Vector Autoregressive Models

Time Series Final | ‘Flavor 2’

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Abstract

This is our abstract, blah blah blah

Introduction

(matt) - Motivated by the goal of predicting a time series via a correlated partner time series, we will investigate Vector Autoregressive models (AKA VAR(p) models).

- Why VAR models exist, what they're better at than AR models. In predictions they don't just include a variable at time t, but take all previous observations into account.

Model

(marko)

The basic structure of a VAR model is quite similar to that of an AR model, however there are two fundamental differences. The first is that there are multiple equations, one for each item in the vector of time series considered. Secondly, rather than have a time point x_t of a given time series being regressed on its own lags, it is also regressed on the lags of the other time series. The order of the model (number of lags) is typically indicated by the p in VAR(p).

There are three main forms of VAR models: reduced form, recursive, and structural. The reduced form defines each variable in the vector of time series as the function of its own lags and the lags of all other variables, with an error term. The error terms from each equation can theoretically be correlated, *but only within the same time period.* Correlation across equations across different time periods would imply autocorrelated errors within a single time series, which is not an assumption of the model.

Recursive VAR models are constructed so that the error terms for a variable are uncorrelated to the prior variables' errors. This is done by adding the preceding variables' current values to the typical VAR equation.

The structural VAR models are distinguished in that they make assumptions of the "causal structure" of the the data allow shocks to be indentified. These shocks would otherwise be incorporated into error terms in recursive and reduced models.

A reduced VAR(1) model with three variables can be modeled as such:

$$\begin{aligned} x_{t,1} &= \alpha_1 + \beta_{1,1}x_{t-1,1} + \beta_{1,2}x_{t-1,2} + \beta_{1,3}x_{t-1,3} + \epsilon_{t,1} \\ x_{t,2} &= \alpha_2 + \beta_{2,1}x_{t-1,1} + \beta_{2,2}x_{t-1,2} + \beta_{2,3}x_{t-1,3} + \epsilon_{t,2} \\ x_{t,1} &= \alpha_3 + \beta_{3,1}x_{t-1,1} + \beta_{3,2}x_{t-1,2} + \beta_{3,3}x_{t-1,3} + \epsilon_{t,3} \end{aligned} \tag{1}$$

We can generalize the previous equations for a VAR(p) model with k variables using matrix algebra:

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \\ \vdots \\ x_{k,t} \end{bmatrix} = \begin{bmatrix} \alpha_1^1 & \alpha_1^2 \\ \alpha_2 & \alpha_2^2 \\ \vdots & \vdots \\ \alpha_k^1 & \alpha_k^2 \end{bmatrix} \begin{bmatrix} 1 \\ t \end{bmatrix} + \begin{bmatrix} \beta_{1,1}^1 & \cdots & \beta_{1,k}^1 \\ \beta_{2,1}^1 & \cdots & \beta_{2,k}^1 \\ \vdots & \ddots & \vdots \\ \beta_{k,1}^1 & \cdots & \beta_{k,k}^1 \end{bmatrix} \begin{bmatrix} x_{t-1,1} \\ x_{t-1,2} \\ \vdots \\ x_{t-1,k} \end{bmatrix} + \cdots + \begin{bmatrix} \beta_{1,1}^p & \cdots & \beta_{1,k}^p \\ \beta_{2,1}^p & \cdots & \beta_{2,k}^p \\ \vdots & \ddots & \vdots \\ \beta_{k,1}^p & \cdots & \beta_{k,k}^p \end{bmatrix} \begin{bmatrix} y_{t-p,1} \\ y_{t-p,2} \\ \vdots \\ y_{t-p,k} \end{bmatrix} + \begin{bmatrix} \epsilon_{t,1} \\ \epsilon_{t,2} \\ \vdots \\ \epsilon_{t,3} \end{bmatrix} \tag{2}$$

or, abstracting the matrices

$$\mathbf{x_t} = \mathbf{Au} + \mathbf{B^1 x_{t-1}} + ... + \mathbf{B^p x_{t-p}} + \epsilon_\mathbf{t} \tag{3}$$

where $\mathbf{x_t}$ is an (k x 1) vector of time series variables, and $\mathbf{x_{t-i}}$ are the vectors of the time series variables at various lags determined by p. $\mathbf{A}$ is the coefficient matrix on the constant and trend term (both optional) represented in $\mathbf{u}$. $\mathbf{B^i x_{t-1}}$ are the (k x k) coefficient matrices; there are p of these matrices dependent on the order of the VAR(p) model. $\epsilon_\mathbf{t}$ is the vector of error terms.

Estimation

(marko) - how its estimed

- "adding variables to the VAR creates complications, because the number of VAR parameters increases as the square of the number of variables: a nine-variable, four-lag VAR has 333 unknown coefficients" https://pubs.aeaweb.org/doi/pdf/10.1257/jep.15.4.101
- interpretation of coeffs

Diagnostics

(matt)

Forecasting

(marko)

Forecasting in VAR is somewhat straightforward and similar to typical AR models. A one-step ahead forecast made at time *t+1* can be expressed as:

$$\mathbf{x_{t+1|t}} = \mathbf{Au} + \mathbf{B^1 x_t} + ... + \mathbf{B^p x_{t-p+1}} \tag{4}$$

Predicting more than one step ahead is done recursively in what can be referred to as the chain-rule of forecasting:

$$\mathbf{x_{t+1|t}} = \mathbf{Au} + \mathbf{B^1 x_t} + ... + \mathbf{B^p x_{t-p+1}} \tag{5}$$

Data Example

(matt)

Discussion

References

Pfaff, Bernhard. VAR, SVAR and SVEC Models: Implementation Within R Package vars. Journal of Statistical Software 27(4), 2008. https://cran.r-project.org/web/packages/vars/vignettes/vars.pdf.

`PennState Eberly College of Science. Vector autoregressive models VAR(p) models.`

https://online.stat.psu.edu/stat510/lesson/11/11.2.

Shumway, Robert H., and David S. Stoffer. Time Series Analysis and Its Applications. Springer Texts in Statistics, Springer International Publishing, 2017, pp. 273–279. https://link.springer.com/content/pdf/10.1007%2F978-3-319-52452-8.pdf.

Code Appendix

```r
# reading in data

btc <- read_csv("BTC-USD.csv")
```

```
##
## -- Column specification --------------------------------------------------------
## cols(
##   Date = col_date(format = ""),
##   Open = col_double(),
##   High = col_double(),
##   Low = col_double(),
##   Close = col_double(),
##   `Adj Close` = col_double(),
##   Volume = col_double()
## )

## Warning: 24 parsing failures.
##  row       col expected actual          file
## 1423 Open      a double   null 'BTC-USD.csv'
## 1423 High      a double   null 'BTC-USD.csv'
## 1423 Low       a double   null 'BTC-USD.csv'
## 1423 Close     a double   null 'BTC-USD.csv'
## 1423 Adj Close a double   null 'BTC-USD.csv'
## .... ......... ........ ...... .............
## See problems(...) for more details.
```

```r
eth <- read_csv("ETH-USD.csv")
```

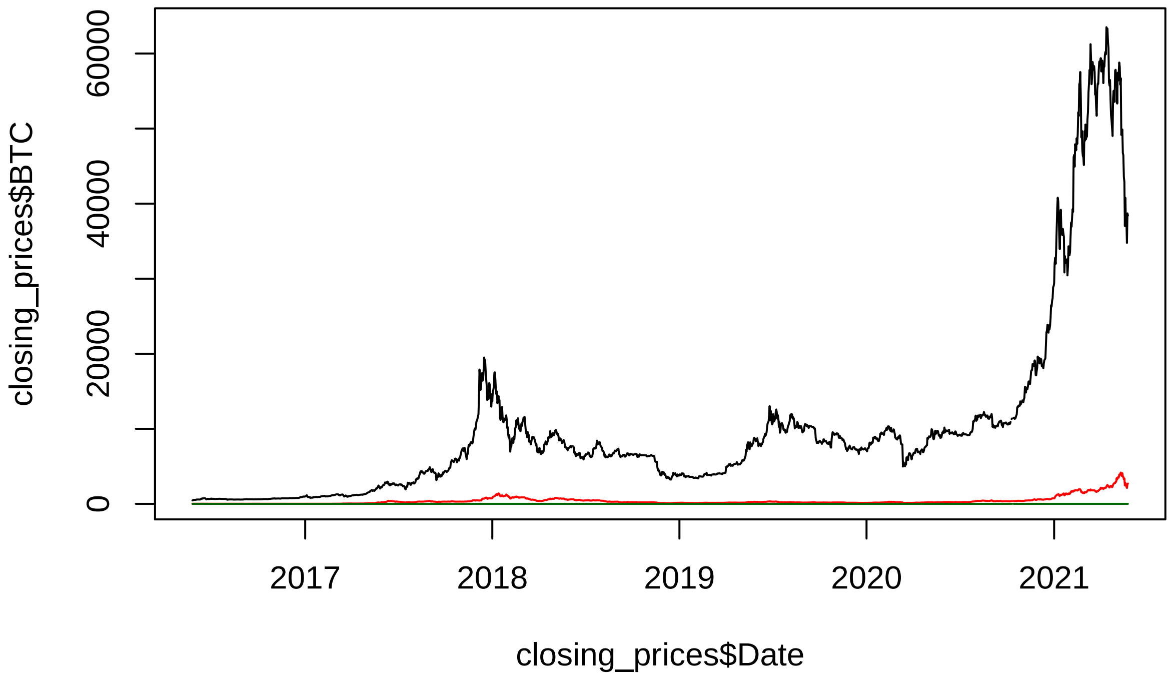
```
##
## -- Column specification --------------------------------------------------------
## cols(
##   Date = col_date(format = ""),
##   Open = col_double(),
##   High = col_double(),
##   Low = col_double(),
##   Close = col_double(),
##   `Adj Close` = col_double(),
##   Volume = col_double()
## )

## Warning: 24 parsing failures.
##  row       col expected actual          file
## 1423 Open      a double   null 'ETH-USD.csv'
## 1423 High      a double   null 'ETH-USD.csv'
## 1423 Low       a double   null 'ETH-USD.csv'
## 1423 Close     a double   null 'ETH-USD.csv'
## 1423 Adj Close a double   null 'ETH-USD.csv'
## .... ......... ........ ...... .............
## See problems(...) for more details.
```

```r
doge <- read_csv("DOGE-USD.csv")
```

