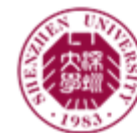




## 实验2：平面最近点对

---

报告人：郑杨 陈敏涵



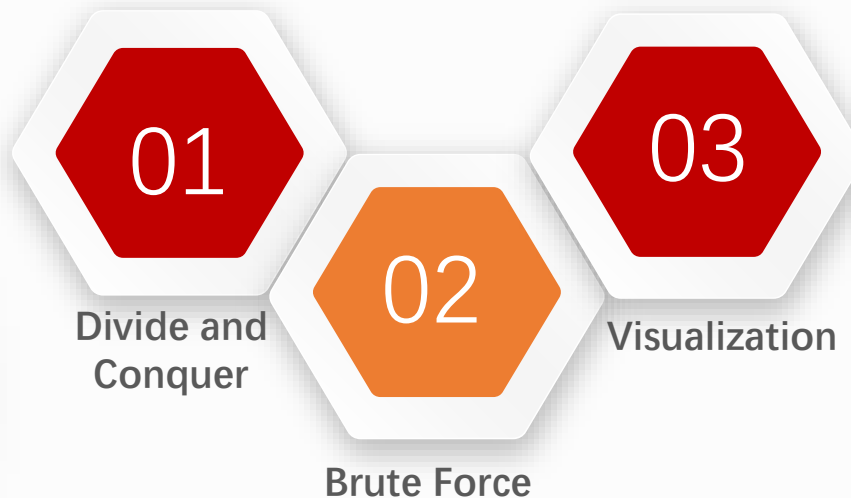
深圳大学  
SHENZHEN UNIVERSITY

# Outline

---



深圳大学  
SHENZHEN UNIVERSITY





深圳大学  
SHENZHEN UNIVERSITY

01

PART 01

SECTION 1

# Divide and Conquer

# Divide and Conquer

---



深圳大学  
SHENZHEN UNIVERSITY

- *Algorithm Process*
- *Complexity Analysis*
- *Experiment*

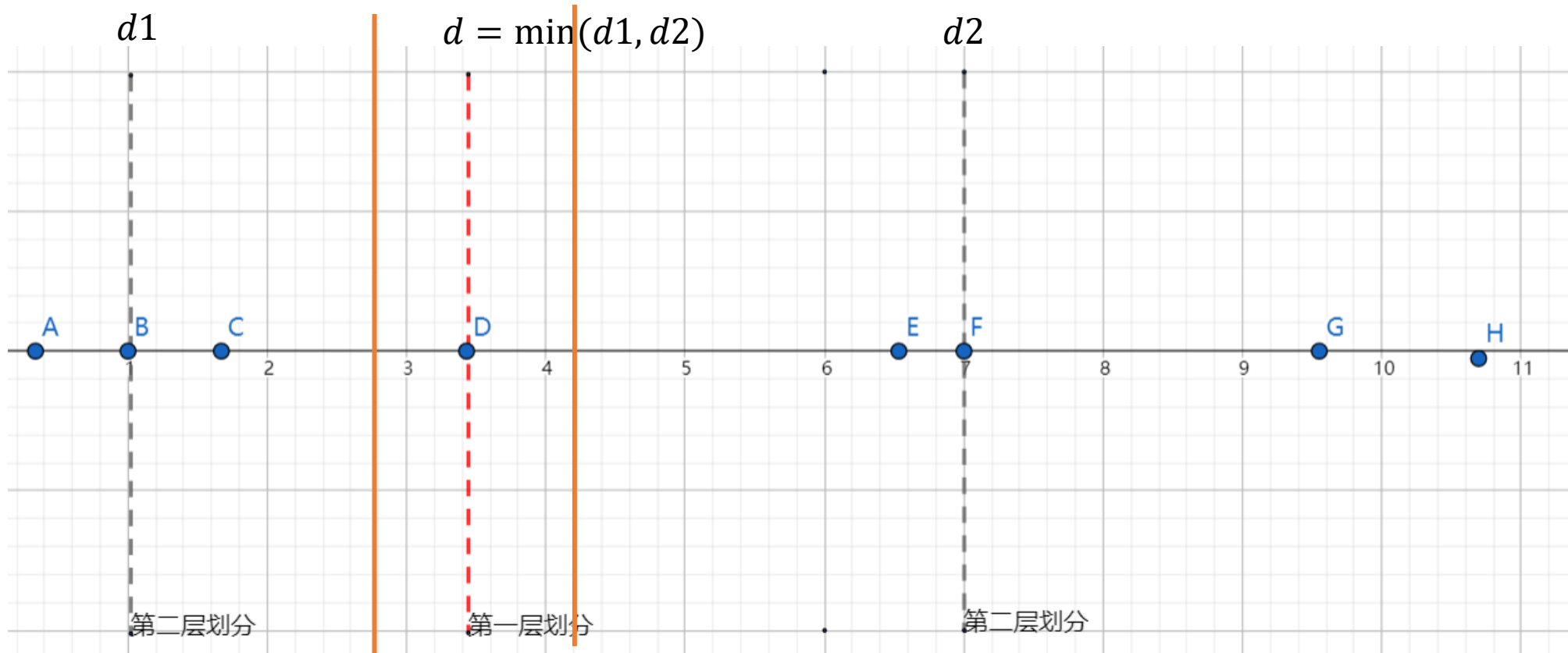
# Divide and Conquer



深圳大学  
