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A minimal introduction to probability theory, statistical inference & hypothesis testing

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Contents

Preface

This is a minimal example of a probability and statistics book. The purpose of this preface is simply to allow the document to compile cleanly.

Introduction

Even fire obeys the laws of numbers.
— J.B. Joseph Fourier

A bit of history

As one might expect, the origins of probability and related concepts can be traced back to very ancient times. Civilizations such as the Babylonians, Egyptians, and Greeks already encountered uncertainty in various aspects of life, including commerce, games of chance, and divination. Consequently, notions of randomness and stochasticity have deep historical roots. For instance, archaeological findings suggest that the earliest known dice date back over 5,000 years, reflecting humanity's early fascination with chance and unpredictability [**?**]. Although these cultures had not yet developed a formal mathematical theory of probability, they recognized recurring patterns in random events and attempted to anticipate outcomes through either empirical observation or superstition. For a detailed historical overview, see Florence Nightingale's 1962 manuscript *"Games, Gods and Gambling"* [**?**].

While classical Greek and Roman philosophers frequently discussed the nature of chance, necessity, and determinism, their inquiries remained primarily philosophical rather than mathematical. Thinkers such as Cicero distinguished between events occurring by chance and those determined by fate, foreshadowing later developments in probability theory [**?**]. These early ideas, though lacking quantitative formalism, provided the intellectual foundation for later scientific inquiry into randomness and causality.

A significant shift occurred during the late medieval and early Renaissance periods, when more rigorous mathematical ideas began to shape. Italian mathematician and gambler Gerolamo Cardano (1501–1576) made substantial contributions to the mathematical analysis of chance. His work *"Liber de Ludo Aleae"* (*"Book on Games of Chance"*) [**?**], posthumously published in 1663, is one of the earliest known texts to explore probability through the analysis of gambling problems. However, Cardano's reasoning, while insightful, lacked the symbolic clarity and mathematical rigour of modern probability theory. Readers consulting the original manuscript will notice an ambiguous and sometimes inconsistent symbolic system, quite unlike the formal structures we use nowadays.

The formalization of probability as a mathematical discipline did not occur until the 17th century, most notably through the seminal correspondence between Blaise Pascal and Pierre de Fermat. Their work, motivated by problems such as finding a fair division of stakes in interrupted games of chance, introduced foundational concepts such as combinatorics, expected value, and variance [**?**]. These developments paved the way for later contributions by Christiaan Huygens, who in 1657 wrote the first published textbook on probability *"De Ratiociniis in Ludo Aleae"* [**?**], and Jacob Bernoulli, whose 1713 *"Ars Conjectandi"* remains among the most influential early texts in the field. Their works, along with many others, collectively laid the groundwork for the probabilistic and statistical methods that foreshadow modern scientific reasoning [?, ?].

During the 19th century, probability theory began to intertwine more deeply with statistics and the emerging mathematical analysis of physical phenomena. Florence Nightingale, best known for her pioneering role in modern nursing, also made significant contributions to statistical methodology and graphical representation of data. Her use of polar area diagrams and her advocacy for statistical reasoning in public health policy helped popularize quantitative approaches to uncertainty and variation. Around the same period, Joseph Fourier's work on heat conduction introduced Fourier series and integral transforms, tools that would later become indispensable for studying random processes, including the analysis of signals, noise, and diffusion phenomena. Although Nightingale and Fourier approached problems of uncertainty from very different perspectives—one through empirical data on human wellbeing, the other through mathematical physics—their contributions expanded the reach of probabilistic thinking and prepared the ground for future developments in stochastic analysis.

A further conceptual leap occurred in the early 20th century with the work of Andrey Markov. Motivated partly by a desire to extend the law of large numbers beyond the assumption of independent trials, Markov developed what are now known as Markov chains, thereby inaugurating the study of dependence structures in stochastic processes. His investigations demonstrated that long-run statistical regularities could emerge even when successive events were not independent, a discovery that profoundly influenced both theoretical probability and its applications in fields as diverse as statistical mechanics, linguistics, economics, and modern machine learning.

