

SIR-Based Model of COVID-19 Transmission at Duke

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1. Abstract

In an effort to control the spread of the Severe Acute Respiratory Syndrome Coronavirus (SARS-CoV-2), the CDC and other national health protection agencies have developed mathematical models to inform decisions about pandemic planning, resource allocation, and implementation of social distancing measures and other interventions (CDC, 2020). These modeling techniques have proved crucial when determining reopening plans for college campuses, as they are particularly prone to the rapid spread of COVID-19. This paper seeks to model the spread of COVID-19 on the Duke University campus using an SIR-based model in hopes of determining the safest and most effective reopening plan for the 2021 spring semester. Previous SIR-based models from other universities were modified to reflect Duke's unique on-campus and off-campus populations. The effects of varying exogenous shocks, inter-student interaction, screening rate, and vaccination rate were analyzed using a parameter sweep to determine the most important factors in limiting spread of the virus. It was concluded that inter-student interaction has the greatest impact on the size of the infected population, and it is vital to keep this number at the lowest possible level; and that screening rate has a significant impact on the size of the infected population, and a minimum of 2 tests/week would be required to keep the infections at a manageable level. Increasing exogenous shocks does not produce an exponential growth in peak infection levels, but does lead to significant differences in infected/recovered on- and off-campus populations. Finally, vaccination must occur rapidly and be distributed to 83% of students in order to nearly eliminate new cases.

2. Background

The novel SARS-CoV-2 discovered in 2019 created a pandemic that has changed the world. With over 1 million deaths caused by the virus, countries have gone into lockdown, education systems have shut down, and wearing masks has become the norm.

SARS-CoV-2 is a coronavirus which causes COVID-19, a disease that can trigger a respiratory tract infection and affect the sinuses, nose, throat, windpipe, and lungs. The virus mainly spreads by respiratory droplets and aerosols, especially in crowded, inadequately ventilated spaces (Centers, 2020). Research shows that the incubation period for the disease is around 14 days (Lauer et al., 2020).

Many new measures are being put in place in order to reduce the transmission of SARS-CoV-2. Since the coronavirus is spread through respiratory droplets, mask usage can reduce the R_t value, or the reproduction number of the virus. Furthermore, to reduce contact with droplets from infected individuals, people are being advised to social distance by maintaining a minimum of 6 feet apart from each other in public spaces and to not frequent indoor places, especially in settings where someone's mask is likely to be taken off. Various testing methods, such as RT-PCR tests, antigen tests, and antibody tests, have been developed for the virus, so infected people can be immediately put into isolation if tested positive (Commissioner, 2020). At Duke University, students are tested weekly and are required to report symptoms daily on the Sym Mon app. Contact tracing is being used to identify students who have been in contact with infected people, and they are quarantined for 14 days (Duke United, 2020).

In order to better understand the epidemiology of COVID-19 at Duke, an SIR-based model that computes the theoretical number of infected people in a population over time was developed. SIR models are compartment models which consist of susceptible, infected, and recovered individuals. Yale University developed an SIR-based model to predict which SARS-CoV-2 screening and isolation programs would minimize danger for US residential universities. The study evaluated campus screening using varied testing frequency (daily-weekly), R_t values (1.5, 2.5, 3.5), and exogenous shock factors that accounted for importation of additional infections. The results of the study showed that across all scenarios, testing frequency had the most impact on outcomes.

Studying such models are crucial for university administrators to determine the safest and most efficient way for students and faculty to return to campus. This involves considering factors such as testing requirements, on- and off-campus student interaction, and organizing facilities required for isolation and quarantine procedures.

Given that Duke University is currently grappling with many of the same dilemmas, this paper aims to use an SIR-based model and kinetics-based equations to investigate the network dynamics between student populations in order to effectively emulate the current COVID-19 pandemic at Duke.

3. Development of Model

3.1 Compartment Model

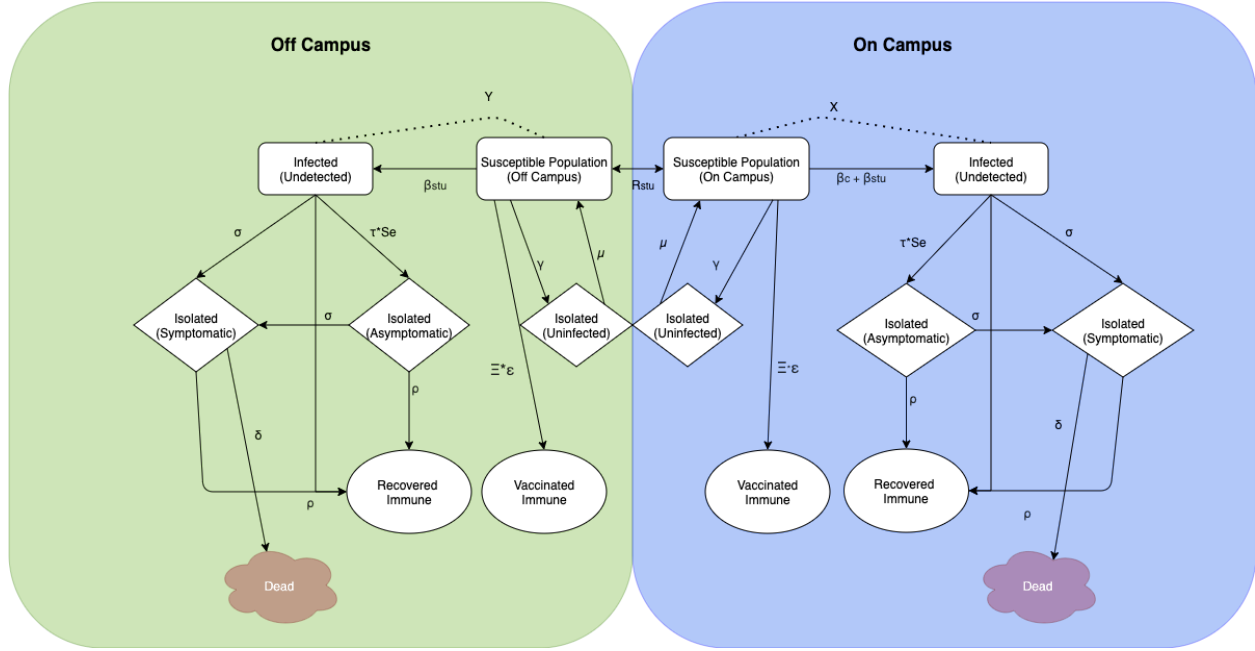


Figure 0. Compartment Model

The compartment model developed in this paper is based on the SIR model by Paltiel et al. (2020) at Yale. Taking inspiration from the model designed by Lopman et al. (2020) at Emory, student populations were split into “on-campus” and “off-campus”, as data from the *Duke Today* demonstrated significantly different infection rates between these populations. Symmetric compartment models were made for both populations with a rate of infection between on- and off-campus susceptible populations. The rate at which general students in the susceptible population become infected by the infectious population was taken to be β_{stu} , while the rate at which on-campus susceptible students become infected by other infected on-campus students constitutes an additional β_c rate. This β_c rate accounts for the higher rate of inter-student interaction for on-campus students. Although Duke’s report numbers show that off-campus students had a higher rate of infection, we assumed that this number is the result of higher exogenous shocks, not a greater level of inter-student interaction. This exogenous shock event factor accounts for the possibility of student interactions with the surrounding community. As such, the off-campus exogenous shock factor was increased.

For both on- and off-campus, the susceptible population accounts for students who are uninfected and unisolated. Students who are uninfected but isolated due to contact tracing protocol are moved to the isolated, uninfected population at a rate of γ ; this was done to best

emulate the contact tracing and isolation process at Duke. Students move back from the isolated, uninfected population to the susceptible population at a rate of μ . Students from the susceptible population are infected at a rate of β , depending on on- or off-campus, and moved to the infected, undetected pool. Exogenous shock events will also move a small proportion of students from the susceptible population to the infected, undetected population weekly.

