A Unified Micromechanical Model for the Mechanical Properties of Two Constituent Composite Materials. Part III: Strength Behavior

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ABSTRACT: This series of papers reports a new, general, and unified micromechanical model for estimating the three-dimensional mechanical properties of a composite made from two constituent materials, i.e., continuous fiber and matrix. The linear elastic and elasto-plastic behaviors have been studied in Parts I and II respectively. The present paper investigates the tensile strength and failure mode of the composite. The tensile strength of the composite is predicted based on the ultimate stresses in the constituent phases, as the states of stresses in both phases are explicitly represented as the function of the applied overall stress field. The maximum normal stress theory for isotropic materials is incorporated with this prediction. As long as the maximum normal stress in either the fibers or the matrix attains its ultimate value, the composite is considered to fail, and the corresponding strength and failure mode are thus automatically defined. This makes determination of the composite strength as well as the failure mode extremely general, yet particularly easy. The composite can be subjected to any kind of load combination, whereas the corresponding strength behavior is determined through the same simple procedure. Comparisons between predicted and measured tensile strengths for several unidirectional composites under off-axial loads and for a plain weft-knitted fabric-reinforced composite under different uniaxial loads confirm that the present micromechanical strength theory is very efficient.

KEY WORDS: fiber composite materials, failure theories, unified micromechanical model, tensile strengths, failure modes.

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1. INTRODUCTION

STRENGTH IS AN important mechanical behavior of composite materials. One must be confident in their load-carrying capacity before making efficient use of them. The purpose of any stress analysis is to predict the conditions under which the materials will fail and to determine the allowable external loads for a desired margin of safety. The ability to estimate accurately the strength of a particular composite is necessary for design.

Failure occurs in a material when the applied load reaches a threshold that is the limit of its load-carrying capacity. "Failure" can be defined in different ways according to the mechanical and deformation characteristics of the subject material and the surviving load requirements. For metals, "failure" is defined in most cases either as "yield" (for ductile materials) or "fracture" (for brittle materials). In any case, however, failure is directly related to the corresponding strength of the materials. The yield failure is controlled by the yield stress (yield strength) of the materials, whereas the fracture failure is usually controlled by the ultimate stress (ultimate strength) of the material (an unstable crack-induced fracture problem is out of the scope of the present concern). The occurrence of failure depends on the material properties and also on the loading configuration (uniaxial vs. multiaxial), loading function (e.g., quasi-static, cyclic), loading rate, temperature, etc. On the other hand, strength is an intrinsic material property. For a composite, strength depends on the strength of the constituent materials as well as on the internal microstructural geometry of the constituents.

When an isotropic material is subjected to uniaxial tensile loading, only one principal stress component exists, and failure can be defined experimentally by a simple tensile test, which provides a characteristic stress-strain curve for the material. Failure is predicted to occur when the maximum normal stress reaches the yield or the ultimate point on the stress-strain curve. When the material is subjected to multiaxial states of stress, however, the interaction among the various stress components makes prediction of failure much more difficult. The number of multiaxial tests required, accounting for the entire range of all stress components and the cost and complexity of such testing, is prohibitive in reaching a definitive experimental characterization of failure. In some cases, experimental duplication of actual loading conditions is not even possible. Therefore, strength theories have to be developed to predict the failure of materials under multiaxial state of stress conditions. Fortunately, principles for governing isotropic material failures are well established.

As for composite materials, most strength theories have been developed based on macromechanics considerations similar to those for isotropic materials. These theories treat composites as general anisotropic materials. They can be considered more or less as generalizations from the corresponding failure theories of isotropic materials. Several comprehensive surveys of these phenomenological strength theories exist in the literature [3–7]. Unlike in isotropic cases, extensive experiments including, possibly, biaxial tests, which may be difficult or expensive to conduct [8], have to be performed to determine the strength coefficients involved in the phenomenological theories whenever they are applied to any particular composite. Even with the same constituent materials, different composites having different fiber volume fractions still require different testing. Another drawback with these theories is that they generally cannot predict the failure mode of the composite. The theories are hardly able to indicate which of the constituent phases initiates the failure of the composite or how strong the other phase is. From a designer's point of view, the micromechanical failure mechanisms of the composite are important. The quantitative relationship of the overall failure of the composite with the respective strengths of the constituent materials and with the composition geometry is definitely valuable for the choice of an existing composite and for the design of a new composite. An accurate micromechanical approach to the failure of the composite can also reduce costs in experiments, which a macromechanical approach requires.