SHENZHEN UNIVERSITY

- *Algorithm Process*

- *one dimension*



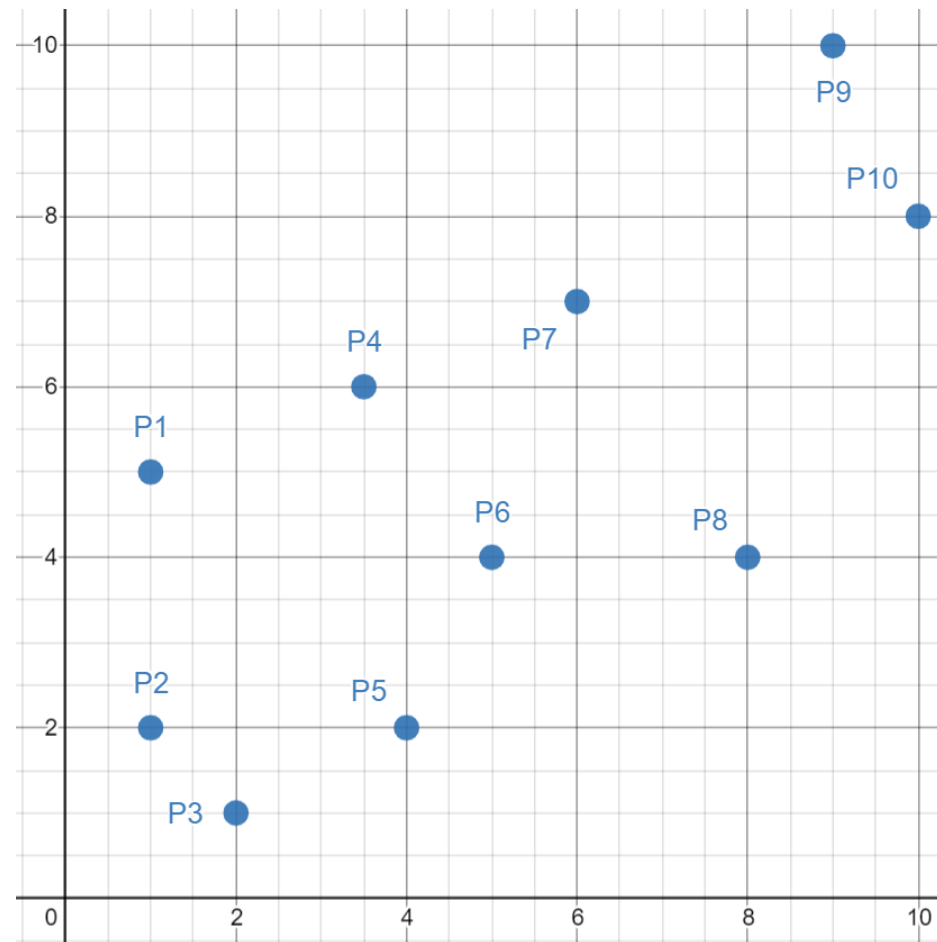
# Divide and Conquer



深圳大学  
SHENZHEN UNIVERSITY

- *Algorithm Process*

- *one dimension*

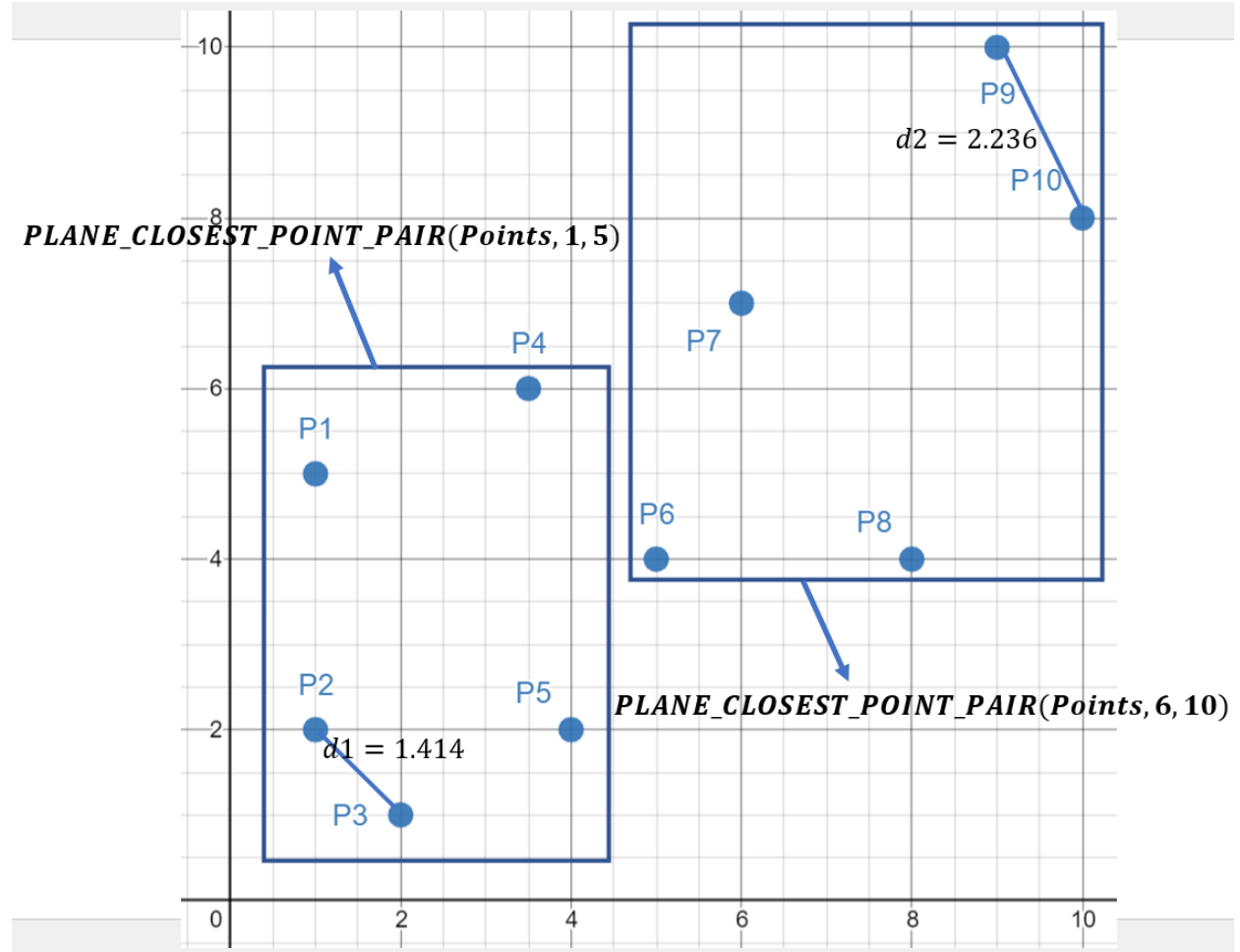


# Divide and Conquer



深圳大学  
SHENZHEN UNIVERSITY

- *Algorithm Process*
- *two dimension*

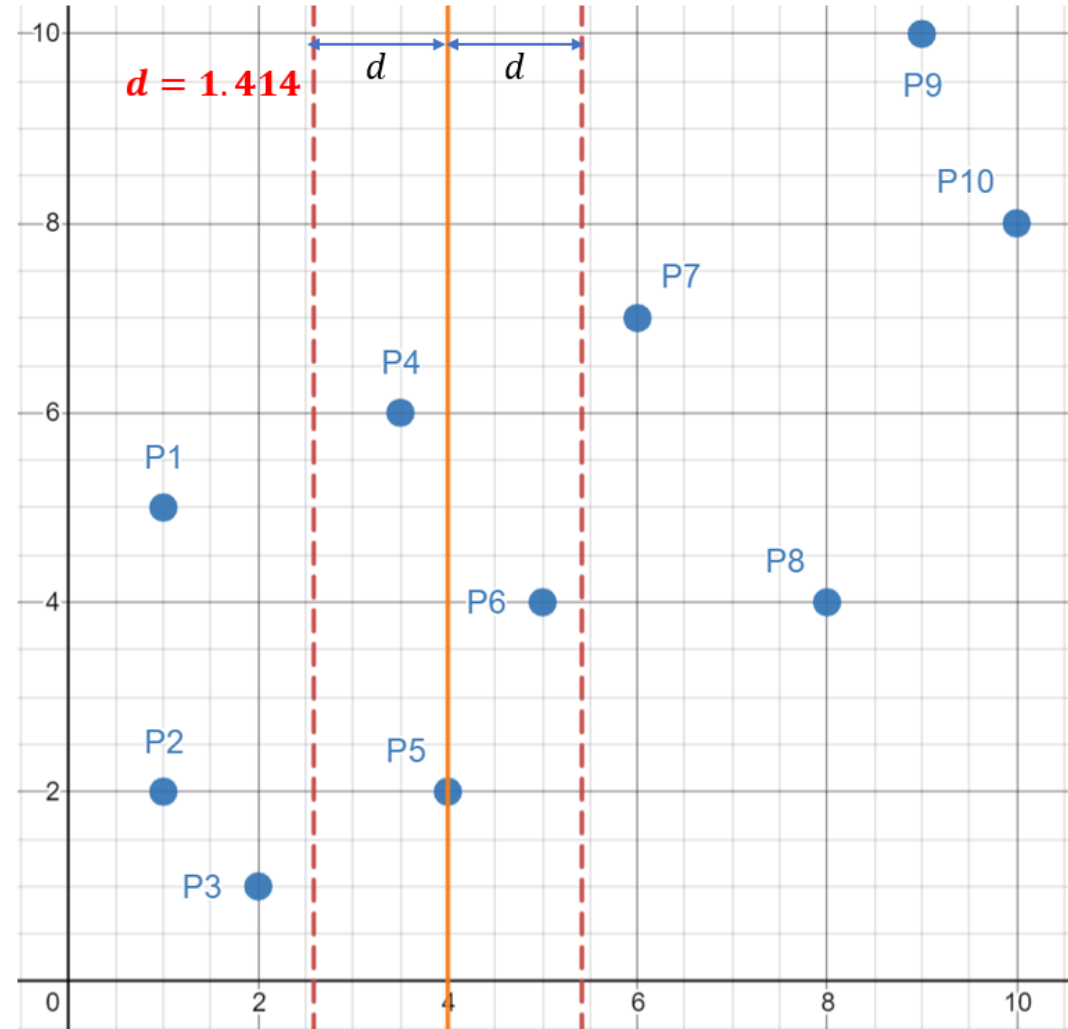


# Divide and Conquer



深圳大学  
SHENZHEN UNIVERSITY

- *Algorithm Process*
