



# Distribution of true stellar rotational velocities

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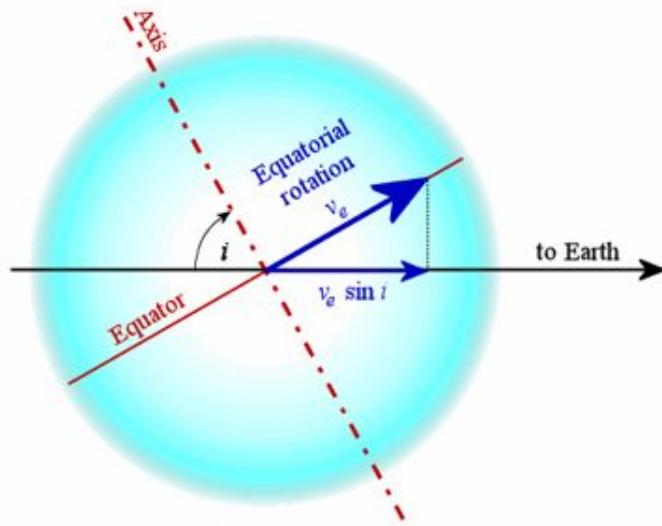


COLLABORATORS:



# Introduction

If we have only access to  $v \sin(i)$  data, how we could get the values of real velocity rotational velocity  $v$ ?



→ Observed data sample is "contaminated" by this geometric projection effect

# Introduction

## Nomenclature Convention

True Rotation Velocity

$$x = v$$

Projected Rotation Velocity

$$y = x \sin i$$

$$y = v \sin i$$

*Apparent Rotation Velocity*

$$\underbrace{f_Y(y)}_{\mathbf{Y}} = \underbrace{\int_y^{\infty} dx \frac{1}{x} \frac{1}{\sqrt{x^2 + y^2}}}_{\mathbf{A}} \underbrace{f_X(x)}_{\mathbf{x}}$$

$$\mathbf{Y} = \mathbf{AX}$$

## Isotropic velocities

Assumed a random orientation of rotational axes

$$f_Y(y) = \int_y^{\infty} \frac{1}{x} \frac{1}{\sqrt{x^2 - y^2}} f_X(x) dx$$

Analytic solution by Abel's integration

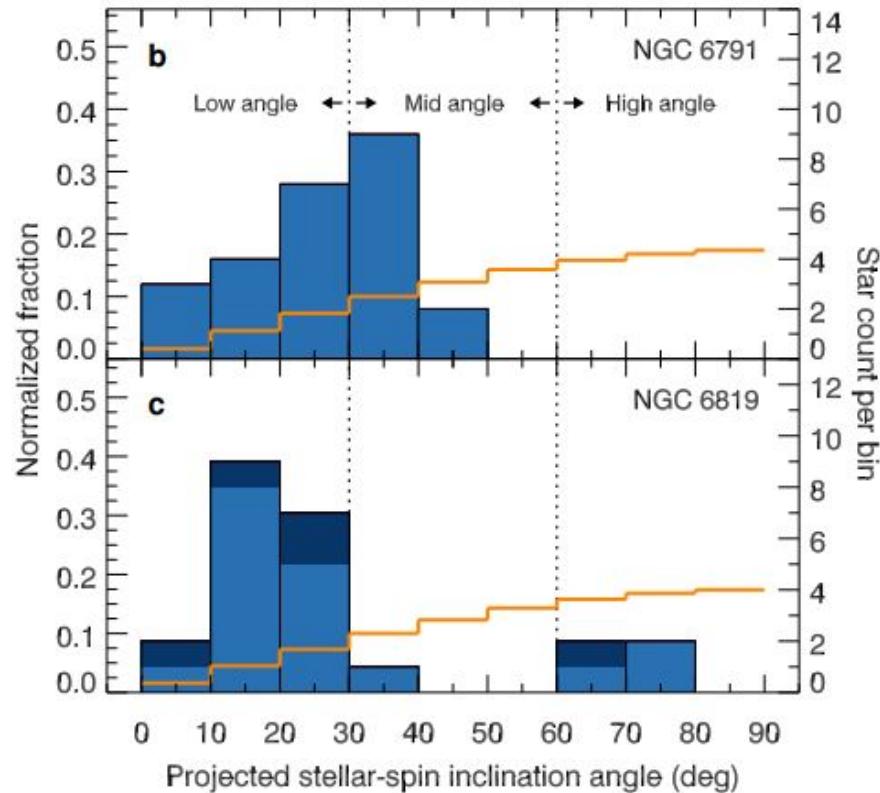
$$f_X(x) = -\frac{2}{\pi} x^2 \frac{\partial}{\partial x} x \int_x^{\infty} \frac{f_Y(y)}{y^2 \sqrt{y^2 - x^2}} dy$$

[S: Chandrasekhar and G. Münch \(1949\)](#)

