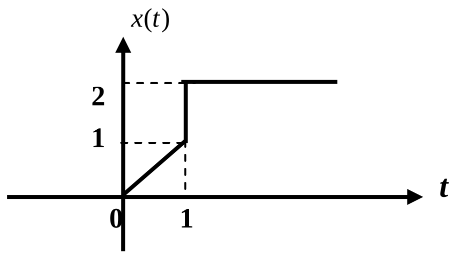
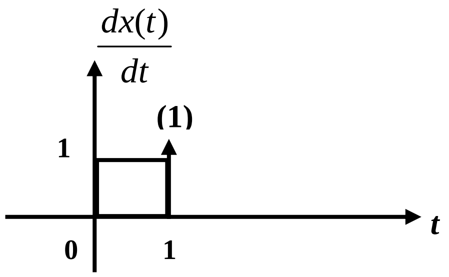
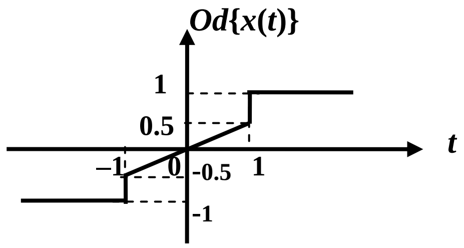


Figure 1

Solution:





(2) (8 points) Consider a continuous-time causal LTI system with system function $H(s)=\frac{1}{s+2}$.

If input $x(t)=\sin 2t$ **, determine the corresponding output** ***y*****(*****t*****).**

Solution: $H(j\omega)=\frac{1}{j\omega+2}$, **The response to frequency** $\omega=2$ **contained in** ***x*****(*****t*****) is:**

$$\left|H(j2)=\frac{1}{j2+2}\right|=\frac{1}{2\sqrt{2}} \qquad \arg H(j2)=-\arg(j2+2)=-\frac{\pi}{4}$$

Thus

$$y(t)=\frac{1}{2\sqrt{2}}\sin\left(2t-\frac{\pi}{4}\right)$$

(3) (8 points) Determine the inverse z-transform ***x*****[*****n*****] of** $X(z)=\frac{z^3+2z^2-3.5z}{(z+1)(z-0.5)(z-2)}, 1<|z|<2$.

Solution: $X(z)=\frac{z}{z-0.5}-\frac{z}{z+1}+\frac{z}{z-2}, 1<|z|<2$

$$0.5^n u[n]\leftrightarrow\frac{z}{z-0.5},\quad |z|>0.5 \qquad -(-1)^n u[n]\leftrightarrow\frac{-z}{z+1},\quad |z|>1 \qquad -2^n u[-n-1]\leftrightarrow\frac{z}{z-2}, |z|<2$$

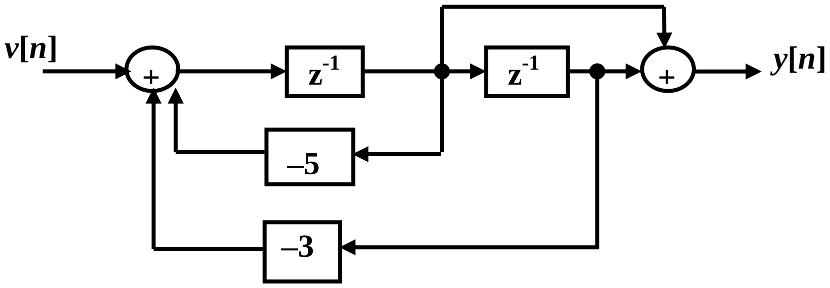
$$x[n]=\left[0.5^n-(-1)^n\right]u[n]-2^n u[-n-1]$$

(4) (8 points) A discrete-time causal LTI system is described by the input-output equation

$$y[n]+5y[n-1]+3y[n-2]=v[n-1]+v[n-2]$$

Where ***v*****[*****n*****] is the input,** ***y*****[*****n*****] is the output. Draw a direct-form block diagram（直接型框图）with least number（最少数量） of unit delayer（单位延时器） and construct state model based on this diagram.**

Solution:



Take the output of two unit delayers as state variables $x_1[n]$ **and** $x_2[n]$**, we can construct following state model: (4 points for state model)**

$$x_1[n+1]=x_2[n]$$
$$x_2[n+1]=-3x_1[n]-5x_2[n]+v[n]$$
$$y[n]=x_1[n]+x_2[n]$$

Matrix form:

$$\begin{bmatrix} x_1[n+1] \\ x_2[n+1] \end{bmatrix}=\begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix}\begin{bmatrix} x_1[n] \\ x_2[n] \end{bmatrix}+\begin{bmatrix} 0 \\ 1 \end{bmatrix}v[n]$$

$$y[n]=\begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} x_1[n] \\ x_2[n] \end{bmatrix}$$

4. (10 points) Consider an ideal low-pass filter with frequency response $H(j\omega)=\begin{cases} e^{-j\omega} & |\omega|<\omega_c \\ 0 & |\omega|>\omega_c \end{cases}$**, where** ω_c **is the cut-off frequency（截止频率）. Try to find the value of** ω_c **if we want to obtain an output signal** ***y*(*t*)** **with its total energy 50% of that of the input** $x(t)=2e^{-t}u(t)$**. （要求输出信号的总能量为输入信号总能量的 50%）**

Solution: **the energy of the output is**

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}|Y(j\omega)|^2d\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty}|X(j\omega)H(j\omega)|^2d\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty}|X(j\omega)|^2|H(j\omega)|^2d\omega=\frac{1}{2\pi}\int_{-\omega_c}^{\omega_c}|X(j\omega)|^2d\omega$$