The infected population can move into two groups: isolated, symptomatic and isolated, asymptomatic. The assumption was made that all symptomatic students who report their symptoms via the Sym Mon application are immediately isolated, prior to testing, at a rate of σ , or the rate of symptom onset for infected individuals (Paltiel et al., 2020). It was assumed that all infected, asymptomatic students discover their infection through testing, and are subsequently put into isolation. These students are deemed infected and then isolated at a rate of $\tau \cdot Se$, with τ being the rate at which individuals in the testing pool are screened for infection (Paltiel et al., 2020), and Se being the sensitivity of the screening test. Those isolated, asymptomatic students who eventually develop symptoms enter the isolated, symptomatic pool at the symptom onset rate of σ . After polling on-campus and off-campus students, it was concluded that screening occurs at the same rate for all individuals, and therefore, the same rates of isolation were used for the on-campus and off-campus populations. Infected, undetected students can move from the infected group to the recovered population at a recovery rate of ρ (Paltiel et al., 2020).

Students in the isolated, infected population move to the recovered population at a recovery rate of ρ . This rate is the same for on-campus and off-campus populations. Isolated, symptomatic individuals die at a death rate of δ and are moved to the dead population. It was also assumed that all recovered students become immune, and hence cannot be reinfected, because there have only been three reported repeat COVID-19 cases worldwide (Mandavilli, 2020).

A hypothetical vaccine strategy was incorporated into our model in the case that a vaccine becomes readily available. Students will be vaccinated, starting from day 0, from the susceptible population and enter the vaccinated population at a vaccination rate times effectiveness of vaccine, or $\Xi^* \epsilon$. We assume that 90% of those who receive the vaccine will become immune, based on recent data released from Pfizer predicting 90% efficacy of the vaccine ("Pfizer", 2020). It was also assumed that there would be an equal amount of vaccines available to the on- and off-campus student populations.

3.2 Kinetic equations

3.2.1 Susceptible

Susceptible (t+1) = Susceptible(t) -New Infected-New IUs +Returning IUs -Exogenous Shocks -New Vaccinated

$$S_i(t+1) = S_i(t) - \lambda_i \cdot S_i(t) - S_i(t-1) \cdot \tau \cdot (1 - Sp) + \mu \cdot IU_i(t) - Z_i \cdot S_i(t) - \Xi \cdot \varepsilon$$

$$\text{where } \lambda_{on-campus} = \beta_{stu} \cdot \frac{U_{total}(t)}{S_{total}(t) + U_{total}(t)} - \beta_c \cdot \frac{U_{on-campus}(t)}{S_{on-campus}(t) + U_{on-campus}(t)},$$

$$\lambda_{off-campus} = \beta_{stu} \cdot \frac{U_{total}(t)}{S_{total}(t) + U_{total}(t)},$$

$$Z_{on-campus} = X, \text{ and } Z_{off-campus} = Y$$

The susceptible compartment represents the uninfected individuals who are capable of being infected at rate λ . These uninfected individuals are isolated as a result of false positive tests at rate $\tau \cdot (1 - Sp)$. Those isolated in this manner return to campus at rate μ . X proportion of students are infected each week due to exogenous shocks. In the trials with vaccine availability, $\Xi \cdot \varepsilon$ (the vaccination rate multiplied by its efficacy) students are removed from the susceptible group weekly.

3.2.2 Infected, Undetected

Undetected(t+1)= Undetected(t) -Symptomatic -Recoveries +New Infections -New IA +Exogenous Shocks

$$U_i(t+1) = U_i(t) - \sigma \cdot U_i(t) - \rho \cdot U_i(t) + \lambda_i \cdot U_i(t) - U_i(t-1) \cdot \tau \cdot Se + Z_i \cdot S_i(t)$$

$$\text{where } \lambda_{on-campus} = \beta_{stu} \cdot \frac{U_{total}(t)}{S_{total}(t) + U_{total}(t)} - \beta_c \cdot \frac{U_{on-campus}(t)}{S_{on-campus}(t) + U_{on-campus}(t)},$$

$$\lambda_{off-campus} = \beta_{stu} \cdot \frac{U_{total}(t)}{S_{total}(t) + U_{total}(t)},$$

$$Z_{on-campus} = X, \text{ and } Z_{off-campus} = Y$$

The infected, undetected population represents individuals who have contracted the virus, but have not been identified via testing. This population increases with new infections at rate λ_i . Students develop symptoms at rate σ , and are promptly isolated. Those who remain asymptomatic are isolated at rate $\tau \cdot Se$ through true positive test results. Individuals in this compartment recover at rate ρ .

3.2.3 Isolated, Uninfected

Isolated_{uninfected}(t+1)= Isolated_{uninfected}(t) - Returning IUs + New IUs

$$IU_i(t+1) = IU_i(t) - \mu \cdot IU_i(t) + S_i(t-1) \cdot \tau \cdot (1 - Sp)$$

The isolated, uninfected population represents the individuals who have not contracted the virus, but are placed in isolation at rate $\tau \cdot (1 - Sp)$ due to false positive tests. These students leave isolation and return to normal activity at rate μ .

3.2.4 Isolated, Asymptomatic

Isolated_{asymptomatic}(t+1)= Isolated_{asymptomatic}(t) - Symptomatic - Recoveries + New IAs

$$IA_i(t+1) = IA_i(t) - \sigma \cdot IA_i(t) - \rho \cdot IA_i(t) + U_i(t-1) \cdot \tau \cdot Se$$

The isolated, asymptomatic population represents students who have been isolated at rate $\tau \cdot Se$ as a result of true positive tests. Individuals who develop symptoms at rate σ remain isolated, but are placed in the isolated, symptomatic compartment. These individuals recover at rate ρ .

3.2.5 Isolated, Symptomatic

Isolated_{symptomatic}(t+1)= Isolated_{symptomatic}(t) - Recoveries - Deaths + New Symptomatic

$$IS_i(t+1) = IS_i(t) - \rho \cdot IS_i(t) - \delta \cdot IS_i(t) + \sigma[IA_i(t) + U_i(t)]$$

The isolated, symptomatic population represents students who have been isolated at rate σ as a result of symptom onset. These individuals recover at rate ρ and die at rate δ .

3.2.6 Recovered

Recovered(t+1)= Recovered(t) + New Recoveries

$$R_i(t+1) = R_i(t) + \rho[IA_i(t) + U_i(t) + IS_i(t)]$$

The recovered population represents students from the isolated, asymptomatic; undetected; and isolated, symptomatic populations who have been infected and subsequently recover at rate ρ .

3.2.7 Vaccinated

Vaccinated(t+1)= Vaccinated(t) +New Vaccinations

$$V_i(t+1) = V_i(t) + \Xi \cdot \varepsilon$$

The vaccinated population represents students who gain immunity with $\Xi \cdot \varepsilon$ newly vaccinated individuals each week.

3.2.8 Deaths

Deaths(t+1)= Deaths(t) + New Deaths

$$D_i(t+1) = D_i(t) + \delta \cdot IS_i(t)$$

The dead population increases at death rate δ .

3.2.9 Total Population

$$P = \sum_i S_i + U_i + IU_i + IA_i + IS_i + R_i + D_i + V_i$$

The sum of all the populations described above, both on-campus and off-campus, make up the total population of interest: Duke students.

3.3 Universal Constants

In order to better compare the variables over which we will sweep, some factors must be kept constant. Initial population was set to be 4000 students spread evenly across on and off campus, 11 of which are infected but undetected (“Duke COVID Testing Tracker”, 2020) (6 on campus, 5 off). Baseline calculations for death rate, recovery rate, symptom onset rate, as well as sensitivity and specificity of COVID-19 tests are cited in Table 1 in the Appendix.