Numerous micromechanical investigations into composite failures have been conducted [9–14]. However, most of these approaches were focused mainly on the nature of the local environment (i.e., the "weak links" of the system such as the fraction of unbroken fibers in the fiber bundle, fiber pull-out, interface debonding, etc.) and were developed by assuming a priori deformation modes. Very few treated the overall failure of a composite in terms of the average responses of its constituent phases, like the approaches to the overall elastic stiffness/compliance of the composites. Aboudi [14] was among these few investigators. He estimated the strength of unidirectional fiber composites based on his method-of-cells model. However, in his prediction, the recovered strengths of both the fiber and matrix phases were higher than the measured transverse strengths of the composites [14], which might be impossible in general. From a phenomenological point of view, when a composite is made "ideally" from a fiber and a matrix and if there are no residual stresses involved in the constituent materials, the tensile strength of the composite in any direction could not be smaller than the minimum of the tensile strengths of the constituent materials.

In general, the failure, especially fracture, of the composite occurs when most matrix or fibers fail. It seems that some debonding between the fiber and the matrix does not correspond to the onset of the composite fracture. The continuous transfer of the stress and strain between two constituent phases may still be considered to exist as a whole. In fact, most debonding takes place when the composite breaks. Supposing that both fiber and matrix are isotropic, it is reasonable to assume that the well-established strength theories for isotropic materials are still applicable to the fiber and the matrix phases in the composite before it ruptures. We can thus postulate that the composite fails if either the stress of the fiber or the stress of the matrix reaches its limit value. The central issue is then transformed to determine the state of stress in each constituent material when the composite is subjected to an external load. However, with the unified micromechanical model developed in References [1] and [2], the stresses generated in both constituents are explicitly expressed in terms of the overall stress of the composite. Thus, the application of this model to predict the strength of the composite is straightforward. Combining the unified micromechanical model and the strength criterion for isotropic materials yields a new strength theory that can be used to predict the strengths of composites under any state of stress. The theory has been applied to estimate the off-axial tensile strengths of several unidirectional composites and a weft-knitted glass-fiber fabric-reinforced epoxy-matrix composite. Very good agreement has been found between the predictions and the corresponding experiments.

2. STRENGTH THEORIES OF ISOTROPIC MATERIALS

As the new theory is based on knowledge of isotropic materials, some of the most widely used strength theories for these materials are reviewed in this section.

All the strength theories of isotropic materials are expressed in terms of principal stresses σ^1, σ^2, and σ^3 with $\sigma^1 \geq \sigma^2 \geq \sigma^3$. For a general stress state (σ_{xx}, σ_{yy}, σ_{zz}, σ_{yz}, σ_{xz}, σ_{xy}) generated in a material, the principal stresses σ^1, σ^2, and σ^3 are the solutions to the following eigen-value equation:

$$\det\begin{vmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} - \sigma[I] \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{vmatrix} = 0 \tag{1}$$

With these principal stresses, the two most widely used strength theories are expressed below.

2.1 Maximum Normal Stress Theory

This theory postulates that a material fails as soon as the maximum normal stress, σ^1, generated in the material reaches its ultimate value, no matter whether the material is under uniaxial or multiaxial stress. The failure criterion can be expressed as

$$\sigma^1 \geq \sigma_u \tag{2}$$

where σ_u is the ultimate tensile stress of the material obtained from a uniaxial test.

Historically, this theory was developed for governing the strength of brittle materials. For this kind of material, there is no noticeable plastic deformation. Under a uniaxial tensile load, the stress-strain curve of the material is monotonically increased to a peak point that corresponds to the ultimate stress and ultimate strain of the material. If we agree to accept that the failure occurs when $dP/d\varepsilon = 0$ [15], where P is the load applied on the material and ε is the resulting strain, i.e., when the material is unable to take any more load, we can easily appreciate the background of this theory.

However, even for ductile materials for which $dP/d\varepsilon = 0$ may occur at another point, the yield point, on the stress-strain curve, we can still use Equation (2) to predict the ultimate strength of the materials after yield failure, since the σ^1 involved does not depend on material properties. Therefore, Equation (2) can be used to govern the ultimate strength of any isotropic material.

2.2 Maximum Distorted Energy Theory (Von Mises-Hencky Theory)

This theory was proposed for governing the yield failure of ductile materials such as ductile metals. When such materials yield, a phenomenon called plastic incompressibility is involved. The volumetric strain of the materials in the plastic regime is nearly zero. Hence, no plastic work can be done by the hydrostatic component (mean normal stresses) of the applied stress field. If we separate the total strain energy into two parts, the volume strain energy part and the distorted strain energy part, we can say that only the distorted strain energy part results in the material yield.