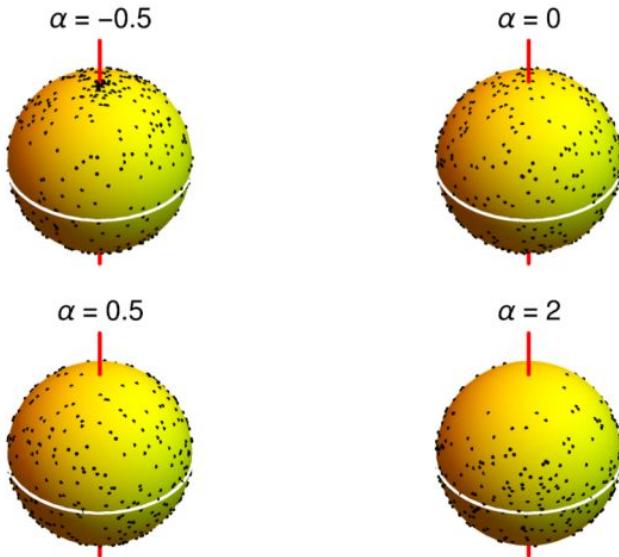
# Introduction

The case of 48 red giants of NGC 6791  
and NGC 6819

The stellar spin axes are *not* randomly oriented



# Introduction



## Non-Isotropic Velocities

$$f_Y(y; \alpha) = c_\alpha \int_y^\infty \frac{y^{2\alpha+1}}{x^{2\alpha+1}} \frac{1}{\sqrt{x^2 - y^2}} f_X(x; \alpha) dx$$

Factor of Normalization  $c_\alpha = \frac{2\Gamma(\alpha + 3/2)}{\Gamma(1/2)\Gamma(\alpha + 1)}$

[Solar M. 2021](#)

$\alpha$  describes how the stellar spin axes are oriented in space:

- $\alpha < 0$ : *Polar alignment*
- $\alpha > 0$ : *Equatorial concentration*

$$\underbrace{f_Y(y; \alpha)}_{\mathbf{Y}} = \underbrace{c_\alpha \int_y^\infty dx \frac{y^{2\alpha+1}}{x^{2\alpha+1}} \frac{1}{\sqrt{x^2 - y^2}}}_{\mathbf{A}} \underbrace{f_X(x; \alpha)}_{\mathbf{X}}$$

# Introduction

Recover the true distribution of rotational velocities ( $f_x(x)$ ) from the observed data ( $f_y(y)$ ) by solving an integral equation with an  $\alpha$ -dependent kernel, using Tikhonov regularization to obtain stable solutions.



# Methodology

# Methodology

## Kernel

Gaussian Kernel (Symmetric Kernel)

$$\hat{f}_h(x) = \frac{1}{nh} \sum_{i=1}^n \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x - x_i)^2}{2h^2}\right)$$

## Regularization

Tikhonov's Regularization

$$\min_{\mathbf{X}} (\|\mathbf{Y} - \mathbf{AX}\|^2 + \lambda \|\mathbf{X}\|^2)$$

We assumed in all cases  $\lambda = 0.1$

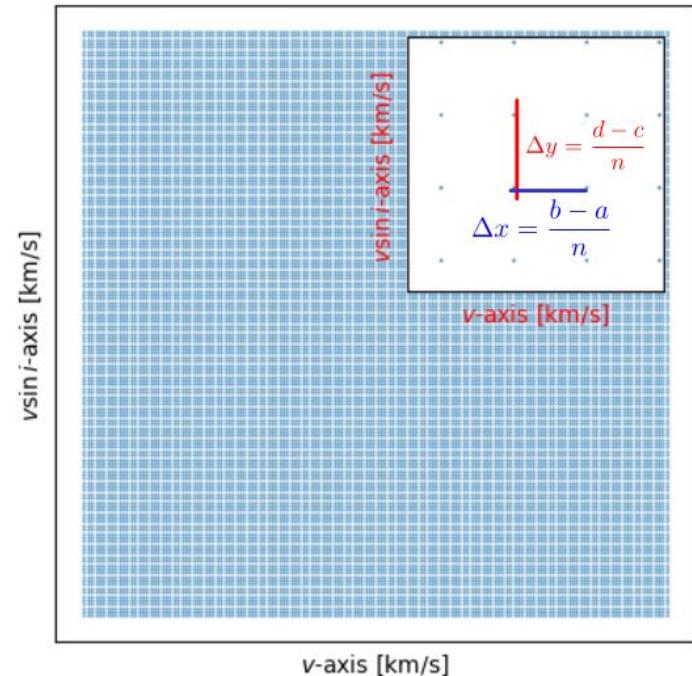
## Metric

Root Mean Square Error

$$\text{RMSE}(\alpha) = \sqrt{\sum_{i=1}^n \frac{(f_Y(y_i; \alpha) - \hat{f}_Y(y_i; \alpha))^2}{n}}$$

