Panel estimates of the gender earnings gap

Individual-specific intercept and individual-specific slope models*

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This paper develops individual-specific slope (as opposed to individual-specific intercept) models to account for unobserved heterogeneity in estimating male-female wage differentials. Estimates from these models are compared to traditional OLS, as well as to fixed- and random-effects individual-specific intercept approaches. We find unambiguously that unobserved heterogeneity accounts for about 50% of the male-female wage gap, with individual-specific slope models explaining a slightly smaller proportion of the wage gap. Based on inferences from life-cycle earnings models, we conclude that most individual-specific differences manifest themselves early in one's work career, probably even in the type of schooling received.

Key words: Panel data; Fixed effects; Gender; Earnings; Discrimination

JEL classification: C23; J16; J31; J7

1. Introduction

The question of gender pay differentials is important for policy purposes. If male-female wage differentials emerge because of unequal opportunities caused by discrimination, then the economy is failing to fully and appropriately utilize its highly productive employees. Accordingly such inefficiencies can justify

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government intervention. On the other hand, governmental intervention could lead to distortions in resource allocation when unequal economic outcomes result from differing individual choices rather than discrimination. In this case rather than helping the disadvantaged, government policy can hamper efficiency so that in the long run all end up suffering. Thus, an understanding of male–female pay differences is important.

Measures of the gender wage gap entail computing what women would earn had they the same characteristics as males. The difference between what females would earn with male characteristics and what women actually earn constitutes the unexplained gender wage gap, often referred to as discrimination. The problem is that this discrimination measure might be biased. For example, if because of differences in lifetime work patterns men's and women's work motivations differ in ways that are difficult to measure directly [Polachek (1975) and Becker (1985)], estimates of the unexplained wage gap that fail to account for such motivational differences would be inaccurate because they might be picking up differences in motivation.

Standard econometric practice handles differences in motivation as a fixed individual-specific effect: earnings of the more motivated individual are assumed higher, implying an earnings function containing individual-specific *intercepts*. The usual approach is via mean-deviation (MD) or first-difference (FD) estimation models. In contrast, earnings function *slopes*, that is the gradient with respect to each earnings function determinant, are assumed common and *independent of motivation*. The problem is that this assumption of common earnings gradients doesn't square with life-cycle human capital theory [Polachek (1975), Weiss and Gronau (1981)]. Such models show that if the more motivated workers are the ones who work more continuously over the life cycle, then these workers are not necessarily the ones with the higher age-earnings profile, but instead are the workers who invest more in themselves, and hence have *steeper* profiles. However, as indicated, current fixed-effects models equate motivation with intercept *instead* of the slope.

This paper's major contribution is to rectify this problem by developing individual-specific slope earnings function estimates. Both random- and fixed-effects models are considered, and applied to estimate 'unexplained' male-female earnings differences. To anchor the results to past studies as well as our own earlier work [Kim and Polachek (1991)], these estimates are compared to traditional OLS estimates, between-groups (BG) estimates (to account for possible variable measurement errors), as well as to fixed- and random-effects estimates that assume individual-specific intercepts.

We find unambiguously that about 50% of the unexplained male-female wage gap can be attributed to unmeasured individual differences. These results emerge both from individual-specific intercept and individual-specific slope specifications, with little difference between the two types of estimation models. This leads us to conclude that at least with the PSID sample used here most

individual-specific differences manifest themselves early in the work career, possibly in the type of schooling received and *not* in the rate of on-the-job training. Thus, assuming common experience gradients in correcting for heterogeneity does not prove to affect estimates of unexplained gender wage differences appreciably. Nevertheless, because gender differences in younger cohort's lifetime work behavior have converged considerably, future research should stratify the data by age to see if these conclusions hold for both young and old cohorts.

2. Computing the unexplained male-female wage gap: Cross-sectional approaches

Becker (1957) defines the proportion of the male-female wage gap unexplained by individual characteristics to constitute discrimination. One approach to measure this is to fit wage functions jointly for men and women. Introducing a gender dummy variable enables one to discern how wages differ holding individual characteristics constant. The usual model is

$$w_{it} = x_{it}^* \beta + G_i \delta + \varepsilon_{ti}, \tag{1}$$

where w_{it} is the logarithm of wage and x_{it}^* is a vector of K regressors (including a constant) composed of education, experience, and other human capital measures constituting the determinants of earnings; G_i is the dummy gender variable; and ε_{it} is a normally distributed error. The coefficient δ depicts gender differences in earnings holding individual characteristics x_{it}^* constant. Typically (1) is estimated by OLS with cross-sectional data.

Many [e.g., Corcoran, Duncan, and Ponza (1983), Gronau (1988)] have argued that this work is deficient because statistical problems can mar the validity of OLS estimates. One such bias is heterogeneity.² Heterogeneity biases arise because unobserved worker characteristics such as motivation can be correlated with a regressor such as work continuity. If the more intermittent worker is less motivated (for example, because intermittency reduces work

¹Alternatively there are other approaches to estimate the unexplained wage gap [e.g., Oaxaca (1978), Neumark (1988)]. However, given that we wish to concentrate on the effect of heterogeneity biases, it is not crucial which discrimination measure we consider as long as we are consistent in definition across estimators. As such, rather than deal with what constitutes the appropriate discrimination measure, this paper will concentrate on how robust a representative discrimination measure is given alternative estimation schemes.