Subtracting the strain energy caused by the change of volume from the total energy expression, we obtain the distorted energy (in terms of principal stresses) as

$$\sigma_{eq} = \sqrt{\frac{1}{2}[(\sigma^1 - \sigma^2)^2 + (\sigma^2 - \sigma^3)^2 + (\sigma^3 - \sigma^1)^2]} \tag{3}$$

where the subscript "*eq*" stands for "equivalent." The maximum distorted energy theory postulates that no matter whether a ductile material is under uniaxial or multiaxial stress, the yield failure of the material occurs if its equivalent stress, defined by Equation (3), attains a limit value, σ_Y. The failure criterion is thus

$$\sigma_{eq} \geq \sigma_Y \tag{4}$$

where σ_Y is the yield strength of the material corresponding to the uniaxial tensile test.

In addition to the above two strength theories, there are several others for isotropic materials [4,5], but none is as powerful and general as the above two. The most important feature of these theories is that as long as the involved limit value [such as σ_u in Equation (2) or σ_Y in Equation (4)] is determined from a single test, usually a uniaxial tensile test, the theories are considered applicable to any other load condition.

3. STRENGTH THEORY FOR COMPOSITES

In a composite made of two-phase materials, the fiber and the matrix may be ductile or brittle or a mixture of both. In the case of two ductile phases, the stress-strain curve will exhibit three regimes: where both phases are linear elastic, where one is plastic/nonlinear elastic and the other remains linear elastic, and where both are plastic/nonlinear elastic. The strength of the composite is obviously controlled by the strength of the fiber, or the strength of the matrix, or the strengths of both. If the fiber volume fraction is moderate, either one of the constituent failures will generally result in the overall failure of the composite. Based on these considerations, the present strength theory postulates that except for the two extreme cases in which the fiber volume fraction is either extremely small or extremely large, the composite is considered to fail whenever the fiber or the matrix attains its failure stress. According to this theory, the tensile strength of the composite is immediately given by

$$\sigma_u^c = \begin{cases} \sigma_c^1(\sigma_u^f), & \textit{if} \quad (\sigma^1)^f \geq \sigma_u^f \\ \sigma_c^1(\sigma_u^m), & \textit{if} \quad (\sigma^1)^m \geq \sigma_u^m \end{cases} \tag{5}$$

where σ_u^c, σ_u^f, and σ_u^m are respectively the tensile strengths (ultimate stresses) of the composite, the fiber, and the matrix; $(\sigma^1)^f$ and $(\sigma^1)^m$ are the resulting maximum normal stresses in the fiber and the matrix phases respectively; and σ_c^1 is the overall maximum normal stress of the composite. The expression $\sigma_c^1(\sigma_u^f)$ indicates that σ_u^c is determined by the corresponding σ_c^1 when the resulting maximum normal stress in the fiber, $(\sigma^1)^f$, reaches its ultimate value σ_u^f. The expression $\sigma_c^1(\sigma_u^m)$ has a similar meaning. It is noted that σ_u^c, $(\sigma^1)^f$, and $(\sigma^1)^m$ all depend on the load direction. Suppose that the composite under consideration is a unidirectional lamina. If the fiber and the matrix are both elastic brittle materials, $(\sigma^1)^f$ and $(\sigma^1)^m$ are evaluated according to Equation (1) by using the averaged stresses $\{\sigma_i^f\}$ and $\{\sigma_i^m\}$ given by Equations (7.1) and (7.2) in Reference [1] respectively. If, however, any one of the materials is ductile (elasto-plastic), $\{\sigma_i^f\}$ and $\{\sigma_i^m\}$ will be determined by means of the procedure described in Section 3 of Reference [2].

It should be pointed out that the accuracy of the ultimate strengths of the fiber and the matrix phases, σ_u^f and σ_u^m, directly influences the prediction of σ_u^c. The values of σ_u^f and σ_u^m in the composite may be different from those when the materials are in bulk form. They should be obtained based on the material forms used in fabricating the composite. If not available from individual tests, these values can be back recovered from the overall strengths of the composite in two different directions, such as the longitudinal and the transverse strengths of a unidirectional composite.

Unlike the tensile strength, which is defined by Equation (5), the definition for the yield failure (strength) of the composite is somewhat ambiguous. A natural ductile material almost always reveals some plastic hardening behavior. When an isotropic material yields, the condition $dP/d\varepsilon = 0$ can be satisfied. However, when one phase of the constituent materials begins to yield but the other phase remains linearly elastic, the overall response of the composite may not satisfy the condition $dP/d\varepsilon = 0$. On the other hand, in some direction or if the fiber volume fraction takes some particular values, the overall response of the composite may be considered as plastic under the same load condition. Nevertheless, the composite yield can always be considered to occur above a stress state corresponding to which one of the constituent phases yields, and below a stress state under which both constituent materials yield. As the entire stress-strain curve of the composite in any direction can be predicted using the procedure developed in Reference [2], in which the yield failure condition (2) was incorporated, the corresponding yield strength may be measured from the curve and will not be discussed in the present paper. The following sections are concerned only with the ultimate strength of a composite.

