Multi-Assembly Pin-By-Pin Flux Solution Using Stochastic and Deterministic Methods

ENU 6106 : Project Description

1.0 Overview

This is a semester long project that will be completed as part of homework and project assignments. This project will take a significant amount of time to complete so start working on it now.

In this project you will model the one-dimensional two-group flux distribution in fuel assemblies. For this project we will use two approaches that are commonly used to model neutronic behavior in general:

1. Monte Carlo Methods (stochastic) (Project Part 1)
2. Finite Difference Methods (deterministic) (Project Part 2)

Write a program to model two adjacent fuel assemblies of varying material. The results will demonstrate the moderating effect of the surrounding water channels and the effect on neutron flux due to differing scatter and absorption cross sections. Your program will simulate the multigroup flux, in one dimension, for a UO_2 and a MOX fuel assembly placed in different configurations. Each assembly will have the following dimensions as shown in Figure 1:

Number of Fuel Rods: $N = 17$

Fuel Rod Diameter: $D_{rod} = 0.94\ cm$

Fuel Rod Pitch: $P = 1.262\ cm$

Assembly Size: $L_{assembly} = 21.42\ cm$

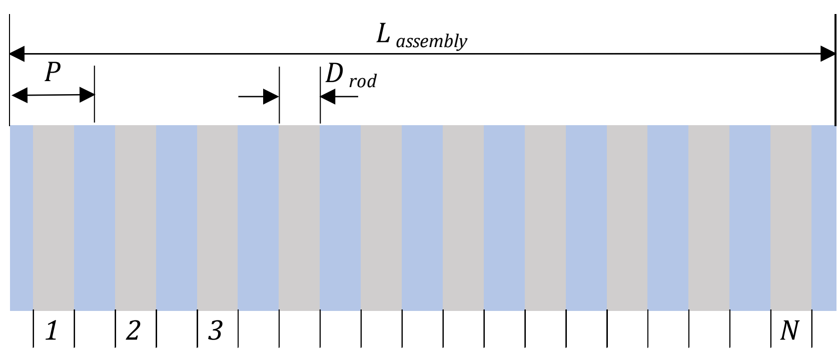

Figure 1: Assembly without control rods.

2.0 Verification of Code Results, Mesh, and Convergence Study

For this part of the project, you will use the following two configurations to verify the accuracy of your results, perform a mesh sensitivity study, and show proper convergence of your solution. For this part of the project use only ***Test Cross Section Set A, Table 1.*** Assembly Configuration 1 & 2, shown in Figure 2 and Figure 3, represent an infinite set of UO_2 and MOX assemblies respectively. For both the Monte Carlo

Method and the Finite difference methods, run your code for these two configurations and show that your code is getting reasonable flux and multiplication factor values. Show that your discretization is reasonable, and the solution has converged.

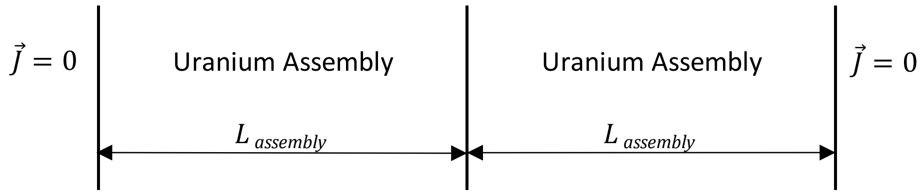

Figure 2: Assembly configuration 1 for part 1 of the project.

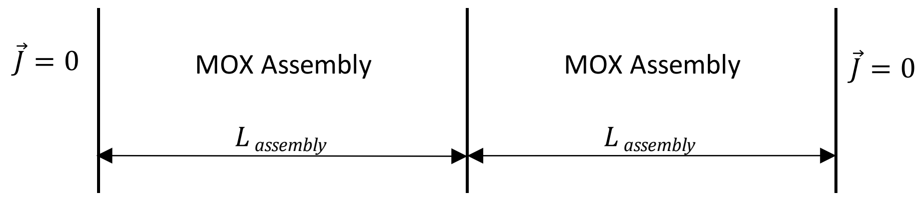

Figure 3: Assembly configuration 2 for part 1 of the project.

