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Fuzzy model-based observers for fault detection in CSTR

Hazael Ballesteros-Moncada [a], Enrique J. Herrera-López [b], Juan Anzurez-Marín [a,*]

[a] Facultad de Ingeniería Eléctrica, Universidad Michoacana de San Nicolás de Hidalgo, Morelia Michoacán, Mexico
[b] Biotecnología Industrial, Centro de Investigación y Asistencia en Tecnología y Diseño del Estado de Jalisco A.C., Avenida Normalistas 800, Colinas de la Normal, C.P. 44270 Guadalajara, Jalisco, Mexico

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ABSTRACT

Under the vast variety of fuzzy model-based observers reported in the literature, what would be the properone to be used for fault detection in a class of chemical reactor? In this study four fuzzy model-based observers for sensor fault detection of a Continuous Stirred Tank Reactor were designed and compared. The designs include (i) a Luenberger fuzzy observer, (ii) a Luenberger fuzzy observer with sliding modes, (iii) a Walcott–Zak fuzzy observer, and (iv) an Utkin fuzzy observer. A negative, an oscillating fault signal, and a bounded random noise signal with a maximum value of ± 0.4 were used to evaluate and compare the performance of the fuzzy observers. The Utkin fuzzy observer showed the best performance under the tested conditions.

1. Introduction

The objective of a chemical process is to convert raw materials to desired products using available sources of energy in the most economical way [1]. The continuous stirred tank reactor (CSTR) is one of the most used reactors, since it can be operated under steady conditions of pressure, composition and temperature. To keep the CSTR in the steady state it is necessary to monitor and control the process. However, nonlinearities in chemical reactions complicate the design of robust controllers [2]. Moreover, a fault in a sensor will propagate through the control loops and affect the plant operation [3]. In a chemical process, faulty sensors may cause process performance degradation or even fatal accidents [4]. Therefore, monitoring strategies must detect faults quickly and accurately so that automatic correction action can be used to return the process to the desired operating conditions [5].

Three methodologies for fault diagnosis in chemical processes have been reported in the literature: (i) statistical analysis, (ii) input–output information knowledge, and (iii) model-based methods. Statistical analysis has been used for industrial process monitoring and fault diagnosis based on univariate and multivariate techniques such as simple thresholding [6], Principal Component Analysis [7–9], probabilistic decision models [10] and Bayesian methods [11]. The input–output information knowledge method is used for primary classification, where Artificial Neural Networks (ANN) are widely used since they are capable of modeling uncertain nonlinear processes. Diverse applications of ANN for fault diagnosis can be found in the literature [12–17].

The third method is the model-based approach which determines faults appearing in a dynamic system comparing available measurements with prior information, represented by a mathematical model. From an adequate mathematical model and if the system is observable, the output of a process can be estimated by an observer. Comparison signals known as residuals can be generated from the difference between the real measurement and the observer providing information about the faults in the system [18]. In this context, diverse model-based observers for fault diagnosis have been applied to chemical processes for instance: the Luenberger observer [19,20], the Kalman filter [21,22], the sliding mode observer [23], the H_∞ based observer [24], the adaptive observer [4], high-gain observer based on Fliess's generalized observability canonical form [25] and unknown input observer [26,27], just to cite some of them.

Better methods for fault diagnosis have become necessary to accomplish the scientific and industrial development of modern processes. A scarcely explored observer model-based method for fault diagnosis in chemical processes is fuzzy logic. Fuzzy logic was first introduced by Zadeh [28] and is now widely used to model

* Corresponding author. Tel.: +52 443 3223500; fax: +52 443 3279728.
E-mail address: janzurez@umich.mx (J. Anzurez-Marín).

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and control nonlinear systems. Although fuzzy logic belongs to the fault detection input–output information knowledge methods, it can also be considered as a model-based method since nonlinear systems can be represented by Takagi–Sugeno fuzzy models [29], and from these models fuzzy observers can be designed.

The purpose of this study was to design and compare the performance of four fuzzy observers for sensor fault detection in a chemical process. The approach was to build Takagi–Sugeno fuzzy models of the chemical process, and then design fuzzy observers to estimate the system output. From the difference between the real and estimated output, a diagnostic residual signal is generated, which is insensitive to parametric variations in the neighborhood of the nominal parameter values, and sensitive only to the fault signal. This paper is organized as follows: the next section provides a brief background on fault detection, and fuzzy systems. Section 3 introduces the Luenberger fuzzy observer, the Luenberger fuzzy observer with sliding modes, the Walcott–Zak fuzzy observer, and the Utkin fuzzy observer. Section 4 describes the mathematical model of the chemical process. The performance of the four fuzzy model-based observers for fault detection is compared and discussed in Section 5. Finally, Section 6 states the main conclusions.

2. Background

2.1. Fault detection

A fault is an unpermitted deviation of at least one characteristic property or parameter of the system from the acceptable/usual/standard condition, and fault detection is the determination of the faults present in a system and the time of detection [30]. Fault detection requires sensitivity to small faults, fast response, and robustness. A fuzzy model-based observer approach for fault detection consists of estimating the output of a system with a fuzzy observer and comparing it with the real measurement of a process sensor in order to generate a residual signal represented by $r(t)=y(t)-\hat{y}(t)$, where $y(t)$ is the nonlinear system output and $\hat{y}(t)$ is the estimated output. From the value of $r(t)$ the fault can be detected. A scheme for residual generation for a fuzzy model-based observer is shown in Fig. 1. In this scheme the system output only depends on the components related to the fault denoted as $f_a(t)$. An operation is defined as normal or faulty as follows:

$$|r(t)| \approx 0 \quad \text{if } f_a(t)=0 \text{ (normal operation)}$$

$$|r(t)| \gg 0 \quad \text{if } f_a(t) \neq 0 \text{ (faulty operation).}$$

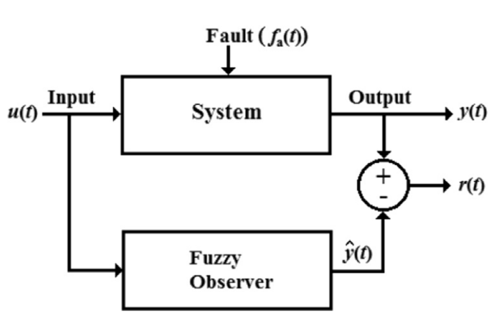

Fig. 1. Representation of a fuzzy model-based observer for residual generation.

