Univerza v Ljubljani
Fakulteta za matematiko in fiziko

Reševanje linearnih elastostatičnih problemov z brezmrežnimi metodami

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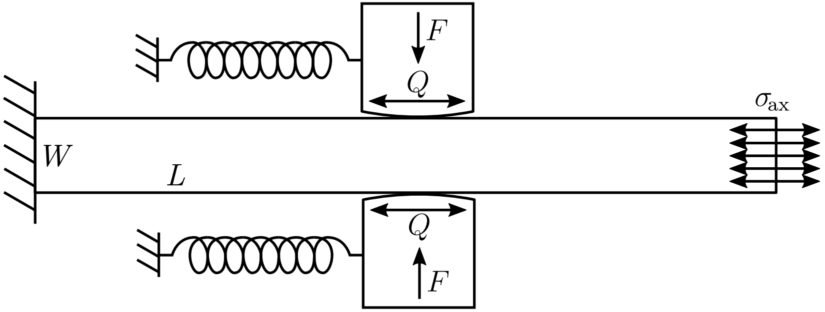
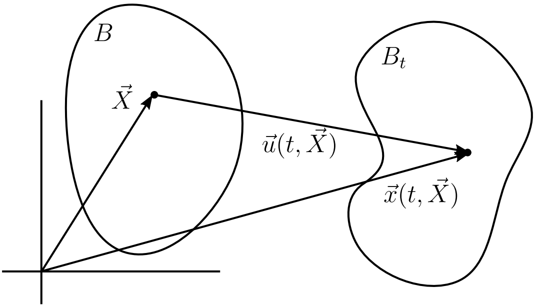
Ljubljana, 5. 9. 2017

Ciljni primer

- Želimo poznati napetosti in pomike v okolici stika



Opis gibanja in aksiomi



Aksioma o gibalni in vrtilni količini

$$\frac{d}{dt}\int_B \dot{\vec{x}}\,dm = \int_B \vec{f}\,dV + \int_{\partial B} \vec{t}\,dS$$

$$\frac{d}{dt}\int_B \dot{\vec{x}} \times \vec{x}\,dm = \int_B \vec{f} \times \vec{x}\,dV + \int_{\partial B} \vec{t} \times \vec{x}\,dS$$

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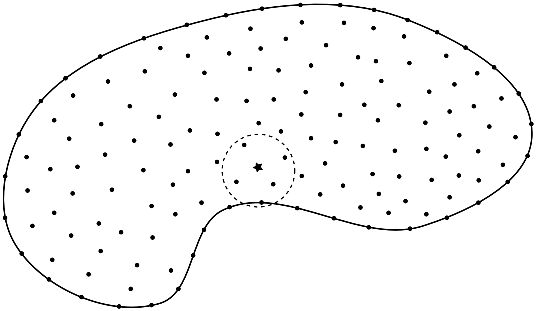
stacionarna enačba, robni pogoji?

- Rešujemo enačbo

$$
\begin{aligned}
\mathcal{L}u &= f \quad \text{na} \quad \Omega,\\
\mathcal{R}u &= g \quad \text{na} \quad \partial\Omega.
\end{aligned}
$$

- Brezmrežna metoda, v močni obliki.



- Ideja: $$(\mathcal{L}u)(x) \approx (\mathcal{L}\hat{u})(x), \quad \hat{u}(x) = \sum_{i=1}^{m} \alpha_i b_i(x)$$
- Zahtevamo interpolacijo v sosednjih točkah

$$\hat{u}(x_i) = u(x_i), \quad \forall x_i \text{ sosed } x$$

Aproksimacija linearnega parcialnega diferencialnega operatorja

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$\Rightarrow$ sledi

$$\begin{bmatrix} b_1(x_1) & \dots & b_m(x_1) \\ \vdots & \ddots & \vdots \\ b_1(x_n) & \dots & b_m(x_n) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_m \end{bmatrix} = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$$

Aproksimacija linearnega parcialnega diferencialnega operatorja

- Sistem $B\alpha = \boldsymbol{u}$
- Rešitev $\alpha = (WB)^{+} W\boldsymbol{u}$

