

Laboratoire 5: Optimisation avec contraintes et calcul variationnel

[illegible]

```
using Krylov, LinearAlgebra, Logging, NLPModels, NLPModelsIpopt, Printf, SolverCore, Test
```

```
In [ ]: using PDENLPModels, Gridap

WARNING: Method definition testargs(Gridap.Arrays.PosNegReindex{A, B} where B where A, Integer) in module Arrays at /home/julien/.julia/package
s/Gridap/EZQEK/src/Arrays/PosNegReindex.jl:10 overwritten in module PDENLPModels at /home/julien/.julia/packages/PDENLPModels/pW0Iv/src/PDENLPM
odels.jl:16.
ERROR: Method overwriting is not permitted during Module precompilation. Use `__precompile__(false)` to opt-out of precompilation.
WARNING: method definition for _compute_hess_structure at /home/julien/.julia/packages/PDENLPModels/pW0Iv/src/hessian_struct_nnz_h_functions.jl:
70 declares type variable T but does not use it.
WARNING: method definition for _compute_hess_structure at /home/julien/.julia/packages/PDENLPModels/pW0Iv/src/hessian_struct_nnz_h_functions.jl:
74 declares type variable T but does not use it.
WARNING: method definition for _compute_hess_structure at /home/julien/.julia/packages/PDENLPModels/pW0Iv/src/hessian_struct_nnz_h_functions.jl:
70 declares type variable T but does not use it.
WARNING: method definition for _compute_hess_structure at /home/julien/.julia/packages/PDENLPModels/pW0Iv/src/hessian_struct_nnz_h_functions.jl:
74 declares type variable T but does not use it.
```

```
In [ ]: using BenchmarkTools, SolverCore, LinearOperators
using JSOSolvers, NLPModels #
using SolverBenchmark
using LinearAlgebra, NLPModels, Printf
```

Quelques commentaires en Julia

Les kwargs: choix optionnels

Dans le projet du dernier labo, une des questions demandait d'ajouter une option pour utiliser la fonction `lsmr` ou `lsqr`. C'est le cas typique d'arguments optionnels:

- On veut proposer un choix par défaut à l'utilisateur, par exemple `lsqr`;
- On veut laisser la possibilité à l'utilisateur de changer;
- On voudrait aussi pouvoir ajouter d'autres par la suite (sans avoir à tout modifier).

```
In [ ]: function dsol(A, b, ε; solver :: Function = lsqr)
    (d, stats) = solver(A, b, atol = ε)
    return d
end

dsol (generic function with 1 method)
```

A noter que l'on donne des valeurs par défaut aux arguments qui apparaissent après le `;`.

Exercice 1: Pénalité quadratique pour les ADNLPModels

Dans cet exercice, on va étudier une version simple d'une méthode de pénalité quadratique pour les problèmes d'optimisation avec contraintes d'égalité.

$$\begin{aligned} &\text{min} \quad f(x) \\ &\text{s.t.} \quad c(x) = 0. \end{aligned}$$

Dans les labos précédents, on a déjà utilisé un NLPModel particulier, le ADNLPModel:

```
In [ ]: using ADNLPModels, LinearAlgebra, Test
fH(x) = (x[2]+x[1].^2-11)^2 + (x[1]+x[2].^2-7)^2
x0H = [10., 20.]
cH(x) = [x[1]-1]
himmelblau = ADNLPModel(fH, x0H, cH, [0.], [0.]);
```

Attention: dans toute la suite de l'exercice on suppose que les bornes sur les contraintes `nlp.meta.lcon` et `nlp.meta.ucon` sont 0 pour simplifier.

Question 1: Transformer un ADNLPModel en un problème pénalisé

Coder la fonction `quad_penalty_adnlp` qui prend en entrée un ADNLPModel, et un paramètre p et qui retourne un nouveau ADNLPModel qui correspond au problème sans contrainte:

$$\min_x f(x) + \frac{\rho}{2} \|c(x)\|^2.$$

Remarque: on peut accéder aux fonctions f et c par `NLPModels.obj()` et `NLPModels.cons()`.

```
In [ ]: using NLPModels, NLPModelsIpopt

function quad_penalty_adnlp(nlp :: ADNLPModel, p :: Real)
    """
    Returns a new NLPModel with a quadratic penalty term added to the objective function.
    """
    f = x -> obj(nlp,x) + p/2 .* norm(cons(nlp,x))^2
    nlp_quad = ADNLPModel(f, nlp.meta.x0)
    return nlp_quad
end

quad_penalty_adnlp (generic function with 1 method)
```

```
In [ ]: #Faire des tests pour vérifier que ça fonctionne.

himmelblau_quad = quad_penalty_adnlp(himmelblau, 1)
@test himmelblau_quad.meta.ncon == 0
@test obj(himmelblau_quad, zeros(2)) == 170.5

stats = ipopt(himmelblau_quad);
```

This program contains Ipopt, a library for large-scale nonlinear optimization.
Ipopt is released as open source code under the Eclipse Public License (EPL).
For more information visit <https://github.com/coin-or/Ipopt>

This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

```
Number of nonzeros in equality constraint Jacobian...:    0
Number of nonzeros in inequality constraint Jacobian.:    0
Number of nonzeros in Lagrangian Hessian.....:        3

Total number of variables.....:        2
      variables with only lower bounds:        0
      variables with lower and upper bounds:      0
      variables with only upper bounds:        0
Total number of equality constraints.....:        0
Total number of inequality constraints.....:        0
      inequality constraints with only lower bounds:    0
      inequality constraints with lower and upper bounds: 0
      inequality constraints with only upper bounds:    0

iter   objective    inf_pr  inf_du lg(mu)  ||d||  lg(rg) alpha_du  alpha_pr  ls
   0   1.7433050e+05  0.00e+00  1.00e+02  -1.0  0.00e+00   -  0.00e+00  0.00e+00   0
   1   3.3498968e+04  0.00e+00  2.95e+01  -1.0  6.65e+00   -  1.00e+00  1.00e+00f   1
   2   6.2387553e+03  0.00e+00  8.63e+00  -1.0  4.39e+00   -  1.00e+00  1.00e+00f   1
   3   1.0981700e+03  0.00e+00  2.49e+00  -1.0  2.87e+00   -  1.00e+00  1.00e+00f   1
   4   1.8219071e+02  0.00e+00  6.96e-01  -1.0  1.84e+00   -  1.00e+00  1.00e+00f   1
   5   2.8534326e+01  0.00e+00  1.91e-01  -1.7  1.17e+00   -  1.00e+00  1.00e+00f   1
   6   4.5704179e+00  0.00e+00  4.69e-02  -2.5  6.78e-01   -  1.00e+00  1.00e+00f   1
   7   2.0605966e+00  0.00e+00  7.67e-03  -2.5  3.05e-01   -  1.00e+00  1.00e+00f   1
   8   1.9683212e+00  0.00e+00  4.00e-04  -3.8  7.35e-02   -  1.00e+00  1.00e+00f   1
   9   1.9680523e+00  0.00e+00  1.31e-06  -5.7  4.25e-03   -  1.00e+00  1.00e+00f   1
iter   objective    inf_pr  inf_du lg(mu)  ||d||  lg(rg) alpha_du  alpha_pr  ls
  10   1.9680523e+00  0.00e+00  1.41e-11  -8.6  1.40e-05   -  1.00e+00  1.00e+00f   1
```

Number of Iterations....: 10

	(scaled)	(unscaled)
Objective.....:	6.0633813587165422e-03	1.9680523214122152e+00
Dual infeasibility.....:	1.4118583374320126e-11	4.5826097916368261e-09
Constraint violation....:	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....:	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....:	1.4118583374320126e-11	4.5826097916368261e-09

```
Number of objective function evaluations      = 11
Number of objective gradient evaluations      = 11
Number of equality constraint evaluations      = 0
Number of inequality constraint evaluations    = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations     = 10
Total seconds in IPOPT                       = 1.599
```

EXIT: Optimal Solution Found.

```
In [ ]: stats.solution

2-element Vector{Float64}:
 2.9678888605539164
 2.0186523682501782

Ajout d'un autre test similaire.
```

```
In [ ]: #Ajouter au moins un autre test similaire avec des contraintes.
using JuMP
using NLPModels, LinearAlgebra, NLPModelsIpopt

# modifié à partir du code de test_set.jl
@testset "Simple problem v2" begin
    n = 10
    nlp = ADNLPModel(x->dot(x, x), zeros(n),
                     x->[sum(x) - 1], zeros(1), zeros(1))

    quad_nlp = quad_penalty_adnlp(nlp, 1)
    stats = ipopt(quad_nlp)

    dual, primal, status = stats.dual_feas, stats.primal_feas, stats.status
    @test sum(stats.solution) - 1 < 1e-6
    @test dual < 1e-6
    @test primal < 1e-6
    @test status == :first_order
end
```

This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

```
Number of nonzeros in equality constraint Jacobian...:      0
Number of nonzeros in inequality constraint Jacobian.:      0
Number of nonzeros in Lagrangian Hessian.....:      55

Total number of variables.....:      10
      variables with only lower bounds:      0
      variables with lower and upper bounds:      0
      variables with only upper bounds:      0
Total number of equality constraints.....:      0
Total number of inequality constraints.....:      0
      inequality constraints with only lower bounds:      0
      inequality constraints with lower and upper bounds:      0
      inequality constraints with only upper bounds:      0
```

```
iter   objective    inf_pr  inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0   5.0000000e-01  0.00e+00  1.00e+00  -1.0  0.00e+00    -  0.00e+00  0.00e+00   0
   1   8.3333333e-02  0.00e+00  5.55e-17  -1.7  8.33e-02    -  1.00e+00  1.00e+00f  1
```

Number of Iterations....: 1

	(scaled)	(unscaled)
Objective.....:	8.333333333333315e-02	8.333333333333315e-02
Dual infeasibility.....:	5.5511151231257827e-17	5.5511151231257827e-17
Constraint violation....:	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....:	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....:	5.5511151231257827e-17	5.5511151231257827e-17

```
Number of objective function evaluations      = 2
Number of objective gradient evaluations      = 2
Number of equality constraint evaluations      = 0
Number of inequality constraint evaluations    = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations     = 1
Total seconds in IPOPT                       = 2.219
```

EXIT: Optimal Solution Found.
Test Summary: | **Pass** **Total** **Time**
Simple problem v2 | **4** **4** 2.9s
Test.DefaultTestSet("Simple problem v2", Any[], 4, false, false, true, 1.71113805407431e9, 1.711138056928362e9, false, "/home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb")

Question 2: KKT

Coder une fonction `KKT_eq_constraint(nlp :: AbstractNLPModel, x, λ)` qui vérifie si le point `x` avec multiplicateur de Lagrange `λ` satisfait les conditions KKT d'un problème avec contraintes d'égalités.

```
In [ ]: using ForwardDiff

function KKT_eq_constraint(nlp :: AbstractNLPModel, x, λ, tol=1e-3)
    """
    Check the KKT conditions for a given NLPModel at a given point (x,λ).
    """
    constr = y -> cons(nlp, y)
    ∇c= ForwardDiff.jacobian(constr, x)
    print(∇c)

    ∇f = grad(nlp, x)
    print(∇f)

    # Stationarity
    stationarity = ∇f + ∇c' * λ
    print(λ)
    print(stationarity)

    # Primal feasibility
    primal_feasibility = constr(x)
    print(primal_feasibility)

    # Check all conditions
    if all(abs.(stationarity) .< tol) && all(abs.(primal_feasibility) .< tol)
        kkt_bool= true
    else
        kkt_bool= false
    end
    return kkt_bool
end
```

KKT_eq_constraint (generic function with 2 methods)

```
In [ ]: #test
@testset "Simple problem v2" begin
    n = 10
    nlp = ADNLPModel(x->dot(x, x), zeros(n),
                     x->[sum(x) - 1], zeros(1), zeros(1))

    stats = ipopt(nlp)
    @test KKT_eq_constraint(nlp, stats.solution, stats.multipliers)

    dual, primal, status = stats.dual_feas, stats.primal_feas, stats.status
    @test sum(stats.solution) - 1 < 1e-6
    @test dual < 1e-6
    @test primal < 1e-6
    @test status == :first_order
end
```

This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...	10
Number of nonzeros in inequality constraint Jacobian..	0
Number of nonzeros in Lagrangian Hessian.....	55
Total number of variables.....	10
variables with only lower bounds:	0
variables with lower and upper bounds:	0
variables with only upper bounds:	0
Total number of equality constraints.....	1
Total number of inequality constraints.....	0
inequality constraints with only lower bounds:	0
inequality constraints with lower and upper bounds:	0
inequality constraints with only upper bounds:	0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	0.0000000e+00	1.00e+00	0.00e+00	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	1.0000000e-01	1.11e-16	0.00e+00	-1.7	1.00e-01	-	1.00e+00	1.00e+00h	1

```
Number of Iterations....: 1
```

	(scaled)	(unscaled)
Objective.....:	9.999999999999964e-02	9.999999999999964e-02
Dual infeasibility.....:	0.000000000000000e+00	0.000000000000000e+00
Constraint violation....:	1.1102230246251565e-16	1.1102230246251565e-16
Variable bound violation:	0.000000000000000e+00	0.000000000000000e+00
Complementarity.....:	0.000000000000000e+00	0.000000000000000e+00
Overall NLP error.....:	1.1102230246251565e-16	1.1102230246251565e-16

Number of objective function evaluations	= 2
Number of objective gradient evaluations	= 2
Number of equality constraint evaluations	= 2
Number of inequality constraint evaluations	= 0
Number of equality constraint Jacobian evaluations	= 2
Number of inequality constraint Jacobian evaluations	= 0
Number of Lagrangian Hessian evaluations	= 1
Total seconds in IPOPT	= 1.726

```
EXIT: Optimal Solution Found.  
[1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0][0.1999999999999998, 0.1999999999999998, 0.1999999999999998, 0.1999999999999998, 0.1999999999999998,  
0.1999999999999998, 0.1999999999999998, 0.1999999999999998, 0.1999999999999998, 0.1999999999999998][[-0.1999999999999998][0.0, 0.0, 0.  
0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0][[-1.1102230246251565e-16]]Test Summary: | Pass Total Time  
Simple problem v2 |      5      5 3.2s  
Test.DefaultTestSet("Simple problem v2", Any[], 5, false, false, true, 1.711138061195937e9, 1.71113806444558e9, false, "/home/julien/Documents/  
code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb")
```

Question 3: méthode de pénalité quadratique

```
In [ ]: using NLPModelsIpopt
```

```
In [ ]: function quad_penalty(nlp      :: AbstractNLPModel,
                             x        :: AbstractVector;
                             ε        :: AbstractFloat = 1e-3,
                             η        :: AbstractFloat = 1e6,
                             σ        :: AbstractFloat = 2.0,
                             ρ        :: AbstractFloat = 1.0,
                             max_eval :: Int = 1000,
                             max_time :: AbstractFloat = 30.,
                             max_iter :: Int = 150
                             )
    """
    Solve a nonlinear program with a quadratic penalty method.
    """

    ##### Initialiser cx et gx au point x;
    cx = cons(nlp,x) # TODO Initialiser la violation des contraintes
    gx = grad(nlp, x) # TODO Initialiser le gradient
    #####
    normcx = normcx_old = norm(cx)

    iter = 0

    el_time = 0.0
    tired   = neval_cons(nlp) > max_eval || el_time > max_time
    status  = :unknown

    start_time = time()
    too_small  = false
    normdual   = norm(gx) #exceptionnellement on ne va pas vérifier toute l'optimalité au début.
    optimal    = max(normcx, normdual) ≤ ε

    nlp_quad   = quad_penalty_adnlp(nlp, ρ)

    @info log_header([:iter, :nf, :primal, :status, :nd, :Δ],
                    [Int, Int, Float64, String, Float64, Float64],
                    hdr_override=Dict{:nf => "#F", :primal => "||F(x)||", :nd => "||d||"})

    while !(optimal || tired || too_small)

        #Appeler Ipopt pour résoudre le problème pénalisé en partant du point x0 = x.
        #utiliser l'option print_level = 0 pour enlever les affichages d'ipopt.
        stats = ipopt(nlp_quad) #...
        #####

        if stats.status == :first_order
            ##### Mettre à jour cx avec la solution renvoyé par Ipopt
            """
            https://jso.dev/SolverCore.jl/dev/reference/#SolverCore.GenericExecutionStats
            """
            x = stats.solution #TODO
            cx = cons(nlp,x) #TODO
            #####
            normcx_old = normcx
            normcx = norm(cx)
        end

        if normcx_old > 0.95 * normcx
            ρ *= σ
        end
    end
```

```
@info log_row(Any[iter, neval_cons(nlp), normcx, stats.status])

nlp_quad    = quad_penalty_adnlp(nlp, ρ)

el_time     = time() - start_time
iter      += 1
many_evals  = neval_cons(nlp) > max_eval
iter_limit  = iter > max_iter
tired       = many_evals || el_time > max_time || iter_limit || ρ ≥ η
##### Utiliser la réalisabilité dual renvoyé par Ipopt pour `normdual`
normdual    = norm(stats.dual_feas) # TODO
#####
optimal     = max(normcx, normdual) ≤ ε
end

status = if optimal
      :first_order
elseif tired
      if neval_cons(nlp) > max_eval
        :max_eval
      elseif el_time > max_time
        :max_time
      elseif iter > max_iter
        :max_iter
      else
        :unknown_tired
      end
elseif too_small
      :stalled
else
      :unknown
end

return GenericExecutionStats(nlp, status = status, solution = x,
                             objective = obj(nlp, x),
                             primal_feas = normcx,
                             dual_feas = normdual,
                             iter = iter,
                             elapsed_time = el_time,
                             solver_specific = Dict(:penalty => ρ))

end
```

quad_penalty (generic function with 1 method)

```
In [ ]: #Faire des tests pour vérifier que ça fonctionne.
stats = quad_penalty(himmelblau, x0H)
@test stats.status == :first_order
@test stats.solution ≈ [1.0008083416169895, 2.709969135758311] atol=1e-2
@test norm(cons(himmelblau, stats.solution)) ≈ 0. atol=1e-3
```

```
└ Info:   iter    #F    ||F(x)||      status    ||d||      Δ
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:34
└ Info:     0      69    2.0e+00    first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
```


