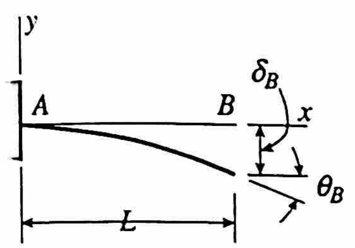
Deflections and Slopes of Beams

Table H-1

Deflections and Slopes of Cantilever Beams



Notation:

v = deflection in the y direction (positive upward)

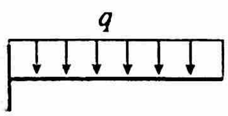
$v' = dv/dx$ = slope of the deflection curve

$\delta_B = -v(L)$ = deflection at end B of the beam (positive downward)

$\theta_B = -v'(L)$ = angle of rotation at end B of the beam (positive clockwise)

EI = constant

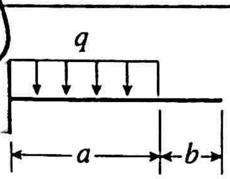
1



$$v = -\frac{qx^2}{24EI}(6L^2 - 4Lx + x^2) \qquad v' = -\frac{qx}{6EI}(3L^2 - 3Lx + x^2)$$

$$\delta_B = \frac{qL^4}{8EI} \qquad \theta_B = \frac{qL^3}{6EI}$$

2



$$v = -\frac{qx^2}{24EI}(6a^2 - 4ax + x^2) \qquad (0 \le x \le a)$$

$$v' = -\frac{qx}{6EI}(3a^2 - 3ax + x^2) \qquad (0 \le x \le a)$$

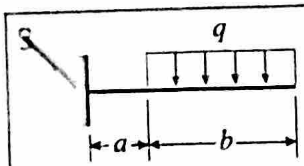
$$v = -\frac{qa^3}{24EI}(4x - a) \qquad v' = -\frac{qa^3}{6EI} \qquad (a \le x \le L)$$

$$\text{At } x = a: v = -\frac{qa^4}{8EI} \qquad v' = -\frac{qa^3}{6EI}$$

$$\delta_B = \frac{qa^3}{24EI}(4L - a) \qquad \theta_B = \frac{qa^3}{6EI}$$

Table H-1 (Continued)

3



$$v = -\frac{qbx^2}{12EI}(3L + 3a - 2x) \qquad (0 \le x \le a)$$

$$v' = -\frac{qbx}{2EI}(L + a - x) \qquad (0 \le x \le a)$$

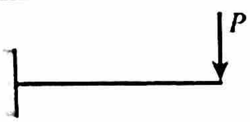
$$v = -\frac{q}{24EI}(x^4 - 4Lx^3 + 6L^2x^2 - 4a^3x + a^4) \qquad (a \le x \le L)$$

$$v' = -\frac{q}{6EI}(x^3 - 3Lx^2 + 3L^2x - a^3) \qquad (a \le x \le L)$$

At $x = a$: $v = -\dfrac{qa^2b}{12EI}(3L + a) \qquad v' = -\dfrac{qabL}{2EI}$

$$\delta_B = \frac{q}{24EI}(3L^4 - 4a^3L + a^4) \qquad \theta_B = \frac{q}{6EI}(L^3 - a^3)$$

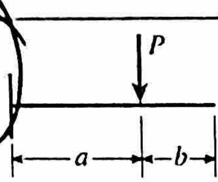
4



$$v = -\frac{Px^2}{6EI}(3L - x) \qquad v' = -\frac{Px}{2EI}(2L - x)$$

$$\delta_B = \frac{PL^3}{3EI} \qquad \theta_B = \frac{PL^2}{2EI}$$

5



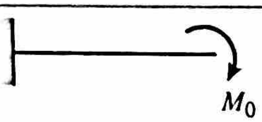
$$v = -\frac{Px^2}{6EI}(3a - x) \qquad v' = -\frac{Px}{2EI}(2a - x) \qquad (0 \le x \le a)$$

$$v = -\frac{Pa^2}{6EI}(3x - a) \qquad v' = -\frac{Pa^2}{2EI} \qquad (a \le x \le L)$$

At $x = a$: $v = -\dfrac{Pa^3}{3EI} \qquad v' = -\dfrac{Pa^2}{2EI}$

$$\delta_B = \frac{Pa^2}{6EI}(3L - a) \qquad \theta_B = \frac{Pa^2}{2EI}$$

6

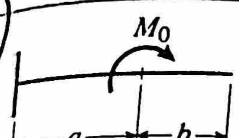


$$v = -\frac{M_0x^2}{2EI} \qquad v' = -\frac{M_0x}{EI}$$

$$\delta_B = \frac{M_0L^2}{2EI} \qquad \theta_B = \frac{M_0L}{EI}$$

Table H-1 (Continued)