4. APPLICATIONS TO UNIDIRECTIONAL COMPOSITES

The strength theory developed in the previous section is now applied to estimate the tensile strengths of three different unidirectional fiber-reinforced composites under off-axial load conditions. One is a boron/epoxy composite; another is a glass/epoxy composite; and the last is a boron/aluminum composite. Experiments have been performed independently by previous investigators [16–18] to determine the strength behavior of these composites under off-axial tensile loads. In the present simulation, the fiber and the matrix of the first composite are considered as brittle elastic until rupture, whereas the fiber phase of the other two composites are taken as elastic and the matrix phase as elasto-plastic. As all the ultimate strengths of the constituents are not available, they have to be inversely determined from the overall longitudinal (0°-direction) and transverse (90°-direction) strengths, X and Y, of the corresponding composites respectively. Details are incorporated with the following examples.

Let us first consider a unidirectional boron fiber-reinforced epoxy-matrix composite. The given material parameters [14] are $E_f = 400$ GPa, $E_m = 3.45$ GPa, $\nu_f = 0.2$, $\nu_m = 0.35$ and $V_f = 0.5$. The ultimate strengths of the boron and the epoxy were not reported. However, the overall tensile strengths of the composite in the 0° and 90° directions were given as [16], respectively, $X = 1296$ MPa and $Y = 62$ MPa. In general, the composite strength in the longitudinal (fiber axial) direction is controlled by the ultimate stress of the fiber, whereas the composite strength in the

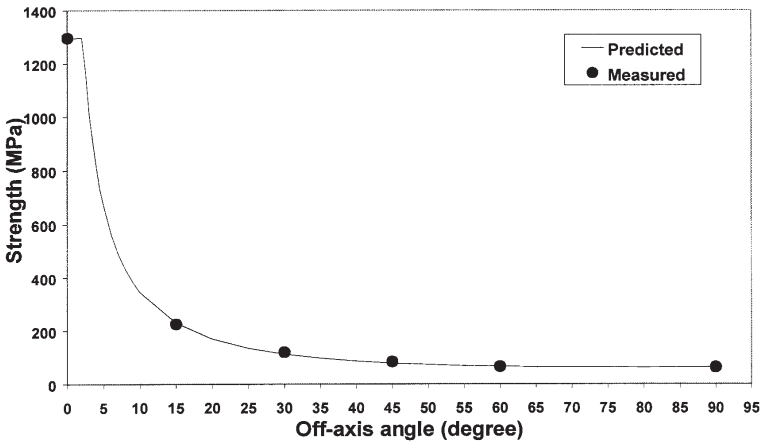

Figure 1a. *Predicted and measured [16] off-axis strength of a unidirectional boron/epoxy composite. The parameters used are* $E_f = 400$ *GPa,* $E_m = 3.45$ *GPa,* $\nu_f = 0.2$*,* $\nu_m = 0.35$*,* $\sigma_u^f = 2569$ *MPa,* $\sigma_u^m = 41.7$ *MPa,* $V_f = 0.5$*.*

transverse direction is governed by that of the matrix. Suppose that this is true. Substituting the formulae given in Section 4 of Reference [1], i.e.,

$$\sigma_1^m = a_{11}\sigma_1^f + a_{12}\sigma_2^f,\ \sigma_2^m = a_{22}\sigma_2^f,\ a_{11} = E_m/E_f$$

$$a_{22} = \frac{1}{2}\left(1 + \frac{E_m}{E_f}\right),\ a_{12} = \frac{S_{12}^f - S_{12}^m}{S_{11}^f - S_{11}^m}(a_{11} - a_{22}) = \frac{\nu_m E_f - \nu_f E_m}{E_m - E_f}(a_{11} - a_{22})$$

into the relationships $\sigma_1 = V_f\sigma_1^f + V_m\sigma_1^m$ and $\sigma_2 = V_f\sigma_2^f + V_m\sigma_2^m$ respectively, it is found that $\sigma_u^f = 2569$ MPa and $\sigma_u^m = 41.6$ MPa. (When Y is applied alone, one has $\sigma_2 = Y = [V_f(a_{22})^{-1} + V_m]\sigma_u^m$, from which σ_u^m is obtained. σ_u^f can be calculated similarly.)

The predicted ultimate off-axial tensile strength versus the off-axial angle (in degree) was plotted in Figure 1a. For comparison, the experimental data taken from Reference [16] were also shown. It is seen that the correlation between the predicted results and the experimental data is very high.