  - *two dimension*



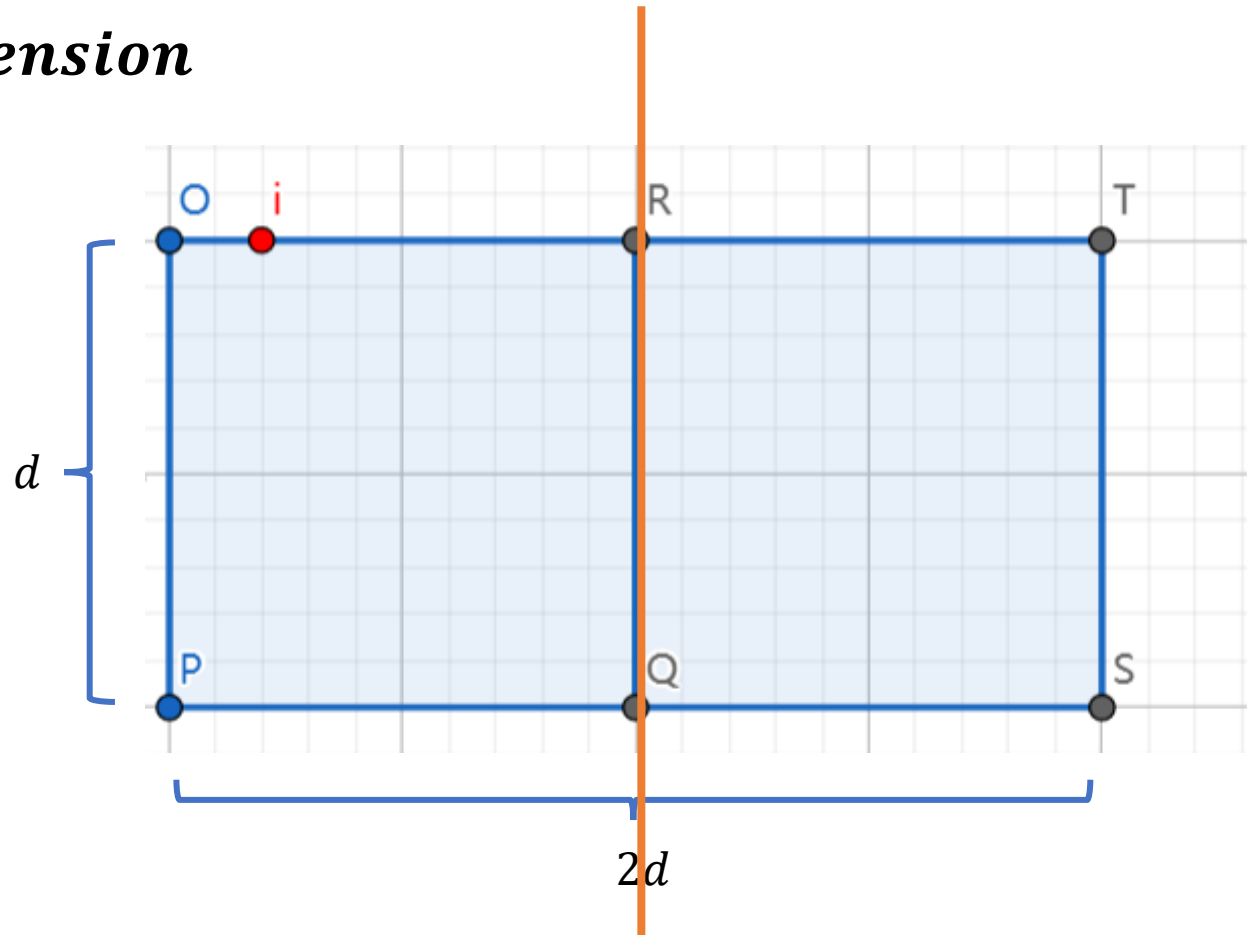


# Divide and Conquer



深圳大学  
SHENZHEN UNIVERSITY

- *Algorithm Process*
  - *two dimension*

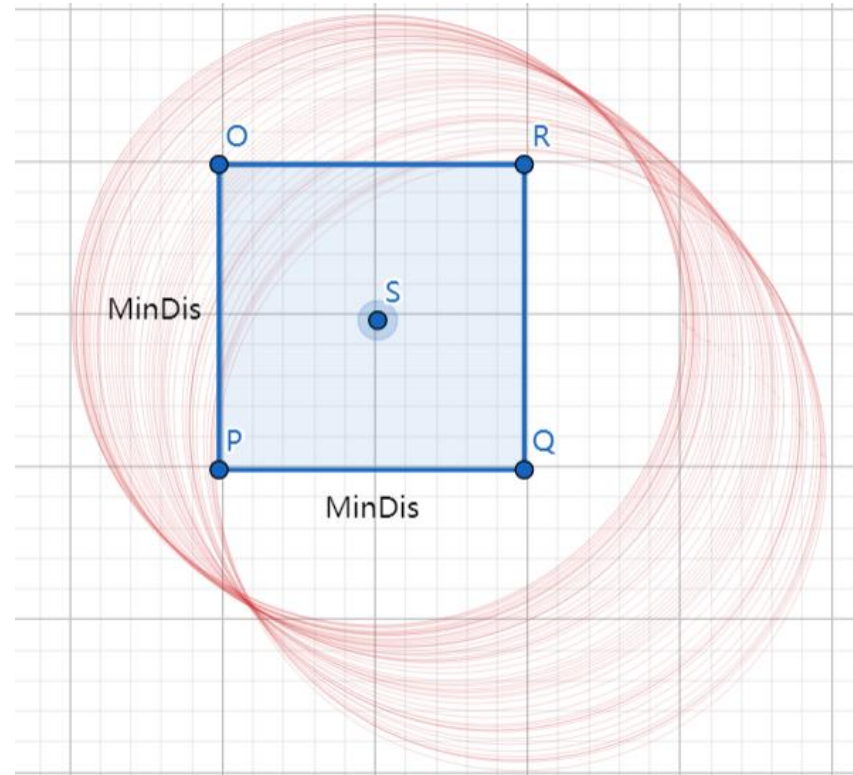
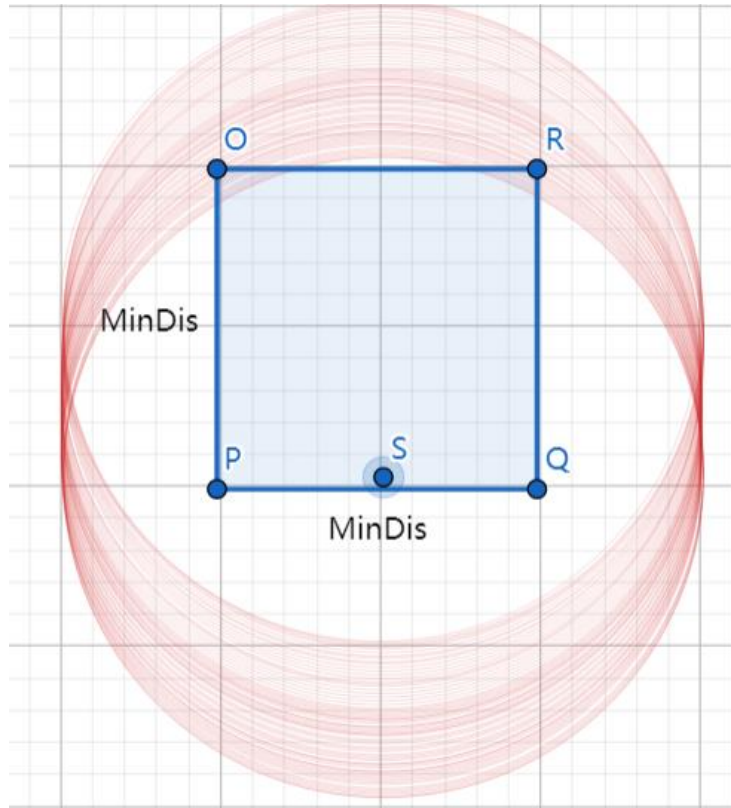


# Divide and Conquer



深圳大学  
SHENZHEN UNIVERSITY

- *Algorithm Process*
- *two dimension*

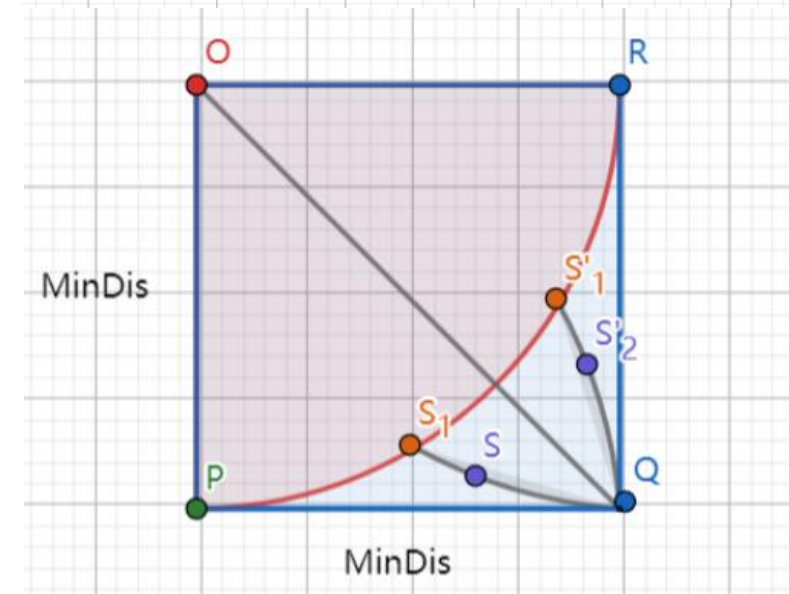
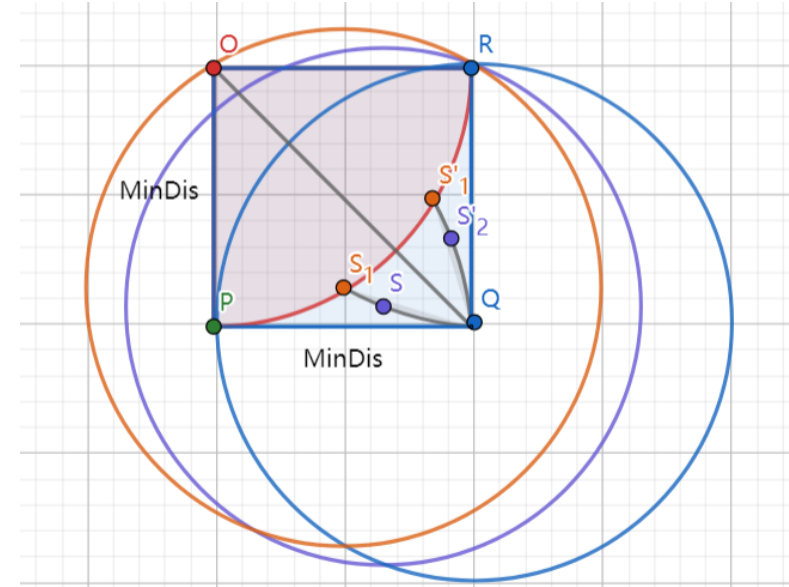
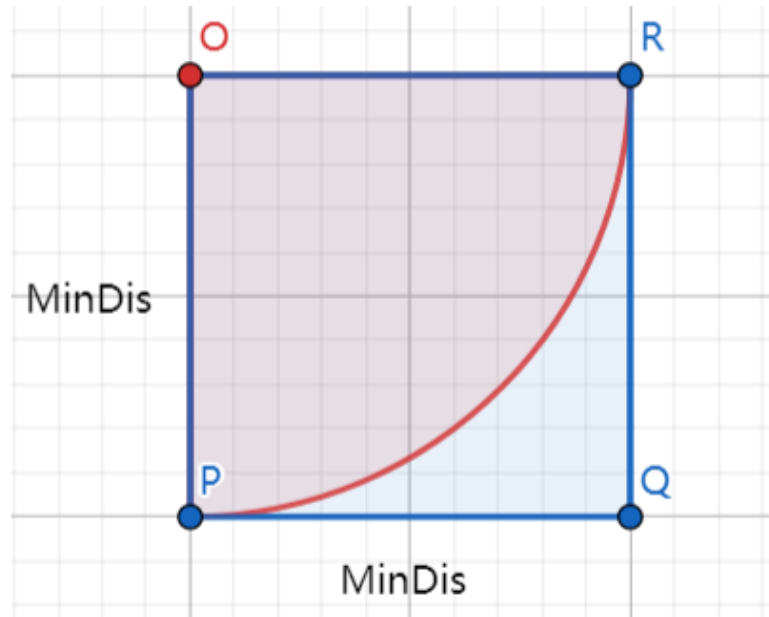