The modern axiomatic formulation of probability was introduced in the early 20th century by the Russian mathematician Andrey Kolmogorov. In his 1933 monograph *"Grundbegriffe der Wahrscheinlichkeitsrechnung"* [?], Kolmogorov synthesized classical and frequentist ideas into a rigorous mathematical framework based on measure theory. His axioms remain the standard foundation for probability theory to this day. It may seem surprising that a concept with such ancient origins was not formally axiomatized until relatively recent times, and we will return to Kolmogorov's formulation and its implications in greater detail in Chapter 5. Nevertheless, philosophical discussions about the interpretation of probability and its relation to the physical sciences—especially in the context of determinism, epistemology, and modern topics such as quantum mechanics—predate Kolmogorov's formulation and continue to evolve to this day.

Chapter 1

Descriptive statistics

Statistics is the grammar of science.

— Karl Pearson

1.1 Population and sampling

1.2 Central tendency and variation

1.3 Data visualization

Exercises

1. Exercise [...].

2. Exercise [...].

3. Exercise [...].

Solutions

1. Solution [...].

2. Solution [...].

3. Solution [...].

Chapter 2

Foundations of Probability

It is through the calculation of probabilities that the divine order becomes visible.

— Jacob Bernoulli

2.1 What is probability?

2.2 Discrete random variables

2.2.1 Bernoulli events

2.2.2 Binomial distribution

2.2.3 Poisson distribution

2.2.4 Discrete uniform distribution

2.3 Continuous random variables

2.3.1 Continuous uniform distribution

2.3.2 Exponential distribution

2.3.3 Gaussian distribution

2.4 Expectation values

Exercises

1. Exercise [...].

2. Exercise [...].

3. Exercise [...].

Solutions

1. Solution [...].

2. Solution [...].

3. Solution [...].

Chapter 3

Estimation, prediction and inference

Numbers have an important story to tell, if given a voice.

— Florence Nightingale

3.1 Prediction vs inference

3.2 The Law of Large Numbers

3.3 The Central Limit Theorem

3.4 Application to Generalized Linear Models

Exercises

1. Exercise [...].

2. Exercise [...].

3. Exercise [...].

Solutions

1. Solution [...].

2. Solution [...].

3. Solution [...].

Chapter 4

Introduction to hypothesis testing

The object of statistical science is the reduction of data to relevant information.

— Ronald A. Fisher

4.1 Prediction vs inference revisted

4.2 General approach to hypothesis testing

4.3 Statistical tests: some examples

4.3.1 Compare sample mean with hypothesized value - One sample t-test

4.3.2 Compare sample means of two independent groups - Two sample t-test

4.3.3 Compare variation on two groups - Fisher's exact test

4.3.4 Compare variation o multiple groups - Fisher's ANOVA

4.3.5 Compare distributions and testing for normality - χ^2 test

4.4 Parametric and non-parametric tests

4.5 Comparing data and normalization

Exercises

1. Exercise [...].

2. Exercise [...].

3. Exercise [...].

Solutions

1. Solution [...].

2. Solution [...].

3. Solution [...].

Chapter 5

Modeling, dependency and correlation

The theory of probabilities is at bottom nothing but common sense reduced to calculation.
— Pierre-Simon Laplace

Chapter 6

Introduction to conditional probability

Probability statements are just summaries of repeated observations.

— W. V. Quine

6.1 Motivation and philosophy

6.2 Dependent and independent events

6.3 Some examples of conditional probability

Exercises

1. Exercise [...].

2. Exercise [...].

3. Exercise [...].

Solutions

1. Solution [...].

2. Solution [...].

3. Solution [...].

Chapter 7

Stochasticity and Markov Processes

The development of mathematics is a continuous process of abstraction.

— Emmy Noether

7.1 Motivation and philosophy

7.2 Mathematical definition

7.3 Some examples of conditional probility

7.4 Stochasticity and Markov processes

Exercises

1. Exercise [...].

2. Exercise [...].

3. Exercise [...].

Solutions

1. Solution [...].

2. Solution [...].

3. Solution [...].

Appendix A

Appendix A

Additional examples and computations may be placed here.

Appendix B

Appendix B

Additional examples and computations may be placed here.

Appendix C

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Additional examples and computations may be placed here.

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