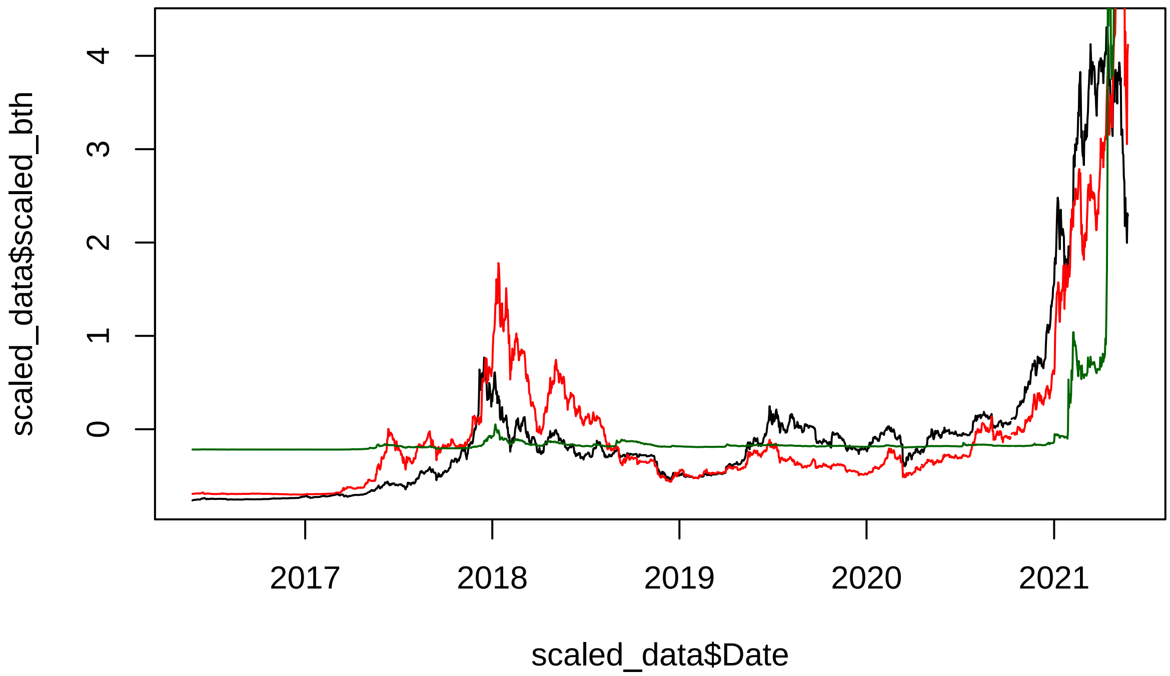
```
##
## -- Column specification --------------------------------------------------------
## cols(
##   Date = col_date(format = ""),
##   Open = col_double(),
##   High = col_double(),
##   Low = col_double(),
##   Close = col_double(),
##   `Adj Close` = col_double(),
##   Volume = col_double()
## )

## Warning: 24 parsing failures.
##  row       col expected actual           file
## 1423 Open      a double   null 'DOGE-USD.csv'
## 1423 High      a double   null 'DOGE-USD.csv'
## 1423 Low       a double   null 'DOGE-USD.csv'
## 1423 Close     a double   null 'DOGE-USD.csv'
## 1423 Adj Close a double   null 'DOGE-USD.csv'
## .... ......... ........ ...... ..............
## See problems(...) for more details.
```

```r
eth <- eth[-1827, ] # matching length
```

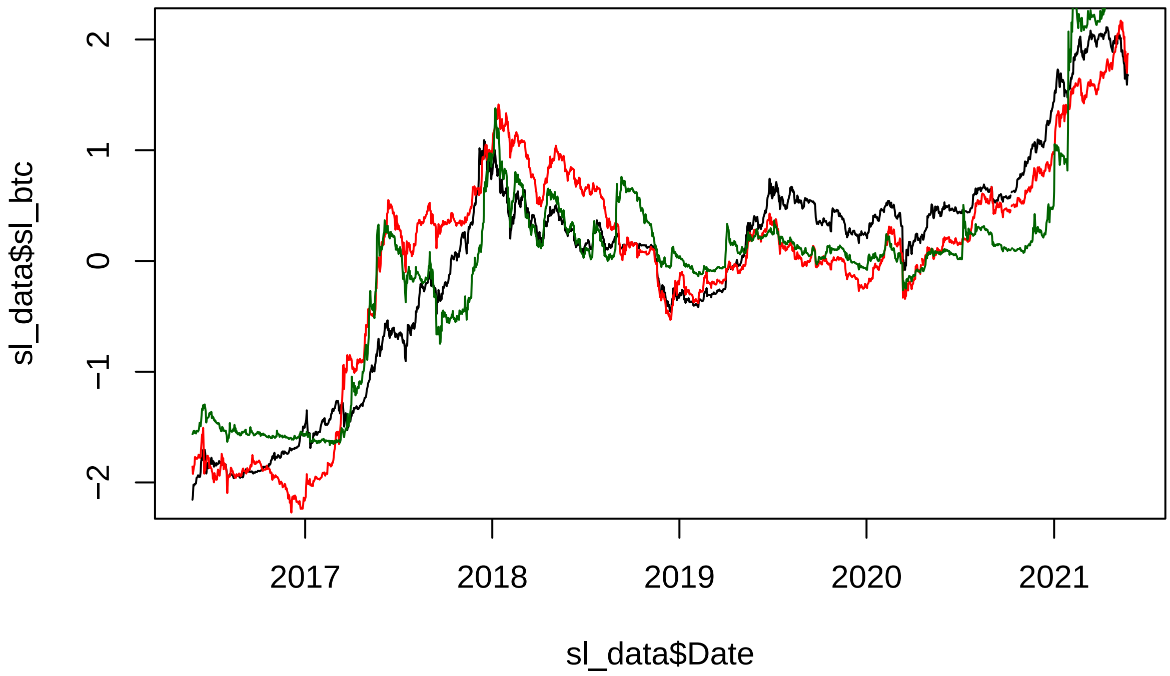
```r
# creating closing price dataframe

closing_prices <- as.data.frame(cbind(btc$Close, eth$Close, doge$Close))

closing_prices$Date <- btc$Date

colnames(closing_prices) <- c("BTC", "ETH", "DOGE", "Date")

plot(closing_prices$Date, closing_prices$BTC, type = "l")
lines(closing_prices$Date, closing_prices$ETH, type = "l", col = "red")
lines(closing_prices$Date, closing_prices$DOGE, type = "l", col = "dark green")
```



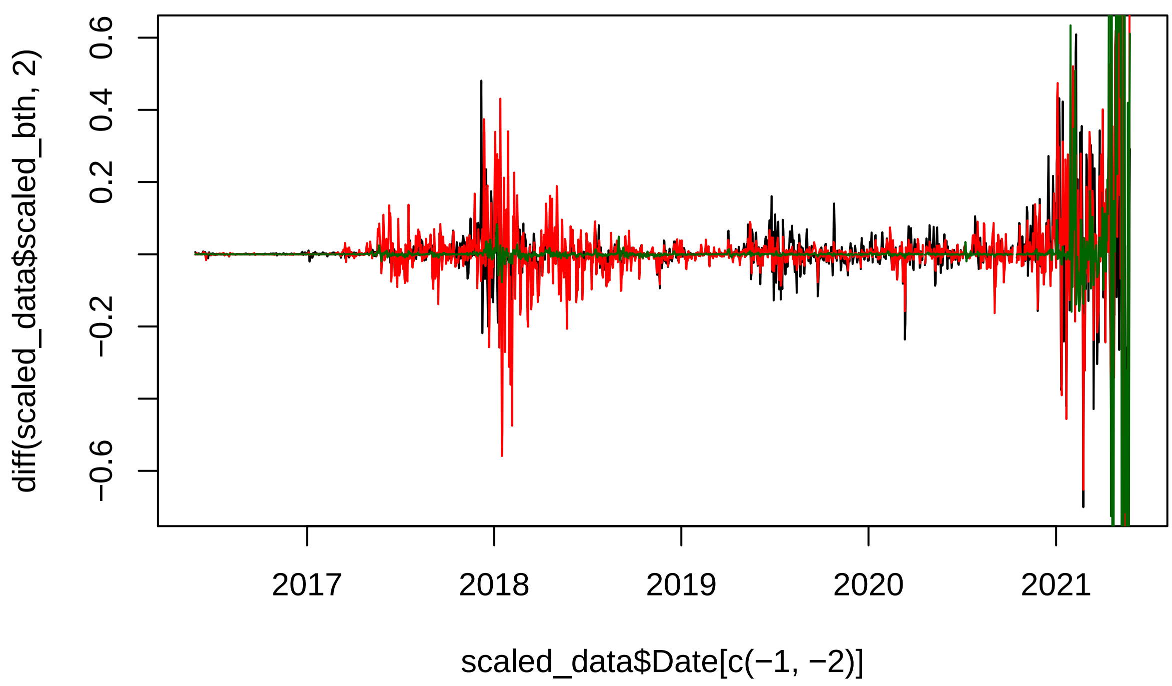

```r
# creating scaled data dataframe

btc_scale <- scale(closing_prices$BTC)
doge_scale <- scale(closing_prices$DOGE)
eth_scale <- scale(closing_prices$ETH)

scaled_data <- data.frame(
  Date = closing_prices$Date,
  scaled_bth = btc_scale[, 1],
  scaled_eth = eth_scale[, 1],
  scaled_doge = doge_scale[, 1]
)

plot(scaled_data$Date, scaled_data$scaled_bth, type = "l")
lines(scaled_data$Date, scaled_data$scaled_eth, type = "l", col = "red")
lines(scaled_data$Date, scaled_data$scaled_doge, type = "l", col = "dark green")
```




```
# creating scaled log dataframe
# resolving NAs
# na2lag1(c(closing_prices$BTC,closing_prices$DOGE,closing_prices$ETH))

Date <- closing_prices$Date
sl_btc <- scale(log((closing_prices$BTC)))[, 1]
sl_doge <- scale(log(as.numeric(closing_prices$DOGE)))[, 1]
sl_eth <- scale(log(as.numeric(closing_prices$ETH)))[, 1]
sl_data <- data.frame(Date, sl_btc, sl_doge, sl_eth)

plot(sl_data$Date, sl_data$sl_btc, type = "l")
lines(sl_data$Date, sl_data$sl_eth, type = "l", col = "red")
lines(sl_data$Date, sl_data$sl_doge, type = "l", col = "dark green")
```