This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 3

Total number of variables.....: 2
 variables with only lower bounds: 0
 variables with lower and upper bounds: 0
 variables with only upper bounds: 0
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
 inequality constraints with only lower bounds: 0
 inequality constraints with lower and upper bounds: 0
 inequality constraints with only upper bounds: 0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	1.7433050e+05	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	3.3498968e+04	0.00e+00	2.95e+01	-1.0	6.65e+00	-	1.00e+00	1.00e+00f	1
2	6.2387553e+03	0.00e+00	8.63e+00	-1.0	4.39e+00	-	1.00e+00	1.00e+00f	1
3	1.0981700e+03	0.00e+00	2.49e+00	-1.0	2.87e+00	-	1.00e+00	1.00e+00f	1
4	1.8219071e+02	0.00e+00	6.96e-01	-1.0	1.84e+00	-	1.00e+00	1.00e+00f	1
5	2.8534326e+01	0.00e+00	1.91e-01	-1.7	1.17e+00	-	1.00e+00	1.00e+00f	1
6	4.5704179e+00	0.00e+00	4.69e-02	-2.5	6.78e-01	-	1.00e+00	1.00e+00f	1
7	2.0605966e+00	0.00e+00	7.67e-03	-2.5	3.05e-01	-	1.00e+00	1.00e+00f	1
8	1.9683212e+00	0.00e+00	4.00e-04	-3.8	7.35e-02	-	1.00e+00	1.00e+00f	1
9	1.9680523e+00	0.00e+00	1.31e-06	-5.7	4.25e-03	-	1.00e+00	1.00e+00f	1
iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
10	1.9680523e+00	0.00e+00	1.41e-11	-8.6	1.40e-05	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 10

	(scaled)	(unscaled)
Objective.....	6.0633813587165422e-03	1.9680523214122152e+00
Dual infeasibility.....	1.4118583374320126e-11	4.5826097916368261e-09
Constraint violation....	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....	1.4118583374320126e-11	4.5826097916368261e-09

Number of objective function evaluations = 11
Number of objective gradient evaluations = 11
Number of equality constraint evaluations = 0
Number of inequality constraint evaluations = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations = 10
Total seconds in IPOPT = 0.376

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 3

Total number of variables.....: 2
 variables with only lower bounds: 0
 variables with lower and upper bounds: 0
 variables with only upper bounds: 0
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
 inequality constraints with only lower bounds: 0
 inequality constraints with lower and upper bounds: 0
 inequality constraints with only upper bounds: 0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	1.7437100e+05	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	3.3508306e+04	0.00e+00	2.95e+01	-1.0	6.65e+00	-	1.00e+00	1.00e+00f	1
2	6.2409455e+03	0.00e+00	8.63e+00	-1.0	4.39e+00	-	1.00e+00	1.00e+00f	1
3	1.0990139e+03	0.00e+00	2.49e+00	-1.0	2.87e+00	-	1.00e+00	1.00e+00f	1
4	1.8321284e+02	0.00e+00	6.96e-01	-1.0	1.84e+00	-	1.00e+00	1.00e+00f	1
5	3.0118995e+01	0.00e+00	1.90e-01	-1.7	1.16e+00	-	1.00e+00	1.00e+00f	1
6	6.3964831e+00	0.00e+00	4.65e-02	-2.5	6.74e-01	-	1.00e+00	1.00e+00f	1
7	3.9584822e+00	0.00e+00	7.44e-03	-2.5	2.99e-01	-	1.00e+00	1.00e+00f	1
8	3.8731860e+00	0.00e+00	3.67e-04	-3.8	7.01e-02	-	1.00e+00	1.00e+00f	1
9	3.8729648e+00	0.00e+00	1.06e-06	-5.7	3.81e-03	-	1.00e+00	1.00e+00f	1
iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
10	3.8729648e+00	0.00e+00	8.94e-12	-8.6	1.11e-05	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 10

	(scaled)	(unscaled)
Objective.....	1.1932234929502137e-02	3.8729648134178034e+00
Dual infeasibility.....	8.9421822039700225e-12	2.9024534997645897e-09
Constraint violation....	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....	8.9421822039700225e-12	2.9024534997645897e-09

Number of objective function evaluations = 11
Number of objective gradient evaluations = 11
Number of equality constraint evaluations = 0
Number of inequality constraint evaluations = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations = 10
Total seconds in IPOPT = 0.002

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 3

Total number of variables.....: 2
 variables with only lower bounds: 0
 variables with lower and upper bounds: 0
 variables with only upper bounds: 0

```
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
    inequality constraints with only lower bounds: 0
    inequality constraints with lower and upper bounds: 0
    inequality constraints with only upper bounds: 0
```

```
iter   objective   inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
  0   1.7445200e+05  0.00e+00  1.00e+02 -1.0  0.00e+00   -  0.00e+00  0.00e+00  0
  1   3.3526893e+04  0.00e+00  2.95e+01 -1.0  6.65e+00   -  1.00e+00  1.00e+00f  1
  2   6.2452219e+03  0.00e+00  8.63e+00 -1.0  4.39e+00   -  1.00e+00  1.00e+00f  1
  3   1.1005974e+03  0.00e+00  2.49e+00 -1.0  2.87e+00   -  1.00e+00  1.00e+00f  1
  4   1.8512475e+02  0.00e+00  6.95e-01 -1.0  1.83e+00   -  1.00e+00  1.00e+00f  1
  5   3.3124594e+01  0.00e+00  1.88e-01 -1.7  1.16e+00   -  1.00e+00  1.00e+00f  1
  6   9.8733329e+00  0.00e+00  4.57e-02 -2.5  6.67e-01   -  1.00e+00  1.00e+00f  1
  7   7.5711712e+00  0.00e+00  7.01e-03 -2.5  2.89e-01   -  1.00e+00  1.00e+00f  1
  8   7.4980811e+00  0.00e+00  3.10e-04 -3.8  6.40e-02   -  1.00e+00  1.00e+00f  1
  9   7.4979301e+00  0.00e+00  7.05e-07 -5.7  3.08e-03   -  1.00e+00  1.00e+00f  1
iter   objective   inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
 10   7.4979301e+00  0.00e+00  3.68e-12 -8.6  7.05e-06   -  1.00e+00  1.00e+00f  1
```

Number of Iterations....: 10

```

                                (scaled)                (unscaled)
Objective.....: 2.3100406937738620e-02  7.4979300838512017e+00
Dual infeasibility.....: 3.6827628054566818e-12  1.1953511513951298e-09
Constraint violation....: 0.0000000000000000e+00  0.0000000000000000e+00
Variable bound violation: 0.0000000000000000e+00  0.0000000000000000e+00
Complementarity.....: 0.0000000000000000e+00  0.0000000000000000e+00
Overall NLP error.....: 3.6827628054566818e-12  1.1953511513951298e-09
```

```
Number of objective function evaluations      = 11
Number of objective gradient evaluations      = 11
Number of equality constraint evaluations      = 0
Number of inequality constraint evaluations    = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations     = 10
Total seconds in IPOPT                       = 0.002
```

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

```
Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 3
```

```
Total number of variables.....: 2
    variables with only lower bounds: 0
    variables with lower and upper bounds: 0
    variables with only upper bounds: 0
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
    inequality constraints with only lower bounds: 0
    inequality constraints with lower and upper bounds: 0
    inequality constraints with only upper bounds: 0
```

```
iter   objective   inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
  0   1.7461400e+05  0.00e+00  1.00e+02 -1.0  0.00e+00   -  0.00e+00  0.00e+00  0
  1   3.3563719e+04  0.00e+00  2.95e+01 -1.0  6.65e+00   -  1.00e+00  1.00e+00f  1
  2   6.2533666e+03  0.00e+00  8.63e+00 -1.0  4.39e+00   -  1.00e+00  1.00e+00f  1
  3   1.1033652e+03  0.00e+00  2.49e+00 -1.0  2.86e+00   -  1.00e+00  1.00e+00f  1
  4   1.8845496e+02  0.00e+00  6.92e-01 -1.0  1.83e+00   -  1.00e+00  1.00e+00f  1
  5   3.8508295e+01  0.00e+00  1.86e-01 -1.7  1.15e+00   -  1.00e+00  1.00e+00f  1
  6   1.6154313e+01  0.00e+00  4.41e-02 -2.5  6.53e-01   -  1.00e+00  1.00e+00f  1
  7   1.4095165e+01  0.00e+00  6.23e-03 -2.5  2.70e-01   -  1.00e+00  1.00e+00f  1
  8   1.4040889e+01  0.00e+00  2.24e-04 -3.8  5.37e-02   -  1.00e+00  1.00e+00f  1
  9   1.4040816e+01  0.00e+00  3.25e-07 -5.7  2.06e-03   -  1.00e+00  1.00e+00f  1
iter   objective   inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
 10   1.4040816e+01  0.00e+00  6.88e-13 -8.6  3.00e-06   -  1.00e+00  1.00e+00f  1
```

Number of Iterations....: 10

```

                                (scaled)                (unscaled)
Objective.....: 4.3258415310307836e-02  1.4040816441419718e+01
Dual infeasibility.....: 6.8838068935813052e-13  2.2343460415186200e-10
Constraint violation....: 0.0000000000000000e+00  0.0000000000000000e+00
Variable bound violation: 0.0000000000000000e+00  0.0000000000000000e+00
Complementarity.....: 0.0000000000000000e+00  0.0000000000000000e+00
Overall NLP error.....: 6.8838068935813052e-13  2.2343460415186200e-10
```

```
Number of objective function evaluations      = 11
Number of objective gradient evaluations      = 11
Number of equality constraint evaluations      = 0
Number of inequality constraint evaluations    = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations     = 10
Total seconds in IPOPT                       = 0.002
```

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

```
Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 3
```

```
Total number of variables.....: 2
    variables with only lower bounds: 0
    variables with lower and upper bounds: 0
    variables with only upper bounds: 0
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
    inequality constraints with only lower bounds: 0
    inequality constraints with lower and upper bounds: 0
    inequality constraints with only upper bounds: 0
```

```
iter   objective   inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
  0   1.7493800e+05  0.00e+00  1.00e+02 -1.0  0.00e+00   -  0.00e+00  0.00e+00  0
  1   3.3636002e+04  0.00e+00  2.95e+01 -1.0  6.65e+00   -  1.00e+00  1.00e+00f  1
  2   6.2680831e+03  0.00e+00  8.63e+00 -1.0  4.39e+00   -  1.00e+00  1.00e+00f  1
```



```
3 1.1074468e+03 0.00e+00 2.48e+00 -1.0 2.86e+00 - 1.00e+00 1.00e+00f 1
4 1.9342361e+02 0.00e+00 6.88e-01 -1.0 1.82e+00 - 1.00e+00 1.00e+00f 1
5 4.7005476e+01 0.00e+00 1.81e-01 -1.7 1.12e+00 - 1.00e+00 1.00e+00f 1
6 2.6267967e+01 0.00e+00 4.12e-02 -2.5 6.26e-01 - 1.00e+00 1.00e+00f 1
7 2.4597613e+01 0.00e+00 5.00e-03 -2.5 2.38e-01 - 1.00e+00 1.00e+00f 1
8 2.4566285e+01 0.00e+00 1.23e-04 -3.8 3.89e-02 - 1.00e+00 1.00e+00f 1
9 2.4566265e+01 0.00e+00 8.00e-08 -5.7 9.97e-04 - 1.00e+00 1.00e+00f 1
iter objective inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
10 2.4566265e+01 0.00e+00 3.41e-14 -8.6 6.51e-07 - 1.00e+00 1.00e+00f 1
```

Number of Iterations....: 10

	(scaled)	(unscaled)
Objective.....:	7.5686319057403931e-02	2.4566265439652167e+01
Dual infeasibility.....:	3.4114610753593757e-14	1.1072920358401461e-11
Constraint violation....:	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....:	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....:	3.4114610753593757e-14	1.1072920358401461e-11

Number of objective function evaluations	= 11
Number of objective gradient evaluations	= 11
Number of equality constraint evaluations	= 0
Number of inequality constraint evaluations	= 0
Number of equality constraint Jacobian evaluations	= 0
Number of inequality constraint Jacobian evaluations	= 0
Number of Lagrangian Hessian evaluations	= 10
Total seconds in IPOPT	= 0.002

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...	0
Number of nonzeros in inequality constraint Jacobian..	0
Number of nonzeros in Lagrangian Hessian.....:	3

Total number of variables.....:	2
variables with only lower bounds:	0
variables with lower and upper bounds:	0
variables with only upper bounds:	0
Total number of equality constraints.....:	0
Total number of inequality constraints.....:	0
inequality constraints with only lower bounds:	0
inequality constraints with lower and upper bounds:	0
inequality constraints with only upper bounds:	0

```
iter objective inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
0 1.7558600e+05 0.00e+00 1.00e+02 -1.0 0.00e+00 - 0.00e+00 0.00e+00 0
1 3.3775243e+04 0.00e+00 2.95e+01 -1.0 6.65e+00 - 1.00e+00 1.00e+00f 1
2 6.2916872e+03 0.00e+00 8.62e+00 -1.0 4.39e+00 - 1.00e+00 1.00e+00f 1
3 1.1108594e+03 0.00e+00 2.48e+00 -1.0 2.85e+00 - 1.00e+00 1.00e+00f 1
4 1.9865272e+02 0.00e+00 6.82e-01 -1.0 1.80e+00 - 1.00e+00 1.00e+00f 1
5 5.7090329e+01 0.00e+00 1.73e-01 -1.7 1.09e+00 - 1.00e+00 1.00e+00f 1
6 3.8813060e+01 0.00e+00 3.62e-02 -2.5 5.80e-01 - 1.00e+00 1.00e+00f 1
7 3.7634148e+01 0.00e+00 3.46e-03 -2.5 1.94e-01 - 1.00e+00 1.00e+00f 1
8 3.7621358e+01 0.00e+00 4.64e-05 -3.8 2.31e-02 - 1.00e+00 1.00e+00f 1
9 3.7621356e+01 0.00e+00 8.65e-09 -5.7 3.16e-04 - 1.00e+00 1.00e+00f 1
```

Number of Iterations....: 9

	(scaled)	(unscaled)
Objective.....:	1.1590780612061255e-01	3.7621355710628421e+01
Dual infeasibility.....:	8.6470348589794690e-09	2.8066545745275558e-06
Constraint violation....:	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....:	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....:	8.6470348589794690e-09	2.8066545745275558e-06

Number of objective function evaluations	= 10
Number of objective gradient evaluations	= 10
Number of equality constraint evaluations	= 0
Number of inequality constraint evaluations	= 0
Number of equality constraint Jacobian evaluations	= 0
Number of inequality constraint Jacobian evaluations	= 0
Number of Lagrangian Hessian evaluations	= 9
Total seconds in IPOPT	= 0.002

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...	0
Number of nonzeros in inequality constraint Jacobian..	0
Number of nonzeros in Lagrangian Hessian.....:	3