3.4 Parameter Sweeping

The following four parameters were swept across base, best, and worse case infection scenarios to investigate the significance of each parameter change on the infected student population: number of exogenous shocks (X and Y for on-campus and off-campus populations respectively), interaction factor between students (R_{stu}), screening rate (τ), and vaccination rate (Ξ). The results of this parameter sweeping may reveal vital insights for administrators to take into consideration when policy making. Table 1 in the Appendix summarizes the parameters used in the model, along with their calculated values, and the method behind their calculation. The figures displayed track a sum of both on- and off-campus infections, detected and undetected separately.

4. Model Results

4.1 Inter-student interaction

Increased inter-student interaction leads to a greater peak infection number and increases the time necessary for the infection to peak; however it does not lead to significant differences in on- and off-campus infected and recovered populations. In order to see this effect, three inter-student R values (R_{stu}) were used, 1.0, 1.5, and 3.0, with an additional constant R_c of .5 between on-campus students. When $R_{stu} = 3.0$, it takes the longest amount of time to reach the peak infection at approximately 50 days, and the shortest time to peak infection occurs when $R_{stu} = 1.0$ at around 8 days. As R_{stu} increases, the maximum number of concurrent infected isolated individuals increases. As the likelihood for infection increases, there is a significant increase in the overall infected students, measured by accounting for those who recovered both on and off campus. The $R_{stu} = 3.0$ case resulted in a total recovered student count 22x greater than the baseline $R_{stu} = 1.5$ case, suggesting that R_{stu} has a significant impact on infection levels. In contrast, the best case, $R_{stu} = 1$, resulted in a 33% reduction in recovered cases from the baseline. As time increases, the infected population oscillates about a steady figure whose value increases as R_{stu} increases. This data shows that having low R_{stu} values will keep the rate of infection low, allowing for a more manageable amount of cases. This low R_{stu} value can likely be achieved through mask wearing, social distancing, and isolation (Chu et al., 2020). If R_{stu} increases due to a lack of safety measures, infections are predicted to increase rapidly. This will lead to increased cost for beds, treatment, and contact tracing. Therefore, it is essential to keep R_{stu} low, so that the infections on campus can be managed and the semester can continue to take place in person. In the event of a total failure to enforce those practices which can minimize inter-student viral transmission, it is possible that the entire student population could become infected (Figure 3).

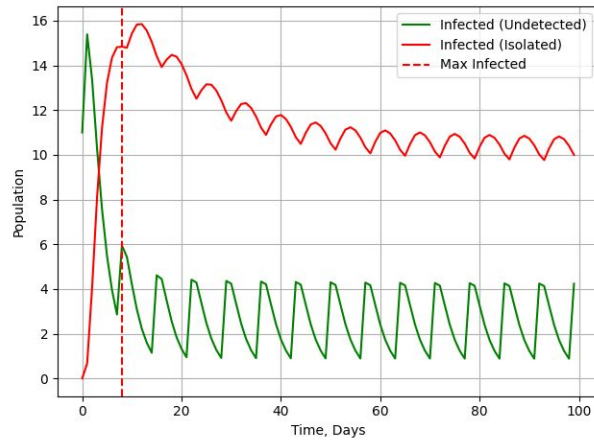


Figure 1. $R_{stu} = 1.0$

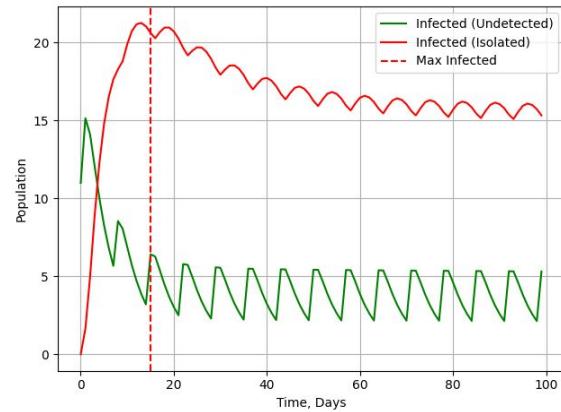


Figure 2. $R_{stu} = 1.5$

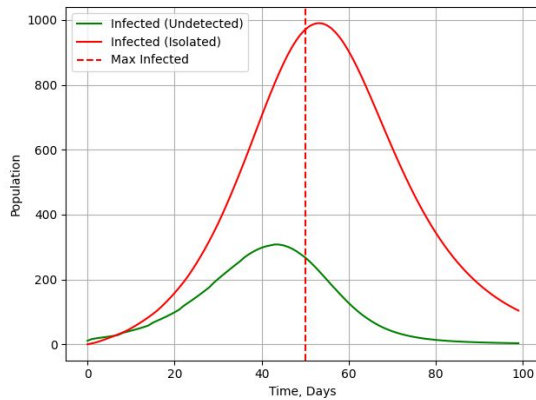


Figure 3. $R_{stu} = 3.0$

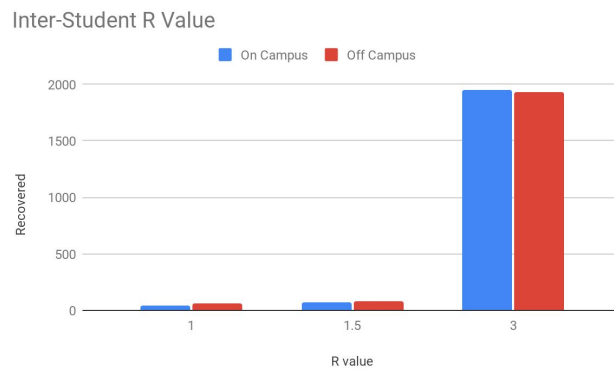


Figure 4. Recovered populations for on/off campus

4.2 Testing Frequency

Graphs for screening occurring once, twice and thrice a week were recorded, and the data shows that increasing screening rate was found to have an extremely significant effect on decreasing peak infection levels. When screening was conducted once a week, the number of infections continued to increase even at the 100 day mark. However, when three tests were conducted weekly, the number of infections peaked at 8 days. The number of recovered individuals was 574 total with only one screening per week, compared to 113 individuals when three tests were conducted per week. Testing twice or three times a week results in greater infections off campus, while testing once a week not only results in an outbreak, but one that affects on-campus students disproportionately (Figure 8). This is caused by the additional R_c between on-campus students, so as student cases rise, on-campus students are at greater risk. As a result, it can be argued that testing frequency is one of the most important factors in preventing

infection outbreak, although increasing screening rate would involve increases in costs, both in terms of testing and bedding for false positives. Our model shows that to maintain infection levels at manageable numbers, a minimum screening rate of 2 tests/week is required.