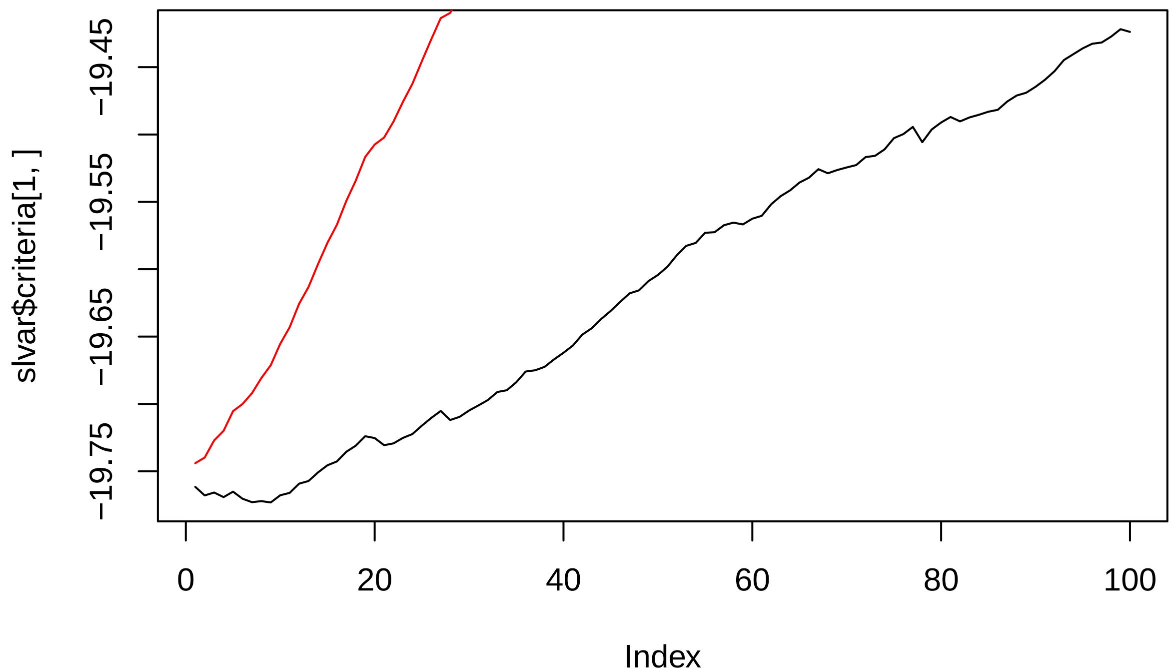

```r
# differenced scaled (unlogged) data plot
plot(scaled_data$Date[c(-1, -2)], diff(scaled_data$scaled_bth, 2), type = "l")
lines(scaled_data$Date[c(-1, -2)], diff(scaled_data$scaled_eth, 2), type = "l", col = "red")
lines(scaled_data$Date[c(-1, -2)], diff(scaled_data$scaled_doge, 2), type = "l", col = "dark green")
```




```r
# varselect for scaled & logged data

slvar <- VARselect(na.omit(sl_data[,-1]), lag.max = 100, type = "both")
slvar$selection
```

```
## AIC(n)  HQ(n)  SC(n) FPE(n)
##      9      1      1      9
```

```
plot(slvar$criteria[1,], type = 'l')
lines(slvar$criteria[2,], type = 'l', col = "red")
```



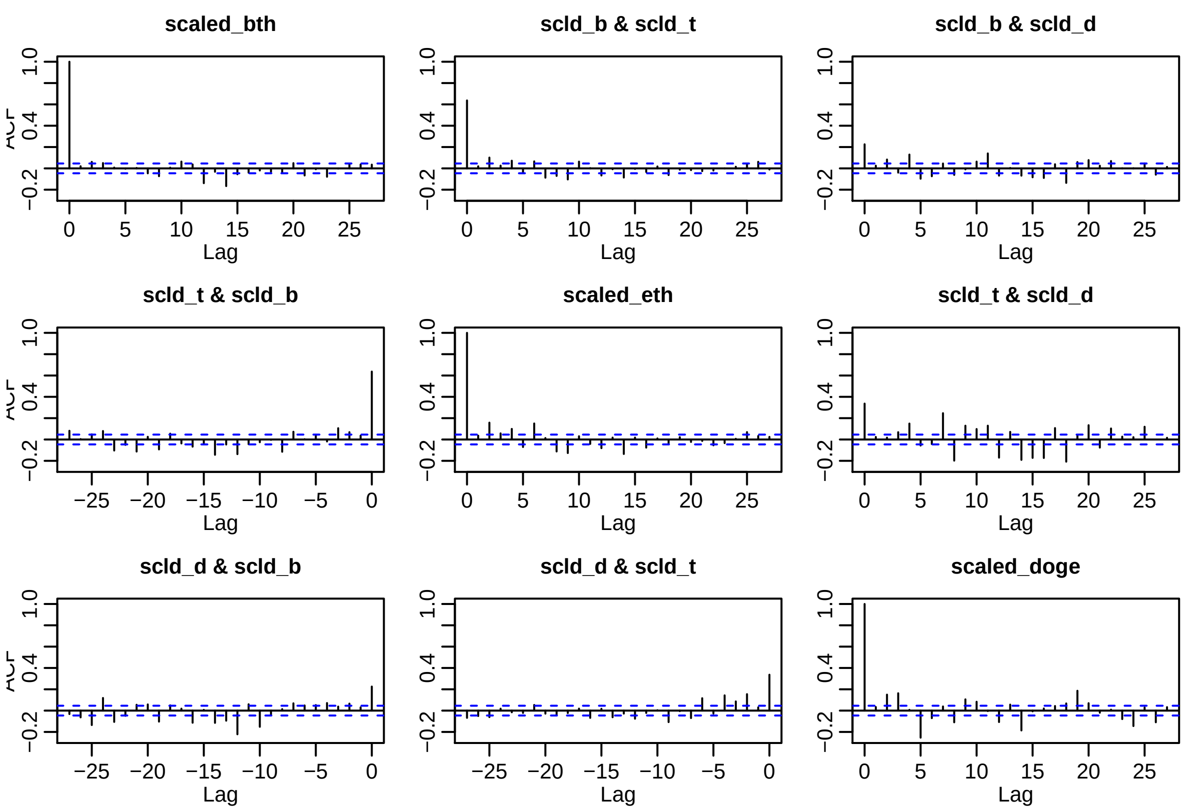

```
# prepping data for vars package
scaled_data_clean <- scaled_data[, -1] %>% na.omit()
og_data_clean <- na.omit(closing_prices)
og_data_clean <- og_data_clean[, 4]

# fitting var model
fitvar <- VAR(scaled_data_clean, p = 2, type = "both")

summary(fitvar)
```

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: scaled_bth, scaled_eth, scaled_doge
## Deterministic variables: both
## Sample size: 1820
## Log Likelihood: 5919.566
## Roots of the characteristic polynomial:
## 0.9964 0.9964 0.9671 0.1379 0.1379 0.05064
## Call:
## VAR(y = scaled_data_clean, p = 2, type = "both")
##
##
## Estimation results for equation scaled_bth:
## ===========================================
```

```
## scaled_bth = scaled_bth.l1 + scaled_eth.l1 + scaled_doge.l1 + scaled_bth.l2 + scaled_eth.l2 + scaled_
##
##                  Estimate Std. Error t value Pr(>|t|)
## scaled_bth.l1   1.082e+00  3.032e-02  35.675  < 2e-16 ***
## scaled_eth.l1  -1.757e-01  2.068e-02  -8.499  < 2e-16 ***
## scaled_doge.l1 -2.315e-03  8.160e-03  -0.284   0.7767
## scaled_bth.l2  -6.694e-02  3.051e-02  -2.194   0.0283 *
## scaled_eth.l2   1.657e-01  2.055e-02   8.065 1.32e-15 ***
## scaled_doge.l2 -5.012e-03  8.246e-03  -0.608   0.5434
## const           3.502e-03  3.450e-03   1.015   0.3102
## trend          -1.704e-06  3.503e-06  -0.486   0.6267
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.05506 on 1812 degrees of freedom
## Multiple R-Squared: 0.997,   Adjusted R-squared: 0.997
## F-statistic: 8.549e+04 on 7 and 1812 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation scaled_eth:
## ============================================
## scaled_eth = scaled_bth.l1 + scaled_eth.l1 + scaled_doge.l1 + scaled_bth.l2 + scaled_eth.l2 + scaled_
##
##                  Estimate Std. Error t value Pr(>|t|)
## scaled_bth.l1   1.907e-01  4.609e-02   4.137 3.68e-05 ***
## scaled_eth.l1   7.024e-01  3.143e-02  22.351  < 2e-16 ***
## scaled_doge.l1  1.454e-02  1.240e-02   1.172  0.24126
## scaled_bth.l2  -1.512e-01  4.637e-02  -3.261  0.00113 **
## scaled_eth.l2   2.617e-01  3.123e-02   8.379  < 2e-16 ***
## scaled_doge.l2 -5.748e-03  1.253e-02  -0.459  0.64654
## const           1.633e-02  5.244e-03   3.114  0.00188 **
## trend          -1.457e-05  5.324e-06  -2.736  0.00628 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.08369 on 1812 degrees of freedom
## Multiple R-Squared: 0.993,   Adjusted R-squared: 0.993
## F-statistic: 3.686e+04 on 7 and 1812 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation scaled_doge:
## =============================================
## scaled_doge = scaled_bth.l1 + scaled_eth.l1 + scaled_doge.l1 + scaled_bth.l2 + scaled_eth.l2 + scalec
##
##                  Estimate Std. Error t value Pr(>|t|)
## scaled_bth.l1  -2.497e-01  9.083e-02  -2.749 0.006038 **
## scaled_eth.l1  -2.083e-01  6.193e-02  -3.363 0.000786 ***
## scaled_doge.l1  8.762e-01  2.444e-02  35.852  < 2e-16 ***
## scaled_bth.l2   2.872e-01  9.137e-02   3.143 0.001699 **
## scaled_eth.l2   2.088e-01  6.154e-02   3.392 0.000709 ***
## scaled_doge.l2  9.866e-02  2.470e-02   3.995 6.74e-05 ***
## const           2.294e-02  1.033e-02   2.220 0.026519 *
```

```
## trend          -2.038e-05  1.049e-05  -1.943 0.052188 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.1649 on 1812 degrees of freedom
## Multiple R-Squared: 0.9729,  Adjusted R-squared: 0.9728
## F-statistic:  9305 on 7 and 1812 DF,  p-value: < 2.2e-16
##
##
##
## Covariance matrix of residuals:
##             scaled_bth scaled_eth scaled_doge
## scaled_bth    0.003032   0.002936    0.002050
## scaled_eth    0.002936   0.007004    0.004646
## scaled_doge   0.002050   0.004646    0.027199
##
## Correlation matrix of residuals:
##             scaled_bth scaled_eth scaled_doge
## scaled_bth      1.0000     0.6370      0.2257
## scaled_eth      0.6370     1.0000      0.3366
## scaled_doge     0.2257     0.3366      1.0000
```