## Distribution

Point Estimate

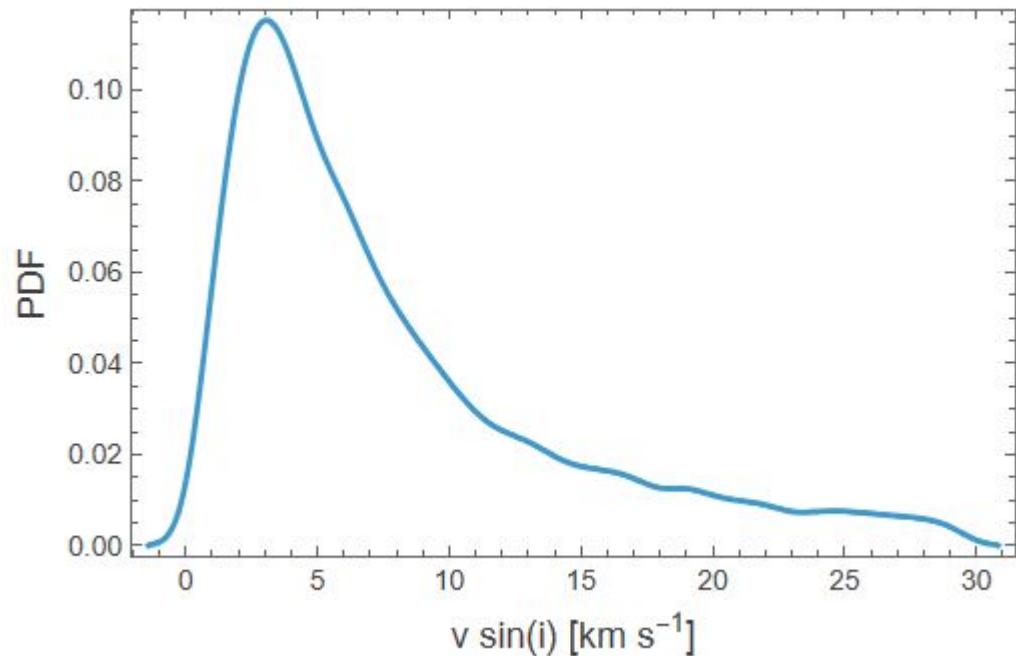


# Methodology - The Geneva Data

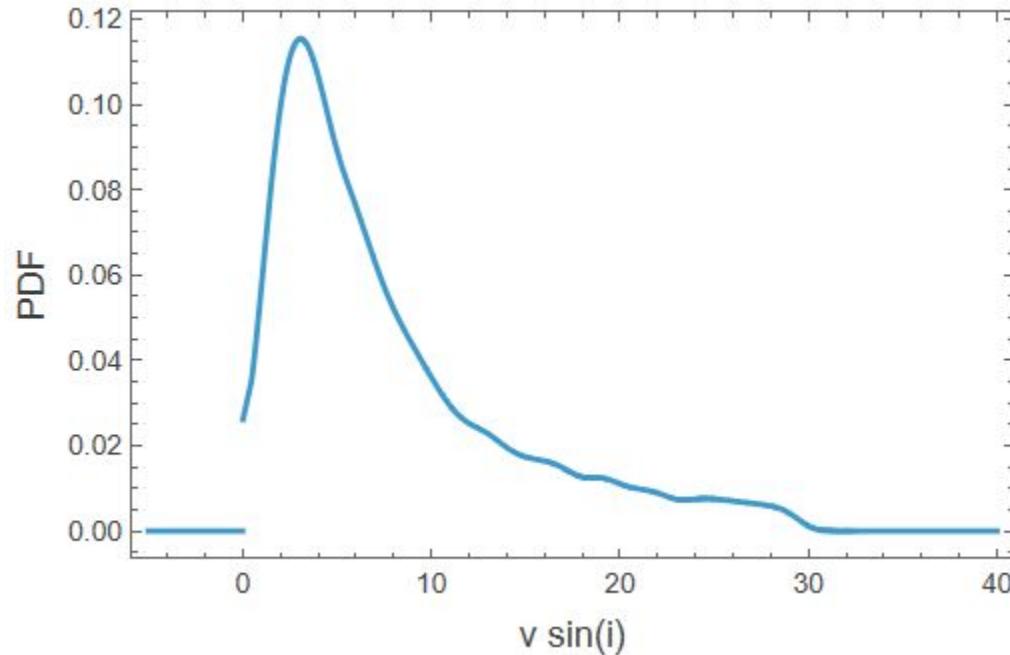
## Dataset

- Geneva - Copenhagen Survey
- 11818 data of  $v \sin i$ .
- Field stars.
- F and G dwarfs.

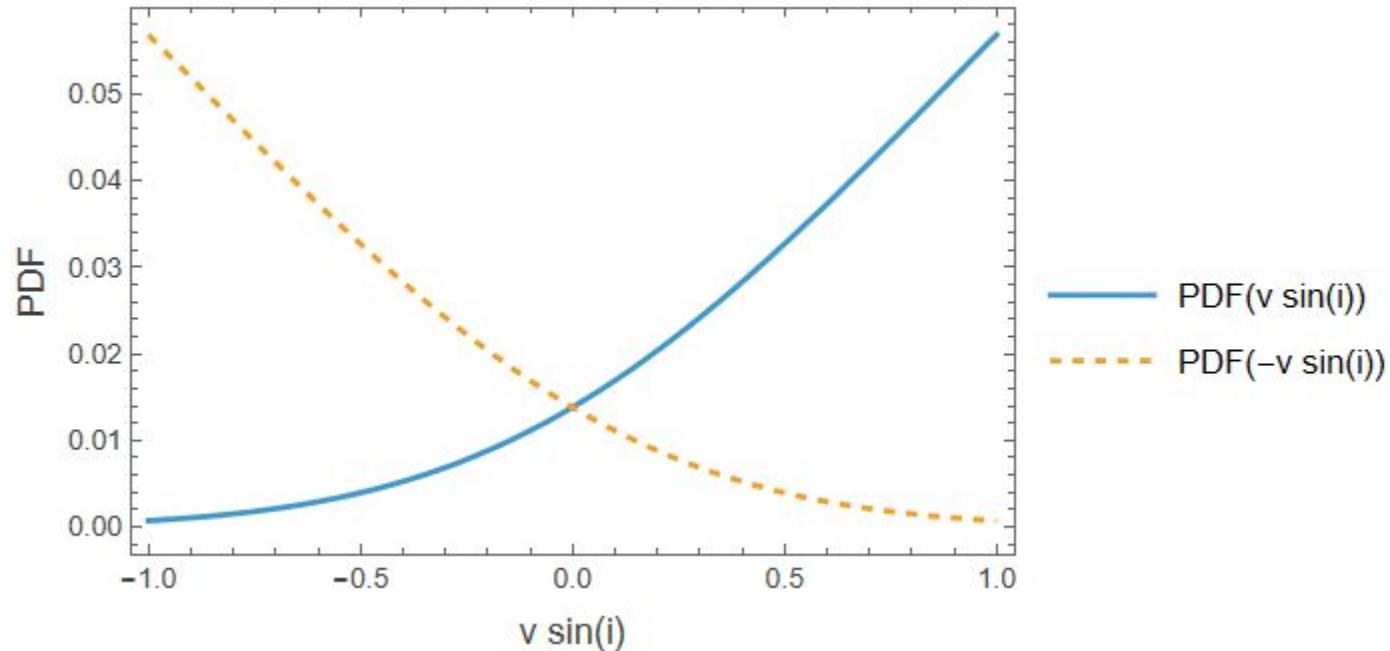
[J. Holmberg et al. 2009](#)



