Lab 6:
Graph’s Bridges

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Instructor： Yanran Li

Catalog

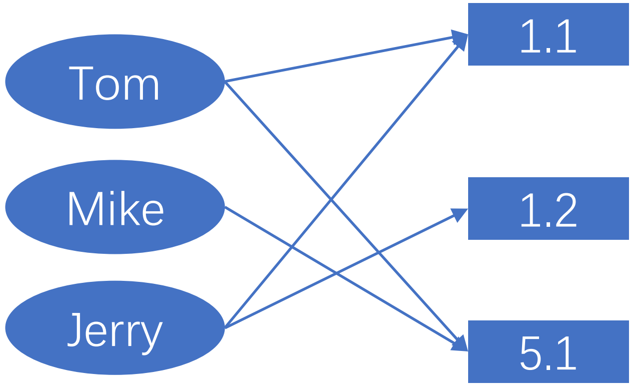
part 1

Problem and Model

Problem

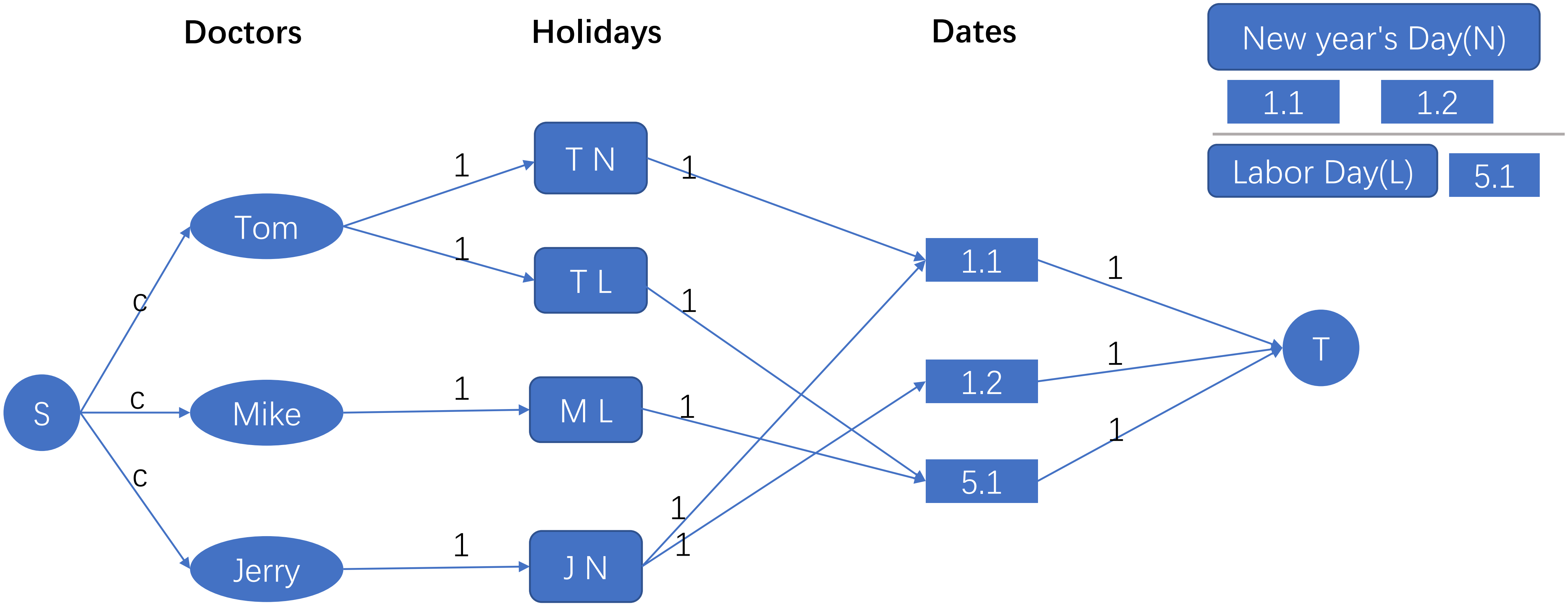
$k\ holidays (k = 2)$

$n\ doctors (n = 3)$



$c: Maximum\ days\ on\ duty$

Model



New year's Day(N)
1.1
1.2
Labor Day(L)
5.1
Doctors
Holidays
Dates
S
c
c
c
Tom
Mike
Jerry
1
1
1
1
T N
T L
M L
J N
1
1
1
1
1
1.1
1.2
5.1
1
1
1
T