Total number of variables.....:	2
variables with only lower bounds:	0
variables with lower and upper bounds:	0
variables with only upper bounds:	0
Total number of equality constraints.....:	0
Total number of inequality constraints.....:	0
inequality constraints with only lower bounds:	0
inequality constraints with lower and upper bounds:	0
inequality constraints with only upper bounds:	0

```
iter objective inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
0 1.7688200e+05 0.00e+00 1.00e+02 -1.0 0.00e+00 - 0.00e+00 0.00e+00 0
1 3.4033627e+04 0.00e+00 2.95e+01 -1.0 6.65e+00 - 1.00e+00 1.00e+00f 1
2 6.3190145e+03 0.00e+00 8.61e+00 -1.0 4.38e+00 - 1.00e+00 1.00e+00f 1
3 1.1058603e+03 0.00e+00 2.47e+00 -1.0 2.84e+00 - 1.00e+00 1.00e+00f 1
4 2.0143272e+02 0.00e+00 6.76e-01 -1.0 1.79e+00 - 1.00e+00 1.00e+00f 1
5 6.4290874e+01 0.00e+00 1.68e-01 -1.7 1.07e+00 - 1.00e+00 1.00e+00f 1
6 4.8155662e+01 0.00e+00 3.13e-02 -2.5 5.30e-01 - 1.00e+00 1.00e+00f 1
7 4.7348699e+01 0.00e+00 2.28e-03 -2.5 1.53e-01 - 1.00e+00 1.00e+00f 1
8 4.7343951e+01 0.00e+00 1.57e-05 -3.8 1.30e-02 - 1.00e+00 1.00e+00f 1
9 4.7343951e+01 0.00e+00 7.58e-10 -8.6 9.04e-05 - 1.00e+00 1.00e+00f 1
```

Number of Iterations....: 9

	(scaled)	(unscaled)
Objective.....:	1.4586219495380107e-01	4.7343951238104751e+01

Dual infeasibility.....: 7.5829008896574595e-10 2.4612579707650179e-07
Constraint violation.....: 0.0000000000000000e+00 0.0000000000000000e+00
Variable bound violation: 0.0000000000000000e+00 0.0000000000000000e+00
Complementarity.....: 0.0000000000000000e+00 0.0000000000000000e+00
Overall NLP error.....: 7.5829008896574595e-10 2.4612579707650179e-07

Number of objective function evaluations = 10
Number of objective gradient evaluations = 10
Number of equality constraint evaluations = 0
Number of inequality constraint evaluations = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations = 9
Total seconds in IPOPT = 0.002

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 3

Total number of variables.....: 2
 variables with only lower bounds: 0
 variables with lower and upper bounds: 0
 variables with only upper bounds: 0
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
 inequality constraints with only lower bounds: 0
 inequality constraints with lower and upper bounds: 0
 inequality constraints with only upper bounds: 0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	1.7947400e+05	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	3.4478663e+04	0.00e+00	2.94e+01	-1.0	6.64e+00	-	1.00e+00	1.00e+00f	1
2	6.3173660e+03	0.00e+00	8.59e+00	-1.0	4.37e+00	-	1.00e+00	1.00e+00f	1
3	1.0839922e+03	0.00e+00	2.46e+00	-1.0	2.83e+00	-	1.00e+00	1.00e+00f	1
4	2.0334390e+02	0.00e+00	6.75e-01	-1.0	1.79e+00	-	1.00e+00	1.00e+00f	1
5	6.7634580e+01	0.00e+00	1.67e-01	-1.7	1.07e+00	-	1.00e+00	1.00e+00f	1
6	5.2349549e+01	0.00e+00	2.93e-02	-2.5	5.06e-01	-	1.00e+00	1.00e+00f	1
7	5.1690316e+01	0.00e+00	1.80e-03	-2.5	1.34e-01	-	1.00e+00	1.00e+00f	1
8	5.1687580e+01	0.00e+00	8.54e-06	-3.8	9.38e-03	-	1.00e+00	1.00e+00f	1
9	5.1687580e+01	0.00e+00	1.95e-10	-8.6	4.49e-05	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 9

	(scaled)	(unscaled)
Objective.....	1.5924449947797065e-01	5.1687579640559711e+01
Dual infeasibility.....	1.9496811720834430e-10	6.3282751483484390e-08
Constraint violation.....	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....	1.9496811720834430e-10	6.3282751483484390e-08

Number of objective function evaluations = 10
Number of objective gradient evaluations = 10
Number of equality constraint evaluations = 0
Number of inequality constraint evaluations = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations = 9
Total seconds in IPOPT = 0.002

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 3

Total number of variables.....: 2
 variables with only lower bounds: 0
 variables with lower and upper bounds: 0
 variables with only upper bounds: 0
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
 inequality constraints with only lower bounds: 0
 inequality constraints with lower and upper bounds: 0
 inequality constraints with only upper bounds: 0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	1.8465800e+05	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	3.5138849e+04	0.00e+00	2.94e+01	-1.0	6.63e+00	-	1.00e+00	1.00e+00f	1
2	6.2140325e+03	0.00e+00	8.56e+00	-1.0	4.35e+00	-	1.00e+00	1.00e+00f	1
3	1.0649659e+03	0.00e+00	2.45e+00	-1.0	2.83e+00	-	1.00e+00	1.00e+00f	1
4	2.0539422e+02	0.00e+00	6.78e-01	-1.0	1.81e+00	-	1.00e+00	1.00e+00f	1
5	6.9087791e+01	0.00e+00	1.67e-01	-1.7	1.06e+00	-	1.00e+00	1.00e+00f	1
6	5.4068824e+01	0.00e+00	2.86e-02	-2.5	4.97e-01	-	1.00e+00	1.00e+00f	1
7	5.3457942e+01	0.00e+00	1.65e-03	-2.5	1.27e-01	-	1.00e+00	1.00e+00f	1
8	5.3455731e+01	0.00e+00	6.76e-06	-3.8	8.27e-03	-	1.00e+00	1.00e+00f	1
9	5.3455731e+01	0.00e+00	1.15e-10	-8.6	3.42e-05	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 9

	(scaled)	(unscaled)
Objective.....	1.6469200545712953e-01	5.3455731131275101e+01
Dual infeasibility.....	1.1514704266657277e-10	3.7374427108716184e-08
Constraint violation.....	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....	1.1514704266657277e-10	3.7374427108716184e-08

Number of objective function evaluations = 10
Number of objective gradient evaluations = 10
Number of equality constraint evaluations = 0
Number of inequality constraint evaluations = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations = 9

Total seconds in IPOPT = 0.002
EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 3

Total number of variables.....: 2
 variables with only lower bounds: 0
 variables with lower and upper bounds: 0
 variables with only upper bounds: 0
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
 inequality constraints with only lower bounds: 0
 inequality constraints with lower and upper bounds: 0
 inequality constraints with only upper bounds: 0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	1.9502600e+05	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	3.5854654e+04	0.00e+00	2.94e+01	-1.0	6.61e+00	-	1.00e+00	1.00e+00f	1
2	6.0040660e+03	0.00e+00	8.54e+00	-1.0	4.34e+00	-	1.00e+00	1.00e+00f	1
3	1.0660586e+03	0.00e+00	2.45e+00	-1.0	2.84e+00	-	1.00e+00	1.00e+00f	1
4	2.0663872e+02	0.00e+00	6.81e-01	-1.0	1.82e+00	-	1.00e+00	1.00e+00f	1
5	6.9770402e+01	0.00e+00	1.68e-01	-1.7	1.06e+00	-	1.00e+00	1.00e+00f	1
6	5.4831245e+01	0.00e+00	2.84e-02	-2.5	4.93e-01	-	1.00e+00	1.00e+00f	1
7	5.4237653e+01	0.00e+00	1.59e-03	-2.5	1.24e-01	-	1.00e+00	1.00e+00f	1
8	5.4235618e+01	0.00e+00	6.16e-06	-3.8	7.86e-03	-	1.00e+00	1.00e+00f	1
9	5.4235618e+01	0.00e+00	9.36e-11	-8.6	3.07e-05	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 9

	(scaled)	(unscaled)
Objective.....	1.6709476161800405e-01	5.4235617725971757e+01
Dual infeasibility.....	9.3579729378456282e-11	3.0374108561659341e-08
Constraint violation....	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....	9.3579729378456282e-11	3.0374108561659341e-08

Number of objective function evaluations = 10
Number of objective gradient evaluations = 10
Number of equality constraint evaluations = 0
Number of inequality constraint evaluations = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations = 9
Total seconds in IPOPT = 0.002

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 3

Total number of variables.....: 2
 variables with only lower bounds: 0
 variables with lower and upper bounds: 0
 variables with only upper bounds: 0
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
 inequality constraints with only lower bounds: 0
 inequality constraints with lower and upper bounds: 0
 inequality constraints with only upper bounds: 0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	2.1576200e+05	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	3.6158906e+04	0.00e+00	2.93e+01	-1.0	6.59e+00	-	1.00e+00	1.00e+00f	1
2	5.8600797e+03	0.00e+00	8.53e+00	-1.0	4.34e+00	-	1.00e+00	1.00e+00f	1
3	1.0706840e+03	0.00e+00	2.46e+00	-1.0	2.86e+00	-	1.00e+00	1.00e+00f	1
4	2.0741593e+02	0.00e+00	6.83e-01	-1.0	1.82e+00	-	1.00e+00	1.00e+00f	1
5	7.0117999e+01	0.00e+00	1.68e-01	-1.7	1.06e+00	-	1.00e+00	1.00e+00f	1
6	5.5190413e+01	0.00e+00	2.83e-02	-2.5	4.92e-01	-	1.00e+00	1.00e+00f	1
7	5.4602949e+01	0.00e+00	1.57e-03	-2.5	1.23e-01	-	1.00e+00	1.00e+00f	1
8	5.4600979e+01	0.00e+00	5.94e-06	-3.8	7.71e-03	-	1.00e+00	1.00e+00f	1
9	5.4600979e+01	0.00e+00	8.61e-11	-8.6	2.94e-05	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 9

	(scaled)	(unscaled)
Objective.....	1.6822040509506808e-01	5.4600979085757196e+01
Dual infeasibility.....	8.6055995105263689e-11	2.7932054891266486e-08
Constraint violation....	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....	8.6055995105263689e-11	2.7932054891266486e-08

Number of objective function evaluations = 10
Number of objective gradient evaluations = 10
Number of equality constraint evaluations = 0
Number of inequality constraint evaluations = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations = 9
Total seconds in IPOPT = 0.002

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 3

Total number of variables.....: 2
 variables with only lower bounds: 0
 variables with lower and upper bounds: 0
 variables with only upper bounds: 0
Total number of equality constraints.....: 0

```
Total number of inequality constraints.....: 0
inequality constraints with only lower bounds: 0
inequality constraints with lower and upper bounds: 0
inequality constraints with only upper bounds: 0
```

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	2.5723400e+05	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	3.5609181e+04	0.00e+00	2.93e+01	-1.0	6.94e+00	-	1.00e+00	1.00e+00f	1
2	5.8347829e+03	0.00e+00	8.54e+00	-1.0	4.36e+00	-	1.00e+00	1.00e+00f	1
3	1.0737730e+03	0.00e+00	2.47e+00	-1.0	2.87e+00	-	1.00e+00	1.00e+00f	1
4	2.0791795e+02	0.00e+00	6.84e-01	-1.0	1.82e+00	-	1.00e+00	1.00e+00f	1
5	7.0303219e+01	0.00e+00	1.68e-01	-1.7	1.06e+00	-	1.00e+00	1.00e+00f	1
6	5.5365277e+01	0.00e+00	2.83e-02	-2.5	4.92e-01	-	1.00e+00	1.00e+00f	1
7	5.4779704e+01	0.00e+00	1.56e-03	-2.5	1.23e-01	-	1.00e+00	1.00e+00f	1
8	5.4777759e+01	0.00e+00	5.86e-06	-3.8	7.65e-03	-	1.00e+00	1.00e+00f	1
9	5.4777759e+01	0.00e+00	8.32e-11	-8.6	2.88e-05	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 9

	(scaled)	(unscaled)
Objective.....	1.6876504577070162e-01	5.4777758556254334e+01
Dual infeasibility.....	8.3231555988459215e-11	2.7015298442734089e-08
Constraint violation....	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....	8.3231555988459215e-11	2.7015298442734089e-08

```
Number of objective function evaluations      = 10
Number of objective gradient evaluations      = 10
Number of equality constraint evaluations      = 0
Number of inequality constraint evaluations    = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations      = 9
Total seconds in IPOPT                        = 0.002
```

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

```
Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 3
```

```
Total number of variables.....: 2
variables with only lower bounds: 0
variables with lower and upper bounds: 0
variables with only upper bounds: 0
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
inequality constraints with only lower bounds: 0
inequality constraints with lower and upper bounds: 0
inequality constraints with only upper bounds: 0
```

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	3.4017800e+05	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	3.4388008e+04	0.00e+00	2.26e+01	-1.0	7.73e+00	-	1.00e+00	1.00e+00f	1
2	5.8438070e+03	0.00e+00	6.60e+00	-1.0	4.37e+00	-	1.00e+00	1.00e+00f	1
3	1.0760978e+03	0.00e+00	1.91e+00	-1.0	2.87e+00	-	1.00e+00	1.00e+00f	1
4	2.0830294e+02	0.00e+00	5.29e-01	-1.0	1.82e+00	-	1.00e+00	1.00e+00f	1
5	7.0412536e+01	0.00e+00	1.30e-01	-1.7	1.06e+00	-	1.00e+00	1.00e+00f	1
6	5.5452379e+01	0.00e+00	2.19e-02	-2.5	4.92e-01	-	1.00e+00	1.00e+00f	1
7	5.4866647e+01	0.00e+00	1.21e-03	-3.8	1.23e-01	-	1.00e+00	1.00e+00f	1
8	5.4864706e+01	0.00e+00	4.51e-06	-5.7	7.63e-03	-	1.00e+00	1.00e+00f	1
9	5.4864706e+01	0.00e+00	6.37e-11	-8.6	2.87e-05	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 9

	(scaled)	(unscaled)
Objective.....	1.3053701107579643e-01	5.4864705755157239e+01
Dual infeasibility.....	6.3678834061857115e-11	2.6764213956198546e-08
Constraint violation....	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....	6.3678834061857115e-11	2.6764213956198546e-08

```
Number of objective function evaluations      = 10
Number of objective gradient evaluations      = 10
Number of equality constraint evaluations      = 0
Number of inequality constraint evaluations    = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations      = 9
Total seconds in IPOPT                        = 0.002
```

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

```
Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 3
```

```
Total number of variables.....: 2
variables with only lower bounds: 0
variables with lower and upper bounds: 0
variables with only upper bounds: 0
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
inequality constraints with only lower bounds: 0
inequality constraints with lower and upper bounds: 0
inequality constraints with only upper bounds: 0
```

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	5.0606600e+05	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	3.3109579e+04	0.00e+00	1.20e+01	-1.0	8.28e+00	-	1.00e+00	1.00e+00f	1
2	5.8538853e+03	0.00e+00	3.52e+00	-1.0	4.38e+00	-	1.00e+00	1.00e+00f	1
3	1.0779167e+03	0.00e+00	1.02e+00	-1.0	2.87e+00	-	1.00e+00	1.00e+00f	1
4	2.0860758e+02	0.00e+00	2.82e-01	-1.0	1.82e+00	-	1.00e+00	1.00e+00f	1
5	7.0481685e+01	0.00e+00	6.93e-02	-1.7	1.06e+00	-	1.00e+00	1.00e+00f	1
6	5.5496463e+01	0.00e+00	1.17e-02	-2.5	4.92e-01	-	1.00e+00	1.00e+00f	1
7	5.4909767e+01	0.00e+00	6.44e-04	-3.8	1.23e-01	-	1.00e+00	1.00e+00f	1

8	5.4907823e+01	0.00e+00	2.40e-06	-5.7	7.63e-03	-	1.00e+00	1.00e+00f	1
9	5.4907823e+01	0.00e+00	3.40e-11	-8.6	2.87e-05	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 9

	(scaled)	(unscaled)
Objective.....:	6.9596956567547774e-02	5.4907822914401137e+01
Dual infeasibility.....:	3.3959962436708391e-11	2.6792372764816715e-08
Constraint violation....:	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....:	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....:	3.3959962436708391e-11	2.6792372764816715e-08

Number of objective function evaluations	= 10
Number of objective gradient evaluations	= 10
Number of equality constraint evaluations	= 0
Number of inequality constraint evaluations	= 0
Number of equality constraint Jacobian evaluations	= 0
Number of inequality constraint Jacobian evaluations	= 0
Number of Lagrangian Hessian evaluations	= 9
Total seconds in IPOPT	= 0.002

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...	0
Number of nonzeros in inequality constraint Jacobian..	0
Number of nonzeros in Lagrangian Hessian.....	3

