



**Machine Learning and
Astrostatistics School:**
Applications to Massive Stars

Distribution of true stellar rotational velocities

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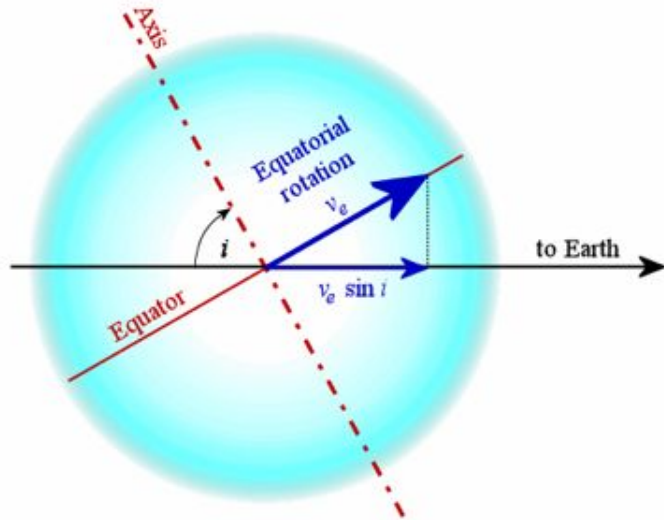


COLLABORATORS:



Introduction

If we have only access to $v \sin(i)$ data, how we could get the values of real velocity rotational velocity v ?



→ Observed data sample is "contaminated" by this geometric projection effect

Introduction

Nomenclature Convention

True Rotation Velocity

$$x = v$$

Projected Rotation Velocity

$$y = x \sin i$$

$$y = v \sin i$$

Apparent Rotation Velocity

Isotropic velocities

Assumed a random orientation of rotational axes

$$f_Y(y) = \int_y^\infty \frac{1}{x} \frac{1}{\sqrt{x^2 - y^2}} f_X(x) dx$$

Analytic solution by Abel's integration

$$f_X(x) = -\frac{2}{\pi} x^2 \frac{\partial}{\partial x} x \int_x^\infty \frac{f_Y(y)}{y^2 \sqrt{y^2 - x^2}} dy$$

[S: Chandrasekhar and G. Münch \(1949\)](#)

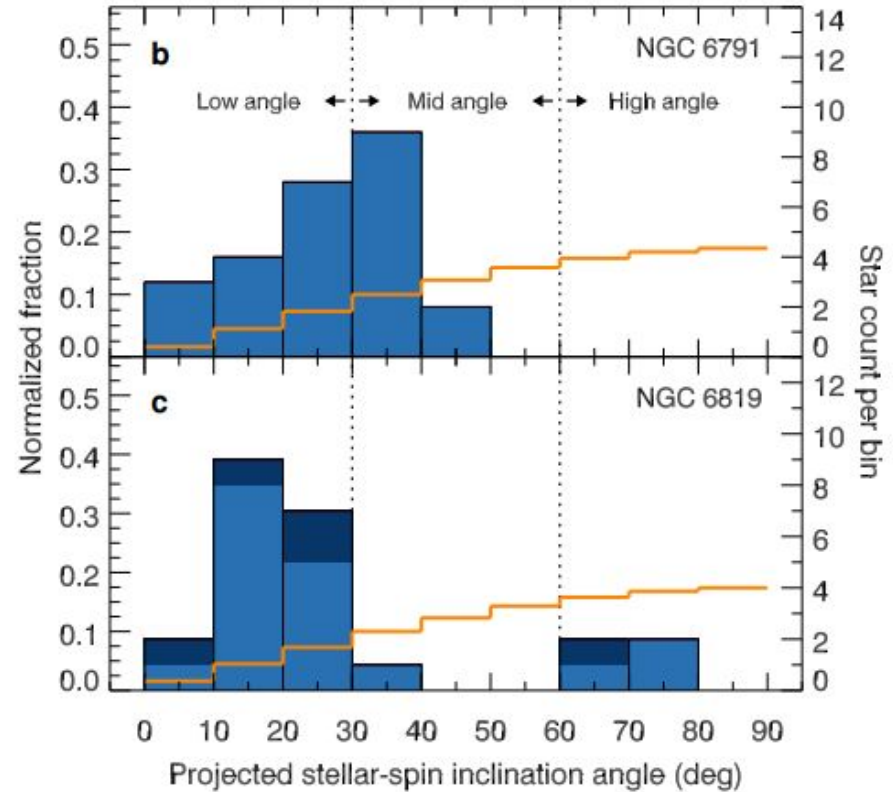
$$\underbrace{f_Y(y)}_{\mathbf{Y}} = \underbrace{\int_y^\infty dx \frac{1}{x} \frac{1}{\sqrt{x^2 - y^2}}}_{\mathbf{A}} \underbrace{f_X(x)}_{\mathbf{X}}$$