2.2. Takagi–Sugeno fuzzy systems

A nonlinear system can be represented by the following sets of differential equations:

$$\begin{aligned}\dot{x}(t)&=f(x(t))+g(x(t))u(t)\\ y(t)&=h(x(t))\end{aligned} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $u(t)\in\mathbb{R}^m$ is the input vector, $y(t)\in\mathbb{R}^p$ is the system output, and $f(\cdot)$, $g(\cdot)$, and $h(\cdot)$ are smooth functions. The system (1), can also be represented by the Takagi–Sugeno (T–S) fuzzy model, which is described by fuzzy IF–THEN rules representing the local linear input–output relationship of a nonlinear system. The main feature of a T–S fuzzy model is that it expresses the local dynamics of each rule by a linear system. The ith rule of the T–S fuzzy model is of the following form [31]:

Rule i:

IF $z_1(t)$ is M_{1i} and … and $z_p(t)$ is M_{pi}

$$\text{THEN}\ \begin{cases}\dot{x}(t)=A_ix(t)+B_iu(t),\\ y(t)=C_ix(t),\end{cases}\qquad i=1,2,\dots,r$$

where $z_1(t), z_2(t), \dots, z_p(t)$ are known premise variables that may be functions of the state, and/or external disturbances. M_{ij} is a fuzzy set and r is the number of rules, $A_i\in\mathbb{R}^{n\times n}$, $B_i\in\mathbb{R}^{n\times m}$ and $C_i\in\mathbb{R}^{q\times n}$. Each linear consequent equation represented by $A_ix(t)+B_iu(t)$ is called a subsystem.

Given a pair of $(x(t), u(t))$, the final outputs of the fuzzy system are inferred as follows:

$$\begin{aligned}\dot{x}(t)&=\frac{\sum_{i=1}^{r}\omega_i(z(t))\{A_ix(t)+B_iu(t)\}}{\sum_{i=1}^{r}\omega_i(z(t))}\\ \dot{x}(t)&=\sum_{i=1}^{r}\mu_i(z(t))\{A_ix(t)+B_iu(t)\}\end{aligned} \tag{2}$$

$$\begin{aligned}y(t)&=\frac{\sum_{i=1}^{r}\omega_i(z(t))C_ix(t)}{\sum_{i=1}^{r}\omega_i(z(t))}\\ y(t)&=\sum_{i=1}^{r}\mu_i(z(t))C_ix(t)\end{aligned} \tag{3}$$

where

$$\omega_i(z(t))=\prod_{j=1}^{p}M_{ij}(z_j(t)),\qquad \mu_i(z(t))=\frac{\omega_i(z(t))}{\sum_{i=1}^{r}\omega_i(z(t))}$$

for all t. The term M_{ij} is the grade of membership of $z_j(t)$ in M_{ij}. Since

$$\begin{cases}\sum_{i=1}^{r}\omega_i(z(t))>0,\\ \omega_i(z(t))\geq 0,\end{cases}\qquad i=1,2,\dots,r$$

we have

$$\begin{cases}\sum_{i=1}^{r}\mu_i(z(t))=1,\\ \mu_i(z(t))\geq 0,\end{cases}\qquad i=1,2,\dots,r$$

for all t.

3. Fuzzy model-based observer design

Fuzzy logic offers several challenges and opportunities in detecting faults based on observers built from T–S fuzzy models. Therefore, this section shows the design of the four fuzzy model-based observers that will be applied to the chemical reactor. The first fuzzy observer is explained below.

3.1. Luenberger fuzzy observer (LFO)

The ith rule for a continuous Luenberger fuzzy observer can be represented by

Observer rule i:
IF $z_1(t)$ is M_{1i} and … and $z_p(t)$ is M_{pi}
THEN

$$\begin{cases} \dot{\hat{x}} = \sum_{i=1}^{r} \mu_i(z(t))\{A_i\hat{x}(t)+B_iu(t)+K_i(y(t)-\hat{y}(t))\} \\ \hat{y}(t) = \sum_{i=1}^{r} \mu_i(z(t))C_i\hat{x}(t), \end{cases} \tag{4}$$

for $i=1,2,\ldots,r$. K_i is the observer gain for the ith subsystem. This observer is known as the Luenberger fuzzy observer (LFO). Defining

$$\dot{e}(t) = \dot{x} - \dot{\hat{x}} \tag{5}$$

and substituting systems (2) and (4) into Eq. (5) give

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(z(t))\{A_ix(t)+B_iu(t)\} - \sum_{i=1}^{r} \mu_i(z(t))\{A_i\hat{x}(t)+B_iu(t) + K_i(y(t)-\hat{y}(t))\} \tag{6}$$

then

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(z(t)) \sum_{j=1}^{r} \mu_j(z(t))(A_i-K_iC_j)e(t) \tag{7}$$

after some algebraic manipulation it can be found that

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(z(t))\mu_i(z(t))\{(A_i-K_iC_i)\boldsymbol{e}(t)\} + 2\sum_{i=1}^{r}\sum_{i<j} \mu_i(z(t))\mu_j(z(t))\left\{\frac{(A_i-K_iC_j)+(A_j-K_jC_i)}{2}\boldsymbol{e}(t)\right\} \tag{8}$$

Theorem 1. *If for the Takagi–Sugeno model (4) there exist positive definite matrices N and P which satisfy the linear matrix inequalities (LMI's) (9), then it will be possible to find a gain matrix, which guarantees asymptotic convergence to zero of the estimation error given by (5) [32]:*

$$\begin{aligned} &P>0 \\ &A_i^TP+PA_i-C_i^TN_i^T-N_iC_i<0 \\ &A_i^TP+PA_i-C_j^TN_i^T-N_iC_j+A_j^TP+PA_j-C_i^TN_j^T-N_jC_i<0 \\ &i<j\le r \end{aligned} \tag{9}$$

Then the LFO given by (4) with $K_i=P^{-1}N_i$ guarantees asymptotic convergence to zero of the estimation error, where P is a common definite positive matrix and K is the observer gain.

