Quantitative Risk Management - Assignment 1

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1 Introduction

Value at Risk (VaR) and Expected Shortfall (ES) are two popular measures which are used for risk management purposes. Over time many different VaR and ES estimation methods have been suggested. In this report we estimate risk measures using various estimation methods. We start with implementation of the Variance-Covariance method, Historical simulation method, Constant Conditional Correlation method and Filtered Historical simulation. We present the risk measures calculated and compare them among each other. Next, we back-test our estimation methods, statistically test the results and complement the analysis with the square root of time estimation. We conclude our report with stress testing of the estimation methods and provide our recommendations for using risk estimation methods.

2 Implementation of VaR and ES system

In this section we implement various VaR and ES estimation methods, we discuss shortly the implementation, present and interpret the results for each method. For our estimation we use 10-years of past data starting at 23-03-2010 untill 19-03-2020. Before diving into the methods we discuss our financial portfolio. Our portfolio consists of the assets Gold, STOXX50, SP500, Nikkei, Swiss Market Index and bonds with underlying Euribor rates. Each asset compiles 20% of the portfolio, however the Swiss Market Index compiles 15% and the bonds compile 5%. Our portfolio is not leveraged and has a total initial value of €100,000,000.

2.1 Variance-Covariance method

Using the Variance-Covariance method we calculate the risk measures using closed form formulas specific to the loss distribution we assume. We assume that all asset dependencies in the portfolio are captured by the covariance matrix.

Multivariate normal distribution Assuming a multivariate normal distribution for our portfolio asset returns we can calculate the portfolio VaR and the ES by the following formulas.

$$VaR_\alpha = \mu + Z_\alpha * \sigma \tag{1}$$

$$ES_\alpha = \mu + \sigma * \frac{f(F^{-1}(\alpha)}{1-\alpha} \tag{2}$$

So to calculate the risk measures we only need a portfolio volatility estimate and a portfolio mean return estimate. Multiplying the VaR and ES estimates by the portfolio value gives us the monetary VaR and ES. Table 1 provides the risk estimates for a 0.975% and 0.99% confidence level. To asses if these estimates are any good, we can start by looking at Figure 1, which shows the QQ-plot of our portfolio returns compared to the normal distribution quantiles. As we can see, the portfolio returns deviate substantial from the normal distribution at both ends of the distribution. This indicates that assuming normality in returns underestimates tail risk and therefore underestimates the VaR and ES.

Table 1: Table providing risk measures under normality assumption of returns for multiple confidence levels.

α	VaR	ES
97.5%	€1,517,041	€1,813,121
99%	€1,804,145	€2,069,686

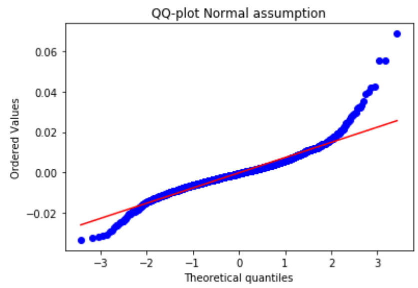


Fig. 1: Figure with QQ-plot of the portfolio returns assuming normality of returns. Red line denoting normal quantiles and blue dots denoting true portfolio return quantiles

Student-t distribution Assuming a multivariate student-t distribution for our portfolio asset returns we can calculate the portfolio VaR and the ES by the following formulas. Only now in eq. 3, T_α is the percent point function evaluated at α and depend on the degrees of freedom.

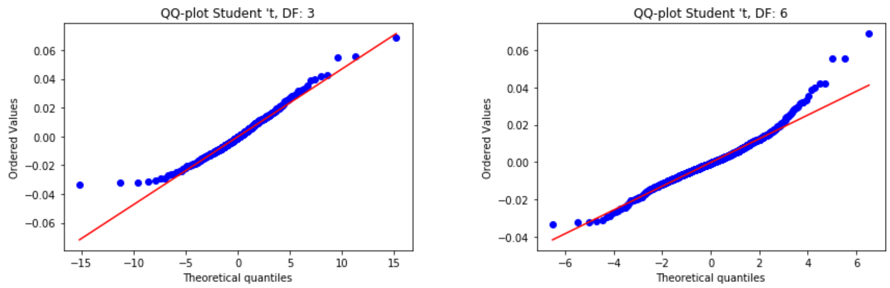
$$VaR_\alpha = \mu + T_\alpha * \sigma \tag{3}$$

$$ES_\alpha = \mu + \sigma * \frac{f(T_\alpha)}{1-\alpha} * \frac{(\nu + T_\alpha)^2}{\nu - 1} \tag{4}$$

Compared to the normality closed formulas we have an additional parameter which is the degrees of freedom (ν). This parameter defines the fatness of the distribution tails. Table 2 provides the risk estimates for a 0.975 and 0.99 confidence level and for 3, 4, 5 and 6 degrees of freedom. To test again the goodness of the distribution assumption we show the QQ-plots (figure 2) of the student-t with 3 and 6 degrees of freedom. As we can see from the figure, using 3 degrees of freedom is over-estimating the probability of huge losses. Using 6 degrees of freedom is a better fit for the loss tail. Looking at the results of the risk estimations we see a decreasing VaR and ES amount as the degrees of freedom increase. This is due to the heaviness of the tails assumed. Fatter tails increase big losses probability and therefore increases risk.

Table 2: Table providing risk measures under student-t assumption of returns for multiple confidence levels and degrees of freedom.