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$$\hat{u}(x) = b(x)^{\mathsf{T}}\alpha = b(x)^{\mathsf{T}}(WB)^{+} W\boldsymbol{u}$$

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$$(\mathcal{L}u)(x) \approx (\mathcal{L}\hat{u})(x) = \underbrace{(\mathcal{L}b)(x)^\mathsf{T}(WB)^+ W}_{\varphi(x)} u$$

Numerično reševanje

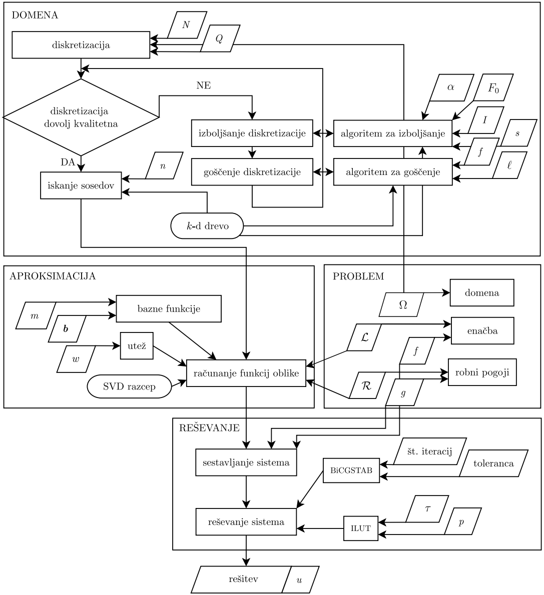
1. Izračunamo $\varphi(x)$ za vsak x in nastavimo enačbe

$$\varphi(x)u = f$$

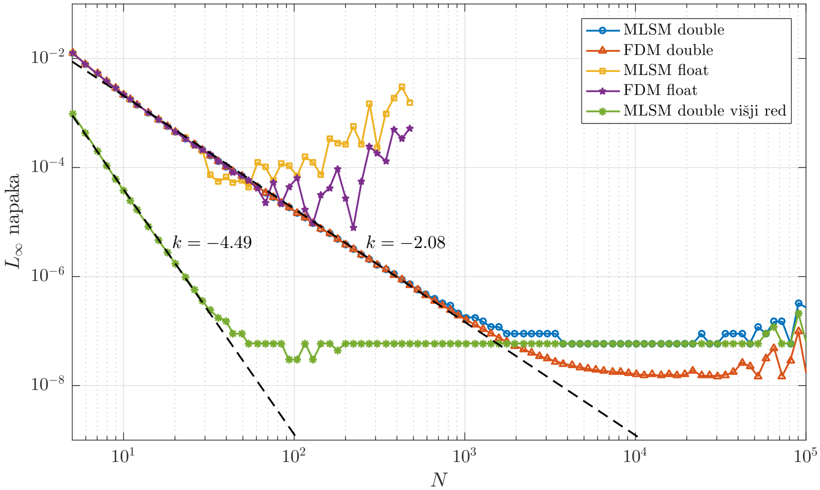
2. Zložimo v matriko A in nastavimo robne pogoje