Total number of variables.....	2
variables with only lower bounds:	0
variables with lower and upper bounds:	0
variables with only upper bounds:	0
Total number of equality constraints.....	0
Total number of inequality constraints.....	0
inequality constraints with only lower bounds:	0
inequality constraints with lower and upper bounds:	0
inequality constraints with only upper bounds:	0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	8.3784200e+05	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	3.2161183e+04	0.00e+00	6.22e+00	-1.0	8.62e+00	-	1.00e+00	1.00e+00f	1
2	5.8607553e+03	0.00e+00	1.82e+00	-1.0	4.39e+00	-	1.00e+00	1.00e+00f	1
3	1.0791453e+03	0.00e+00	5.27e-01	-1.0	2.87e+00	-	1.00e+00	1.00e+00f	1
4	2.0881436e+02	0.00e+00	1.46e-01	-1.7	1.82e+00	-	1.00e+00	1.00e+00f	1
5	7.0523349e+01	0.00e+00	3.59e-02	-2.5	1.06e+00	-	1.00e+00	1.00e+00f	1
6	5.5518845e+01	0.00e+00	6.03e-03	-2.5	4.92e-01	-	1.00e+00	1.00e+00f	1
7	5.4931241e+01	0.00e+00	3.33e-04	-3.8	1.23e-01	-	1.00e+00	1.00e+00f	1
8	5.4929293e+01	0.00e+00	1.25e-06	-5.7	7.64e-03	-	1.00e+00	1.00e+00f	1
9	5.4929293e+01	0.00e+00	1.76e-11	-8.6	2.88e-05	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 9

	(scaled)	(unscaled)
Objective.....:	3.5990416134754580e-02	5.4929292913185130e+01
Dual infeasibility.....:	1.7611962014066064e-11	2.6879728665107908e-08
Constraint violation....:	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....:	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....:	1.7611962014066064e-11	2.6879728665107908e-08

Number of objective function evaluations	= 10
Number of objective gradient evaluations	= 10
Number of equality constraint evaluations	= 0
Number of inequality constraint evaluations	= 0
Number of equality constraint Jacobian evaluations	= 0
Number of inequality constraint Jacobian evaluations	= 0
Number of Lagrangian Hessian evaluations	= 9
Total seconds in IPOPT	= 0.002

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...	0
Number of nonzeros in inequality constraint Jacobian..	0
Number of nonzeros in Lagrangian Hessian.....	3

Total number of variables.....	2
variables with only lower bounds:	0
variables with lower and upper bounds:	0
variables with only upper bounds:	0
Total number of equality constraints.....	0
Total number of inequality constraints.....	0
inequality constraints with only lower bounds:	0
inequality constraints with lower and upper bounds:	0
inequality constraints with only upper bounds:	0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	1.5013940e+06	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	3.1577858e+04	0.00e+00	3.16e+00	-1.0	8.80e+00	-	1.00e+00	1.00e+00f	1
2	5.8647994e+03	0.00e+00	9.27e-01	-1.0	4.39e+00	-	1.00e+00	1.00e+00f	1
3	1.0798728e+03	0.00e+00	2.68e-01	-1.7	2.87e+00	-	1.00e+00	1.00e+00f	1
4	2.0893706e+02	0.00e+00	7.43e-02	-1.7	1.82e+00	-	1.00e+00	1.00e+00f	1
5	7.0546700e+01	0.00e+00	1.83e-02	-2.5	1.06e+00	-	1.00e+00	1.00e+00f	1
6	5.5530166e+01	0.00e+00	3.07e-03	-3.8	4.92e-01	-	1.00e+00	1.00e+00f	1
7	5.4941957e+01	0.00e+00	1.70e-04	-3.8	1.23e-01	-	1.00e+00	1.00e+00f	1
8	5.4940006e+01	0.00e+00	6.34e-07	-5.7	7.64e-03	-	1.00e+00	1.00e+00f	1
9	5.4940006e+01	0.00e+00	8.98e-12	-8.6	2.88e-05	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 9

	(scaled)	(unscaled)
Objective.....:	1.8308575048419630e-02	5.4940005833796654e+01
Dual infeasibility.....:	8.9807927170583881e-12	2.6949383169494467e-08
Constraint violation....:	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....:	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....:	8.9807927170583881e-12	2.6949383169494467e-08

Number of objective function evaluations = 10
Number of objective gradient evaluations = 10
Number of equality constraint evaluations = 0
Number of inequality constraint evaluations = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations = 9
Total seconds in IPOPT = 0.002

EXIT: Optimal Solution Found.

```
└ Info:      1      103      1.9e+00      first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:      2      137      1.9e+00      first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:      3      171      1.7e+00      first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:      4      205      1.5e+00      first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:      5      236      1.1e+00      first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:      6      267      5.5e-01      first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:      7      298      2.4e-01      first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:      8      329      1.1e-01      first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:      9      360      5.4e-02      first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:     10      391      2.6e-02      first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:     11      422      1.3e-02      first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:     12      453      6.5e-03      first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:     13      484      3.2e-03      first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:     14      515      1.6e-03      first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:     15      546      8.1e-04      first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
```

Test Passed

```
In [ ]: print(stats.multipliers)

print((stats.solution[:,1]))

[7.1777085169e-313][1.0008083416168554, 2.709969136165859]
```

Vérifier que la solution rendue vérifie les conditions KKT avec la fonction de la question précédente.

```
In [ ]: fH(x) = (x[2]+x[1].^2-11)^2 + (x[1]+x[2].^2-7)^2
x0H = [10., 20.]
cH(x) = [x[1]-1]
himmelblau = ADNLPMModel(fH, x0H, cH, [0.], [0.]);
stats = ipopt(himmelblau)
KKT_eq_constraint(himmelblau, stats.solution, stats.multipliers )
```

This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...: 2
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 3

Total number of variables.....: 2
variables with only lower bounds: 0
variables with lower and upper bounds: 0
variables with only upper bounds: 0
Total number of equality constraints.....: 1
Total number of inequality constraints.....: 0
inequality constraints with only lower bounds: 0
inequality constraints with lower and upper bounds: 0
inequality constraints with only upper bounds: 0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	1.7429000e+05	9.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	3.0904536e+04	0.00e+00	2.92e+01	-1.0	9.00e+00	-	1.00e+00	1.00e+00f	1
2	5.8693387e+03	0.00e+00	8.57e+00	-1.0	4.39e+00	-	1.00e+00	1.00e+00f	1
3	1.0806931e+03	0.00e+00	2.48e+00	-1.0	2.87e+00	-	1.00e+00	1.00e+00f	1
4	2.0907560e+02	0.00e+00	6.88e-01	-1.0	1.82e+00	-	1.00e+00	1.00e+00f	1
5	7.0572121e+01	0.00e+00	1.69e-01	-1.7	1.06e+00	-	1.00e+00	1.00e+00f	1
6	5.5541596e+01	0.00e+00	2.84e-02	-2.5	4.92e-01	-	1.00e+00	1.00e+00f	1
7	5.4952659e+01	0.00e+00	1.57e-03	-2.5	1.23e-01	-	1.00e+00	1.00e+00f	1
8	5.4950704e+01	0.00e+00	5.88e-06	-3.8	7.65e-03	-	1.00e+00	1.00e+00f	1
9	5.4950704e+01	0.00e+00	8.33e-11	-8.6	2.88e-05	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 9

	(scaled)	(unscaled)
Objective.....	1.6929787436250029e-01	5.4950704060580350e+01
Dual infeasibility.....	8.3309050636453264e-11	2.7040451655580000e-08
Constraint violation....	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....	8.3309050636453264e-11	2.7040451655580000e-08

Number of objective function evaluations = 10
Number of objective gradient evaluations = 10
Number of equality constraint evaluations = 10
Number of inequality constraint evaluations = 0
Number of equality constraint Jacobian evaluations = 10
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations = 9
Total seconds in IPOPT = 0.627

EXIT: Optimal Solution Found.

[1.0 0.0][-26.469565896135332, 2.704045165558e-8][26.469565897798233][1.6629009280677565e-9, 2.704045165558e-8][0.0]
true

Les contraintes de KKT sont respectées.

Devoir - Question 1 - Un problème ADNLProblems

```
In [ ]: using ADNLModels, Random

In [ ]: using OptimizationProblems
using OptimizationProblems.ADNLProblems
meta = OptimizationProblems.meta
names_pb_vars = meta[(meta.has_equalities_only .== true), [:nvar, :name, :has_equalities_only]];

In [ ]: n = 5
nlp = ADNLProblems.hs49(n = n)
stats = quad_penalty(nlp, nlp.meta.x0);
```

┌ Info:	iter	#F	F(x)	status	d	Δ
└ @ Main	/home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:34					
┌ Info:	0	65	8.7e-01	first_order		
└ @ Main	/home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61					

This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 15

Total number of variables.....: 5
 variables with only lower bounds: 0
 variables with lower and upper bounds: 0
 variables with only upper bounds: 0
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
 inequality constraints with only lower bounds: 0
 inequality constraints with lower and upper bounds: 0
 inequality constraints with only upper bounds: 0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	3.0850006e+02	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	5.3509646e+01	0.00e+00	3.33e+01	-1.0	7.16e+00	-	1.00e+00	1.00e+00f	1
2	1.1348680e+01	0.00e+00	9.86e+00	-1.0	1.62e+00	-	1.00e+00	1.00e+00f	1
3	3.2269595e+00	0.00e+00	2.92e+00	-1.0	1.13e+00	-	1.00e+00	1.00e+00f	1
4	1.6428861e+00	0.00e+00	8.65e-01	-1.0	7.85e-01	-	1.00e+00	1.00e+00f	1
5	1.3301283e+00	0.00e+00	2.56e-01	-1.7	5.27e-01	-	1.00e+00	1.00e+00f	1
6	1.2683490e+00	0.00e+00	7.60e-02	-1.7	3.51e-01	-	1.00e+00	1.00e+00f	1
7	1.2561456e+00	0.00e+00	2.25e-02	-2.5	2.34e-01	-	1.00e+00	1.00e+00f	1
8	1.2537351e+00	0.00e+00	6.67e-03	-3.8	1.56e-01	-	1.00e+00	1.00e+00f	1
9	1.2532590e+00	0.00e+00	1.98e-03	-3.8	1.04e-01	-	1.00e+00	1.00e+00f	1
iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
10	1.2531649e+00	0.00e+00	5.86e-04	-3.8	6.94e-02	-	1.00e+00	1.00e+00f	1
11	1.2531463e+00	0.00e+00	1.73e-04	-5.7	4.62e-02	-	1.00e+00	1.00e+00f	1
12	1.2531427e+00	0.00e+00	5.14e-05	-5.7	3.08e-02	-	1.00e+00	1.00e+00f	1
13	1.2531419e+00	0.00e+00	1.52e-05	-5.7	2.06e-02	-	1.00e+00	1.00e+00f	1
14	1.2531418e+00	0.00e+00	4.51e-06	-8.6	1.37e-02	-	1.00e+00	1.00e+00f	1
15	1.2531418e+00	0.00e+00	1.34e-06	-8.6	9.13e-03	-	1.00e+00	1.00e+00f	1
16	1.2531418e+00	0.00e+00	3.96e-07	-8.6	6.09e-03	-	1.00e+00	1.00e+00f	1
17	1.2531418e+00	0.00e+00	1.17e-07	-8.6	4.06e-03	-	1.00e+00	1.00e+00f	1
18	1.2531418e+00	0.00e+00	3.48e-08	-8.6	2.71e-03	-	1.00e+00	1.00e+00f	1
19	1.2531418e+00	0.00e+00	1.03e-08	-8.6	1.80e-03	-	1.00e+00	1.00e+00f	1
iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
20	1.2531418e+00	0.00e+00	3.05e-09	-9.0	1.20e-03	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 20

	(scaled)	(unscaled)
Objective.....	5.4962357586802446e-01	1.2531417529790958e+00
Dual infeasibility.....	3.0537225420637178e-09	6.9624873959052777e-09
Constraint violation....	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....	3.0537225420637178e-09	6.9624873959052777e-09

Number of objective function evaluations = 21
Number of objective gradient evaluations = 21
Number of equality constraint evaluations = 0
Number of inequality constraint evaluations = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations = 20
Total seconds in IPOPT = 1.442

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 15

Total number of variables.....: 5
 variables with only lower bounds: 0
 variables with lower and upper bounds: 0
 variables with only upper bounds: 0
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
 inequality constraints with only lower bounds: 0
 inequality constraints with lower and upper bounds: 0
 inequality constraints with only upper bounds: 0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	3.5100006e+02	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	5.3524303e+01	0.00e+00	3.79e+01	-1.0	7.16e+00	-	1.00e+00	1.00e+00f	1
2	1.1538711e+01	0.00e+00	1.12e+01	-1.0	1.57e+00	-	1.00e+00	1.00e+00f	1
3	3.4640085e+00	0.00e+00	3.33e+00	-1.0	1.13e+00	-	1.00e+00	1.00e+00f	1
4	1.8805313e+00	0.00e+00	9.87e-01	-1.0	7.87e-01	-	1.00e+00	1.00e+00f	1
5	1.5677735e+00	0.00e+00	2.92e-01	-1.7	5.27e-01	-	1.00e+00	1.00e+00f	1
6	1.5059941e+00	0.00e+00	8.66e-02	-1.7	3.51e-01	-	1.00e+00	1.00e+00f	1
7	1.4937908e+00	0.00e+00	2.57e-02	-2.5	2.34e-01	-	1.00e+00	1.00e+00f	1
8	1.4913803e+00	0.00e+00	7.60e-03	-3.8	1.56e-01	-	1.00e+00	1.00e+00f	1
9	1.4909041e+00	0.00e+00	2.25e-03	-3.8	1.04e-01	-	1.00e+00	1.00e+00f	1
iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
10	1.4908101e+00	0.00e+00	6.68e-04	-3.8	6.94e-02	-	1.00e+00	1.00e+00f	1
11	1.4907915e+00	0.00e+00	1.98e-04	-5.7	4.62e-02	-	1.00e+00	1.00e+00f	1
12	1.4907878e+00	0.00e+00	5.86e-05	-5.7	3.08e-02	-	1.00e+00	1.00e+00f	1
13	1.4907871e+00	0.00e+00	1.74e-05	-5.7	2.06e-02	-	1.00e+00	1.00e+00f	1
14	1.4907870e+00	0.00e+00	5.14e-06	-8.6	1.37e-02	-	1.00e+00	1.00e+00f	1
15	1.4907869e+00	0.00e+00	1.52e-06	-8.6	9.13e-03	-	1.00e+00	1.00e+00f	1
16	1.4907869e+00	0.00e+00	4.52e-07	-8.6	6.09e-03	-	1.00e+00	1.00e+00f	1
17	1.4907869e+00	0.00e+00	1.34e-07	-8.6	4.06e-03	-	1.00e+00	1.00e+00f	1
18	1.4907869e+00	0.00e+00	3.97e-08	-8.6	2.71e-03	-	1.00e+00	1.00e+00f	1
19	1.4907869e+00	0.00e+00	1.17e-08	-8.6	1.80e-03	-	1.00e+00	1.00e+00f	1
iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
20	1.4907869e+00	0.00e+00	3.48e-09	-9.0	1.20e-03	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 20

	(scaled)	(unscaled)
Objective.....	7.4539345984100414e-01	1.4907869196820083e+00
Dual infeasibility.....	3.4812436978986599e-09	6.9624873957973199e-09
Constraint violation....	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....	3.4812436978986599e-09	6.9624873957973199e-09

Number of objective function evaluations = 21
Number of objective gradient evaluations = 21
Number of equality constraint evaluations = 0
Number of inequality constraint evaluations = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations = 20
Total seconds in IPOPT = 0.003