ν	VaR		ES	
	97.5%	99%	97.5%	99%
3	€ 4,300,600	€ 6,144,100	€ 6,821,220	€ 9,486,200
4	€ 3,058,040	€ 4,133,550	€ 4,406,840	€ 5,766,630
5	€ 2,581,700	€ 3,385,290	€ 3,542,760	€ 4,485,450
6	€ 2,329,560	€ 2,997,290	€ 3,106,210	€ 3,851,320

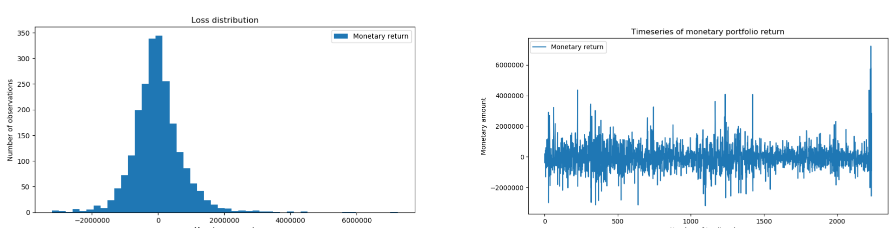


(a) ν: 3 (b) ν: 6

Fig. 2: Figure with QQ-plots of the portfolio returns assuming student-t distribution of returns. Red line denoting normal quantiles and blue dots denoting true portfolio return quantiles

2.2 Historical simulation method

To implement the historical simulation method we take the return timeseries of each asset. We calculate the portfolio return timeseries using all assets and the portfolio weights. We take this historical realized return timeseries and order each of the returns from biggest to lowest loss. With the loss distribution obtained we can calculate the VaR and ES by taking either the α-quantile of the distribution or taking the mean of all losses which exceeded the VaR α-quantile. Figure 3 shows the loss distribution of our portfolio using historical simulation. Table 3 provides the risk measures using the historical simulation method.



(a) Distribution (b) Timeseries

Fig. 3: Figure with the loss-distribution obtained using the historical simulation method and the timeseries of our portfolio returns. In both plots we use the loss distribution, positive values are portfolio losses.

Table 3: Table providing risk measures using historical simulation method for multiple confidence levels.

α	VaR	ES
97.5%	€1,547,310	€2,471,967
99%	€2,340,814	€3,380,936

2.3 Constant Conditional Correlation method

In this section we will show the CCC-method, by using a GARCH(1,1)-model. The same methodology as the variance-covariance method will be used. Only in this case, the covariance matrix will be given by a GARCH-estimate. We presuppose the correlation matrix fixed, and simply use the historical correlation of the portfolio components. To get to the final covariance matrix, we use the following formula:

$$Cov_t(i,j) = \rho_{i,j} * \sigma_{t,i} * \sigma_{t,j} \tag{5}$$

Where we used:

$$\sigma_{t,i}^2 = \omega_i + \phi\sigma_{t-1,i}^2 + \alpha\epsilon_{t-1,i}^2 \tag{6}$$

If we want to compute the VaR- and ES-estimates, we use Equations 1 and 2, only now the portfolio volatility estimate σ will be derived from the abovementioned covariance matrix, instead of a simple historical covariance matrix.
The results of this computations are shown in Table 4.

Table 4: The VaR- and ES- estimates using the Constant Correlation Method.

α	VaR	ES
97.5%	€4,965,883	€5,926,825
99%	€5,965,883	€6,759,518

2.4 Filtered historical simulation method

For the filtered historical simulation we estimate the portfolio variance over time using the EWMA-model. This way, we have a representation of the variance on that day in our dataset. We scale the returns, by dividing it by the found variance, to obtain the standardized returns. With the historical distribution of standardized returns we can calculate the VaR and ES estimates. The procedure is very similar to the historical simulation method however with the filtered historical simulation method we make our VaR and ES dependent on the variance at this moment, which is also given by the EWMA-model. The formulas to obtain the return distribution are shown below.

$$\sigma_t^2 = \lambda * \sigma_{t-1}^2 + (1-\lambda) * R_{t-1}^2 \tag{7}$$

$$\tilde{Z}_t = \frac{R_t}{\sigma_t} \tag{8}$$

$$\tilde{R}^{(1)}_{t+1} = \tilde{Z}_t * \tilde{\sigma}_{t+1} \tag{9}$$

In short, we forecast volatility using eq. 7, then we calculate standardised residuals by dividing return and volatility, eq. 8, and we compile a distribution of standardised returns by multiplying the standardised residuals and forecasted volatility, eq. 9. For our calculation purposes we set λ equal to 0.94. Table 5 present the VaR and ES risk estimates for multiple confidence levels.

Table 5: Table providing risk measures using filtered historical simulation method with EWMA variance estimation for multiple confidence levels.

α	VaR	ES
97.5%	€ 5,094,021	€ 7,488,099
99%	€ 7,210,143	€ 9,607,769

2.5 Comparison of risk estimation

The Variance-Covariance and the historical simulation risk estimation methods are static estimation procedures. This leads to slow varying risk estimations which do not account for higher risk in volatile periods and low risk in less volatile periods. Therefore we compare these 3 risk estimation methods among each other. In this section we look at the risk measures when using 10-years and 5-years of data. We also look at risk measures using only non-stressed periods. Table 6 shows the risk estimations for both VaR and ES calculated over different horizons.

As the table shows us, the static risk measures are not very sensitive to the length of the return series. 5 years of data is already a lot and including more data-points does not add much more value. We however see that the risk measures are slightly lower using only the last 5 years of data. As we see from the timeseries plot (figure 3b) leaving our the first 5 years means excluding a volatile period. As the data overall becomes less volatile the risk measures have a lower estimate. When we look at the third time-frame, the non-stressed period, we see it substantially decreases the risk measures. A straightforward result as we excluded the first 500 observations and the last 20 observations. The first excluded observations include periods during the European Sovereign Debt Crisis and the last excluded observations include the Coronacrisis period. Excluding these observations decreases the overall volatility and therefore leads to overall lower risk estimates.

Comparing the results the table shows that the risk measures of the historical simulation method are the most sensitive to the estimation window. Due to the

Table 6: Table providing risk measures for 0.99 confidence level. Each estimation method provides 10Y, 5Y and non-stressed period risk measures. 10Y (2010-03-23, 2020-03-19), 5Y (2015-03-23, 2020-03-19) and non-stressed (2012-01-04, 2020-02-21).