3.2. Luenberger fuzzy observer with sliding modes (LFOSM)

Recently, sliding mode observers have received attention as a tool for estimating parameters in nonlinear uncertain systems. Sliding mode observers are highly robust since they are insensitive to disturbances. They have been used to monitor chemical processes [33] and for fault diagnosis [34]. For T–S fuzzy systems a Luenberger fuzzy observer with sliding modes (LFOSM) can be designed for each subsystem [35], as follows:

Observer rule i:
IF $z_1(t)$ is M_{1i} and … and $z_p(t)$ is M_{pi}
THEN

$$\dot{\hat{x}} = \sum_{i=1}^{r} \mu_i(z(t))\{\{A_i\hat{x}(t)+B_iu(t)+K_i(y(t)-\hat{y}(t))\}+\varphi_i(t)\} \tag{10}$$

The output for the observer is defined as given in (4), and $\varphi(t)$ is a discontinuous function given by

$$\varphi_i(t) = P^{-1}\overline{C}_i^T\frac{\overline{C}_ie(t)}{\|\overline{C}_ie(t)\|}.$$

where $\overline{C}=\sum_{i=1}^{r} w_iC_i$. To study the convergence of this observer the systems (2) and (10) are substituted into Eq. (5). After simplifying this becomes

$$\dot{e}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(z(t))\mu_j(z(t))\{(A_i-K_iC_j)e(t)-\varphi_i(t)\} \tag{11}$$

In addition to Theorem 1 the following one is used for the LFOSM [35]:

Theorem 2. *If the function $\varphi_i(t)$ satisfies*

$$\varphi_i(t) = \begin{cases} k_i\dfrac{P^{-1}\overline{C}^T\overline{C}e(t)}{\|\overline{C}e(t)\|} & \text{if} \|\overline{C}e(t)\|>\varepsilon \\ k_i\dfrac{P^{-1}\overline{C}^T\overline{C}e(t)}{\varepsilon} & \text{if} \|\overline{C}e(t)\|\le\varepsilon \end{cases}$$

with $k_i>0$, and $0<\varepsilon<1$, the observer given by (13), with $K_i=P^{-1}N_i$, guarantees asymptotic convergence to zero of the estimation error [18].

3.3. Walcott–Zak fuzzy observer (WZFO)

Another method for sliding mode observers was proposed by Walcott and Zak [36]. Consider the linear system given by

$$\begin{aligned} \dot{x}(t) &= Ax(t)+Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{12}$$

The problem consists of estimating the states of a system assuring that the error is quadratically stable. A Walcott–Zak observer requires that there exists a matrix $G\in\mathbb{R}^{n\times p}$ such that $A_0=A-GC$ has stable eigenvalues and there exist Lyapunov matrices (P, Q) for A_0 [37], such that

$$C_i^TF_i^T = PB_i \tag{13}$$

is satisfied for the matrix $F_i\in\mathbb{R}^{m\times p}$. Under these considerations the ith rule for a fuzzy Walcott–Zak fuzzy observer (WZFO) is described as

Observer rule i:
IF $z_1(t)$ is M_{1i} and … and $z_p(t)$ is M_{pi}
THEN

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(z(t))\left\{A_i\hat{x}(t)+B_iu(t)-G_i\left\{\sum_{j=1}^{r}\mu_j(z(t))(C_j\hat{x}(t)-C_jx(t))\right\} +P^{-1}C_i^TF_i^Tv_i\right\} \tag{14}$$

and

$$v_i = \begin{cases} -\rho\dfrac{FCe}{\|FCe\|} & \text{if } FCe\neq 0 \\ 0 & \text{elsewhere} \end{cases} \tag{15}$$

where ρ is a scalar function, and the output for the observer is defined as in Eq. (4). To analyze the convergence of the WZFO, the observer error is defined as $\dot{e}(t)=\dot{\hat{x}}-\dot{x}$ then the error derivative is given as

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(z(t)) \left\{ A_i\hat{x}(t) + B_i u(t) - G_i \left\{ \sum_{j=1}^{r} \mu_j(z(t))(C_j\hat{x}(t) - C_j x(t)) \right\} + P^{-1} C_i^T F_i^T v_i \right\} - \sum_{i=1}^{r} \mu_i(z(t))\{A_i x(t) + B_i u(t)\} \tag{16}$$

simplifying and using (13) give

$$\dot{e}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(z(t))\mu_j(z(t))\{(A_i - G_iC_j)e(t) + B_i v_i\} \tag{17}$$

Proposition 1. *For the Walcott–Zak fuzzy observer (17) there exists a Lyapunov candidate function*

$$\dot{v}(t) = e^T(t)P\dot{e}(t) + \dot{e}^T(t)Pe(t) \tag{18}$$

that guarantees quadratic stability.

Proof.

$$\dot{v}(t) = e^T(t)P\left\{ \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(z(t))\mu_j(z(t))\{(A_i - G_iC_j)e(t) + B_i v_i\} \right\} + \left\{ \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(z(t))\mu_j(z(t))\{(A_i - G_iC_j)e(t) + B_i v_i\} \right\}^T (t)Pe(t) \square \tag{19}$$

Since $A_{oij} = A_i - G_iC_j$, simplifying it is found that

$$\dot{v}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(z(t))\mu_j(z(t)) \cdot \{e^T(t)(PA_{oij} + A_{oij}^T P)e(t) + 2e^T(t)PB_i v_i\} \tag{20}$$

The term $(PA_{oij} + A_{oij}^T P)$ in Eq. (20) is negative definite, then Eq. (19) can be expressed as

$$\dot{v}(t) \le -\gamma \| e(t) \|^2 + \sum_{i=1}^{r} \mu_i(z(t))\{2e^T(t)PB_i v_i\} \tag{21}$$

considering that the discontinuous term is $-\rho\frac{F_iC_ie}{\|F_iC_ie\|}$ then Eq. (19) can be written as

$$\dot{v}(t) \le -\gamma \| e(t) \|^2 - \sum_{i=1}^{r} \mu_i(z(t))\left[2e^T(t)PB_i\right]\rho\frac{F_iC_ie}{\|F_iC_ie\|} \tag{22}$$

From expression (13), it is obtained that

$$\dot{v}(t) \le -\gamma \| e(t) \|^2 - 2\rho \sum_{i=1}^{r} \mu_i(z(t)) \| F_iC_ie(t) \| \tag{23}$$

choosing $\rho > 0$, the Lyapunov function is negative definite.

