1 Exam

1. A y
2. C y
3. B y
4. A y
5. B y
6. B y
7. B y
8. A y
9. A y
10. C y
11. C y
12. C y
13. D y
14. C y
15. B y
16. B y
17. B y
18. C y
19. D y
20. D y
21.

a.
sert truth table

y b. No, becuase the row [x] has a different value for both / Yes because every row underneith the final columns has the same value.y

c. Take all values that are false. Then do $\neg((row[p] \wedge row[q] \wedge row[r]) \vee (nextrow[p] \wedge nextrow[q] \wedge nextrow[r]) \vee ...) andthensimplifyoutusingdeMorganslawuntilatCNF.y22.$
$a.$
$D = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$
$E = 0, 2, 4, 6, 8, 10$
$P = 2, 3, 5, 7$
$S = 0, 1, 4, 9$
$R = (9, 10), (10, 9), (8, 10), (10, 8), (9, 9), (10, 10)$
$b.Yes, namelyx = 2$

$23.a.Toprove : \forall n \geq 1, n \in N : \sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3} basecase : for \text{n=1}, \sum_{i=1}^{1} i(i+1) = \frac{1(1+1)(1+2)}{3} 1(1+1) = \frac{6}{3} 2 = 2 Thebasecaseholds.$

I.H: We assume, for an arbitrary $n \in N, n \geq 1$:

$$\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$$

We want to prove:

$$\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3} \to \sum_{i=1}^{n+1} i(i+1) = \frac{(n+1)(n+1+1)(n+2+1)}{3}$$

$$\sum_{i=1}^{n+1} i(i+1) = \frac{(n+1)(n+1+1)(n+2+1)}{3}$$

$$\sum_{i=1}^{n+1} i(i+1) = \frac{(n+1)(n+2)(n+3)}{3}$$

$$(n+1)(n+2) + \sum_{i=1}^{n} i(i+1) = \frac{(n+1)(n+2)(n+3)}{3}$$

By the I.H.:

$$(n+1)(n+2) + \frac{n(n+1)(n+2)}{3} = \frac{(n+1)(n+2)(n+3)}{3}$$

$$\frac{3(n+1)(n+2)}{3} + \frac{n(n+1)(n+2)}{3} = \frac{(n+1)(n+2)(n+3)}{3}$$

$$\frac{3(n+1)(n+2) + n(n+1)(n+2)}{3} = \frac{(n+1)(n+2)(n+3)}{3}$$

$$\frac{(3+n)(n+1)(n+2)}{3} = \frac{(n+1)(n+2)(n+3)}{3}$$

$$\frac{(n+1)(n+2)(n+3)}{3} = \frac{(n+1)(n+2)(n+3)}{3}$$

Which is what we wanted to prove. n was arbitrarily chosen, therefore, our Proof by Induction is finished, and we have proven that

$$\forall n \geq 1, n \in N : \sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$$

□ y b. i.
Okay.

$$x = \sum_{i=0}^{0} i$$

$$x = 0$$

, which is true. ii.
It holds before the loop, so

$$x = \sum_{i=0}^{c} i$$

After the loop:

$$x + (c+1) = \sum_{i=0}^{c+1} i$$

$$x + (c+1) = c + 1 + \sum_{i=0}^{c} i$$

$$x + (c+1) = c + 1 + x$$

, which is correct.
y iii. n is a finite number. c starts at 0 and always grows, so at some point, c will be bigger than n.

24.
a. We can rewrite the first part as:
$(x \in A \oplus x \in B) \wedge (x \in C)$
$((x \in A \vee x \in B) \wedge \neg(x \in A \wedge x \in B)) \wedge (x \in C)$
$((x \in A \vee x \in B) \wedge (x \notin A \vee x \notin B)) \wedge (x \in C)$