Measure	VaR	ES
Normal 10Y	€ 1,804,145	€ 2,069,686
Normal 5Y	€ 1,784,806	€ 2,044,195
Normal non-stressed	€ 1,541,541	€ 1,771,270
student-t 10Y, DF:3	€ 6,144,100	€ 9,486,200
student-t 5Y, DF:3	€ 6,024,220	€ 9,288,880
student-t non-stressed, DF:3	€ 5,296,190	€ 8,187,550
student-t 10Y, DF:4	€ 4,133,550	€ 5,766,630
student-t 5Y, DF:4	€ 4,060,240	€ 5,655,490
student-t non-stressed, DF:4	€ 3,556,790	€ 4,969,620
student-t 10Y, DF:5	€ 3,385,290	€ 4,485,450
student-t 5Y, DF:5	€ 3,329,320	€ 4,403,990
student-t non-stressed, DF:5	€ 2,909,440	€ 3,861,230
student-t 10Y, DF:6	€ 2,997,290	€ 3,851,320
student-t 5Y, DF:6	€ 2,950,310	€ 3,784,550
student-t non-stressed, DF:6	€ 2,573,770	€ 3,312,620
Hist sim 10Y	€ 2,340,814	€ 3,380,936
Hist sim 5Y	€ 2,447,239	€ 3,921,413
Hist sim non-stressed	€ 1,771,024	€ 2,391,143

method taking the ”true” return series into account it depends highly on the volatility and return of the return series. This is less important for the Var-Cov methods using normality or student-t assumption. This result is not in favor of using these risk measures as they do not adequately capture current market conditions. The result is however easily understood by looking at the closed form formula. As we assume a certain distribution, almost all parameters are derived from the distribution. Only our portfolio variance and mean return are derived from the underlying ”true” return distribution. As both have a minor effect on the calculation the outcome is not very sensitive to changes in these measures and thus the estimation horizon.

3 Backtest of VaR and ES system

In this section, we will backtest the methods we used in the previous sections. We will do this for the VaR as well as the ES methodology.

3.1 Backtest VaR-methods, total and yearly violations

Let us start by looking at Figure 4. We can clearly see the difference in estimation

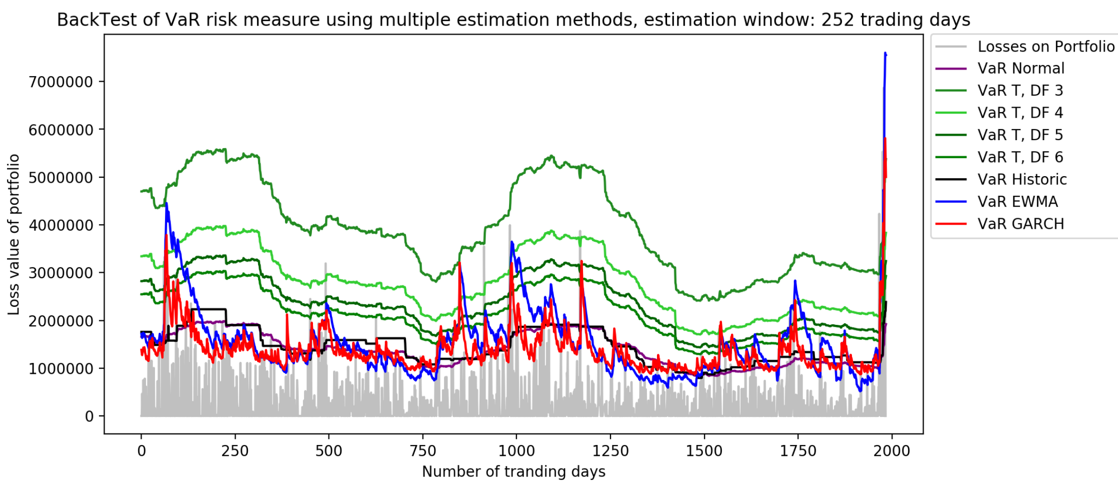


Fig. 4: Backtest results of VaR risk measures. The 97.5% VaR values are used here.

methods. The EWMA and CCC-GARCH methods are much more sensitive to the current developments in the market, while the other measures are relatively static. We can also observe that the VaR-values with the student-t distribution differ quite a lot depending on the degrees of freedom. With 3 degrees of freedom, the risk seems wildly overshot. On the other hand, the higher the degrees of freedom, and the normal distribution seem to underestimate the risk. We can also see the recent high volatility causing a huge spike at the end of the dataset. In Table 7 we can observe these differences as well.

We can see that the Student-T with three degrees of freedom has the lowest amount of violations. Besides not wanting too many violations, we also do not want to overestimate the risk, as is clearly the case with the Student-T (3). This is why we use a two-sided binomial significance test. The historical simulation, the EWMA and CCC-GARCH-methods produce around the same total number of violations for the 97.5% VaR. All three seem equaly appropriate, with the EWMA performing slightly better, and being the only one statistically not different from the expected number of violations. The other methods result in

Table 7: The number of violations per year, the first period starts at 2011-03-23 and ends in 2012-03-23 and so forth. For the total we conducted a 2-sided Binomial test for significance.