$$\mathbf{Y} = \mathbf{A}\mathbf{X}$$

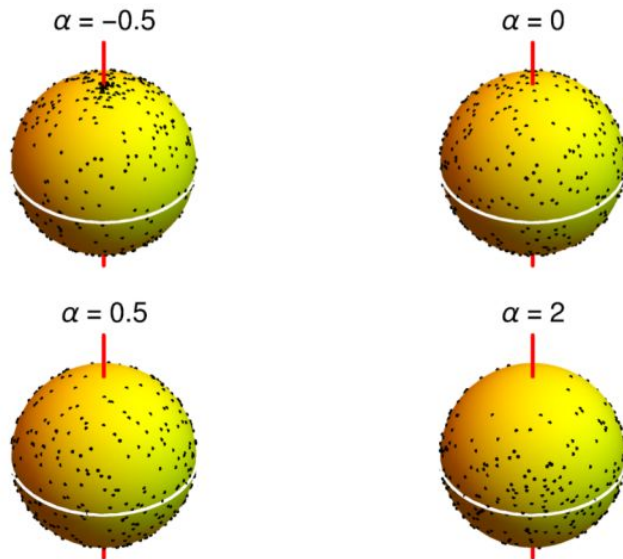
Introduction

The case of 48 red giants of NGC 6791
and NGC 6819

The stellar spin axes are *not* randomly oriented



Introduction



Non-Isotropic Velocities

$$f_Y(y; \alpha) = c_\alpha \int_y^\infty \frac{y^{2\alpha+1}}{x^{2\alpha+1}} \frac{1}{\sqrt{x^2 - y^2}} f_X(x; \alpha) dx$$

Factor of Normalization $c_\alpha = \frac{2\Gamma(\alpha + 3/2)}{\Gamma(1/2)\Gamma(\alpha + 1)}$

[Solar M. 2021](#)

α describes how the stellar spin axes are oriented in space:

- $\alpha < 0$: Polar alignment
- $\alpha > 0$: Equatorial concentration

$$\underbrace{f_Y(y; \alpha)}_Y = \underbrace{c_\alpha \int_y^\infty dx \frac{y^{2\alpha+1}}{x^{2\alpha+1}} \frac{1}{\sqrt{x^2 - y^2}}}_A \underbrace{f_X(x; \alpha)}_X$$

Introduction

Recover the true distribution of rotational velocities ($f_x(x)$) from the observed data ($f_y(y)$) by solving an integral equation with an α -dependent kernel, using Tikhonov regularization to obtain stable solutions.

The background is a high-resolution astronomical image of a nebula, likely the Carina Nebula. It features intricate, swirling patterns of gas and dust in shades of blue, purple, and brown, set against a dark cosmic background filled with numerous stars of varying brightness. The word "Methodology" is centered in a clean, white, sans-serif font.

Methodology

Methodology

Kernel

Gaussian Kernel (Symmetric Kernel)

$$\hat{f}_h(x) = \frac{1}{nh} \sum_{i=1}^n \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x - x_i)^2}{2h^2}\right)$$

Regularization

Tikhonov's Regularization

$$\min_{\mathbf{X}} (||\mathbf{Y} - \mathbf{A}\mathbf{X}||^2 + \lambda ||\mathbf{X}||^2)$$

We assumed in all cases $\lambda = 0.1$

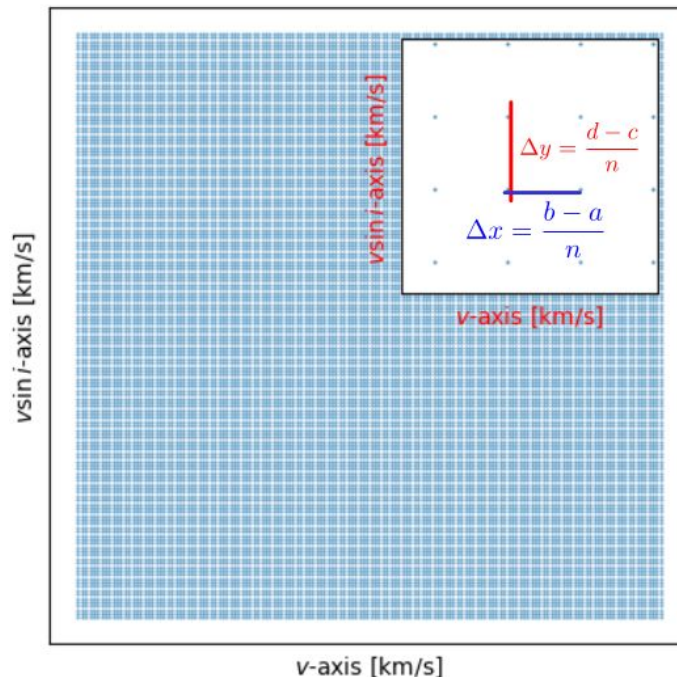
Metric

Root Mean Square Error

$$\text{RMSE}(\alpha) = \sqrt{\sum_{i=1}^n \frac{\left(f_Y(y_i; \alpha) - \hat{f}_Y(y_i; \alpha)\right)^2}{n}}$$