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 15

Total number of variables.....: 5
 variables with only lower bounds: 0
 variables with lower and upper bounds: 0
 variables with only upper bounds: 0
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
 inequality constraints with only lower bounds: 0
 inequality constraints with lower and upper bounds: 0
 inequality constraints with only upper bounds: 0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	4.3600006e+02	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	5.3531665e+01	0.00e+00	5.27e+01	-1.0	7.16e+00	-	1.00e+00	1.00e+00f	1
2	1.1687412e+01	0.00e+00	1.56e+01	-1.0	1.52e+00	-	1.00e+00	1.00e+00f	1
3	3.6362962e+00	0.00e+00	4.62e+00	-1.0	1.13e+00	-	1.00e+00	1.00e+00f	1
4	2.0529254e+00	0.00e+00	1.37e+00	-1.0	7.88e-01	-	1.00e+00	1.00e+00f	1
5	1.7401676e+00	0.00e+00	4.06e-01	-1.0	5.27e-01	-	1.00e+00	1.00e+00f	1
6	1.6783883e+00	0.00e+00	1.20e-01	-1.7	3.51e-01	-	1.00e+00	1.00e+00f	1
7	1.6661850e+00	0.00e+00	3.56e-02	-2.5	2.34e-01	-	1.00e+00	1.00e+00f	1
8	1.6637744e+00	0.00e+00	1.06e-02	-2.5	1.56e-01	-	1.00e+00	1.00e+00f	1
9	1.6632983e+00	0.00e+00	3.13e-03	-3.8	1.04e-01	-	1.00e+00	1.00e+00f	1
iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
10	1.6632042e+00	0.00e+00	9.27e-04	-3.8	6.94e-02	-	1.00e+00	1.00e+00f	1
11	1.6631856e+00	0.00e+00	2.75e-04	-5.7	4.62e-02	-	1.00e+00	1.00e+00f	1
12	1.6631820e+00	0.00e+00	8.14e-05	-5.7	3.08e-02	-	1.00e+00	1.00e+00f	1
13	1.6631812e+00	0.00e+00	2.41e-05	-5.7	2.06e-02	-	1.00e+00	1.00e+00f	1
14	1.6631811e+00	0.00e+00	7.15e-06	-5.7	1.37e-02	-	1.00e+00	1.00e+00f	1
15	1.6631811e+00	0.00e+00	2.12e-06	-8.6	9.13e-03	-	1.00e+00	1.00e+00f	1
16	1.6631811e+00	0.00e+00	6.27e-07	-8.6	6.09e-03	-	1.00e+00	1.00e+00f	1
17	1.6631811e+00	0.00e+00	1.86e-07	-8.6	4.06e-03	-	1.00e+00	1.00e+00f	1
18	1.6631811e+00	0.00e+00	5.51e-08	-8.6	2.71e-03	-	1.00e+00	1.00e+00f	1
19	1.6631811e+00	0.00e+00	1.63e-08	-8.6	1.80e-03	-	1.00e+00	1.00e+00f	1
iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
20	1.6631811e+00	0.00e+00	4.84e-09	-9.0	1.20e-03	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 20

	(scaled)	(unscaled)
Objective.....	1.1549868454069627e+00	1.6631810573860264e+00
Dual infeasibility.....	4.8350656254193936e-09	6.9624945006039264e-09
Constraint violation....	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....	4.8350656254193936e-09	6.9624945006039264e-09

Number of objective function evaluations = 21
Number of objective gradient evaluations = 21
Number of equality constraint evaluations = 0
Number of inequality constraint evaluations = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations = 20
Total seconds in IPOPT = 0.003

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 15

Total number of variables.....: 5
 variables with only lower bounds: 0
 variables with lower and upper bounds: 0
 variables with only upper bounds: 0
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
 inequality constraints with only lower bounds: 0
 inequality constraints with lower and upper bounds: 0
 inequality constraints with only upper bounds: 0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	6.0600006e+02	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	5.3535354e+01	0.00e+00	3.16e+01	-1.0	7.16e+00	-	1.00e+00	1.00e+00f	1
2	1.1785200e+01	0.00e+00	9.36e+00	-1.0	1.49e+00	-	1.00e+00	1.00e+00f	1
3	3.7449813e+00	0.00e+00	2.77e+00	-1.0	1.14e+00	-	1.00e+00	1.00e+00f	1
4	2.1616322e+00	0.00e+00	8.22e-01	-1.0	7.89e-01	-	1.00e+00	1.00e+00f	1
5	1.8488744e+00	0.00e+00	2.44e-01	-1.7	5.27e-01	-	1.00e+00	1.00e+00f	1
6	1.7870951e+00	0.00e+00	7.22e-02	-1.7	3.51e-01	-	1.00e+00	1.00e+00f	1
7	1.7748918e+00	0.00e+00	2.14e-02	-2.5	2.34e-01	-	1.00e+00	1.00e+00f	1
8	1.7724812e+00	0.00e+00	6.34e-03	-3.8	1.56e-01	-	1.00e+00	1.00e+00f	1
9	1.7720051e+00	0.00e+00	1.88e-03	-3.8	1.04e-01	-	1.00e+00	1.00e+00f	1
iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
10	1.7719110e+00	0.00e+00	5.56e-04	-3.8	6.94e-02	-	1.00e+00	1.00e+00f	1
11	1.7718925e+00	0.00e+00	1.65e-04	-5.7	4.62e-02	-	1.00e+00	1.00e+00f	1
12	1.7718888e+00	0.00e+00	4.88e-05	-5.7	3.08e-02	-	1.00e+00	1.00e+00f	1
13	1.7718881e+00	0.00e+00	1.45e-05	-5.7	2.06e-02	-	1.00e+00	1.00e+00f	1
14	1.7718879e+00	0.00e+00	4.29e-06	-8.6	1.37e-02	-	1.00e+00	1.00e+00f	1
15	1.7718879e+00	0.00e+00	1.27e-06	-8.6	9.13e-03	-	1.00e+00	1.00e+00f	1
16	1.7718879e+00	0.00e+00	3.76e-07	-8.6	6.09e-03	-	1.00e+00	1.00e+00f	1
17	1.7718879e+00	0.00e+00	1.12e-07	-8.6	4.06e-03	-	1.00e+00	1.00e+00f	1
18	1.7718879e+00	0.00e+00	3.30e-08	-8.6	2.71e-03	-	1.00e+00	1.00e+00f	1
19	1.7718879e+00	0.00e+00	9.79e-09	-8.6	1.80e-03	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 19

	(scaled)	(unscaled)
Objective.....:	7.3829252565870207e-01	1.7718878863643921e+00
Dual infeasibility.....:	9.7910703097715200e-09	2.3498380754901698e-08
Constraint violation....:	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....:	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....:	9.7910703097715200e-09	2.3498380754901698e-08

Number of objective function evaluations = 20
Number of objective gradient evaluations = 20
Number of equality constraint evaluations = 0
Number of inequality constraint evaluations = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations = 19
Total seconds in IPOPT = 0.003

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 15

Total number of variables.....: 5
 variables with only lower bounds: 0
 variables with lower and upper bounds: 0
 variables with only upper bounds: 0
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
 inequality constraints with only lower bounds: 0
 inequality constraints with lower and upper bounds: 0
 inequality constraints with only upper bounds: 0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	9.4600006e+02	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	5.3537201e+01	0.00e+00	1.58e+01	-1.0	7.16e+00	-	1.00e+00	1.00e+00f	1
2	1.1842273e+01	0.00e+00	4.68e+00	-1.0	1.47e+00	-	1.00e+00	1.00e+00f	1
3	3.8070711e+00	0.00e+00	1.39e+00	-1.0	1.14e+00	-	1.00e+00	1.00e+00f	1
4	2.2237279e+00	0.00e+00	4.11e-01	-1.0	7.89e-01	-	1.00e+00	1.00e+00f	1
5	1.9109701e+00	0.00e+00	1.22e-01	-1.7	5.27e-01	-	1.00e+00	1.00e+00f	1
6	1.8491907e+00	0.00e+00	3.61e-02	-2.5	3.51e-01	-	1.00e+00	1.00e+00f	1
7	1.8369874e+00	0.00e+00	1.07e-02	-2.5	2.34e-01	-	1.00e+00	1.00e+00f	1
8	1.8345769e+00	0.00e+00	3.17e-03	-3.8	1.56e-01	-	1.00e+00	1.00e+00f	1
9	1.8341007e+00	0.00e+00	9.39e-04	-3.8	1.04e-01	-	1.00e+00	1.00e+00f	1
iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
10	1.8340067e+00	0.00e+00	2.78e-04	-5.7	6.94e-02	-	1.00e+00	1.00e+00f	1
11	1.8339881e+00	0.00e+00	8.24e-05	-5.7	4.62e-02	-	1.00e+00	1.00e+00f	1
12	1.8339844e+00	0.00e+00	2.44e-05	-5.7	3.08e-02	-	1.00e+00	1.00e+00f	1
13	1.8339837e+00	0.00e+00	7.24e-06	-5.7	2.06e-02	-	1.00e+00	1.00e+00f	1
14	1.8339836e+00	0.00e+00	2.14e-06	-8.6	1.37e-02	-	1.00e+00	1.00e+00f	1
15	1.8339835e+00	0.00e+00	6.35e-07	-8.6	9.13e-03	-	1.00e+00	1.00e+00f	1
16	1.8339835e+00	0.00e+00	1.88e-07	-8.6	6.09e-03	-	1.00e+00	1.00e+00f	1
17	1.8339835e+00	0.00e+00	5.58e-08	-8.6	4.06e-03	-	1.00e+00	1.00e+00f	1
18	1.8339835e+00	0.00e+00	1.65e-08	-8.6	2.71e-03	-	1.00e+00	1.00e+00f	1
19	1.8339835e+00	0.00e+00	4.90e-09	-9.0	1.80e-03	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 19

	(scaled)	(unscaled)
Objective.....:	3.8208142817820184e-01	1.8339835192919476e+00
Dual infeasibility.....:	4.8955185300066625e-09	2.3498394950076199e-08
Constraint violation....:	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....:	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....:	4.8955185300066625e-09	2.3498394950076199e-08

Number of objective function evaluations = 20
Number of objective gradient evaluations = 20
Number of equality constraint evaluations = 0
Number of inequality constraint evaluations = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations = 19
Total seconds in IPOPT = 0.003

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 15

Total number of variables.....: 5
 variables with only lower bounds: 0
 variables with lower and upper bounds: 0
 variables with only upper bounds: 0
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
 inequality constraints with only lower bounds: 0
 inequality constraints with lower and upper bounds: 0
 inequality constraints with only upper bounds: 0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	1.6260001e+03	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	5.3538125e+01	0.00e+00	7.90e+00	-1.0	7.16e+00	-	1.00e+00	1.00e+00f	1
2	1.1873274e+01	0.00e+00	2.34e+00	-1.0	1.46e+00	-	1.00e+00	1.00e+00f	1
3	3.8404375e+00	0.00e+00	6.94e-01	-1.0	1.14e+00	-	1.00e+00	1.00e+00f	1
4	2.2570963e+00	0.00e+00	2.06e-01	-1.7	7.89e-01	-	1.00e+00	1.00e+00f	1
5	1.9443384e+00	0.00e+00	6.09e-02	-1.7	5.27e-01	-	1.00e+00	1.00e+00f	1
6	1.8825591e+00	0.00e+00	1.80e-02	-2.5	3.51e-01	-	1.00e+00	1.00e+00f	1
7	1.8703558e+00	0.00e+00	5.35e-03	-3.8	2.34e-01	-	1.00e+00	1.00e+00f	1
8	1.8679453e+00	0.00e+00	1.58e-03	-3.8	1.56e-01	-	1.00e+00	1.00e+00f	1
9	1.8674691e+00	0.00e+00	4.69e-04	-3.8	1.04e-01	-	1.00e+00	1.00e+00f	1
iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
10	1.8673751e+00	0.00e+00	1.39e-04	-5.7	6.94e-02	-	1.00e+00	1.00e+00f	1
11	1.8673565e+00	0.00e+00	4.12e-05	-5.7	4.62e-02	-	1.00e+00	1.00e+00f	1

12	1.8673528e+00	0.00e+00	1.22e-05	-5.7	3.08e-02	-	1.00e+00	1.00e+00f	1
13	1.8673521e+00	0.00e+00	3.62e-06	-8.6	2.06e-02	-	1.00e+00	1.00e+00f	1
14	1.8673519e+00	0.00e+00	1.07e-06	-8.6	1.37e-02	-	1.00e+00	1.00e+00f	1
15	1.8673519e+00	0.00e+00	3.18e-07	-8.6	9.13e-03	-	1.00e+00	1.00e+00f	1
16	1.8673519e+00	0.00e+00	9.41e-08	-8.6	6.09e-03	-	1.00e+00	1.00e+00f	1
17	1.8673519e+00	0.00e+00	2.79e-08	-8.6	4.06e-03	-	1.00e+00	1.00e+00f	1
18	1.8673519e+00	0.00e+00	8.26e-09	-8.6	2.71e-03	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 18

	(scaled)	(unscaled)
Objective.....:	1.9451621293193161e-01	1.8673519094352549e+00
Dual infeasibility.....:	8.2611710042977759e-09	7.9307083026775353e-08
Constraint violation....:	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....:	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....:	8.2611710042977759e-09	7.9307083026775353e-08

Number of objective function evaluations	= 19
Number of objective gradient evaluations	= 19
Number of equality constraint evaluations	= 0
Number of inequality constraint evaluations	= 0
Number of equality constraint Jacobian evaluations	= 0
Number of inequality constraint Jacobian evaluations	= 0
Number of Lagrangian Hessian evaluations	= 18
Total seconds in IPOPT	= 0.003

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...	0
Number of nonzeros in inequality constraint Jacobian..	0
Number of nonzeros in Lagrangian Hessian.....:	15

Total number of variables.....:	5
variables with only lower bounds:	0
variables with lower and upper bounds:	0
variables with only upper bounds:	0
Total number of equality constraints.....:	0
Total number of inequality constraints.....:	0
inequality constraints with only lower bounds:	0
inequality constraints with lower and upper bounds:	0
inequality constraints with only upper bounds:	0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	2.9860001e+03	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	5.3538587e+01	0.00e+00	3.95e+00	-1.0	7.16e+00	-	1.00e+00	1.00e+00f	1
2	1.1889455e+01	0.00e+00	1.17e+00	-1.0	1.45e+00	-	1.00e+00	1.00e+00f	1
3	3.8577605e+00	0.00e+00	3.47e-01	-1.0	1.14e+00	-	1.00e+00	1.00e+00f	1
4	2.2744201e+00	0.00e+00	1.03e-01	-1.7	7.89e-01	-	1.00e+00	1.00e+00f	1
5	1.9616623e+00	0.00e+00	3.04e-02	-2.5	5.27e-01	-	1.00e+00	1.00e+00f	1
6	1.8998830e+00	0.00e+00	9.02e-03	-2.5	3.51e-01	-	1.00e+00	1.00e+00f	1
7	1.8876797e+00	0.00e+00	2.67e-03	-3.8	2.34e-01	-	1.00e+00	1.00e+00f	1
8	1.8852691e+00	0.00e+00	7.92e-04	-3.8	1.56e-01	-	1.00e+00	1.00e+00f	1
9	1.8847930e+00	0.00e+00	2.35e-04	-5.7	1.04e-01	-	1.00e+00	1.00e+00f	1
iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
10	1.8846989e+00	0.00e+00	6.95e-05	-5.7	6.94e-02	-	1.00e+00	1.00e+00f	1
11	1.8846803e+00	0.00e+00	2.06e-05	-5.7	4.62e-02	-	1.00e+00	1.00e+00f	1
12	1.8846767e+00	0.00e+00	6.10e-06	-5.7	3.08e-02	-	1.00e+00	1.00e+00f	1
13	1.8846760e+00	0.00e+00	1.81e-06	-8.6	2.06e-02	-	1.00e+00	1.00e+00f	1
14	1.8846758e+00	0.00e+00	5.36e-07	-8.6	1.37e-02	-	1.00e+00	1.00e+00f	1
15	1.8846758e+00	0.00e+00	1.59e-07	-8.6	9.13e-03	-	1.00e+00	1.00e+00f	1
16	1.8846758e+00	0.00e+00	4.70e-08	-8.6	6.09e-03	-	1.00e+00	1.00e+00f	1
17	1.8846758e+00	0.00e+00	1.39e-08	-8.6	4.06e-03	-	1.00e+00	1.00e+00f	1
18	1.8846758e+00	0.00e+00	4.13e-09	-9.0	2.71e-03	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 18

	(scaled)	(unscaled)
Objective.....:	9.8160294708390994e-02	1.8846757737234485e+00
Dual infeasibility.....:	4.1305813679716300e-09	7.9307082957893019e-08
Constraint violation....:	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....:	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....:	4.1305813679716300e-09	7.9307082957893019e-08

Number of objective function evaluations	= 19
Number of objective gradient evaluations	= 19
Number of equality constraint evaluations	= 0
Number of inequality constraint evaluations	= 0
Number of equality constraint Jacobian evaluations	= 0
Number of inequality constraint Jacobian evaluations	= 0
Number of Lagrangian Hessian evaluations	= 18
Total seconds in IPOPT	= 0.003

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...	0
Number of nonzeros in inequality constraint Jacobian..	0
Number of nonzeros in Lagrangian Hessian.....:	15

Total number of variables.....:	5
variables with only lower bounds:	0
variables with lower and upper bounds:	0
variables with only upper bounds:	0
Total number of equality constraints.....:	0
Total number of inequality constraints.....:	0
inequality constraints with only lower bounds:	0
inequality constraints with lower and upper bounds:	0
inequality constraints with only upper bounds:	0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	5.7060001e+03	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	5.3538818e+01	0.00e+00	1.98e+00	-1.0	7.16e+00	-	1.00e+00	1.00e+00f	1
2	1.1897724e+01	0.00e+00	5.85e-01	-1.0	1.45e+00	-	1.00e+00	1.00e+00f	1
3	3.8665903e+00	0.00e+00	1.73e-01	-1.7	1.14e+00	-	1.00e+00	1.00e+00f	1
4	2.2832503e+00	0.00e+00	5.14e-02	-2.5	7.89e-01	-	1.00e+00	1.00e+00f	1
5	1.9704925e+00	0.00e+00	1.52e-02	-2.5	5.27e-01	-	1.00e+00	1.00e+00f	1
6	1.9087132e+00	0.00e+00	4.51e-03	-3.8	3.51e-01	-	1.00e+00	1.00e+00f	1

