LMECA2550 Aircraft Propulsion Systems

Homework 1: Blade Element Momentum Theory

Hand-out: Oct 20, 2025
Hand-in: Nov 17, 2025 at 23:59

Guidelines

Evaluation: The evaluation in a 10 minutes presentation in front of the course teaching assistant, followed by a discussion and some questions. You are asked to present your results (figures and/or tables) along with their analysis (no need to provide a recap of the course). Given the oral nature of the evaluation, emphasis will be placed on the clarity and the readability of your results, particularly the quality of the plots. Ensure that your plots are vectorized, feature a grid, proper labels and units, and relevant x and y axis limits.

Individual work: The project is to be carried out alone.

Submission: Submit your **python code** and the **presentation** file (PDF) on the Moodle website. Name your code using the nomenclature : *"YOURNAME_CODENAME.py"*. If you submit multiple codes, compress them in a single `.zip` archive.

Questions: Questions can be asked via the dedicated forum on Moodle. If needed, a Q&A session could be organized, as well.

---

1 Blade Element Momentum theory

Implement your own BEM code in Python. Use the algorithm presented in class and the following ingredients.

- Write your own non-linear solver for a and a'. The equations for these induction factors can be easily re-written into an iterative procedure.

$$a_{new} = \frac{\sigma c_N (1+a)}{2(1-\cos 2\phi)} \tag{1}$$

$$a'_{new} = \frac{\sigma c_T (1-a')}{2 \sin 2\phi} \tag{2}$$

- Make this non-linear solver **robust** by under-relaxing every step:

$$a^k = (1-\omega)a^{k-1} + \omega a_{new}$$

  where ω is the relaxation factor, chosen here < 1, e.g. 0.3.

- Be efficient. The results ($a(r_i)$, $a'(r_i)$,...) for a given r_i and a given advance ratio J will be good guesses for the next point in your curves.

2 Verification

Verify your code on the simplified propeller configuration provided in Table 1. Plot the characteristic curves k_T,k_P, k_Q, and η_P vs J, over the a valid range of advance ratios *i.e.* for which $k_T > 0$. You can check your results against reference data provided on Moodle.

Parameter	**Value**	**Remarks**
Diameter	1 m	
Hub diameter	0.25 m	
Number of blades	2	
Chord	0.15 m	Constant over r
Constant pitch angle along the radius, β	25°	
Airfoil	$C_L = 2\pi\alpha$, $C_D = 0$	Ideal polar for small α

Table 1: Data for the simplified propeller

3 Application: P-51D Mustang

We consider the North-American P-51D Mustang WWII aircraft (P51D) that was studied during the class.

Figure 1: P-51D Mustang aircraft (www.wikipedia.org).

The following characteristics are provided below :

- Propeller
- Internal combustion engine
- Aircraft aerodynamics

3.1 Hamilton-Standard propeller

The data of the propeller is summarized in Table 2. This propeller has a constant geometric pitch $p = p_{\mathrm{ref},0}$, with the reference airfoil angle $\beta_{\mathrm{ref},0} = 15°$ at a radius $r = 0.75R$. The geometric pitch is then given by

$$p_{\mathrm{ref},0} = 2\,\pi\,0.75\,R\ \tan(\beta_{\mathrm{ref},0})$$

and we can write for the rest of the blade

$$\beta(r) = \arctan\left(\frac{p_{\mathrm{ref},0}}{2\,\pi\,r}\right) + \Delta\beta,$$

with $\Delta\beta = \beta_{\mathrm{pitch}} - \beta_{\mathrm{ref},0}$. Indeed, the propeller is a variable-pitch propeller which means that the blade pitch can be changed. Setting the blade pitch to a value β_{pitch} consists in rotating each blade around its own axis so that the resulting blade angle at $r = 0.75R$ is equal to β_{pitch}. The propeller also features a Constant Speed Unit (CSU) that automatically controls the blade pitch so as to maintain the rotation speed to the value set by the pilot.

Parameter	Value	Remarks
Diameter	3.4 m	
Hub diameter	0.45 m	
Number of blades	4	
Chord	0.25 m	Constant over r
Pitch angel at $r = 0.75R$, $\beta_{\text{ref},0}$	15°	Reference pitch angle
Airfoil	NACA 16-509	Python routine on Moodle
Propeller speed reduction	0.477	The propeller rotation speed is reduced to 0.477 that of the engine through a gearbox.