```r
acf(residuals(fitvar))
```

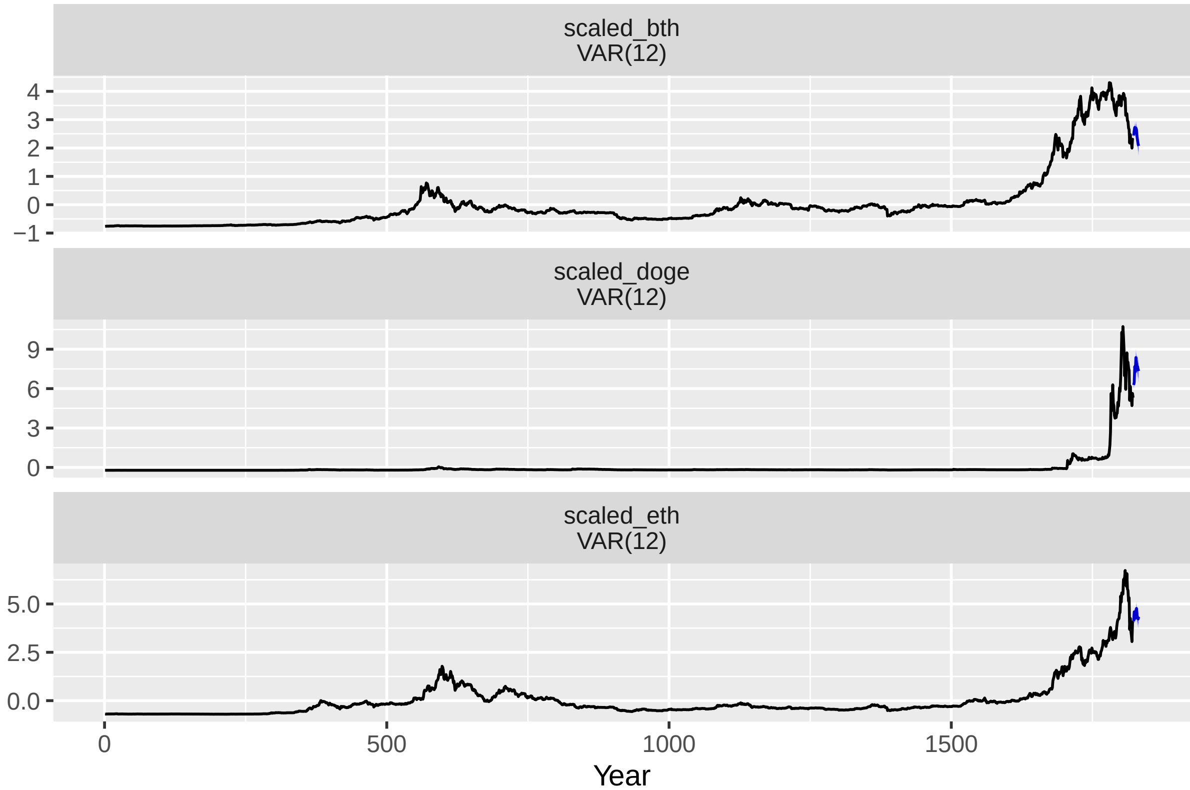



```r
# UNSURE; TO BE LABELED
ts_obj <- as.ts(scaled_data_clean)
```

```r
var12 <- VAR(ts_obj, p = 12, type = "const")
serial.test(var12, lags.pt = 12, type = "PT.asymptotic")
```

```
##
##  Portmanteau Test (asymptotic)
##
## data:  Residuals of VAR object var12
## Chi-squared = 244.24, df = 0, p-value < 2.2e-16
```

```r
# forecasts

forecast(var12) %>%
  autoplot() + xlab("Year")
```




```r
# should we do an ADF test??

# granger test for causality COMMENTED OUT RN

grangertest(sl_btc ~ sl_doge, data = sl_data, order = 2)
```

```
## Granger causality test
##
## Model 1: sl_btc ~ Lags(sl_btc, 1:2) + Lags(sl_doge, 1:2)
## Model 2: sl_btc ~ Lags(sl_btc, 1:2)
##   Res.Df Df      F Pr(>F)
## 1   1815
## 2   1817 -2 1.8576 0.1563
```

```r
#grangertest(BTC ~ ETH, data = og_data_clean)

  #grangertest(DOGE ~ BTC, order = 50, data = og_data_clean)
```

```r
# removing NAs
sl_data$sl_btc<-na2lag1(sl_data$sl_btc)
sl_data$sl_doge<-na2lag1(sl_data$sl_doge)
sl_data$sl_eth<-na2lag1(sl_data$sl_eth)


# using results of VARselect to fit two models than compare them
fitvar_p9 <- VAR(sl_data[,-1], p = 9, type = "both")
fitvar_p1 <- VAR(sl_data[,-1], p = 1, type = "both")

summary(fitvar_p9)
```

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: sl_btc, sl_doge, sl_eth
## Deterministic variables: both
## Sample size: 1817
## Log Likelihood: 10290.539
## Roots of the characteristic polynomial:
## 0.997 0.994 0.994 0.7428 0.7428 0.738 0.738 0.7244 0.7244 0.7173 0.7173 0.7138 0.7138 0.7036 0.7036 (
## Call:
## VAR(y = sl_data[, -1], p = 9, type = "both")
##
##
## Estimation results for equation sl_btc:
## =======================================
## sl_btc = sl_btc.l1 + sl_doge.l1 + sl_eth.l1 + sl_btc.l2 + sl_doge.l2 + sl_eth.l2 + sl_btc.l3 + sl_dog
##
##              Estimate Std. Error t value Pr(>|t|)
## sl_btc.l1   1.027e+00  3.213e-02  31.944  < 2e-16 ***
## sl_doge.l1  1.938e-02  1.766e-02   1.097 0.272668
## sl_eth.l1  -1.024e-01  2.808e-02  -3.648 0.000272 ***
## sl_btc.l2  -4.302e-02  4.693e-02  -0.917 0.359383
## sl_doge.l2  1.428e-02  2.513e-02   0.568 0.569856
## sl_eth.l2   1.400e-01  4.084e-02   3.428 0.000621 ***
## sl_btc.l3   2.967e-02  4.701e-02   0.631 0.528062
## sl_doge.l3 -3.598e-02  2.509e-02  -1.434 0.151776
## sl_eth.l3  -4.533e-02  4.095e-02  -1.107 0.268475
## sl_btc.l4  -7.042e-02  4.703e-02  -1.497 0.134513
## sl_doge.l4  1.452e-02  2.498e-02   0.581 0.561149
## sl_eth.l4   8.543e-02  4.085e-02   2.091 0.036627 *
## sl_btc.l5   9.714e-02  4.699e-02   2.067 0.038878 *
## sl_doge.l5  2.108e-03  2.503e-02   0.084 0.932877
## sl_eth.l5  -1.079e-01  4.087e-02  -2.640 0.008374 **
## sl_btc.l6  -1.438e-02  4.695e-02  -0.306 0.759389
## sl_doge.l6 -1.789e-02  2.496e-02  -0.717 0.473616
## sl_eth.l6   4.203e-02  4.092e-02   1.027 0.304575
## sl_btc.l7  -3.603e-02  4.685e-02  -0.769 0.441951
```

```
## sl_doge.l7 -1.629e-03  2.518e-02  -0.065 0.948421
## sl_eth.l7  -4.891e-02  4.101e-02  -1.192 0.233239
## sl_btc.l8   4.882e-02  4.672e-02   1.045 0.296242
## sl_doge.l8  2.609e-03  2.529e-02   0.103 0.917825
## sl_eth.l8  -3.192e-02  4.105e-02  -0.778 0.436846
## sl_btc.l9  -4.384e-02  3.190e-02  -1.374 0.169515
## sl_doge.l9 -2.041e-03  1.787e-02  -0.114 0.909087
## sl_eth.l9   7.474e-02  2.824e-02   2.646 0.008213 **
## const      -4.756e-03  3.680e-03  -1.292 0.196422
## trend       7.232e-06  3.899e-06   1.855 0.063766 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.03533 on 1788 degrees of freedom
## Multiple R-Squared: 0.9987,  Adjusted R-squared: 0.9987
## F-statistic: 5.098e+04 on 28 and 1788 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation sl_doge:
## ==========================================
## sl_doge = sl_btc.l1 + sl_doge.l1 + sl_eth.l1 + sl_btc.l2 + sl_doge.l2 + sl_eth.l2 + sl_btc.l3 + sl_do
##
##             Estimate Std. Error t value Pr(>|t|)
## sl_btc.l1  -5.736e-02  4.889e-02  -1.173 0.240917
## sl_doge.l1  1.034e+00  2.687e-02  38.485  < 2e-16 ***
## sl_eth.l1  -2.622e-02  4.273e-02  -0.614 0.539500
## sl_btc.l2   5.784e-02  7.140e-02   0.810 0.418065
## sl_doge.l2  5.321e-03  3.824e-02   0.139 0.889338
## sl_eth.l2   5.424e-02  6.213e-02   0.873 0.382799
## sl_btc.l3  -2.513e-02  7.153e-02  -0.351 0.725374
## sl_doge.l3  2.994e-02  3.818e-02   0.784 0.433079
## sl_eth.l3   7.165e-03  6.231e-02   0.115 0.908476
## sl_btc.l4  -1.586e-02  7.157e-02  -0.222 0.824687
## sl_doge.l4 -1.036e-01  3.800e-02  -2.726 0.006479 **
## sl_eth.l4   5.564e-02  6.215e-02   0.895 0.370815
## sl_btc.l5   1.281e-01  7.150e-02   1.792 0.073333 .
## sl_doge.l5 -3.818e-02  3.808e-02  -1.002 0.316240
## sl_eth.l5  -4.973e-02  6.218e-02  -0.800 0.424021
## sl_btc.l6  -1.429e-01  7.144e-02  -2.000 0.045635 *
## sl_doge.l6  1.406e-01  3.798e-02   3.701 0.000221 ***
## sl_eth.l6   1.438e-02  6.227e-02   0.231 0.817429
## sl_btc.l7   1.337e-01  7.129e-02   1.876 0.060880 .
## sl_doge.l7 -7.247e-02  3.832e-02  -1.891 0.058769 .
## sl_eth.l7  -1.490e-01  6.240e-02  -2.388 0.017044 *
## sl_btc.l8  -1.425e-01  7.109e-02  -2.005 0.045165 *
## sl_doge.l8  3.073e-03  3.847e-02   0.080 0.936351
## sl_eth.l8   6.342e-02  6.245e-02   1.016 0.309990
## sl_btc.l9   7.253e-02  4.854e-02   1.494 0.135304
## sl_doge.l9 -3.976e-03  2.719e-02  -0.146 0.883764
## sl_eth.l9   2.890e-02  4.297e-02   0.672 0.501354
## const       4.376e-03  5.600e-03   0.781 0.434613
## trend      -2.155e-06  5.932e-06  -0.363 0.716427
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.05376 on 1788 degrees of freedom
## Multiple R-Squared: 0.9971,  Adjusted R-squared: 0.9971
## F-statistic: 2.217e+04 on 28 and 1788 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation sl_eth:
## =======================================
## sl_eth = sl_btc.l1 + sl_doge.l1 + sl_eth.l1 + sl_btc.l2 + sl_doge.l2 + sl_eth.l2 + sl_btc.l3 + sl_dog
##
##              Estimate Std. Error t value Pr(>|t|)
## sl_btc.l1  -5.852e-02  3.604e-02  -1.624   0.1046
## sl_doge.l1  3.849e-02  1.981e-02   1.943   0.0521 .
## sl_eth.l1   9.844e-01  3.149e-02  31.258   <2e-16 ***
## sl_btc.l2   9.314e-02  5.263e-02   1.770   0.0769 .
## sl_doge.l2 -2.771e-02  2.818e-02  -0.983   0.3256
## sl_eth.l2   3.852e-02  4.580e-02   0.841   0.4005
## sl_btc.l3  -5.005e-02  5.273e-02  -0.949   0.3426
## sl_doge.l3 -1.512e-02  2.814e-02  -0.537   0.5911
## sl_eth.l3   3.707e-04  4.593e-02   0.008   0.9936
## sl_btc.l4   2.991e-02  5.275e-02   0.567   0.5708
## sl_doge.l4  1.082e-02  2.801e-02   0.386   0.6993
## sl_eth.l4  -1.456e-02  4.581e-02  -0.318   0.7507
## sl_btc.l5   2.373e-02  5.270e-02   0.450   0.6525
## sl_doge.l5  2.517e-02  2.807e-02   0.897   0.3700
## sl_eth.l5  -2.930e-02  4.584e-02  -0.639   0.5227
## sl_btc.l6  -3.450e-02  5.266e-02  -0.655   0.5125
## sl_doge.l6 -4.787e-02  2.800e-02  -1.710   0.0875 .
## sl_eth.l6   7.411e-02  4.590e-02   1.615   0.1065
## sl_btc.l7  -2.740e-02  5.255e-02  -0.521   0.6021
## sl_doge.l7  5.345e-02  2.824e-02   1.892   0.0586 .
## sl_eth.l7  -7.923e-02  4.600e-02  -1.722   0.0852 .
## sl_btc.l8   5.884e-02  5.240e-02   1.123   0.2616
## sl_doge.l8 -5.500e-02  2.836e-02  -1.939   0.0526 .
## sl_eth.l8  -4.759e-03  4.603e-02  -0.103   0.9177
## sl_btc.l9  -3.139e-02  3.578e-02  -0.877   0.3803
## sl_doge.l9  1.679e-02  2.004e-02   0.838   0.4023
## sl_eth.l9   2.711e-02  3.168e-02   0.856   0.3922
## const       2.280e-03  4.127e-03   0.552   0.5807
## trend      -6.635e-07  4.372e-06  -0.152   0.8794
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.03963 on 1788 degrees of freedom
## Multiple R-Squared: 0.9984,  Adjusted R-squared: 0.9984
## F-statistic: 4.068e+04 on 28 and 1788 DF,  p-value: < 2.2e-16
##
##
##
## Covariance matrix of residuals:
##            sl_btc   sl_doge    sl_eth
```

```
## sl_btc  0.0012485 0.0008596 0.0009070
## sl_doge 0.0008596 0.0028906 0.0008759
## sl_eth  0.0009070 0.0008759 0.0015704
##
## Correlation matrix of residuals:
##         sl_btc sl_doge sl_eth
## sl_btc  1.0000  0.4525 0.6478
## sl_doge 0.4525  1.0000 0.4111
## sl_eth  0.6478  0.4111 1.0000
```

```
summary(fitvar_p1)
```