Year	2011	2012	2013	2014	2015	2016	2017	2018	2019	All
Trading Days	210	228	227	225	225	229	228	214	199	1985
Results for 97.5% VaR										
Expected no. of violations	5.3	5.7	5.7	5.6	5.6	5.7	5.7	5.4	5.0	49.6
Hist. VaR. Norm	8	0	8	8	10	1	10	10	12	67 (0.02)
Hist. VaR. Student-T (3)	0	0	0	0	0	0	0	0	4	4 (0.00)
Hist. VaR. Student-T (4)	2	0	1	0	2	1	0	0	6	12 (0.00)
Hist. VaR. Student-T (5)	3	0	2	0	4	1	2	2	6	20 (0.00)
Hist. VaR. Student-T (6)	3	0	2	1	5	1	3	3	8	26 (0.00)
Hist. Simulation	8	1	8	7	11	1	11	6	10	63 (0.06)
EWMA	4	4	8	7	7	5	9	7	9	60 (0.15)
CCC-GARCH	9	6	7	4	10	3	5	7	11	62 (0.08)
Results for 99% VaR										
Expected no. of violations	2.1	2.3	2.3	2.3	2.3	2.3	2.3	2.1	2.0	19.9
Hist. VaR. Norm	6	0	7	4	7	1	4	6	10	45 (0.00)
Hist. VaR. Student-T (3)	0	0	0	0	0	0	0	0	2	2 (0.00)
Hist. VaR. Student-T (4)	0	0	0	0	1	0	0	0	4	5 (0.00)
Hist. VaR. Student-T (5)	0	0	1	0	2	0	0	0	5	8 (0.00)
Hist. VaR. Student-T (6)	2	0	1	0	2	1	0	1	6	13 (0.14)
Historical Simulation	3	0	7	4	7	1	4	5	7	38 (0.00)
EWMA	2	2	4	4	3	2	6	1	6	30 (0.03)
CCC-GARCH	7	2	5	2	6	2	3	3	4	34 (0.00)

too many or too few violations. For the 99% VaR, the results are almost the same, with the exception of the Student-T (6)- variance covariance method. It shows the highest significance out of all the methods. We can also observe that the var-covariance methods seem to product unevenly spread violations. This we will investigate further with the models that perform best in this first part.

3.2 Sample window length sensitivity

The estimation above is done with a sample window of 252 days. Now we will make our backtest window such a length that it will produce the first outcome at 2013-03-22, so we add 438 trading days to our existing 252 to get a sample period of 690 days.

In Figure 5 we see the difference clearly, the VaR based on a simple variance-covariance method or historical simulation, barely change through time because of the prolonged estimation window we adopted here. Similarly as before, these

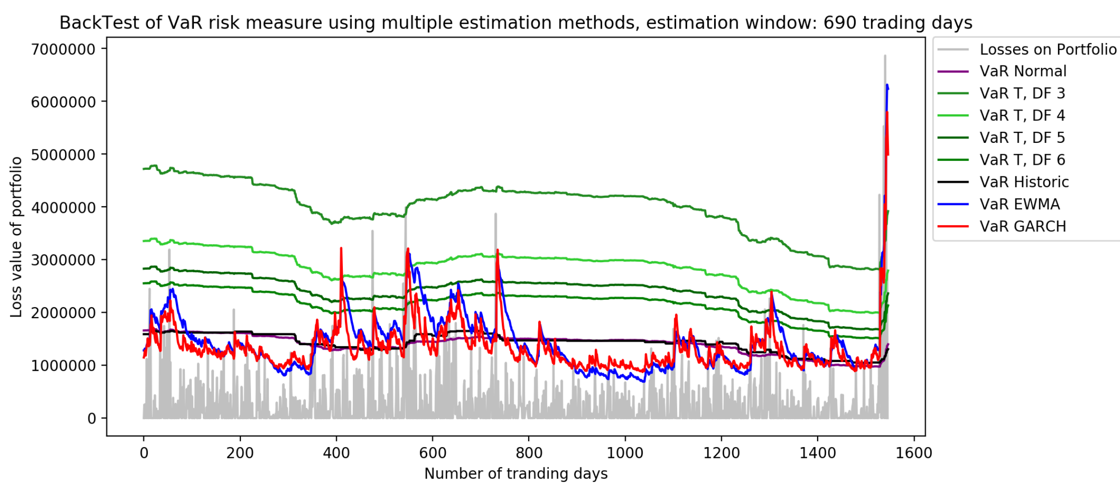


Fig. 5: The backtest results, when a estimation window of 690 days is used. The 97.5% VaR values are shown here.

results can be observed as well in the table of yearly violations (Table 8). The better method seems to be the EWMA-method, which performed good as well with the previous sample size. This time, it is tied with the historical simulation, and the CCC-GARCH methodology, all having 44 violations in the 97.5% VaR. With the 99% VaR violations the EWMA-model really shows superiority over the others by scoring the highest.

Therefore we conclude that the window sensitivity is fairly important for the accuracy of the models, but quite consistently the EWMA-method scores well. The CCC-GARCH method, historical simulation, and the variance-covariance

Table 8: The number of violations per year for the different sample size of 690 days, the first period starts at 2013-03-22 and ends in 2014-03-24 and so forth. With the total we conducted a 2-sided Binomial test for significance.

