Matematika 1-B, 2014/15
rezultati računskega dela izpita
29. januar 2015

vpisna št.	točke	vpisna št.	točke	vpisna št.	točke
26110692	50	26111008	10	26203670	40
26110697	35	26203647	5	26300407	20
26110708	5	26203651	5	26300421	15
26110877	20	26203652	10	26300422	15
26110906	40	26203653	25	26300436	20
26110910	20	26203655	5	26300447	0
26110919	40	26203656	5	26300448	20
26110924	5	26203658	15	26300456	20
26110926	35	26203659	20	26300457	10
26110932	15	26203661	30	26300459	15
26110936	10	26203664	15	26300462	20
26110937	10	26203669	15		

7 LIMITE IN ZVEZNOST FUNKCIJ

1. Izračunajte limiti:

$$(a)\quad \lim_{x\to 0}\frac{\sin(2x)}{\sqrt{2x+9}-3};\qquad (b)\quad \lim_{x\to\infty}\left(\frac{x^3}{3x^2+2}-\frac{2x^2}{6x+1}\right)$$

Rešitev: (a) 6; (b) $\frac{1}{18}$

a) $$\lim_{x\to 0}\frac{\sin 2x}{\sqrt{2x+9}-3}=\lim\frac{\sin 2x\left(\sqrt{2x+9}+3\right)}{\left(\sqrt{2x+9}-3\right)\left(\sqrt{2x+9}+3\right)}=$$

$$=\lim_{x\to 0}\frac{\sin 2x\left(\sqrt{2x+9}+3\right)}{2x+9-9}=\lim_{x\to 0}\frac{\sin 2x\left(\sqrt{2x+9}+3\right)}{2x}=$$

$$=\sqrt{9}+3=6$$

b) $$\lim_{x\to\infty}\left(\frac{x^3}{3x^2+2}-\frac{2x^2}{6x+1}\right)=\lim_{x\to\infty}\left(\frac{x^3(6x+1)-2x^2(3x^2+2)}{(3x^2+2)(6x+1)}\right)$$

$$=\lim_{x\to\infty}\frac{6x^4+x^3-6x^4-4x^2}{18x^3+3x^2+12x+2}=\lim_{x\to\infty}\frac{x^3-4x^2}{18x^3+3x^2+12x+2}=$$

$$=\frac{1}{18}$$

2. Določite a tako, da bo funkcija

$$f(x)=\begin{cases}\frac{\sqrt{1+6x}-5}{\sqrt{x}-2} & x>4\\ a & x\leq 4\end{cases}$$

zvezna.

Rešitev:

Funkcija bo zvezna, če bo $\lim_{x\searrow 4}f(x)=\lim_{x\nearrow 4}f(x)=f(4)$. Z množenjem števca in imenovalca z izrazom $(\sqrt{1+6x}+5)(\sqrt{x}+2)$ ali z uporabo l'Hospitalovega pravila dobimo $\lim_{x\searrow 4}=\frac{12}{5}$. Ker je $\lim_{x\nearrow 4}f(x)=f(4)=a$, bo funkcija zvezna za $a=\frac{12}{5}$.

- $$\lim_{x\uparrow 4}f(x)=\lim_{x\downarrow 4}f(x)=f(4)$$

- $$\lim_{x\downarrow 4}\frac{\sqrt{1+6x}-5}{\sqrt{x}-2}=\lim_{x\downarrow 4}\frac{\left(\sqrt{1+6x}-5\right)\left(\sqrt{1+6x}+5\right)\left(\sqrt{x}+2\right)}{\left(\sqrt{x}-2\right)\left(\sqrt{1+6x}+5\right)\left(\sqrt{x}+2\right)}=$$

$$=\lim_{x\downarrow 4}\frac{(1+6x-25)(\sqrt{x}+2)}{(x-4)\left(\sqrt{1+6x}+5\right)}=\lim_{x\downarrow 4}\frac{(6x-24)(\sqrt{4}+2)}{(x-4)\left(\sqrt{1+6x}+5\right)}=$$

$$=\lim_{x\downarrow 4}\frac{6(x-4)(\sqrt{4}+2)}{(x-4)\left(\sqrt{1+6x}+5\right)}=\lim_{x\downarrow 4}\frac{6(\sqrt{4}+2)}{\left(\sqrt{1+6x}+5\right)}=$$

$$=\frac{6(\sqrt{4}+2)}{\left(\sqrt{1+6\cdot 4}+5\right)}=\frac{24}{5+5}=\frac{24}{10}=\frac{12}{5}$$