²Endogeneity and selectivity are other biases, but we consider these latter biases in Kim and Polachek (1991). Also see Gronau (1988), Licht and Steiner (1991), and Korenman and Neumark (1991). Although not explicitly considered in this paper, these latter biases can interact with heterogeneity since each type bias essentially occurs because the error term is correlated with some of the regressors. On the other hand, adjusting for either one of these can decrease the importance of the other. Again, see Kim and Polachek (1991).

effort's remuneration), he or she will earn less. To the extent societal forces cause women to have more familial responsibilities so that they tend to be more intermittent than men, estimates of the gender dummy variable might be picking up resulting differences in motivation, thus biasing the unexplained wage gap estimates. Heterogeneity biases thus can occur when unobserved worker characteristics are correlated both with wages and at least one exogenous regressor such as labor market intermittency.³

Unobserved individual effects such as motivation can be dispersed among a population according to a known distribution or they may be spread randomly following no particular distribution. For example, individual-specific effects might be distributed normally if unobservable characteristics as motivation are normally distributed. If, on the other hand, unobservable individual-specific effects cannot be simply modeled, then they might follow no known distribution.

3. Panel estimation of the male-female wage gap: Individual-specific intercepts

The standard response to heterogeneity entails using panel data. If one has reason to believe that individual effects follow a specific distribution, perhaps because ability or motivation might be normally distributed, then this information can be used in specifying the individual-specific term. In this case the normally distributed person-specific error is added to the normally distributed general error to obtain a two-component normal distribution, thus transforming the problem to an error components model in which gender differences can be obtained by the dummy gender coefficient after controlling for normally distributed person-specific unobserved heterogeneity. Since the individual-specific effects are correlated with individual characteristics, this entails augmenting the GLS estimator with instrumental variables (IV).

Alternatively, if motivation follows no known distribution, the typical approach is to follow a given individual over time. Following a given individual holds constant nonchanging individual characteristics, thereby enabling the researcher to obtain unbiased estimates of labor market effects. With these resulting unbiased earnings function parameters, male and female earnings can be compared and appropriate gender differences can be computed. This latter approach is comparable to estimating a fixed-effects (FE) model containing a person-specific dummy variable which controls for unobserved factors that affect each person's wage.

To implement these panel data approaches we respecify eq. (1) to include individual-specific effects α_i and break up x^* into x and z vectors in order to

³Note we do not claim that women are less motivated than men, per se. (That would be discriminatory.) Instead we argue that on average women's work behavior differs from men's. Differences in work behavior cause average motivation to differ, which when omitted from the wage regression biases estimates of the unexplained gender wage gap.

distinguish between time-varying and time-invariant elements of x^* :

$$w_{it} = x_{it}\beta + Z_i\delta + \alpha_i + \varepsilon_{it}. \tag{2}$$

As before, w_{it} depicts wages (in logarithms) for individual i and index t denotes the specific year (t) when earnings are realized. Since not all regressors vary over time, we divide the regressors into time-varying x_{it} and time-invariant z_i . As indicated, α_i depicts unobservable individual-specific effects and ε_{it} represents unobservable effects varying both across individuals and over time. As is standard, we assume that ε and α are independent, that ε is serially uncorrelated.⁴ and that ε has a zero mean:

$$E(\varepsilon_{it} \alpha_i) = 0$$
 for all $i, t,$
 $E(\varepsilon_{it} \varepsilon_{js}) = 0$ unless $i = j, t = s,$
 $E(\varepsilon_{it}) = 0$ for all $i, t.$

Finally, as was indicated, two cases are considered: First where α_i is assumed normally distributed across the population, and second where α_i has no known distribution. When α_i is normal, the GLS estimator is appropriate. However, one uses a fixed-effects (FE) estimator when the distribution of α_i is unknown.

The GLS estimator can be obtained from (2) by projecting the block-diagonal covariance matrix to the W_{ii} and $[X_{ii} Z_i]$ variable matrices, providing that σ_{ε}^2 and σ_{α}^2 are known. The $\text{cov}(\alpha_i + \varepsilon_{ii} | X_{ii}, z_i)$ is a block-diagonal matrix and given by $\Phi = \sigma_{\varepsilon}^2 I_{NT} + \sigma_{\alpha}^2 [I_N \otimes j_T j_T']$. In the absence of correlations between α_i and $[X_{ii} Z_i]$, this GLS estimator is best linear unbiased, and given by

$$b_{GLS} = (V'\Phi^{-1}V)^{-1}V'\Phi^{-1}W,$$
(3)

where V = [XZ]. If α_i is correlated with a regressor such as labor market intermittency, this estimate is inconsistent. To correct this problem one should use an instrumental variable, two-stage, generalized least-squares approach with variables orthogonal to X serving as instruments, though as illustrated in Kim and Polachek (1991) this is more difficult than might be expected because the results turn out to be sensitive to the choice of instrumental variables.

The fixed-effects estimator can be obtained from eq. (2) by transforming each observation either by a mean deviation or a first-difference operator. In effect these operators sweep the unobserved person-specific effects. However, as we shall see, they also eliminate the time-invariant variables so that a second-step analysis of residuals is necessary to obtain time-invariant coefficient estimates.

⁴See Kim (1992) for models relaxing this assumption.

The mean deviation approach is essentially an OLS regression obtained by subtracting each individual's mean variable values from each observation,

$$(W_{it} - W_{i.}) = (X_{it} - X_{i.})\beta + \varepsilon_{it} - \varepsilon_{i.}, \tag{4}$$

where W_i is the individual mean of time-variant W_{it} ($W_{i.} = \sum_{t=1}^{T} W_{it}/T$). Note that the α_i and Z_i terms cancel out since they are time-invariant individual-specific. One can write (4) in matrix form

$$D_{NT}W_{it} = D_{NT}X_{it}\beta + D_{NT}\varepsilon_{it}, (5)$$

where $D_{NT} = I_{NT} - [I_N \otimes j_T j_T'/T]$ which produces a vector of deviations from the group mean is an idempotent matrix, j_T denotes a T vector of ones, and \otimes is the Kronecker product.

The mean-deviation fixed-effects estimator denoted b_w can be expressed as

$$b_{w} = (X'_{it}D_{NT}X_{it})^{-1}X'_{it}D_{NT}W_{it}. (6)$$

Its covariance matrix is given by $\sigma_{\varepsilon}^2(X'D_{NT}X)^{-1}$.