We can rewrite the second part as:
$(x \in A \wedge x \in C) \oplus (x \in B \wedge x \in C)$
$((x \in A \wedge x \in C) \vee (x \in B \wedge x \in C)) \wedge \neg((x \in A \wedge x \in C) \wedge (x \in B \wedge x \in C))$
$((x \in A \vee x \in B) \wedge x \in C)) \wedge \neg((x \in A \wedge x \in C) \wedge (x \in B \wedge x \in C))$
$(x \in A \vee x \in B) \wedge x \in C \wedge (\neg(x \in A \wedge x \in C) \vee \neg(x \in B \wedge x \in C))$
$(x \in A \vee x \in B) \wedge x \in C \wedge ((x \notin A \vee x \notin C) \vee (x \notin B \vee x \notin C))$
$(x \in A \vee x \in B) \wedge x \in C \wedge (x \notin A \vee x \notin B \vee x \notin C)$
$(x \in A \vee x \in B) \wedge x \in C \wedge (x \notin A \vee x \notin B)$
$(x \in A \vee x \in B) \wedge (x \notin A \vee x \notin B) \wedge x \in C$

And therefore, we can see that these two statements are the same, which is what we wanted to prove. $\square$
y

b is not true, with as counterexample:
A = 1
B = 1,2
C = 2
y 25.
a.
I : $120 \in S$
II: $(x \cdot y = z \wedge z \in S) \rightarrow x, y \in S$
III: $(x, y \in S) \rightarrow (x \cdot y \in S)$
Iv: There is nothing else in S.

b.
Define a function f(x) that takes in a word x and returns the amount of a in that word. To prove: $x \in S \rightarrow 2 \nmid f(x)$
We will use a proof by structural induction.
Base case: $a \in S$. $f(a) = 1$, which is odd, and therefore the base case holds.

Induction hypothesis: $x \in S \wedge 2 \nmid f(x)$ We assume this is true for some arbitrary $x \in S$.

Inductive step 1: $x \in S \rightarrow xi \in S$
We know that since $x \in S$, $f(x) = 2y + 1$ for some integer y. $f(xi) = f(x) + f(i) = 2y + 1 + 0 = 2y + 1$, which is odd, which is what we wanted to prove.

Inductive step 2: $x \in S \rightarrow axa \in S$
We know that since $x \in S$, $f(x) = 2y + 1$ for some integer y. $f(axa) = f(a) + f(x) + f(a) = 1 + 2y + 1 + 1 = 2(y + 1) + 1$, which is odd, which is what we wanted to prove.
Inductive step 3: $x, y \in S \rightarrow ixiyixi \in S$

We know that since $x \in S$, $f(x) = 2a + 1$ for some integer a, and similarly, since $y \in S$, $f(y) = 2b + 1$ for some integer b.

$f(ixiyixi) = f(i) + f(x) + f(i) + f(y) + f(i) + f(x) + f(i) = 0 + 2a + 1 + 0 + 2b + 1 + 0 + 2a + 1 + 0 = 2(2a + b + 1) + 1$, which is odd, which is what we wanted to prove.

And since we have proven this all for some arbitrary element x in S, this concludes our proof by structural induction and we have proven that if x is in S, then the number of a in x is odd.
□

26.a
Every function is a relation, but not every relation is a function. For example, $f(X) = \pm x$ is a relation but not a function. This is because functions can only have 1 output per input, while relations do not have that constraint.

b.
i. This one does, namely $f(x) = \frac{x-26}{18}$
ii. No, the cardinality of Z and R are different, they are not binjective, and therefore they do not have an inverse. iii. Sure, $h^{-1} = \{(b, a), (d, c), (f, e), (h, g), (j, i)\}$
iv. Sure, namely $l^{-1} = \{(1, 1), (2, 2), (3, 3)\}$
c.
i. is true, A triangle is always similar to itself, if triangle a is similar to triangle b, and triangle b is similar to triangle c, then triangle a is similar to triangle

c. And lastly, if triangle a is similar to triangle b, then triangle b is similar to triangle a. Similarity has all the properties of an equivalence relationship, and therefore it is an equivalence relationship.

ii. suppose there is a person x, with an age of 1 and a wage of 2, and there is person y with an age of 100 and a wage of 1.
First, we can deduce (x,y) is in b, because 3¡=101.
To be an equivalence relationship, (x,x) should be in B. Therefore, 2+2¡=1+1, so 4¡=2. However, this does not hold, so B is not an equivalence relationship.