3.4. Utkin fuzzy observer (UTFO)

The Utkin observer was proposed by Utkin [38]. In this observer it is convenient to make a change in coordinates, assuring that system outputs are the last states of the transformed system

$$T_{ci} = \begin{bmatrix} Nc_i^T \\ C_i \end{bmatrix} \tag{24}$$

where $Nc_i \in \mathbb{R}^{n\times(n-p)n}$ is the null space of the matrix C_i. If $C_1 = C_2 = \cdots = C_r$ then the matrix transformation will be unique [38] and the system (1) can be expressed as

$$\dot{\mathbf{z}}(t) = \sum_{i=1}^{r} \mu_i(z(t))\{A_{ni}\mathbf{z}(t) + B_{ni}u(t)\}$$

$$y_u(t) = \sum_{i=1}^{r} \mu_i(z(t))C_{ni}x(t) \tag{25}$$

where $A_{ni} = T_{ci}A_iT_{ci}^{-1}$, $B_{ni} = T_{ci}B_i$, $C_{ni} = C_iT_{ci}^{-1}$ and

$$z(t) = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \begin{matrix} \updownarrow n-p \\ \updownarrow p \end{matrix} \tag{26}$$

then (25) can be written as

$$\dot{\mathbf{z}}(t) = \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \end{bmatrix} = \sum_{i=1}^{r} \mu_i(z(t))\{A_{ni}\mathbf{z}(t) + B_{ni}u(t)\}$$

Now the ith rule for a Utkin fuzzy observer (UTFO) is written as

Observer rule i:

IF $z_1(t)$ is M_{1i} and … and $z_p(t)$ is M_{pi}
THEN

$$\dot{\hat{x}}_1(t) = \sum_{i=1}^{r} \mu_i(z(t))\{A_{11i}\hat{x}_1(t) + A_{12i}\hat{y}_1(t) + B1_i u(t) + L_i v\}$$

$$\dot{\hat{y}}_1(t) = \sum_{i=1}^{r} \mu_i(z(t))\{A_{21i}\hat{x}_1(t) + A_{22i}\hat{y}_1(t) + B2_i u(t) - v\} \tag{27}$$

where $(\hat{x}_1, \hat{y}_1)$ stands for the estimates of (x_1, y_1), the variable $L_i \in \mathbb{R}^{(n-p)\times p}$ is a constant gain matrix for every subsystem, and v is a discontinuous vector given by

$$v = M\ \mathrm{sgn}(\hat{y}_1 - y) \tag{28}$$

where $M \in \mathbb{R}^+$. To design the matrix L we should consider that the error between the estimated and the real values is described by $e_1 = \hat{x}_1 - x_1$ and $e_{y1} = \hat{y}_1 - y_1$, consequently the error dynamics are given by $\dot{e}_1 = \dot{\hat{x}}_1 - \dot{x}_1$ and $\dot{e}_{y1} = \dot{\hat{y}}_1 - \dot{y}_1$ respectively, using Eqs. (25) and (27) gives

$$\dot{e}_1 = \sum_{i=1}^{r} \mu_i(z(t))\{A_{11i}\hat{x}_1(t) + A_{12i}\hat{y}_1(t) + B1_i u(t) + L_i v\} - \sum_{i=1}^{r} \mu_i(z(t))\{A_{11i}x_1(t) + A_{12i}y_1(t) + B1_i u(t)\}$$

$$\dot{e}_{y1} = \sum_{i=1}^{r} \mu_i(z(t))\{A_{21i}\hat{x}_1(t) + A_{22i}\hat{y}_1(t) + B2_i u(t) - v\} - \sum_{i=1}^{r} \mu_i(z(t))\{A_{21i}x_1(t) + A_{22i}y_1(t) + B2_i u(t)\}$$

Reducing terms, the system can be expressed as

$$\dot{e}_1(t) = \sum_{i=1}^{r} \mu_i(z(t))\{A_{11}e_1(t) + A_{12}e_{y1}(t) + L_i v\}$$

$$\dot{e}_{y1}(t) = \sum_{i=1}^{r} \mu_i(z(t))\{A_{21}e_1(t) + A_{22}e_{y1}(t) - v\} \tag{29}$$

As the pair (A_i, C_i) is observable, then the pair (A_{11i}, A_{21i}) is also observable, therefore it is possible to define the matrix L_i such that the eigenvalues of the matrix

$$A_{11i} + L_iA_{21i} \tag{30}$$

are $\mathbb{C}^-$. Propounding a new matrix $K = -L$, then (30) can be written as

$$A_{11i} - K_iA_{21i} \tag{31}$$

Applying Theorem 1 the following LMIs are obtained:

$$P > 0$$
$$A_{111}^T P + PA_{111} - A_{211}^T N_1^T - N_1A_{211} < 0$$
$$A_{112}^T P + PA_{112} - A_{212}^T N_2^T - N_2A_{212} < 0$$

$$A_{111}^T P + PA_{111} - A_{212}^T N_1^T - N_1 A_{212} + A_{112}^T P + PA_{112} - A_{211}^T N_2^T - N_2 A_{211} < 0 \quad (32)$$

where $A_i = A_{11i}$ and $C_i = A_{21i}$.

This section presented the fundamental theory for each of the four fuzzy observers. In the next section the LFO, the LFOSM, the WZFO, and the UTFO will be applied as fuzzy model-based observers for fault detection in a continuous stirred tank reactor.

