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IoT Challenge #1
Optimizing Sink Position
in a
Wireless Sensor Network
Exercise PDF

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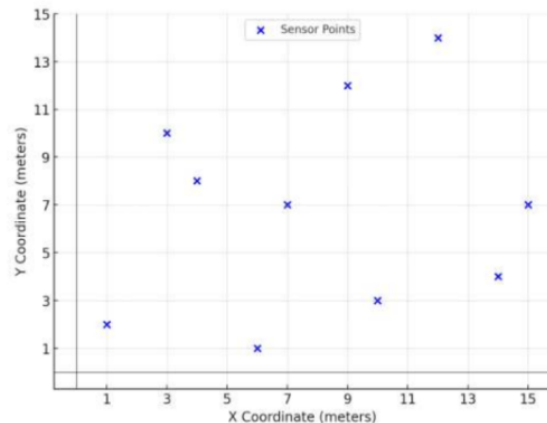
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1 Optimizing Sink Position in a Wireless Sensor Network

In the parking lot, there are 10 sensors that monitor the parking spaces. Each sensor has a fixed position (x,y) in the parking lot reported in Table.

Sensor	Coordinates (x,y)
1	(1, 2)
2	(10, 3)
3	(4, 8)
4	(15, 7)
5	(6, 1)
6	(9, 12)
7	(14, 4)
8	(3, 10)
9	(7, 7)
10	(12, 14)



1.1 Data and specifications

- Each sensor transmits a status update every **10 minutes**.
- The packet size is $b = 2000$ bits, and the initial energy per sensor is $E_b = 5$ mJ.

The energy consumption for transmission depends on the distance between the sensor and the sink:

- **Energy for the TX/RX circuitry:** $E_c = 50$ nJ/bit
- **Energy for transmission:**

$$E_{tx}(d) = k \cdot d^2 \quad (\text{nJ/bit})$$

where d is the distance from the sensor to the sink, and $k = 1$ nJ/bit/m².

1.2 Objectives

A. System Lifetime with Fixed Sink Position

Determine the system lifetime when the sink is placed at the fixed position:

$$(x_s, y_s) = (20, 20)$$

The lifetime is defined as the time until the first sensor's battery depletes, based on its energy consumption.

B. Optimal Sink Position

Find the optimal coordinates (x_s, y_s) that maximize the system lifetime. This corresponds to minimizing the energy consumption of the worst-case sensor (the one that depletes the fastest).

C. Trade-offs Between Fixed and Dynamic Sink Positions

Discuss the trade-offs of using a fixed sink position versus a dynamically moving sink. Consider the impact on

- System lifetime
- Energy consumption per sensor

2 A - Lifetime with fixed position

To determine the best position for the sink within the system, we will calculate the distances between the position (20, 20) and all the positions of the other sensors. The lifetime will be determined by the sensor that uses the most energy. The sensor that is farthest from the sink will be the one that consumes the most energy.

Sensor number	Distance (m)	Coordinates of Points (x, y)	Energy per sensor (mJ)
1	26.17	(1, 2)	1.47
2	19.72	(10, 3)	0.88
3	20.00	(4, 8)	0.90
4	13.93	(15, 7)	0.49
5	23.60	(6, 1)	1.21
6	13.60	(9, 12)	0.47
7	17.09	(14, 4)	0.68
8	19.72	(3, 10)	0.88
9	18.38	(7, 7)	0.78
10	10.00	(12, 14)	0.30

As observed in the table, sensor 1 is the furthest from the sink, meaning that data transmission will be the most energy consuming. Therefore, the system's lifetime is primarily determined by how long Sensor 1's battery lasts.

For this sensor, the energy consumed per transmission would be:

$$\begin{aligned}
 E_{\text{total } 1} &= E_c \cdot b + E_{\text{tx}}(d) \cdot b \\
 E_{\text{tx}}(26.17) &= k \cdot d^2 \quad (\text{nJ/bit}) = 1 \text{ nJ/bit/m}^2 \cdot (26.17 \text{ m})^2 = 6.85 \times 10^{-7} \text{ J/bit} \\
 E_{\text{total } 1} &= b \cdot (E_c + E_{\text{tx}}(d)) \quad E_{\text{total } 1} = 2000 \cdot (50 \text{ nJ/bit} + 6.85 \times 10^{-7} \text{ J/bit}) = 1.47 \text{ mJ}
 \end{aligned}$$

So we can calculate the number of cycles that the sensor will be able to endure before the battery is discharged.

$$N = \frac{E_0}{E_{\text{total } 1}} = \frac{5}{1.47} \text{ mJ} \simeq 3 \text{ cycles}$$

The sensor will have enough energy to transmit 3 times but not 4. So, considering the 10 minutes time between each transmission, we obtain a lifetime of $N \times T = 30$ minutes