Distribution

Point Estimate

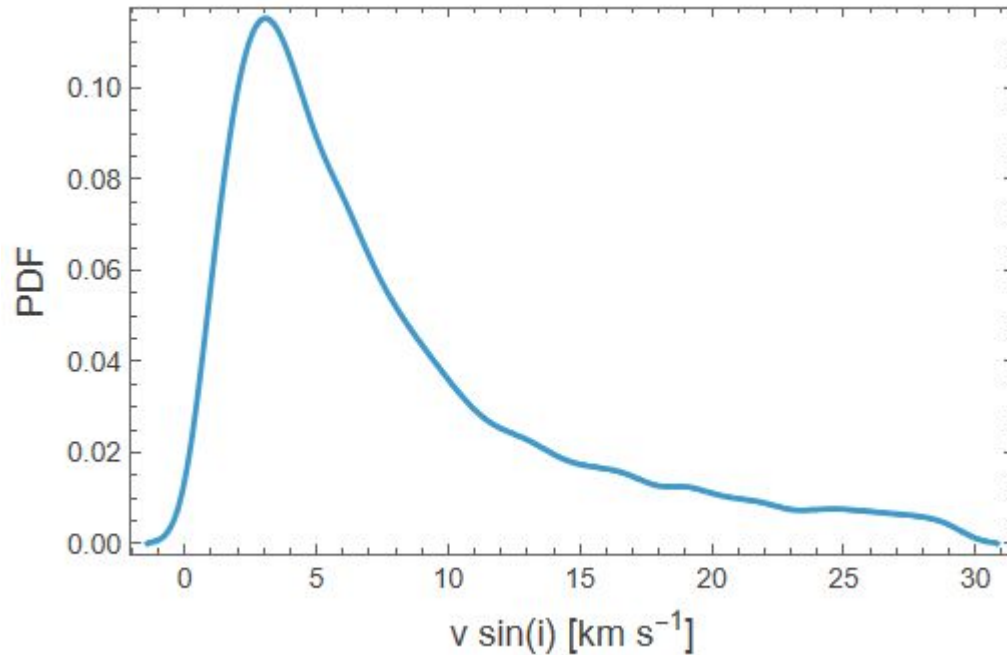


Methodology - The Geneva Data

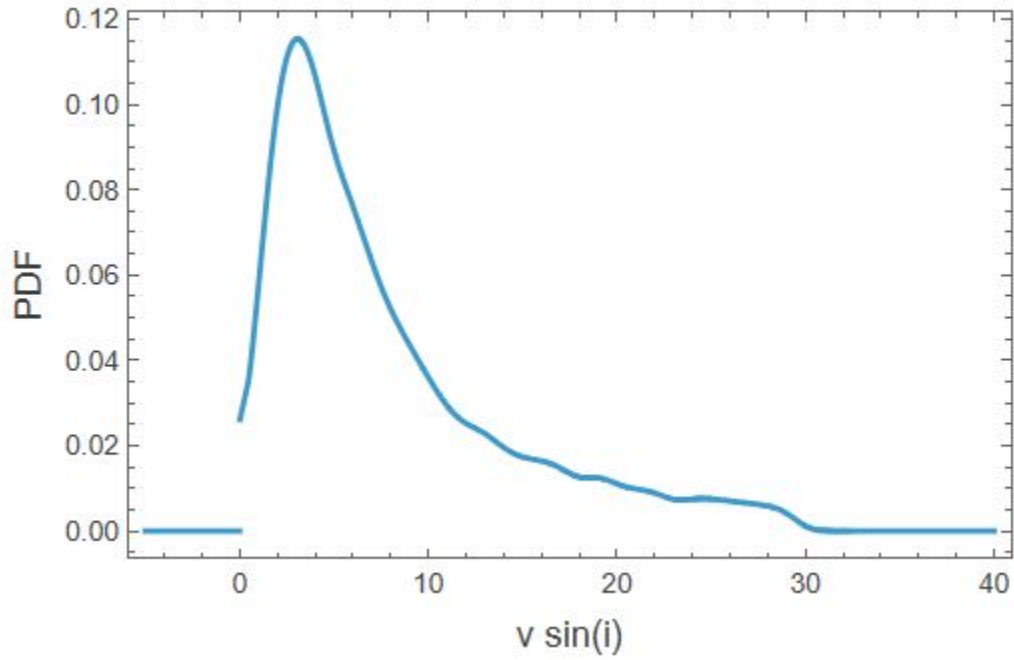
Dataset

- Geneva - Copenhagen Survey
- 11818 data of $v \sin i$.
- Field stars.
- F and G dwarfs.

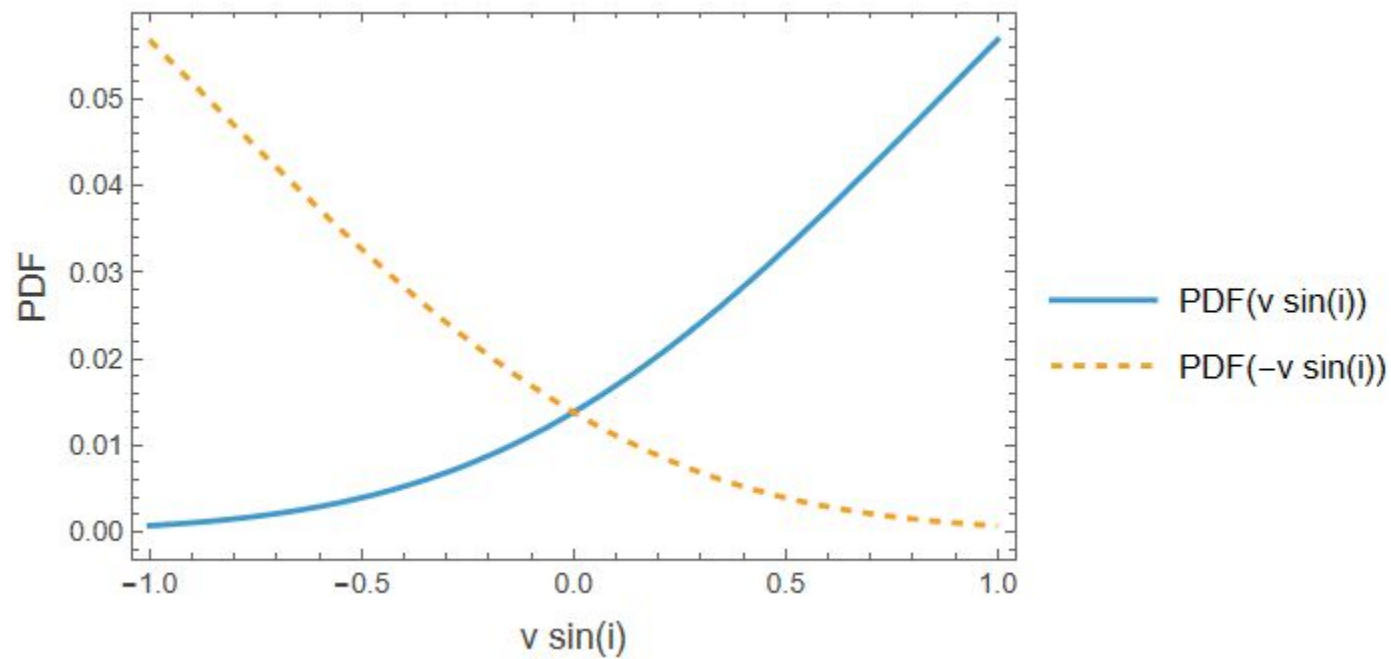
[J. Holmberg et al. 2009](#)