4. Case study: the chemical reactor model

Consider the model of a continuous stirred tank reactor (CSTR) which has the following isothermal first order chemical reaction:

$$P_r \to Q \to R$$

where P_r and Q are highly acidic reactants and R is neutral. The system is described by the following equations [39]:

$$\begin{aligned} \dot{x}_1 &= -(1+D_{a1})x_1 + u \\ \dot{x}_2 &= D_{a1}x_1 - x_2 - D_{a2}x_2^2 \\ y &= x_1 + x_2 \end{aligned} \quad (33)$$

where $x_1 = C_p/C_{P0}$ and $x_2 = C_Q/C_{P0}$, C_P and C_Q are respectively the concentration of species P_r and Q (mol/m^3) in the reactor, C_{P0} is the desired concentration of species P_r and Q (mol/m^3). The constants D_{a1} and D_{a2} are dilution rates and can be defined as $D_{a1} = k_1V/F$ and $D_{a2} = k_2VC_{P0}/F$ respectively, where k_1 (1/s) and k_2 (m^3/mol s) are the first order and second order rate constants respectively. V is the volume in the reactor (m^3) and F is the volumetric feed rate (m^3/s). Finally $u = \frac{N_{PF}}{FC_{Po}}$, where N_{PF} is the molar feed rate of species P_r (mol/s). In order to design the observers the system (33) is rewritten as in (1):

$$\begin{aligned} f(x(t)) &= \begin{bmatrix} -(1+D_{a1})x_1 \\ D_{a1}x_1 - x_2 - D_{a2}x_2^2 \end{bmatrix} \\ g(x(t)) &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ h(x(t)) &= [x_1 + x_2] \end{aligned} \quad (34)$$

Setting the region of operation for the state variables as $x_1 \in [0, 6]$ and $x_2 \in [0, 2]$, it is possible to find the following operational points: $[\overline{x}_1, \overline{x}_2]_1 = [0, 0]$, and $[\overline{x}_1, \overline{x}_2]_2 = [6, 2]$ [40]. Defining $x = [x - \overline{x}]$ and $u = [u - \overline{u}]$ and linearizing the system (34) about $[\overline{x}_1, \overline{x}_2]_1$ with

$$A = \frac{\partial f}{\partial x}(x, u)\bigg|_{\substack{x = \overline{x} \\ u = \overline{u}}}, \quad B = \frac{\partial f}{\partial x}(x, u)\bigg|_{\substack{x = \overline{x} \\ u = \overline{u}}},$$

$$C = \frac{\partial h}{\partial x}(x)\bigg|_{x = \overline{x}}.$$

the following linear matrices are obtained:

$$A_1 = \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_1 = [1 \ \ 1]$$

To calculate $[\overline{x}_1, \overline{x}_2]_2$ the following relationship is used [41]:

$$\begin{cases} a_i^j = \nabla df_i(x_0^i) + \dfrac{f_j(x_0^i) - x_0^i \nabla f_j(x_0^i)}{\| (x_0^i) \|^2} x_0^i \\ B_i = g(x_0^i) \end{cases} \quad (35)$$

then A_2 is given by

$$A_2 = \begin{bmatrix} -2 & 0 \\ 1.6 & -4.8 \end{bmatrix}$$

$B_2 = B_1$ and $C_2 = C_1$. The rules for the T–S fuzzy model are set as

Rule 1:

IF $x_1(t)$ is 0 and x_2 is 0

$$\text{THEN} \begin{cases} \dot{x}(t) = A_1x(t) + B_1u(t) \\ y(t) = C_1x(t) \end{cases}$$

Rule 2:

IF $x_1(t)$ is 6 and x_2 is 2

$$\text{THEN} \begin{cases} \dot{x}(t) = A_2x(t) + B_2u(t) \\ y(t) = C_2x(t) \end{cases}$$

The membership functions are of the form reported in [40], then

$$\begin{aligned} \mu_1(x_1) &= 1 - \frac{x_1}{6} \\ \mu_2(x_1) &= \frac{x_1}{6} \end{aligned} \quad (36)$$

5. Performance of the fuzzy model-based observers

In order to evaluate the performance of the fuzzy model-based observers, two different test scenarios were used:

- No fault is induced in the output sensor (S_1).
- Inducing multiple faults and noise in the output sensor (S_2).

5.1. No fault is induced in the output sensor

In a first instance the performance of the LFO, the LFOSM, the WZFO, and the UTFO were evaluated as plain observers without inducing a fault [42]. Fig. 2(a)–(d) shows that the four fuzzy observers converged to the desired output. The gain for the LFO and the LFO were calculated using the theory from Section 3, Theorem (1), and the LMIs (9). Feasible solutions for P, N_1 and N_2

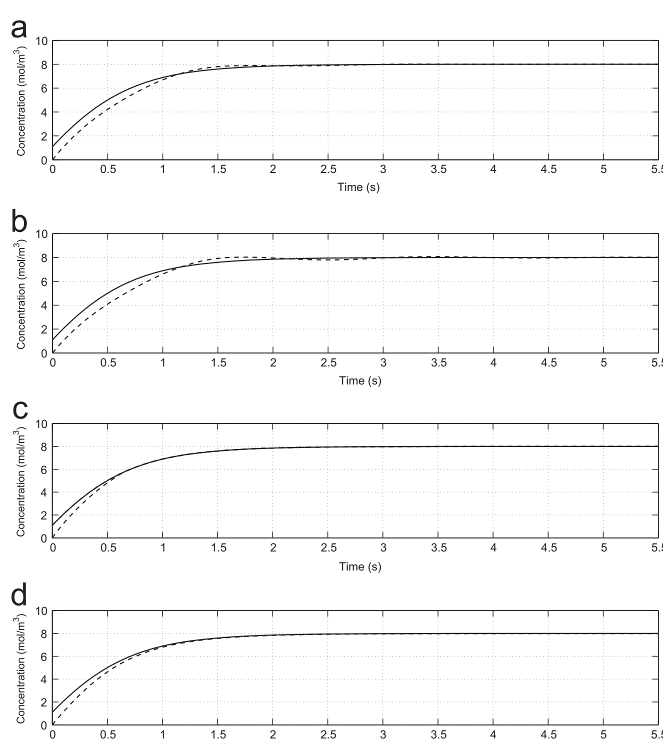

Fig. 2. Comparison between (a) the Luenberger fuzzy observer, (b) Luenberger fuzzy observer with sliding modes, (c) the Walcott–Zak fuzzy observer and (d) the Utkin fuzzy observer without induced fault in the output sensor. Solid line represents the systems output and the dashed line stands for the fuzzy observers output.