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: sl_btc, sl_doge, sl_eth
## Deterministic variables: both
## Sample size: 1825
## Log Likelihood: 10254.74
## Roots of the characteristic polynomial:
## 0.9969 0.9955 0.9955
## Call:
## VAR(y = sl_data[, -1], p = 1, type = "both")
##
##
## Estimation results for equation sl_btc:
## =======================================
## sl_btc = sl_btc.l1 + sl_doge.l1 + sl_eth.l1 + const + trend
##
##              Estimate Std. Error t value Pr(>|t|)
## sl_btc.l1   9.960e-01  4.069e-03 244.781   <2e-16 ***
## sl_doge.l1 -3.777e-03  2.103e-03  -1.796   0.0727 .
## sl_eth.l1   3.967e-03  2.995e-03   1.324   0.1855
## const      -3.149e-03  3.626e-03  -0.869   0.3852
## trend       5.738e-06  3.861e-06   1.486   0.1375
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.03564 on 1820 degrees of freedom
## Multiple R-Squared: 0.9987,  Adjusted R-squared: 0.9987
## F-statistic: 3.573e+05 on 4 and 1820 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation sl_doge:
## ========================================
## sl_doge = sl_btc.l1 + sl_doge.l1 + sl_eth.l1 + const + trend
##
##              Estimate Std. Error t value Pr(>|t|)
## sl_btc.l1   8.445e-03  6.181e-03   1.366    0.172
## sl_doge.l1  9.963e-01  3.194e-03 311.899   <2e-16 ***
## sl_eth.l1  -1.971e-03  4.551e-03  -0.433    0.665
## const       5.322e-03  5.508e-03   0.966    0.334
## trend      -2.837e-06  5.866e-06  -0.484    0.629
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.05414 on 1820 degrees of freedom
## Multiple R-Squared: 0.9971,  Adjusted R-squared: 0.9971
## F-statistic: 1.547e+05 on 4 and 1820 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation sl_eth:
## =======================================
## sl_eth = sl_btc.l1 + sl_doge.l1 + sl_eth.l1 + const + trend
##
##              Estimate Std. Error t value Pr(>|t|)
## sl_btc.l1   4.834e-03  4.536e-03   1.066    0.287
## sl_doge.l1 -1.001e-05  2.344e-03  -0.004    0.997
## sl_eth.l1   9.955e-01  3.340e-03 298.076   <2e-16 ***
## const       3.875e-03  4.042e-03   0.959    0.338
## trend      -2.007e-06  4.305e-06  -0.466    0.641
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.03973 on 1820 degrees of freedom
## Multiple R-Squared: 0.9984,  Adjusted R-squared: 0.9984
## F-statistic: 2.875e+05 on 4 and 1820 DF,  p-value: < 2.2e-16
##
##
##
## Covariance matrix of residuals:
##            sl_btc   sl_doge    sl_eth
## sl_btc  0.0012699 0.0008696 0.0009107
## sl_doge 0.0008696 0.0029308 0.0008757
## sl_eth  0.0009107 0.0008757 0.0015786
##
## Correlation matrix of residuals:
##         sl_btc sl_doge sl_eth
## sl_btc  1.0000  0.4508 0.6432
## sl_doge 0.4508  1.0000 0.4071
## sl_eth  0.6432  0.4071 1.0000
```

```r
# diagnostic tests

# Asymptotic Portmanteau test

# we fail to find evidence of autocorrelation in the residuals of the VAR(9) model at the default of 16
serial.test(fitvar_p9, lags.pt = 50)
```

```
##
##  Portmanteau Test (asymptotic)
##
## data:  Residuals of VAR object fitvar_p9
## Chi-squared = 387.05, df = 369, p-value = 0.2487
```

```r
# we find evidence of autocorrelation in the residuals of the VAR(1) model
serial.test(fitvar_p1, lags.pt = 40)
```

```
##
##  Portmanteau Test (asymptotic)
##
## data:  Residuals of VAR object fitvar_p1
## Chi-squared = 503.99, df = 351, p-value = 1.494e-07
```

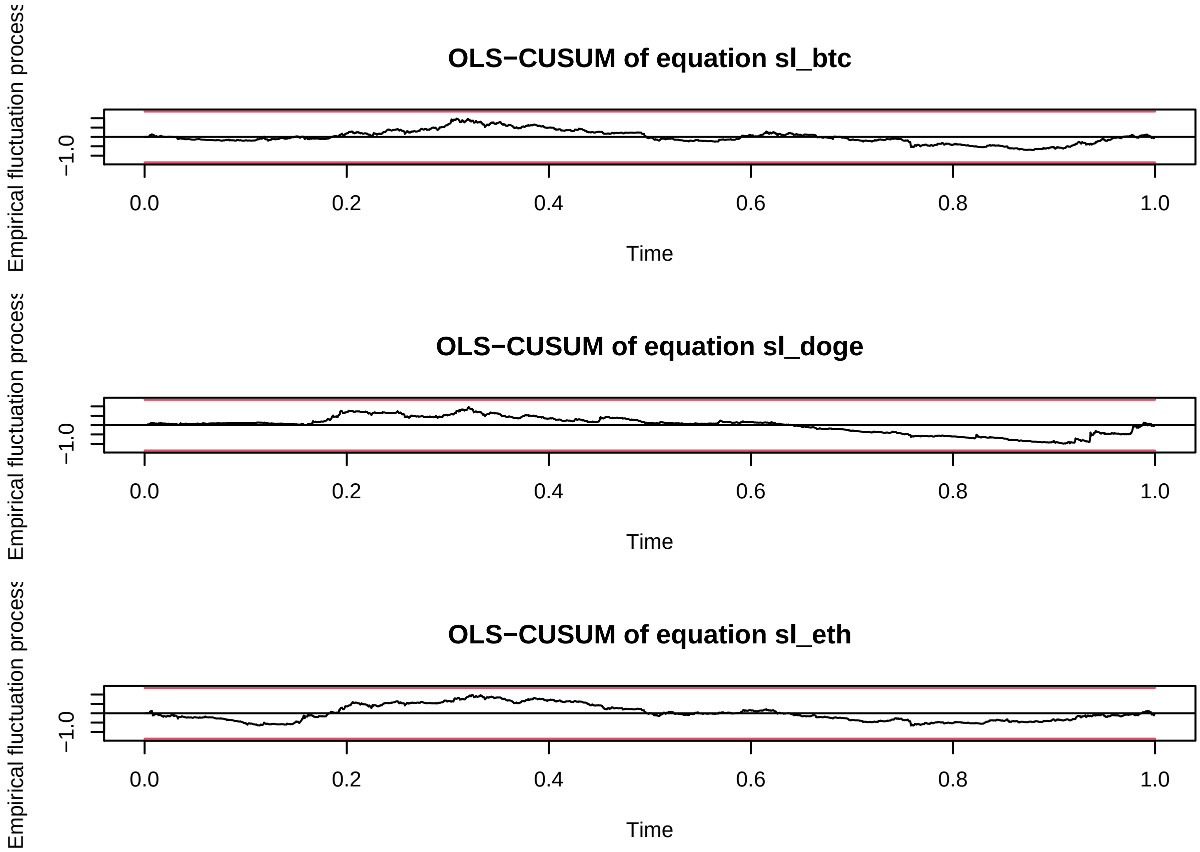
```r
# Breusch-Godfrey test