The most important feature of the present strength theory is that the state of stress in each constituent phase is explicitly known. The failure mode, which is considered as a source to initiate the composite failure, is thus clearly seen. In the present theory, a failure mode can be either fiber fracture, matrix fracture, or both. Figure 1b shows the relationship of the maximum normal stresses in the compos-

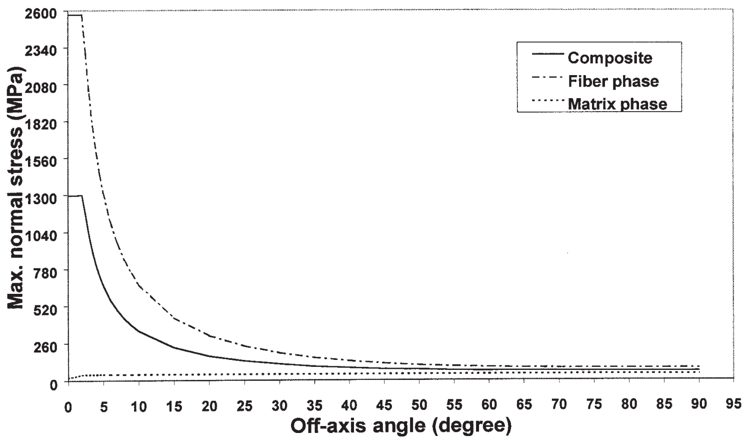

Figure 1b. *Maximum normal stresses in different phases vs off-axis angle for unidirectional boron/epoxy composite.*

ite, the fiber, and the matrix versus the off-axial angle when the composite attains the corresponding tensile strength. The figure clearly indicates that except for a small neighborhood of angle = 0° (up to 2.5°), the composite failure (tensile strength) in any other direction is controlled by the matrix strength. When the applied external load has an off-axial angle within this neighborhood, the composite failure is caused by the fiber fracture. The figure also indicates that the maximum normal stress, or the tensile strength, of the matrix in any off-axial direction is always smaller than the corresponding strength of the composite. It is found that with a reinforcement of 50% fiber volume fraction, the strength of the composite increases by at least 50% over that of the pure matrix. This is reasonable but different from Aboudi's prediction [14].

The second example is a unidirectional glass-fiber-reinforced epoxy-matrix composite. The experimentally measured values of off-axial tensile strength for this composite were reported in Reference [17]. The material data used are [14] E_f = 73 GPa, E_m = 3.45 GPa, ν_f = 0.22, ν_m = 0.35 and V_f = 0.6. No other material parameters except the off-axial strengths of the composite are available. As in the first example, the tensile strengths in the longitudinal and transverse directions, X = 1236 MPa and Y = 28.45 MPa, are used to recover the other involved parameters of the constituent materials. However, in the present case, if we assumed that both materials were linearly elastic until rupture, we could not recover the ultimate stresses of the glass and the epoxy, σ_u^f and σ_u^m, by virtue of the given X and Y. Otherwise, the strengths of the composite in both the longitudinal and transverse directions would be governed by the matrix strength only, while the predicted longitudinal strength would be smaller than the measured data, X = 1236 MPa. This means that the given two materials are not both linearly elastic. Supposing that the glass fiber is elastic and the epoxy matrix is elasto-plastic, the problem can be addressed. The recovered ultimate stresses of the fiber and the matrix are $\sigma_u^f = 2052$ MPa and $\sigma_u^m = 18.5$ MPa.

It should be noted that there are two other material parameters involved in the elasto-plastic behavior of the matrix, i.e., the hardening modulus E_T^m and the yield stress σ_Y^m. They are pre-assumed by trial-and-error due to no other information. However, as long as the longitudinal strength is determined by the ultimate stress of the fiber, and the transverse strength is obtained based on that of the matrix, the particular values of E_T^m and σ_Y^m have little influence on the overall tensile strength of the composite. This may be attributed to the fact that the ultimate strength of the composite is dependent mainly on the ultimate stresses, not on the yield stress or hardening modulus, of the constituent materials, although the ultimate strain and the entire stress-strain curve of the composite do depend on them.

The predicted tensile off-axial strengths of the composite versus off-axial angle with four different combinations of E_T^M and σ_Y^m are plotted in Figure 2a. The four curves are seen to coincide exactly with each other. The experimental data taken from Reference [17] are also shown in the figure. Good agreement exists between the predicted results and the measured data. Figure 2b indicates the corresponding maximum normal stresses generated in the fiber and in the matrix when the off-axial tensile stress of the composite reaches its ultimate value (based on matrix plastic parameters of $E_T^m = 201$ MPa and $\sigma_Y^m = 10.5$ MPa). The quantitative results are similar to those shown in Figure 1b. Hence, the same conclusion can be made for this type of composite.