were obtained as

$$P=\begin{bmatrix}3044.9 & 1700.2\\ 1700.2 & 1671.5\end{bmatrix}>0 \tag{37}$$

$$N_1=\begin{bmatrix}-3153.6\\ -435.6\end{bmatrix},\quad N_2=\begin{bmatrix}-2165\\ -6819\end{bmatrix} \tag{38}$$

where P is positive definite since its eigenvalues are $\lambda=[524.6,\ 4191.9]$. From N_1 and N_2 the gains are calculated as $K_i=P^{-1}N_i$, then

$$K_1=\begin{bmatrix}-2.0605\\ 1.8353\end{bmatrix},\quad K_2=\begin{bmatrix}3.6268\\ -7.7685\end{bmatrix} \tag{39}$$

From the gains obtained in (39) it is possible to simulate the behavior of the LFO and the LFOSM. The initial conditions for all the fuzzy observers were $x(0)=0$. The LFO and the LFOSM had similar behaviors. Both observers had little overshoot, although it was greater for the LFOSM. The LFO converged to the desired signal in about 2.9 s (Fig. 2a), while the LFOSM settled down in about 5 s (Fig. 2b). For the WZFO it was found that $G_1=K_1$ and $G_2=K_2$, the same gains obtained for the LFO and the LFOSM and then $F_1=F_2=2372.6$. The WZFO reached the output signal in about 0.7 s, without any overshoot (Fig 2c). The UTFO gains were obtained from (32) with $G_1=K_1$ and $G_2=K_2$. The additional parameters to guarantee convergence were $L_1=-1.44$, $L_2=-0.80$ and $M=25$. The UTFO converged in about 1.5 s. No overshoot was observed; however, a little offset was always shown (Fig 2d).

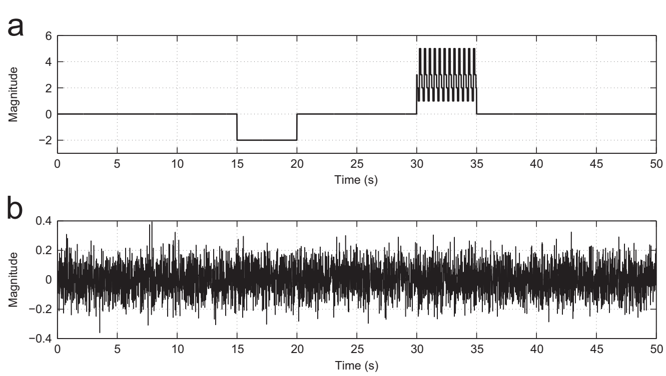

Fig. 3. $f_a(t)$ induced fault (a), and noise (b) signals used to test the four fuzzy model-based observers for fault detection.

5.2. Inducing multiple faults and noise in the output sensor

To evaluate the performance of the fuzzy model-based observers the fault signal shown in Fig. 3a was induced in the output sensor. At the start the signal has a null value, until a negative fault is generated from 15 to 20 s; this effect could represent a malfunction in the sensor or in the transmitter. Later the fault is disabled for 10 s and a positive oscillating periodic signal is switched on from 30 to 35 s. This signal could represent several short malfunction pulses. In addition, a bounded random noise signal with a maximum value of $\pm$ 0.4 was added to the system, Fig. 3b. A value of $|r(t)|\geq 0.3$ units in magnitude was used as the decision threshold for all the conditions tested.

In this section the performance of the fuzzy observers were evaluated when the fault "$f_a(t)$" (Fig. 3a) and the noise (Fig. 3b) signals were induced in the output sensor of the CSTR system. The induced noise signal remained active during all the simulation. The fault signal was not enabled during the first 15 s of the evaluation of the fuzzy observers. In this period the LFO, the LFOSM, the WZFO, and the UTFO converged to the system signal and no fault indicators and false alarms activated, in spite of the noise signal, Figs. 4–7. When the negative fault is induced all the fuzzy observers detected the fault. The LFO, the LFOSM, and the WZFO presented false alarms when the fault disappeared; while the UTFO do not showed false alarms. During the lapse 20–30 s the fault indicator for all the fuzzy observers once again remained inactive, in spite of the noise signal. When the positive fault pulses were induced in the output signal the LFO, the LFOSM, and the WZFO tried to follow the oscillating signal enabling/disabling the fault indicators. Several fault alarms were present for these

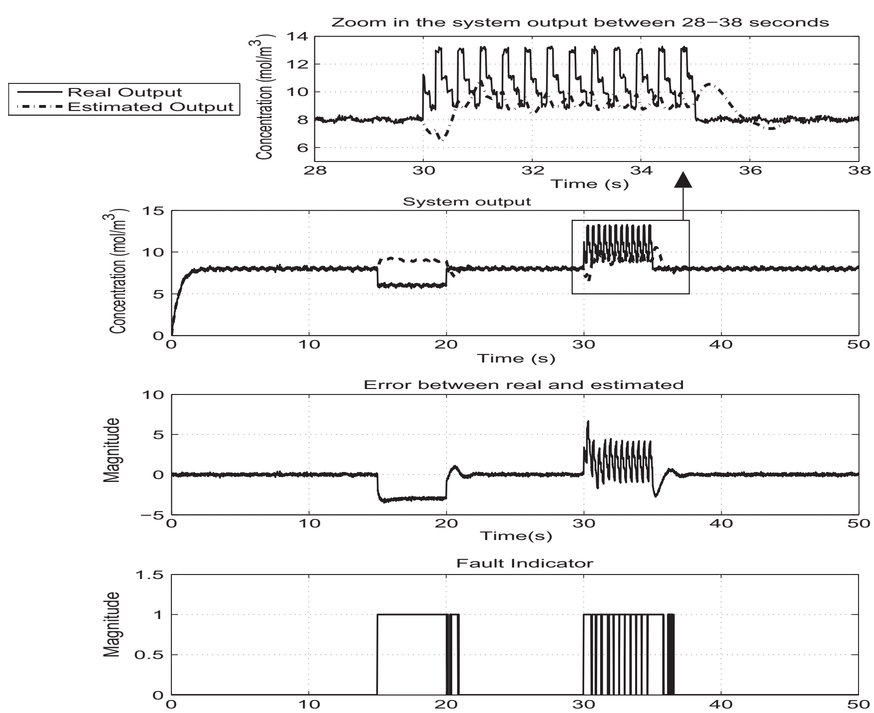

Fig. 4. Performance evaluation of the Luenberger fuzzy model-based observer when the fault and noise signals of Fig. 3 are induced in the nonlinear system.