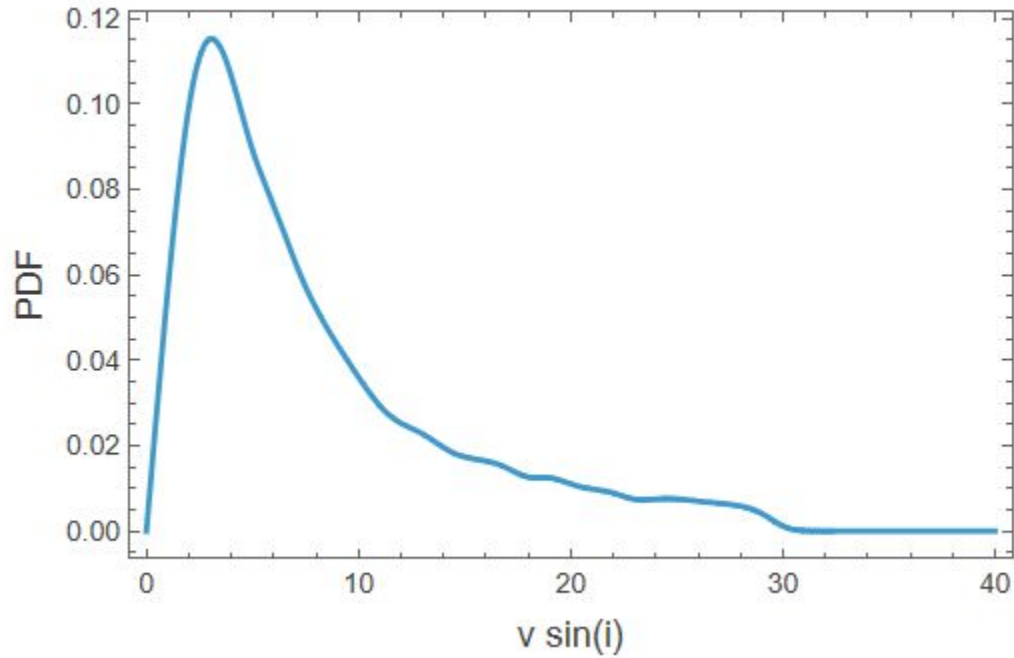
Methodology - The bounding of the solution



Methodology - A trick



Methodology - The distribution of projected velocities



Methodology - Solving for \mathbf{X}

Matricial relationship

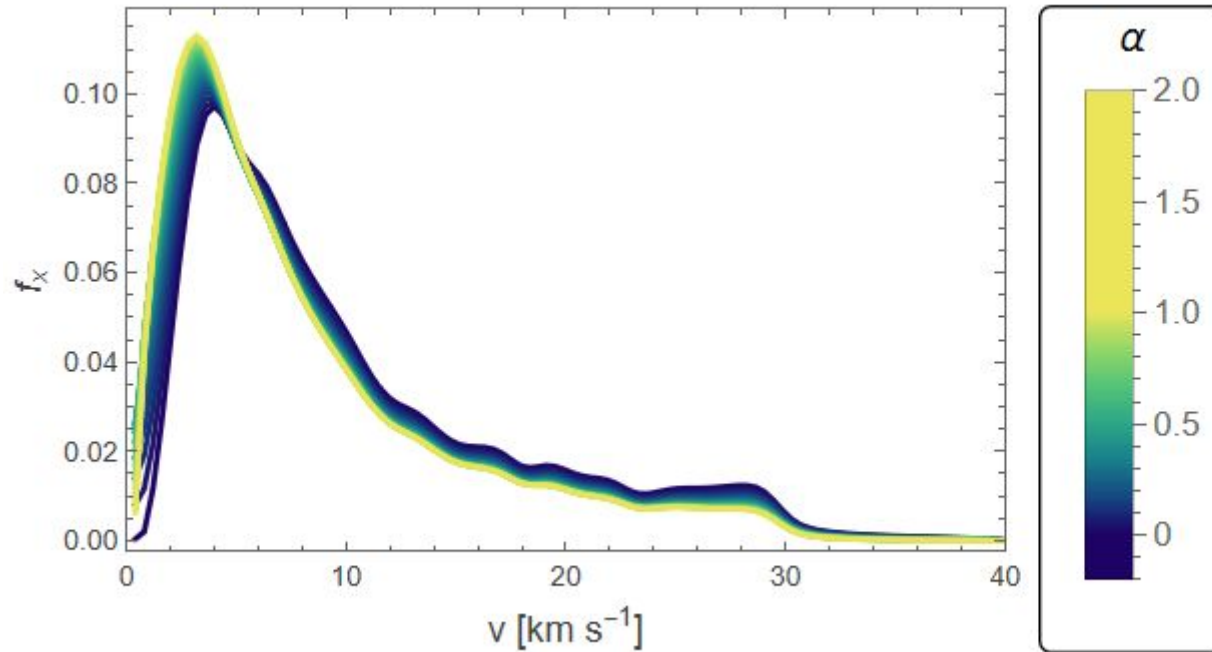
$$\underbrace{f_Y(y; \alpha)}_{\mathbf{Y}} = c_\alpha \underbrace{\int_y^\infty dx \frac{y^{2\alpha+1}}{x^{2\alpha+1}} \frac{1}{\sqrt{x^2 - y^2}}}_{\mathbf{A}} \underbrace{f_X(x; \alpha)}_{\mathbf{X}}$$