- $$\lim_{x\uparrow 4}f(x)=\lim_{x\uparrow 4}a=a=\lim_{x\downarrow 4}f(x)=f(4)=a=\frac{12}{5}$$

3. Določite parametra a in b tako, da bo funkcija

$$f(x) = \begin{cases} \frac{\sqrt{x+5}-3}{4-x} & x < 4 \\ ax+b & 4 \le x \le 6 \\ e^{\frac{1}{6-x}} & x > 6 \end{cases}$$

zvezna.

Rešitev:

Funkcija bo zvezna povsod, če parametra a in b določimo tako, da bo $\lim_{x\nearrow 4} f(x) = \lim_{x\searrow 4} f(x) = f(4)$ in $\lim_{x\nearrow 6} f(x) = \lim_{x\searrow 6} f(x) = f(6)$. Ker je $\lim_{x\nearrow 4} f(x) = -\frac{1}{6}$ in $\lim_{x\searrow 4} f(x) = f(4) = 4a+b$, dobimo pogoj $4a+b=-\frac{1}{6}$. Ker je $\lim_{x\nearrow 6} f(x) = f(6) = 6a+b$ in $\lim_{x\searrow 6} f(x) = 0$, dobimo še pogoj $6a+b=0$. Torej bo funkcija zvezna, če bo $a = \frac{1}{12}$ in $b = -\frac{1}{2}$.

- $\lim\limits_{x\uparrow 4} f(x) = \lim\limits_{x\downarrow 4} f(x) = f(4)$
- $\lim\limits_{x\uparrow 6} f(x) = \lim\limits_{x\downarrow 6} f(x) = f(6)$
- $\lim\limits_{x\uparrow 4} f(x) = \lim\limits_{x\uparrow 4} \frac{(\sqrt{x+5}-3)(\sqrt{x+5}+3)}{(4-x)\ (\sqrt{x+5}+3)} = \lim\limits_{x\uparrow 4} \frac{x+5-9}{(4-x)(\sqrt{x+5}+3)} =$

  $= \lim\limits_{x\uparrow 4} \frac{x-4}{(4-x)(\sqrt{x+5}+3)} = \lim\limits_{x\uparrow 4} \frac{-(4-x)}{(4-x)(\sqrt{x+5}+3)} =$

  $= \frac{-1}{\sqrt{4+5}+3} = \frac{-1}{6}$
- $\lim\limits_{x\downarrow 4} f(x) = \lim\limits_{x\downarrow 4} (ax+b) = 4a+b \quad \left(= \lim\limits_{x\uparrow 4} f(x) = -\frac{1}{6}\right)$
- $\lim\limits_{x\downarrow 6} f(x) = \lim\limits_{x\downarrow 6} e^{\frac{1}{6-x}} = e^{-\infty} = 0$ ker $x>6 \Rightarrow e^{\frac{1}{6-6{,}0001}} = e^{\frac{1}{-0{,}0001}}$
- $\lim\limits_{x\uparrow 6} f(x) = \lim\limits_{x\uparrow 6} (ax+b) = 6a+b = \lim\limits_{x\downarrow 6} f(x) = 0$
- $6a+b=0$ | $4a+b=-\frac{1}{6}$ | $b=-6a$
  $b=-6a$ | $4a-6a=-\frac{1}{6}$ | $b=-6\cdot\frac{1}{12}=-\frac{1}{2}$
  | $-2a=-\frac{1}{6} \ /:(-2)$ |
  | $a=\frac{1}{12}$ |

4. Določite a tako, da bo funkcija

$$f(x) = \begin{cases} \frac{\sin 2x}{\sqrt{9-x}-3} & x < 0 \\ a & x \ge 0 \end{cases}$$

zvezna v točki $x=0$.