7	1.8965099e+00	0.00e+00	1.34e-03	-3.8	2.34e-01	-	1.00e+00	1.00e+00f	1
8	1.8940993e+00	0.00e+00	3.96e-04	-5.7	1.56e-01	-	1.00e+00	1.00e+00f	1
9	1.8936232e+00	0.00e+00	1.17e-04	-5.7	1.04e-01	-	1.00e+00	1.00e+00f	1
iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
10	1.8935291e+00	0.00e+00	3.48e-05	-5.7	6.94e-02	-	1.00e+00	1.00e+00f	1
11	1.8935105e+00	0.00e+00	1.03e-05	-5.7	4.62e-02	-	1.00e+00	1.00e+00f	1
12	1.8935069e+00	0.00e+00	3.05e-06	-8.6	3.08e-02	-	1.00e+00	1.00e+00f	1
13	1.8935061e+00	0.00e+00	9.04e-07	-8.6	2.06e-02	-	1.00e+00	1.00e+00f	1
14	1.8935060e+00	0.00e+00	2.68e-07	-8.6	1.37e-02	-	1.00e+00	1.00e+00f	1
15	1.8935060e+00	0.00e+00	7.94e-08	-8.6	9.13e-03	-	1.00e+00	1.00e+00f	1
16	1.8935060e+00	0.00e+00	2.35e-08	-8.6	6.09e-03	-	1.00e+00	1.00e+00f	1
17	1.8935060e+00	0.00e+00	6.97e-09	-9.0	4.06e-03	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 17

	(scaled)	(unscaled)
Objective.....	4.9310075779693534e-02	1.8935059631867763e+00
Dual infeasibility.....	6.9703466605806783e-09	2.6766117793564211e-07
Constraint violation....	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....	6.9703466605806783e-09	2.6766117793564211e-07

Number of objective function evaluations	= 18
Number of objective gradient evaluations	= 18
Number of equality constraint evaluations	= 0
Number of inequality constraint evaluations	= 0
Number of equality constraint Jacobian evaluations	= 0
Number of inequality constraint Jacobian evaluations	= 0
Number of Lagrangian Hessian evaluations	= 17
Total seconds in IPOPT	= 0.003

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...	0
Number of nonzeros in inequality constraint Jacobian..	0
Number of nonzeros in Lagrangian Hessian.....	15

Total number of variables.....	5
variables with only lower bounds:	0
variables with lower and upper bounds:	0
variables with only upper bounds:	0
Total number of equality constraints.....	0
Total number of inequality constraints.....	0
inequality constraints with only lower bounds:	0
inequality constraints with lower and upper bounds:	0
inequality constraints with only upper bounds:	0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	1.1146000e+04	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	5.3538934e+01	0.00e+00	9.88e-01	-1.0	7.16e+00	-	1.00e+00	1.00e+00f	1
2	1.1901904e+01	0.00e+00	2.93e-01	-1.7	1.45e+00	-	1.00e+00	1.00e+00f	1
3	3.8710484e+00	0.00e+00	8.67e-02	-1.7	1.14e+00	-	1.00e+00	1.00e+00f	1
4	2.2877086e+00	0.00e+00	2.57e-02	-2.5	7.89e-01	-	1.00e+00	1.00e+00f	1
5	1.9749508e+00	0.00e+00	7.61e-03	-3.8	5.27e-01	-	1.00e+00	1.00e+00f	1
6	1.9131714e+00	0.00e+00	2.26e-03	-3.8	3.51e-01	-	1.00e+00	1.00e+00f	1
7	1.9009681e+00	0.00e+00	6.68e-04	-3.8	2.34e-01	-	1.00e+00	1.00e+00f	1
8	1.8985576e+00	0.00e+00	1.98e-04	-5.7	1.56e-01	-	1.00e+00	1.00e+00f	1
9	1.8980814e+00	0.00e+00	5.87e-05	-5.7	1.04e-01	-	1.00e+00	1.00e+00f	1
iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
10	1.8979874e+00	0.00e+00	1.74e-05	-5.7	6.94e-02	-	1.00e+00	1.00e+00f	1
11	1.8979688e+00	0.00e+00	5.15e-06	-8.6	4.62e-02	-	1.00e+00	1.00e+00f	1
12	1.8979651e+00	0.00e+00	1.53e-06	-8.6	3.08e-02	-	1.00e+00	1.00e+00f	1
13	1.8979644e+00	0.00e+00	4.52e-07	-8.6	2.06e-02	-	1.00e+00	1.00e+00f	1
14	1.8979643e+00	0.00e+00	1.34e-07	-8.6	1.37e-02	-	1.00e+00	1.00e+00f	1
15	1.8979642e+00	0.00e+00	3.97e-08	-8.6	9.13e-03	-	1.00e+00	1.00e+00f	1
16	1.8979642e+00	0.00e+00	1.18e-08	-8.6	6.09e-03	-	1.00e+00	1.00e+00f	1
17	1.8979642e+00	0.00e+00	3.49e-09	-9.0	4.06e-03	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 17

	(scaled)	(unscaled)
Objective.....	2.4713082066439775e-02	1.8979642282113989e+00
Dual infeasibility.....	3.4851754175467751e-09	2.6766140515222433e-07
Constraint violation....	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....	3.4851754175467751e-09	2.6766140515222433e-07

Number of objective function evaluations	= 18
Number of objective gradient evaluations	= 18
Number of equality constraint evaluations	= 0
Number of inequality constraint evaluations	= 0
Number of equality constraint Jacobian evaluations	= 0
Number of inequality constraint Jacobian evaluations	= 0
Number of Lagrangian Hessian evaluations	= 17
Total seconds in IPOPT	= 0.003

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...	0
Number of nonzeros in inequality constraint Jacobian..	0
Number of nonzeros in Lagrangian Hessian.....	15

Total number of variables.....	5
variables with only lower bounds:	0
variables with lower and upper bounds:	0
variables with only upper bounds:	0
Total number of equality constraints.....	0
Total number of inequality constraints.....	0
inequality constraints with only lower bounds:	0
inequality constraints with lower and upper bounds:	0
inequality constraints with only upper bounds:	0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	2.2026000e+04	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	5.3538992e+01	0.00e+00	4.94e-01	-1.0	7.16e+00	-	1.00e+00	1.00e+00f	1
2	1.1904006e+01	0.00e+00	1.46e-01	-1.7	1.45e+00	-	1.00e+00	1.00e+00f	1

3	3.8732884e+00	0.00e+00	4.34e-02	-2.5	1.14e+00	-	1.00e+00	1.00e+00f	1
4	2.2899486e+00	0.00e+00	1.28e-02	-2.5	7.89e-01	-	1.00e+00	1.00e+00f	1
5	1.9771908e+00	0.00e+00	3.81e-03	-3.8	5.27e-01	-	1.00e+00	1.00e+00f	1
6	1.9154115e+00	0.00e+00	1.13e-03	-3.8	3.51e-01	-	1.00e+00	1.00e+00f	1
7	1.9032082e+00	0.00e+00	3.34e-04	-5.7	2.34e-01	-	1.00e+00	1.00e+00f	1
8	1.9007977e+00	0.00e+00	9.90e-05	-5.7	1.56e-01	-	1.00e+00	1.00e+00f	1
9	1.9003215e+00	0.00e+00	2.93e-05	-5.7	1.04e-01	-	1.00e+00	1.00e+00f	1
iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
10	1.9002274e+00	0.00e+00	8.69e-06	-5.7	6.94e-02	-	1.00e+00	1.00e+00f	1
11	1.9002089e+00	0.00e+00	2.58e-06	-8.6	4.62e-02	-	1.00e+00	1.00e+00f	1
12	1.9002052e+00	0.00e+00	7.63e-07	-8.6	3.08e-02	-	1.00e+00	1.00e+00f	1
13	1.9002045e+00	0.00e+00	2.26e-07	-8.6	2.06e-02	-	1.00e+00	1.00e+00f	1
14	1.9002043e+00	0.00e+00	6.70e-08	-8.6	1.37e-02	-	1.00e+00	1.00e+00f	1
15	1.9002043e+00	0.00e+00	1.98e-08	-8.6	9.13e-03	-	1.00e+00	1.00e+00f	1
16	1.9002043e+00	0.00e+00	5.88e-09	-9.0	6.09e-03	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 16

	(scaled)	(unscaled)
Objective.....	1.2371123271247031e-02	1.9002042969379769e+00
Dual infeasibility.....	5.8812387110337463e-09	9.0335815309499997e-07
Constraint violation....	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....	5.8812387110337463e-09	9.0335815309499997e-07

Number of objective function evaluations	= 17
Number of objective gradient evaluations	= 17
Number of equality constraint evaluations	= 0
Number of inequality constraint evaluations	= 0
Number of equality constraint Jacobian evaluations	= 0
Number of inequality constraint Jacobian evaluations	= 0
Number of Lagrangian Hessian evaluations	= 16
Total seconds in IPOPT	= 0.003

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...	0
Number of nonzeros in inequality constraint Jacobian..	0
Number of nonzeros in Lagrangian Hessian.....	15

Total number of variables.....	5
variables with only lower bounds:	0
variables with lower and upper bounds:	0
variables with only upper bounds:	0
Total number of equality constraints.....	0
Total number of inequality constraints.....	0
inequality constraints with only lower bounds:	0
inequality constraints with lower and upper bounds:	0
inequality constraints with only upper bounds:	0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	4.3786000e+04	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	5.3539020e+01	0.00e+00	2.47e-01	-1.0	7.16e+00	-	1.00e+00	1.00e+00f	1
2	1.1905060e+01	0.00e+00	7.32e-02	-1.7	1.45e+00	-	1.00e+00	1.00e+00f	1
3	3.8744112e+00	0.00e+00	2.17e-02	-2.5	1.14e+00	-	1.00e+00	1.00e+00f	1
4	2.2910714e+00	0.00e+00	6.42e-03	-3.8	7.89e-01	-	1.00e+00	1.00e+00f	1
5	1.9783136e+00	0.00e+00	1.90e-03	-3.8	5.27e-01	-	1.00e+00	1.00e+00f	1
6	1.9165343e+00	0.00e+00	5.64e-04	-3.8	3.51e-01	-	1.00e+00	1.00e+00f	1
7	1.9043310e+00	0.00e+00	1.67e-04	-5.7	2.34e-01	-	1.00e+00	1.00e+00f	1
8	1.9019204e+00	0.00e+00	4.95e-05	-5.7	1.56e-01	-	1.00e+00	1.00e+00f	1
9	1.9014443e+00	0.00e+00	1.47e-05	-5.7	1.04e-01	-	1.00e+00	1.00e+00f	1
iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
10	1.9013502e+00	0.00e+00	4.35e-06	-8.6	6.94e-02	-	1.00e+00	1.00e+00f	1
11	1.9013317e+00	0.00e+00	1.29e-06	-8.6	4.62e-02	-	1.00e+00	1.00e+00f	1
12	1.9013280e+00	0.00e+00	3.82e-07	-8.6	3.08e-02	-	1.00e+00	1.00e+00f	1
13	1.9013273e+00	0.00e+00	1.13e-07	-8.6	2.06e-02	-	1.00e+00	1.00e+00f	1
14	1.9013271e+00	0.00e+00	3.35e-08	-8.6	1.37e-02	-	1.00e+00	1.00e+00f	1
15	1.9013271e+00	0.00e+00	9.92e-09	-8.6	9.13e-03	-	1.00e+00	1.00e+00f	1

Number of Iterations....: 15

	(scaled)	(unscaled)
Objective.....	6.1892161687342817e-03	1.9013270882022206e+00
Dual infeasibility.....	9.9245737697887941e-09	3.0488288715273010e-06
Constraint violation....	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....	9.9245737697887941e-09	3.0488288715273010e-06

Number of objective function evaluations	= 16
Number of objective gradient evaluations	= 16
Number of equality constraint evaluations	= 0
Number of inequality constraint evaluations	= 0
Number of equality constraint Jacobian evaluations	= 0
Number of inequality constraint Jacobian evaluations	= 0
Number of Lagrangian Hessian evaluations	= 15
Total seconds in IPOPT	= 0.003

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian...	0
Number of nonzeros in inequality constraint Jacobian..	0
Number of nonzeros in Lagrangian Hessian.....	15

Total number of variables.....	5
variables with only lower bounds:	0
variables with lower and upper bounds:	0
variables with only upper bounds:	0
Total number of equality constraints.....	0
Total number of inequality constraints.....	0
inequality constraints with only lower bounds:	0
inequality constraints with lower and upper bounds:	0
inequality constraints with only upper bounds:	0

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	8.7306000e+04	0.00e+00	1.00e+02	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	5.3539035e+01	0.00e+00	1.23e-01	-1.0	7.16e+00	-	1.00e+00	1.00e+00f	1

```
2 1.1905588e+01 0.00e+00 3.66e-02 -2.5 1.45e+00 - 1.00e+00 1.00e+00f 1
3 3.8749732e+00 0.00e+00 1.08e-02 -2.5 1.14e+00 - 1.00e+00 1.00e+00f 1
4 2.2916335e+00 0.00e+00 3.21e-03 -3.8 7.89e-01 - 1.00e+00 1.00e+00f 1
5 1.9788757e+00 0.00e+00 9.52e-04 -3.8 5.27e-01 - 1.00e+00 1.00e+00f 1
6 1.9170964e+00 0.00e+00 2.82e-04 -5.7 3.51e-01 - 1.00e+00 1.00e+00f 1
7 1.9048931e+00 0.00e+00 8.35e-05 -5.7 2.34e-01 - 1.00e+00 1.00e+00f 1
8 1.9024825e+00 0.00e+00 2.48e-05 -5.7 1.56e-01 - 1.00e+00 1.00e+00f 1
9 1.9020064e+00 0.00e+00 7.33e-06 -5.7 1.04e-01 - 1.00e+00 1.00e+00f 1
iter objective inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
10 1.9019123e+00 0.00e+00 2.17e-06 -8.6 6.94e-02 - 1.00e+00 1.00e+00f 1
11 1.9018937e+00 0.00e+00 6.44e-07 -8.6 4.62e-02 - 1.00e+00 1.00e+00f 1
12 1.9018901e+00 0.00e+00 1.91e-07 -8.6 3.08e-02 - 1.00e+00 1.00e+00f 1
13 1.9018893e+00 0.00e+00 5.65e-08 -8.6 2.06e-02 - 1.00e+00 1.00e+00f 1
14 1.9018892e+00 0.00e+00 1.67e-08 -8.6 1.37e-02 - 1.00e+00 1.00e+00f 1
15 1.9018892e+00 0.00e+00 4.96e-09 -9.0 9.13e-03 - 1.00e+00 1.00e+00f 1
```

Number of Iterations....: 15

	(scaled)	(unscaled)
Objective.....:	3.0955228364576925e-03	1.9018891712855674e+00
Dual infeasibility.....:	4.9622897119973231e-09	3.0488307037751923e-06
Constraint violation....:	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....:	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....:	4.9622897119973231e-09	3.0488307037751923e-06

Number of objective function evaluations = 16
Number of objective gradient evaluations = 16
Number of equality constraint evaluations = 0
Number of inequality constraint evaluations = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations = 15
Total seconds in IPOPT = 0.003

EXIT: Optimal Solution Found.

```
└ Info:      1      129    5.5e-01    first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:      2      193    3.2e-01    first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:      3      254    1.7e-01    first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:      4      315    9.0e-02    first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:      5      373    4.6e-02    first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:      6      431    2.3e-02    first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:      7      486    1.2e-02    first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:      8      541    5.9e-03    first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:      9      593    3.0e-03    first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:     10      642    1.5e-03    first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
└ Info:     11      691    7.4e-04    first_order
└ @ Main /home/julien/Documents/code/MTH8408-Hiv24/lab5_JP/Lab5-notebook.ipynb:61
```

Devoir - Question 2 - Benchmark ADNLPproblems

```
In [ ]: ad_problems = (eval(Meta.parse(problem)))() for problem in OptimizationProblems.meta[(meta.has_equalities_only == true), :name]);
#ad_problems = (eval(Meta.parse(problem)))() for problem in OptimizationProblems.meta[!, :name])
```

Base.Generator{Vector{String}, var"#22#23"}(var"#22#23"(), ["BOX2", "aircraft", "argauss", "avion2", "booth", "britgas", "bt1", "catenary", "chain", "channel" ... "hs8", "hs80", "hs81", "hs87", "hs9", "hs99", "marine", "mgh01feas", "polygon2", "zangwil3"])

Rouler cette case vient max out la mémoire réservées aux sorties. Il devient donc impossible d'afficher la suite du TP... Je ne comprends pas pourquoi.