Tikhonov's Regularization

$$\min_{\mathbf{X}} (||\mathbf{Y} - \mathbf{A}\mathbf{X}||^2 + \lambda ||\mathbf{X}||^2)$$

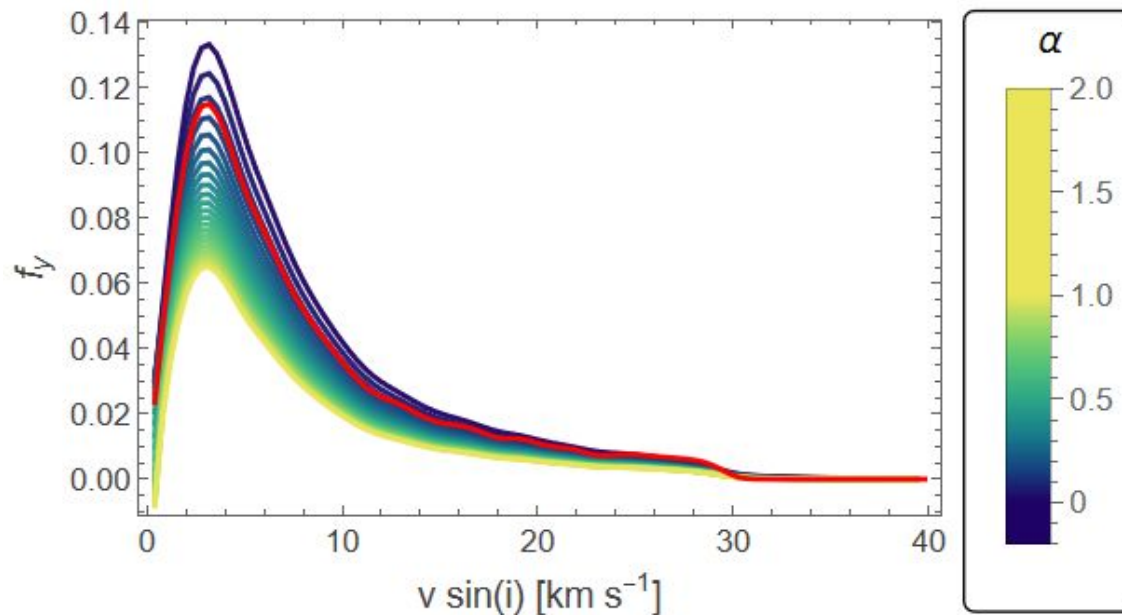
We assumed in all cases $\lambda = 0.1$

Methodology - The distribution of true velocities



Methodology - Reconstruction of f_Y

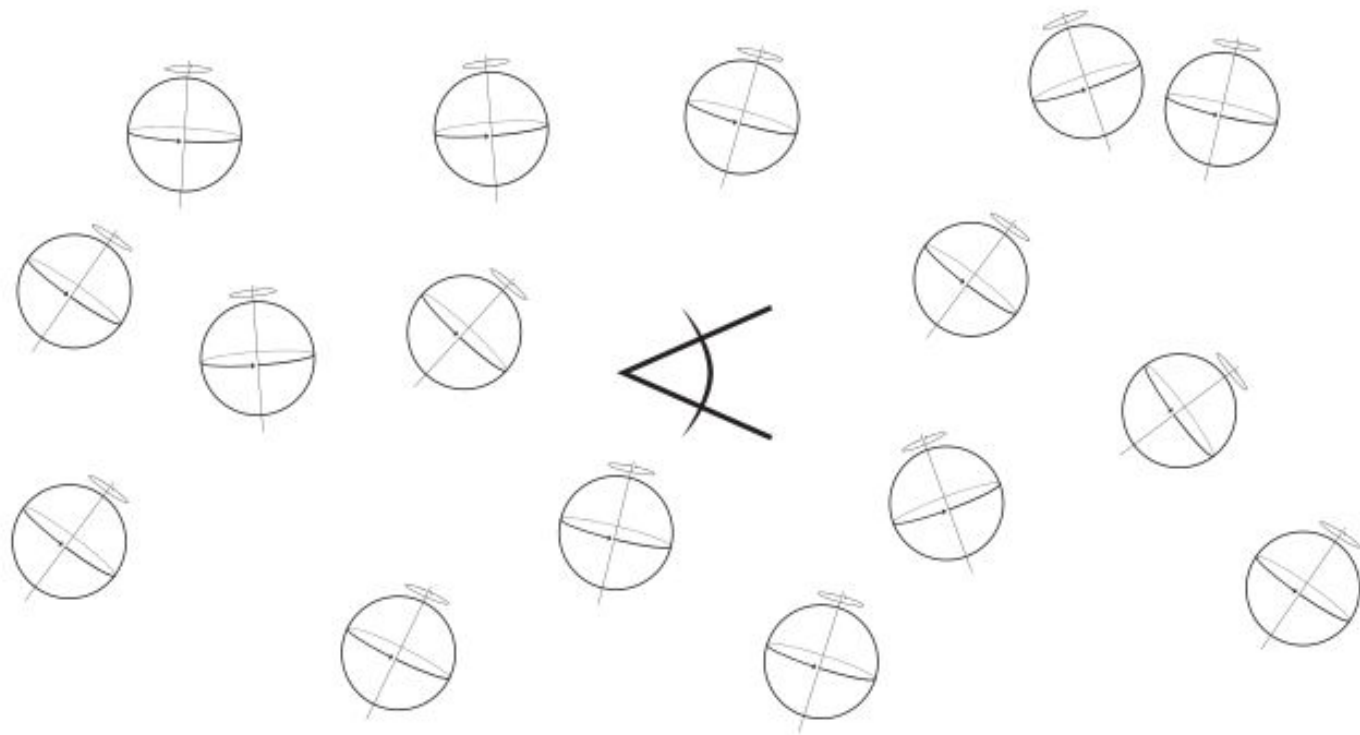
$$f_Y(y; \alpha) = \int_y^\infty \frac{y^{2\alpha+1}}{x^{2\alpha+1}} \frac{1}{\sqrt{x^2 - y^2}} f_X(x; \alpha) dx$$



The background of the slide is a high-resolution astronomical image. It shows a vast field of stars, many of which are bright and have visible diffraction spikes. A large, complex