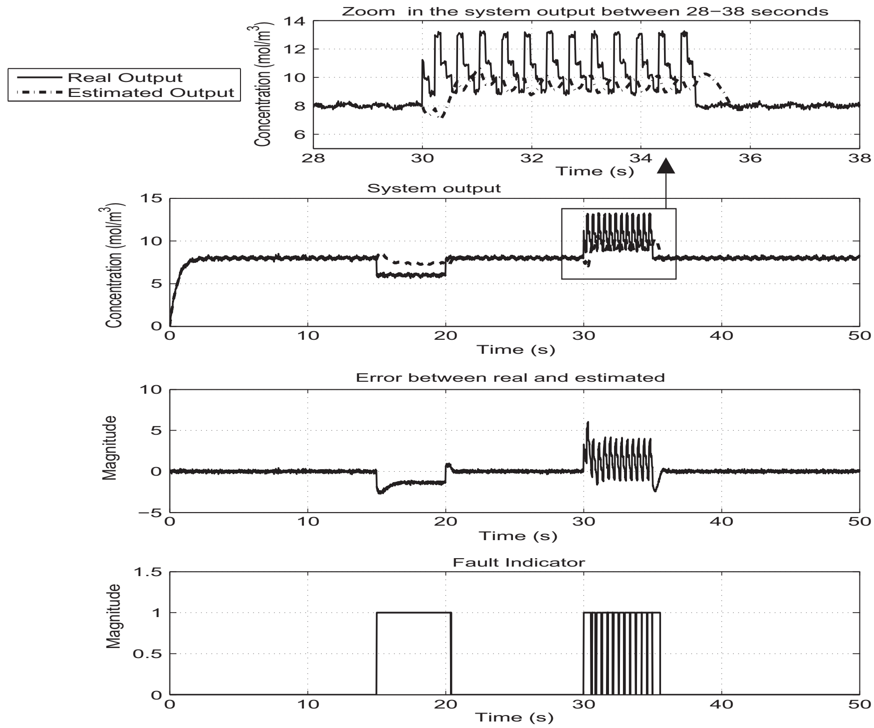

Fig. 5. Performance evaluation of the Luenberger fuzzy model-based observer with sliding modes when the fault and noise signals of Fig. 3 are induced in the nonlinear system.

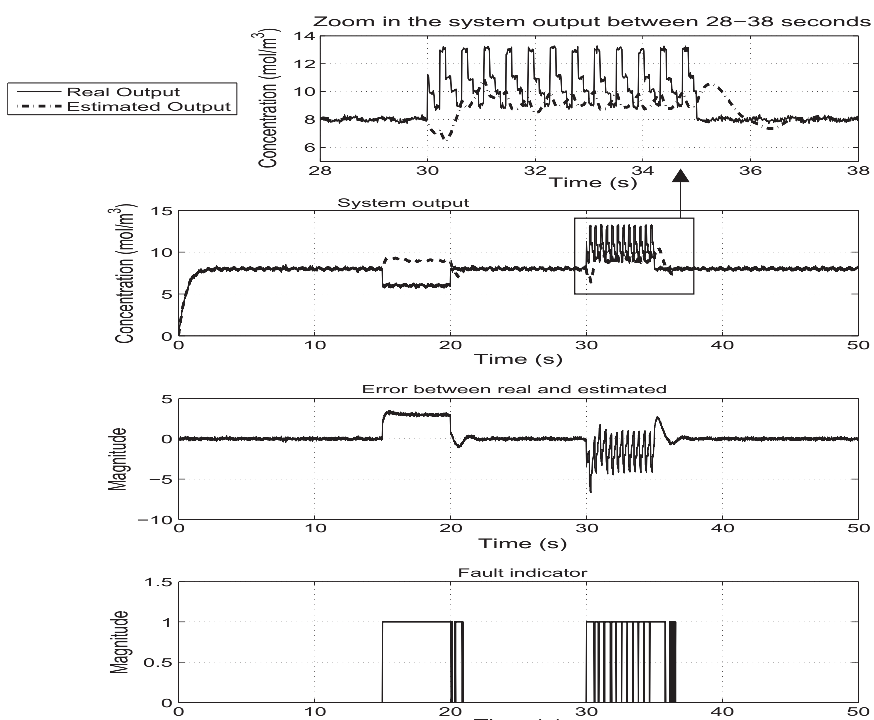

Fig. 6. Performance evaluation of the Walcott–Zak fuzzy model-based observer when the fault and noise signals of Fig. 3 are induced in the nonlinear system.

observers when the fault was disabled. The UTFO do not try to follow the pulses although it was able to detect the fault correctly; moreover, no false alarm was present after the fault disappeared.

