VIS VAJE 10

Dodatek

$r(X,Y)$ - nam dobro meri za linearno odvisnost dveh spremenljivk
$[-1,1]$
negativna premica ← pozitivna linearna odvisnost (premica)

(1)

X \ Y	0	1	3	
0	0,05	0,1	0,2	0,35
1	0,15	0	0,15	0,3
4	0,2	0,1	0,05	0,35
	0,4	0,2	0,4	1

$Cov(X,Y) = E(XY) - E(X)E(Y) = 1,45 - 1,7 \cdot 1,4 = \underline{-0,93}$

če je $Cov = 0$ še ne pomeni da sta neodvisni

$E(X) = 0 \cdot 0,35 + 1 \cdot 0,3 + 4 \cdot 0,35 = 1,7$
$E(Y) = 0 \cdot 0,4 + 1 \cdot 0,2 + 3 \cdot 0,4 = 1,4$

$E(X \cdot Y) = 0 \cdot 0,5 + 0 \cdot 0,1 + 0 \cdot 0,2 + \cancel{1 \cdot 0,15} + \cancel{1 \cdot 0} + \cancel{1 \cdot 0,15} + \cancel{4 \cdot 0}$

$E(X \cdot Y) = 1 \cdot 3 \cdot 0,15 + 4 \cdot 1 \cdot 0,1 + 4 \cdot 3 \cdot 0,05 = 0,45 + 0,4 + 0,6 = 1,45$

$r(X,Y) = \frac{Cov(X,Y)}{\sigma(X) \cdot \sigma(Y)} = \frac{-0,93}{1,735 \cdot 1,356} = \underline{\underline{-0,4}}$

$\sigma(X) = \sqrt{D(X)}$

$D(X) = E(X^2) - E(X)^2$

$\cancel{= 1^2 \cdot 0,3 + 4^2 \cdot 0,35 = 5,9}$

$E(X^2) = 1^2 \cdot 0,3 + 4^2 \cdot 0,35 = 5,9$

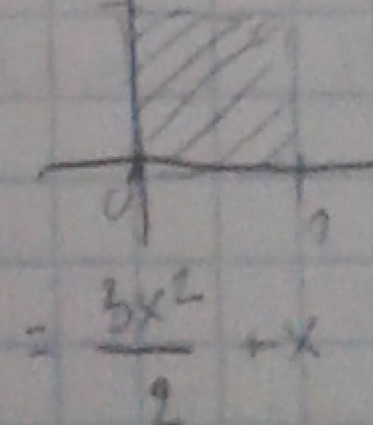
$D(X) = 5,9 - 1,7^2 = 3,01$

$\sigma(X) = \sqrt{3,01} = 1,735$

$E(Y^2) = 1^2 \cdot 0,2 + 3^2 \cdot 0,4 = 3,8$
$D(Y) = 3,8 - 1,4^2 = 1,84$

$\sigma(Y) = \sqrt{1,84} = 1,356$

(2) $p_{X,Y}(x,y) = \begin{cases} 3(x^2y + xy^2); & x,y \in [0,1] \\ 0 & ; \text{sicer} \end{cases}$



~~Cov(X,Y) =~~

- $p_X(x) = \int_0^1 3(x^2y + xy^2)\,dy = \frac{3x^2y^2}{2} + \frac{3xy^3}{3}\Big|_0^1 = \frac{3x^2}{2} + x$

- $p_Y(y) = \begin{cases} \frac{3y^2}{2} + y, & y \in [0,1] \\ 0, & \text{sicer} \end{cases}$ $\quad p_X(x) = \begin{cases} \frac{3x^2}{2} + x, & x \in [0,1] \\ 0, & \text{sicer} \end{cases}$

- $E(XY) = \int_0^1\int_0^1 x \cdot y \cdot p_{X,Y}(x,y)\,dx\,dy =$
$= \int_0^1\int_0^1 3xy(x^2y + xy^2)\,dx\,dy = \int_0^1\int_0^1 (3x^3y^2 + 3x^2y^3)\,dx\,dy =$
$= \int_0^1 \frac{3x^4y^2}{4} + x^3y^3 \Big|_0^1 dy = \int_0^1 \left(\frac{3}{4}y^2 + y^3\right) dy = \frac{3}{4}\frac{y^3}{3} + \frac{y^4}{4}\Big|_0^1 = \frac{1}{4} + \frac{1}{4} = \underline{\underline{\frac{1}{2}}}$

- $E(X) = \int_0^1 x \, p_X(x) = \int_0^1 x\left(\frac{3x^2}{2} + x\right)dx$
$= \int_0^1 \left(\frac{3x^3}{2} + x^2\right)dx = \frac{3x^4}{8} + \frac{x^3}{3} = \frac{17}{24}$

- $E(Y) = E(X)$ - ker imata enako gostoto

- $E(X^2) = E(Y^2) = \int_0^1 x^2 p_X(x)\,dx = \underline{\underline{\frac{11}{20}}}$

- $D(Y) = D(X) = E(X^2) - E(X)^2 = \frac{11}{20} - \left(\frac{17}{24}\right)^2 = 0,04826$