

Problem 2.1: Expected Zero-One-Loss

The expected zero-one-loss is given by:

$$\begin{aligned} E(x, y) &\sim P[I[f_0(x) \neq y]] \\ &= P[Y = +1] \cdot P[f_0(x) \neq +1|Y = +1] \\ &\quad + P[Y = -1] \cdot P[f_0(x) \neq -1|Y = -1] \\ &= p^+ \cdot 0.5 + (1 - p^+) \cdot 0.5 \\ &= 0.5 \end{aligned}$$

So, the expected zero-one-loss for $f_0(x)$ is 0.5.

Problem 2.2: Probability of Zero Training Loss

The probability of zero training loss with N samples is given by:

$$\begin{aligned} P(\text{Zero training loss}) &= [P(X(1) < 0|Y = +1)]^{N/2} \cdot [P(X(1) \geq 0|Y = +1)]^{N/2} \\ &\quad \cdot [P(X(1) < 0|Y = -1)]^{N/2} \cdot [P(X(1) \geq 0|Y = -1)]^{N/2} \\ &= (0.5^{N/2}) \cdot (0.5^{N/2}) \cdot (0.5^{N/2}) \cdot (0.5^{N/2}) \\ &= 0.25^N \end{aligned}$$

So, the probability of zero training loss with N samples is 0.25^N .

Problem 2.3: Evolution of Probability with Increasing N

As N increases to infinity, the probability of zero training loss, $P(\text{Zero training loss})$, approaches zero. This is because as N becomes very large, the probability of all the samples satisfying the conditions for zero training loss becomes extremely low, and it becomes increasingly unlikely to obtain zero training loss as N goes to infinity.