Alternatively the first-difference approach sweeps out α_i by subtracting lagged variable values from each observation according to

$$(W_{it} - W_{i,t-1}) = (X_{it} - X_{i,t-1})\beta_d + (\varepsilon_{it} - \varepsilon_{i,t-1}). \tag{7}$$

Again the Z_i term is eliminated.

In matrix form this can be written

$$F_{\star}W_{ii} = F_{\star}X_{ii}\beta + F_{\star}\varepsilon_{ii}, \tag{8}$$

where F_{τ} produces the vector of first differences for each individual. The estimator for β denoted by b_d is

$$b_d = (X'F'FX)^{-1}X'F'FW, (9)$$

where its covariance is given by $(X'F'FX)^{-1}$.

To identify δ one substitutes the consistent estimate of β (either b_w or b_d) into (2), averaged for each individual over time, to obtain

$$w_{i.} = x_{i.}b + x_{i.}(\beta - b) + z_{i}\delta + \alpha_{i} + \varepsilon_{i.}, \tag{10}$$

which can be written as

$$w_{i,} - x_{i,}b = z_{i}\delta + x_{i,}(\beta - b) + \alpha_{i} + \varepsilon_{i,} = z_{i}\delta + \eta_{i}, \tag{11}$$

where $\eta_i = x_i (\beta - b) + \alpha_i + \varepsilon_i$. If η_i is uncorrelated with z_i , then (11) can be estimated by OLS, or GLS if η_i is heteroscedastic.⁵

To date many studies apply these types of fixed-effects models to estimate earnings functions. Early studies [Mincer and Polachek (1978), Mincer and Ofek (1982), Corcoran, Duncan, and Ponza (1983)] estimating models akin to (4) concentrate on wage effects of intermittency. Recent analyses [Kim and Polachek (1991), Licht and Steiner (1991), Light and Ureta (1992)] deal more specifically with estimating the gender wage gap. All assume that unobserved heterogeneity manifests itself via individual-specific intercepts. The innovation of this paper is to introduce individual-specific slopes. We do so within the context of both a random- and fixed-effects framework.

4. Panel estimation of the male-female wage gap: Individual-specific slopes

One problem with past fixed-effects procedures is that they assume the individual-specific effect – motivation – to affect earnings only through the earnings function intercept. In short, the models assume motivation affects earnings levels but *not* growth. However, this assumption may be inconsistent with human capital theory if, as one would expect, motivation is related to one's lifetime work commitment. Human capital theory is clear: while greater ability may raise earnings throughout life, more motivation and effort increase monetary gains from investing in human capital, which in turn steepen earnings profiles [Polachek (1975), Weiss and Gronau (1981)]. The current panel data earnings function models just outlined consider individual-specific intercepts but fail to address this issue of individual-specific slopes. In what follows we modify panel estimation techniques to consider individual-specific slopes.

Respecify eq. (2) as follows:

$$w_{it} = x_{it}\beta_i + Z_i\delta + \alpha_i + \varepsilon_{it}, \tag{12}$$

where all terms are as previously defined, except β_i which now varies by individual i.

Again, as before, there are two possibilities: (1) to assume random-effects models govern the individual-specific slopes, or (2) to assume fixed-effects models govern the individual-specific slopes. We consider each in turn.

⁵Instrumental variable estimation should be used if η_i is correlated with z_i . See Hausman and Taylor (1981). However, this is not the case in our representation since unmeasured motivation is correlated with intermittency, not gender.

4.1. Random-effects individual-specific slope model

Without loss of generality we simplify the specification of (12) to incorporate α_i into β_i ,

$$w_{it} = x_{it}\beta_i + Z_i\delta + \varepsilon_{it}, \tag{13}$$

where now X_{it} includes an initial vector of ones so that the first parameter of β_i is α_i .⁶ Assume β_i can be broken down into a common (β) and random component (β^*) such that

$$\beta_i = \beta + \beta_i^*. \tag{14}$$

Rewrite (13) decomposing the $x_{it}\beta_i$ into stochastic and nonstochastic components,

$$w_{it} = x_{it}\beta + Z_i\delta + \varepsilon_{it} + x_{it}\beta_i^*$$

$$= x_{it}\beta + Z_i\delta + \eta_i^*,$$
(15)

where $\eta_{it}^* = \varepsilon_{it} + x_{it}\beta_i^*$. $E(\eta_{it}^*) = 0$ and $E(\eta_{it}^*\eta_{it}^{*'}) = var(\varepsilon_{it} + X_{it}\beta_i^*) = \sigma_i^2 I_T + X_{it}\Delta^2 X_{it}'$ has the heteroscedastic residual covariance, and where σ_i^2 is the individual-specific variance of ε_{it} , I_T is the identity matrix, and Δ^2 is individual-specific variance of β_i .

Generalized least squares is used to obtain an efficient estimator for β and δ ,

$$\begin{bmatrix} \hat{\beta} \\ \hat{\delta} \end{bmatrix} = \left[\sum_{i} (X_{it} : Z_{it})' (\sigma_{i}^{2} I_{T} + (X_{it} : Z_{i}) \Delta^{2} (X_{it} : Z_{i})')^{-1} (X_{it} : Z_{i}) \right]^{-1} \times \left[\sum_{i} (X_{it} : Z_{i})' (\sigma_{i}^{2} I_{T} + (X_{it} : Z_{i}) \Delta^{2} (X_{it} : Z_{i})')^{-1} W_{it} \right],$$
(16)

where $\left[\sum_{i}(X_{it} : Z_{i})'(\sigma_{i}^{2}I_{T} + (X_{it} : Z_{i})\Delta^{2}(X_{it} : Z_{i})')^{-1}(X_{it} : Z_{i})\right]^{-1}$ is the covariance matrix [Swamy (1970), Hsiao (1986)]. Since σ_{i}^{2} and Δ^{2} are unknown, they must be estimated for each individual. One can obtain least-squares