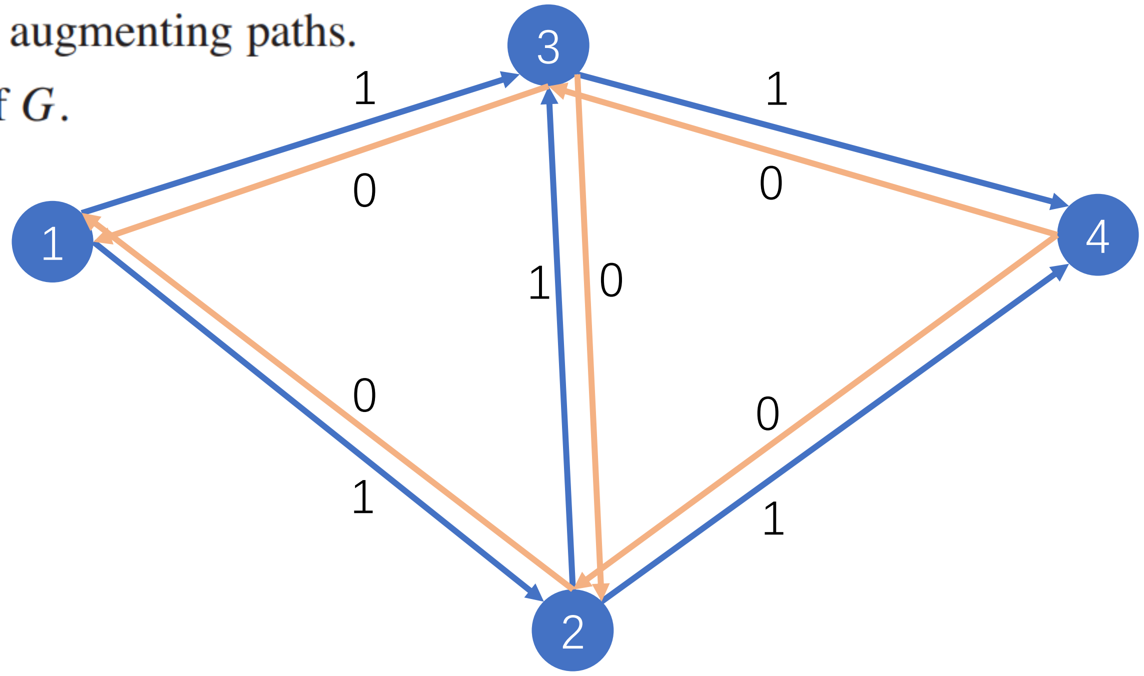
Ford-Fulksonff

Ford-Fulksonff

- Idea

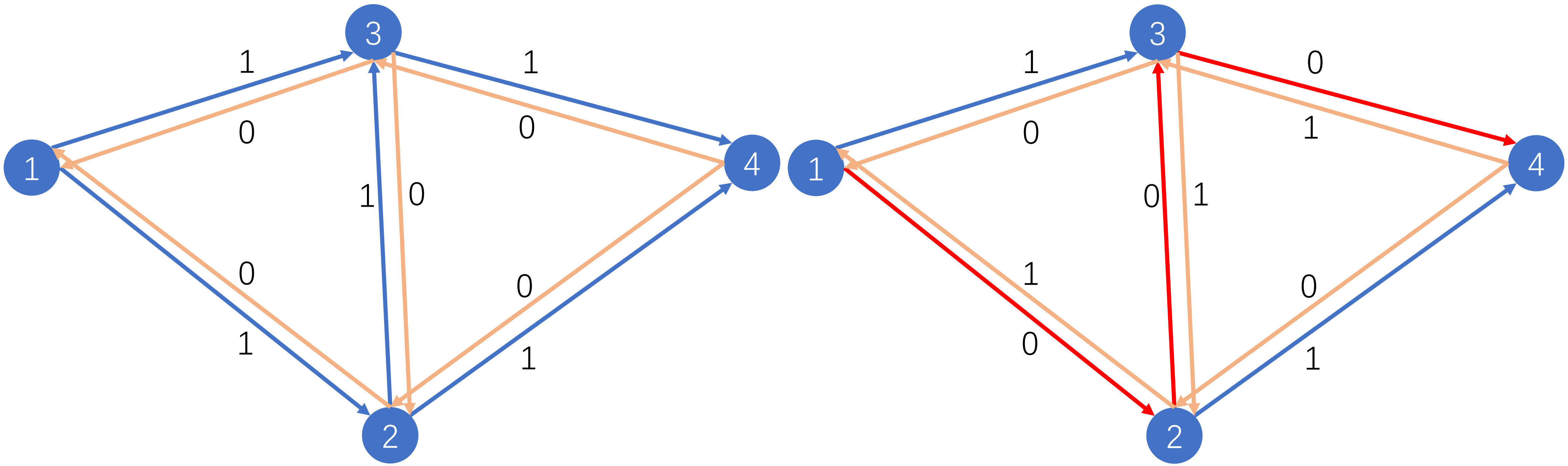
Theorem 26.6 (Max-flow min-cut theorem)
If f is a flow in a flow network $G = (V, E)$ with source s and sink t, then the following conditions are equivalent:

1. f is a maximum flow in G.
2. The residual network G_f contains no augmenting paths.
3. $|f| = c(S, T)$ for some cut (S, T) of G.