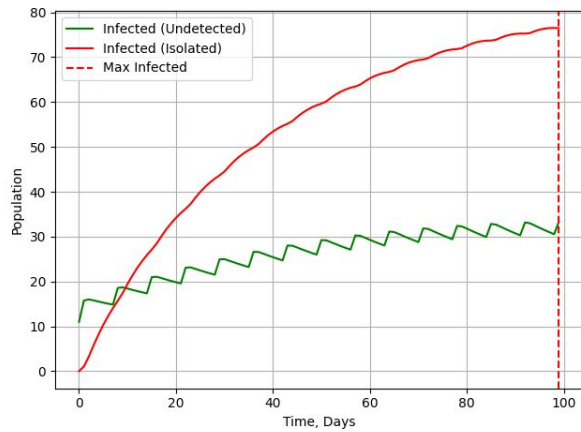


Figure 5. 1 test per week

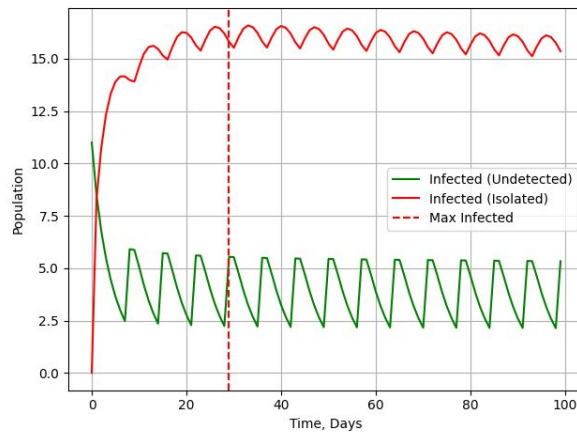


Figure 6. 2 tests per week

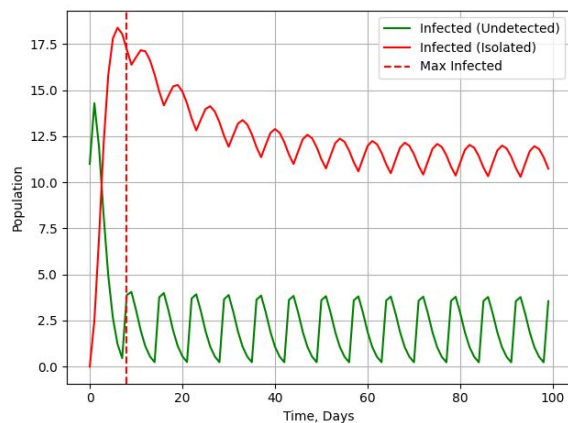


Figure 7. 3 test per week

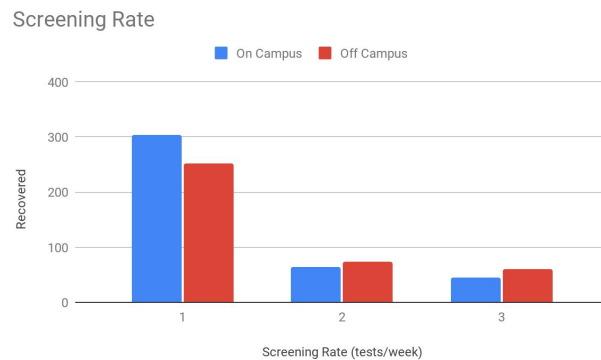


Figure 8. Recovered populations for on/off campus

4.3 Exogenous Shocks

Increasing exogenous shocks creates an increase in infection oscillations but does not lead to an exponential growth in peak infection numbers. To determine this, three different sets of exogenous shock values were analyzed: no exogenous shocks, 1 per week on campus and 3 per week off campus, and 2 per week on campus and 6 per week off campus. As the number of exogenous shocks increases, the time taken for infection levels to peak is increased. It only takes 6 days for maximum infections to occur with zero weekly shocks, while it takes 29 days with the maximum number of weekly shocks. Although the maximum number of infected isolated individuals increases with exogenous shocks, there is not an exponential increase in the number

of infected individuals. Therefore, it can be concluded that the number of exogenous shocks, while likely to cause an increase in the number of average cases, would not prompt an outbreak. There are also no significant changes in the number of infected, undetected individuals across the three cases. However, there is a large impact on the number of recovered students, with 233 more individuals total in the maximum shock case than in the no shock case. There are more recovered individuals from the off-campus population in all cases except when there are no exogenous shocks. The effect of exogenous shocks on off-campus versus on-campus student populations was compared. In the best case scenario (exogenous shocks=0), there were approximately 17 and 13 recovered individuals for on-campus and off-campus populations respectively. In the worst case scenario (~ 2 /week on-campus and ~ 6 /week off-campus), there were approximately 122 and 144 recovered individuals in the on- and off-campus populations respectively (see Figures A1-A4). These results demonstrate that community infections have a greater impact on the off-campus population, which can introduce more cases to the student body overall. However, overall, community infections are unlikely to cause significant outbreak among students.

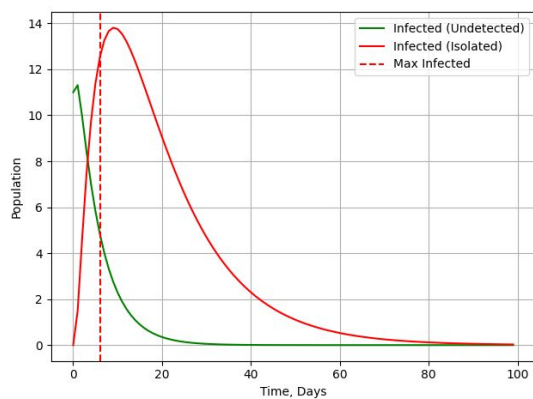


Figure 9. No exogenous shocks

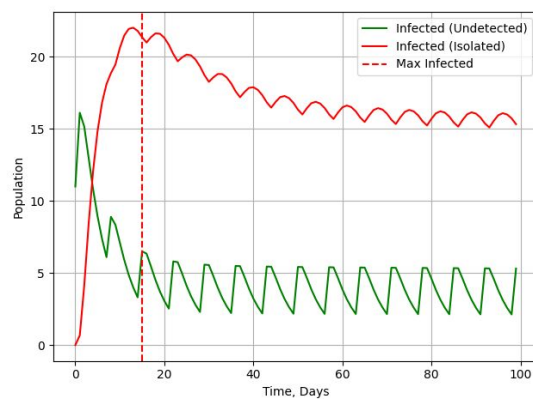


Figure 10. 1/week on campus + 3/week off campus

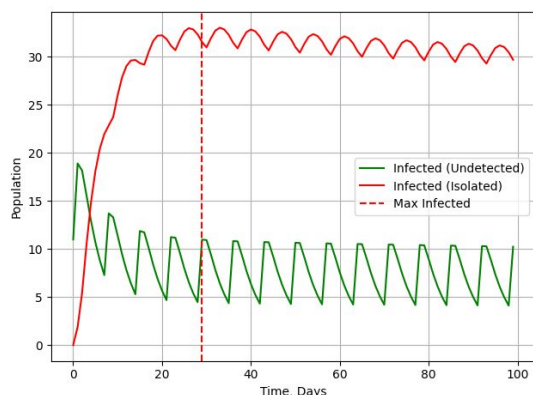


Figure 11. 2/week on campus + 6/week off campus

Varied Exogenous Shocks

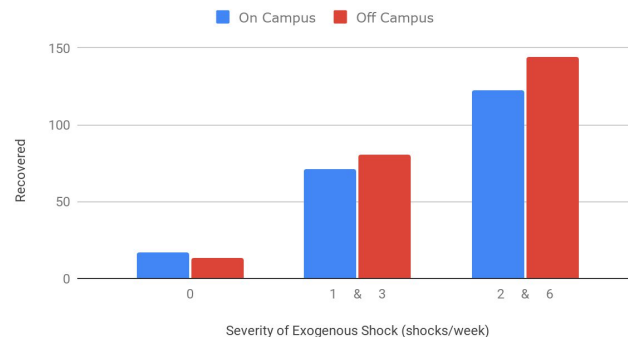


Figure 12. Recovered populations for on/off campus

4.4 Vaccine Distribution

Presence of a vaccine is hailed by many as an end to the COVID-19 pandemic, but even as vaccines seem to be on the horizon, their availability and distribution will be certain to pose challenges. Analysis of the results of varying availability of vaccines, 100, 200, and 300 doses per week, has provided some insight into what to expect once a vaccine is produced. Assuming that vaccines are distributed among students randomly, and at a continuous and constant rate from the beginning of the semester, they may prove to decrease student infections. Recognizing the high demand for vaccines, as well as the relatively low risk of college students, it is reasonable to expect a fairly low availability, so it would be unlikely for Duke's undergraduate population to reach herd immunity until a significant time after the beginning of the semester.