⁶In matrix forms this is
$$[w_{it}] = \begin{bmatrix} 1 & x_{it} \end{bmatrix} \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} + \begin{bmatrix} z_i \end{bmatrix} \delta + \begin{bmatrix} \varepsilon_{it} \end{bmatrix}$$
.

estimates of β_i ,

$$\tilde{\beta}_{i} = (X'_{it}X_{it})^{-1}X'_{it}W_{it}, \tag{17}$$

using panel data for each individual. Similarly, residuals $u_{it} = W_{it} - X_{it} \beta_i$ can be computed for each individual, and estimates of σ_i^2 and Δ^2 obtained as follows:

$$\tilde{\sigma}_i^2 = u_{it}' u_{it} / (T - K), \tag{18}$$

where K_1 is the number of time-varying X regressors, and

$$\tilde{\Delta}^{2} = (1/N) \sum_{i} \left(\tilde{\beta}_{i} - N^{-1} \sum_{i} \tilde{\beta}_{i} \right) \left(\tilde{\beta}_{i} - N^{-1} \sum_{i} \tilde{\beta}_{i} \right)' - (1/N) \sum_{i} \tilde{\sigma}_{i}^{2} (X'_{it} X_{it})^{-1},$$

$$(19)$$

where N is the number of individuals. Noting that K_2 is the number of time-invariant Z regressors,

$$\Delta^2 = \begin{bmatrix} \tilde{\Delta}_{K_1 \times K_1}^2 & O_{K_1 \times K_2} \\ O_{K_2 \times K_1} & O_{K_2 \times K_2} \end{bmatrix}. \tag{20}$$

Substituting (18) and (20) into (16) yields the common coefficient β and the δ coefficients in (15).

The advantage of this random-coefficients approach is that it allows individuals-specific components for each β_i . The disadvantage is that it restricts these β_i so that $E(\beta_i^*) = 0$. Fixed-effects models impose no such restrictions on β_i . However, to identify β the fixed-effects models that we have developed to date restrict the number of time-varying parameters. We now turn to these models.

4.2. Fixed-effects individual-specific slope models

We consider two genres of FE models. The simplest and most straightforward is one in which only the earnings profile slope (i.e., the experience gradient) varies with unobservable individual motivation. Though less general, this makes most sense from the human capital perspective since, as already indicated, increased motivation increases both work continuity and effort, which increase investment and hence wage growth.

The second genre is more general in that it allows both for individual-specific intercepts and individual-specific slopes, but it is computationally more

⁷We wish to thank Bong Yoon for suggesting this approach.

difficult because it requires regressions for each individual. We discuss each in turn.

4.2.1. Fixed-effects individual-specific experience model

Respecify eq. (12) to consider the case of a common intercept, but allowing for one other coefficient (in our case, experience) to vary across individuals,

$$w_{it} = x_{oit}\beta_{oi} + x_{it}\beta + Z_i\delta + \varepsilon_{it}, \tag{21}$$

where x_{oit} is experience for individual i at time t, and x_{it} is a matrix such that $x_{it} = \{1, x_{1it}, x_{2it}, \dots, x_{kit}\}$. All other variables are as previously defined. To rid ourselves of the individual-specific parameters divide all terms by x_{oit} , yielding

$$\frac{w_{it}}{x_{oit}} = \beta_{oi} + \frac{x_{it}}{x_{oit}} \beta + \frac{z_i}{x_{oit}} \delta + \frac{\varepsilon_{it}}{x_{oit}}, \tag{22}$$

where $E(\varepsilon_{it}/x_{oit}) = 0$, since $E(\varepsilon_{it})$ was assumed equal to zero. Delete β_{oi} either by taking the first difference or mean deviation, yielding

$$\frac{w_{it}}{x_{oit}} - \left(\frac{\bar{w}_{it}}{x_{oit}}\right)_{i} = \left[\frac{x_{it}}{x_{oit}} - \left(\frac{\bar{x}_{it}}{x_{oit}}\right)\right] \beta + \left[\frac{Z_{i}}{x_{oit}} - \left(\frac{\bar{Z}_{i}}{x_{oit}}\right)\right] \delta + \frac{\varepsilon_{it}}{x_{oit}} - \left(\frac{\bar{\varepsilon}_{it}}{x_{oit}}\right)_{i},$$
(23)

which can be written as

$$\tilde{w}_{it} = \tilde{\chi}_{it}^* \beta + \tilde{Z}_{it} \delta \tilde{\varepsilon}_{it}, \tag{24}$$

where

$$\tilde{w}_{it} = \frac{w_{it}}{x_{oit}} - \left(\frac{\bar{w}_{it}}{x_{oit}}\right)_{i}, \qquad \tilde{x}_{it}^* = \frac{x_{it}}{x_{oit}} - \left(\frac{\bar{x}_{it}}{x_{oit}}\right)_{i},$$

$$\widetilde{Z}_{it} = \left(\frac{Z_i}{x_{oit}}\right) - \left(\frac{\overline{Z}_i}{x_{oit}}\right)_{i.}, \quad \widetilde{\varepsilon}_{it} = \frac{\varepsilon_{it}}{x_{oit}} - \left(\frac{\overline{\varepsilon}_{it}}{x_{oit}}\right)_{i.}$$

⁸In this model specify x_{it} with a vector of ones so that the common constant is subsumed in β .

Finally, combining \tilde{x}_{ii}^* and \tilde{Z}_{ii} such that $\tilde{x}_{it} = [\tilde{x}_{it} \tilde{Z}_{it}]$, one can express (24) as

$$\tilde{w}_{it} = \tilde{x}_{it}\beta^* + \varepsilon_{it},\tag{25}$$

where

$$\tilde{x}_{it} = [\tilde{x}_{it}^* Z_{it}]$$
 and $\beta^* = \begin{bmatrix} \beta \\ \delta \end{bmatrix}$.