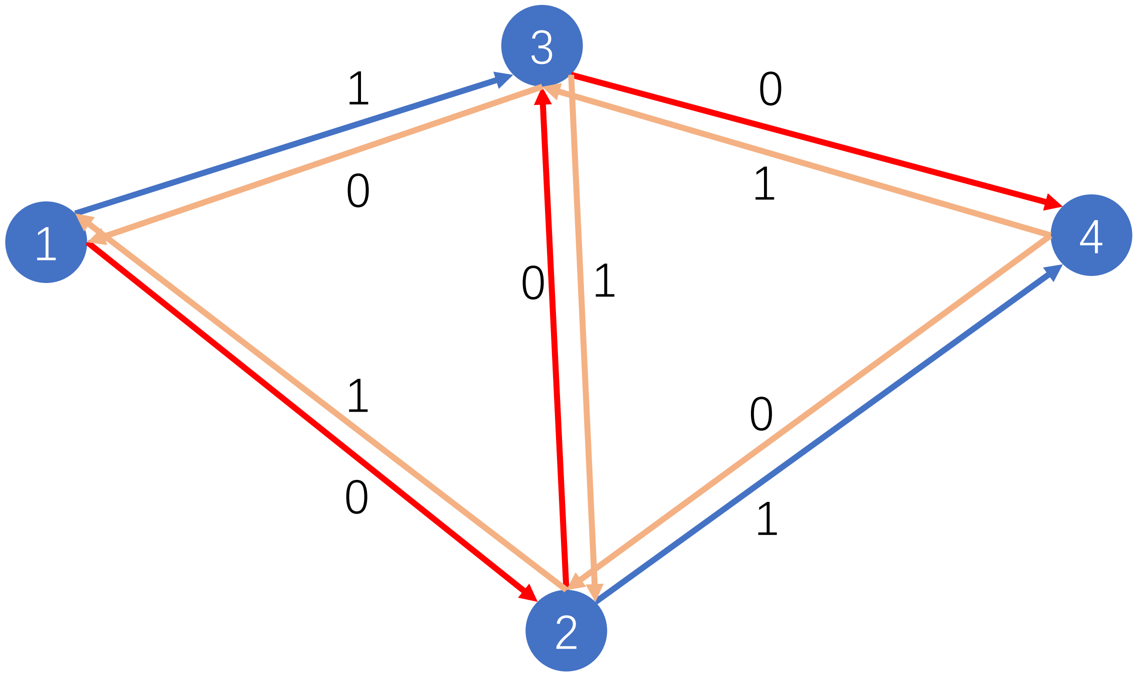
Ford-Fulksonff

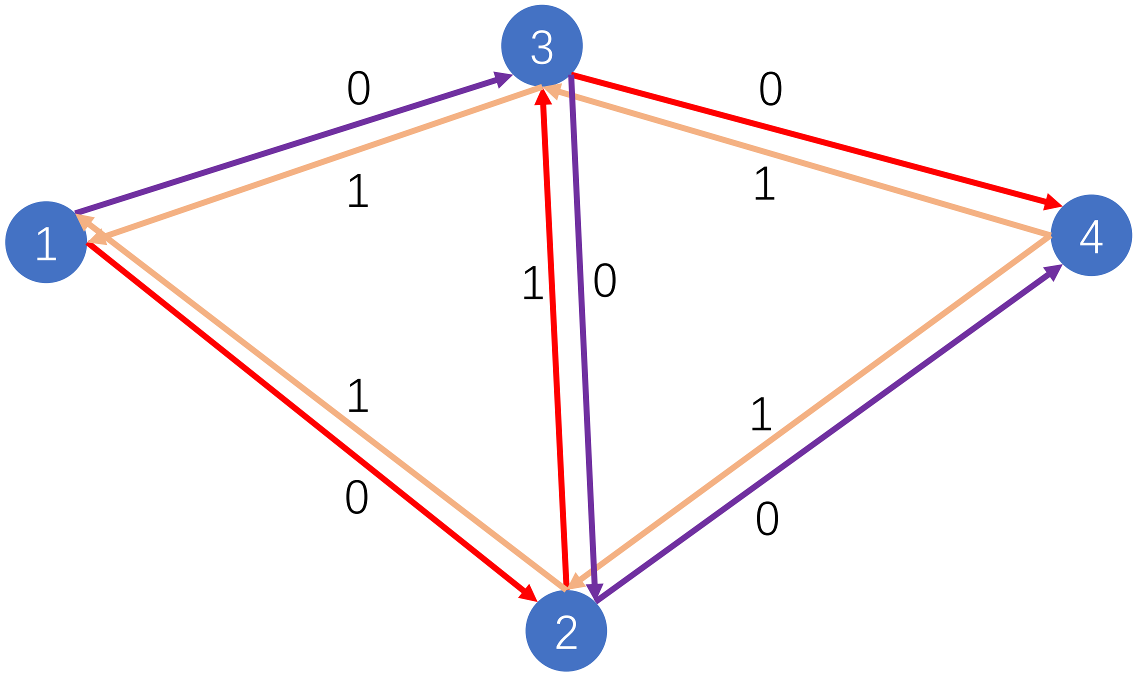
- Process



Ford-Fulksonff

- Process

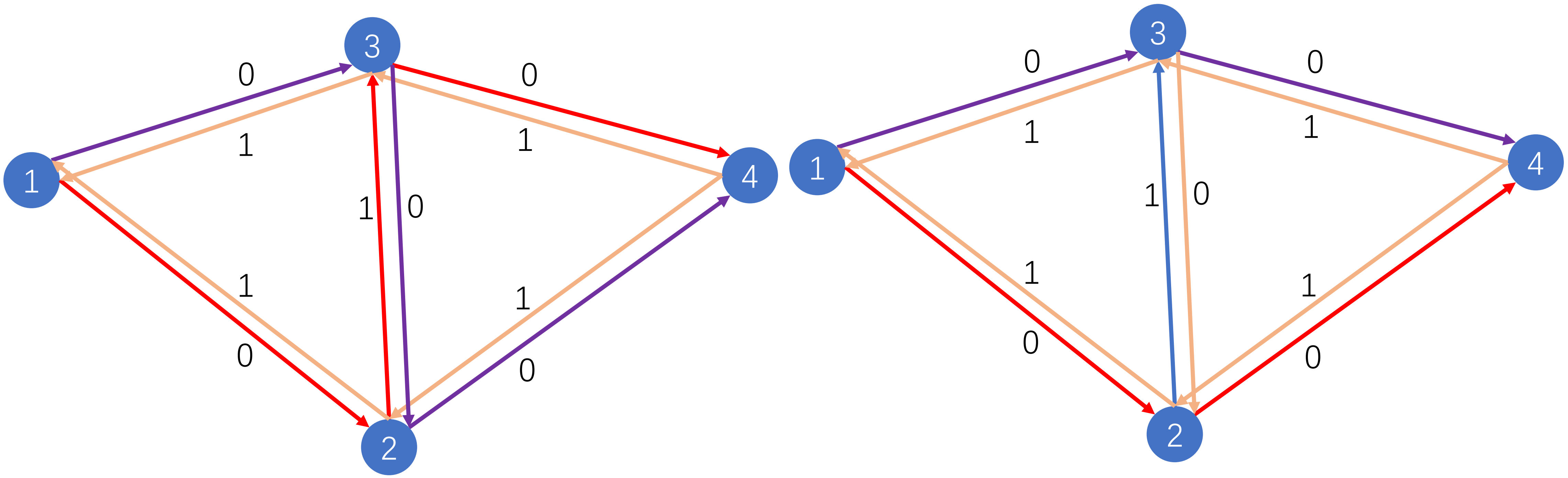




Ford-Fulksonff

- Process

New distribution scheme



Ford-Fulksonff

- Time Complexity

$find\ an\ augmented\ path\ in\ the\ residual\ network: O(V+E') = O(E)$

$Least\ increased\ flow\ per\ time:\ 1$

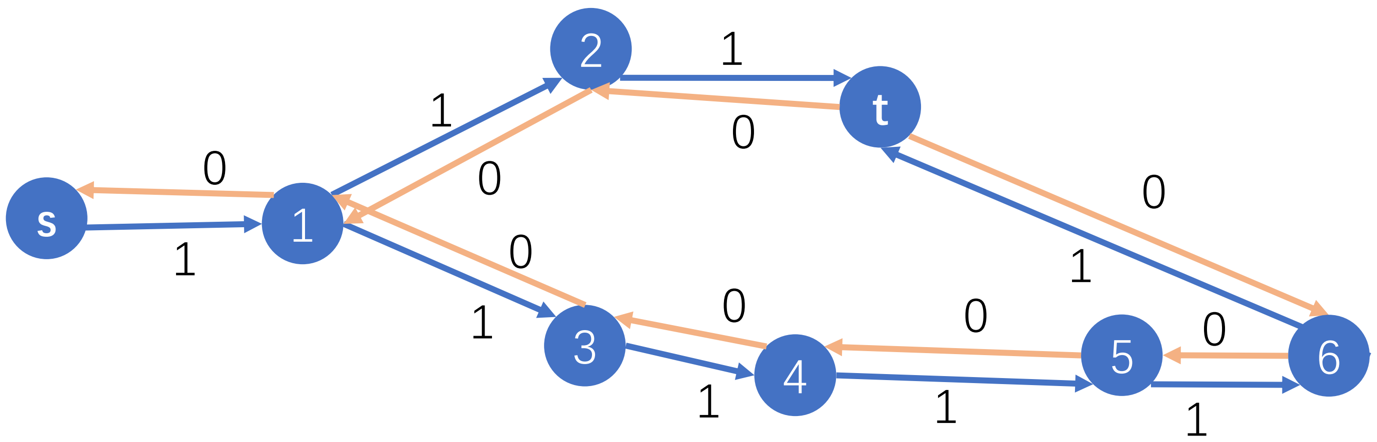
$Max\ searching\ times: |f^*|$

$$O(E|f^*|)$$

Edmonds-Karp

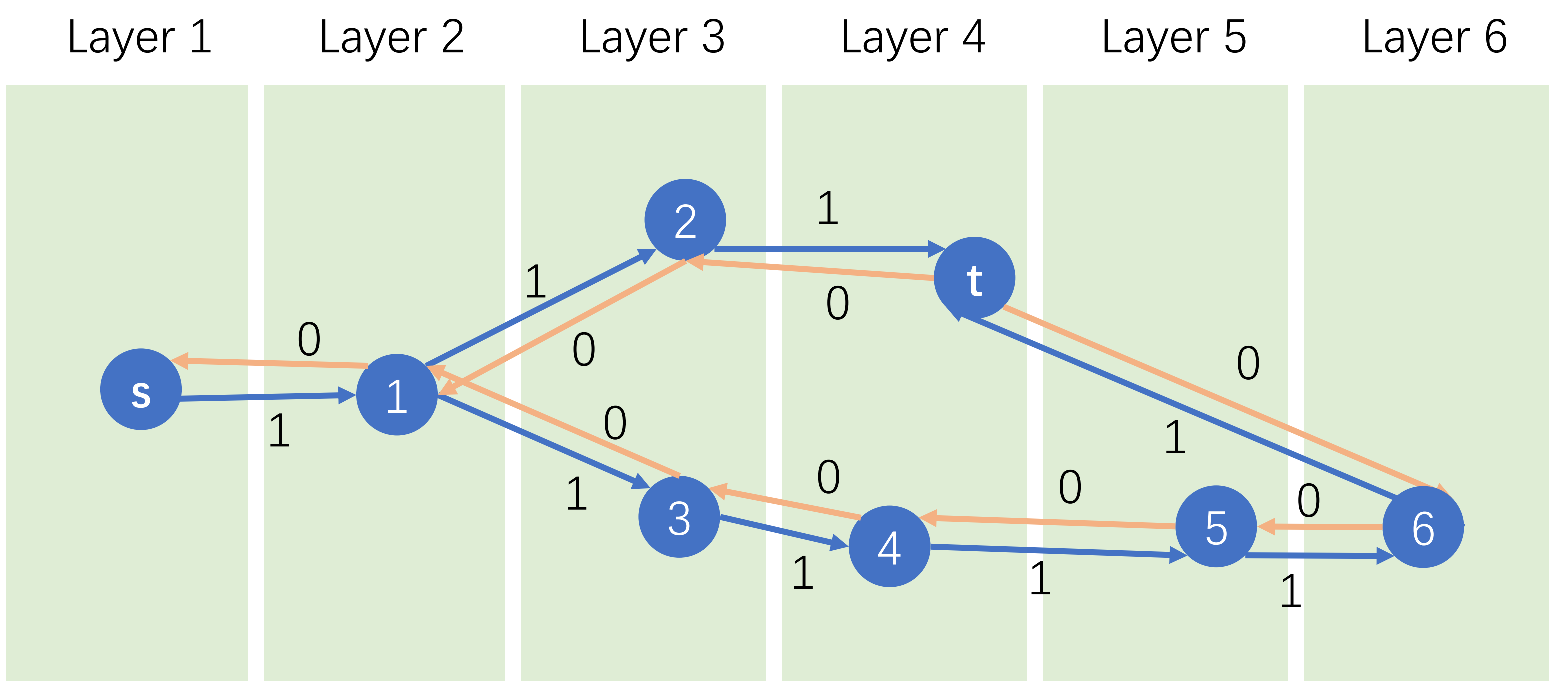
Edmonds-Karp

- FF's Problem



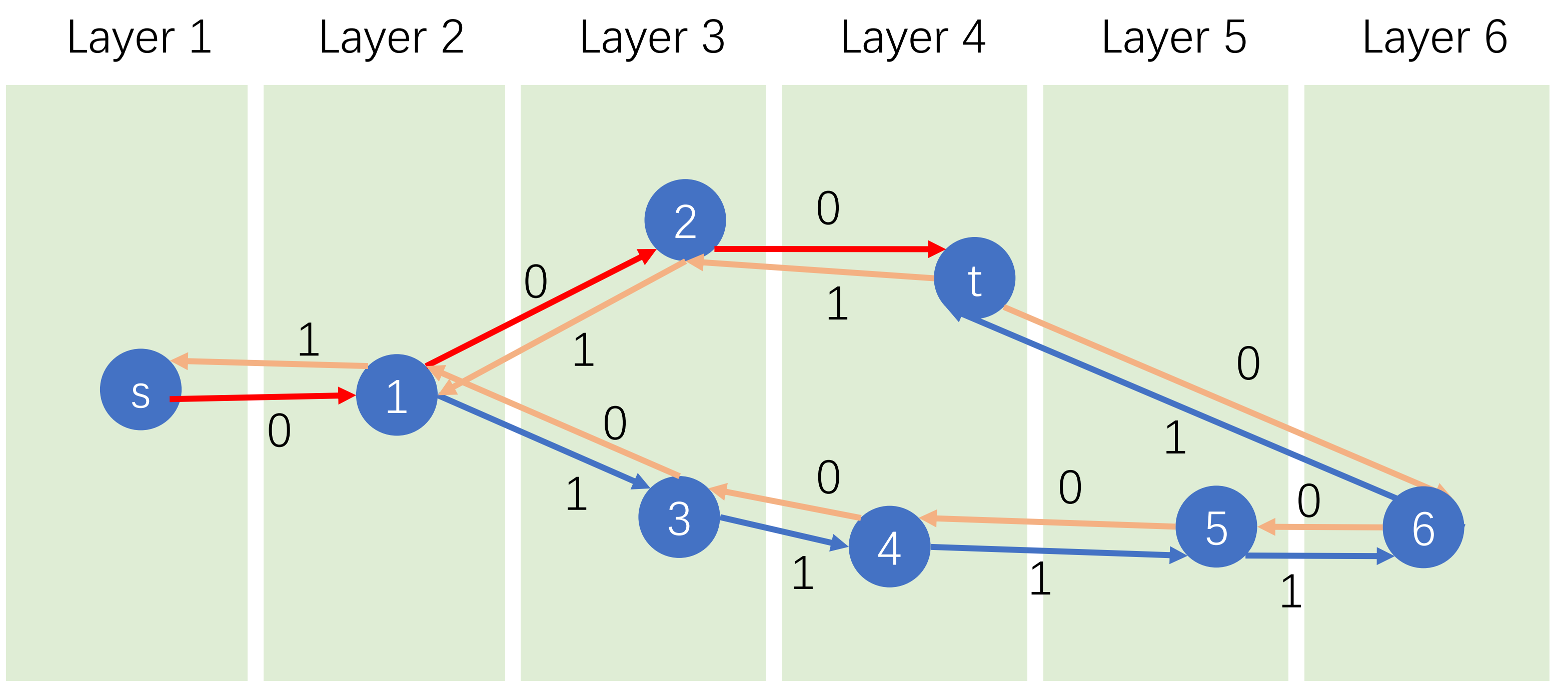
Edmonds-Karp