3 B - Optimal Sink position

Let there be N sensors located at coordinates (x_i, y_i) for $i = 1, \dots, N$. The sink is to be placed at the coordinates (x_s, y_s) , which are decision variables. Each sensor transmits data packets of size b (in bits) and consumes energy per packet given by:

$$E_{\text{total},i} = b \cdot (E_c + k \cdot d_i^2),$$

where

- E_c is the energy consumed by the TX/RX circuitry (in nJ/bit),
- k is a constant (in nJ/bit/m²),
- d_i is the Euclidean distance between sensor i and the sink, i.e.,

$$d_i = \sqrt{(x_s - x_i)^2 + (y_s - y_i)^2}.$$

The objective is to maximize the lifetime of the system, which is determined by the sensor with the highest energy consumption. This can be formulated as a minimization problem of the distance:

$$\min_{x_s, y_s} d_i.$$

3.0.1 Code Sample

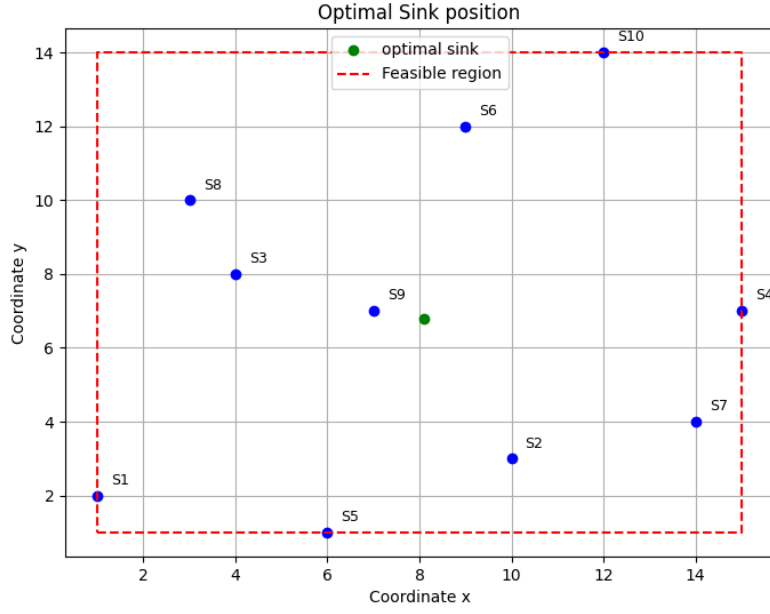
```
# Sensor coordinates (sample data)
sensors = [(1, 2), (10, 3), (4, 8), (15, 7), (6, 1), (9, 12), (14, 4), (3, 10), (7, 7), (12, 14)]

# Extract x and y coordinates separately
x_coords = [p[0] for p in sensors]
y_coords = [p[1] for p in sensors]

# Function to minimize: sum of Euclidean distances between point (sx, sy) and all sensors
def objective(p):
    sx, sy = p # Sink coordinate (central sensor)
    dist_sum = 0
    for (x, y) in sensors:
        dist_sum = min(dist_sum, (sx - x)**2 + (sy - y)**2) # Euclidean distance
    return dist_sum

# Initial condition: midpoint among all sensors
initial_guess = [np.mean(x_coords), np.mean(y_coords)]

# Solve the minimization problem
result = minimize(objective, initial_guess, method='Nelder-Mead')
```



The optimal position for the sink is (8.10, 6.80). The coordinates (xs, ys) of the sink that minimize the energy consumption for the worst-case sensor are actually the same as the coordinates of Sensor 1, which are (1, 2). Finally, we can compute the energy consumption corresponding to the best sink position.

Sensor number	Distance (m)	Coordinates of Points (x, y)	Energy (nJ/bit)
1	8.57	(1,2)	123.45
2	4.25	(10,3)	68.05
3	4.27	(4,8)	68.25
4	6.90	(15,7)	97.65
5	6.17	(6,1)	88.05
6	5.28	(9,12)	77.85
7	6.53	(14,4)	92.65
8	6.02	(3,10)	86.25
9	1.12	(7,7)	51.25
10	8.19	(12,14)	117.05

The system lifetime is extended to approximately:

$$E_{\text{total } 1} = 123.45 \times 10^{-9} \text{ nJ/bit} \times 2000 \text{ bit} = 0.25 \text{ mJ}$$

$$N = \frac{E_0}{E_{\text{total } 1}} = \frac{5}{0.25} \text{ mJ} \simeq 20 \text{ cycles}$$

4 C. Trade-offs Between Fixed and Dynamic Sink Positions

The choice between a fixed sink and a dynamically moving sink in a wireless sensor network (WSN) significantly affects energy consumption, network lifetime, and system complexity.

A fixed sink position simplifies system design, reducing complexity and implementation effort. However, this approach often leads to higher energy consumption, particularly for nodes located farther from the sink, as they must transmit data over longer distances.

On the other hand, a dynamic sink position optimizes energy distribution across the network, ensuring more balanced power consumption among nodes. Although this method improves system efficiency and extends the life of the network, it introduces greater computational and communication overhead, since the nodes must continuously determine the optimal position of the sink.

Aspect	Fixed Sink Position	Dynamic Sink Position
Complexity	Low complexity, easy to implement	High complexity, requires adaptive control
Energy Consumption	Higher for distant nodes, leading to early depletion	More balanced across nodes, reducing energy hotspots
Network Lifetime	Shorter due to uneven energy drain	Longer due to energy balancing
Implementation	Simple and static deployment	Requires continuous sink repositioning
Communication Overhead	Minimal, as the sink remains in one place	Increased, due to frequent position updates

Table 1: Comparison of Fixed vs. Dynamic Sink Positioning