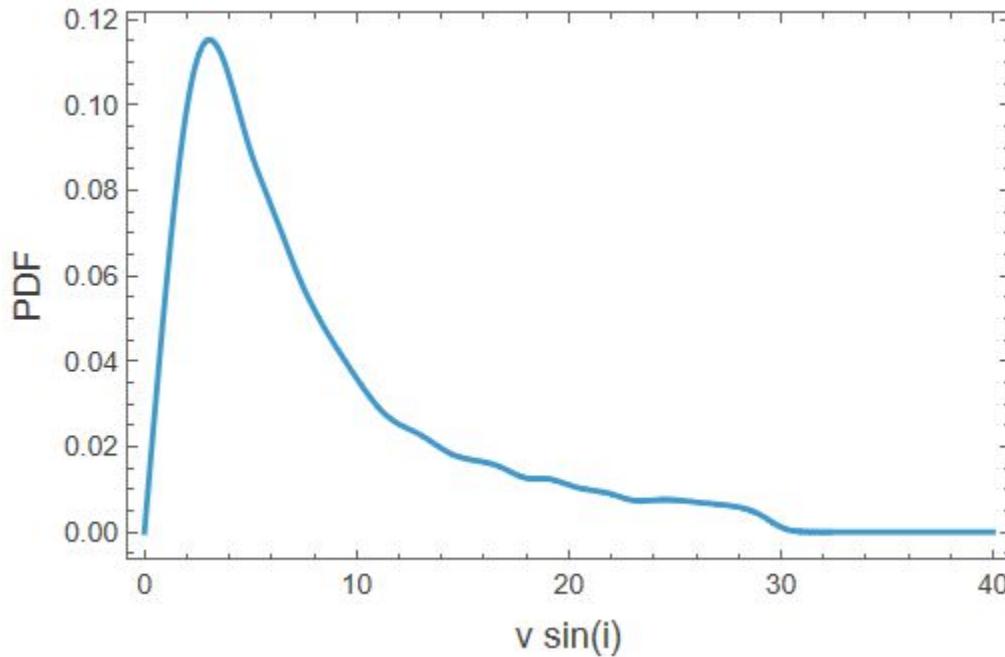
## Methodology - The bounding of the solution



# Methodology - A trick



## Methodology - The distribution of projected velocities



# Methodology - Solving for X

Matricial relationship

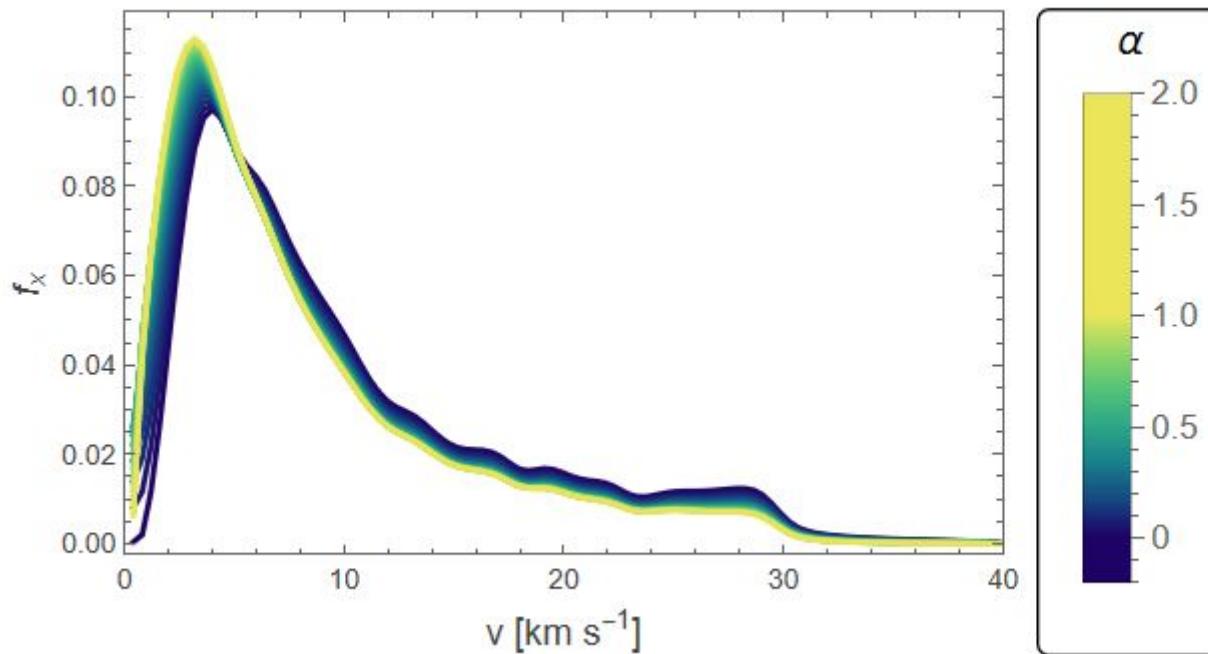
$$\underbrace{f_Y(y; \alpha)}_{\mathbf{Y}} = \underbrace{c_\alpha \int_y^\infty dx \frac{y^{2\alpha+1}}{x^{2\alpha+1}} \frac{1}{\sqrt{x^2 - y^2}}}_{\mathbf{A}} \underbrace{f_X(x; \alpha)}_{\mathbf{X}}$$