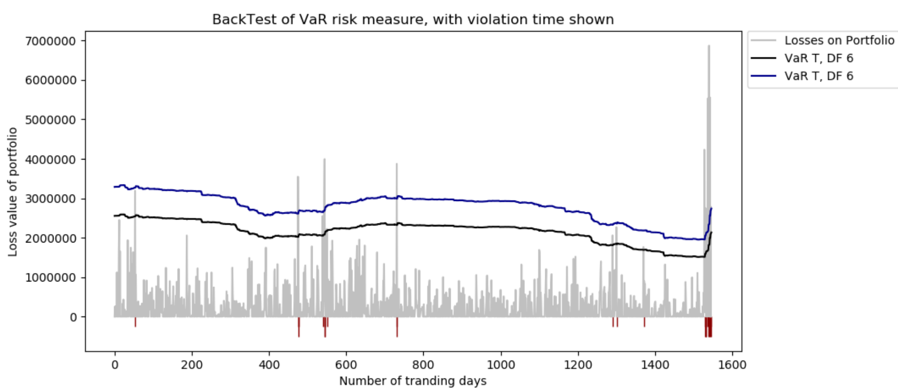
Year	2013	2014	2015	2016	2017	2018	2019	All
Trading Days	227	225	225	229	228	214	199	1547
Results for 97.5% VaR								
Expected no. of violations	5.7	5.6	5.6	5.7	5.7	5.4	5.0	38.7
Hist. VaR. Norm	6	4	15	2	2	5	13	47 (0.19)
Hist. VaR. Student-T (3)	0	0	1	0	0	0	4	5 (0.00)
Hist. VaR. Student-T (4)	0	0	3	1	0	0	7	11 (0.00)
Hist. VaR. Student-T (5)	1	0	4	1	0	2	8	16 (0.00)
Hist. VaR. Student-T (6)	1	0	6	1	0	2	9	19 (0.00)
Hist. Simulation	7	3	14	1	2	5	12	44 (0.37)
EWMA	6	5	7	2	9	7	8	44 (0.37)
CCC-GARCH	6	4	10	3	5	7	9	44 (0.37)
Results for 99% VaR								
Expected no. of violations	2.3	2.3	2.3	2.3	2.3	2.1	2.0	15.5
Hist. VaR. Norm	4	1	11	1	1	2	11	31 (0.00)
Hist. VaR. Student-T (3)	0	0	0	0	0	0	4	4 (0.00)
Hist. VaR. Student-T (4)	0	0	1	0	0	0	4	5 (0.00)
Hist. VaR. Student-T (5)	0	0	2	1	0	0	6	9 (0.12)
Hist. VaR. Student-T (6)	0	0	3	1	0	0	7	11 (0.30)
Historical Simulation	1	1	7	1	0	2	9	21 (0.15)
EWMA	3	2	4	1	2	2	4	18 (0.52)
CCC-GARCH	5	1	6	2	3	3	4	24 (0.04)

Student-T (6)-methods also performed somewhat better than the other, and we will keep these for the next subsection to investigate clustering. The performance was better with the longer (690 days) sample period, thus we will keep this window-size on in this section.

3.3 Clustering of VaR-violations

The violations should not be clustered, but should occur randomly through time. For this we check by looking at the time between violations. We will not show graphs for all the methods, since that will be quite overwhelming, but we take the best performing three, which show a quintessential difference between the methods. The easiest way to compare these are by highlighting the times where a VaR-violation takes place. Figure 6, 7 and 8 try to illustrate these differences.

Fig. 6: The backtested violations of the variance Covariance method with a Student-T (6) distribution. The red lines below represent a violation in the VaR. The small bins show the 97.5% violations, and the large bins the 99% violations.



What we can conclude, is that the variance-covariance method using a Student-T (6) distribution has clustered violations, especially seen at the end of the dataset. The GARCH- and EWMA-methods still seem to have a few clustered violations, but in general they are more spread out evenly.

Fig. 7: The backtested violations of the filtered historical simulation EWMA-method. The red lines below represent a violation in the VaR. The small bins show the 97.5% violations, and the large bins the 99% violations.

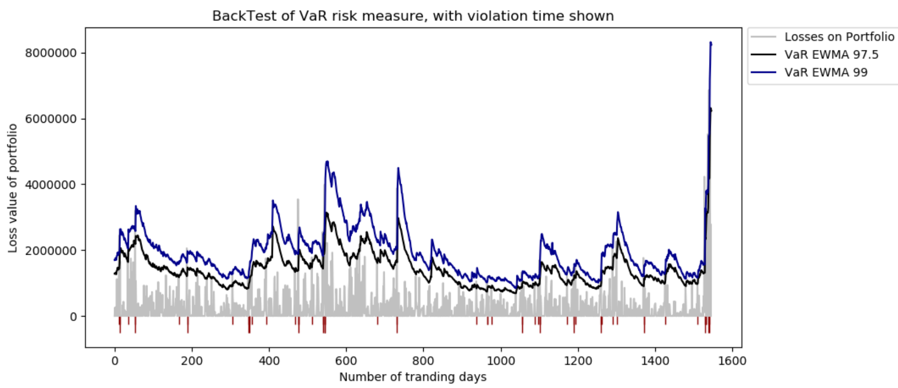
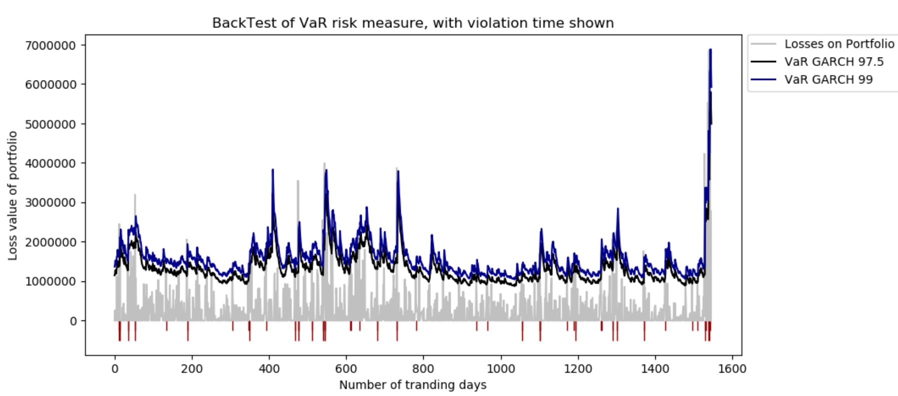


Fig. 8: The backtested violations of the CCC-GARCH method. The red lines below represent a violation in the VaR. The small bins show the 97.5% violations, and the large bins the 99% violations.



3.4 Backtest ES-methods

We will now look at the Expected Shortfall, and how the various methods have performed. We will test the real shortfalls each year with the average expected shortfall in that period. We first compute the VaR of the realized loss distribution. After this, all the values that exceed this amount will be stored. We take the average of these values and take this as our 'realized' shortfalls. We will test how well the ES-methodologies can approximate this amount. We compute similarily as with the VaR-computation, the ES for each day in the dataset (starting from day 690). This we take the average of, and compare this with the 'realized' average shortfall. The outcomes of these computations are shown in Table 9

Table 9: The real yearly expected shortfalls compared with the expected shortfall based on all of the methodologies for the two different confidence intervals.