In each case below, the amount of infected students slopes downwards as more are vaccinated, but only in the best case scenario, 300 vaccinations per week, does the new inter-student infection count drop below 1 per week within the period analyzed. This is achieved after 11 weeks of continuous vaccinating, when approximately 83% of the student population has been immunized, as well as about 2.5% of the population having developed immunity from contracting the virus. As is clear in Figure 16, widespread immunization levels can decrease the overall number of cases in a semester, but immunizing over the course of the semester has a limited effect on the overall case count. The best case scenario still results in 106 students becoming infected with COVID-19, as compared to approximately 151 in the base, no-vaccine case. Our model may be limited, however, in its prediction of vaccine efficacy. Literature predicts that with an R_{stu+c} of 2, we should expect herd immunity to begin at approximately 50% vaccination (Anderson et al., 2020; Miao et al., 2010). Because our model does not account for

interaction between the vaccinated immune population and the susceptible population, we are unable to draw sound conclusions about herd immunity.

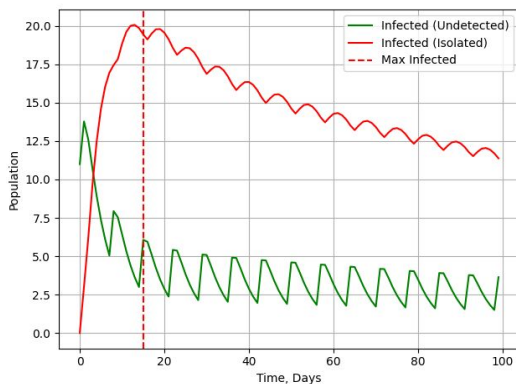


Figure 13. 100 vaccinations/week

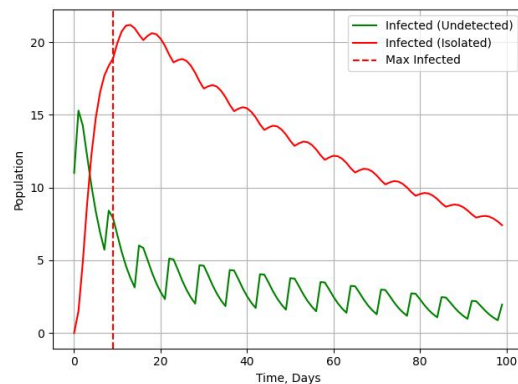


Figure 14. 200 vaccinations/week

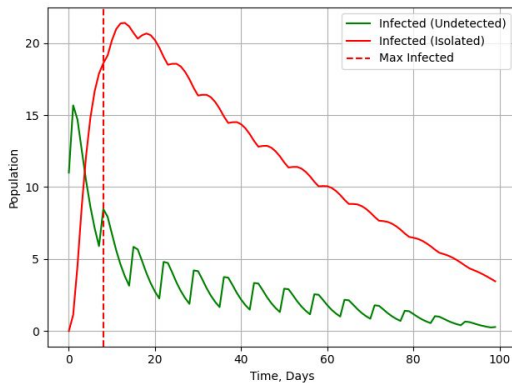


Figure 15. 300 vaccinations/week

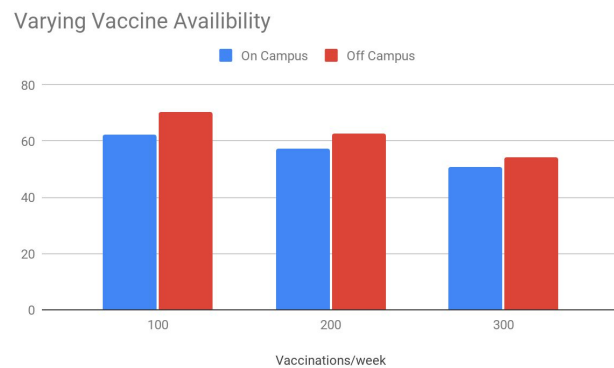


Figure 16. Recovered populations for on/off campus

5. Reflection

In evaluating the model and its results, several strengths and weaknesses were identified. Among its strengths, the most important is the separation of the student population into on- and off-campus groups, while also modeling the interaction between the two. This is a more accurate representation of the actual student population at Duke and at several other colleges where student residential options are more varied than regular dorm-style living. Accounting for this separation has allowed us to tailor our containment plan to each population's unique circumstances and susceptibility to infection spread.

Another strength of the model is that it analyzes the effects of several different parameters. These include the screening rate, the interaction between students (which would vary depending on campus protocols enforced by administration), the number of exogenous shocks

(which would vary based on local community infection levels), and finally vaccine distribution, which has not been analyzed in many models so far. Evaluating the effect of changing these parameters helped achieve the goals specified by the initial motivation behind creating the model: to help the administration consider how to weigh the impact of these factors when implementing an infection containment plan. Sweeping across different values has allowed for the successful identification of increasing rates of screening and limiting student interaction levels as most crucial in curbing infection outbreak and spread, whereas reducing exogenous shocks does not have as great of an impact relative to the effect of varying other parameters. These results are supported by findings in Paltiel's model, which demonstrated that keeping inter-student R value below 2.5 and implementing a rapid, even poorly sensitive ($> 70\%$), test conducted at least every two days, would produce a modest number of containable infections (Paltiel et al., 2020). In addition, the effect of exogenous shocks would be lessened if this testing strategy was implemented (Paltiel et al., 2020).

However, there are demerits to the model as well. First, the model does not consider the faculty population, which would potentially be an important source of infection since the faculty live primarily off campus and are more likely to be affected by exogenous shocks. Faculty also would not be subjected to the same rates of testing as students, which would mean that positive cases among faculty could go undetected for a longer amount of time. Additionally, the model did not take into account the financial and material resources required to sufficiently implement the required screening rates and accommodate the estimated isolated/quarantined population. Furthermore, our model does not account for the efficacy of pandemic control measures (apart from regulating in-person/student interaction) such as mask wearing or regular sanitization on limiting infection rate. While these measures would likely be more difficult to incorporate in the model due to their nuanced nature, it is possible that they could have a potent effect in reducing infection numbers, especially considering the strong advocacy for mask wearing by public health officials. Lastly, the model does not account for interaction between the vaccinated immune population and the susceptible population. Reaching herd immunity is dependent upon pre-existing immunity of a high proportion of individuals who interact with the susceptible population; thus, we are unable to make predictions regarding herd immunity from our model.

6. Conclusion

Increased inter-student interaction (R_{stu}) increases the time taken to reach peak infection levels and produces exponentially greater maximum infection levels. The R_{stu} value has the greatest impact on the size of the infected population, and it is vital to keep this factor at the lowest possible level. This can be done through use of masks, social distancing, and self-isolation. Screening rate also has an extremely significant effect on the size of the infected population. Results indicate that a minimum of 2 tests per week would be required to keep the infections at a manageable level. Exogenous shocks, while increasing the average number of infections and the severity of infection oscillations, did not produce an exponential growth in peak infection levels. They did, however, lead to significant differences in on- and off-campus infected and recovered populations, demonstrating that community infections have a greater impact on the off-campus population but are unlikely to cause significant outbreak among students. Increased screening for off-campus students is recommended to account for this discrepancy, which would eliminate the excess costs involved with increased screening for *all* individuals. Finally, vaccination would need to take place rapidly and be distributed to approximately 83% of students in order to nearly eliminate new cases. The current model could be further improved by adding the faculty population, determining the efficacy of pandemic control measures on limiting infection spread, and allowing for interaction between vaccinated immune individuals and the susceptible population. The model could also be extended to estimate average costs of testing and isolation/quarantine facilities required to help the administration create infection containment plans.