7



$$v = -\frac{M_0 x^2}{2EI} \qquad v' = -\frac{M_0 x}{EI} \qquad (0 \le x \le a)$$

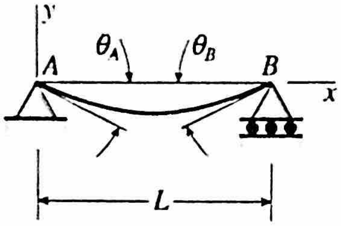
$$v = -\frac{M_0 a}{2EI}(2x - a) \qquad v' = -\frac{M_0 a}{EI} \qquad (a \le x \le L)$$

$$\text{At } x = a\text{:} \quad v = -\frac{M_0 a^2}{2EI} \qquad v' = -\frac{M_0 a}{EI}$$

$$\delta_B = \frac{M_0 a}{2EI}(2L - a) \qquad \theta_B = \frac{M_0 a}{EI}$$

Table H-2

Deflections and Slopes of Simple Beams



Notation:

v = deflection in the y direction (positive upward)

$v' = dv/dx$ = slope of the deflection curve

$\delta_C = -v(L/2)$ = deflection at midpoint C of the beam (positive downward)

x_1 = distance from support A to point of maximum deflection

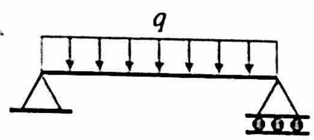
$\delta_{max} = -v_{max}$ = maximum deflection (positive downward)

$\theta_A = -v'(0)$ = angle of rotation at left-hand end of the beam (positive clockwise)

$\theta_B = v'(L)$ = angle of rotation at right-hand end of the beam (positive counterclockwise)

EI = constant

1

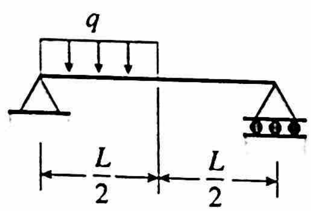


$$v = -\frac{qx}{24EI}(L^3 - 2Lx^2 + x^3)$$

$$v' = -\frac{q}{24EI}(L^3 - 6Lx^2 + 4x^3)$$

$$\delta_C = \delta_{max} = \frac{5qL^4}{384EI} \qquad \theta_A = \theta_B = \frac{qL^3}{24EI}$$

2



$$v = -\frac{qx}{384EI}(9L^3 - 24Lx^2 + 16x^3) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

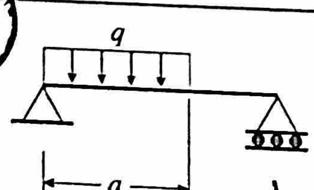
$$v' = -\frac{q}{384EI}(9L^3 - 72Lx^2 + 64x^3) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$v = -\frac{qL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3) \qquad \left(\frac{L}{2} \le x \le L\right)$$

$$v' = -\frac{qL}{384EI}(24x^2 - 48Lx + 17L^2) \qquad \left(\frac{L}{2} \le x \le L\right)$$

$$\delta_C = \frac{5qL^4}{768EI} \qquad \theta_A = \frac{3qL^3}{128EI} \qquad \theta_B = \frac{7qL^3}{384EI}$$

3



$$v = -\frac{qx}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 2a^2x^2 - 4aLx^2 + Lx^3) \qquad (0 \le x \le a)$$

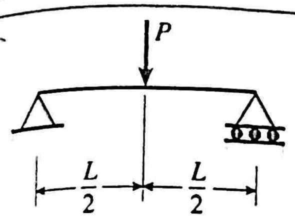
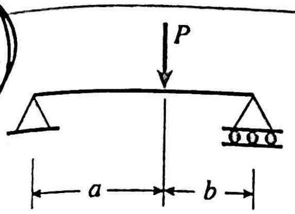
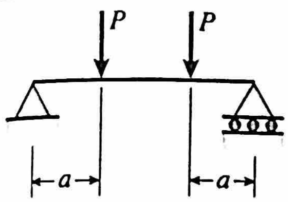
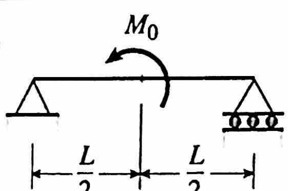
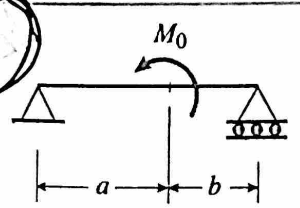
$$v' = -\frac{q}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 6a^2x^2 - 12aLx^2 - 4Lx^3) \qquad (0 \le x \le a)$$