Table 1: Test Cross Section Set A

Energy Group 1	Σ_{tr}^1	$\Sigma_{s,0}^{1\to1}$	$\Sigma_{s,0}^{1\to2}$	Σ_a^1	Σ_f^1	ν_f^1	χ^1
Uranium Fuel	0.2	0.185	0.015	0	0	0	1
MOX Fuel	0.2	0.185	0.015	0	0	0	1
Water	0.2	0.17	0.03	0	0	0	0
Energy Group 2	Σ_{tr}^2	$\Sigma_{s,0}^{2\to2}$	$\Sigma_{s,0}^{2\to1}$	Σ_a^2	Σ_f^2	ν_f^2	χ^2
Uranium Fuel	1.0	0.8	0	0.2	0.18	1.4	0
MOX Fuel	1.2	0.8	0	0.4	0.3	1.5	0
Water	1.1	1.1	0	0	0	0	0

At a minimum, you should:

a. **Verification:** To verify your results, compare your calculated multiplication factor with the actual multiplication factor for each configuration.
b. **Verification:** Plot the fast and thermal flux for the two assemblies to verify the solution looks ok. You may want to include additional analysis to help verify that the solution is correct. For example, you can calculate the average flux in each assembly and compare those values.
c. **Mesh Convergence Study:** You should perform a mesh sensitivity study by changing the mesh size until the change of your results are at an acceptable level. *NOTE: the finite difference results will be affected more by the mesh size. However, you should still perform this study with the Monte Carlo Method to determine an appropriate number of tally regions.*
d. **Numerical Convergence Study:** For the Monte Carlo method you will have to determine the appropriate number of histories (particles simulated) and number of generations. You should vary these to determine the optimal values of each. Once you have determined the number of

generations, show that the error in your Monte Carlo solution varies as $1/\sqrt{histories}$ (i.e. show that to reduce the error in the solution by an order of magnitude, you will have to increase the number of histories by two orders of magnitude.)

e. **Numerical Convergence Study:** For the finite difference method you will be given convergence criteria. You should verify that these are appropriate values by varying them and looking at how your solution changes.

3.0 Flux Analysis of Varying Assemblies and Energy Group Structure

Flux Magnitude

Now that your code is verified and is working properly, you have an appropriate mesh size, and your solutions converge, we are going to make one last modification to the code before you perform the following analysis. Determine the magnitude of the flux for all analysis in section 3.0 by applying the power condition to your solution. Use a reactor power of $3565\ MW_{th}$ and assume $200\ MeV/fission$. Document any additional assumptions that you make. Compare the value you obtained in your analysis to that of a standard power reactor. Provide all necessary calculations and plots.

Energy Group Analysis

You can now start to do some analysis with your code. You will use the following three configurations Figure 4, Figure 5, and Figure 6 to investigate how the flux shapes and multiplication factors change for different configurations and different number of energy groups. For this part of the project use Table 2: Test Cross Section Set B and Table 3: Test Cross Section Set C.

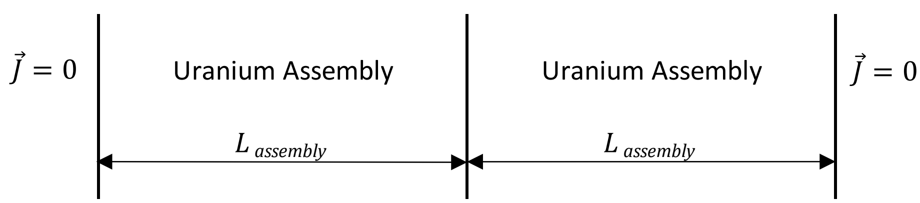

Figure 4: Assembly configuration 1 for part 2 of the project.

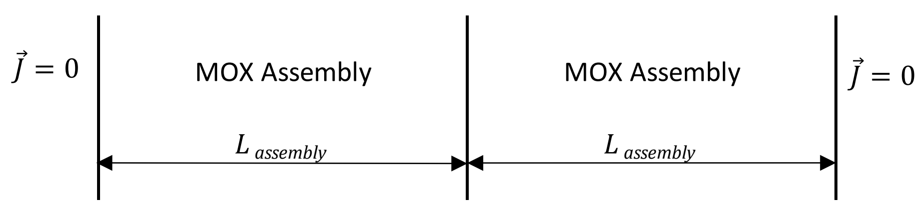

Figure 5: Assembly configuration 2 for part 2 of the project.