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Appendix

Model Equations

Table 1. Model Parameters

	Definition	Est. Value	Calc. Method
β_c	Rate at which susceptible on campus population is contacted and infected by infectious population	0.204	$\beta_c = (\rho + \sigma) \cdot R_c$ (Paltiel et al.)
β_{stu}	Basal rate at which students are contacted and infected by other students	Best: 0.204 Base: 0.306 Worse: 0.612	$\beta_{stu} = (\rho + \sigma) \cdot R_{stu}$ (Paltiel et al.)
τ	Screening rate	Worst Case: 0.143 tests/day (1 test/week) Base Case: 0.2857 tests/day (2 tests/week) Best Case: 0.428 tests/day (3 tests/week)	Total no. of tests (not including entry tests) administered/no of weeks
σ	Rate of symptomatic onset	0.06122	(Paltiel et al.) $\frac{\sigma}{\sigma + \rho}$
δ	Death rate of symptomatic individuals	0.0092	(Benneyan et al.)
ρ	Recovery rate	1/14 days	1/length of infection in days (Paltiel et al.)

μ	Rate of false positives returning to susceptible compartment (uninfected compartment)	1/14	1/(Time to false positive return)
γ	Rate of false positives going from susceptible to isolated (uninfected) compartment	0.2857 tests/day	$\tau \cdot Se$
Se	Sensitivity	80%	(from Paltiel et al. base)
Sp	Specificity	98%	(from Paltiel et al. base)
X	Proportion of on-campus population infected each week due to exogenous shocks	Best case: 0 (≈ 0 cases/week) Base case: 0.0035 (≈ 1 cases/week) Worst case: 0.007 (≈ 2 cases/week)	$X = \frac{\text{cases per week}}{S_{\text{on-campus}}(0)}$ *calculated to have initial 0,1,2 cases per week Assumption
Y	Proportion of off-campus population infected each week due to exogenous shocks	Best case: 0 (≈ 0 cases/week) Base case: 0.0105 (≈ 3 cases/week) Worst case: 0.021 (≈ 6 cases/week)	$Y = \frac{\text{cases per week}}{S_{\text{off-campus}}(0)}$ *calculated to have initial 0,3,6 cases per week Assumption
R_c	Average number of on-campus susceptible students who become infected by on-campus infectious students	.5	Assumption

R_{stu}	Average number of off-campus susceptible students who become infected by on-campus infectious students and vice versa	Best case: 1 Base case: 1.5 Worst case: 3	Best and worst case are drawn from Paltiel et al. - R_c , and the base case was found to approximate true results of the past semester
Ξ	Number of vaccinations administered per week to each population (on-campus and off-campus)	Best case: 100/week Base case: 200/week Worst case: 300/week	Estimation
ε	Effectiveness of vaccine	0.9	(Pfizer)

Additional Figures

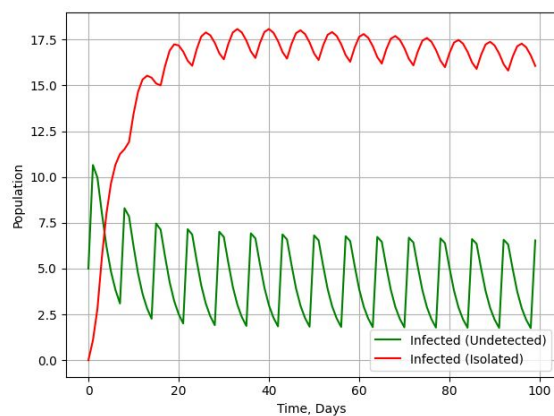


Fig A1. Off Campus Worst Case Exogenous Shock 2/week

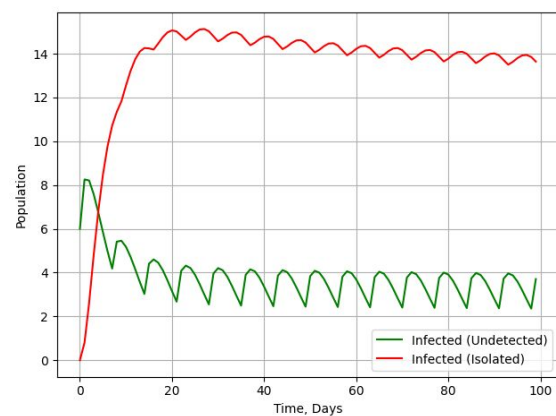


Fig A2. On Campus Worst Case Exogenous Shocks 6/week

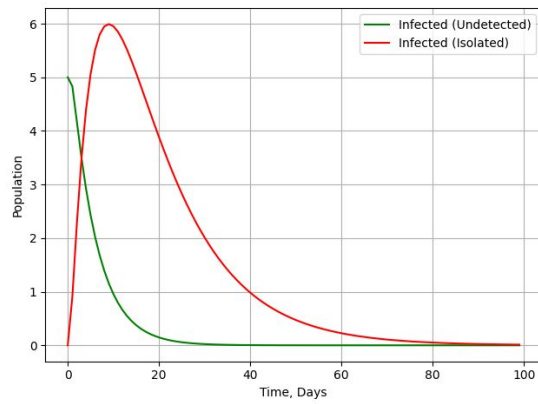


Fig A3. Off Campus Best Case Exogenous Shock 0/week

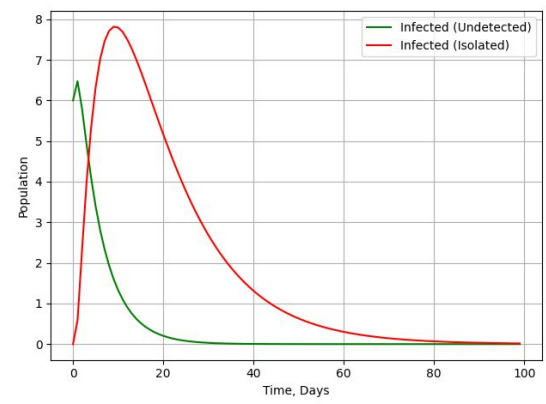


Fig A4. On Campus Best Case Exogenous Shock 0/week

Table 2. Compiled Raw Data for Parameter Sweeps

Total Recovered	On Campus	Off Campus	R_{stu}	τ (per week)	X	Y	Weekly Vaccinations	Max. Infected Time (days)	Parameter
102.0999655	43.9806569 8	58.1193085 6	1	2	0.0005	0.0015	0	8	R_{stu}
150.2507059	70.4962807 7	79.7544251 2	1.5	2	0.0005	0.0015	0	15	R_{stu}
3880.477049	1947.86541 9	1932.61163 1	3	2	0.0005	0.0015	0	50	R_{stu}
556.0831099	304.087340 6	251.995769 3	1.5	1	0.0005	0.0015	0	99	τ
138.0297126	63.3824010 4	74.6473115 8	1.5	2	0.0005	0.0015	0	29	τ
104.4448914	44.5799529 2	59.8649384 8	1.5	3	0.0005	0.0015	0	8	τ
30.34445865	17.2292142 2	13.1152444 4	1.5	2	0	0	0	6	Exogenous
151.8790115	71.4242427 9	80.4547686 9	1.5	2	0.0005	0.0015	0	15	Exogenous
266.1330764	122.125095 4	144.007981	1.5	2	0.001	0.003	0	29	Exogenous
132.5627304	62.4083029 2	70.1544274 4	1.5	2	0.0005	0.0015	100	15	Vaccine
119.8658217	57.1800049 2	62.6858168 2	1.5	2	0.0005	0.0015	200	9	Vaccine

105.237671	50.8475386 5	54.3901323 5	1.5	2	0.0005	0.0015	300	8	Vaccine
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```