Tikhonov's Regularization

$$\min_{\mathbf{X}} (||\mathbf{Y} - \mathbf{AX}||^2 + \lambda ||\mathbf{X}||^2)$$

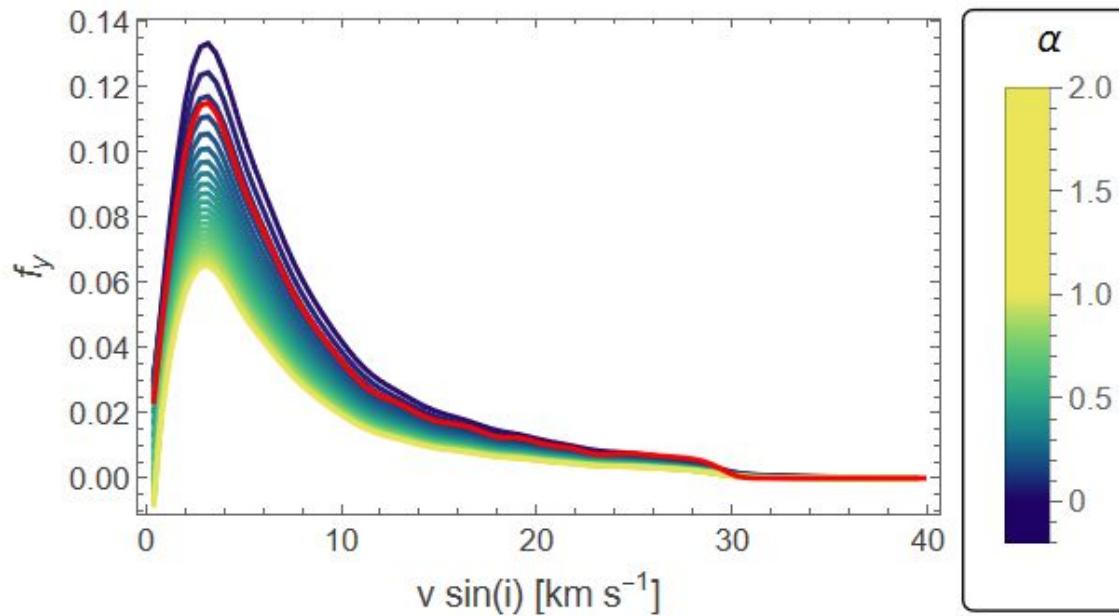
We assumed in all cases  $\lambda = 0.1$

