by Jack L. Treynor

Market Efficiency and the Bean Jar Experiment

MARKET EFFICIENCY is a premise, not a conclusion.[1] The finance literature offers no proof of market efficiency; indeed, the only rationale offered in that literature is inconsistent with risk aversion. The rationale asserts that investors aware of a discrepancy between price and value will expand their positions until the discrepancy disappears. The problem is that, as those positions expand, portfolio risk increases faster than portfolio return. Beyond a certain point, further expansion is irrational if the investors in question are risk-averse.

The standard rationale has another problem. It assumes that those investors who know the true value of a security expand their positions when that value exceeds the market price, while those investors with a mistaken estimate of value don't. But the latter also perceive a discrepancy between price and their estimate of value. In effect, the rationale assumes that those investors who are right know they are right, while those investors who are wrong know they are wrong—an unlikely state of affairs.

Where does the accuracy of market prices come from, if not from a few determined investors who know they are right? It comes from the faulty opinions of a large number of investors who err independently. If their errors are wholly independent, the standard error in equilibrium price declines with roughly the square root of the number of investors.[1]

But what assurance do we have that investors' errors are really independent? Are errors *ever* independent across the population of investors? Who is to say whether GM or AT&T is correctly priced or mispriced? Fortunately, the mechanism whereby a large number of error-prone judgments are pooled to achieve a more accurate "consensus" is not confined to finance, or even economics.

The Bean Jar

The mechanism is present even in traditional "bean jar" contests, where observers are asked to guess the number of beans filling a jar. How accurate is the mean of the guesses? How much more accurate than the average guess? Do shared errors creep into the guesses, hence into the mean?

Results of bean jar experiments conducted in the author's investment classes indicate that the mean estimate has been close to the true value. In the first experiment, the jar held 810 beans; the mean estimate was 841, and only two of 46 guesses were closer to the true value. In the second experiment, the jar held 850 beans, and the mean estimate was 871; only one of 56 guesses was closer to the true value.

These results suggest that, in situations where the subjects have not been schooled in a "correct" approach, the bulk of the individual errors will be independent, rather than shared. Apparently it doesn't take knowledge of beans, jars or packing factors for a group of students to make an accurate estimate of the number of beans in a jar. All it takes is independence.

In a second set of bean jar experiments, the observers were cautioned to allow (after recording their original guesses) for first, air space at the top of the bean jar and, second, the fact that the jar, being plastic rather than glass, had thinner walls than a conventional jar, hence more capacity for the same external dimensions. The means of the guesses after the first and second "warnings" were 952.6 and 979.2, corresponding respectively to errors of 102.6 and 129.2. Although the cautions weren't intended to be misleading, they seem to have caused some shared error to creep into the estimates.

Published Research and Market Prices

Shared errors may be as common in appraising companies as in appraising bean jars. There is

1. Footnotes appear at end of article.

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one class of shared errors that is particularly important to asset prices—the shared errors created by published research. When a piece of investment research is published, some investors will be persuaded by it and some (including those who haven't read it, or abstracts of it) won't. The accuracy of value estimates based on this research will presumably be higher than the average accuracy of the individual estimates it replaces. If other things were equal, this effect would increase the accuracy of the mean estimate, hence the price.

But other things aren't equal. Publication of an estimate may replace many independent estimates with a single estimate. Any error in the published estimate will be reflected in the estimates of all the investors persuaded by it. The impact of any published research will thus depend on both its accuracy and its persuasiveness.

Consider the case of a totally persuasive piece of research, one that reaches and persuades every investor. Any error implicit in this research will be fully reflected in equilibrium price. Suppose the publishing analyst (who is, after all, only human) makes an error equivalent to 10 per cent of a security's true value. Suppose, further, that he is five times more accurate than the average investor in his audience. And suppose, finally, that before he published, the investors in his audience had 10,000 independent opinions. If their opinions had roughly equal weights, then the error in the stock's price after publication of the research will be 20 times as large as the error before publication.[2] In general, the more persuasive a published opinion is, the more damage it does to market efficiency.

The number of investors persuaded by a piece of research increases with time elapsed since its publication up to an asymptotic limit that may take several months (or less) to reach. The accuracy of market price first rises following publication of research, then begins to fall. It keeps on falling until persuasion has gone as far as it can go. The calculations presented in the appendix suggest that the point of minimum error is likely to be reached before a published opinion has propagated very far.