5.3. Discussion

When no fault was induced (see scenario S_1 in Fig. 2) the LFO and the WZFO properly converged to the estimated output signal without overshoot. In this case the WZFO had the fastest response reaching to the desired signal in about 0.7 s. The UTFO required 1.5 to converge to the sensor signal which is twice as long as the time needed for the WZFO. However, the UTFO always presented a little offset along the simulation. Regarding the LFO and the LFOSM both had overshoot and crossed the reference signal in about 2.2 s; although the LFOSM required 5 s to converge, while the LFO took 2.9 s. When the negative fault was induced to the CSTR system the

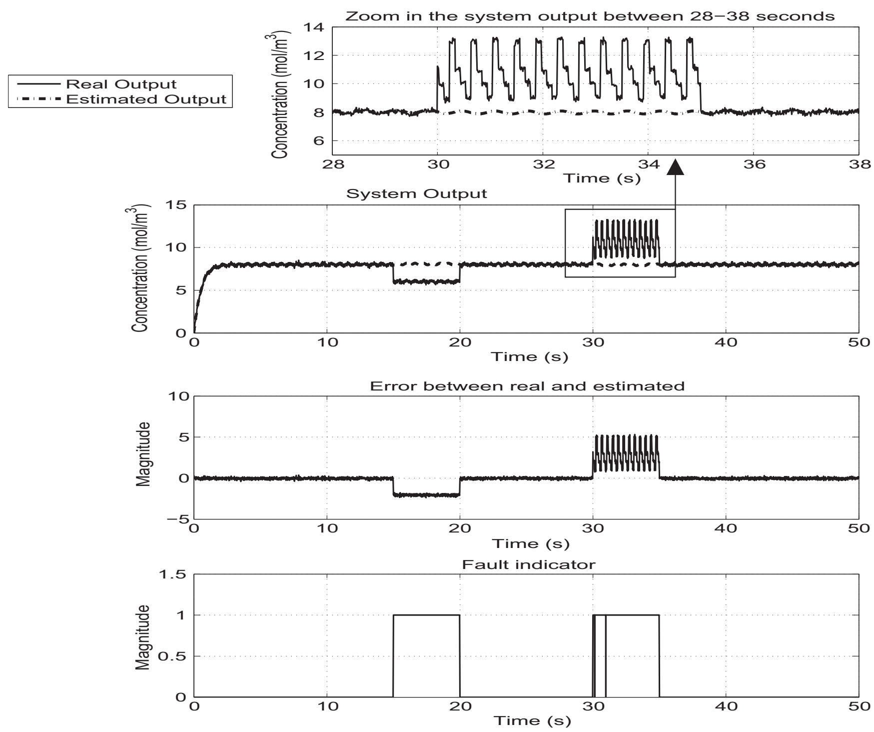

Fig. 7. Performance evaluation of the Utkin fuzzy model-based observer when the fault and noise signals of Fig. 3 are induced in the nonlinear system.

LFO and the WZFO had similar responses increasing the steady value from 8.0 to 9.35 for the LFO (Fig. 4) and 9.3 for the WZFO (Fig. 6). These values remained constant until the fault was disabled. It is worth to remark the interesting behavior for these observers, resulting in a negative value for the LFO and a positive for the WZFO. This behavior can be explained easily remembering that the error for the WZFO was calculated as $\dot{e}(t)=\dot{\hat{x}}-\dot{x}$, which is opposite to the error defined for the LFO in Eq. (5). After the fault disappeared the LFO took 2 s to settle down to the output system, while the WZFO required 1.5 s. Both fuzzy observers generated at least 2 false alarms. The LFOSM during the negative fault increased its value to 8.6 and rapidly decreased to 7.7 staying there until the fault disappeared, then smoothly converged to the sensor signal in 2.2 s, triggering only one brief false alarm (Fig. 5). The UTFO was not apparently affected by the induced fault; however, a residual signal enabled the fault indicator. When the fault disappeared the UTFO remained in its steady value avoiding any false alarm (Fig. 7). Table 1 compares the response of the four fuzzy observers on the diverse tested conditions.

The second part of the fault signal (Fig. 3a) is composed of 12 pulse periodic signals with a peak to peak amplitude of 4 units, an offset of 3 units and a frequency of approximately of 2.4 Hz. All the fuzzy observers were able to detect the induced fault. The LFO, the LFOSM and the WZFO responded to the pulsating signal decreasing and then slowly increasing their values trying to follow the oscillating signal without achieving convergence. Once again the LFO (Fig. 4) and the WZFO (Fig. 6) had similar behaviors enabling/disabling 12 times the fault indicator. After the fault disappeared it took 2.2 s for the LFO and 1.8 s for the WZFO to settle down generating fast oscillating false alarms. When the fault was disabled the LFOSM (Fig. 5) settled down in 0.6 s consequently less false alarms were generated. The UTFO (Fig. 7) do not try to follow the induced fault pulsating signal; although, indeed it was able to detect the fault without the oscillating behavior characteristic of the LFO, the LFOSM and the WZFO during the simulations. Moreover, the UTFO converged to the sensor signal immediately after the fault was disabled; therefore this fuzzy observer does not generate false alarms. Table 1 compares the response of the four fuzzy observers on the diverse tested conditions.

Table 1
Response characteristics of the fuzzy observers under the tested conditions S_1 and S_2.

Type of	Scenario S_1	Scenario S_2 (Negative Fault)		Scenario S_2 (oscillating fault)	
observer	Settling time (s)	FA_1	RT_1 (s)	FD_2	RT_2 (s)
LFO	2.9	4	2.0	5	2.2
LFOSM	5	1	0.6	2	0.6
WZ	0.7	4	1.5	5	1.8
UTFO	1.5	0	0	2	0

FA_1 and FA_2: number of false alarms for the Negative Fault for the scenario S_2. RT_1 and RT_2: restoration time (s) for the Oscillating Fault for the scenario S_2.

6. Conclusion

All the fuzzy observers designed were able to follow the desired output sensor when no fault was present in the system; however, the WZFO converged faster than the other observers. When the negative and oscillating faults were induced in the CSTR all the fuzzy observers detected the faults correctly, even in the presence of noise. The LFO and the WZFO had similar behaviors regarding overshoot and convergence time. When the faults were disabled the LFO, the WZFO and the LFOSM generated false alarms; nevertheless, the last observer converged faster than the LFO and than the WZFO to the output signal. The UTFO was the best observers since it detected all the faults correctly without causing false alarms. The LFO, the LFOSM, the WZFO and the UTFO have potential to be used as a fuzzy model-based observer, but they need to be tested in more complex chemical and biological processes. This will be addressed in future research.

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