- Optimization---BFS to find the shortest paths



Edmonds-Karp

- Optimization---BFS to find the shortest paths

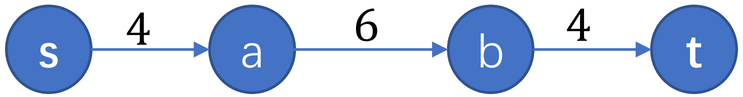


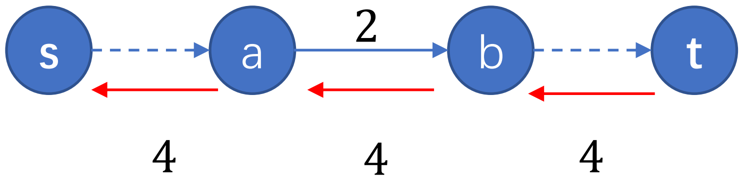
Edmonds-Karp

- Time Complexity

G_f $\quad c_f(p) = c_f(u,v) \Rightarrow (u,v)$ *is* **critical** $\quad c_f(p) = c_f(u,v) = 4$

p





Find an augmented path in the $G_f: O(V+E') = O(E)$

Lemma 26.7

If the Edmonds-Karp algorithm is run on a flow network $G = (V, E)$ with source s and sink t, then for all vertices $v \in V - \{s, t\}$, the shortest-path distance $\delta_f(s, v)$ in the residual network G_f increases monotonically with each flow augmentation.