1 import numpy as np
2 import matplotlib.pyplot as plt
3 import pandas as pd
4 from scipy.integrate import odeint
5
6 T_end = 100  # In Days
7
8 # ON CAMPUS VARIABLES
9 Susceptible_c = np.zeros(T_end)
10 Susceptible_c[0] = 1996
11
12 Undetected_c = np.zeros(T_end)
13 Undetected_c[0] = 6
14
15 Iso_un_c = np.zeros(T_end)  # false pos
16 Iso_in_c = np.zeros(T_end)  # true pos
17 Sympt_c = np.zeros(T_end)  # Symptomatic
18 Vaccinated_c = np.zeros(T_end)
19 Recovered_c = np.zeros(T_end)
20 Deaths_c = np.zeros(T_end)
21
22 # OFF CAMPUS VARIABLES
23 Susceptible_o = np.zeros(T_end)
24 Susceptible_o[0] = 1995
25
26 Undetected_o = np.zeros(T_end)
27 Undetected_o[0] = 5
28
29 Iso_un_o = np.zeros(T_end)  # false pos
30 Iso_in_o = np.zeros(T_end)  # true pos
31 Sympt_o = np.zeros(T_end)  # Symptomatic
32 Recovered_o = np.zeros(T_end)
33 Vaccinated_o = np.zeros(T_end)
34 Deaths_o = np.zeros(T_end)
35
36 totSusceptible = np.zeros(T_end)
37 totUndetected = np.zeros(T_end)
38
39 death = 0.00004
40 recovery_rate = (1 / 14)
41 symptom_onset = 0.06122
42 mu = (1 / 14)  # false positives returned
43 sensitivity = .8
44 specificity = .98
45 vaccineEfficacy = .9  #
46 vaccineDate = 0
47
48 # SWEEP VARIABLES
49 R_stu_range = np.array([1, 1.5, 2.5])
50 R_c = .5

```



```

51
52 screening_range = np.array([1, 2, 3]) / 7 # days per week
53
54 X_range = np.array([0, .0035, .007]) / 7 # infections per
    week
55 Y_range = np.array([0, .0105, .021]) / 7
56
57 VaccineDist = np.array([50,100,150, 250]) / 7 # Daily
    vaccine distribution
58
59 dataFile = open("CompiledData.csv", "w")
60 dataFile.write("Total Recovered, On Campus, Off Campus,
    R_stu, Screening, X, Y, Vaccinations, Max Infected Time \n
    ")
61
62 def InitializeVars():
63     # ON CAMPUS VARIABLES
64     Susceptible_c = np.zeros(T_end)
65     Susceptible_c[0] = 2000
66
67     Undetected_c = np.zeros(T_end)
68     Undetected_c[0] = 6
69
70     Iso_un_c = np.zeros(T_end) # false pos
71     Iso_in_c = np.zeros(T_end) # true pos
72     Sympt_c = np.zeros(T_end) # Symptomatic
73     Vaccinated_c = np.zeros(T_end)
74     Recovered_c = np.zeros(T_end)
75     Deaths_c = np.zeros(T_end)
76
77     # OFF CAMPUS VARIABLES
78     Susceptible_o = np.zeros(T_end)
79     Susceptible_o[0] = 2000
80
81     Undetected_o = np.zeros(T_end)
82     Undetected_o[0] = 5
83
84     Iso_un_o = np.zeros(T_end) # false pos
85     Iso_in_o = np.zeros(T_end) # true pos
86     Sympt_o = np.zeros(T_end) # Symptomatic
87     Recovered_o = np.zeros(T_end)
88     Vaccinated_o = np.zeros(T_end)
89     Deaths_o = np.zeros(T_end)
90
91     totSusceptible = np.zeros(T_end)
92     totUndetected = np.zeros(T_end)
93     print("cleared")
94
95
96 def plot(Susceptible, Undetected, Symptomatic, Recovered,

```

```

96 Dead, Fp, Tp, t, name, dir, sweptVar, maxInfected):
97     fig = plt.figure(num=1, clear=True)
98     ax = fig.add_subplot(1, 1, 1)
99     # Plot using red circles
100    # ax.plot(t, G, 'b-', label='Oral OP Concentration ( $\mu$ g
    /L)', markevery=10)
101
102    # ax.plot(t, U, 'g-', label='Uninfected')
103    """"
104    ax.plot(t, A, 'b-', label='Undetected (Asymptomatic)')
105    ax.plot(t, S, 'r-', label='Isolated (Symptomatic)')
106    #ax.plot(t, R, 'm-', label='Recovered')
107    #ax.plot(t, Fp, 'c-', label='Isolated (Uninfected)')
108    ax.plot(t, Tp, 'y-', label='Isolated (Asymptomatic)')
109    ax.plot(t, D, 'k-', label='Dead')
110    """"
111
112    totalInfectedIsolated = np.array([Symptomatic, Tp]).
    sum(axis=0)
113    totalVaccinated = np.array([Vaccinated_o, Vaccinated_c
    ]).sum(axis=0)
114    totalSusceptible = np.array([Susceptible_c,
    Susceptible_o]).sum(axis=0)
115    ax.plot(t, Undetected, 'g-', label='Infected (
    Undetected)')
116    ax.plot(t, totalInfectedIsolated, 'r-', label='
    Infected (Isolated)')
117    if maxInfected:
118        plt.axvline(x=maxInfected, color='r', ls="--",
    label="Max Infected")
119    """"ax.plot(t, totalVaccinated, 'b-', label="Vaccinated
    ")
120    ax.plot(t, totalSusceptible, 'k-', label="Susceptible
    ")""""
121
122    # Set labels and turn grid on
123    ax.set(xlabel='Time, Days', ylabel=r'Population')
124    ax.grid(True)
125    ax.legend(loc='best')
126    # Use space most effectively
127    fig.tight_layout()
128    fig.savefig("{}_{}.png".format(dir, name.replace(
    ' ', '_'), sweptVar))
129    fig.show()
130
131
132 def OnCampus(t, R_stu, screening, X, vaccineDist):
133     infectedFrac = (
134         totUndetected[t] / (totSusceptible[t] +
    totUndetected[t])) # Fraction of non-isolated students

```

```

134 infected
135
136     infectedFrac_c = Undetected_c[t] / (Susceptible_c[t
137 ] + Undetected_c[t]) # Fraction of on-campus non-isolated
138 students infected
139
140     Beta_stu = (recovery_rate + symptom_onset) * R_stu #
141 Each student to each student
142
143     Beta_c = (recovery_rate + symptom_onset) * R_c #
144 Between on campus students
145
146     if t % 7 == 0 and Susceptible_c[t] >= X:
147         x = X
148         # print("New Infections On Campus: {:.3}".format(
149 Iso_in_c[t]+Sympt_c[t]))
150     else:
151         x = 0
152
153     # print(infectedFrac, infectedFrac_c)
154
155     Susceptible_c[t + 1] = Susceptible_c[t] * (1 -
156 Beta_stu * infectedFrac - Beta_c * infectedFrac_c) \
157     - Susceptible_c[t - 1] *
158 screening * (1 - specificity) + mu * Iso_un_c[t] - x*
159 Susceptible_c[t] \
160     - vaccineDist *
161 vaccineEfficacy * (t>vaccineDate)
162
163     #print(totSusceptible[t])
164     Undetected_c[t + 1] = Undetected_c[t] * (1 -
165 symptom_onset - recovery_rate) \
166     + Susceptible_c[t] * (Beta_stu
167 * infectedFrac + Beta_c * infectedFrac_c)\
168     - Undetected_c[t - 1] *
169 screening * sensitivity + x*Susceptible_c[t]
170
171     Iso_un_c[t + 1] = Iso_un_c[t] * (1 - mu) +
172 Susceptible_c[t - 1] * screening * (1 - specificity)
173
174     Iso_in_c[t + 1] = Iso_in_c[t] * (1 - symptom_onset -
175 recovery_rate) + Undetected_c[t - 1] * screening *
176 sensitivity
177
178     Sympt_c[t + 1] = Sympt_c[t] * (1 - recovery_rate -
179 death) + symptom_onset * (Iso_in_c[t] + Undetected_c[t])
180
181     Vaccinated_c[t + 1] = Vaccinated_c[t] + vaccineDist *
182 vaccineEfficacy* (t>vaccineDate)
183
184     Recovered_c[t + 1] = Recovered_c[t] + recovery_rate

```