# same reversial present for the BG test...
serial.test(fitvar_p9,lags.bg = 40, type = 'BG')
```

```
##
##  Breusch-Godfrey LM test
##
## data:  Residuals of VAR object fitvar_p9
## Chi-squared = 529.28, df = 360, p-value = 1.434e-08
```

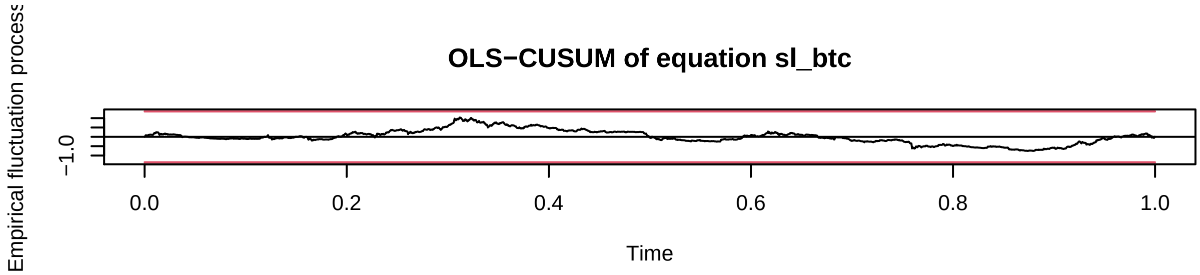
```r
serial.test(fitvar_p1,lags.bg = 40, type = 'BG')
```

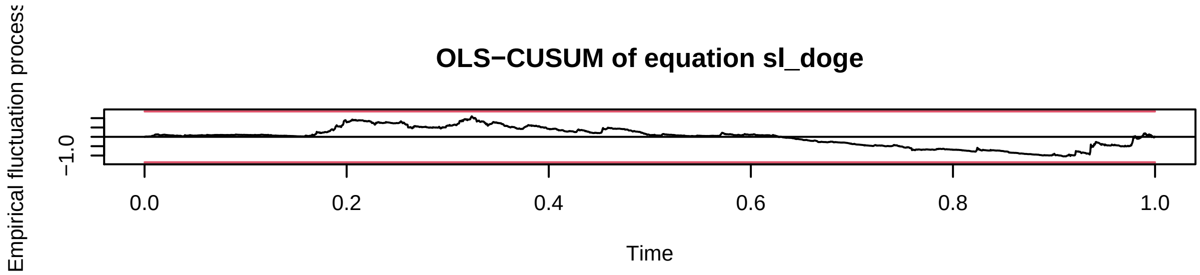
```
##
##  Breusch-Godfrey LM test
##
## data:  Residuals of VAR object fitvar_p1
## Chi-squared = 508.58, df = 360, p-value = 3.849e-07
```

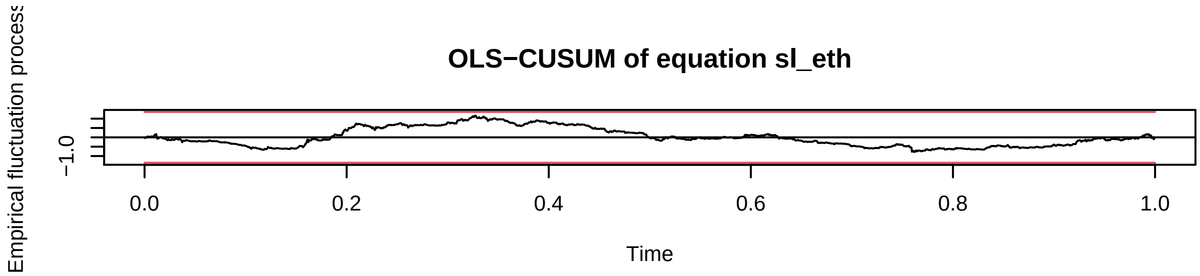
```r
# more diagnostics
stablity_test_p9 <-stability(fitvar_p9, type = c("OLS-CUSUM"))
stablity_test_p1 <-stability(fitvar_p1, type = c("OLS-CUSUM"))