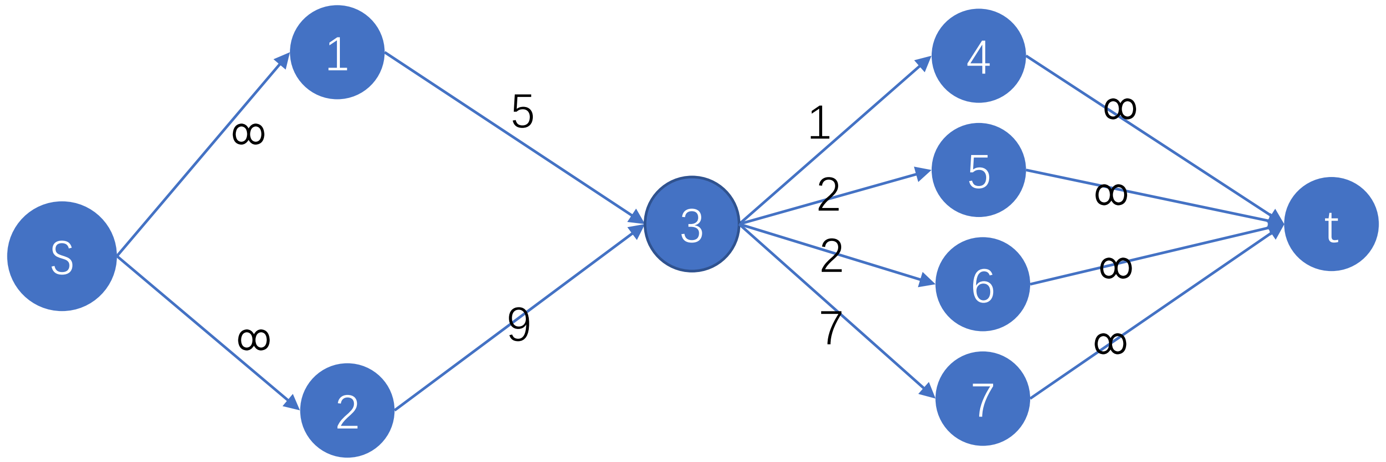
Consequently, from the time (u, v) becomes critical to the time when it next becomes critical, the distance of u from the source increases by at least 2.