3. Rešimo razpršen sistem $Au = f$

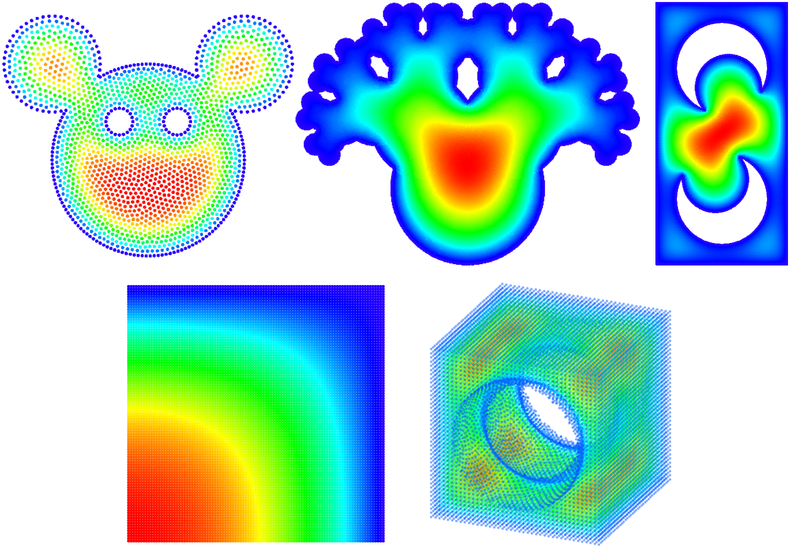
Implementacija



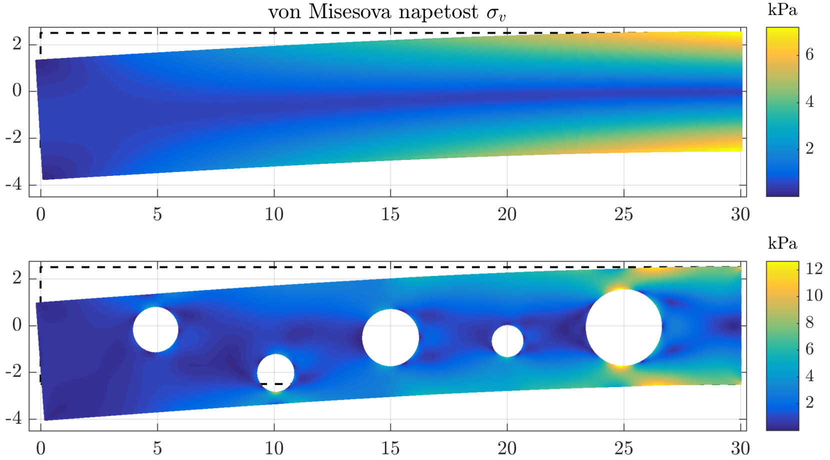
Primerjava s FDM



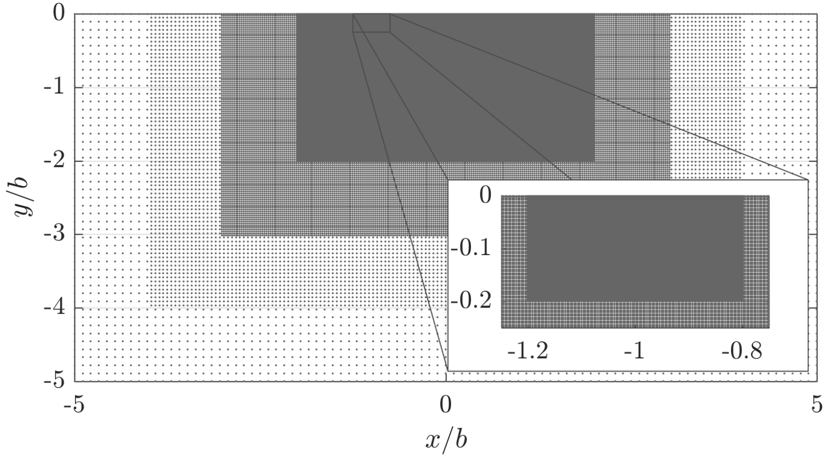
Difuzijska enačba



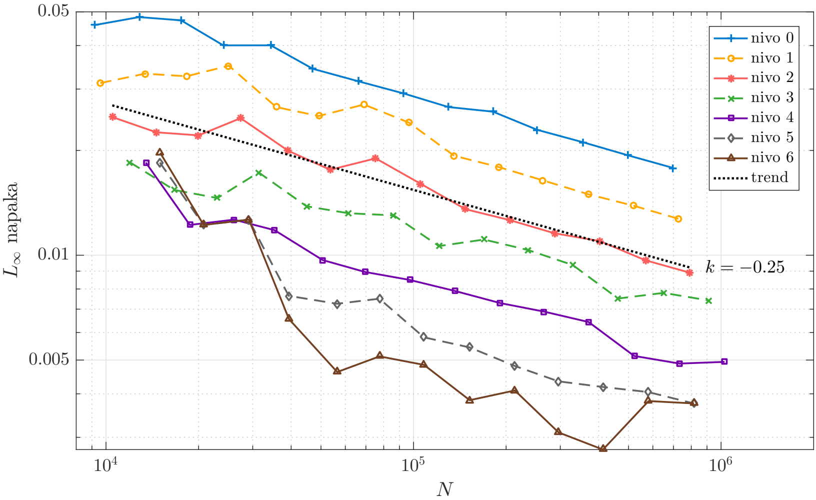
Vpet nosilec – rešitev



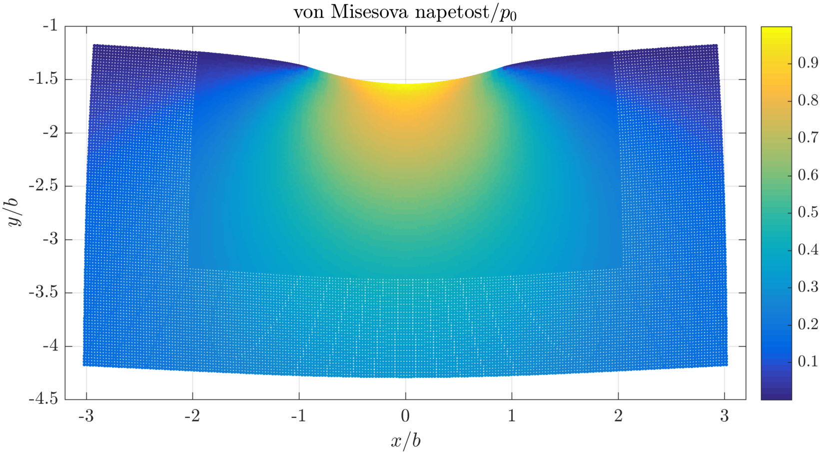
Hertzev kontakt – domena