Table 2: Data for the Hamilton-Standard 24D50 - 6813

For a given blade geometry, the BEM algorithm is an aerodynamic model that provides an estimation of the propeller power and thrust as a function of flight speed u_0, rotation speed Ω, blade pitch β_{pitch} and altitude z (assuming standard atmosphere conditions):

$$\begin{aligned} T_{\text{propeller}} &= T_{\text{propeller}}(u_0, \Omega, \beta_{\text{pitch}}, z) \\ P_{\text{propeller}} &= P_{\text{propeller}}(u_0, \Omega, \beta_{\text{pitch}}, z) \ . \end{aligned}$$

3.2 Packard Merlin V-1650 engine

The V-1650 is a version of the Rolls-Royce Merlin V-12 aircraft engine (see 2) featuring a two-speed two-stage gear-driven supercharger with automatic boost control. The *Automatic Boost Control* device regulates the intake gas throttle valve opening so as to maintain the manifold pressure to the value set by the pilot.

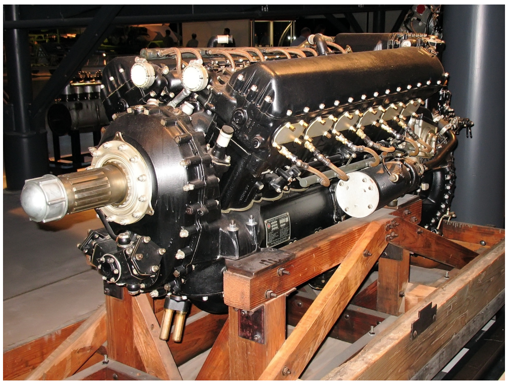

Figure 2: Packard Merlin V-1650-7 engine (www.wikipedia.org).

The V-1650-3 engine performance charts are shown in 3 for *low blower* operation (medium supercharger gear setting for lower altitudes) and 4 for *high blower* operation (full supercharger gear setting for higher altitudes). For the sake of illustration in this homework, we will use these charts although they correspond to the P-51B aircraft (P-51D was actually powered by the V-1650-7 version of the engine).

Considering a fixed supercharger gear setting (*low blower* mode or *high blower* mode) and a given combination of rotation speed and manifold pressure setting[1], each of these charts provide the maximum altitude[2] and the resulting shaft power that the internal combustion engine can operate at. In other words, the manifold pressure cannot be kept maintained above a certain altitude, even when fully opening the throttle valve. Further increasing the altitude while keeping the same engine settings results in a drop in manifold pressure to drop anyway (this limit is represented in the charts by the full throttle 3000 RPM line).

[1]Expressed here in *inHg*, i.e. inch of mercury.
[2]Assuming again standard atmosphere conditions.

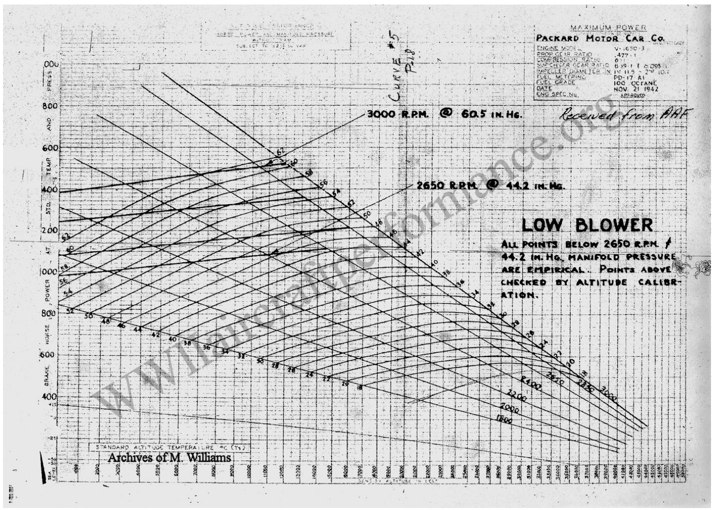

Figure 3: Low blower engine performance charts along with measurement data from a high speed flight test on Nov. 21, 1942 (refer to the following website for the high resolution figure : www.wwiiaircraftperformance.org/mustang/V-1650-3-low-blower-curve.jpg).

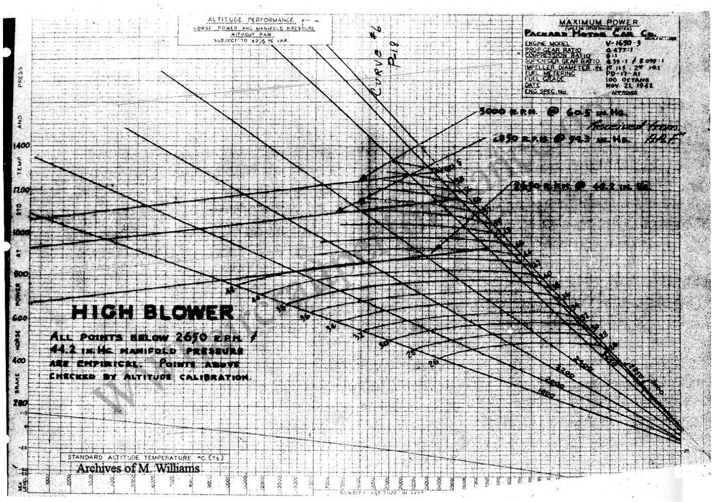

Figure 4: High blower engine performance charts along with measurement data from a high speed flight test on Nov. 21, 1942 (refer to the following website for the high resolution figure : www.wwiiaircraftperformance.org/mustang/V-1650-3-high-blower-curve.jpg).