# Divide and Conquer



深圳大学  
SHENZHEN UNIVERSITY

- *Algorithm Process*
- *two dimension*



# Divide and Conquer

---



深圳大学  
SHENZHEN UNIVERSITY

- *Algorithm Process*
  - *two dimension*

# Divide and Conquer



深圳大学  
SHENZHEN UNIVERSITY

- *Complexity Analysis*

$$T(n) = O(n \log n) + t(n)$$

**PLANE\_CLOSEST\_POINT\_PAIR**(Points, l, r)  $\leftarrow$

mindist = INF  $\leftarrow$

if l == r return mindist  $\leftarrow$

if l == r - 1 return dist(l, r)  $\leftarrow$

if l == r - 2 return min (dist(l, r), dist(l + 1, r), dist(l, l + 1))  $\leftarrow$

mid = (l + r) / 2  $\leftarrow$

d1 = PLANE\_CLOSEST\_POINT\_PAIR(Points, l, mid)  $\leftarrow$

d2 = PLANE\_CLOSEST\_POINT\_PAIR(Points, mid + 1, r)  $\leftarrow$

d3 = Solve(Points, l, r)  $\leftarrow$   **$O(n)$**

mindist = min (d1, d2, d3)  $\leftarrow$   **$O(1)$**

**$O(1)$**

**$2t(n/2)$**

$$t(n) = \begin{cases} 2t(n/2) + O(n), & n > 3 \\ O(1), & n \leq 3 \end{cases}$$

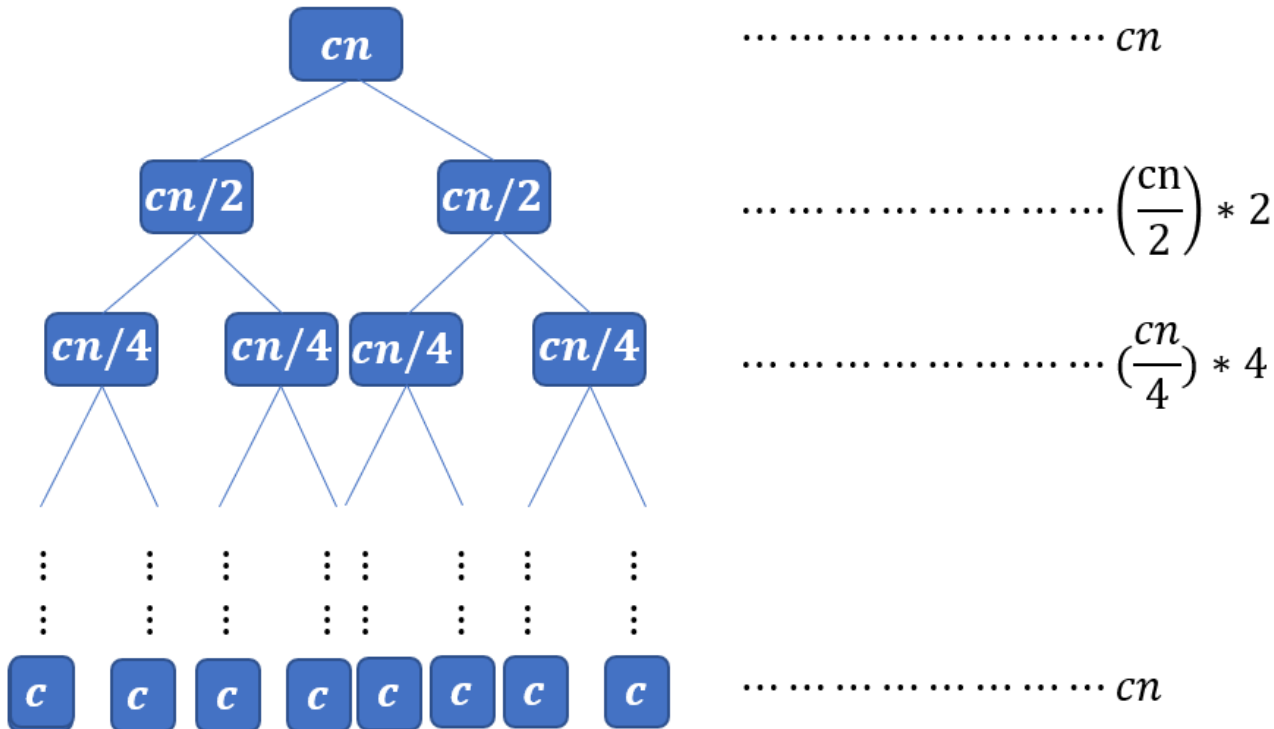
# Divide and Conquer



深圳大学  
SHENZHEN UNIVERSITY

## • Complexity Analysis

**Time:  $O(n \log n)$     Space:  $O(n)$**



$$t(n) = (1 + \log n) * cn = c(n + n \log n)$$

$$t(n) = \begin{cases} 2t(n/2) + O(n), & n > 3 \\ O(1), & n \leq 3 \end{cases}$$

$$\log n \quad t(n) = \begin{cases} 2t(n/2) + cn, & n > 3 \\ c, & n \leq 3 \end{cases}$$

$$t(n) = c(n + n \log n) = O(n \log n)$$

$$T(n) = O(n \log n) + t(n) = O(n \log n)$$

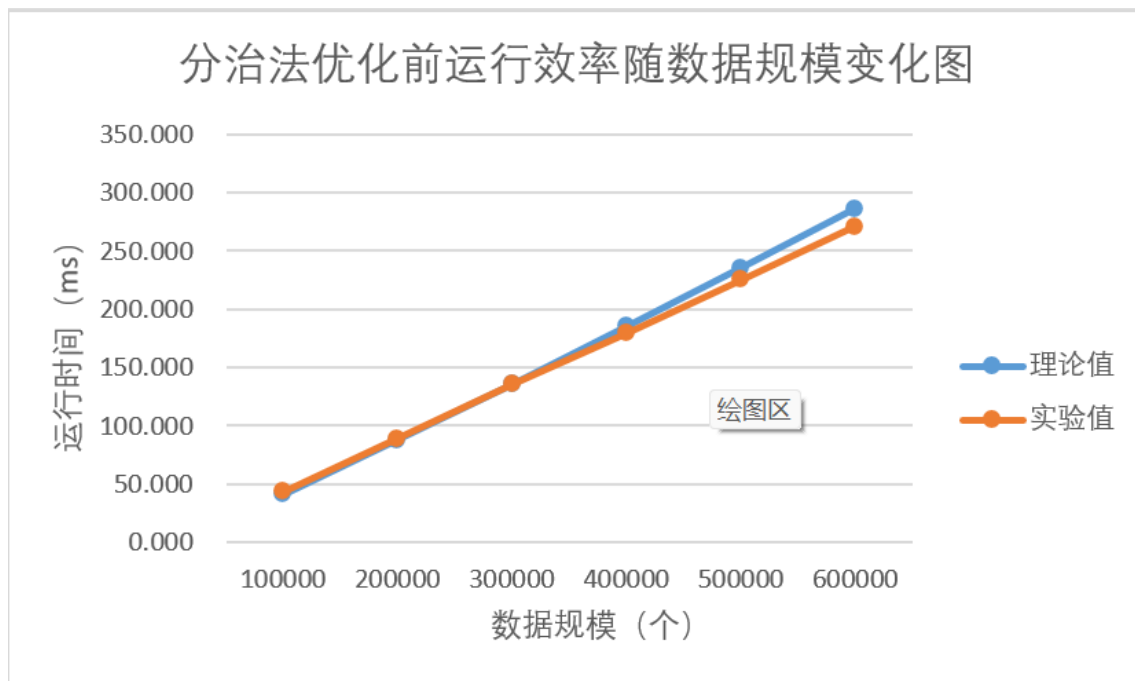
# Divide and Conquer



深圳大学  
SHENZHEN UNIVERSITY

- *Experiment*  
*before optimization*  
100000~600000

数据规模 (个)	100000	200000	300000	400000	500000	600000
理论值 (ms)	41.272	87.514	135.631	184.967	235.208	286.172
实验值 (ms)	43.120	88.994	135.631	179.285	225.171	270.779
误差	4.476%	1.692%	0.000%	-3.072%	-4.268%	-5.379%



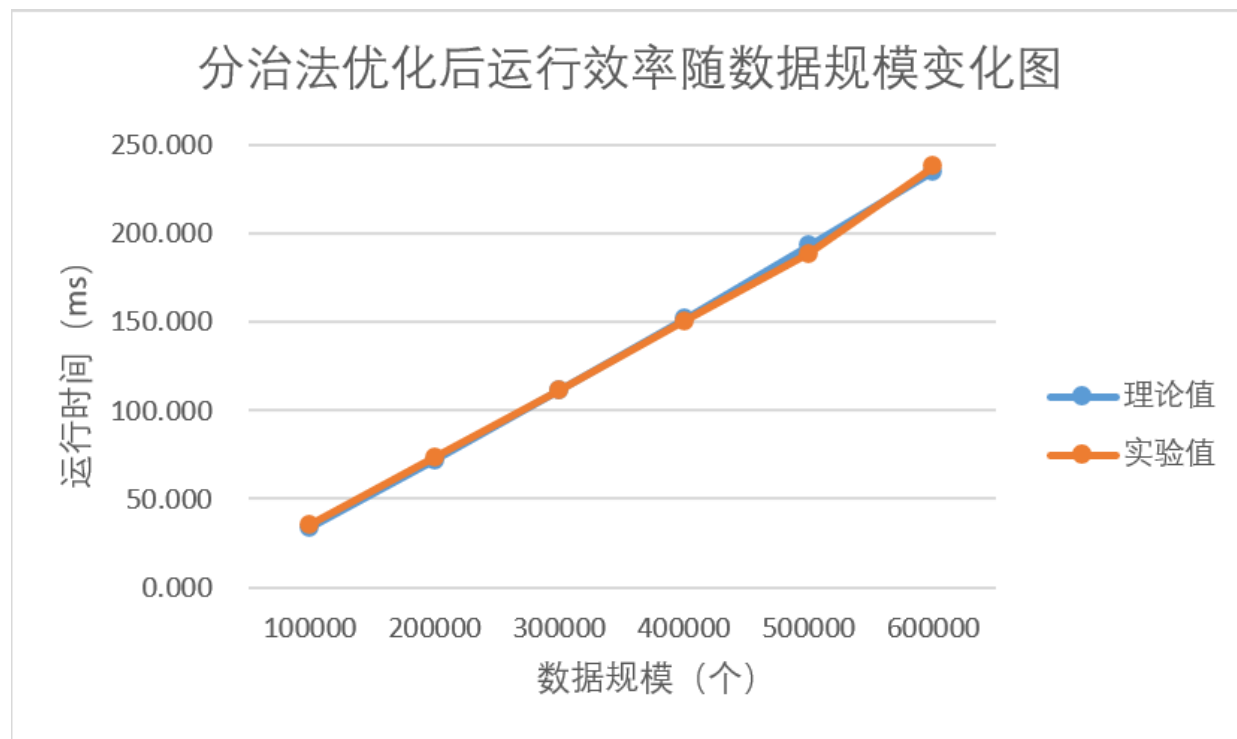
# Divide and Conquer



深圳大学  
SHENZHEN UNIVERSITY

- *Experiment*  
after optimization  
100000~600000

数据规模 (个)	100000	200000	300000	400000	500000	600000
理论值 (ms)	33.856	71.788	111.259	151.729	192.942	234.748
实验值 (ms)	35.286	73.418	111.259	150.563	188.152	237.662
误差	4.224%	2.271%	0.000%	-0.769%	-2.483%	1.241%