$$v = -\frac{qa^2}{24LEI}(-a^2L + 4L^2x + a^2x - 6Lx^2 + 2x^3) \qquad (a \le x \le L)$$

$$v' = -\frac{qa^2}{24LEI}(4L^2 + a^2 - 12Lx + 6x^2) \qquad (a \le x \le L)$$

$$\theta_A = \frac{qa^2}{24LEI}(2L - a)^2 \qquad \theta_B = \frac{qa^2}{24LEI}(2L^2 - a^2)$$

Table H-2 (Continued)

		$v = -\frac{Px}{48EI}(3L^2 - 4x^2)$ $\quad v' = -\frac{P}{16EI}(L^2 - 4x^2)$ $\quad \left(0 \le x \le \frac{L}{2}\right)$ $\delta_C = \delta_{max} = \frac{PL^3}{48EI}$ $\quad \theta_A = \theta_B = \frac{PL^2}{16EI}$
		$v = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2)$ $\quad v' = -\frac{Pb}{6LEI}(L^2 - b^2 - 3x^2)$ $\quad (0 \le x \le a)$ $\theta_A = \frac{Pab(L+b)}{6LEI}$ $\quad \theta_B = \frac{Pab(L+a)}{6LEI}$ If $a \ge b$, $\delta_C = \frac{Pb(3L^2 - 4b^2)}{48EI}$ $\quad$ If $a \le b$, $\delta_C = \frac{Pa(3L^2 - 4a^2)}{48EI}$ If $a \ge b$, $\quad x_1 = \sqrt{\frac{L^2 - b^2}{3}}$ and $\delta_{max} = \frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}LEI}$
6		$v = -\frac{Px}{6EI}(3aL - 3a^2 - x^2)$ $\quad v' = -\frac{P}{2EI}(aL - a^2 - x^2)$ $\quad (0 \le x \le a)$ $v = -\frac{Pa}{6EI}(3Lx - 3x^2 - a^2)$ $\quad v' = -\frac{Pa}{2EI}(L - 2x)$ $\quad (a \le x \le L - a)$ $\delta_C = \delta_{max} = \frac{Pa}{24EI}(3L^2 - 4a^2)$ $\quad \theta_A = \theta_B = \frac{Pa(L-a)}{2EI}$
7		$v = -\frac{M_0 x}{6LEI}(2L^2 - 3Lx + x^2)$ $\quad v' = -\frac{M_0}{6LEI}(2L^2 - 6Lx + 3x^2)$ $\delta_C = \frac{M_0 L^2}{16EI}$ $\quad \theta_A = \frac{M_0 L}{3EI}$ $\quad \theta_B = \frac{M_0 L}{6EI}$ $x_1 = L\left(1 - \frac{\sqrt{3}}{3}\right)$ and $\delta_{max} = \frac{M_0 L^2}{9\sqrt{3}EI}$
8		$v = -\frac{M_0 x}{24LEI}(L^2 - 4x^2)$ $\quad v' = -\frac{M_0}{24LEI}(L^2 - 12x^2)$ $\quad \left(0 \le x \le \frac{L}{2}\right)$ $\delta_C = 0$ $\quad \theta_A = \frac{M_0 L}{24EI}$ $\quad \theta_B = -\frac{M_0 L}{24EI}$
		$v = -\frac{M_0 x}{6LEI}(6aL - 3a^2 - 2L^2 - x^2)$ $\quad (0 \le x \le a)$ $v' = -\frac{M_0}{6LEI}(6aL - 3a^2 - 2L^2 - 3x^2)$ $\quad (0 \le x \le a)$ At $x = a$: $\quad v = -\frac{M_0 ab}{3LEI}(2a - L)$ $\quad v' = -\frac{M_0}{3LEI}(3aL - 3a^2 - L^2)$ $\theta_A = \frac{M_0}{6LEI}(6aL - 3a^2 - 2L^2)$ $\quad \theta_B = \frac{M_0}{6LEI}(3a^2 - L^2)$

(Continued)