nebula is visible, characterized by swirling patterns of blue and white gas, with darker, dustier regions interspersed. The overall color palette is dominated by deep blues, blacks, and the warm whites and yellows of the stars.

Results

Isotropic

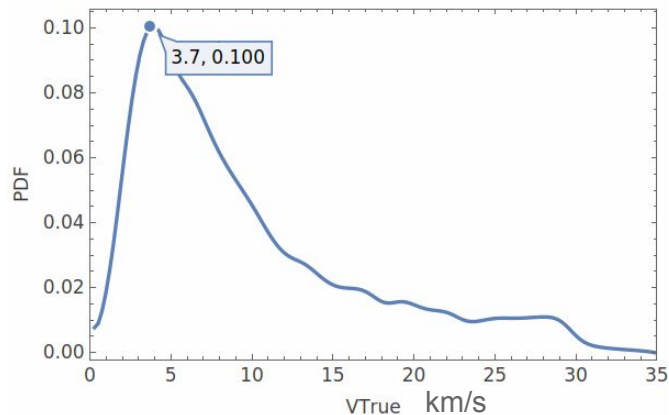
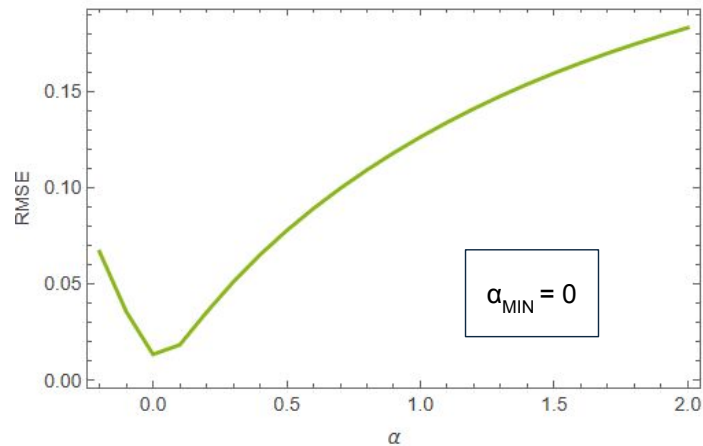


Geneva Database

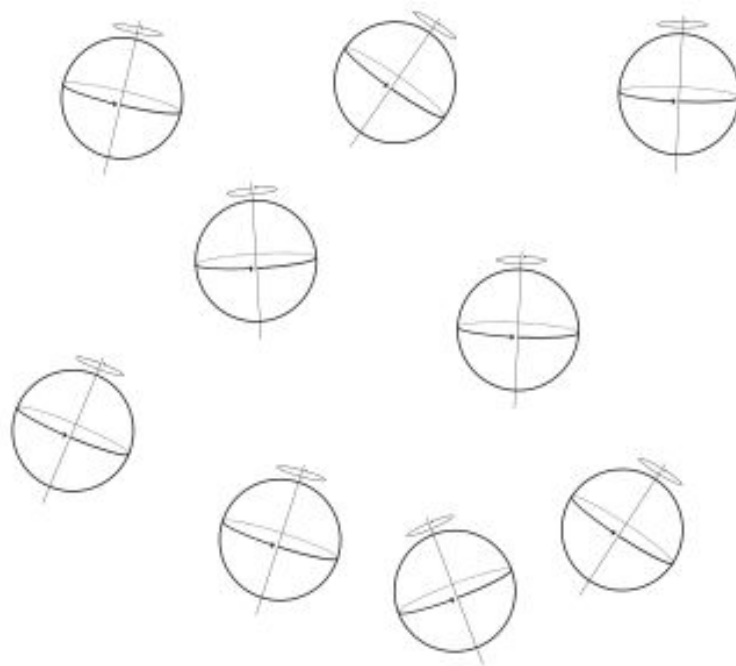
$$f_Y(y; \alpha) = c_\alpha \int_y^\infty \frac{y^{2\alpha+1}}{x^{2\alpha+1}} \frac{1}{\sqrt{x^2 - y^2}} f_X(x; \alpha) dx$$

$$\text{RMSE}(\alpha) = \sqrt{\sum_{i=1}^n \frac{\left(f_Y(y_i; \alpha) - \hat{f}_Y(y_i; \alpha)\right)^2}{n}}$$