En faisant varier les hyperparamètres, on peut aboutir à une combinaison qui diminue le nombre de calculs. Il faudrait utiliser une méthode d'optimisation de boîte noire pour éviter d'y aller par essai-erreur.

```
In [ ]: solvers = Dict{
    :sig_2_rho_1 => model -> quad_penalty(model, model.meta.x0, epsilon=1e-6, eta=1e6, sigma=2.0, rho = 1.0),
    :sig_2_rho_10 => model -> quad_penalty(model, model.meta.x0, epsilon=1e-6, eta=1e6, sigma=2.0, rho = 10.0),
}

stats = bmark_solvers(
    solvers, ad_problems,
    skipif=prob -> (!(equality_constrained(prob.meta))) || get_nvar(prob) > 12 || get_nvar(prob) < 5)); # plus haute dimension fait crash mon ordi.
```

```
In [ ]: cols = [:id, :name, :nvar, :objective, :dual_feas, :neval_obj, :neval_grad, :neval_hess, :iter, :elapsed_time, :status,]
header = Dict{
    :nvar => "n",
    :objective => "f(x)",
    :dual_feas => "||∇f(x)||",
    :neval_obj => "# f",
    :neval_grad => "# ∇f",
    :neval_hess => "# ∇²f",
    :elapsed_time => "t",
}
for solver in keys(solvers)
    pretty_stats(stats[solver][!, cols], hdr_override=header)
end
```

Exercice 2: Calcul Variationnel

Dans cet exercice, on considère le problème de calcul variationnel suivant:

$$\min \int_0^1 (\dot{x}(t)^2 + 2x(t)^2)e^t dt, \quad x(0) = 0, x(1) = e - e^{-2}$$

modélisé avec `PDENLPModels`.

```
In [ ]: function cv_model(n :: Int)

    domain = (0,1) # set the domain
```



```
partition = n
model = CartesianDiscreteModel(domain,partition) # set discretization

labels = get_face_labeling(model)
add_tag_from_tags!(labels,"diri1",[2])
add_tag_from_tags!(labels,"diri0",[1]) # boundary conditions

order=1
valuetype=Float64
reffe = ReferenceFE(lagrangian, valuetype, order)
V0 = TestFESpace(model, reffe; conformity=:H1, dirichlet_tags=["diri0","diri1"])
U = TrialFESpace(V0,[0., exp(1)-exp(-2)])

trian = Triangulation(model)
degree = 2
dΩ = Measure(trian,degree) # integration machinery

# Our objective function
w(x) = exp(x[1])
function f(y)
    ∫((∇(y)⊙∇(y) + 2 * y * y) * w) * dΩ
end

xin = zeros(Gridap.FESpaces.num_free_dofs(U))
nlp = GridapPDENLPModel(xin, f, trian, U, V0)
return nlp
end
```

cv_model (generic function with 1 method)

Question 1: Résoudre

Résoudre le NLPModel généré par la fonction cv_model pour n = 16 avec ipopt et afficher la solution (attention la solution rendue ne contient pas les valeurs aux bords qu'il faut rajouter).

```
In [ ]: # TODO
n = 16
stats = ipopt(cv_model(n))
```

This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

```
Number of nonzeros in equality constraint Jacobian...:      0
Number of nonzeros in inequality constraint Jacobian.:      0
Number of nonzeros in Lagrangian Hessian.....:      44

Total number of variables.....:      15
      variables with only lower bounds:      0
      variables with lower and upper bounds:      0
      variables with only upper bounds:      0
Total number of equality constraints.....:      0
Total number of inequality constraints.....:      0
      inequality constraints with only lower bounds:      0
      inequality constraints with lower and upper bounds:      0
      inequality constraints with only upper bounds:      0

iter   objective    inf_pr  inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0   2.8202747e+02  0.00e+00  1.00e+02   -1.0  0.00e+00    -  0.00e+00  0.00e+00   0
   1   2.0987074e+01  0.00e+00  3.02e-14   -1.0  2.40e+00    -  1.00e+00  1.00e+00f  1

Number of Iterations....: 1
```

```
                                (scaled)                                (unscaled)
Objective.....:     9.6484800135597375e+00      2.0987074475792042e+01
Dual infeasibility.....:  3.0216136547373433e-14      6.5725203057809267e-14
Constraint violation....:  0.0000000000000000e+00      0.0000000000000000e+00
Variable bound violation:  0.0000000000000000e+00      0.0000000000000000e+00
Complementarity.....:    0.0000000000000000e+00      0.0000000000000000e+00
Overall NLP error.....:    3.0216136547373433e-14      6.5725203057809267e-14
```

```
Number of objective function evaluations = 2
Number of objective gradient evaluations = 2
Number of equality constraint evaluations = 0
Number of inequality constraint evaluations = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations = 1
Total seconds in IPOPT = 2.853
```

```
EXIT: Optimal Solution Found.
"Execution stats: first-order stationary"
```

```
In [ ]: sol = stats.solution
sol = vcat(0, sol, exp(1) - exp(-2) )
println(sol)
println(stats.objective)
```

[0.0, 0.181963067700128, 0.35428142029136056, 0.5188461331646761, 0.6773749095244438, 0.831435610236028, 0.9824672217813843, 1.1317985765449068, 1.2806651035693912, 1.4302238561349259, 1.5815670345204873, 1.7357341976668703, 1.8937233357803989, 2.056500956855748, 2.2250113233500324, 2.400184960541827, 2.5829465452224323]
20.987074475792042

Devoir - Question 3: Convergence en n

Afficher sur un même graphique la solution obtenue par ipopt pour plusieurs valeurs de n .

```
In [ ]: hist_obj = []
hist_sol = []
hist_n = []
for n in [15,63,127]
    stats = ipopt(cv_model(n))
    append!(hist_obj, stats.objective)
    append!(hist_n, n)
    sol = stats.solution
    sol = vcat(0, sol, exp(1) - exp(-2) )
    append!(hist_sol, [sol])
end
```

This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

```
Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 41

Total number of variables.....: 14
    variables with only lower bounds: 0
    variables with lower and upper bounds: 0
    variables with only upper bounds: 0
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
    inequality constraints with only lower bounds: 0
    inequality constraints with lower and upper bounds: 0
    inequality constraints with only upper bounds: 0

iter   objective   inf_pr   inf_du lg(mu)  ||d|| lg(rg) alpha_du alpha_pr ls
  0   2.6395311e+02  0.00e+00  1.00e+02  -1.0  0.00e+00   -   0.00e+00  0.00e+00  0
  1   2.0987227e+01  0.00e+00  1.48e-14  -1.0  2.39e+00   -   1.00e+00  1.00e+00f  1
```

Number of Iterations....: 1

	(scaled)	(unscaled)
Objective.....	1.0314871972017846e+01	2.0987226900039182e+01
Dual infeasibility.....	1.4841845893178002e-14	3.0198066269804252e-14
Constraint violation....	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....	1.4841845893178002e-14	3.0198066269804252e-14

```
Number of objective function evaluations = 2
Number of objective gradient evaluations = 2
Number of equality constraint evaluations = 0
Number of inequality constraint evaluations = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations = 1
Total seconds in IPOPT = 0.030
```

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

```
Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 185

Total number of variables.....: 62
    variables with only lower bounds: 0
    variables with lower and upper bounds: 0
    variables with only upper bounds: 0
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
    inequality constraints with only lower bounds: 0
    inequality constraints with lower and upper bounds: 0
    inequality constraints with only upper bounds: 0

iter   objective   inf_pr   inf_du lg(mu)  ||d|| lg(rg) alpha_du alpha_pr ls
  0   1.1336967e+03  0.00e+00  1.00e+02  -1.0  0.00e+00   -   0.00e+00  0.00e+00  0
  1   2.0986035e+01  0.00e+00  3.68e-14  -1.0  2.54e+00   -   1.00e+00  1.00e+00f  1
```

Number of Iterations....: 1

	(scaled)	(unscaled)
Objective.....	2.3912701385293924e+00	2.0986034775831953e+01
Dual infeasibility.....	3.6838323111835286e-14	3.2329694477084564e-13
Constraint violation....	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....	3.6838323111835286e-14	3.2329694477084564e-13

```
Number of objective function evaluations = 2
Number of objective gradient evaluations = 2
Number of equality constraint evaluations = 0
Number of inequality constraint evaluations = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations = 1
Total seconds in IPOPT = 0.028
```

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

```
Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 0
Number of nonzeros in Lagrangian Hessian.....: 377

Total number of variables.....: 126
    variables with only lower bounds: 0
    variables with lower and upper bounds: 0
    variables with only upper bounds: 0
Total number of equality constraints.....: 0
Total number of inequality constraints.....: 0
    inequality constraints with only lower bounds: 0
    inequality constraints with lower and upper bounds: 0
    inequality constraints with only upper bounds: 0

iter   objective   inf_pr   inf_du lg(mu)  ||d|| lg(rg) alpha_du alpha_pr ls
  0   2.2942372e+03  0.00e+00  1.00e+02  -1.0  0.00e+00   -   0.00e+00  0.00e+00  0
  1   2.0985981e+01  0.00e+00  4.26e-14  -1.0  2.56e+00   -   1.00e+00  1.00e+00f  1
```

Number of Iterations....: 1

	(scaled)	(unscaled)
Objective.....	1.1814172902345206e+00	2.0985980505142805e+01
Dual infeasibility.....	4.2600418838398981e-14	7.5672801358450670e-13
Constraint violation....	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity.....	0.0000000000000000e+00	0.0000000000000000e+00
Overall NLP error.....	4.2600418838398981e-14	7.5672801358450670e-13

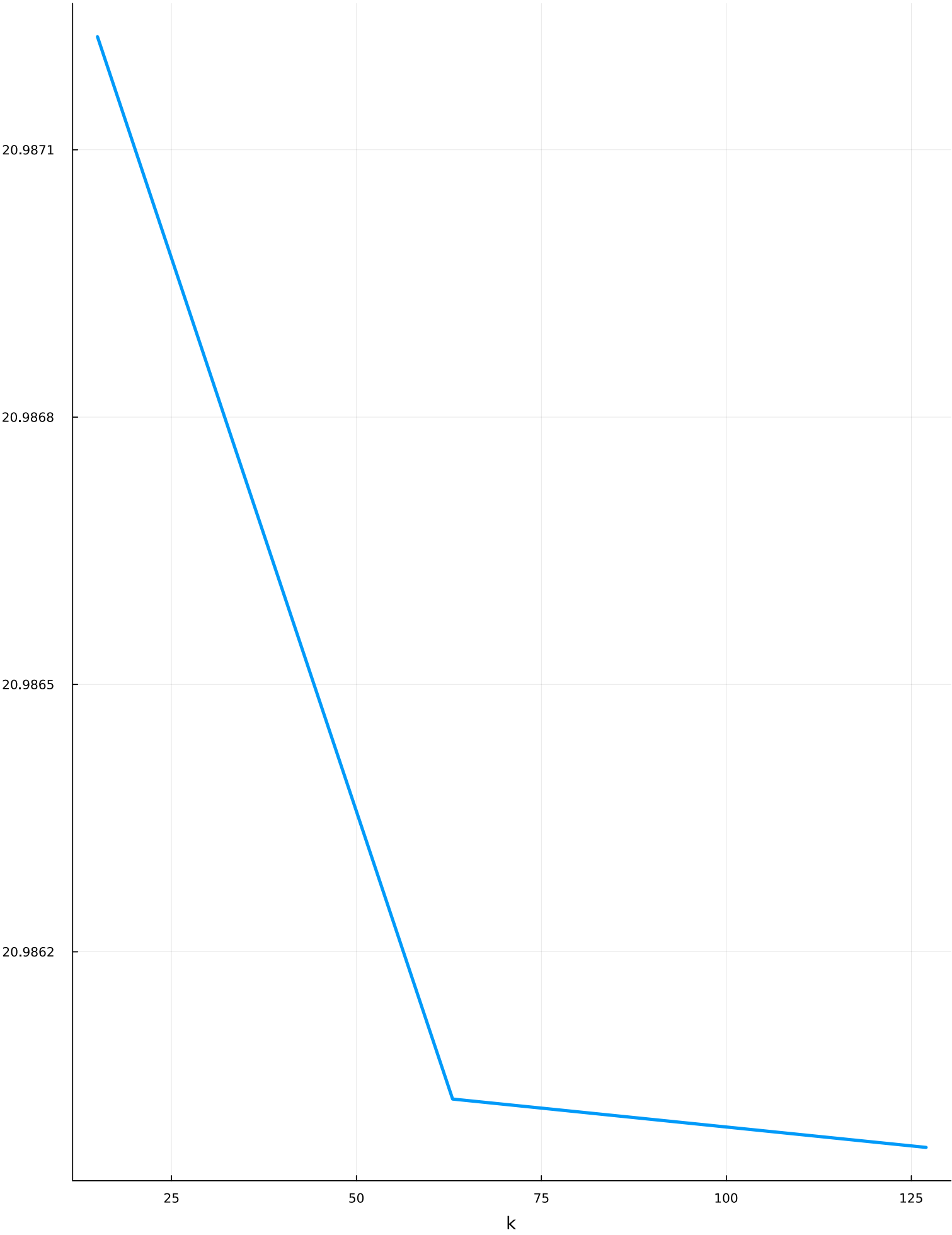
```
Number of objective function evaluations      = 2
Number of objective gradient evaluations     = 2
Number of equality constraint evaluations    = 0
Number of inequality constraint evaluations  = 0
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations    = 1
Total seconds in IPOPT                      = 0.055
```

EXIT: Optimal Solution Found.

In []: **using** Plots

```
p1=plot(hist_n, hist_obj, title="Objective according to n", label="", linewidth=3, xlabel="k",left_margin=3Plots.mm)
plot!(size=(900,1200))
plot(p1, layout=(1,1), legend=true)
```