Max searching times of v to be critical: $\frac{|V|}{2} - 1$ $\qquad O(VE^2)$

Dinic

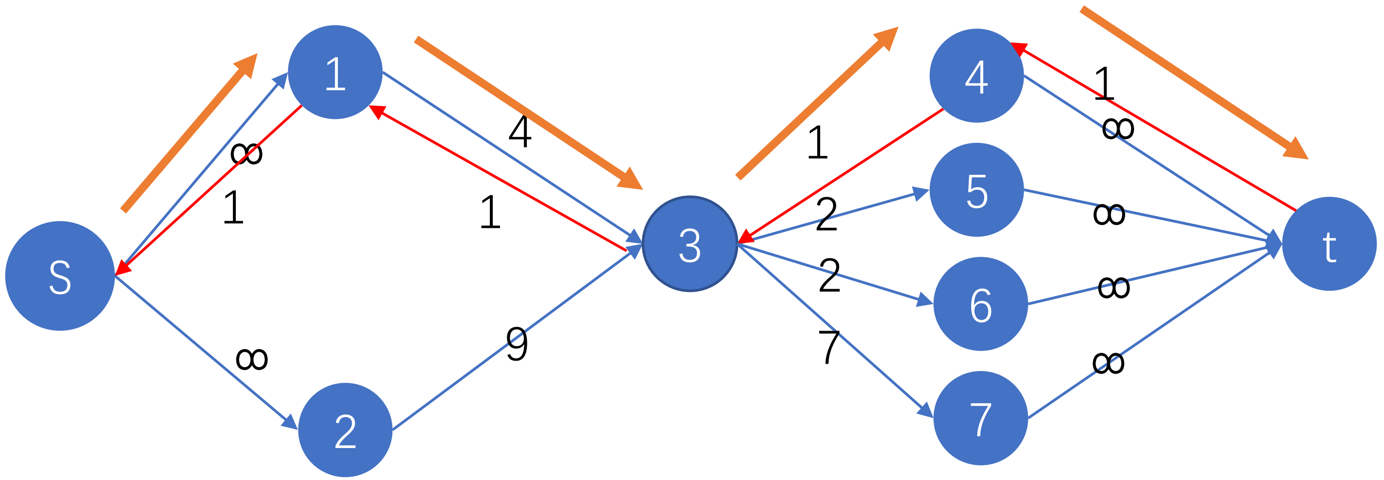
Dinic

- EK's problem



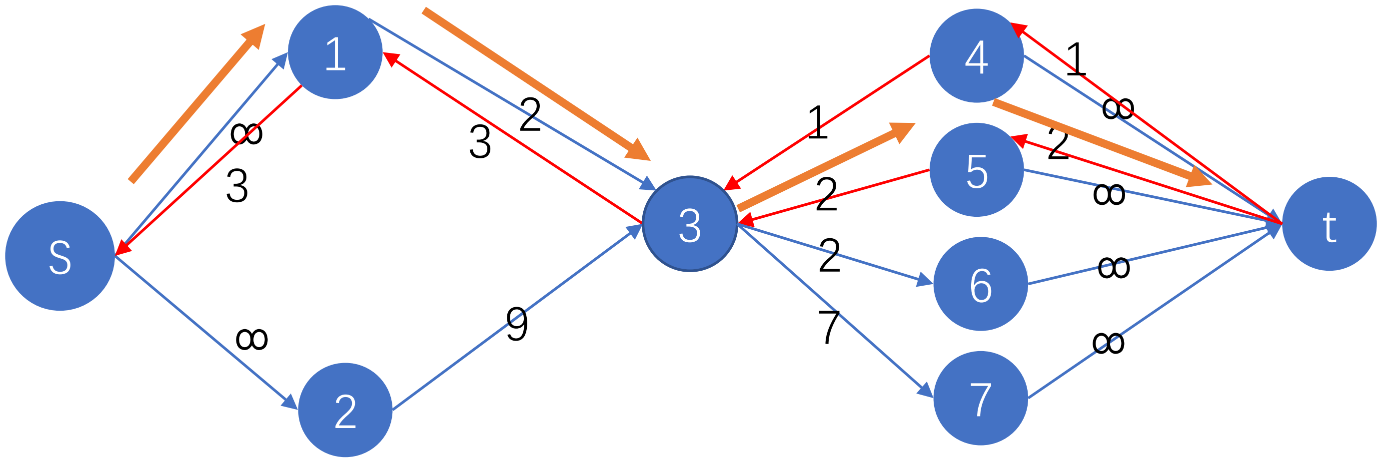
Dinic

- EK’s problem



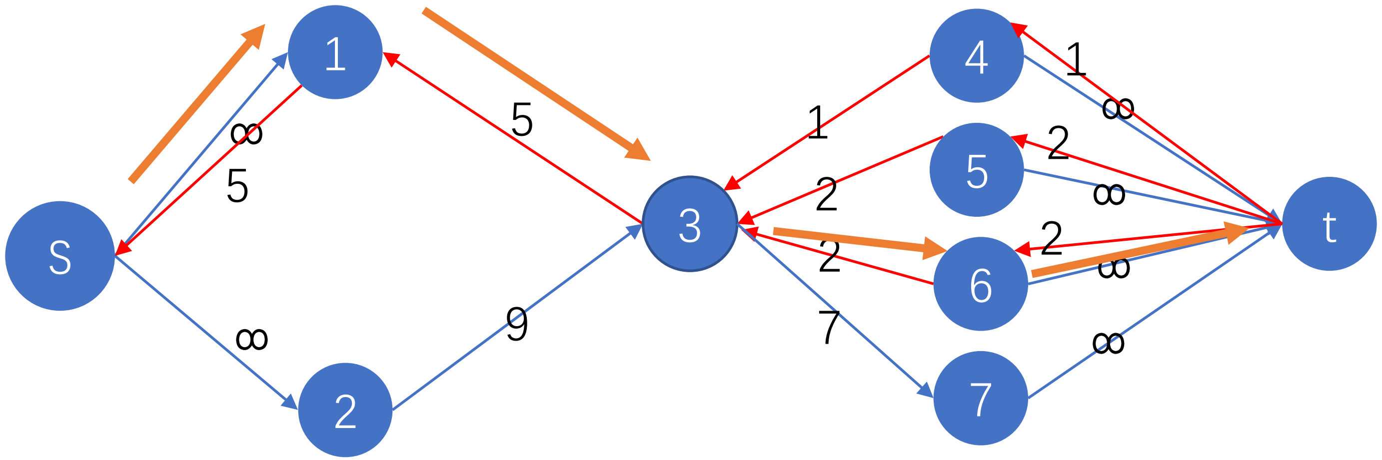
Dinic

- EK's problem



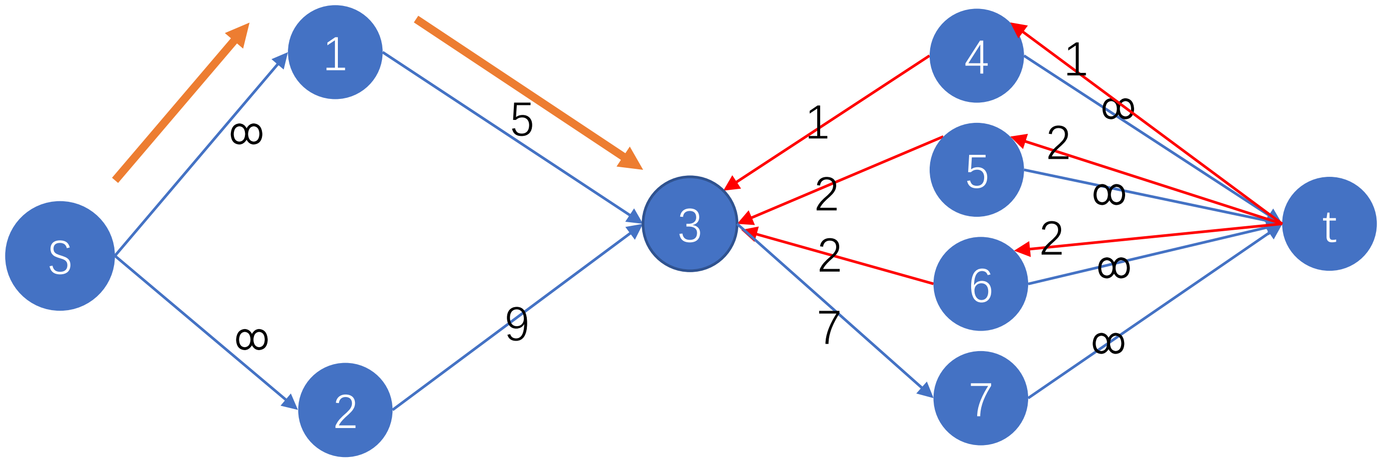
Dinic

- EK's problem



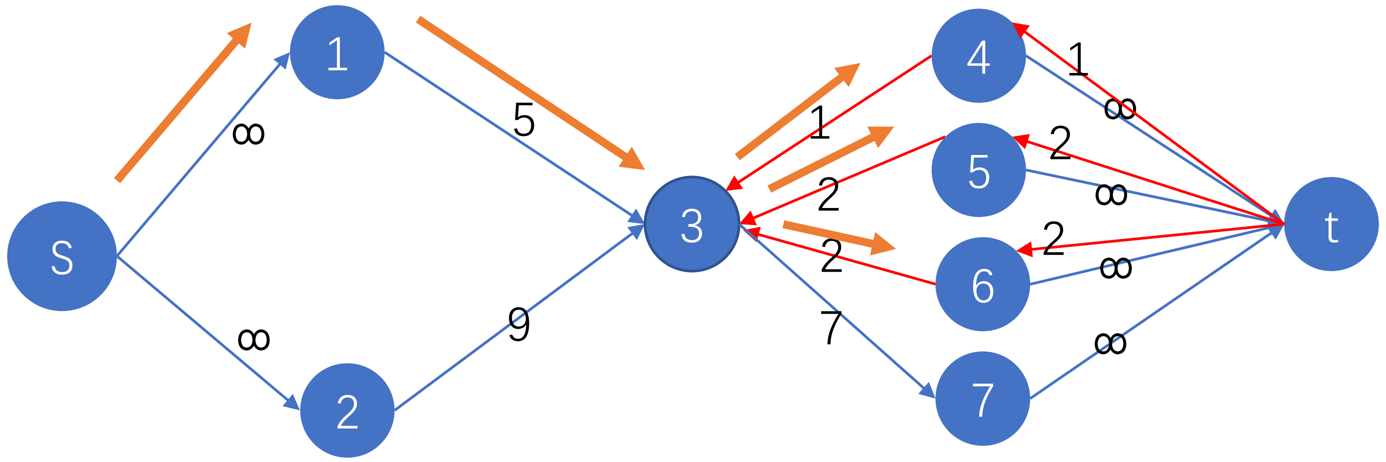
Dinic

- EK's problem



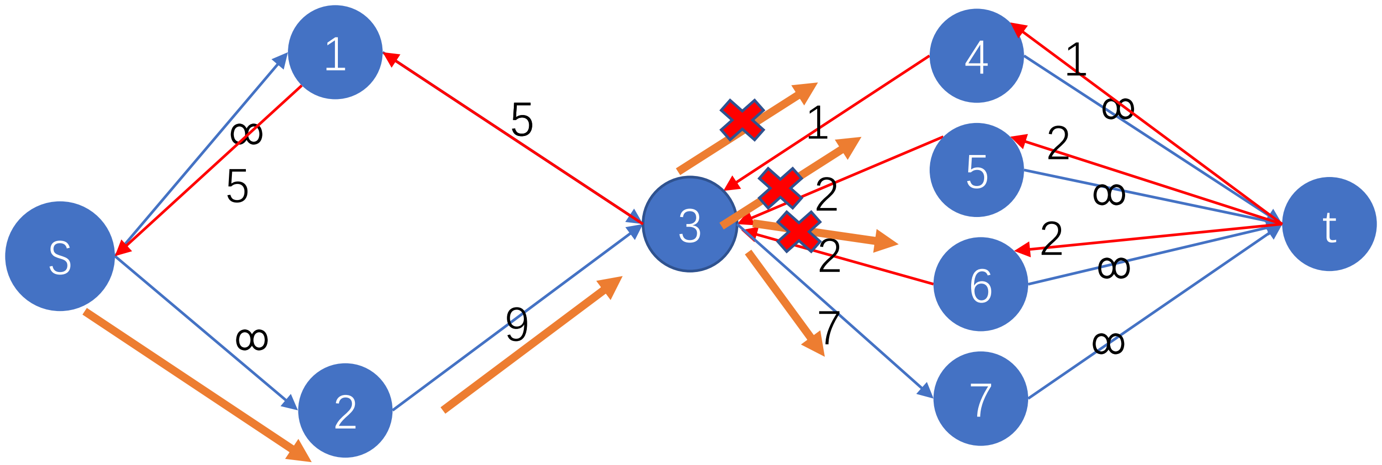
Dinic

- Optimization