# Methodology - The distribution of true velocities



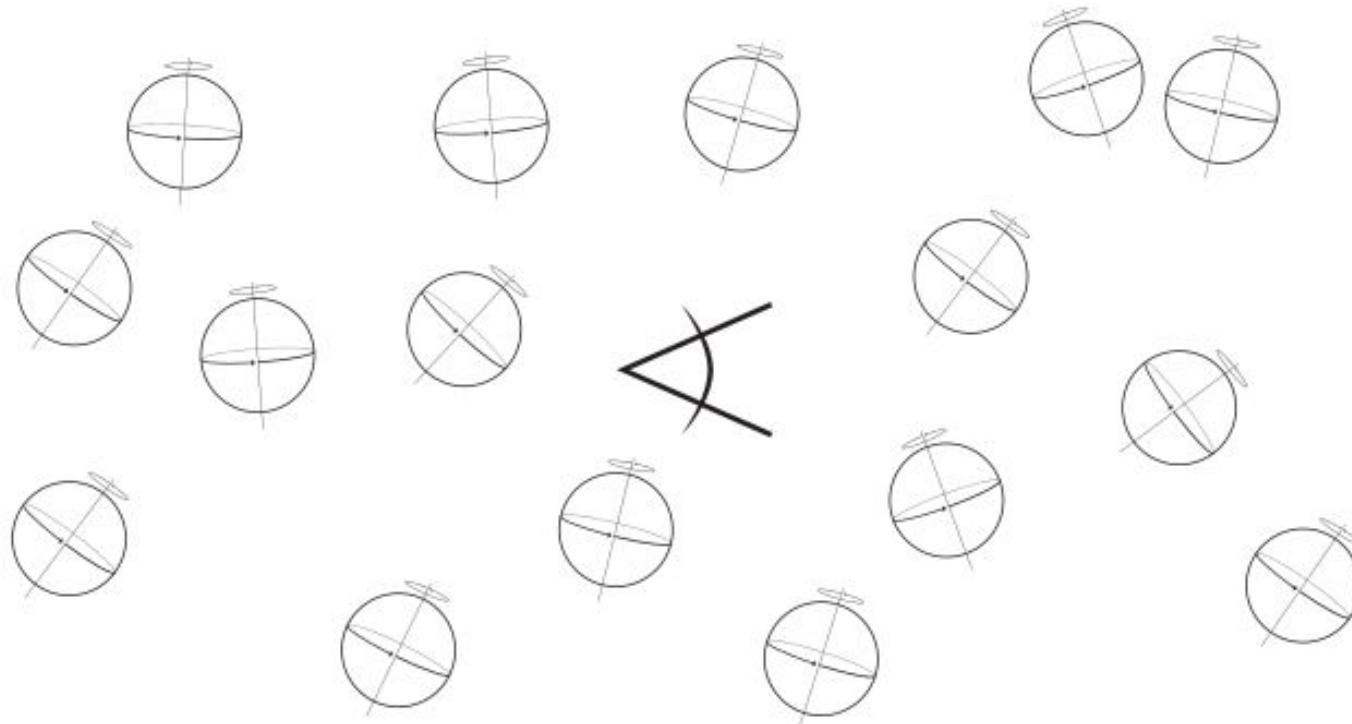
# Methodology - Reconstruction of $f_Y$

$$f_Y(y; \alpha) = \int_y^{\infty} \frac{y^{2\alpha+1}}{x^{2\alpha+1}} \frac{1}{\sqrt{x^2 - y^2}} f_X(x; \alpha) dx$$



# Results

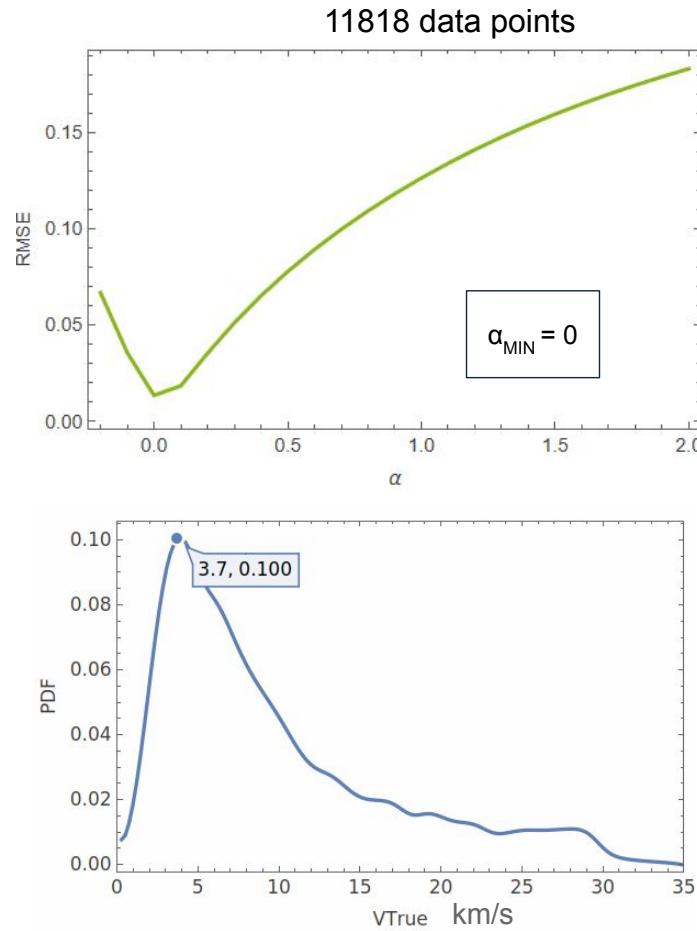
# Isotropic



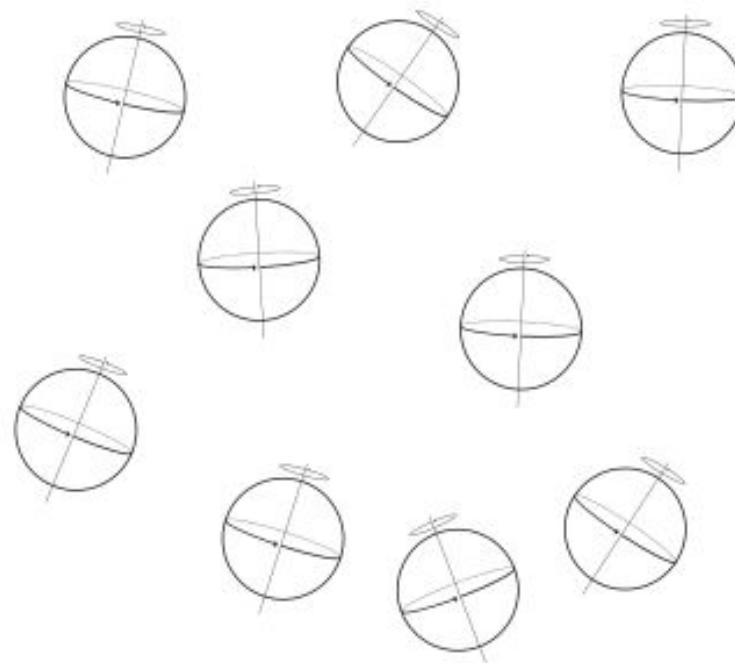
# Geneva Database

$$f_Y(y; \alpha) = c_\alpha \int_y^\infty \frac{y^{2\alpha+1}}{x^{2\alpha+1}} \frac{1}{\sqrt{x^2 - y^2}} f_X(x; \alpha) dx$$

$$\text{RMSE}(\alpha) = \sqrt{\sum_{i=1}^n \frac{(f_Y(y_i; \alpha) - \hat{f}_Y(y_i; \alpha))^2}{n}}$$



