

Recursion

- Using pure functions we don't have access to variables. So how do we loop? Recursion!

- In F# we can define recursive functions using let rec ... =

- Recursive functions have two cases:

- The recursive case, which "loops" further by calling itself

- The base case which terminates the recursion, and starts unwinding

- But isn't recursion terribly inefficient?

Yes and no! Regular recursion is quite expensive in terms of memory, stack and function calls.

But! We can often rearrange recursion into a form where tail call optimization can be performed. \rightarrow recursion becomes equivalent to imperative loops.

(How can we recurse in pure λ -calculus?
The λ -combinator!)

let rec fact n =
 if $n = 0$ then
 1
 else
 $n * \text{fact } (n - 1)$

With TCO:

let fact n =
 let facacc n =
 if $n = 0$ then
 1
 else
 $\lambda. (n * \text{acc}) (\text{facacc } (n - 1))$
 $\lambda. \text{on}$

How does TCO work? (using mutation)

$\lambda. \text{acc } n =$
 top:
 if $n = 1$
 acc
 else
 $\lambda. \text{acc} \leftarrow n * \text{acc}$
 $\lambda. n \leftarrow n - 1$
 $\lambda. \text{goto top}$

\Rightarrow acc $\leftarrow 1$
 while $n > 0$:
 acc $\leftarrow n * \text{acc}$
 $n \leftarrow n - 1$

Recursive data structures

- Data types can also be recursive :

```
type List<'a> =  
| Cons of 'a * List<'a>  
| Nil
```

This is the canonical representation of a linked list using product and sum types.

Note how elegant it is! No pointers or references to the next element. Just clean recursion!

"To loop is human, to recurse is divine"

- We can construct a list like this:

```
let l = Cons (1, Cons (2, Cons (3, Nil)))
```

and we destructure it using recursion and pattern matching

```
let rec sum x = function  
| Cons (n, l) → sum (x+n), l  
| Nil → x
```

Mutual recursion

- In F# we must always define things before we use them. But what if we can't?

let $f\ x = \text{if } x > 0 \text{ then } g(x-1) \text{ else } x$

let $g\ x = \text{if } x > 0 \text{ then } f(x-1) \text{ else } x$

\Rightarrow let rec $f\ x = \dots$
and $g\ x = \dots$

- For types:

type Folder = {

path : string

files : File list

}

and File = {name: string; folder: Folder}

- Sometimes when we have many mutually recursive cases it's easier to define a recursive module:

module rec Stuff = ...

This makes everything in the module mutually recursive. Use with care!!

Collection types

List : a linked list which can be appended to with (:) (cons)

Array : A fixed size array. Much more performant than lists, and much less flexible APIs.

Map : A immutable dictionary (key, value)

Set : A collection of unique elements stored in a binary tree.
Fast lookups.

Seq : Seq<'T> is an alias for
IEnumerable<'T>

All of above are instances of seq!

- All collection types are well documented,
also in IntelliSense / FSAC

- Notable functions :

seq.empty
seq.singleton

seq.of List
seq.of Array

Looping part 2

- The most common cause for looping is traversing and transforming data structures (collections)
- For this purpose we have 3 work horses for collection types: map, filter, fold
- map $f \colon [\dots] \rightarrow [\dots]$
map transforms, while retaining shape
two? / false?
- filter $p \colon [\dots] \rightarrow [\dots]$
filter removes elements which do not satisfy the predicate p
- fold $(\text{func} \text{ acc } x \rightarrow \dots) m [\dots]$
 - acc \downarrow accumulator
 - $m \downarrow$ initial value and type
 - $x \downarrow$ next element

fold is very general and can map, filter, reduce, expand, contract data structures!

 - The "normal" fold is a left fold.
 - There is also a right fold:

fold Back $(\text{func } x \text{ acc} \rightarrow \dots) [\dots] m$

 - Right folds are primitive recursive and can encode any recursion!

Mutable state (be ware!)

- In F# mutable variables are declared and modified explicitly:

```
let mutable v = 0
```

```
v ← 1
```

- Always think twice before using mutables! We do need them, sometimes for performance, but most often we do not.

- F# also have reference cells:

```
let r = ref 42
```

```
r := 42 + 1
```

```
printfn "%d" !r
```

- Reference cells are defined like this

```
type Ref<'a> = { mutable v : 'a }
```

```
let ref x = { v = x }
```

- What is the difference:

```
let a = ref 5
```

```
let b = a
```

```
b := 10 // modifies both
```

```
let mutable a = 5
```

```
let mutable b = a
```

```
b ← 10 // only b
```

Reference cells are actual values!