plot(stablity_test_p9)
```



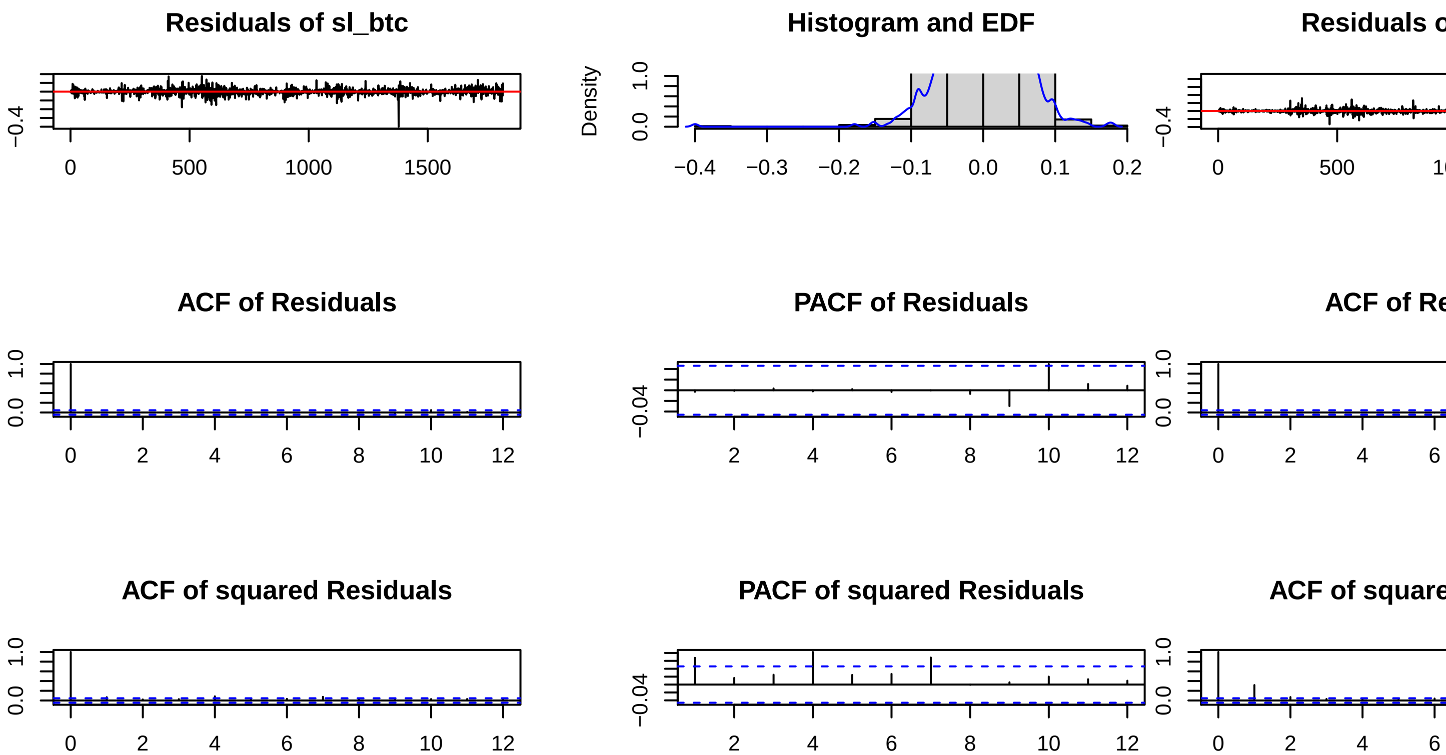

```
plot(stablity_test_p1)
```

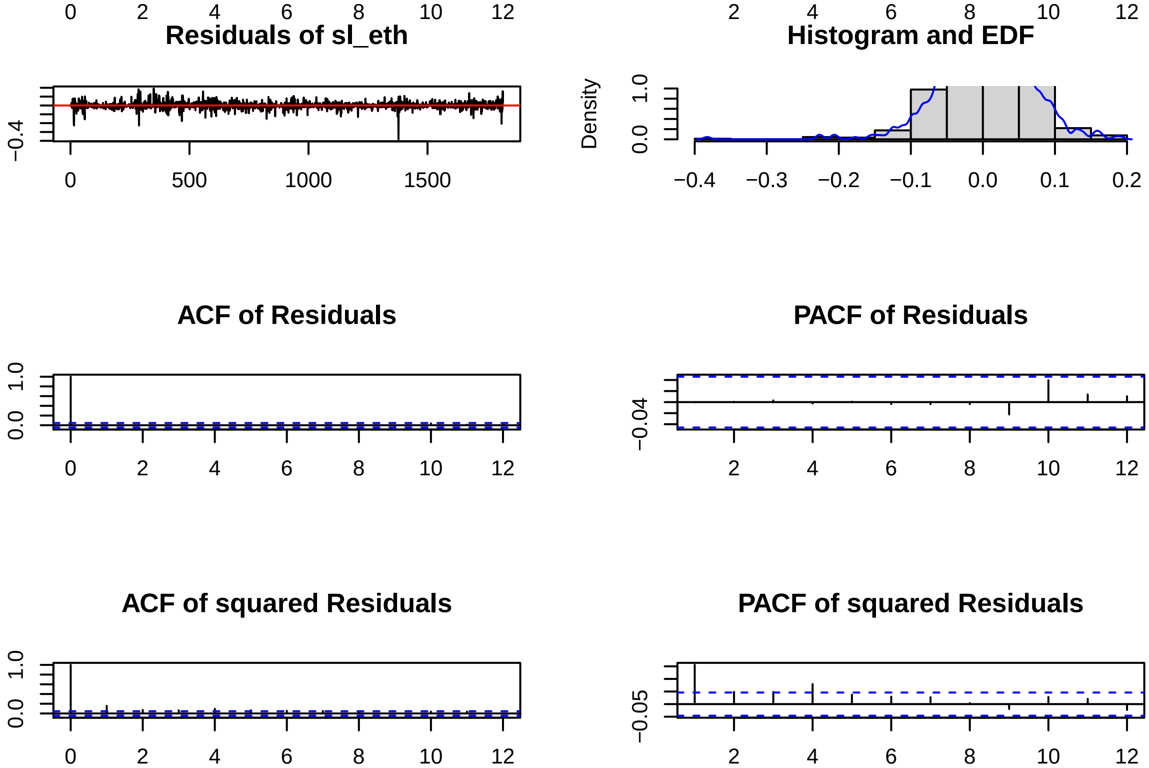




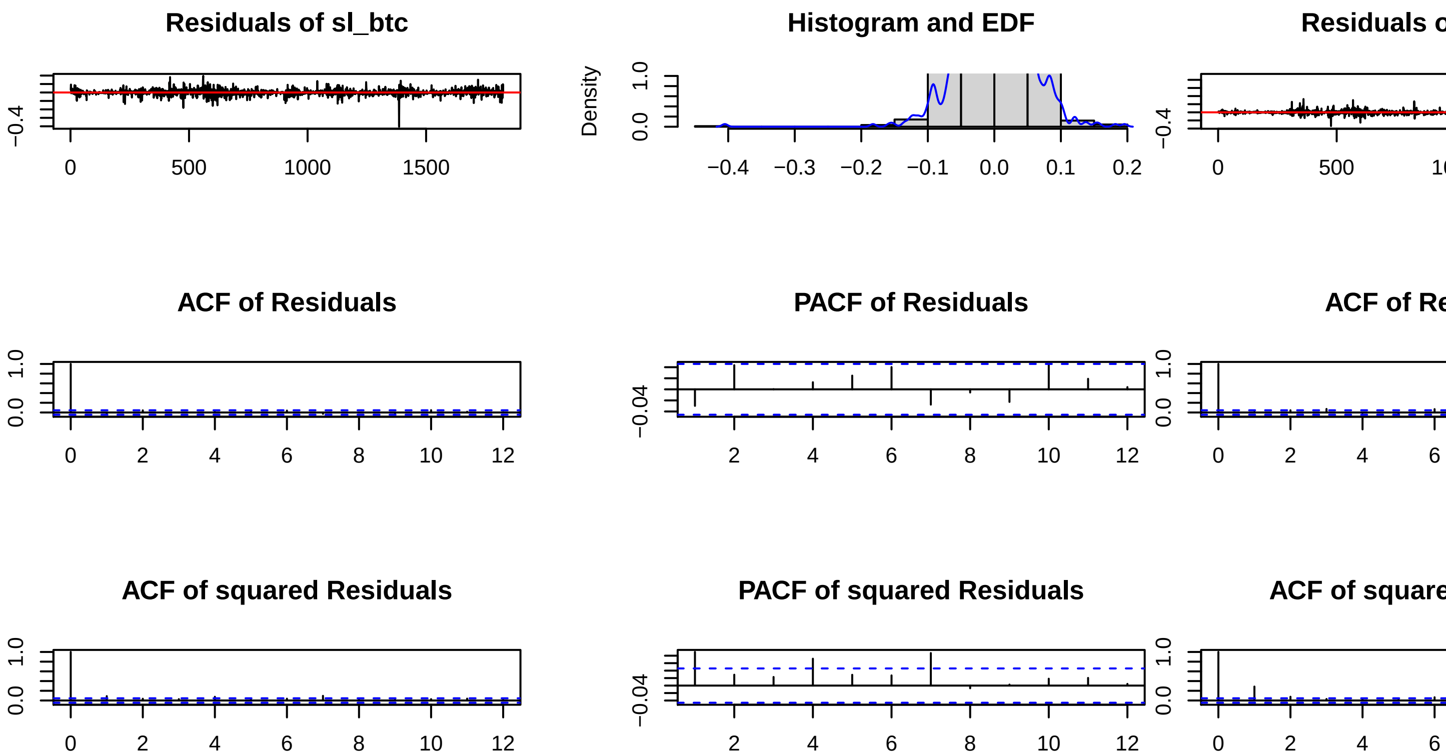


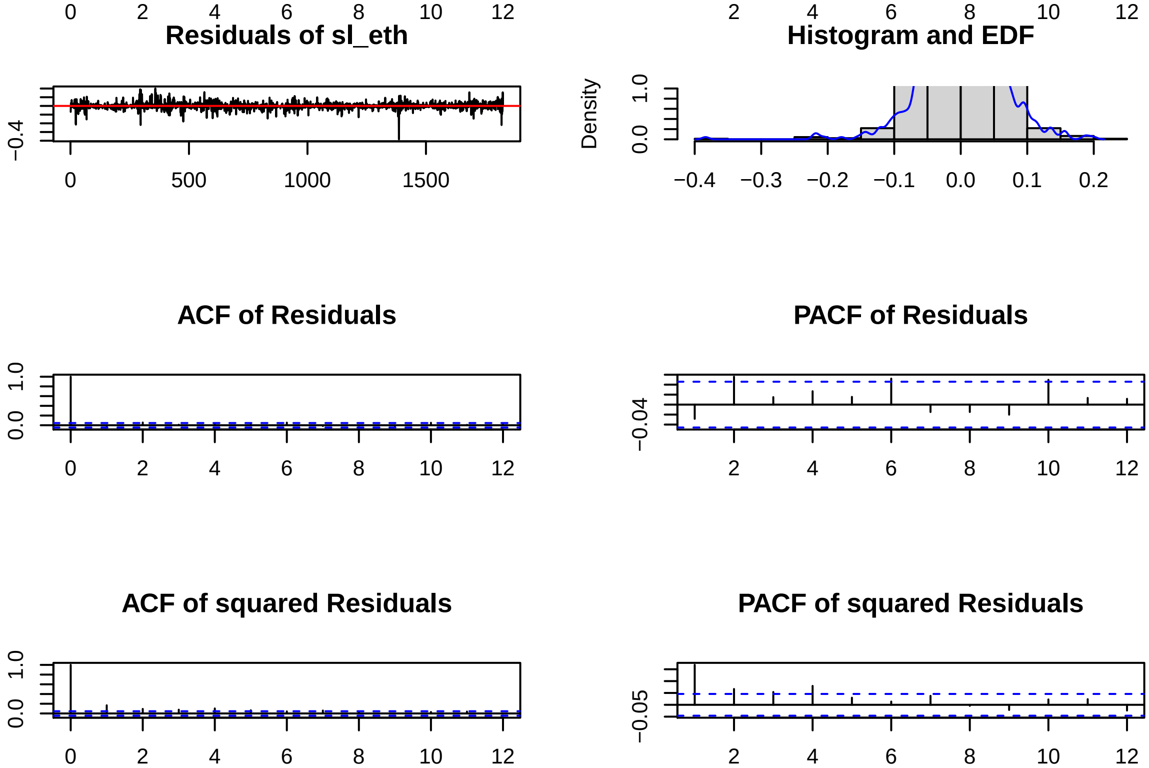

```
# Jarque-Bera test is a goodness-of-fit test of whether sample data have the skewness and kurtosis matc
p9_normal <- normality.test(fitvar_p9)

plot(p9_normal)
```






```
p1_normal <- normality.test(fitvar_p1)

plot(p1_normal)
```






```r
# Portmanteau Q and test for the null hypothesis that the residuals of a ARIMA model are homoscedastic
arch.test(fitvar_p9)
```

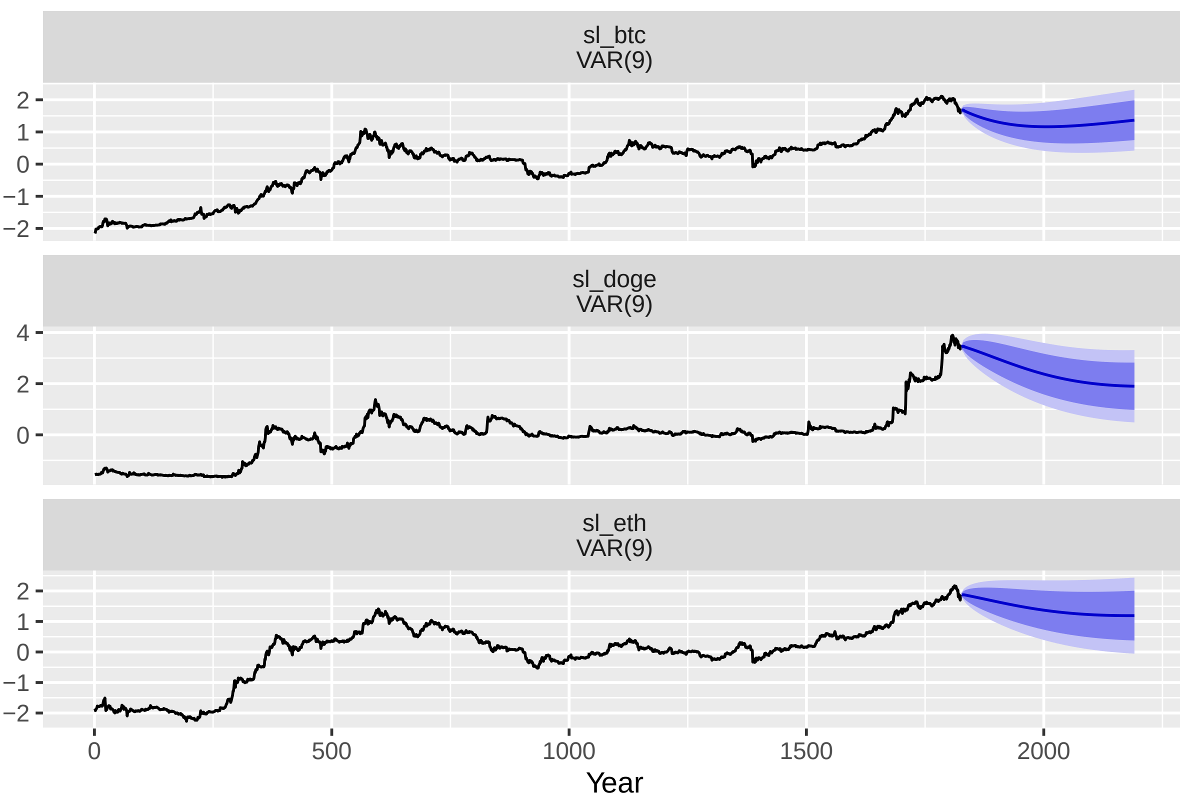
```
##
##  ARCH (multivariate)
##
```