Because the residuals are heteroscedastic, the appropriate estimator is GLS,

$$\hat{\beta}^* = \left[\sum_i \tilde{X}'_{it} \Omega_i^{-1} \tilde{X}_{it}\right]^{-1} \left[\sum_i \tilde{X}'_{it} \Omega_i^{-1} \tilde{W}_{it}\right], \tag{26}$$

where $\Omega = \text{var}(\tilde{\epsilon}_{it})$ is block-diagonal if ϵ_{it} and x_{oit} are independent, and

$$\Omega_i = (1/x_{oi})\sigma_i^2(1/x_{oi})' + (1/x_{oi})'\sigma_i^2/T(1/x_{oi}),$$

$$\hat{\sigma}_i^2 = \hat{\varepsilon}_i' \hat{\varepsilon}_i / (T - K) = 1 / (T - K) w_i' [1 - x_i (x_i' x_i)^{-1} x_i] w_i.$$

Note that this estimator holds whether or not the panel is balanced or unbalanced. In both cases individual-specific Ω'_i s are computed separately and summed according to (26).

4.2.2. Fixed-effects individual-specific intercepts and slopes

If the panel is sufficiently long, then individual-specific intercepts and slopes can be obtained by running regressions for each person. To illustrate modify eq. (12) to contain an individual-specific intercept and individual-specific as well as common slopes, along with a set of time-invariant variables and their respective coefficients,

$$w_{it} = x_{it}\beta + x_{oit}\beta_{oi} + Z_i\delta + \alpha_i + \varepsilon_{it}, \qquad (27)$$

where all variables are as already defined.

For each individual estimate,

$$w_{it} = a_i + x_{it}\beta_i + x_{ait}\beta_{ai} + \varepsilon_{it}, \tag{28}$$

where a_i is an estimate of $\alpha_i + z_i \delta$ and β_i of β . From (27) subtract $x_{oit} \hat{\beta}_{oi}$ observation by observation for each individual,

$$\hat{w}_{it} = w_{it} - x_{oit}\hat{\beta}_{oi} = x_{it}\hat{\beta} + Z_i\delta + \alpha_i + \varepsilon_{it}. \tag{29}$$

To (29) apply either the first-difference or mean-deviation operator to delete α_i and $z_i\delta$,

$$\hat{w}_{it} - \hat{w}_{i.} = (x_{it} - x_{i.})\beta + (\varepsilon_{it} - \varepsilon_{i.}). \tag{30}$$

Substituting $x_i \hat{\beta}$ from (29) identifies δ ,

$$\hat{w}_{it} - x_{i.}\hat{\beta} = z_i\delta + \eta_{i.}. \tag{31}$$

5. The data

We apply these techniques to the Panel Study of Income Dynamics (PSID), one of the longest most complete data panels, originally begun in 1968, consisting of a cross-section of about 5000 families. Of these we concentrate on a panel of 2659 individuals (both male and female and black and white) whom we could follow and obtain complete data in each year between 1976 and 1987. Two samples were chosen: (1) a sample of 1088 denoted as the 'less intermittent sample' of workers who worked at least part-time in at least ten of the possible twelve years between 1976 and 1987, and (2) a sample including all workers even if there were incomplete wage data. In this case all individuals were included even if they exhibited no wage data for up to ten out of the twelve possible time periods. This 'full sample' group is thus the 'more intermittent' of the two, and affords comparison of the effects of intermittency as related to the length of intermittency. Comparing results from both samples enables one to draw a conclusion concerning the effects of selectivity. This second sample contains all 2659 individuals.

The advantage of the first sample is that GLS estimation is more straightforward, given the same number of time periods for each individual. ¹⁰ The main drawback of the first sample is that it contains only individuals who augment their intermittency by no more than three years during a twelve-year period. Biases from such a select sample can be addressed by comparing results with the less restrictive second sample. Characteristics of both samples are given in table 1. Individuals in the more intermittent 'whole' sample are older, obtain a lower wage, have less education, live in smaller cities, and have more children.

⁹We chose 1976 as a beginning year because that was the first year that contained sufficiently detailed information on *both* husbands and wives.

¹⁰When the number of time periods differ across individuals, as in the second sample, one need revert to a block-diagonal covariance error structure for computation as opposed to a quasi-mean deviation approach. The block-diagonal covariance error structure approach takes an inordinate amount of computer memory, and thus becomes unfeasible even with relatively small samples.

	Less intermi	ttent sample	Entire	sample
Variables	Male	Female	Male	Female
AGE (it)	41.31	41.93	43.16	42.52
$RAC\dot{E}(i)$ (white = 1)	0.79	0.71	0.78	0.70
HRSWK (it) ^a	2015.41	1539.58	2012.26	1409.62
LOGWG (it) ^b	1.33	0.83	1.25	0.71
EXPERIÈNCE (it)°	22.39	16.67	24.37	15.68
HOMETIME (it)d	1.85	7.69	2.22	9.62
# OF KIDS (it) ^e	1.33	1.08	1.28	1.16
AGE OF KID (it)	10.83	12.57	11.38	12.25
SMSA SIZE (it) ^f	3.10	2.89	3.13	2.90
YEARS OF SCHOOLING (ED)	12.99	12.80	12.52	12.40
REGION (i) (NORTH & CENTRAL)	0.45	0.47	0.45	0.46
REGION (i) (SOUTH)	0.39	0.38	0.40	0.39
REGION (i) (WEST)	0.16	0.16	0.15	0.15
# OF INDIVIDUALS	727	361	1543	1116
# OF OBSERVATIONS	7270	3610	13439	8456

Table 1 Variable means by sample and gender; PSID data.