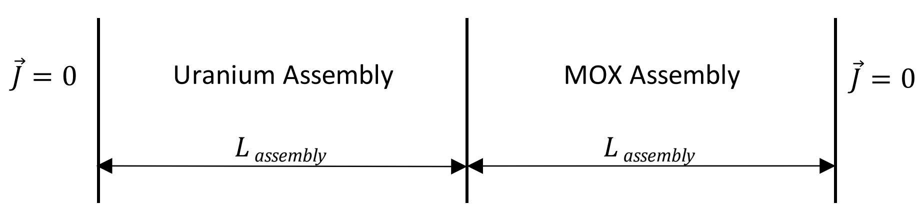

Figure 6: Assembly configuration 3 for Part 2 of the project.

Table 2: Test Cross Section Set B

Energy Group 1	Σ_t^1	Σ_s^1	$\bar{\mu}^1$	Σ_a^1	Σ_f^1	v_f^1	χ^1
Uranium Fuel	0.534002	0.523864	0.412142	0.010100	0.003121	2.535540	1.000000
MOX Fuel	0.524927	0.510744	0.463140	0.014135	0.003584	2.876540	1.000000
Water	0.666894	0.666236	0.825072	0.000657	0.000000	0.000000	0.000000
Energy Group 2	Σ_t^2	Σ_s^2	$\bar{\mu}^2$	Σ_a^2	Σ_f^2	v_f^2	χ^2
Uranium Fuel	1.331240	1.232240	0.345351	0.099006	0.067817	2.436200	0.000000
MOX Fuel	1.462150	1.195500	0.332065	0.266653	0.152776	2.858410	0.000000
Water	1.946990	1.927130	0.235732	0.019862	0.000000	0.000000	0.000000

UO2 Scattering Matrix		g	
		1	2
g’	1	0.507186	0.001769
	2	0.016678	1.230470

MOX Scattering Matrix		g	
		1	2
g’	1	0.498446	0.004290
	2	0.012298	1.191210

H2O Scattering Matrix		g	
		1	2
g’	1	0.625799	0.000334
	2	0.040438	1.926800

Table 3: Test Cross Section Set C

Energy Group 1	Σ_t^1	Σ_s^1	$\bar{\mu}^1$	Σ_a^1	Σ_f^1	v_f^1	χ^1
Uranium Fuel	0.207790	0.203713	0.108199	0.003929	0.003246	2.805800	0.698645
MOX Fuel	0.208364	0.203567	0.170009	0.004616	0.003885	2.913020	0.709543
Water	0.176541	0.176196	0.640062	0.000345	0.000000	0.000000	0.000000
Energy Group 2	Σ_t^2	Σ_s^2	$\bar{\mu}^2$	Σ_a^2	Σ_f^2	v_f^2	χ^2
Uranium Fuel	0.599466	0.593633	0.403134	0.005833	0.001185	2.439000	0.301355
MOX Fuel	0.591052	0.584549	0.411846	0.006503	0.001237	2.870630	0.290456
Water	0.745980	0.745900	0.745836	0.000080	0.000000	0.000000	0.000000
Energy Group 3	Σ_t^3	Σ_s^3	$\bar{\mu}^3$	Σ_a^3	Σ_f^3	v_f^3	χ^3

Uranium Fuel	0.833793	0.797272	0.457012	0.036521	0.010191	2.434030	0.000000
MOX Fuel	0.849673	0.784157	0.475422	0.065516	0.013182	2.857790	0.000001
Water	1.078190	1.075750	0.566563	0.002441	0.000000	0.000000	0.000000
Energy Group 4	$\boldsymbol{\Sigma_t^4}$	$\boldsymbol{\Sigma_s^4}$	$\boldsymbol{\bar{\mu}^4}$	$\boldsymbol{\Sigma_a^4}$	$\boldsymbol{\Sigma_f^4}$	$\boldsymbol{\nu_f^4}$	$\boldsymbol{\chi^4}$
Uranium Fuel	1.330170	1.231070	0.345974	0.099101	0.067901	2.436200	0.000000
MOX Fuel	1.462940	1.196590	0.339269	0.266341	0.152613	2.858430	0.000000
Water	1.946930	1.927070	0.234071	0.019861	0.000000	0.000000	0.000000