```
## data:  Residuals of VAR object fitvar_p9
## Chi-squared = 900.25, df = 180, p-value < 2.2e-16
```

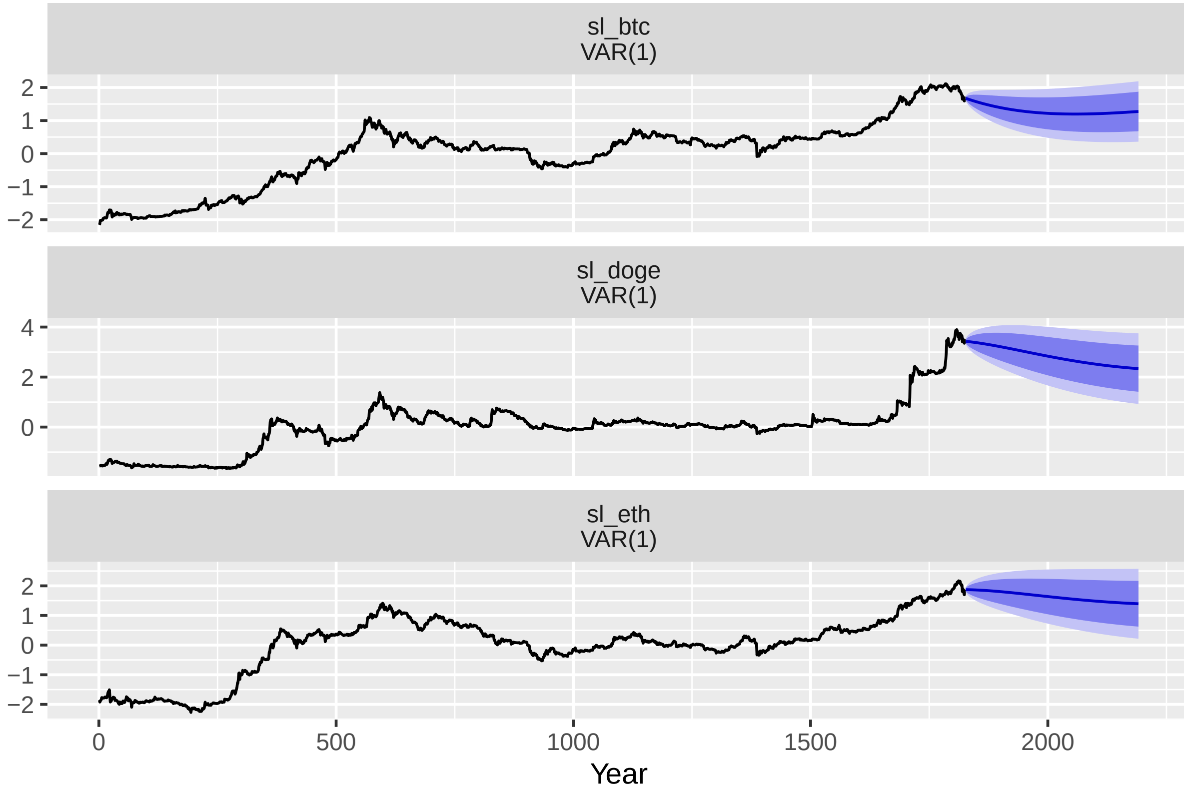
```
arch.test(fitvar_p1)
```

```
##
##  ARCH (multivariate)
##
## data:  Residuals of VAR object fitvar_p1
## Chi-squared = 914.61, df = 180, p-value < 2.2e-16
```

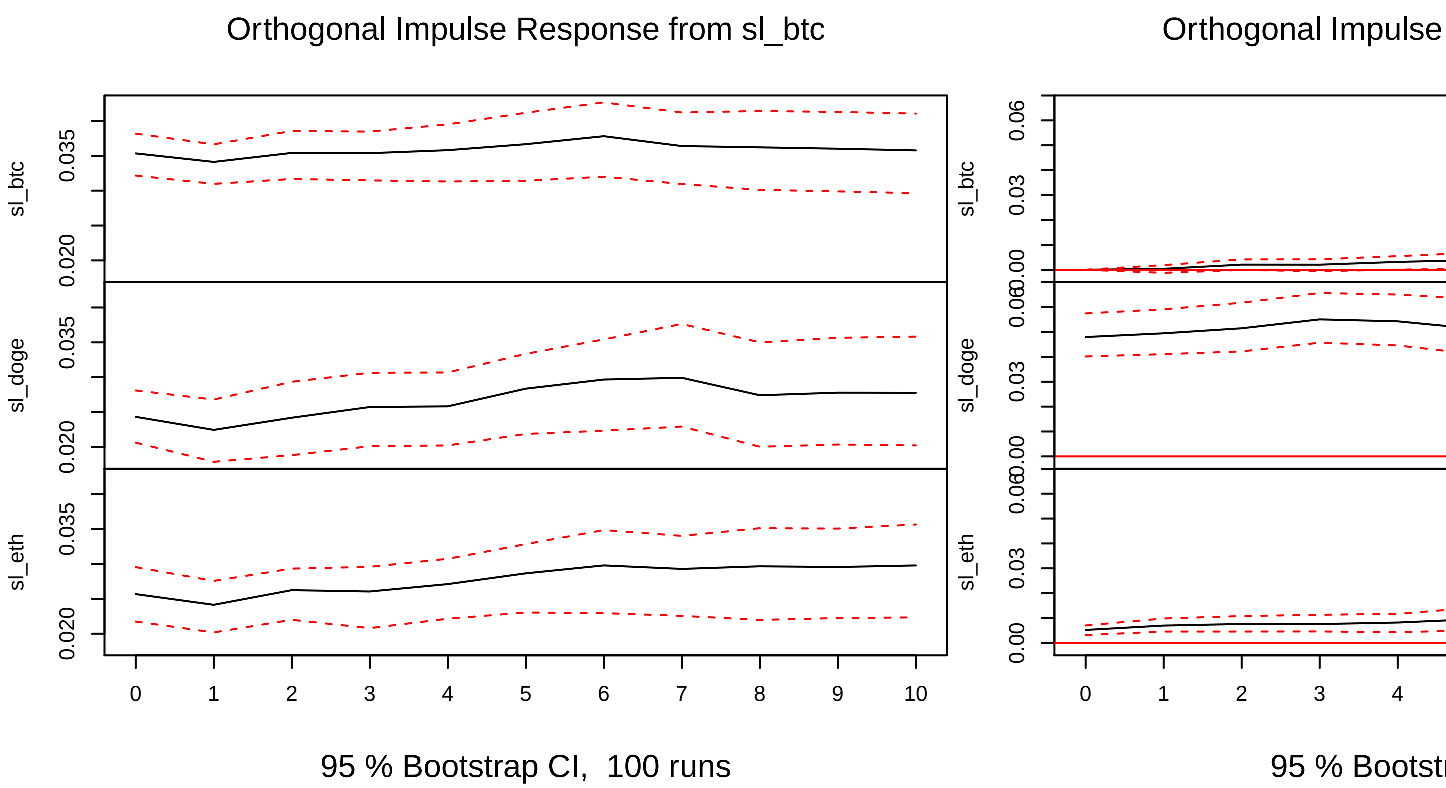
```
#making some forecasts

#need to refit model using slightly differently formatted data
sl_data_as_ts <- as.ts(sl_data[,-1])
fitvar_p9 <- VAR(sl_data_as_ts, p = 9, type = "both")

forecast(fitvar_p9, h = 365) %>%
  autoplot() + xlab("Year")
```

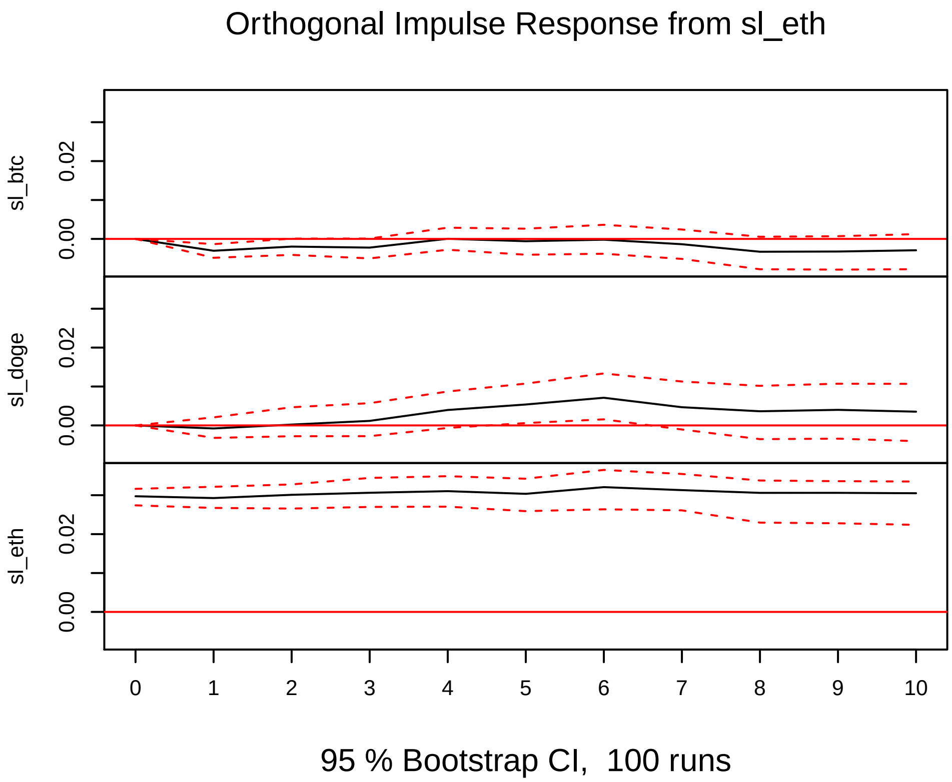



```
fitvar_p1 <- VAR(sl_data_as_ts, p = 1, type = "both")

forecast(fitvar_p1, h = 365) %>%
  autoplot() + xlab("Year")
```




```r
#irf stuff

plot(irf(fitvar_p9))
```




Orthogonal Impulse Response from sl_eth
sl_btc
sl_doge
sl_eth
0.00
0.02
0.00
0.02
0.00
0.02
0 1 2 3 4 5 6 7 8 9 10
95 % Bootstrap CI, 100 runs