Trading on Information

These considerations suggest one approach for investing in published research: Wait until propagation is complete, or almost complete, and then copper it. (Has frequency of reference to the research in brokers' letters and periodicals died down? Has that influential Wall Street analyst known to many of us as Zachary Zilch finally adopted the key arguments in the research?)

In order to profit from fully propagated research, one must of course know the sign of the (net) error in the research; this is tantamount to knowing which way the research moved the price. Unfortunately, many other pieces of information arriving in the interim will also have moved the price, and in combination they may be more important than any error in the research. The induction task facing the investor is considerable.[3]

As the appendix explains, the accuracy of a mean estimate depends critically on the degree of shared (i.e., systematic) error present. Absent shared error, the standard error of the mean will be the average individual standard error divided by the square root of the number of individual errors. As that number increases, the standard error of the mean falls. If the number is large enough, the mean error approaches zero. The practical implication is clear: Analyzing dimensions of value on which other investors form their independent opinions is a "futile and unnecessary endeavor."[4]

To put it the other way round, unless the analyst has reason to think shared error is present, the market price will always be his best source of information on value. Specifically, if a company's value is significantly sensitive to 100 attributes and the analyst can identify shared errors in two, his estimating advantage in these two will usually be overwhelmed by his disadvantage in the other 98. If shared errors are rare, or at least hard to identify, then the analyst is better off estimating the incremental impact of the shared errors he can identify, rather than attempting to estimate the overall value of the company.

Conclusion

Stocks, like people, are concrete: They have histories and personalities, unique strengths and weaknesses. We all like to believe we can judge people, and some of us like to believe we can judge stocks. But attributes are abstractions. Many of us are diffident about our ability to evaluate abstractions, deferring instead to experts. For this reason, shared error (i.e., the expert's error) will be far more common in investors' assessment of attributes than in their assessment of stocks. But this means that investment opportunity in attributes will arise more often than investment opportunity in stocks.

Stocks often have common attributes; a shared error present in one such attribute represents an investment opportunity with respect to every stock whose value depends on it. And, if we weight our investment in these stocks appropriately, we can minimize our exposure to their other attributes.

Some students of political science have argued that democracies are more successful electing the right officials than electing the right policies. If so, the reason is not hard to find: Voters defer to experts in their choice of policies, but not in their choice of men. Shared error is consequently less of a problem in the latter than in the former. ■

Appendix

Published Research and Shared Error

In general, published research will have two dimensions—(1) its relative accuracy, or how much it reduces the error of the average investor who is persuaded by it, and (2) its persuasiveness, or what fraction of the investor population abandons their previous view in deference to the published view.

Let the average of the untutored investors' standard valuation errors be σ_1 and the standard error of the published research be σ_2. Let the number of investors be n and the number persuaded by the published view at a point in time be m. Then the error variance of the equilibrium price is:

$$\frac{1}{n^2}\left[(n-m)^2\frac{\sigma_1{}^2}{n-m}+m^2\sigma_2{}^2\right],$$

assuming the untutored opinions are independent of each other and of the published opinion. (The standard error in the consensus of persuaded investors is of course σ_2 m, and the standard error in the consensus of unpersuaded investors is $\sigma_1/\sqrt{n-m}$.

Differentiating the expression for the error variance of the equilibrium price with respect to m, we have:

$$\frac{1}{n^2}(-\sigma_1{}^2+2\sigma_2{}^2m).$$

Table AI

Relative Accuracy (σ_1/σ_2)	*Number of Persuaded Investors at which Error in Price is Minimal*
2	2
10	50
100	5,000

The second derivative is

$$2\left(\frac{\sigma_2}{n}\right)^2 > 0.$$

Thus the error in the price reaches its minimum when

$$m=\frac{1}{2}\left(\frac{\sigma_1}{\sigma_2}\right)^2.$$

As still more investors are persuaded to the published view, accuracy declines. The obvious thing about this expression is that it doesn't depend on n. This fact permits us to compute the sort of results presented in Table AI. As the number of investors persuaded increases up to n, the error variance increases up to $\sigma_2{}^2$. The lowest the variance ever gets is:

$$\frac{\sigma_1{}^2}{n}\left[1-\frac{1}{n}\left(\frac{\sigma_1}{\sigma_2}\right)^2\right].$$

Measuring Standard Error

Assume the individual investor takes a position, k_i, proportional to the discrepancy between a security's price, p, and his perception of its value, v_i, so that:

$$k_i = w_i\left(\frac{v_i-p}{\sigma_i{}^2}\right),$$

where w_i is the investor's factor of proportionality.