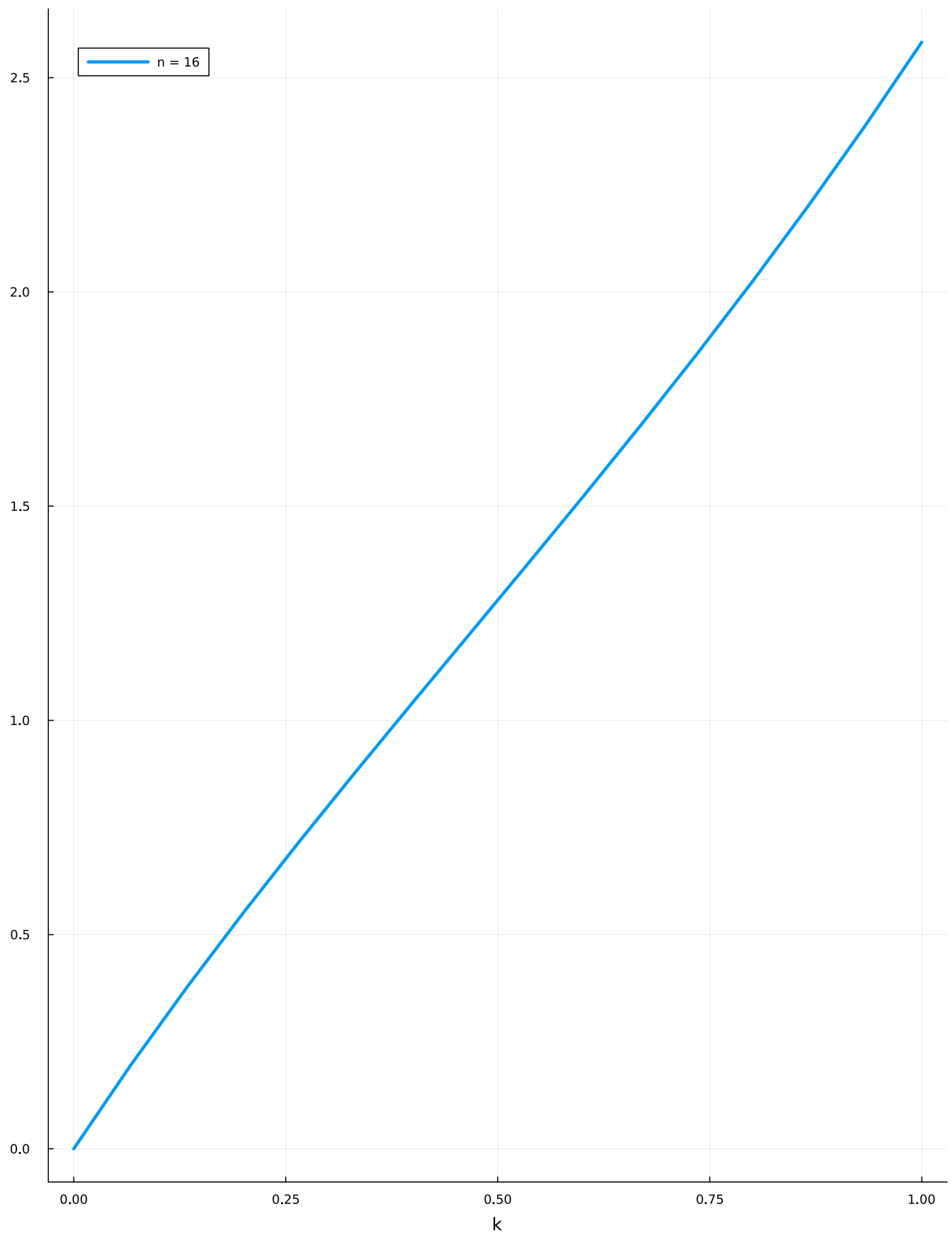
Objective according to n



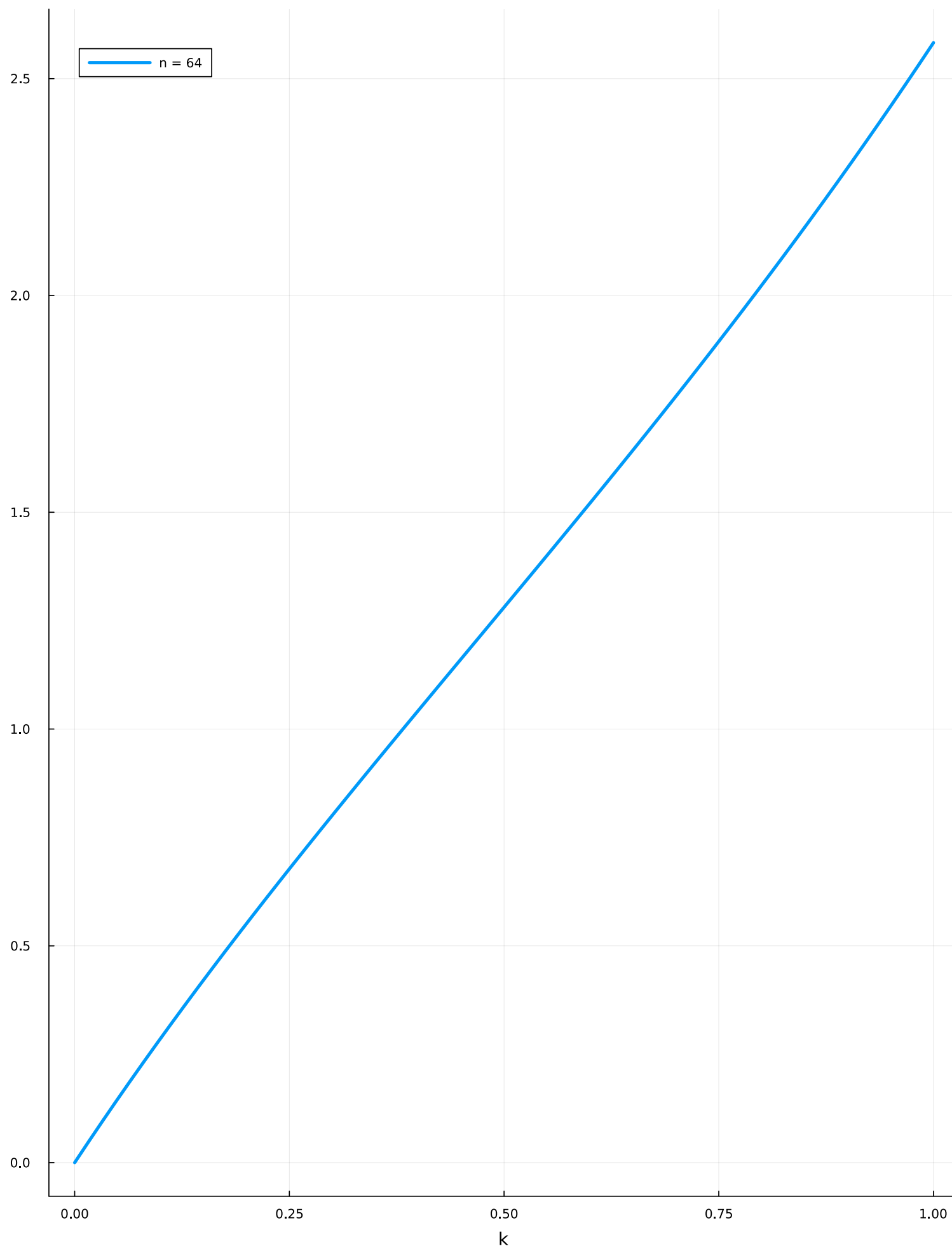
In []: **gr()**

```
for n in 1:length(hist_sol)
    leng = length(hist_sol[n])
    t = range(0, 1, length= leng)
    p = plot(t, hist_sol[n], title="Solution according with n = $leng", label= "n = $leng", linewidth=3, xlabel="k",left_margin=3Plots.mm)
    plot!(size=(900,1200))
    display(plot(p, layout=(1,1), legend=true))
end
```

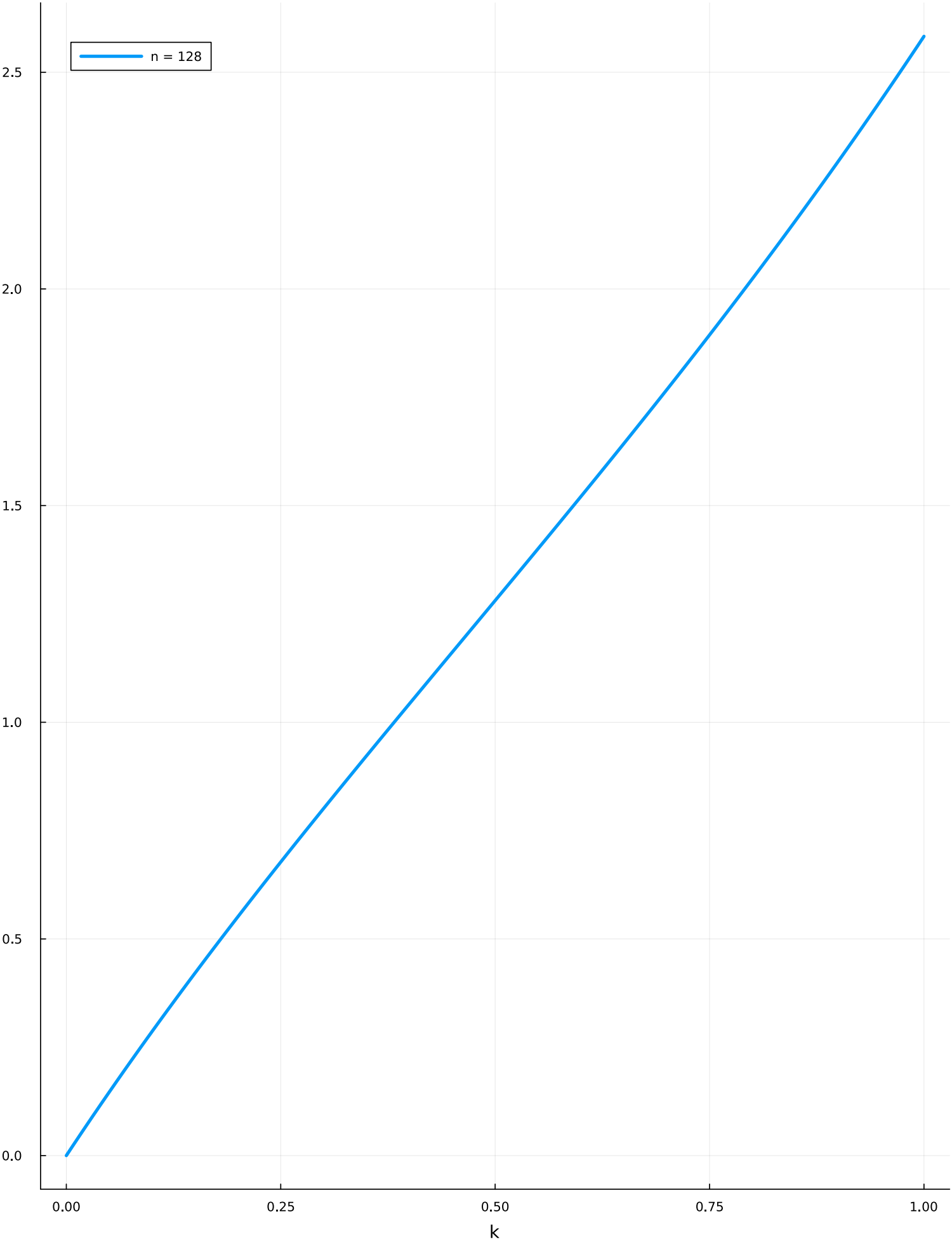
Solution according with $n = 16$



Solution according with $n = 64$



Solution according with n = 128



Je n'arrive pas à mettre chaque courbe sur le même graphique étant donné le nombre de points qui diffère d'une courbe à l'autre.

Question 3: Comparer à la solution exacte

La solution exacte est $x(t) = e^t - e^{-2t}$ et la valeur optimale est $e^3 - 2e^{-3} + 1$.

```
In [ ]: x = t -> exp.(t) .- exp.(-2 .*t)
opt = exp(3) - 2*exp(-3) + 1

println(opt)
println(abs(opt - hist_obj[end]))
```

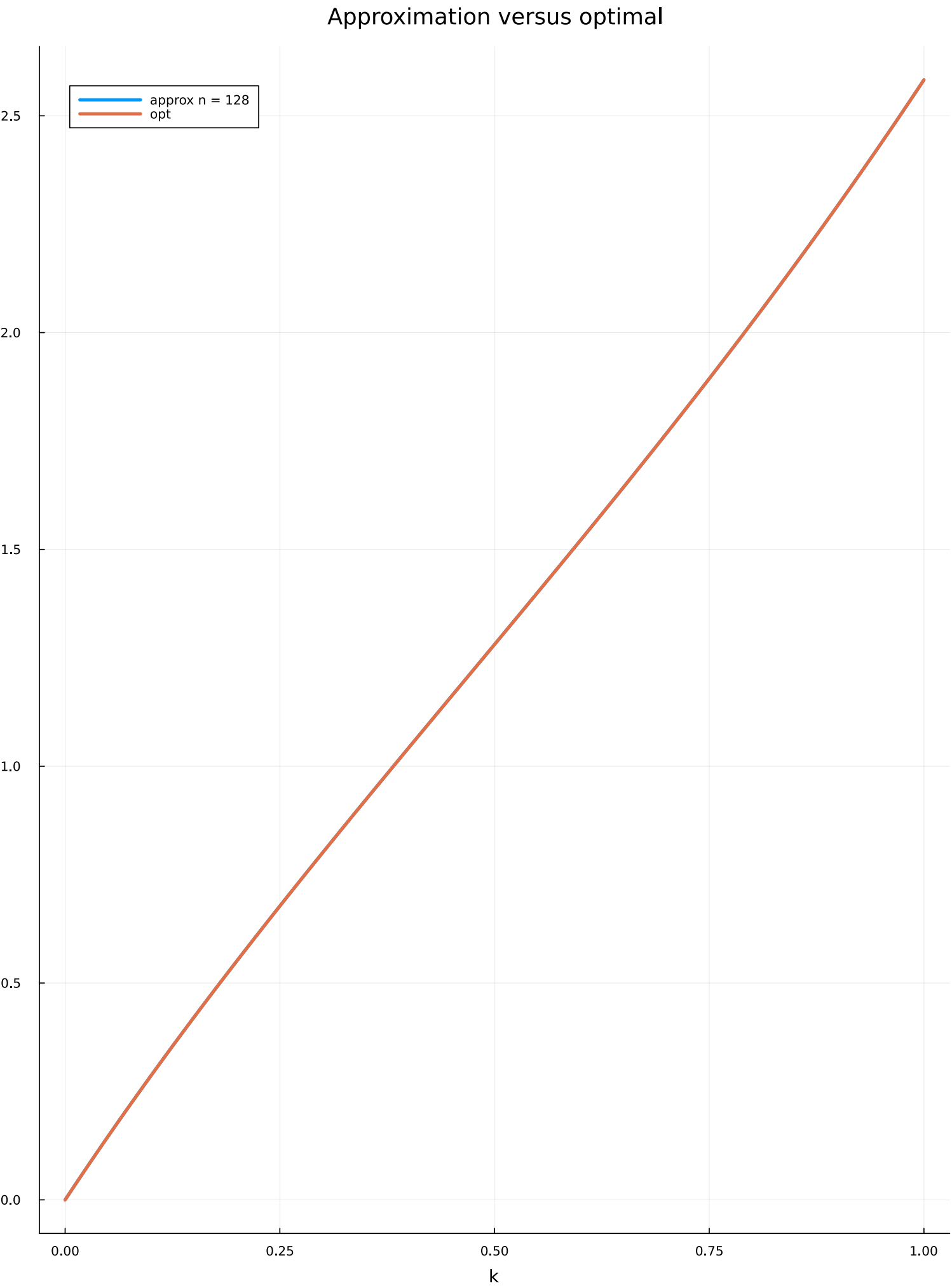
20.98596278645194
1.771869086653055e-5

La solution optimale est presque'identique à la solution avec 128 points.

```
In [ ]: leng = length(hist_sol[end])
t = collect(range(0, 1, length=leng))
print(x(t))
```

[0.0, 0.023529776948816417, 0.04687791384849538, 0.0700487191556386, 0.09304644563216324, 0.11587529130739727, 0.13853940042587098, 0.16104286438103166, 0.18338972263510545, 0.20558396362533304, 0.22762952565679262, 0.24953029778203006, 0.2712901206677064, 0.2929127874484704, 0.31440204456826537, 0.3357615926092695, 0.3569950871086691, 0.37810613936346427, 0.39909831722349565, 0.41997514587288765, 0.4407401086000906, 0.46139664755671017, 0.48194816450530287, 0.5023980215563196, 0.5227495418943702, 0.5430060104939846, 0.5631706748250407, 0.5832467455480261, 0.6032373971993026, 0.623145768866532, 0.6429749648544264, 0.6627280553409828, 0.6824080770243527, 0.7020180337605043, 0.7215608971918286, 0.7410396073668327, 0.7604570733510746, 0.7798161738294764, 0.7991197577001629, 0.8183706446599613, 0.8375716257817065, 0.8567254640834775, 0.875834895089908, 0.8949026273856967, 0.9139313431614499, 0.9329236987519831, 0.9518823251672061, 0.9708098286157171, 0.9897087910212261, 1.00858177053193, 1.0274313020229542, 1.0462598975919795, 1.0650700470481689, 1.0838642183945069, 1.1026448583036608, 1.1214143925874769, 1.1401752266602179, 1.1589297459956456, 1.1776803165780572, 1.1964292853473781, 1.2151789806384081, 1.2339317126143285, 1.2526897736945615, 1.2714554389770838, 1.2902309666552885, 1.309018598429489, 1.3278205599131594, 1.3466390610339987, 1.3654762964299172, 1.3843344458400233, 1.4032156744907067, 1.422122133476901, 1.4410559601386081, 1.4600192784327737, 1.4790141993005905, 1.4980428210303147, 1.5171072296156736, 1.5362094991099424, 1.5553516919757717, 1.5745358594308376, 1.5937640417893917, 1.6130382687997882, 1.632360559978054, 1.6517329249375845, 1.6711573637150263, 1.6906358670924213, 1.7101704169156837, 1.7297629864094732, 1.7494155404885348, 1.7691300360655688, 1.7889084223557008, 1.808752641177609, 1.82866462725138, 1.8486463084931486, 1.8686996063065888, 1.8888264358713123, 1.9090287064282374, 1.9293083215619862, 1.949667179480367, 1.9701071732910025, 1.9906301912751578, 2.0112381171588223, 2.0319328303811086, 2.052716206360008, 2.0735901167555753, 2.0945564297305763, 2.1156170102086644, 2.13677372013013, 2.158028418705278, 2.179382962665475, 2.200839206511928, 2.222399002762236, 2.2440642021947554, 2.265836654090842, 2.287718206475008, 2.309710706353032, 2.3318159999480854, 2.354035932934904, 2.376372350672058, 2.398827098432362, 2.421402021631462, 2.4440989660546544, 2.466919778081967, 2.489866304911548, 2.5129403947814093, 2.5361438971895494, 2.5594786631125164, 2.5829465452224323]

```
In [ ]: p = plot(t, [hist_sol[end] x(t)], title="Approximation versus optimal", label= ["approx n = $leng" "opt"], linewidth=3, xlabel="k",left_margin=
plot!(size=(900,1200))
plot(p, layout=(1,1), legend=true)
```



Les courbes se chevauchent.