The fuel consumption $\dot{m}_f$ of the internal combustion engine is generally a function of the rotation speed, manifold pressure and air/fuel ratio. For the present case, one can show that within a limited range of rotation speed and manifold pressure MP, the following linear regression provides a sufficiently accurate estimation :

$$\dot{m}_f = C_1 + C_2 \, P_{\text{engine}}$$

with the regression coefficient values provided in Table 3.

Supercharger mode	C_1 [gal/h]	C_1 [gal/(W h)]
Low Blower	-36.12	$1.785 \cdot 10^{-4}$
High Blower	-20.13	$1.849 \cdot 10^{-4}$

Table 3: Linear regression coefficients for the engine fuel consumption, estimation valid for 2400 RPM $\leq \Omega \leq$ 3000 RPM and 30 inHg $\leq$ MP $\leq$ 61 inHg.

3.3 P-51D aerodynamics

We assume here that the aircraft mass remains constant for all considered flight conditions. Considering a climb angle θ and further assuming that the aircraft/propeller axis is aligned with the flight speed vector $\mathbf{u}_0$, the lift L is equal to $Mg\cos\theta$. Using the definition of the lift coefficient, we can write :

$$C_L = \frac{Mg\cos\theta}{\frac{1}{2}\rho u_0^2\, A_{\mathrm{wing}}} \ ,$$

with A_{wing} the wing area. One can observe that the aircraft lift coefficient C_L must change with the flight speed u_0 and the altitude z in order to maintain the adequate lift. In practice, this operation is performed by the pilot by properly trimming the aircraft (elevator trim). Increasing the lift coefficient to compensate for a drop of dynamic pressure $\frac{1}{2}\rho u_0^2$ (either coming from a reduced flight speed or from an increased altitude) is typically done by increasing the angle of attack of the aircraft (pitching up) and, at low speed, by extending the flaps, which increases the effective camber of the wing. The lift coefficient is hence an adjustment variable and we will assume that it adapts to the flight conditions. Yet, there is a stall limit beyond which the wing is not able to provide additional lift, typically $C_{L,\mathrm{max}} = 2.0$. The stall speed limit $u_{0,\mathrm{min}}$ is a function of the altitude resulting from the upper bound constraint $C_L \leq C_{L,\mathrm{max}}$.

For a flight Mach number M_0 that is not too high, the drag coefficient C_D can be modelled as follows :

$$C_D = C_{D0} + KC_L^2 \ ,$$

with C_{D0} the zero-lift drag coefficient (friction and pressure drag) and KC_L^2 the lift-induced drag. K is a constant given by :

$$K = \frac{1}{\pi\,\mathrm{AR}\,e} \ ,$$

where e is the Oswald efficiency ($e = 1$ for an elliptical wing) and $\mathrm{AR} = b^2/A_{\mathrm{wing}}$ is the wing aspect ratio, b being the wing span. Using the definition of the drag coefficient, we can write:

$$D = \frac{1}{2}\rho u_0^2\, A_{\mathrm{wing}}\, C_D = \frac{1}{2}\rho u_0^2\, A_{\mathrm{wing}} \left[C_{D0} + K \left(\frac{Mg\cos\theta}{\frac{1}{2}\rho u_0^2\, A_{\mathrm{wing}}} \right)^2 \right] \ .$$

The aircraft aerodynamics data for the P-51D Mustang is provided in Table 4.

Parameter	**Value**
M	8430 [lb]
A_{wing}	21.83 [m^2]
b	11.28 [m]
$C_{D,0}$	0.0163
e	0.8

Table 4: Aircraft aerodynamics data

3.4 P-51D Mustang aircraft flight model

Considering steady flight, we must have a force equilibrium between propeller thrust, aircraft drag and aircraft weight, as well as a power balance between propeller and engine shaft power :

$$\begin{aligned} T_{\mathrm{propeller}} &= D_{\mathrm{aircraft}} + Mg\sin\theta \\ P_{\mathrm{propeller}} &= P_{\mathrm{engine}} \ . \end{aligned}$$

For a given engine rotation speed Ω, engine shaft power P_{engine} and altitude z, this is a non-linear system of 2 equations for the 2 unknowns u_0 and β_{pitch}. In order to solve this system numerically, you can use several existing solvers such as `scipy.fsolve`. To assure convergence, make sure to choose initial guesses that are not too far from the solution and check the convergence. Depending on the cases, some trial and error may be required.