Rešitev:

Funkcija bo zvezna, če bo $\lim_{x\nearrow 0} f(x) = \lim_{x\searrow 0} f(x) = f(0)$. Ker je $\lim_{x\searrow 0} f(x) = f(0) = a$ in z uporabo l'Hospitalovega pravila dobimo $\lim_{x\nearrow 0} = -12$, bo funkcija zvezna za $a=-12$.

- $\lim\limits_{x\uparrow 0} f(x) = \lim\limits_{x\downarrow 0} f(x) = \lim(0)$
- $\lim\limits_{x\uparrow 0} f(x) = \lim\limits_{x\uparrow 0} \frac{\sin 2x\ (\sqrt{9-x}+3)}{\sqrt{9-x}-3\ (\sqrt{9-x}+3)} = \lim\limits_{x\uparrow 0} \frac{\sin 2x(\sqrt{9-x}+3)}{9-x-9} =$

  $= \lim\limits_{x\uparrow 0} \frac{\sin 2x(\sqrt{9-x}+3)}{-x} \frac{(2)}{(2)} = \lim\limits_{x\uparrow 0} \frac{2\ \sin 2x(\sqrt{9-x}+3)}{-2x} =$

  $= \lim\limits_{x\uparrow 0} \frac{2\ (\sqrt{9-x}+3)}{-1} = -2\ (\sqrt{9}+3) = -2(6) = -12$
- $\lim\limits_{x\downarrow 0} f(x) = \lim\limits_{x\downarrow 0} a = a = \lim\limits_{x\uparrow 0} f(x) = -12 = a$

5. Določi parametra a in b tako, da bo funkcija

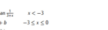
$$f(x) = \begin{cases} \arctan\frac{1}{3+x} & x < -3 \\ ax+b & -3 \le x \le 0 \\ \frac{\sin(3x)}{\sqrt{x+4}-2} & x > 0 \end{cases}$$

zvezna.

Rešitev:

Funkcija bo zvezna, če parametra a in b določimo tako, da bo $\lim_{x\nearrow -3} f(x) = \lim_{x\searrow -3} f(x) = f(-3)$ in $\lim_{x\nearrow 0} f(x) = \lim_{x\searrow 0} f(x) = f(0)$. Ker je $\lim_{x\nearrow -3} f(x) = -\frac{\pi}{2}$ in $\lim_{x\searrow -3} f(x) = f(-3) = -3a+b$, dobimo pogoj $-3a+b=-\frac{\pi}{2}$. Ker je $\lim_{x\nearrow 0} f(x) = f(0) = b$ in $\lim_{x\searrow 0} f(x) = 12$, dobimo še pogoj $b=12$. Torej bo funkcija zvezna, če bo $a = \frac{24+\pi}{6}$.

- $\lim\limits_{x\uparrow -3} f(x) = \lim\limits_{x\downarrow -3} f(x) = f(-3)$



  $\lim\limits_{x\uparrow 0} f(x) = \lim\limits_{x\downarrow 0} f(x) = f(0)$

  Da je $f(x)$ zvezna (neprekinjena) more tu držat.
- $\lim\limits_{x\uparrow -3} f(x) = \lim\limits_{x\uparrow 3} \arctan\frac{1}{3+x} = \arctan -\infty = -\frac{\pi}{2}$

  ker $x<-3 \Rightarrow \frac{1}{3+(-3{,}00001)} = \frac{1}{-0{,}00001} = -\infty$
- $\lim\limits_{x\downarrow -3} f(x) = \lim\limits_{x\downarrow -3} (ax+b) = -3a+b \quad \left(= \lim\limits_{x\uparrow -3} f(x) = -\frac{\pi}{2}\right)$
- $\lim\limits_{x\downarrow 0} f(x) = \lim\limits_{x\downarrow 0} \frac{(\sin 3x)\ (\sqrt{x+4}+2)}{(\sqrt{x+4}-2)\ (\sqrt{x+4}+2)} = \lim\limits_{x\downarrow 0} \frac{\sin 3x\ (\sqrt{x+4}+2)}{x+4-4} =$