1. SMSA: largest city 500,000 or more 2. SMSA: largest city 100,000–499,999 largest city 50,000–99,000 4. Non-SMSA: largest city 25,000–49,999 5. Non-SMSA: largest city 10,000–24,999 6. Non-SMSA: largest city under 10,000

6. Estimation

Until now the specific variable definitions for time-varying and time-invariant variables have not been precisely specified, but at this time it makes sense to do so. The usual segmented earnings function [Mincer and Polachek (1974)] defines X_{ii} to be composed of experience and hometime. As such let X_{ii} be composed of hometime (defined to be the number of one's potential work years out of the labor force) and experience (the number of years actually worked).¹¹

^a Hours worked per year.

^b Logarithm of average hourly labor income deflated by the Consumer Price Index, using 1967 as the base (1967 = 100).

 $^{^{\}circ}EX_{u}$ is the actual years worked for money since age 18. These were computed as the experience coded for 1976 augmented by hours worked divided by 2000 each subsequent year.

^d HH_{ii} is the number of years not worked since age 18, computed as (AGE - ED - 5 - EX).

^{*} NOKID_{it} is the number of children aged 0-17 in the family unit.

^f SMSA_{it} is a bracketed variable that measures the size of largest city in county of residence:

¹¹Since the PSID data to not contain actual experience for each year, this variable must be computed by augmenting 1974 work experience by the number of hours worked in each succeeding year divided by 2000. As will be indicated later, in order to identify both hometime and experience coefficients this is defined as the sum of the fraction of each year for which individuals do not work full time. Thus, if someone works only 500 hours per year, their hometime is augmented

In addition, we include number of children and SMSA size as time-variant variables. Years of schooling, which is fixed in our data, is considered as a time-invariant earnings determinant. In addition, gender is incorporated as an exogenous and obviously time-invariant variable. Thus $X_{it} = [X_{1it}, X_{2it}, X_{3it}, X_{4it}]$, where $X_1 \equiv$ hometime, $X_2 \equiv$ experience, $X_3 \equiv$ number of children, and $X_4 \equiv$ SMSA size.¹² The time-invariant vector is $Z_1 = [Z_{1i} Z_{2i}]$, where $Z_1 \equiv$ years of schooling and $Z_2 \equiv$ a dummy gender variable. Table 1 contains the sample means.

Our focus is on the latter individual-specific slope models, but to anchor our results to past estimates [Kim and Polachek (1991)] table 2 presents a summary of results for the various models discussed. Reported are the unexplained male-female wage differentials, computed as the coefficient of a dummy gender variable adjusting for other individual wage-related attributes. (Complete results for each model are in the appendix tables.) Begin with the OLS results (row 1) estimated by eq. (1). For the entire sample there is a 41% unexplained wage gap. (Consistent with what one might expect, this unexplained gap is smaller for the more continuously employed, less intermittent sample. (Consistent with across all time periods helps eliminate errors in

by 0.75, that part of the normal work year spent not working. Tenure was not used because the PSID is known to contain many erroneous responses for this variable [Polachek, Wunnava, and Hutchins (1987), Topel (1991)]. We do not use a quadratic functional form because in segmented earnings functions linear approximations yield comparable results [Mincer and Polachek (1974)].

 $^{^{12}} SMSA$ is reported categorically (see table 1 for the definition) . To conserve on degrees of freedom we treat this variable as continuous and to preserve a monotonically rising SMSA size with variable value, multiply the coefficient by (- 1).

¹³No attempt was made to choose the most appropriate set of exogenous regressors to minimize the unexplained wage gap. For example, including an 'expected human capital' variable [Polachek (1975, Goldin and Polachek (1987), Koa, Polachek, and Wunnava (forthcoming)], which yields a very small unexplained wage gap, was not attempted. Instead, our purpose was to adopt a standard approach to see how the unexplained wage gap changes as one adjusts for various manifestations of heterogeneity. Clearly, other biases such as selectivity and endogeneity, to which we have already alluded, may affect the unexplained wage gap estimates. In addition, as indicated, since each bias is essentially related to the correlation of regressors with error terms, these biases may interact with each other. Thus, correcting for selectivity in one particular way may eradicate the other two biases. It is beyond the scope of this paper to measure the extent to which this might occur. Similarly, it is beyond the scope of this paper to nest the models or perform nonnested tests to compare the performance of each.

¹⁴Again this may appear large, since many studies explain a greater proportion of gender wage differences. But, as indicated, our purpose here is not to come up with the 'appropriate' point estimate for the unexplained male-female wage gap, but instead to evaluate how various adjustments for heterogeneity bias affect a representative estimate.

¹⁵Because of greater lifetime work, females in the more continuous group would have relatively more motivation and higher wages. Thus, the smaller unexplained wage gap occurs not because discrimination is smaller (why should firms discriminate less against continuous workers?), but because of more effort and/or motivation.

measurement biases. Estimates of the unexplained wage gap accounting for these are contained in the between-group (BG) results reported in the second row. They are comparable to OLS results. Adjusting for random-effect individual-specific intercepts (the GLS results reported in the third row) yields a smaller unexplained wage gap, as would be expected, ¹⁶ thus illustrating that by controlling for unmeasured differences in work motivation one explains about 20% [(41-34)/41] of the 'unexplained' gap. This makes sense since being shackled with home responsibilities decreases earnings, so that the unexplained wage gap narrows once one accounts for these differences.

Ridding oneself of the assumption that the individual-specific constants need be distributed normally by estimating eq. (11) using β determined by mean-deviation and first-difference fixed-effects estimators (6) and (9), the unexplained wage gap decreases by close to 50% (ISI-MD and ISI-FD rows) to approximately 0.22 from 0.41 (or for the more continuous workers from 0.38 to about 0.22). Again this makes sense because fixed-effect models pose fewer restrictions on person-specific effects than the random-effect model. As such, correcting less restrictively for unobserved motivation yields lower unexplained gender differences.