4 Hamilton-Standard propeller analysis

- Apply your code to Hamilton-standard propeller for a series of pitch settings : $\beta_{\text{pitch}} = [10° \ 20° \ 30° \ 40° \ 50° \ 60°]$ and produce the characteristic curves k_T,k_P, k_Q, and η_P vs J, over the range $[0, 5]$.
- From these results, propose a pitch setting value to fly at maximum propulsive efficiency under the following conditions :

Engine shaft speed:	$rpm_{\text{engine}} = 3000$ [rpm]
Flight Mach number:	$M = 0.5$
Altitude:	$z = 20000$ [ft]

  Compute then the thrust and power of the propeller for this setting.

The standard atmosphere model is provided as a Python routine on the Moodle website.
Remember, we take the blade defined earlier and **rotate** it about the blade axis (we are not **deforming** it!).

5 P-51D flight analysis

A flight test campaign took place on November 21, 1942. The aim of this test campaign was to measure the aircraft performance for different flight conditions[3].

5.1 High speed performance tests

The engine settings, testing parameters and true airspeed measurements (flight speed) are provided in Table 5. These conditions correspond to steady level flight and thus $\theta = 0°$.

Supercharger mode	**RPM**	**MP [inHg]**	P_{engine} **[bhp]**	**Altitude [ft]**	**True airspeed [mph] (measured)**
Low blower	3000	60.5	1450	5000	363
Low blower	3000	60.5	1485	10000	394
Low blower	3000	60.5	1530	16800	425
High blower	3000	60.5	1270	23200	422
High blower	3000	60.5	1275	29800	441
High blower	3000	48.0	985	35000	421
High blower	3000	40.7	815	38000	403

Table 5: High speed performance tests

- Based on the parameters provided in this table, use your model to compute the flight speed and blade pitch for each operating point, i.e. for each row of the table.
- Compare your results with the actual flight speed measurements and explain the differences (if any).
- Discuss the results by analyzing, for example, the pitch setting evolution, advance ratio, flight Mach number, blade tip Mach number, propulsive efficiency, etc.
- Is there a difference in the distribution of angle of attack over the blade when increasing the altitude ?

[3] The detailed report of this test campaign can be found at *https://www.wwiiaircraftperformance.org/mustang/p51b-12093.html*

5.2 Climb performance tests

The engine settings, testing parameters and rate of climb measurements (vertical component of flight speed $u_{0,z}$) are provided in Table 6. The climb angle θ is here a function of the flight speed : $\theta = \arcsin(u_{0,z}/u_0)$.

Supercharger mode	**RPM**	**MP [inHg]**	$P_{\mathbf{engine}}$ **[bhp]**	**Altitude [ft]**	**Rate of climb [ft/min] (measured)**
Low blower	3000	60.5	1500	0 (sea level)	3600
Low blower	3000	60.5	1510	5000	3570
Low blower	3000	60.5	1525	10000	3540
Low blower	3000	60.5	1510	13000	3520
High blower	3000	60.5	1320	17400	2965
High blower	3000	60.5	1310	20000	2915
High blower	3000	60.5	1260	26000	2780
High blower	3000	51.6	1075	30000	2125
High blower	3000	41.8	850	35000	1280
High blower	3000	32.8	630	40000	450

Table 6: Climb performance tests

- Based on the parameters and on the measured climb rate provided in this table, compute the flight speed, climb angle and blade pitch for each operating point, i.e. for each row of the table.
- Considering a piece-wise constant evolution of the flight speed and engine power from one operating point to another, compute the time and total fuel mass required by the aircraft to reach the different altitudes up to the maximum altitude of 40000ft.
- Discuss and analyze your results.

5.3 Take-off

For take-off at sea level, the engine rotation speed is set to 3000 rpm and the manifold pressure to MP = 61 inHg which leads the engine to develop a shaft power of 1400 bhp. The take-off speed of the aircraft is 150 mph, i.e. the speed at which the aircraft can safely become airborne. Which is the typical value for an heavily loaded aircraft.

- Compute the associated lift coefficient and compare it to the stall limit.
- Considering $\theta = 0°$ at the moment when the aircraft becomes airborne, what is the thrust and the blade pitch (asuming the BEM model remains valid despite the flow being unsteady)?
- What is the acceleration of the aircraft compared to g, the gravitational acceleration?