  $= \lim\limits_{x\downarrow 0} \frac{\sin 3x\ (\sqrt{x+4}+2)}{x} \frac{(\cdot 3)}{(\cdot 3)} = \lim\limits_{x\downarrow 0} \frac{\sin 3x\ \ 3(\sqrt{x+4}+2)}{3x} =$

  $= \lim\limits_{x\downarrow 0} 3(\sqrt{x+4}+2) = 3(\sqrt{4}+2) = 3\cdot 4 = 12$
- $\lim\limits_{x\uparrow 0} f(x) = \lim\limits_{x\uparrow 0} (ax+b) = a\cdot 0 + b = b \quad \left(= \lim\limits_{x\downarrow 0} f(x) = 12\right)$
- $-3a+b=-\frac{\pi}{2}$ | $b=12$
  $-3a+12=-\frac{\pi}{2}$
  $-3a=-\frac{\pi}{2}-12 \ /:(-3)$
  $a=\frac{\pi}{6}+4$

6. Določite a in b tako, da bo funkcija

$$f(x) = \begin{cases} \frac{1}{x+2} + \frac{4}{x^2-4} & x < -2 \\ ax + b & -2 \leq x \leq 0 \\ \frac{\cos(3x)\sin\sqrt{x}}{\sqrt{x}} & x > 0 \end{cases}$$

zvezna povsod.

Rešitev:

Funkcija bo zvezna povsod, če parametra a in b določimo tako, da bo $\lim_{x \nearrow -2} f(x) = \lim_{x \searrow -2} f(x) = f(-2)$ in $\lim_{x \nearrow 0} f(x) = \lim_{x \searrow 0} f(x) = f(0)$. Ker je $\lim_{x \nearrow -2} f(x) = -\frac{1}{4}$ in $\lim_{x \searrow -2} f(x) = f(-2) = -2a + b$, dobimo pogoj $-2a + b = -\frac{1}{4}$. Ker je $\lim_{x \nearrow 0} f(x) = f(0) = b$ in $\lim_{x \searrow 0} f(x) = 1$, dobimo še pogoj $b = 1$. Torej bo funkcija zvezna, če bo $a = \frac{5}{8}$.

- $\lim\limits_{x \uparrow -2} f(x) = \lim\limits_{x \downarrow -2} f(x) = f(-2)$

  $\lim\limits_{x \uparrow 0} f(x) = \lim\limits_{x \downarrow 0} f(x) = f(0)$

- $\lim\limits_{x \uparrow -2} f(x) = \lim\limits_{x \uparrow -2} \left(\frac{1\,(x-2)}{x+2\,(x-2)} + \frac{4}{x^2-4} \right) = \lim\limits_{x \uparrow -2} \frac{x-2 \;\; +4}{(x^2-4)} =$

  $= \lim\limits_{x \uparrow -2} \frac{x+2}{(x-2)(x+2)} = \lim\limits_{x \uparrow -2} \frac{1}{(x-2)} = \frac{1}{-2-2} = -\frac{1}{4}$

- $\lim\limits_{x \downarrow -2} f(x) = \lim\limits_{x \downarrow -2} (ax+b) = -2a+b \quad \left(= \lim\limits_{x \uparrow -2} f(x) = -\frac{1}{4} \right)$

- $\lim\limits_{x \uparrow 0} f(x) = \lim\limits_{x \uparrow 0} (ax+b) = a \cdot 0 + b = b$

- $\lim\limits_{x \downarrow 0} f(x) = \lim\limits_{x \downarrow 0} \frac{\cos 3x \cdot \sin\sqrt{x}}{\sqrt{x}} = \lim\limits_{x \downarrow 0} \cos 3x = \cos(3 \cdot 0) =$

  $= \cos 0 = 1 \quad \left(= \lim\limits_{x \uparrow 0} f(x) = b \right)$

- $-2a + b = -\frac{1}{4} \qquad b = 1$

  $-2a + 1 = -\frac{1}{4}$

  $-2a = -\frac{1}{4} - 1$

  $-2a = -\frac{5}{4} \quad / : (-2)$

  $a = \frac{5}{8}$