Hertzev kontakt – konvergenca



Hertzev kontakt – rešitev



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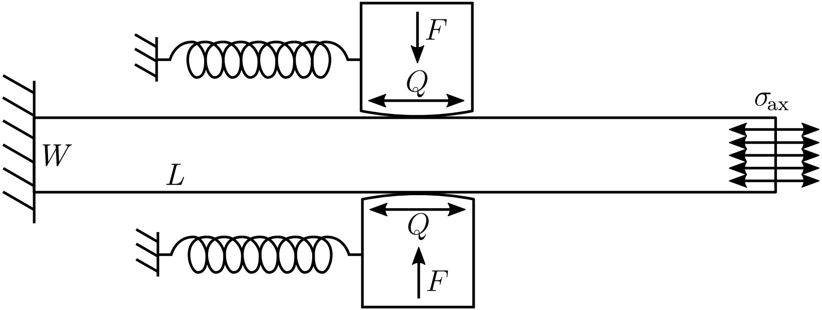

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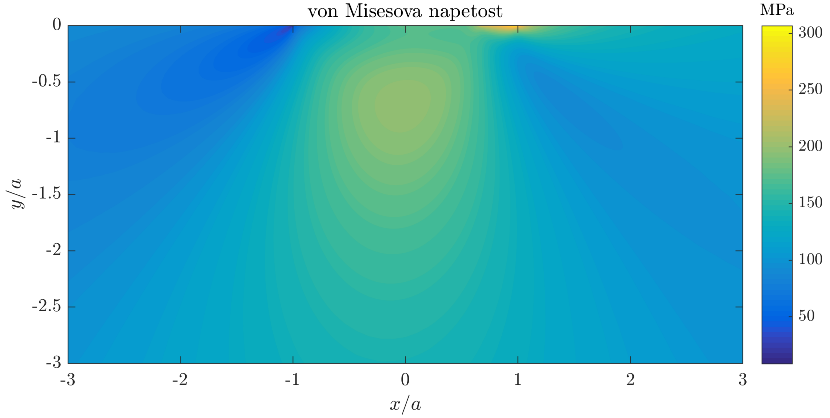

Nauk

Z brezmrežnimi metodami je mogoče uspešno in učinkovito reševati probleme iz linearne elastomehanike.

Hvala za pozornost!