APMA 3100 – EXAM 3 KEY November 20/21, 2014

STUDENT NAME: KEY

SECTION: (circle one as appropriate) 10:00 am 11:00 am

Answer each question in the space provided. Please write clearly and legibly. If the work is deemed illegible by the grader then it will be marked wrong. Show all work in order to receive full credit and <u>clearly identify your final answer</u>. You may use an approved scientific calculator (no graphing calculator allowed). The Gaussian (0,1) (as seen in your text page 123) and the appropriate material from Appendix A of the text are provided and may be used as needed. No other sources may be used.

Please sign the pledge:

On my honor as a student, I have neither given nor received aid on this exam.

Signed: ______________________

1. Let Y be a Poisson random variable with E[Y] = 10.
   a) [5 points] Use the Markov inequality to find $P[Y \geq 18]$.

$$P[Y \geq 18] \leq \frac{E[Y]}{18} = \frac{10}{18} = .\bar{5}$$

   b) [5 points] Use the Chebyshev inequality to find $P[Y \geq 18]$.

$$P[|Y-\mu_Y| \geq 18-10] \leq \frac{Var[Y]}{8^2}$$

Poisson
$Var[Y] = \alpha = 10$
from part (a).

$$= \frac{10}{64} = .15625$$

2. X is a Bernoulli random variable with probability of success p = 0.7.

from Appendix

a) [2 points] Find E[X] and Var[X].

$E[X] = p \qquad Var[X] = p(1-p) = (.7)(.3)$

$E[X] = .7 \qquad Var[X] = .21$

b) [4 points] Find P[$X_{25} \geq 0.75$]. $= 0.7$ since

$\geq .75 \Rightarrow x = 1$

$$P_X(x) = \begin{cases} .3 & x=0 \\ .7 & x=1 \\ 0 & \text{oth.} \end{cases}$$

c) [4 points] Find P[$M_{30}(X) \geq 0.75$]. $= 1 - P[M_{30}(x) < .75]$

$= 1 - P\left[Z_{30} < \frac{.75 - .7}{\sqrt{.21}}\right]$

$= 1 - P\left[Z_{30} < \frac{.05}{.458}\right]$

$= 1 - P[Z < .11]$

$= 1 - .5438 = .4562$

3. Random variable X and Y have joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 6e^{-(2x+3y)} & x \ge 0, y \ge 0 \\ 0 & otherwise \end{cases}$$

a) [2 points] Find $f_Y(y)$. $= \int_0^\infty 6e^{-2x-3y}\,dx$

$= \lim_{t\to\infty} -3e^{-2x-3y}\Big|_0^t = \lim_{t\to\infty} -3e^{-2t-3y} + 3e^{-2(0)-3y}$

$$f_Y(y) = \begin{cases} 3e^{-3y} & y \geq 0 \\ 0 & otherwise \end{cases}$$

b) [4 points] Find $f_{X|Y}(x|y)$.

$$\frac{6e^{-2x-3y}}{3e^{-3y}} = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & otherwise \end{cases}$$

c) [4 points] Are *X* and *Y* independent? Show why or why not.

yes

$f_X(x) = \int_0^\infty 6e^{-2x-3y}\,dy$

$= \lim_{t\to\infty} -2e^{-2x-3y}\Big|_0^t$

$= \lim_{t\to\infty} -2e^{-2x-3t} + 2e^{-2x-3(0)}$

$$f_X(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & otherwise \end{cases}$$

Since $f_X(x) \cdot f_Y(y)$

$= 2e^{-2x} \cdot 3e^{-3y} = 6e^{-2x-3y}$

$= f_{XY}(x,y)$

They are independent

4. Telephone calls can be classified as either voice (V) or data (D). The local telephone company has collected information to generate the following probability models: $P[V] = 0.7$ and $P[D] = 0.3$. Data calls and voice calls occur independently of one another, and the random variable R_n is the number of data calls collected in n phone calls.

a) [2 points] Find $E[R_{100}]$, the expected number of data calls in a set of 100 calls?

$$E[R_{100}] = 100(.3) = 30$$

Similar to QUIZ 10 problems

b) [2 points] Find $\sigma_{R_{100}}$, the standard deviation of data calls in a set of 100 calls?

$$\sigma = \sqrt{30(.7)} = \sqrt{21}$$

Went over in class!

c) [4 points] Use the central limit theorem to estimate $P[\,28 \le R_{100} \le 38]$.

$$P\left[\frac{28-30}{\sqrt{21}} \le Z \le \frac{38-30}{\sqrt{21}}\right]$$

$$= P[-.4367 < Z < 1.7467]$$

$$= .9599 - (1 - .67)$$

$$= .6299$$

d) [4 points] Use the DeMoivre-Laplace formula to estimate $P[\,28 \le R_{100} \le 38]$.

$$P[-.5418 \le Z \le 1.8558]$$

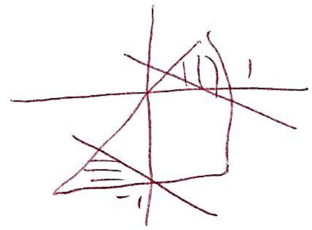
$$= .9686 - (1 - .7088)$$

$$= .6774$$

5. Random variables X and Y have joint PDF $f_{X,Y}(x,y) = 1/2$ for $-1 \le y \le x \le 1$, $f_{X,Y}(x,y) = 0$ otherwise. Let $W = X + Y$.

   a) [4 points] Find the sample space for W AND accurately draw the base region.

$-2 \le w \le 2$



Quiz 9 problem

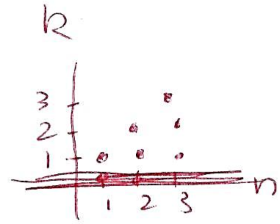
   b) [6 points] Set up ALL the appropriate integrals to find the CDF for the random variable W. **Do not solve the integrals.**

Went over in class!

$$F_W(w) = \begin{cases} 0 & w < -2 \\ \displaystyle\int_{-1}^{w/2} \int_{y}^{w-y} \tfrac{1}{2}\, dx\, dy & -2 \le w \le 0 \\ \displaystyle 1 - \int_{w/2}^{1} \int_{w-x}^{x} \tfrac{1}{2}\, dy\, dx & 0 \le w \le 2 \\ 1 & w > 2 \end{cases}$$

6. The joint PMF of random variables N and K is

$$P_{N,K}(n,k) = \begin{cases} (1-p)^{n-1}p/n & k = 1,2,\ldots,n \\ & n = 1,2\ldots \\ o & \text{otherwise} \end{cases}$$



a. [6 points] Find $P_N(n)$. Simplify your answer.

$$P_N(n) = \sum_{k=1}^{n} \frac{(1-p)^{n-1}p}{n} = n\left(\frac{(1-p)^{n-1}p}{n}\right)$$

$$P_N(n) = \begin{cases} (1-p)^{n-1}p & n=1,2,\ldots \\ 0 & \text{Otherwise} \end{cases}$$

b. [6 points] Find $P_K(k)$ by setting up a sum. You do not need to evaluate the sum.

$$P_K(k) = \begin{cases} \sum_{n=k}^{\infty} \frac{(1-p)^{n-1}p}{n} & k=1,2,\ldots \\ 0 & \text{otherwise} \end{cases}$$