Assuming a random-coefficients individual-specific intercept and individual-specific experience model [eq. (15) estimated by eq. (16) to (20)]¹⁷ adds little to the GLS individual-specific intercept model in terms of increasing the gender wage gap's explanatory power (RC row).¹⁸ On the other hand, the approach has potentially important implications regarding male and female earnings profiles: when one adjusts for person-specific slopes, one finds the relative steepness of female and male earnings profiles to reverse.¹⁹ More specifically, in contrast to the standard fixed-effects models, once one controls for unobserved motivation,

¹⁶Here the GLS procedure [eq. (3)] was used instead of GLS/IV, as is suggested in the text. See Kim and Polachek (1991) for GLS/IV results. The problem with GLS/IV is that the estimates appear to vary considerably (from -0.23 to 0.08), depending on the choice of instruments. For the whole sample, the number of observations per person differs since each worker participates differently in the labor force regarding number of periods worked, thus yielding an unbalanced sample. For this β_{GLS} was estimated using a quasi-mean-deviation approach:

$$w_{it} - \tau w_{i.} = (x_{it} - \tau x_{i.}) \beta_{GLS} + (1 - \tau) \alpha_i + \varepsilon_{it} + \tau \varepsilon_i,$$

where τ is $1 - \sigma_{\epsilon}/\sigma_{1}$, $\sigma_{1}^{2} = (\sigma_{\epsilon}^{2} T \sigma^{2})$ and $(1 - \tau)\alpha_{i} + \varepsilon_{it} - \tau \varepsilon_{i}$ is a scalar-variance term. Since the number of time periods varies by individual, we approximate τ by the average number of observations per person.

¹⁷Having an unbalanced panel implies that for each person x_i is a $K \times T_i$ matrix. Note that having this individually-dimensioned matrix is of no consequence for this model. The GLS estimator is obtained from eq. (16) and uses $x_i x_i'$ which for all individuals is $K \times K$. Thus, β_{GLS} is easily computed even for an unbalanced panel.

¹⁸However, over 100% of the gender gap is explained implying that women have a wage advantage when the intercept, experience, hometime, number of kids, and city size are all allowed individual-specific coefficients.

¹⁹The coefficients for this are available upon request.

Type of estimation	Less intermittent sample	Entire sample
OLS ^b	- 0.38	- 0.41
BG ^c	-0.39	- 0.42
GLS ^d	-0.32	-0.34
ISI-MD ^e	-0.21	-0.22
ISI-FD ^f	-0.23	- 0.15
RC ^g	-0.33	-0.31
ISS-MD ^h	-0.34	-0.37
ISS-FD ⁱ	-0.32	-0.26
ISIS-MD ^j	- 0.21	- 0.21 ^{NS}
ISIS-FD ^k	-0.27	0.26

Table 2
Unexplained gender differences by model and PSID sample.^a

women have steeper earnings profiles than men. (The observed male experience gradient is 0.0142, while women's experience gradient is 0.0219.)

Adopting the fixed-effects individual-specific slope model estimated by eq. (26) assumes a common intercept but experience slopes that vary across individuals. The model entails estimating a mean-deviation (or differenced) GLS on data normalized by experience. The results are contained in the ISS-MD and ISS-FD rows. As can be seen, the unexplained gender difference is between 26% and 37%, which is smaller than the cross-sectional OLS estimates, but larger in magnitude than the estimates generated by individual-specific intercept models.

In a sense this is surprising since human capital theory suggests that if motivation deficiency manifests itself by a decrease in lifetime work behavior, then one has the incentive to decrease human capital investment which results in a flatter earnings profile. To a certain extent this is borne out in the results because the unexplained wage gap declines from cross-sectional OLS. On the other hand, adjusting solely for individual-specific intercept differences explains a greater proportion of the wage gap. Though several reasons are possible,

^a The gender coefficient of a wage function adjusted by experience, hometime, number of children, SMSA size, and education, estimated by the indicated models chosen on the basis of the assumed error structure.

^bOrdinary least-squares fit of eq. (1).

^c Between-group estimation of eq. (1): $w_{i.} = x_{i.}\beta + z_{i}\delta + \varepsilon_{i}$.

^d Generalized least-squares estimator, eq. (3).

^e Individual-specific intercept mean-deviation model.

¹ Individual-specific intercept first-difference model.

⁸ Random-coefficient model, assuming normally distributed intercept and normally distributed experience coefficient, eqs. (13) to (20).

h Individual-specific slope mean-deviation model assuming individual-specific experience gradients, eqs. (21)–(26).

¹Individual-specific slope first-difference model assuming individual-specific experience gradients, eqs. (21)–(26).

^j Individual-specific slope and intercept mean-deviation model assuming individual-specific intercept and experience gradients, eqs. (27)–(31).

^k Individual-specific slope and intercept first-difference model assuming individual-specific intercept and experience gradients, eqs. (27)–(31).

explaining a greater wage gap portion with an individual-specific intercept model is consistent with motivation altering investment behavior early in life (for example in school – perhaps in the subjects chosen) more so than later in life (such as through on-the-job training) so that accounting for individual-specific experience slopes has less explanatory power than one might expect. Nevertheless, it is not clear that these results are definitive as it is possible to experiment further with functional form and other adjustments.

For example, adjusting for both individual-specific intercepts and individual-specific experience coefficients entails fitting eq. (28) separately for each individual, then re-estimating after subtracting the individual-specific component, and finally applying a first-difference or mean-deviation to obtain coefficients for the time-invariant variables. This process yields coefficients (ISIS-MD and ISIS-FD rows) with magnitudes roughly comparable to the individual-specific intercept models. Again, these results are consistent with motivation altering initial human capital investments that affect earnings levels more so than earnings gradients.