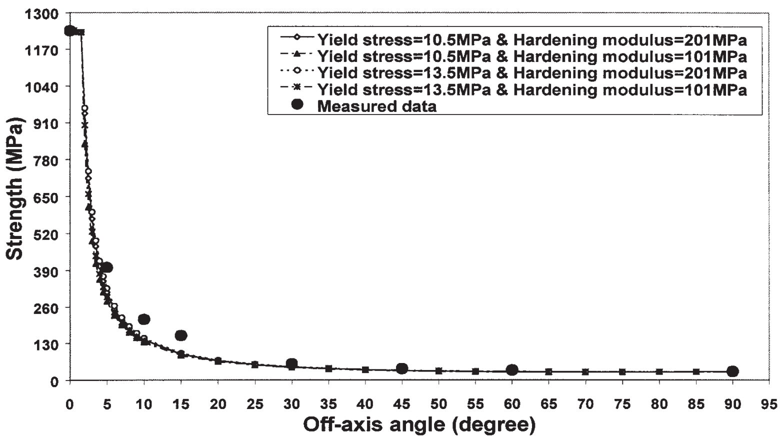

Figure 2a. *Predicted and measured [17] off-axis strength of a unidirectional glass/epoxy composite. The parameters used are* $E^f = E_T^f = 73$ *GPa,* $E^m = 3.45$ *GPa,* $\nu_f = 0.22$, $\nu_m = 0.35$, $\sigma_u^f = \sigma_Y^f = 2052$ *MPa,* $V_f = 0.6$ *and* $\sigma_u^m = 18.5$ *MPa.*

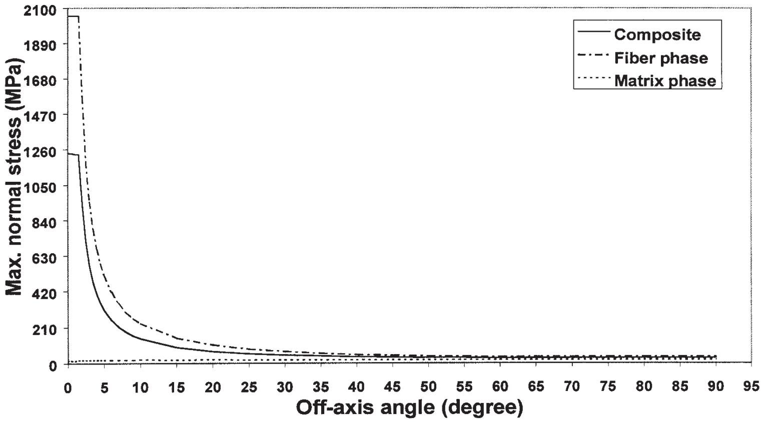

Figure 2b. *Maximum normal stresses in different phases vs off-axis angle for unidirectional glass/epoxy composite.*

The last unidirectional composite under study is made of boron fibers and an aluminum matrix. Experimental measurement for the off-axial strength of this composite was conducted by Becker et al. [18]. Young's moduli and Poisson's ratios of the boron and aluminum are [14] $E_f = 400$ GPa, $E_m = 72.5$ GPa, $\nu_f = 0.2$, and $\nu_m = 0.33$. The fiber volume fraction is $V_f = 0.46$. The measured longitudinal and transverse strengths of the composite (under cyclic load) are $X = 1600$ MPa and $Y = 148$ MPa, respectively.

Similarly, as in the second example, the present composite must be considered as being made from elastic boron and elasto-plastic aluminum. The procedure to recover the other involved constituent parameters from the given longitudinal and transverse strengths is the same as that described in the second example. These parameters are found to be $\sigma_u^f = 3385$ MPa, $\sigma_u^m = 103$ MPa, $E_T^m = 3.2$ GPa, and $\sigma_Y^m = 50$ MPa.

