Distribution of true stellar rotational velocities

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The following problem is considered:
Determine the unknown distribution of the true rotational velocities of stars, $f_x(x)$, from the observed distribution of projected velocities, $f_y(y)$, by solving the following Fredholm integral equation:

$$f_y(y) = c_\alpha \int_y^\infty \frac{y^{2\alpha+1}}{x^{2\alpha+1}} \frac{1}{\sqrt{x^2 - y^2}} f_x(x)\, dx \,, \tag{1}$$

where

$$c_\alpha = \frac{2\Gamma(\alpha + 3/2)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\alpha + 1\right)}, \tag{2}$$

being Γ the gamma function. The parameter α characterises possible non-isotropy in the orientation of stellar axes.

This is a numerically ill-posed inverse problem, as it involves estimating $f_x(x)$ from $f_y(y)$, with both being density functions related by an integral transform whose kernel depends on an unknown parameter.

Upon discretization, the problem takes the matrix form $\mathbf{Y} = \mathbf{A}\mathbf{X}$, where $\mathbf{Y}$ is the observed vector (f_y), $\mathbf{X}$ is the unknown vector (f_v), and $\mathbf{A}$ is a matrix that depends on α.

Due to the problem being ill-posed, Tikhonov regularisation is applied to minimise.

$$\min_{\mathbf{X}} \left(\|\mathbf{Y} - \mathbf{A}\mathbf{X}\|^2 + \lambda \|\mathbf{X}\|^2 \right) , \tag{3}$$

where λ is the regularisation parameter, which must be chosen carefully, for example via generalised cross-validation.

In summary, the challenge is to recover the true distribution of rotational velocities f_x from observed data f_y by solving an integral equation with a kernel that depends on α, which requires regularisation techniques to obtain stable solutions.