From requirement we can write $\frac{1}{2\pi}\int_{-\omega_c}^{\omega_c}|X(j\omega)|^2d\omega=50\%\cdot\frac{1}{2\pi}\int_{-\infty}^{\infty}|X(j\omega)|^2d\omega$ ①

Since $X(j\omega)=\frac{2}{j\omega+1}$**, Thus** $|X(j\omega)|^2=\frac{4}{1+\omega^2}$**, taking into equation ① yielding**

$$\frac{1}{2\pi}\int_{-\omega_c}^{\omega_c}\frac{4}{1+\omega^2}d\omega=\frac{0.5}{2\pi}\int_{-\infty}^{\infty}\frac{4}{1+\omega^2}d\omega$$

So

$$\arctan\omega\Big|_{-\omega_c}^{\omega_c}=0.5\cdot\arctan\omega\Big|_{-\infty}^{\infty}$$

$$2\arctan\omega_c=0.5\left(\frac{\pi}{2}+\frac{\pi}{2}\right)=\frac{\pi}{2}\Rightarrow\arctan\omega_c=\frac{\pi}{4}\Rightarrow\omega_c=1\quad \text{rad/s}$$

5. (15 points) A second-order causal LTI system is composed by two parallel connected causal systems S_1 and S_2. S_1 and S_2 are described by following equations respectively:

$$S_1: \quad \frac{d\,y_1(t)}{d\,t} + y_1(t) = x_1(t) \qquad S_2: \quad \frac{d\,y_2(t)}{d\,t} + 3y_2(t) = x_2(t)$$

Suppose that the initial states of this second-order system are not zero, answer following questions:

(a) Determine the overall $H(s)$ and specify its ROC;

(b) Is the second-order system stable?

(c) It is known that when input $x(t)=u(t)$ the complete response $y(t)=\frac{4}{3}\left[1-e^{-3t}\right]u(t)$, try to find zero-state response $y_{zs}(t)$, zero-input response $y_{zi}(t)$, and initial values of $y(0^-)$, $y'(0^-)$.

(d) Specify natural response（自然响应）$y_h(t)$ and forced response（受迫响应）$y_p(t)$ in above complete response $y(t)=\frac{4}{3}\left[1-e^{-3t}\right]u(t)$.

Solution: **(a)** $H(s)=\frac{1}{s+1}+\frac{1}{s+3}=\frac{2s+4}{s^2+4s+3}$ **ROC: Re{s} > –1**

(b) It's stable.

(c) $Y_{zs}(s)=X(s)H(s)=\frac{2s+4}{s(s+1)(s+3)}=\frac{4/3}{s}+\frac{-1}{s+1}+\frac{-1/3}{s+3}, \quad \mathrm{Re}\{s\}>0$

Thus $y_{zs}(t)=\left[\frac{4}{3}-e^{-t}-\frac{1}{3}e^{-3t}\right]u(t) \qquad y_{zi}(t)=y(t)-y_{zs}(t)=\left[e^{-t}-e^{-3t}\right]u(t)$

$y(0^-)=y_{zi}(0^-)=0$, $\qquad y'(0^-)=y_{zi}'(0^-)=-e^{-t}+3e^{-3t}\big|_{t=0}=2$

(d) $y_h(t)=-\frac{4}{3}e^{-3t}u(t)$, $\qquad y_p(t)=\frac{4}{3}\,u(t)$

6. (15 points) A discrete-time causal and stable LTI system *S* is represented by the following equation

Score	

$$y[n]+\frac{k}{4}y[n-1]=x[n]-\frac{k}{3}x[n-1]$$

Here *k* is a real number.

(a) Find expression of *H*(*z*), specify its ROC and determine the unit sample response *h*[*n*] in terms of *k*（用 *k* 表示）；

(b) Determine the range of value of *k*（确定 *k* 的取值范围）；

(c) If *k* = 2, find the response *y*[*n*] for the input $x[n]=1$**;**

(d) Suppose that *S* is invertible and its inverse system（逆系统）S_1 is also both causal and stable, find the expression of system function $H_1(z)$ of S_1 in terms of *k*. And determine the range of value of *k* by analyzing the locations（位置）of pole and zero of *H*(*z*).

Solution: **(a)** $H(z)=\dfrac{1-\frac{k}{3}z^{-1}}{1+\frac{k}{4}z^{-1}}=\dfrac{z-\frac{k}{3}}{z+\frac{k}{4}}$ $\qquad |z|>\left|\dfrac{k}{4}\right|$

$$h[n]=\left(-\frac{k}{4}\right)^n u[n]-\frac{k}{3}\left(-\frac{k}{4}\right)^{n-1}u[n-1]=\delta[n]+\frac{7}{3}\left(-\frac{k}{4}\right)^n u[n]$$

(b) The only pole is in unit circle, so $\left|\dfrac{k}{4}\right|<1\Rightarrow |k|<4$

(c) When *k* = 2, $H(z)=\dfrac{z-\frac{2}{3}}{z+\frac{2}{4}}$, $|z|>\dfrac{1}{2}$**, then** $y[n]=H(1)\bullet x[n]=\dfrac{1-\frac{2}{3}}{1+\frac{2}{4}}\bullet 1=\dfrac{2}{9}$

(d) $H_1(z)=\dfrac{z+\frac{k}{4}}{z-\frac{k}{3}}$

Because S_1 is also stable, both the pole and zero of *H*(*z*) are located in unit circle, i.e.

$\left|\dfrac{k}{3}\right|<1\Rightarrow |k|<3$