The estimated and measured off-axial strengths of the composite varying with the off-axis angle are graphed in Figure 3a. Again, very good agreement between the results obtained from these two different methods has been found. Stress shares by the fiber and by the matrix are indicated in Figure 3b. In contrast to the previous two composites for which the predicted off-axis strength curve is monotonously decreased from 0 degrees to 90 degrees, an interesting feature for the present composite is that the predicted curve has a local minimum value in the neighborhood of angle = 2.5°. This is because the minimum stress (in terms of the first principal stress) in the matrix phase appears at an off-axis angle of greater than 0°. Namely, the matrix attains its minimum stress when the load direction is inclined with, rather than parallel to, the longitudinal direction. The angle of this inclination depends mainly on the stiffness ratio between the Young's moduli of the two

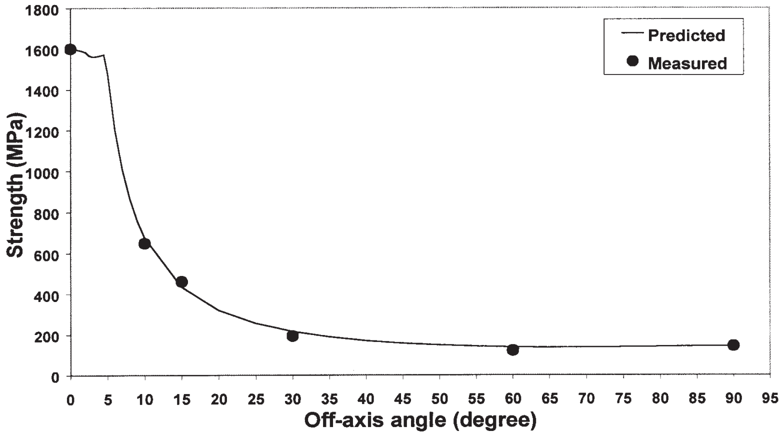

Figure 3a. *Predicted and measured [18] off-axis strength of a unidirectional boron/aluminum composite. The parameters used are* $E^f = 400$ *GPa,* $E^m = 72.5$ *GPa,* $\nu_f = 0.2$*,* $\nu_m = 0.33$*,* $\sigma_u^f = 3385$ *MPa,* $\sigma_Y^m = 50$ *MPa,* $E_T^m = 3.2$ *GPa,* $\sigma_u^m = 103$ *MPa and* $V_f = 0.46$*.*

constituent materials. When the Young's modulus of the fiber is significantly higher than that of the matrix, such as in the first example, the matrix attains its minimum stress under a longitudinal load. In the present example, the stiffness ratio between the Young's modulus of the matrix and that of the fiber is not too small. Hence, the matrix arrives at its minimum stress when the applied load is in a relatively larger inclined angle (about 2.5°) with the longitudinal direction, giving

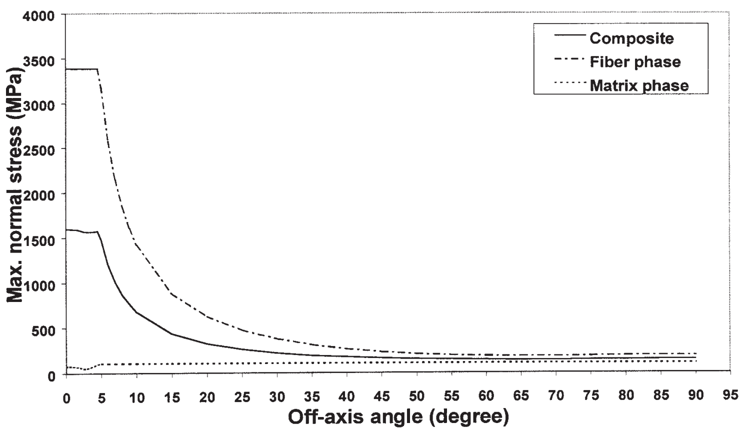

Figure 3b. *Maximum normal stresses in different phases vs off-axis angle for unidirectional boron/aluminum composite.*

rise to a locally minimal off-axis strength for the composite.

In concluding this section, it can be said that the unified model developed in this series of papers provides a powerful technique to micromechanically predict the ultimate strength of a composite by means of the strength behavior of its constituent materials. The model has a strong theoretical and logical background. The only pre-requirement is the continuous transfer of the internal stresses between the two constituent phases, i.e., a perfect bonding between the fibers and the matrix. From a macromechanical point of view, this requirement is considered to be fulfilled by most composite materials just before the failure. It is also worth mentioning that, because the stress share in every constituent phase is given exactly with the present micromechanical model, it is possible to perform an extensive parametric study to obtain an optimum load-carrying composite. Such an investigation is easily carried out based on the present theory.

5. STRENGTH OF PLAIN WEFT-KNITTED GLASS-FIBER FABRIC-REINFORCED EPOXY COMPOSITE

Experimentally measured tensile strengths of the plain weft-knitted glass-fiber fabric-reinforced epoxy-matrix composite were reported in Reference [19]. They are listed in Table 1. Again, there were no reported ultimate stresses of the glass fiber and the epoxy matrix. However, a unidirectional composite made from the same constituent materials has been tested [19], and the measured strengths in the longitudinal and transverse directions are summarized in Table 2. From these data, the ultimate strengths of the fiber and the matrix can be determined, as described in Section 4. Again, the epoxy used must be considered as an elasto-plastic material. As has been verified in the second example of Section 4, the particular values of the plastic parameters (yield stress and hardening modulus) do not significantly influence the ultimate strength of the composite. The recovered properties of the fiber and the epoxy are thus σ_u^f = 1933 MPa, σ_u^m = 31.5 MPa, E_T^m = 480 MPa, and σ_Y^m = 20 MPa.