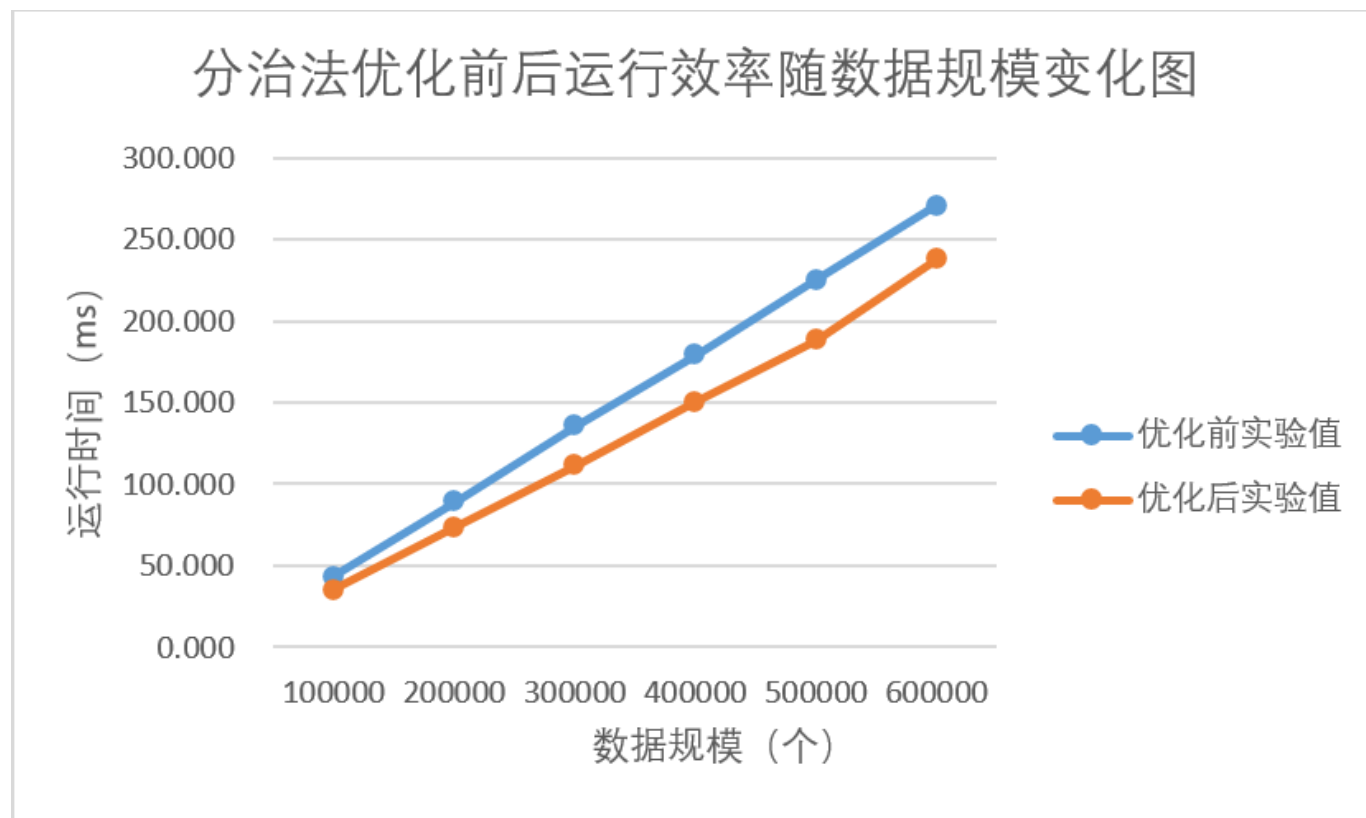


# Divide and Conquer



深圳大学  
SHENZHEN UNIVERSITY

- *Experiment  
comparison*



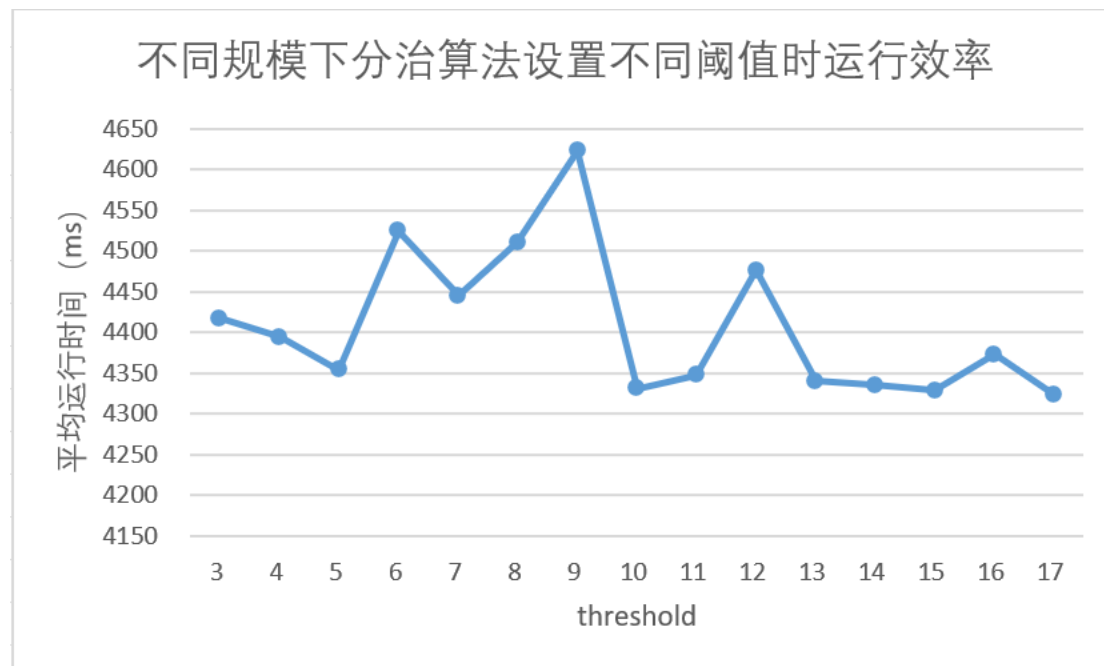
# Divide and Conquer



深圳大学  
SHENZHEN UNIVERSITY

- Experiment*  
*with threshold*

数据规模 (个)	10000000														
threshold	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
平均运行时间 (ms)	4418	4394	4354	4525	4445	4511	4623	4331	4348	4476	4340	4335	<b>4328</b>	4373	<b>4324</b>