11818 data points



Non-Isotropic

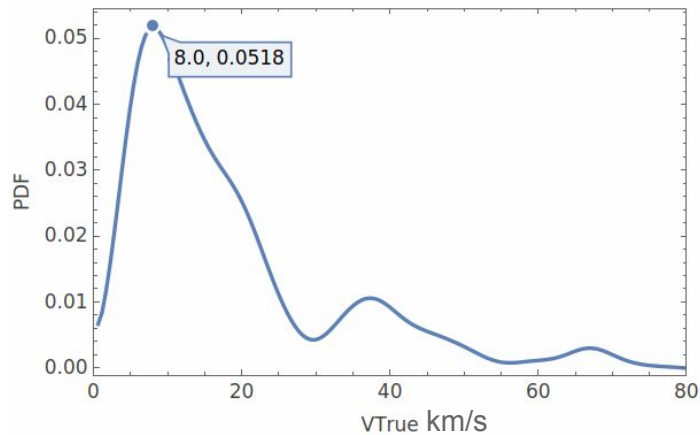
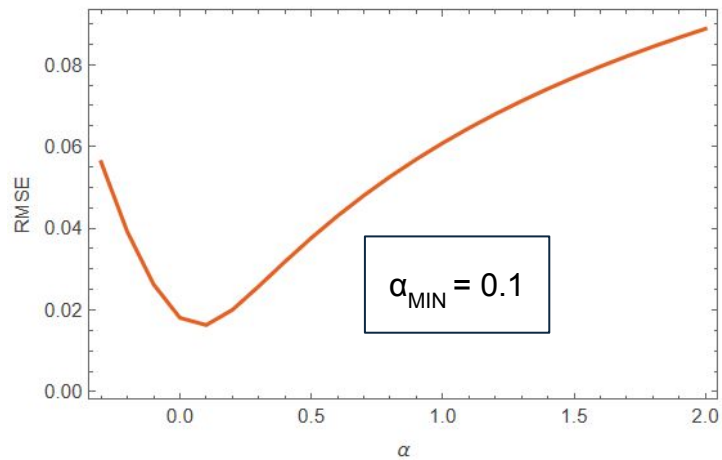


Pleiades



Open Star Cluster
~440 light-years from Earth
Constellation Taurus

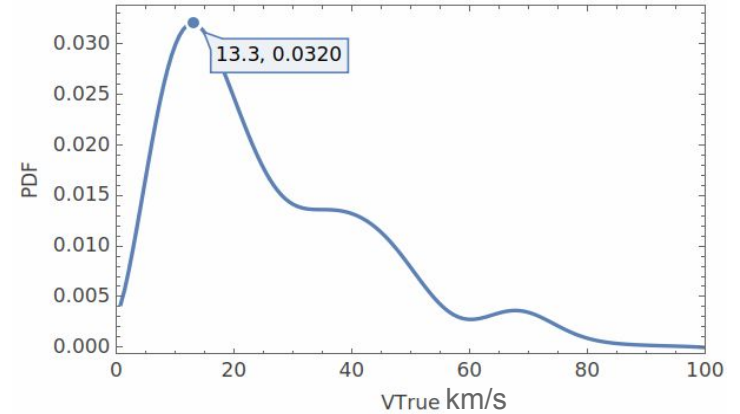
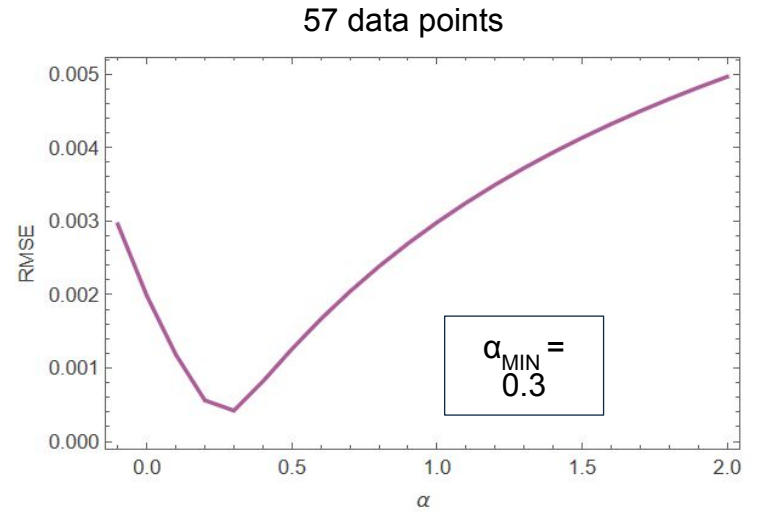
145 data points



Alpha Persei



Open Star Cluster
~570 light-years from Earth
Constellation Perseus



The background is a high-resolution astronomical image of a nebula, likely the Carina Nebula, showing intricate patterns of gas and dust in shades of blue, purple, and brown, with numerous bright stars scattered throughout. The text 'Conclusions and future work' is centered in a bold, white, sans-serif font.