The incremental procedure described in Section 4 of Reference [2] has been applied to evaluate the stresses in the glass fiber and the epoxy matrix. However, the compliance matrix $[S_{ij}]$ and the bridging matrix $[B_{ij}]$ used there must be replaced by $[\overline{S_{ij}}]$ and $[\overline{B_{ij}}]$, which are given in the following formulae, due to yarn orientation in the representative volume element:

$$[\overline{S_{ij}}] = \sum_{n=1}^{N-1} \frac{\left|x_{n+1}^{1st} - x_n^{1st}\right|}{(2L)} [\overline{S_{ij}}]_n^{1st} + \sum_{n=1}^{N-1} \frac{\left|x_{n+1}^{2nd} - x_n^{2nd}\right|}{(2L)} [\overline{S_{ij}}]_n^{2nd} \tag{6.1}$$

$$[\overline{B_{ij}}] = [B_{ij}] \left(\sum_{n=1}^{N-1} \frac{\left|x_{n+1}^{1st} - x_n^{1st}\right|}{(2L)} ([T_{ij}]_s^T)_n^{1st} + \sum_{n=1}^{N-1} \frac{\left|x_{n+1}^{2nd} - x_n^{2nd}\right|}{(2L)} ([T_{ij}]_s^T)_n^{2nd} \right) \tag{6.2}$$

Table 1. Tensile strength of plain knitted fiber fabric composites.

Load Direction	Strength of Composite (MPa)		Maximum Normal Stress (MPa)		Failure	
	Measured	Predicted*	Fiber	Matrix	Fiber	Matrix
Wale	62.83	65.40	408.53	31.51	No	Yes
Course	35.5	37.56	55.36	31.53	No	Yes

*Predicted with $d\sigma = 0.06$ MPa.

Note: The parameters used are $E_f = 74$ GPa, $E_m = 3.6$ GPa, $\nu_f = 0.23$, $\nu_m = 0.35$, $E_T^m = 480$ MPa, $\sigma_Y^m = 20$ MPa, $\sigma_u^f = 1933$ MPa, $\sigma_u^m = 31.5$ MPa, $d = 0.0445$ cm, $D_y = 177.8$, $K = 0.45$, $\rho_f = 254$ g/cm^3, $C = 2.5$ loop/cm, $W = 2$ loop/cm, $t = 0.06$ cm.

Table 2. Tensile strengths of unidirectional glass-fiber epoxy-matrix lamina.

Longitudinal Strength (MPa)		Transverse Strength (MPa)	
Measured	Predicted*	Measured	Predicted*
885	887.18	45	44.8

*Predicted with $d\sigma = 0.148$ MPa.

Note: $E_f = 74$ GPa, $E_m = 3.6$ GPa, $\nu_f = 0.23$, $\nu_m = 0.35$, $E_T^m = 480$ MPa, $\sigma_Y^m = 20$ MPa, $\sigma_u^f = 1933$ MPa, $\sigma_u^m = 31.5$ MPa, $V_f = 0.45$.

All the quantities in (6.1) and (6.2) may refer to Section 5.2 in Reference [1]. The predicted tensile strengths in the wale and course directions are given in Table 2. From these results, it is seen that the predictions are good.

Corresponding to the tensile strengths, the maximum normal stresses in the fiber and in the matrix are recorded and are also given in Table 2. These stresses clearly indicate that the failure of the composite, in both the wale and course directions, results from the failure of the epoxy matrix. This is consistent with the experimental observation [19].

6. CONCLUSION

The newly developed unified micromechanical model is powerful for estimating the tensile strength of a fiber-reinforced composite. The prediction is based on the well-established failure criteria for the isotropic constituent phases, and hence the failure mode of the composite is explicitly implied. The ultimate tensile stresses of the constituent materials must be determined accurately. However, these stresses can be obtained by means of the longitudinal and transverse strengths of the unidirectional composite lamina. The constituent materials can be elastic or elasto-plastic. If with elastic assumptions, the tensile strengths of both the fiber and the matrix cannot be determined by the overall longitudinal and transverse strengths of the composite, an elasto-plastic material assumption has to be made for one or both of the constituent phases. Although the involved yield stress and hardening modulus have to be determined by trial-and-error method, they do not affect the overall strength value of the composite significantly.

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