深圳大学  
SHENZHEN UNIVERSITY

02

PART 02

## SECTION 2 **Brute Force**

# Brute Force

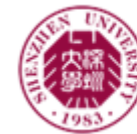
---



深圳大学  
SHENZHEN UNIVERSITY

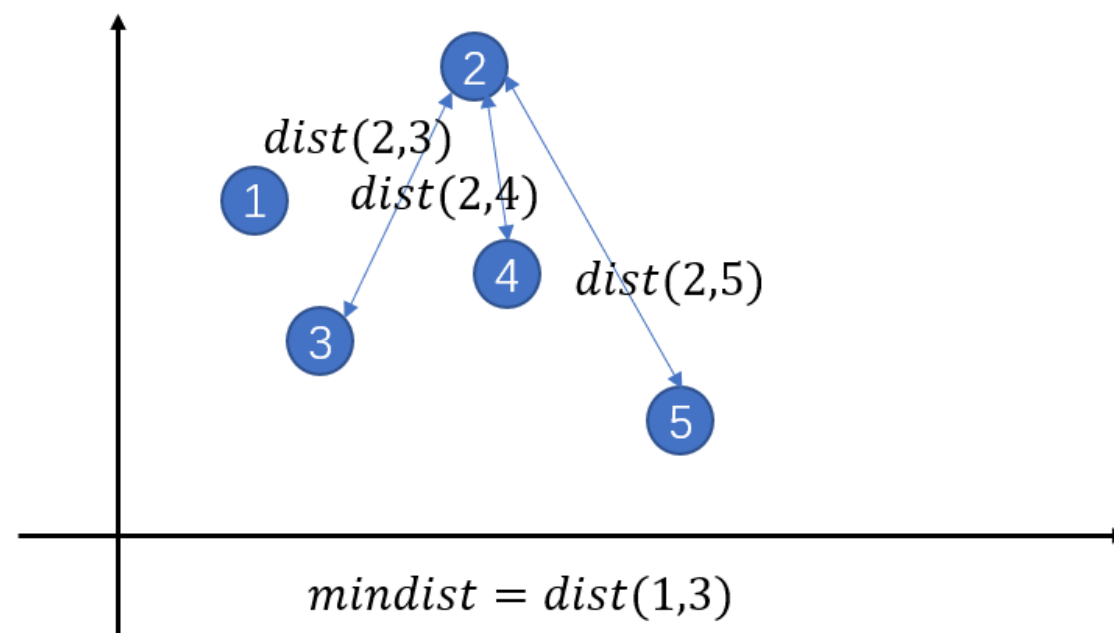
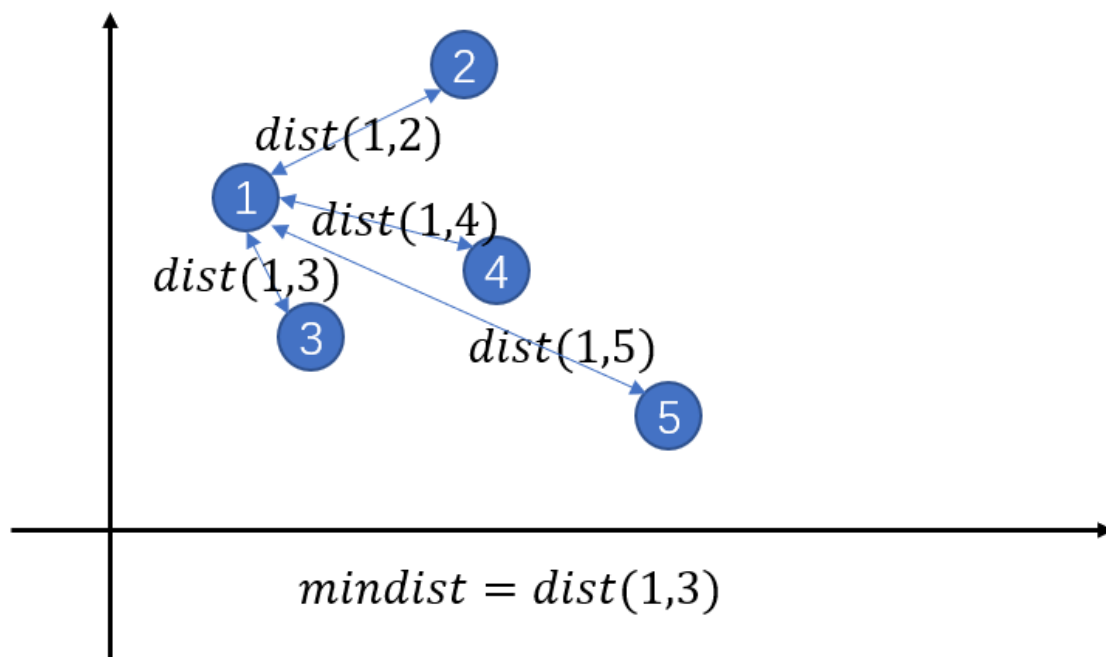
- *Algorithm Process*
- *Complexity Analysis*
- *Experiment*

# Brute Force



深圳大学  
SHENZHEN UNIVERSITY

- *Algorithm Process*



- *Algorithm Process*

*optimization*

$$dist = sqrt((p_i.x - p_j.x)^2 + (p_i.y - p_j.y)^2)$$

→  $dist = (p_i.x - p_j.x)^2 + (p_i.y - p_j.y)^2$

*return ans;*

→ *return sqrt(ans);*

# Brute Force



深圳大学  
SHENZHEN UNIVERSITY

- Complexity Analysis**

**Time:  $O(n^2)$     Space:  $O(1)$**

**BRUTE\_FORCE**(Points, n)

    ans = INF

    for i = 1 to n

        for j = i + 1 to n

            ans = min (ans, dist(Points[i], Points[j]))

Then:  $\leftarrow$

**cost**

**time**  $\leftarrow$

$c_1$

$1 \leftarrow$

$c_2$

$n + 1 \leftarrow$

$c_3$

$\sum_{i=1}^n n - i + 1 \leftarrow$

$c_4$

$\sum_{i=1}^n n - i \leftarrow$

$$T(n) = c_1 + c_2(n+1) + c_3 \left( \sum_{i=1}^n n - i + 1 \right) + c_4 \sum_{i=1}^n n - i$$

$$= c_1 + c_2n + c_2 + c_3 \frac{n(n+1)}{2} + c_4 \frac{n(n-1)}{2} \quad \leftarrow$$

$$= \frac{1}{2}(c_3 + c_4)n^2 + (c_2 + \frac{1}{2}c_3 - \frac{1}{2}c_4)n + c_1 + c_2$$

# Brute Force

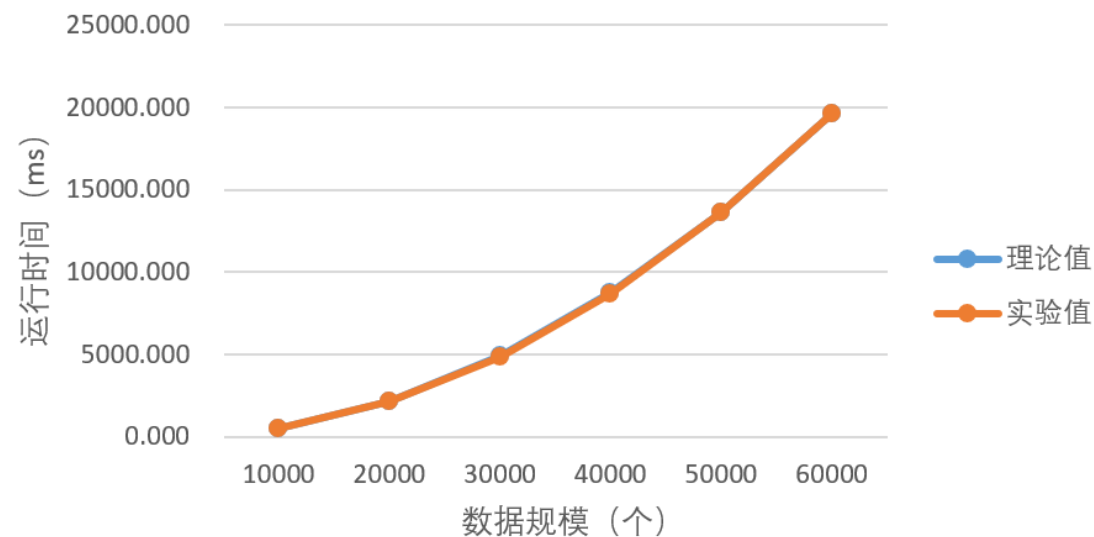


深圳大学  
SHENZHEN UNIVERSITY

- *Experiment*  
*before optimization*  
10000~60000

数据规模 (个)	10000	20000	30000	40000	50000	60000
理论值 (ms)	546.340	2185.359	4917.058	8741.437	13658.495	19668.233
实验值 (ms)	546.340	2178.502	4894.144	8703.998	13606.661	19603.077
误差	0.000%	-0.314%	-0.466%	-0.428%	-0.380%	-0.331%