Period	2013	2014	2015	2016	2017	2018	2019
Results of the 97.5% ES							
Real average shortfall	€2,175,960	€1,402,080	€2,884,740	€1,726,380	€1,328,580	€1,661,970	€4,994,310
Hist. VaR. Norm	€1,951,590	€1,700,170	€1,688,870	€1,800,830	€1,746,370	€1,517,280	€1,248,080
Hist. VaR. Student-T (3)	€7,352,990	€6,445,060	€6,416,990	€6,808,770	€6,598,650	€5,697,180	€4,740,410
Hist. VaR. Student-T (4)	€4,749,010	€4,157,570	€4,137,590	€4,394,470	€4,259,390	€3,682,070	€3,056,780
Hist. VaR. Student-T (5)	€3,818,150	€3,339,860	€3,322,760	€3,531,420	€3,423,170	€2,961,720	€2,454,920
Hist. VaR. Student-T (6)	€3,346,230	€2,925,300	€2,909,660	€3,093,880	€2,999,230	€2,596,520	€2,149,800
Historical Simulation	€2,369,650	€1,968,030	€2,055,530	€2,215,820	€2,176,940	€1,852,650	€1,514,560
EWMA	€2,164,690	€1,820,630	€2,793,130	€2,174,300	€1,384,540	€1,740,690	€1,878,820
CCC-GARCH	€1,697,680	€1,575,260	€1,980,110	€1,611,110	€1,290,940	€1,482,580	€1,592,410
Results of the 99% ES							
Real average shortfall	€2,565,660	€1,545,900	€3,481,340	€2,278,160	€1,545,980	€1,947,560	€6,211,440
Hist. VaR. Norm	€2,228,310	€1,943,250	€1,931,090	€2,057,390	€1,994,960	€1,731,410	€1,426,990
Hist. VaR. Student-T (3)	€10,227,300	€8,969,980	€8,932,980	€9,473,660	€9,180,710	€7,921,450	€6,598,800
Hist. VaR. Student-T (4)	€6,215,590	€5,445,900	€5,421,360	€5,754,220	€5,576,880	€4,816,990	€4,005,010
Hist. VaR. Student-T (5)	€4,833,790	€4,232,050	€4,211,800	€4,473,080	€4,335,560	€3,747,680	€3,111,600
Hist. VaR. Student-T (6)	€4,149,850	€3,631,250	€3,613,120	€3,838,960	€3,721,150	€3,218,410	€2,669,390
Historical Simulation	€3,057,250	€2,446,890	€2,660,590	€2,943,800	€2,950,200	€2,403,910	€1,964,250
EWMA	€2,695,180	€2,223,910	€3,533,490	€2,869,930	€1,826,200	€2,123,100	€2,249,970
CCC-GARCH	€1,938,830	€1,800,840	€2,263,120	€1,841,090	€1,475,740	€1,691,860	€1,819,540

As can be seen, the highest ES-values are with the Student-T distribution, similarily to the VaR-computation. These produce very high ES-values compared to the realized average shortfall. The CCC-Garch is most of the time below the real expected shortfall but is fairly accurate in the 97.5% ES. The historical simulation overestimates the ES. The EWMA-method again shows to be fairly accurate, but does overestimate fairly often.

3.5 Multiple day VaR

The square root of time rule of thumb tells us that we can obtain the 5-day or 10-day VaR and ES measures by multiplying the 1-day VaR and ES measures with the square root of time. This rule however relies heavily on the assumption of a Gaussian return distribution and independent identically distributed observations. Both of these assumptions have already been shown are not supported by our portfolio return distribution (Figure 1 and Figure 3). Table 10 shows empirical 5-day, 10-day VaR and square root of time VaR for both normality assumption and historical simulation. The square root of time rule is commonly applied in practise as using non-overlapping data makes it very hard to make accurate estimates. The tables show that the square root of time rule is inadequate when using historical simulation method, but is an adequate proxy when using the normal Var-Cov method. The rule also performs better for 5-day risk measures than for 10-day risk measures which is due to the lower estimation error for smaller time horizons. For the historical simulation method the square root of time rule mostly underestimates the VaR and ES compared to empirical estimates. This is exactly the situation a regulator does not want and therefore the square root of time rule specifically for more conditional type of risk measuring methods is not good. The poor results of the square root of time rule are mostly due to the violation of the underlying assumptions mentioned earlier. Our portfolio returns do not comply with these assumptions.

Table 10: Table providing empirical 5-day and 10-day VaR and ES estimates including square root of time estimation for comparison. The estimations are calculated using all 10Y of data (2010-03-23, 2020-03-19). estimates above horizontal line are 5-day estimates and below are 10-day estimates.

		Empirical		Square root rule	
α	Measure	VaR	ES	VaR	ES
97.5%	Var-Cov Normal	€ 3,568,853	€ 4,274,906	€ 3,392,207	€ 4,054,262
	Hist sim	€ 4,055,754	€ 5,944,915	€ 3,459,890	€ 5,527,486
99%	Var-Cov Normal	€ 4,253,500	€ 4,886,727	€ 4,034,191	€ 4,627,959
	Hist sim	€ 4,588,969	€ 8,194,448	€ 5,234,219	€ 7,560,003
97.5%	Var-Cov Normal	€ 5,450,022	€ 6,536,679	€ 4,797,305	€ 5,733,592
	Hist sim	€ 6,189,397	€ 9,561,944	€ 4,893,024	€ 7,817,046
99%	Var-Cov Normal	€ 6,503,735	€ 7,478,308	€ 5,705,207	€ 6,544,922
	Hist sim	€ 8,697,929	€ 12,248,607	€ 7,402,304	€ 10,691,458