    - Multichannel Augmentation



Dinic

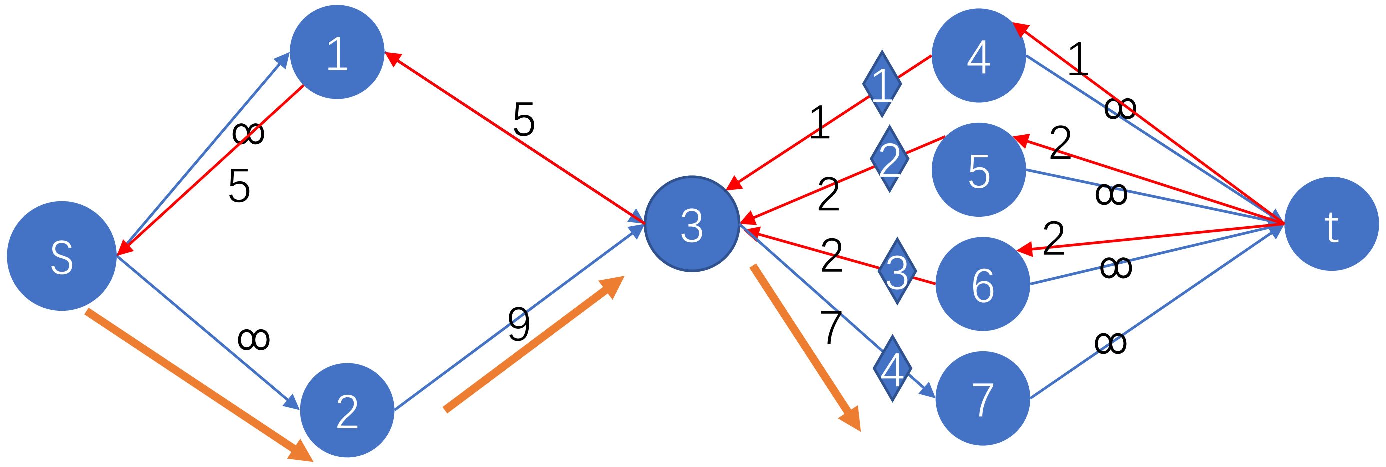
- Problem



Dinic

- Optimization
  - Current Arc Optimization

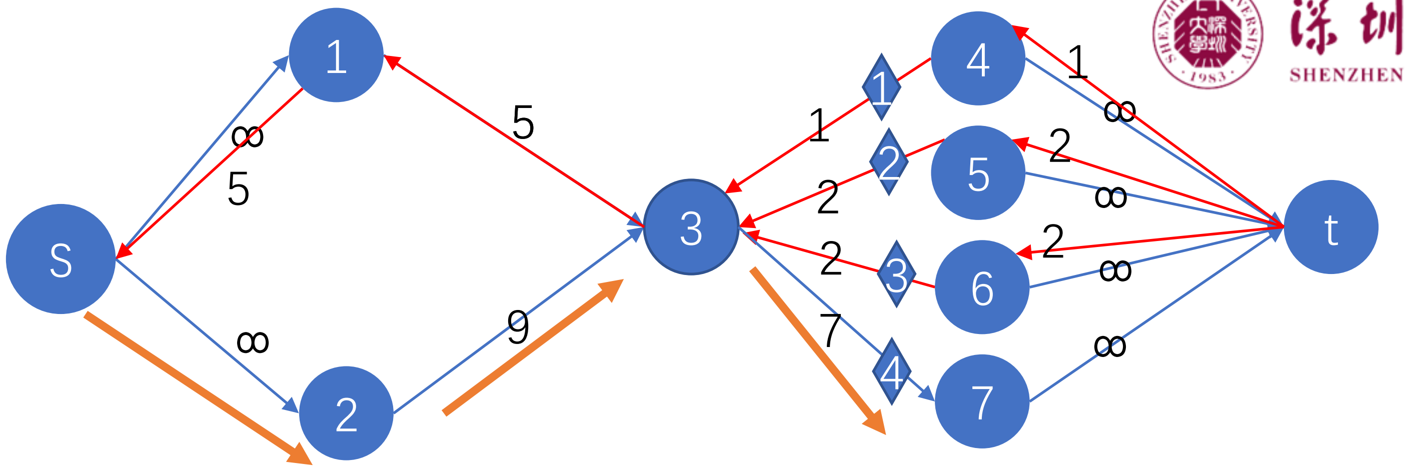
$cur[3] = 4$



Dinic

- Time Complexity

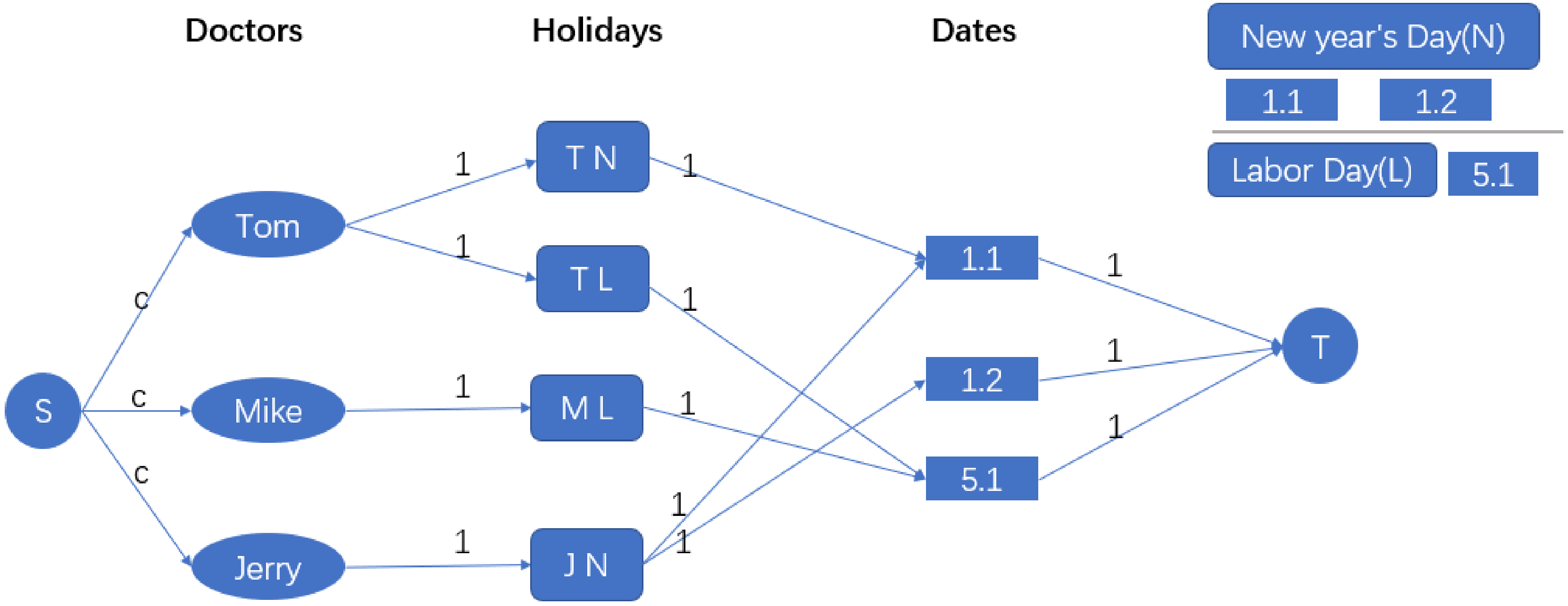
$V\ vertices, E\ edges$



$$\left.\begin{array}{ll} dfs: cur[i]\ most\ change\ E, & O(VE) \\ bfs: depth\ most\ change\ V-1, & O(V) \end{array}\right\} O(V^2E)$$

Experiment

Experiment