# Non-Isotropic

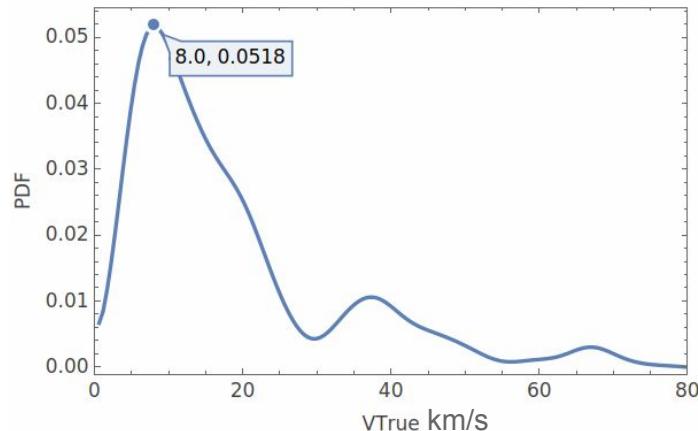
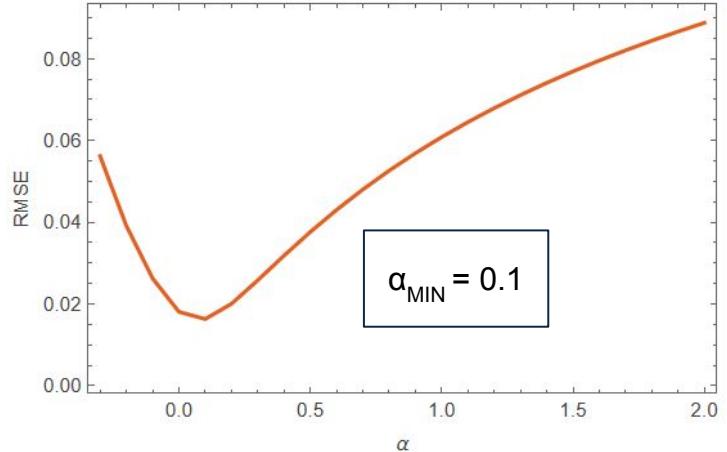


# Pleiades



Open Star Cluster  
~440 light-years from Earth  
Constellation Taurus

145 data points

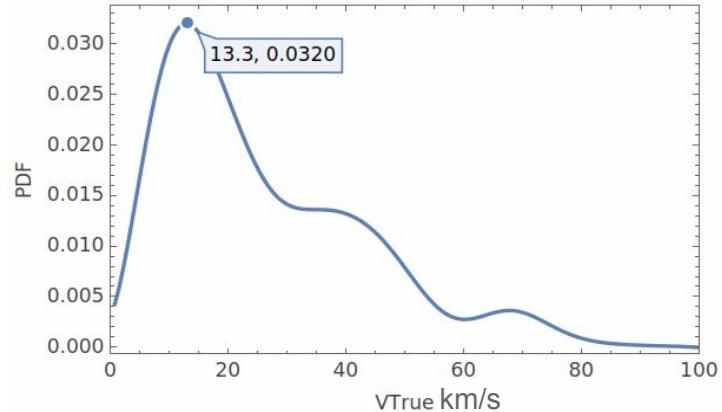
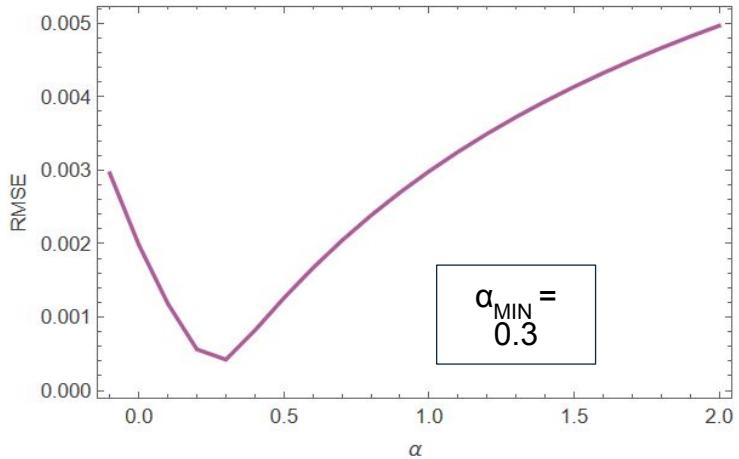