```

166 * (Iso_in_c[t] + Undetected_c[t] + Sympt_c[t])
167
168 Deaths_c[t + 1] = Deaths_c[t] + death * Sympt_c[t]
169
170
171 if Susceptible_c[t+1] <= 0:
172     Susceptible_c[t] = 0
173
174 # if t % 21 == 0:
175 # print("True Pos:{}, Asympt: {}, Sympt: {}".format(
176   Iso_in_c[t], Undetected_c[t], Sympt_c[t]))
177
178 def OffCampus(t, R_stu, screening, Y, vaccineDist):
179     infectedFrac = (
180         totUndetected[t] / (totSusceptible[t] +
181         totUndetected[t])) # Fraction of non-isolated students
182         infected
183
184     Beta_stu = (recovery_rate + symptom_onset) * R_stu #
185     Each student to each student
186
187     if t % 7 == 0 and Susceptible_o[t] >= Y:
188         # print("New Infections Off Campus: {}".format(
189           Iso_in_o[t]+Sympt_o[t]))
190         y = Y
191     else:
192         y = 0
193
194     Susceptible_o[t + 1] = Susceptible_o[t] * (
195         1 - Beta_stu * infectedFrac) - Susceptible_o[t
196         - 1] * screening * (1 - specificity) + \
197         mu * Iso_un_o[t] - y*
198     Susceptible_o[t] - vaccineDist *vaccineEfficacy * (t>
199     vaccineDate)
200
201     Undetected_o[t + 1] = Undetected_o[t] * (1 -
202     symptom_onset - recovery_rate) + \
203     Beta_stu * Susceptible_o[t] *
204     infectedFrac \
205     - Undetected_o[t - 1] *
206     screening * sensitivity + y*Susceptible_o[t]
207
208     Iso_un_o[t + 1] = Iso_un_o[t] * (1 - mu) +
209     Susceptible_o[t - 1] * screening * (1 - specificity)
210
211     Iso_in_o[t + 1] = Iso_in_o[t] * (1 - symptom_onset -
212     recovery_rate) + Undetected_o[t - 1] * screening *
213     sensitivity
214

```

```

202     Sympt_o[t + 1] = Sympt_o[t] * (1 - recovery_rate -
    death) + symptom_onset * (Iso_in_o[t] + Undetected_o[t])
203
204     Vaccinated_o[t + 1] = Vaccinated_o[t] +
    vaccineEfficacy * vaccineDist * (t > vaccineDate)
205
206     Recovered_o[t + 1] = Recovered_o[t] + recovery_rate
    * (Iso_in_o[t] + Undetected_o[t] + Sympt_o[t])
207
208     Deaths_o[t + 1] = Deaths_o[t] + death * Sympt_o[t]
209
210     if Susceptible_o[t+1] <= 0:
211         Susceptible_o[t] = 0
212     if (Susceptible_c[t] * (Beta_stu * infectedFrac)) <= .
    01:
213         print("No new growth when Vaccinated: {} \n
    Recovered: {}".format(Vaccinated_o[t], Recovered_o[t] ))
214     # if t % 21 == 0:
215     # print("True Pos:{}, Asympt: {}, Sympt: {}".format(
    Iso_in_o[t], Undetected_o[t], Sympt_o[t]))
216
217
218
219 def Model (R_stu, screening, X, Y, vaccineDist, dir,
    sweptVar):
220
221     time = range(0, T_end)
222
223     for t in time[0:T_end - 1]:
224
225         totSusceptible[t] = Susceptible_c[t] +
    Susceptible_o[t]
226         totUndetected[t] = Undetected_c[t] + Undetected_o[
    t]
227         OnCampus(t, R_stu, screening, X, vaccineDist)
228         OffCampus(t, R_stu, screening, Y, vaccineDist)
229
230         maxInfected = np.argmax(np.array([Undetected_c,
    Undetected_o,
231                                         Iso_in_o, Iso_in_c,
    Sympt_o, Sympt_c]).sum(axis=0))
232         print("Max Infected at {}".format(maxInfected))
233
234         print("Total Recovered {} On Campus: {}, Off Campus {}
    \n R:{}, Screening:{}, X:{}, Y:{}, Vaccine : {} per week"
235             .format(Recovered_c[99] + Recovered_o[99],
    Recovered_c[99],
236                                     Recovered_o[99],
    R_stu, screening, X, Y, vaccineDist*7))
237         newData = ([Recovered_c[99] + Recovered_o[99],

```

```

237 Recovered_c[99], Recovered_o[99], R_stu, screening*7, X, Y
238     ,
239     vaccineDist * vaccineEfficacy*7,
240     maxInfected])
241     newData = str(newData).replace("[", "").replace("]",
242     "")
243     dataFile.write(newData+"\n")
244     campusData = (Susceptible_c, Undetected_c, Sympt_c,
245     Recovered_c, Deaths_c, Iso_un_c, Iso_in_c)
246     offCampusData = (Susceptible_o, Undetected_o, Sympt_o
247     , Recovered_o, Deaths_o, Iso_un_o, Iso_in_o)
248     plot(Susceptible_c, Undetected_c, Sympt_c, Recovered_c
249     , Deaths_c, Iso_un_c, Iso_in_c, time, "On Campus",
250     dir, sweptVar, 0)
251     plot(Susceptible_o, Undetected_o, Sympt_o, Recovered_o
252     , Deaths_o, Iso_un_o, Iso_in_o, time, "Off Campus",
253     dir, sweptVar, 0)
254     plot(np.add(Susceptible_c, Susceptible_o),
255     np.add(Undetected_c, Undetected_o),
256     np.add(Sympt_c, Sympt_o),
257     np.add(Recovered_c, Recovered_o),
258     np.add(Deaths_c, Deaths_o),
259     np.add(Iso_un_c, Iso_un_o),
260     np.add(Iso_in_c, Iso_in_o),
261     time, "Overall Data", dir, sweptVar, maxInfected)
262     InitializeVars()
263
264
265 swept = int(input("Select Sweep Variable \n 1) Inter-
266 Student R \n 2) Screening Frequency \n 3) Exogenous Shocks
267 \n "
268 "4) Vaccine Availability\n 5)All of the
269 Above\n"))
270
271 def Sweep(sweepVar):
272     if sweepVar == 1:
273         for r in R_stu_range:
274             Model(r, screening_range[1], X_range[1],
275             Y_range[1], 0, "R_Stu", "r_{}".format(r))
276     elif sweepVar == 2:
277         for screen in screening_range:
278             Model(R_stu_range[1], screen, X_range[1],
279             Y_range[1], 0, "Screening", "Screening_{}".format(screen
280             * 7))
281     elif sweepVar == 3:

```

```
274         for shock in range(0, 3):
275             Model(R_stu_range[1], screening_range[1],
276                 X_range[shock], Y_range[shock], 0, "Exogenous",
277                 "ShockLevel_{}".format(shock))
278         elif sweepVar == 4:
279             for vaccineDist in VaccineDist:
280                 print(vaccineDist)
281                 Model(R_stu_range[1], screening_range[1],
282                     X_range[1], Y_range[1], vaccineDist, "Vaccine",
283                     "Vaccine_{}".format(vaccineDist * 7))
284 if swept == 5:
285     for sweepVar in range(1, 5):
286         Sweep(sweepVar)
287 else:
288     Sweep(swept)
289
290 dataFile.close()
291
```