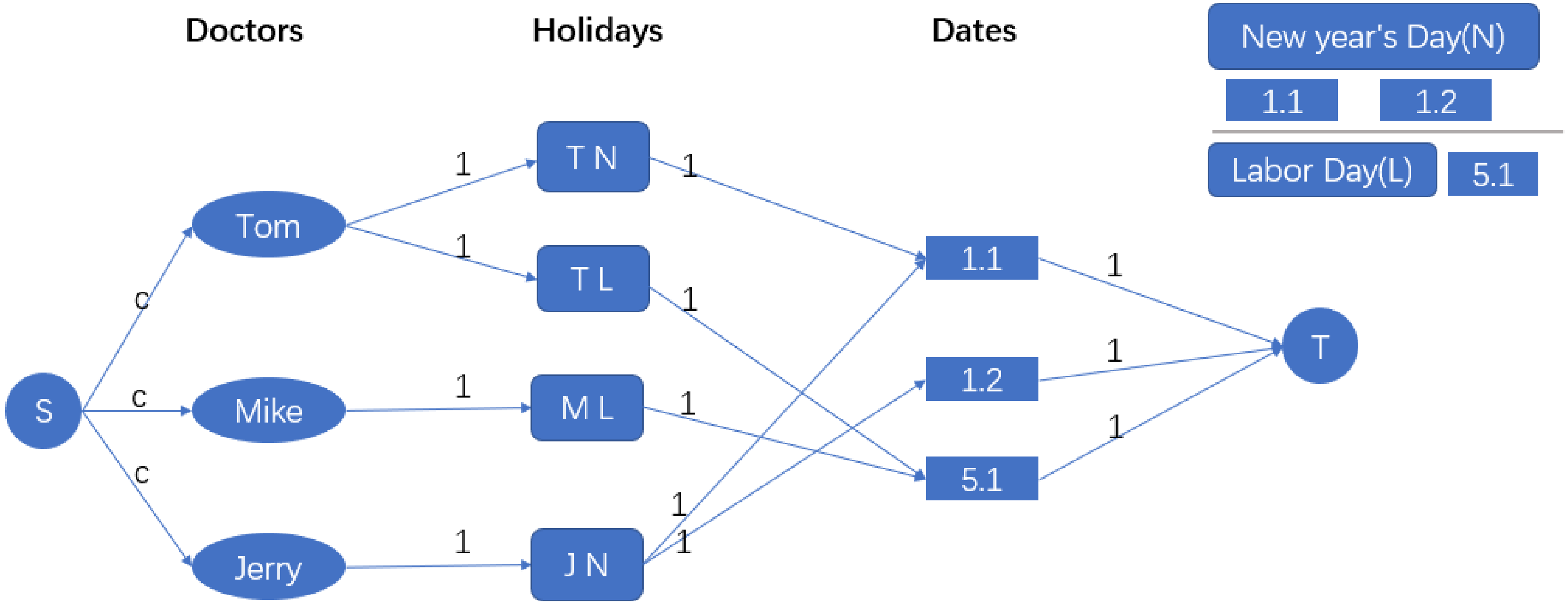



```
duty doctor of date 1: Doctor 1
duty doctor of date 2: Doctor 3
duty doctor of date 3: Doctor 2
请按任意键继续. . .
```

Experiment

$Holiday\ Number: 20, \quad dates\ of\ each\ holiday: 5, \quad Maximum\ number\ of\ shift\ days\ per\ doctor: 5$

DoctorNumber	200	400	600	800	1000
EK(ms)	89.37	177.27	262.23	366.44	443.79
dinic+Multichannel(ms)	23.50	85.96	199.19	377.65	565.35
dinic+CurrentArc(ms)	3.88	6.40	11.18	14.83	17.61



THANKS

2022.6.22