蛮力法优化前运行效率随数据规模变化图





# Brute Force

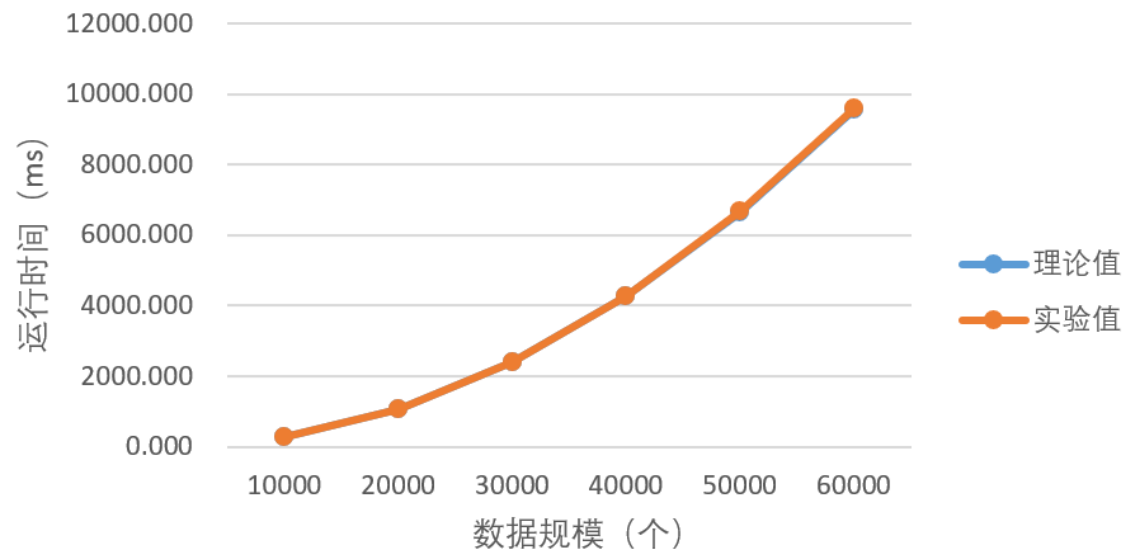


深圳大学  
SHENZHEN UNIVERSITY

- *Experiment*  
after optimization  
10000~60000

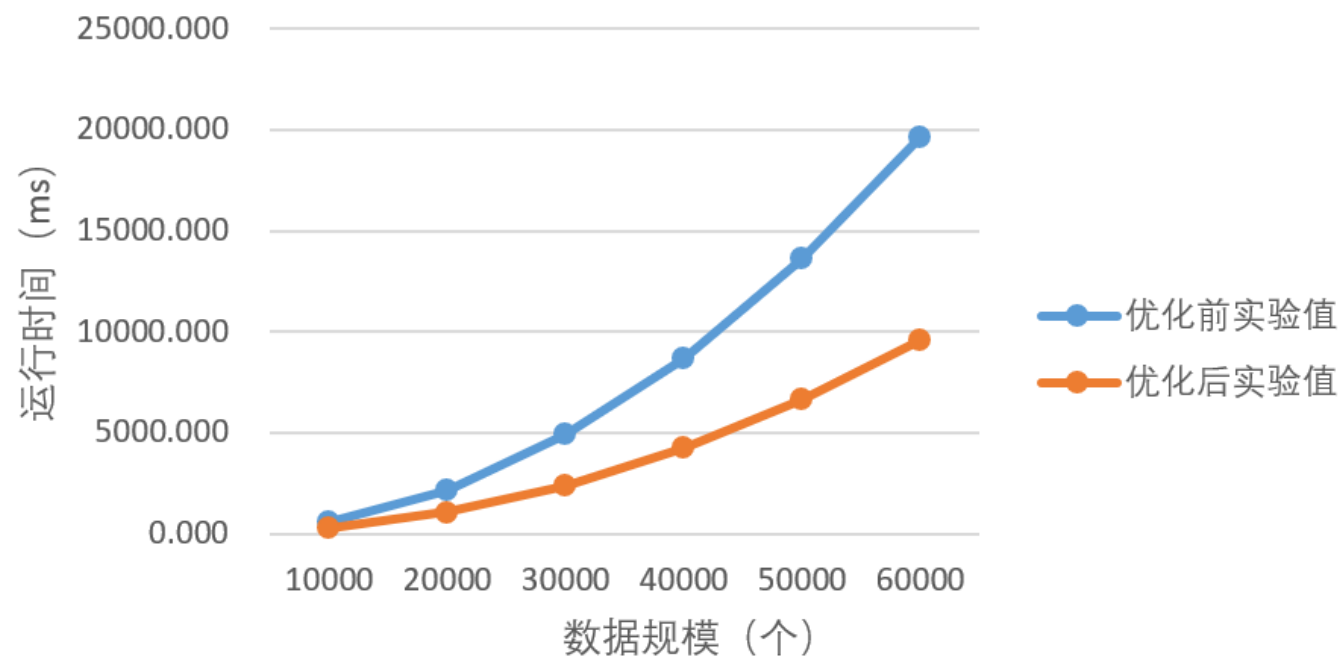
数据规模 (个)	10000	20000	30000	40000	50000	60000
理论值 (ms)	265.960	1063.839	2393.637	4255.355	6648.993	9574.549
实验值 (ms)	265.960	1063.524	2394.907	4258.898	6667.589	9585.261
误差	0.000%	-0.030%	0.053%	0.083%	0.280%	0.112%

蛮力法优化后运行效率随数据规模变化图



- *Experiment comparison*

蛮力法优化前后运行效率随数据规模变化图



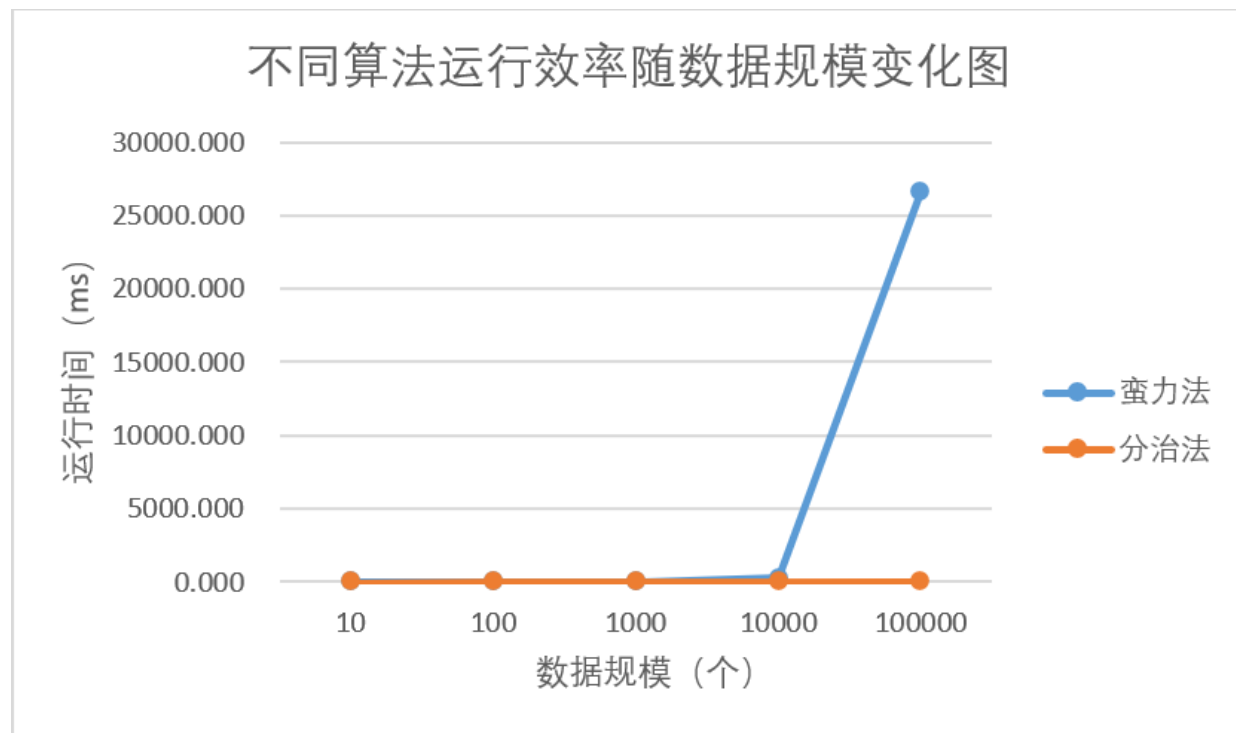
# Divide and Conquer



深圳大学  
SHENZHEN UNIVERSITY

- *Experiment*  
*comparison with Divide*

数据规模 (个)	10	100	1000	10000	100000
蛮力法实验值 (ms)	0.000	0.030	2.765	267.092	26637.267
分治法实验值 (ms)	0.001	0.018	0.231	2.946	35.359