UO2 Scattering Matrix		g			
		1	2	3	4
g’	1	0.136536	0.000000	0.000000	0.000000
	2	0.067172	0.555910	0.000000	0.000000
	3	0.000005	0.037506	0.690652	0.001813
	4	0.000000	0.000217	0.106620	1.229260

MOX Scattering Matrix		g			
		1	2	3	4
g’	1	0.138546	0.000000	0.000000	0.000000
	2	0.065017	0.549150	0.000000	0.000000
	3	0.000004	0.035199	0.695658	0.004219
	4	0.000000	0.000200	0.088499	1.192380

H2O Scattering Matrix		g			
		1	2	3	4
g’	1	0.100009	0.000000	0.000000	0.000000
	2	0.076180	0.669911	0.000000	0.000000
	3	0.000007	0.075530	0.888667	0.000332
	4	0.000000	0.000459	0.187086	1.926740

At a minimum, you should:

a. **Multiplication Factor :** Provide me with the two and four group multiplication factors for all three configurations.
b. **Flux:** Plot the group flux for the two assemblies to verify the solution looks ok. You may want to include additional analysis to help verify that the solution is correct. For example, you can calculate the average flux in each assembly and compare those values.

4.0 Cross Section Homogenization and Energy Collapse

For this part of the project, you will use the converged four group flux distributions calculated above to collapse the four group cross sections to **both two energy groups and single energy group** and homogenize the cross sections for each assembly. This part of the project will only be done for assembly configurations 1 and 2 part 2 of the project, using Table 3: Test Cross Section Set C data.

At a minimum, you should:

a. Provide me with the one and two group homogenized cross sections for both the UO2 and MOX assemblies.
b. Use the flux calculated by both the Monte Carlo method and finite difference method for the cross section collapsing and compare the final cross sections and multiplication factors.
c. Compare the two group cross sections to those given in Table 2: Test Cross Section Set B.

4.1 Extra Credit 1 (Up to 5 points)

Add a water reflector to the right boundary of the two-assembly configuration, Figure 7 and Figure 8. Compare the flux distributions in both cases. Next, replace the water reflector with a vacuum boundary condition. Using the results from these calculations try to estimate the reflector savings for both configurations. Use only Table 2: Test Cross Section Set B**.** Provide all necessary calculations and plots.

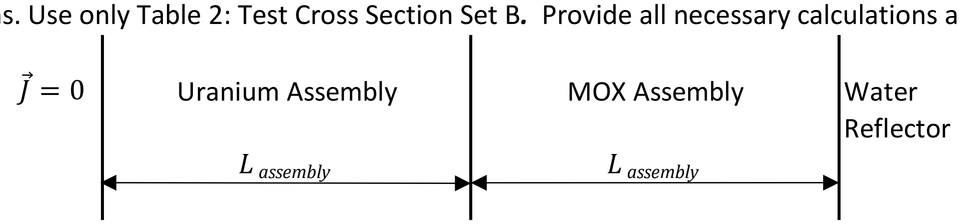

Figure 7: Assembly configuration 1 for water reflector problem.

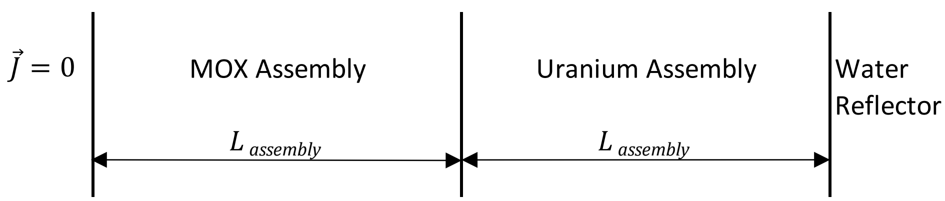

Figure 8: Assembly configuration 2 for water reflector problem.

4.2 Extra Credit 2 (Up to 5 points)

Insert control rods in positions 3 and 15 of the left assembly and rerun the analysis for section 3.0. Compare the results with those without the control rod inserted. Use only Table 2: Test Cross Section Set B**.** Provide all necessary calculations and plots.