4 Stress testing

To finish of our risk system we complement each method with stress testing. As our dataset runs till 19-03-2020 we drop the last observations as otherwise would have very extreme outcomes as we would already have included the Coronacrisis. To assess the impact of extreme scenario's even better we only use the last 5-years of observation. In other words our dataframe runs from 23-03-2015 till 21-02-2020. For the stress testing we run the following testing procedure, we shock the last observation for each risk factor separately. To make the table more readable we only show the stress test results for the student-t with 3 and 6 degrees of freedom. The tables containing the results for the 97.5% confidence level and 99% confidence level can be found in APPENDIX, Tables 11 and 12. Only shocking the last observation does not impact the risk estimates using the Var-Cov methods much. Most values stay relative the same for different shocks. Especially for the commodities, FX and interest rate shocks measures are not sensitive. For the equities shock we already get quite different results. The risk measures change substantially when we shock all equities with -40%. That our risk estimates are sensitive for this shock is easily explained by the portfolio composition. Our portfolio has a monetary weight of €75,000,000 in equity assets. Shocking these assets with a 40% decrease lead us to a theoretical loss of €30,000,000. These findings are present at both 97.5% and 99% confidence levels and are more pronounced for the 99% confidence level.

Let us move to the historical simulation method results. The stress test results show us that for all shocks the VaR and ES values are higher than assuming normality, but are lower than values assuming a student-t with 6 degrees of freedom. What is also very interesting but somewhat counter intuitive is that the VaR estimates do not change depending on the shock we apply. This finding holds for each the ES estimates but only when we apply positive shocks. For negative shocks the ES estimates do change and increase in value. As a positive shock does not influences the mean loss we would experience by a VaR violation the risk measure doesn't change. This effect is only observed for the historical simulation method.

We take EWMA and CCC-Garch together as they both try to capture and forecast volatility making them conditional risk measuring methods. At first glance it is already noticeable that the risk measures are more sensitive to shocked values and take them into account almost immediately. Therefore the risk measures shoot up due to the shock and shall decrease afterwards. Even the student-t with 3 degrees of freedom has lower risk estimates than they provide. We again see that a FX shock and equity shock have the most impact. FX shocks are a bit less important than equity shocks as we have one asset, STOXX 50, which is not sensitive the FX changes. A almost strange observation is the ES estimate for a -40% shock in commodities. The ES estimate shoots up to above 11 million were no other ES estimate is shooting up that much for the same shock. However the ES in general behaves more sensitive using the EWMA method. Looking at the equity shocks we again see that for a -40% shock the EWMA ES estimate for 97.5% confidence level is more than twice as high than the CCC-Garch estimate. This same effect is even more pronounced in the 99% confidence level results, there we see an almost 4 times higher ES estimate.

A general note, interest rates have almost no impact for our VaR and ES risk measures. We can explain this using two arguments. First, our portfolio consists of Euribor linked bonds with a monetary value of €5,000,000 which is 5% of portfolio value. It is therefore of much less importance as a risk factor. Second, the historical return series of interest rates, especially daily return series, has substantial number of zero return days. The return series as a whole is much less volatile than an equity return series. Moving the interests up and down with 2 à 3% doesn't influences much of the loss distribution.

5 Conclusion and recommendations

During this assignment we've looked at various types of risk estimation methods for the VaR and ES. In this section we conclude the assignment and give our recommendation for using a specific risk estimation method for our portfolio. Based on all previous research we recommend to use the Filtered Historical Simulation method using the EWMA volatility forecasting model.

For VaR-estimation, the EWMA filtered historical simulation proves to be the best model by having the highest accuracy in most of the scenario's. We do see that the model has too many violations of the VaR in hindsight. This means that when using the EWMA-model in this exact way will likely lead to an underestimation of risk. By tweaking the model parameter λ, which in our case was set to 0.94, we can influence the model to make it less sensitive to return. We can achieve this by setting λ equal to 0.97, commonly suggested by regulators. Looking at clustering of violations, the EWMA method shows less tendency to show this pattern opposed to the more static methods which do show these patterns more often. The EWMA filtered historical simulation also shows good results for the expected shortfall. There is only a slight overestimation of risk in this case. Checking the stress test results, the EWMA method shows quick and adequate responses to various types of shocks. The CCC-Garch method also shows the same adequate response and can in this way may be used to complement the risk system.

Concluding, the EWMA is in almost all cases the best model. We do still note that for example in the VaR-computation the violations are too high most of the time. This means that we recommend caution when working with this model. We also recommend to use the EWMA method as the main estimation method and complement the risk system with the CCC-Garch model to verify observed results. Specific model adjustments can be made by looking at the λ parameter.

6 APPENDIX

Table 11: Table providing all stress tested VaR and ES values for each risk measuring method. For each risk factor different stress scenario's are calculated. This table provides the risk measures for a 97.5% confidence level.