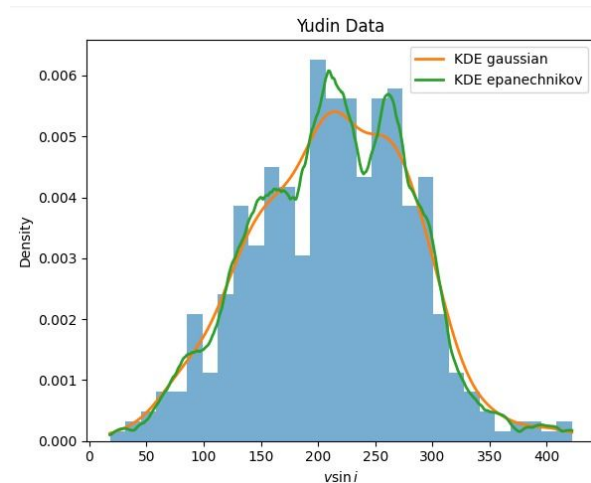
Conclusions and future work

Conclusions

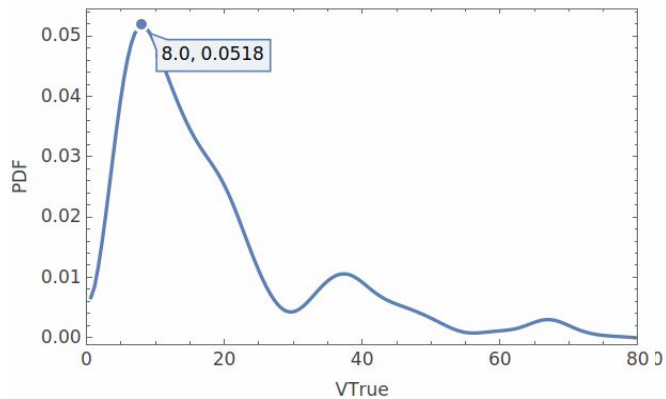
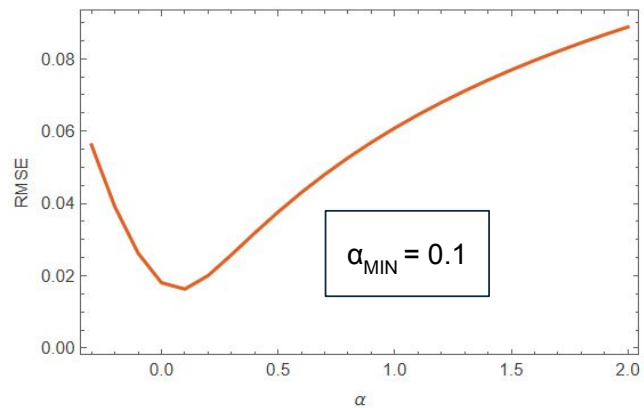
- As expected we found that for field stars $\alpha \approx 0$ as there should be a uniform distribution of axis angles.
- In the cases of Pleiades and Alpha Per which are clusters we found that $\alpha \neq 0$ because there is a privileged direction.

Future work

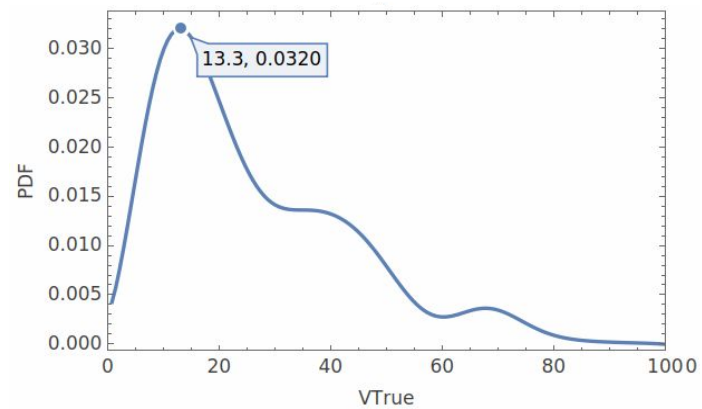
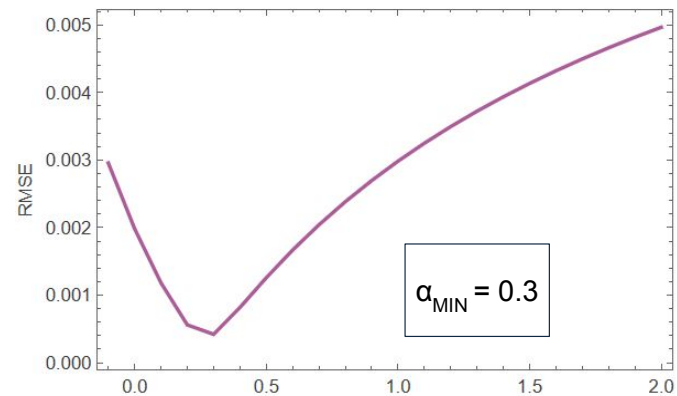
- To explore the space of lambda parameters, in this work we used a fixed value.
- To use more appropriate kernels, like for example asymmetric ones to avoid having to fix by hand the PDF.
- Work with more intelligent binning to find better solutions.



Pleiades



Alpha Persei



References

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