深圳大学  
SHENZHEN UNIVERSITY

03

PART 03

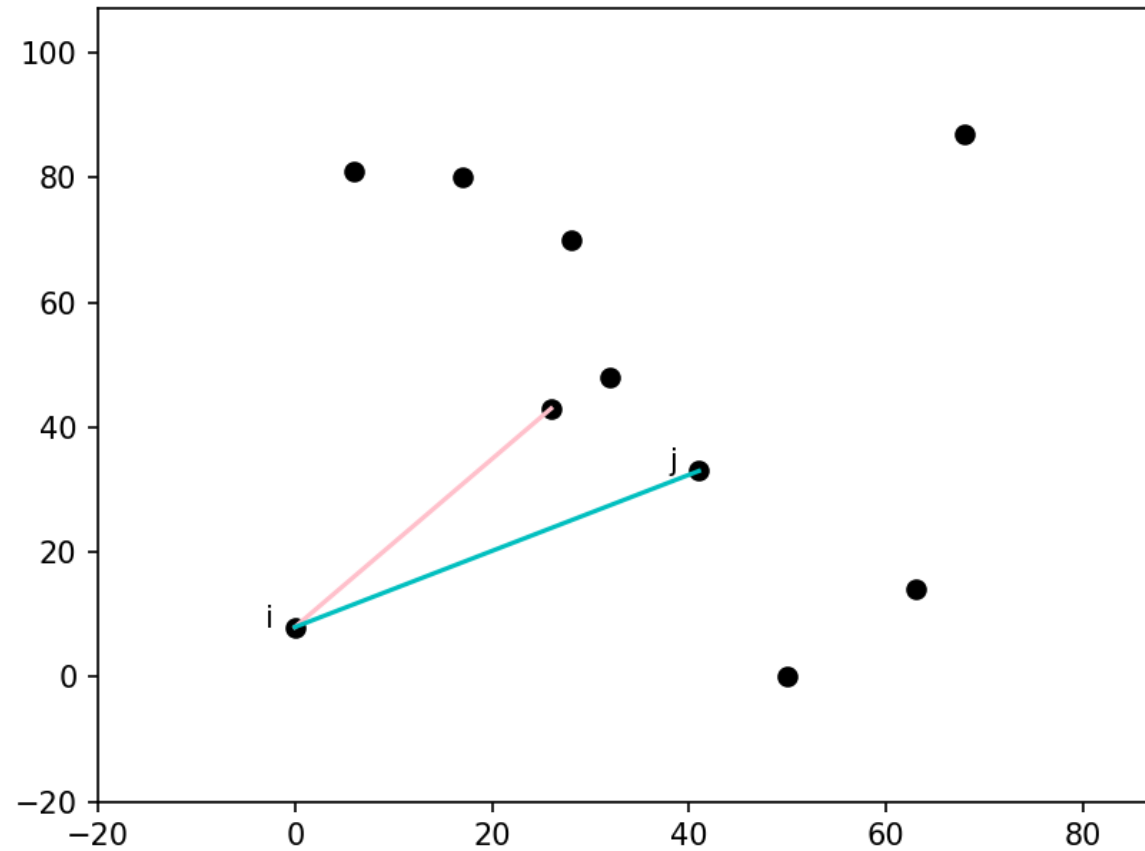
## SECTION 3 **Visualization**

*python + matplotlib*

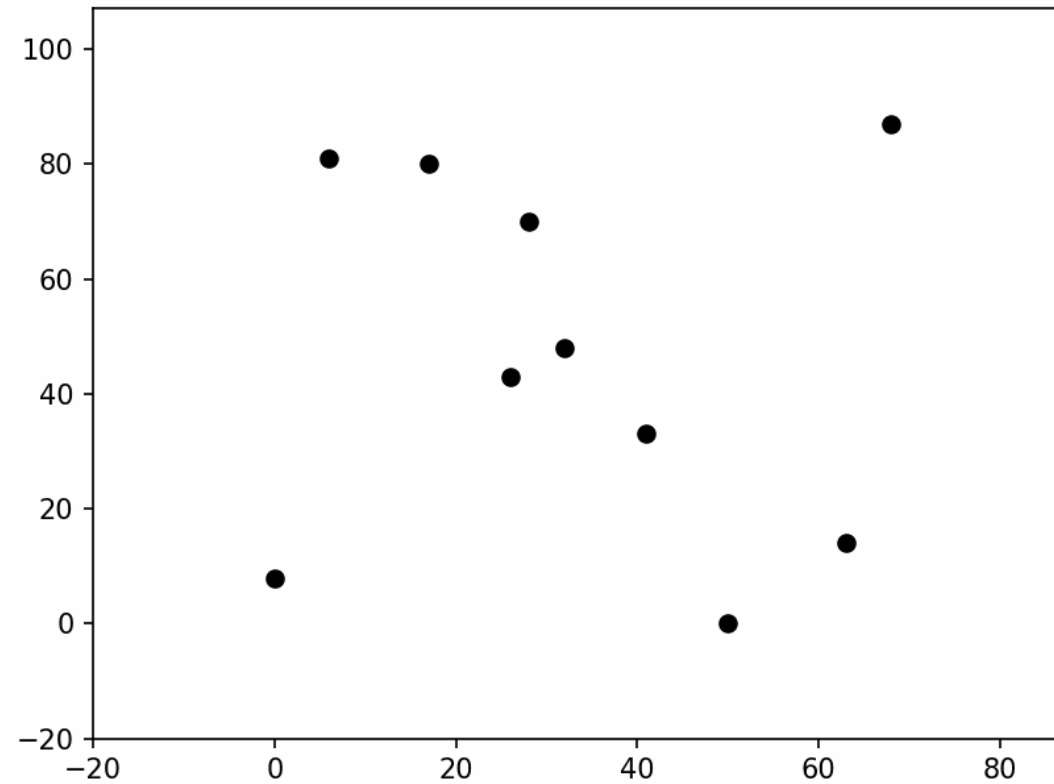
*draw each frame*

- *Brute Force*
- *Divide and Conquer*

- *Brute Force*



- *Brute Force*

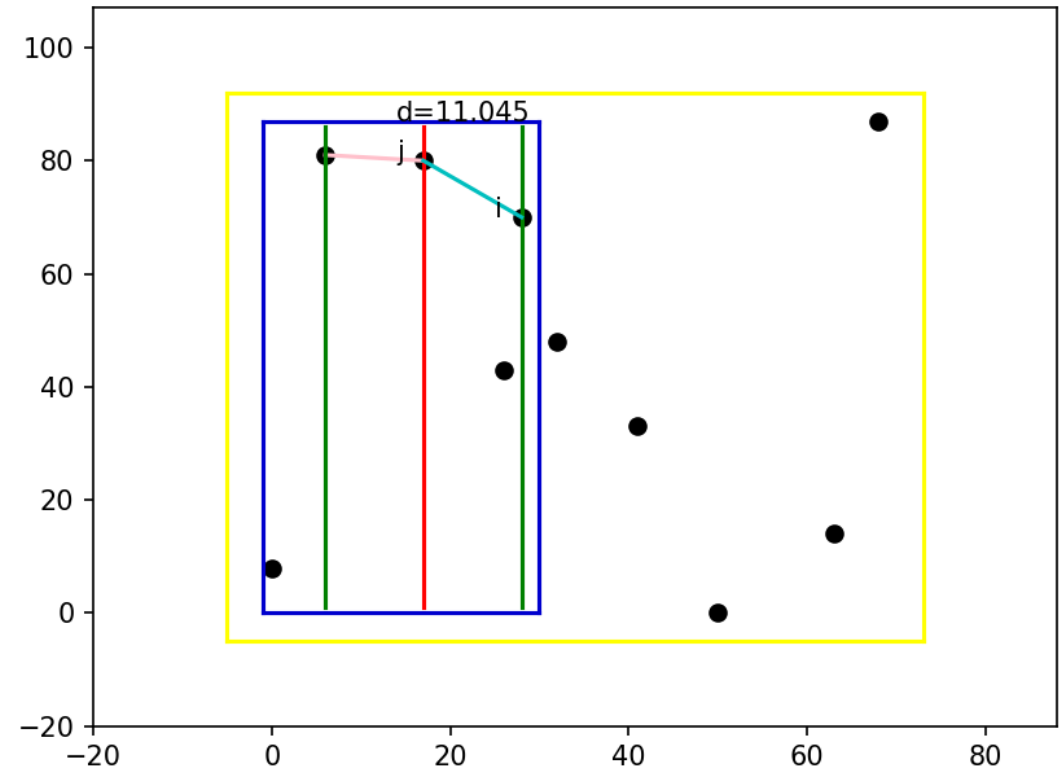
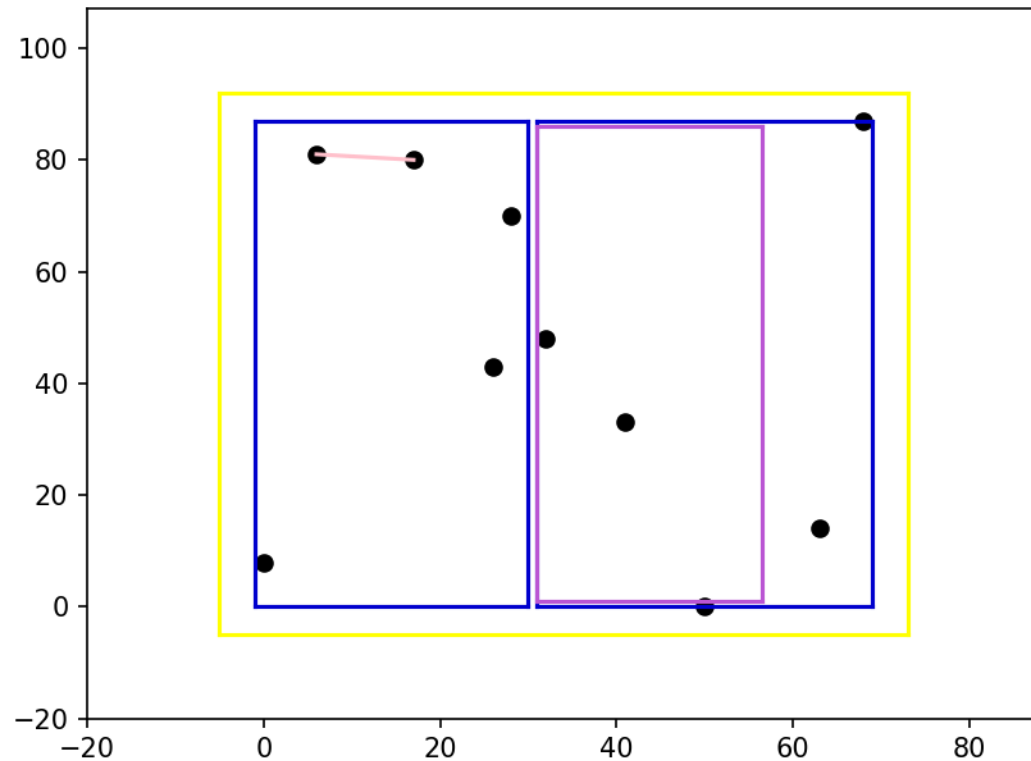


# Visualization



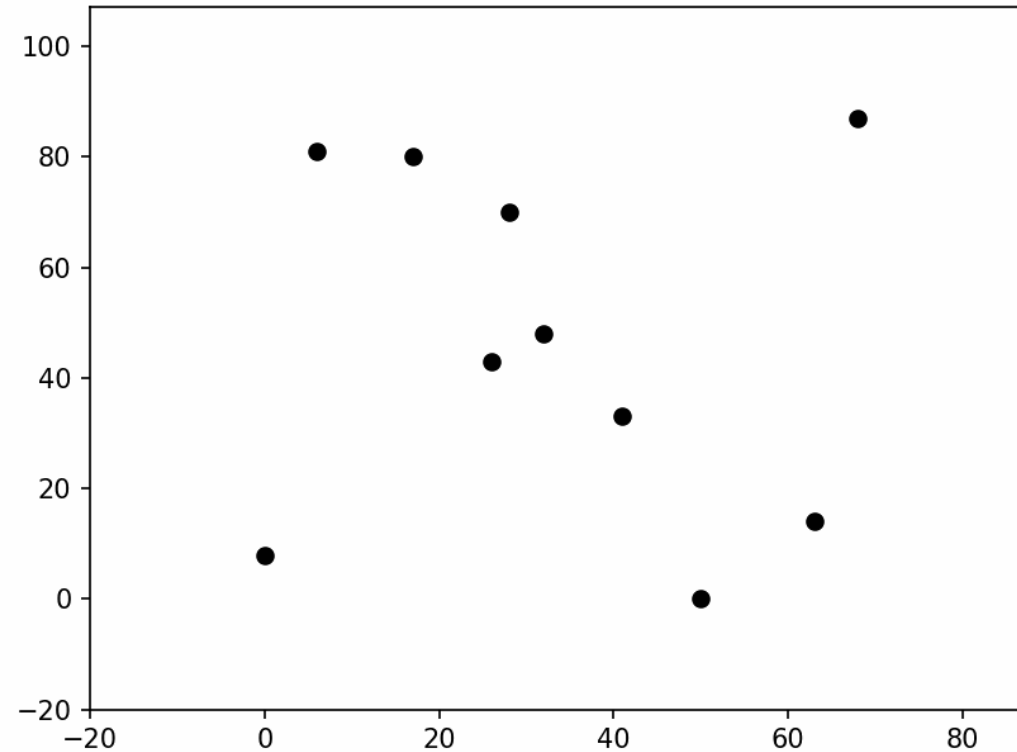
深圳大学  
SHENZHEN UNIVERSITY

- Divide and Conquer*





- *Divide and Conquer*



# Thank You!      Questions?



深圳大学计算机与软件学院

报告人：郑杨 陈敏涵  
指导老师：李炎然

2022年4月11日