Market equilibrium requires that all active positions—i.e., all departures from the market portfolio—must sum to zero. We therefore have:

$$\Sigma_i\, k_i = \Sigma_i \frac{w_i}{\sigma_i{}^2}(v_i-p)=0,$$

$$p=\frac{\Sigma_i\left(\frac{w_i}{\sigma_i{}^2}\right)v_i}{\Sigma_i \frac{w_i}{\sigma_i{}^2}}.$$

Let:

$$W_i = \frac{w_i k_i^2}{\Sigma_i w_i k_i^2}.$$

Then we have:

$$p = \Sigma W_i v_i,$$

$$\Sigma W_i = 1.$$

Clearly, price p is a weighted mean of individual investors' opinions.

Now consider the true value of the security, P. The error, e_p, in the price is:

$$\begin{aligned} e_p &= p - P, \\ &= \Sigma W_i v_i - P, \\ &= \Sigma W_i v_i - \Sigma W_i P. \end{aligned}$$

If we define the individual investor's error, e_i, as follows

$$e_i = v_i - P,$$

we have:

$$e_p = \Sigma W_i e_i.$$

If we assume individual investors' errors are mutually independent, then the variance of the error in the price is:

$$\begin{aligned} \sigma_p^2 &= E(e_p^2) \\ &= E[\Sigma_i (W_i e_i)^2] \\ &= \Sigma_i W_i^2 E(e_i^2) \\ &= \Sigma_i W_i^2 \sigma_i^2, \end{aligned}$$

assuming $E(e_i)$ equals zero for all i.

To simplify, replace the individual investor's error variance σ_i^2 with a representative variance σ^2, and his weight W_i with an average weight W. Then:

$$\begin{aligned} \sigma_p^2 &= \text{variance } (e_p) \\ &= E(e_p^2) \\ &= \sigma^2 \Sigma W^2, \end{aligned}$$

if we assume $E(e_i) = 0$, hence $E(e_p) = \Sigma W_i E(e_i) = 0$. But we know that:

$$\Sigma W = \Sigma W_i = 1.$$

If there are N investors, we have:

$$\Sigma W = NW = 1,$$

$$W = \frac{1}{N},$$

whence,

$$W^2 = \frac{1}{N^2},$$

$$\Sigma_i W^2 = N\left(\frac{1}{N^2}\right) = \frac{1}{N}.$$

We thus have:

$$\text{variance } (e_p) = \sigma^2 \Sigma W^2 = \frac{\sigma^2}{N}.$$

Thus the standard error σ_p is:

$$\begin{aligned} \sigma_p &= \sqrt{\text{variance } (e_p)} \\ &= \sqrt{\frac{\sigma^2}{N}} \\ &= \frac{\sigma}{\sqrt{N}}. \end{aligned}$$

Footnotes

1. Readers interested in this subject are referred to F. Black, "Noise," *Journal of Finance*, July 1986, pp. 529–543.
2. When individual investors bet against the market price, they are in effect betting that the price reflects a shared error. (Betting against the market assumes we are exempt from an error other investors share.) On the other hand, when academics assert that markets are efficient, they are effectively asserting that shared errors are nonexistent, or at least rare.
3. In fact, it is reminiscent of, but more difficult than, the problem addressed in J. Treynor and R. Ferguson, "In Defense of Technical Analysis," *Journal of Finance*, July 1985, pp. 757–773.
4. J.H. Langbein and R.A. Posner, "The Revolution in Trust Investment Law," *American Bar Association Journal* 62, pp. 764–768. The authors' actual words (regarding the general concept of market efficiency) were: "Trustees who ignore the new learning and who underperform the market will be hard pressed to justify their adherence to an investment strategy of demonstrated riskiness, costliness and futility."