# Alpha Persei



Open Star Cluster  
~570 light-years from Earth  
Constellation Perseus

57 data points





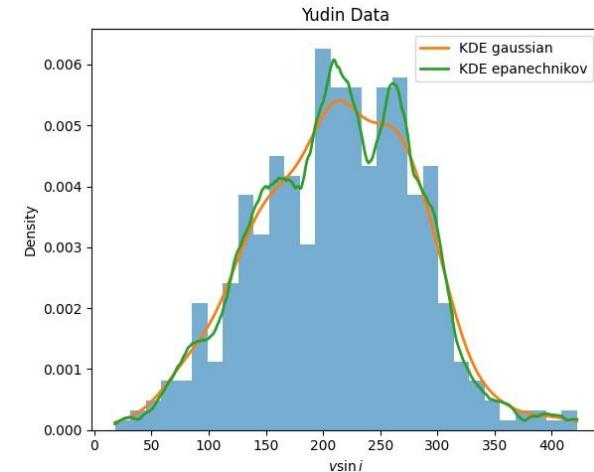
# Conclusions and future work

# Conclusions

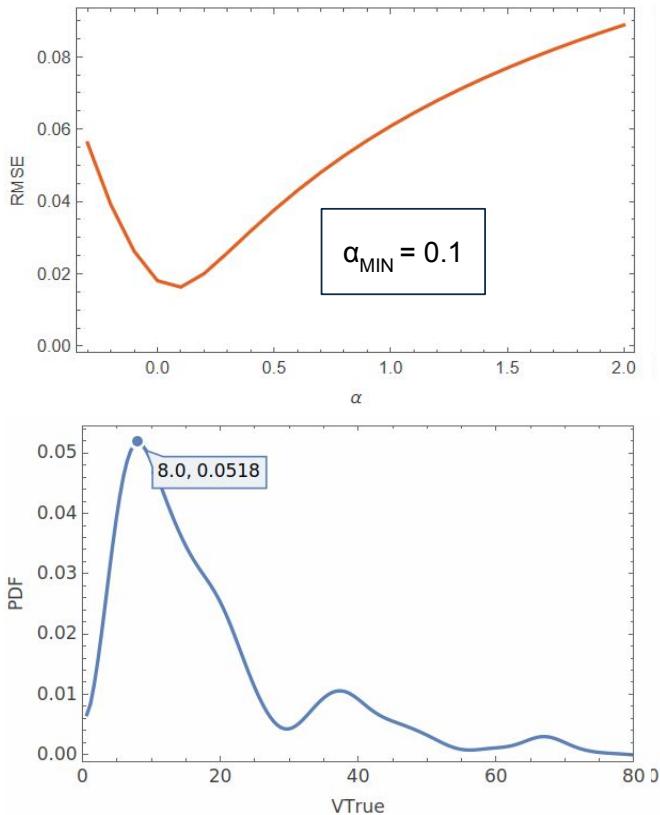
- As expected we found that for field stars  $\alpha \approx 0$  as there should be a uniform distribution of axis angles.
- In the cases of Pleiades and Alpha Per which are clusters we found that  $\alpha \neq 0$  because there is a privileged direction.

# Future work

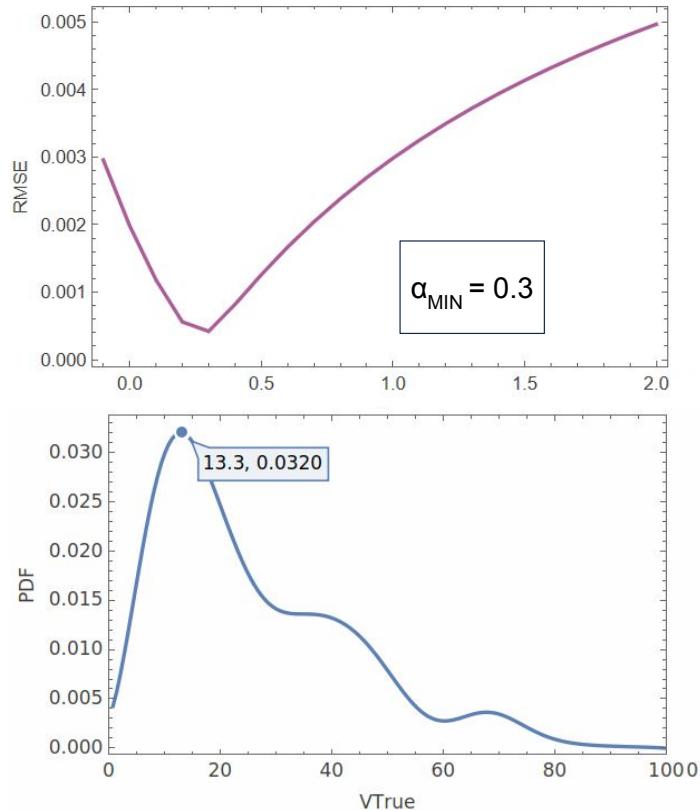
- To explore the space of lambda parameters, in this work we used a fixed value.
- To use more appropriate kernels, like for example asymmetric ones to avoid having to fix by hand the PDF.
- Work with more intelligent binning to find better solutions.



### Pleiades



### Alpha Persei



# References

- Chandrasekhar, S., & Münch, G. (1950). On the integral equation governing the distribution of the true and the apparent rotational velocities of stars. *The Astrophysical Journal*, 111, 142. <https://doi.org/10.1086/145245>
- Corsaro, E., Lee, Y.-N., García, R. A., Hennebelle, P., Mathur, S., Beck, P. G., Mathis, S., Stello, D., & Bouvier, J. (2017). Spin alignment of stars in old open clusters. *Nature Astronomy*, 1, 0064. <https://doi.org/10.1038/s41550-017-0064>
- Holmberg, J., Nordström, B., & Andersen, J. (2009). The Geneva-Copenhagen survey of the solar neighbourhood. III. Improved distances, ages, and kinematics. *Astronomy & Astrophysics*, 501(3), 941–947. <https://doi.org/10.1051/0004-6361/200811191>
- Solar, M. (2021). Non-isotropic distributions of stellar rotational velocities [Tesis de magíster no publicada, Universidad de Valparaíso]. Repositorio Institucional UV. <https://repositoriobibliotecas.uv.cl/items/7c463ec8-453b-42b3-ab49-9fd2b2961a6a/full>