7. Conclusion

Cross-sectional estimates of gender (or any group's) wage differences may be contaminated because each group can differ in not easily measurable ways. Work motivation and hence human capital investment incentives based on lifetime work expectations may be an example of these types of 'unmeasurable' population differences. Not adjusting for such unmeasurable differences yields heterogeneity biases. Standard approaches deal with heterogeneity biases by assuming individual-specific intercepts. For earnings function research individual-specific intercepts need not be the most appropriate solution because unmeasured motivation may cause differences in earnings growth more so than earnings level.

The main point of this paper was to develop individual-specific slope (as opposed to individual-specific intercept) models to account for this phenomena. Estimates from these models are compared to traditional OLS, between-groups, as well as to fixed- and random-effects individual-specific intercept approaches. We found unambiguously that about 50% of unexplained male-female wage gap can be attributed to unmeasured individual differences. These results emerge both from individual-specific intercept and individual-specific slope models, with individual-specific slope models resulting in a slightly smaller unexplained male-female wage gap. At this point this leads us to conclude that most individual-specific differences manifest themselves early in one's work career, possibly even in the type of schooling received. Because of increasing lifetime work expectations among the young, future research should stratify by age to see if these conclusions hold for both young and old cohorts.

Appendix

Table A.1 Coefficient estimates by model for the less intermittent sample (standard errors in parentheses); PSID data; 1976-1987.

	STO	BG	GLS	ISI-MD	ISI-FD	RC	ISS-MD	ISS-FD	ISIS-MD	ISIS-FD
НТ	- 0.0074	- 0.0061 (0.0026)	- 0.0155 (0.0021)	- 0.0290 (0.0035)	- 0.0191 (0.0050)	- 0.0084	0.0070 (0.0241)	0.0103	- 0.0178 (0.0029)	- 0.0296 (0.0051)
EXP	0.0075 (0.0004)	0.0070 (0.0007)	0.0116 (0.0008)	0.0170 (0.0011)	0.0317 (0.0036)	0.0170 (0.0024)	1 !			
KIDS	0.0370 (0.0039)	0.0427	0.0280 (0.0049)	0.0283 (0.0052)	0.0125 (0.0095)	0.0598 (0.0040)	0.0527 (0.0234)	0.0311 (0.0148)	0.0021 (0.0043)	0.0155 (0.0097)
SMSA	0.0394 (0.0027)	0.0463 (0.0073)	0.0182 (0.0037)	0.0114 (0.0042)	0.0023 (0.0064)	0.0446 (0.0029)	0.0235 (0.0177)	0.0333 (0.0182)	0.0024 (0.0035)	0.0048 (0.0065)
ED	0.0954 (0.0010)	0.0970 (0.0026)	0.0859 (0.0018)	0.0772 (0.0004)	0.0509 (0.0006)	0.0914 (0.0020)	0.0996 (0.0216)	0.0823 (0.0147)	0.0744 (0.0026)	0.1018 (0.0005)
GENDER	-0.3831 (0.0124)	-0.3920 (0.0311)	-0.3190 (0.0294)	-0.2138 (0.0087)	-0.2280 (0.0129)	-0.3273 (0.0125)	-0.3388 (0.1019)	-0.3206 (0.0714)	-0.2140 (0.0584)	-0.2735 (0.0108)
OBS	10880	1088	10880	10880	9792	10880	10880	9792	10880	9792
R^2	0.847	0.901	0.477	0.028	0.010	0.298	0.229	0.015	0.004	0.004
F-value	10047.5	1631.5	1652.7	79.2	23.69	924.1	538.7	24.0	12.7	12.4
Root MSE	0.516	0.405	0.337	0.318	0.399	0.527	0.044	0.052	0.270	0.407

41

(0.0056)(0.0039)(0.0079)(0.0004)0.0313 0.1020 0.2625 (0.0087)0.0013 ISIS-FD 62961 0.004 0.0045 (0.0029) (0.0154)(0.3163)ISIS-MD (0.0034)0.0796 - 0.2067 0.0004 21895 0.004 0.314 26.8 Coefficient estimates by model for entire sample (standard errors in parentheses); PSID data; 1976–1987. 0.0197 (0.0054) 0.0421 (0.0039) 0.0728 (0.0053) -0.2601 (0.0228) ISS-FD 62961 0.108 900.0 0.0153 (0.0057) (0.0066)(0.0302)(0.0084)0.3700 0.0829 ISS-MD 0.0181 21895 0.149 574.3 0.0725 (0.0118) 0.0636 (0.0077) 0.0841 (0.0057) 0.3128 (0.0324) 0.0170 (0.0022)21895 0.2800.630 RC (0.0035)0.0267 (0.0008) 0.0066 (0.0054) (0.0005)(0.0096)(0.0074)0.0562 0.0070 -0.0260-0.1467ISI-FD 19236 0.009 0.487 0.0160 (0.0008) 0.0163 (0.0034) 0.0758 (0.0003) 0.2184 (0.0066) (0.0024)(0.0041)0.0199 -0.0253112.79 21895 SI-MD 0.020 0.376 (0.0039)(0.0014)(0.0000)(0.0016)(0.0247)(0.0032)0.3378 0.0197 0.0241 0.0848 1926.8 21895 0.346 0.394 0.0940 (0.0020) -0.0068 (0.0014) 0.0076 (0.0007) 0.0322 (0.0082) (0.0235)(0.0053)0.4233 0.0551 2118.1 2659 0.827 BG (0.0003)0.0291 (0.0031) (0.0021)(0.0008)(0.0100)(0.0007)0.0079 0.4105 0.0453 0.0937 12892.8 21895 0.779 OLS Root MSE GENDER F-value SMSA KIDS EXPOBS HTED

Table A.2

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