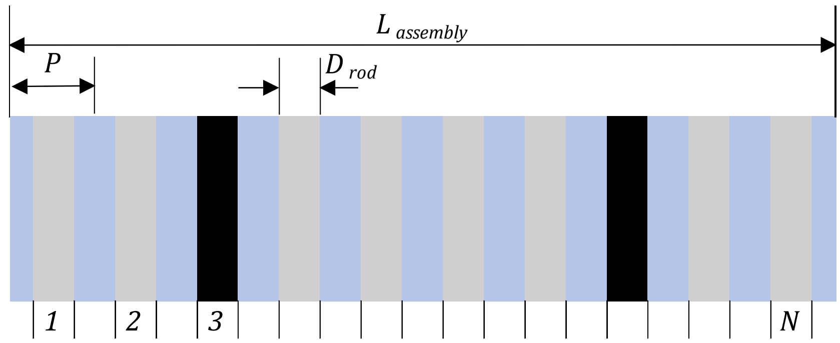

Figure 9: Assembly with control rods in positions 3 and 15.

Table 4: Control rod cross sections for extra credit.

Cross Sections	Σ_{tr}^{1}	$\Sigma_{s,0}^{1\to1}$	$\Sigma_{s,0}^{1\to2}$	Σ_{a}^{1}	Σ_{f}^{1}	ν_{f}^{1}	χ^{1}
Control Rod	0.1	0	0	0.1	0	0	0
Cross Sections	Σ_{tr}^{2}	$\Sigma_{s,0}^{2\to2}$	$\Sigma_{s,0}^{2\to1}$	Σ_{a}^{2}	Σ_{f}^{2}	ν_{f}^{2}	χ^{2}
Control Rod	1.1	0.1	0	1.0	0	0	0

5.0 Report Requirements

Provide me with a report documenting your work. Your report should be standalone (i.e. someone not familiar with this project should be able to read your report and understand the project). The report should be executed in a professional manner. This means that I do not want to see copy and pasts of text or equations from the project outline or other sources. Restate the problem and objectives in your own words and type all equations. At a minimum your report should include:

Title Page

Table of Contents

Table of Figures

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Abstract

One page summary of the project and important results.

Introduction

This should include a project description and show any figures or data needed for the project. Do NOT copy and paste the figures or tables from the project description.

Methods

Summary of the solution methods used to include any equations, flow diagrams, code descriptions, etc. Document your Monte Carlo solution as well as the finite difference solution you used for this project. This should include all assumptions you made, coefficients used, etc.

Results and Analysis

This section should contain a detailed summary and analysis of the results. At a minimum the report should include the following plots and corresponding analysis for each solution method (Monte Carlo and Diffusion) except where noted.

- Describe the mesh chosen for each solution method and your rational for selecting these values. If you changed your mesh size to see how the results changed then include a discussion of this as well. Your mesh may be different between the Monte Carlo and Finite Difference methods.
- Describe the number of histories, number generations, and discarded generations used and you're your rational for your selection. This should include a discussion on how changing the number of histories and generations change the results (Monte Carlo only). You should also show how the error in the Monte Carlo method changes as you change the number of histories.
- Discussion of any numerical issues associated with numerical matrix solutions (diffusion with gauss seidel or SOR method only).
- Plots: You may include any plots you want that help you describe specific aspects of your analysis. However, the report should contain at least the following figures.
  - Plots that show how your simulation results change when you change the number of generations and histories.
  - Plot the multiplication factor as a function of generation (Monte Carlo Only).
  - Plot the fundamental modes for all flux, current, and multiplication factors (Monte Carlo) and the final flux and current (diffusion). Discuss the results for all three different configurations. This discussion should include how and why the flux and current change for each fuel type.
  - Compare your calculated k_∞ results to hand calculated k_∞ values for both solution methods for configurations 1 and 2. Discuss how each of the results compare to the hand calculated value.
- Compare the results for the two methods (Monte Carlo & diffusion) and discuss the results. You may want to include additional plots overlying the results of the two solution methods. You could also calculate the average flux in a unit cell to compare the results (suggestions).
- You should compare the computational speed vs accuracy of the two solution methods. For example, you may want to include computation time in your k_∞ table.

Conclusion

Short summary of your results and major findings/conclusions.

Source code included as a separate file.