97.5%		Normal		Stud-t, 3		Stud-t, 6		Hist sim		EWMA		CCC-Garch	
Risk Factor	Shock	VaR	ES	VaR	ES	VaR	ES	VaR	ES	VaR	ES	VaR	ES
Commodity	20%	€ 1286924	€ 1539917	€ 3665400	€ 5819200	€ 1981200	€ 2644820	€ 1367160	€ 1994914	€ 2164811	€ 2754859	€ 1395580	€ 1669520
	40%	€ 1325807	€ 1586848	€ 3779940	€ 6002250	€ 2042160	€ 2726900	€ 1367160	€ 1994914	€ 3404228	€ 4467541	€ 1245590	€ 1491170
	-20%	€ 1306191	€ 1561447	€ 3705940	€ 5879000	€ 2006670	€ 2676240	€ 1372907	€ 2098879	€ 2215117	€ 3720089	€ 1568410	€ 1874210
	-40%	€ 1422598	€ 1699264	€ 4023630	€ 6378980	€ 2181840	€ 2907560	€ 1372907	€ 2249902	€ 4670522	€ 11680128	€ 2208590	€ 2636780
FX	20%	€ 1492580	€ 1787020	€ 4260660	€ 6767260	€ 2300580	€ 3072910	€ 1367160	€ 1994910	€ 4925000	€ 6452050	€ 6872760	€ 8204370
	-20%	€ 1950630	€ 1634310	€ 4608090	€ 7300960	€ 2502350	€ 3332080	€ 1372910	€ 2503550	€ 6225290	€ 12244500	€ 9865900	€ 11769100
Equities	20%	€ 1492582	€ 1787016	€ 4260660	€ 6767260	€ 2300580	€ 3072910	€ 1367160	€ 1994914	€ 5759478	€ 7757484	€ 7557950	€ 9021660
	40%	€ 1936140	€ 2318150	€ 5527550	€ 8779730	€ 2984460	€ 3986520	€ 1367160	€ 1994910	€ 10675100	€ 14371400	€ 14371400	€ 20034900
	-20%	€ 1634310	€ 1950630	€ 4608090	€ 7300960	€ 2502350	€ 3332080	€ 1372910	€ 2499800	€ 7556070	€ 14204200	€ 10790900	€ 12872400
	-40%	€ 2649470	€ 3157620	€ 7426770	€ 11752800	€ 4043950	€ 5376890	€ 1372910	€ 3051740	€ 16741500	€ 44259900	€ 16414900	€ 19576700
Interest Rates	2%	€ 1273390	€ 1523120	€ 3621190	€ 5747220	€ 1958710	€ 2613780	€ 1367160	€ 1994910	€ 1176750	€ 1549310	€ 1094810	€ 1310120
	3%	€ 1273390	€ 1523120	€ 3621190	€ 5747220	€ 1958710	€ 2613780	€ 1367160	€ 1994910	€ 1176750	€ 1549310	€ 1094810	€ 1310120
	-2%	€ 1273390	€ 1523120	€ 3621190	€ 5747220	€ 1958710	€ 2613780	€ 1367160	€ 1994910	€ 1176750	€ 1549310	€ 1094810	€ 1310110
	-3%	€ 1273390	€ 1523120	€ 3621190	€ 5747220	€ 1958710	€ 2613780	€ 1367160	€ 1994910	€ 1176750	€ 1549310	€ 1094810	€ 1310110

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Table 12: Table providing all stress tested VaR and ES values for each risk measuring method. For each risk factor different stress scenario's are calculated. This table provides the risk measures for a 99% confidence level.

99%		Normal		Stud-t, 3		Stud-t, 6		Hist sim		EWMA		CCC-Garch	
Risk Factor	Shock	VaR	ES	VaR	ES	VaR	ES	VaR	ES	VaR	ES	VaR	ES
Commodity	20%	€ 1532247	€ 1759145	€ 5240630	€ 8096360	€ 2551760	€ 3281500	€ 1806476	€ 2610442	€ 2575756	€ 3291778	€ 1661220	€ 1906900
	40%	€ 1578934	€ 1813049	€ 5405280	€ 8351840	€ 2630880	€ 3383830	€ 1806476	€ 2610442	€ 4376335	€ 5333655	€ 1483720	€ 1703970
	-20%	€ 1553708	€ 1782635	€ 5295260	€ 8176520	€ 2582340	€ 3318610	€ 1905676	€ 2817065	€ 2719612	€ 5624879	€ 1864940	€ 2139200
	-40%	€ 1690877	€ 1939006	€ 5746260	€ 8869220	€ 2805790	€ 3603820	€ 1905676	€ 3187758	€ 6164892	€ 21263958	€ 2623800	€ 3007820
FX	20%	€ 1778090	€ 2042150	€ 6093920	€ 9417430	€ 2964600	€ 3813880	€ 1806480	€ 2610440	€ 6316400	€ 7579200	€ 8164000	€ 9358270
	-20%	€ 2224730	€ 1941040	€ 6577580	€ 10148100	€ 3215720	€ 4128110	€ 1905680	€ 3810340	€ 8581640	€ 19723500	€ 11711400	€ 13418300
Equities	20%	€ 1778089	€ 2042154	€ 6093920	€ 9417430	€ 2964600	€ 3813880	€ 1806476	€ 2610442	€ 7680493	€ 9857214	€ 8977280	€ 10290000
	40%	€ 2306560	€ 2649170	€ 7906100	€ 12218200	€ 3846000	€ 4947880	€ 1806480	€ 2610440	€ 14294500	€ 18375200	€ 18375200	€ 22847200
	-20%	€ 1941040	€ 2224730	€ 6577580	€ 10148100	€ 3215720	€ 4128110	€ 1905680	€ 3801140	€ 10598700	€ 22897400	€ 12809300	€ 14676100
	-40%	€ 3142210	€ 3597950	€ 10590700	€ 16326600	€ 5189960	€ 6655690	€ 1905680	€ 5155900	€ 23458100	€ 82603500	€ 19480800	€ 22316500
Interest Rates	2%	€ 1515550	€ 1739520	€ 5176110	€ 7995010	€ 2521910	€ 3242240	€ 1806480	€ 2610440	€ 1524260	€ 1875680	€ 1303590	€ 1496690
	3%	€ 1515550	€ 1739520	€ 5176110	€ 7995020	€ 2521920	€ 3242240	€ 1806480	€ 2610440	€ 1524260	€ 1875670	€ 1303590	€ 1496690
	-2%	€ 1515550	€ 1739520	€ 5176110	€ 7995010	€ 2521910	€ 3242240	€ 1806480	€ 2610440	€ 1524260	€ 1875680	€ 1303590	€ 1496680
	-3%	€ 1515550	€ 1739520	€ 5176110	€ 7995020	€ 2521920	€ 3242240	€ 1806480	€ 2610440	€ 1524260	€ 1875670	€ 1303590	€ 1496680