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Nonlinear model-predictive control with disturbance rejection property using adaptive neural networks

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Abstract

In this paper, a method is proposed to reject disturbances in the model predictive control (MPC) strategy. In addition, uncertainties in the system parameters (i.e., internal disturbances) are considered as well. To achieve these goals, adaptive neural networks are designed as the predictor model and as the nonlinear disturbance observer, respectively. The disturbances are rejected via the optimization problem of the MPC. Stability of the closed-loop system is studied based on the Input-to-State Stability method. The proposed method is applied to the pH neutralization process and CSTR system and its effectiveness in optimal rejection of the disturbances and satisfying the system constrains is compared with the feed-forward control method.

1. Introduction

Model predictive control (MPC) method has been used in different industries due to its effectiveness in the control of the constraint nonlinear systems [1,2]. This control strategy is a constraint optimal controller, where an on-line optimization problem is solved and an input sequence over a certain time horizon (prediction/control horizon) is generated at every sampling time. Due to the receding horizon policy, only the first input is applied to system and this procedure is repeated during the next sampling instant [3,4].

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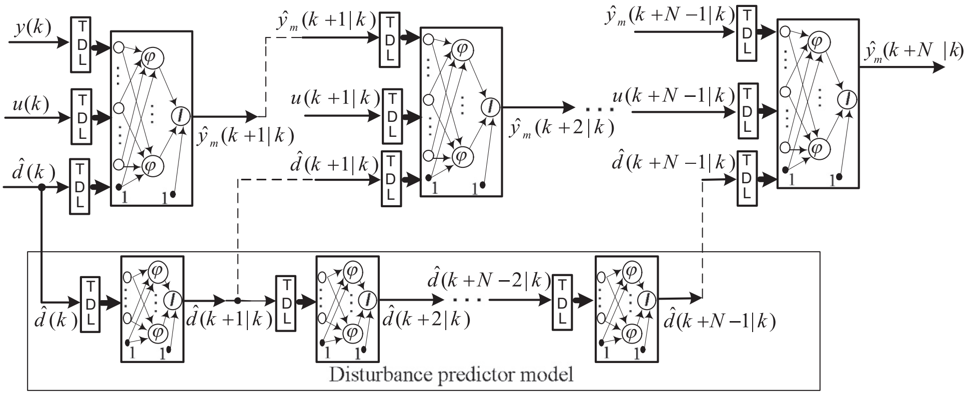

Fig. 1. The proposed NN model.

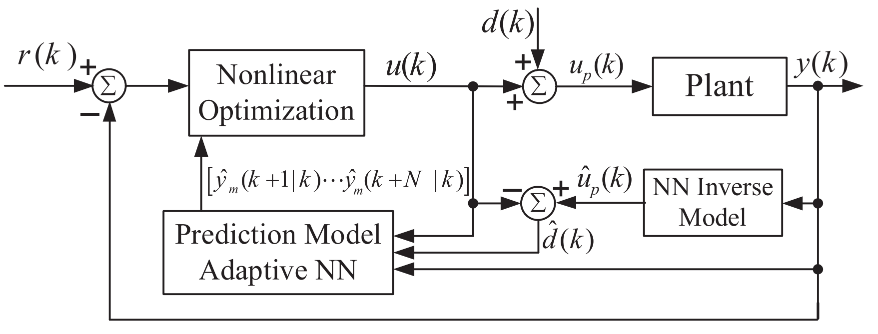

Fig. 2. Block diagram of proposed closed-loop control system.

Modeling the system is an essential part of the MPC. The model is used to predict the system output based on the past and current information. Accuracy of the model has significant effect on the performance and stability of the closed-loop system. Different modeling approaches (e.g., neural networks and fuzzy systems) are used to model the nonlinear systems in the MPC [5,6]. Neural networks (NNs) are known as universal approximators using different learning algorithms. Hence, they are widely used to model nonlinear systems [5,7,8]. For MPCs, the NN provides the system model only based on the input–output data and hence, less knowledge of the system is required. Moreover, dealing with the nonlinear and complex equations of the system is relaxed.

Changes in the system parameters and external disturbances are common in practical applications that can lead to deterioration of the performance of the closed-loop system. Most of the classic controllers have inherent robustness against disturbances but cannot reject severe disturbances fast and properly [9]. Different controllers have been proposed in literature to compensate disturbances and uncertainties [10–14]. Robust control methods can improve the control performance by attenuating the disturbances to a certain limit but it needs information about the bounds of the disturbances and the uncertainties [15,16]. Adaptive control methods are other strategies to deal with disturbances and uncertainties. These methods have the ability of adapting to the variations in the system parameters and in the environment. In most of these methods, an adaptive estimator is used to estimate the uncertainties and the parameters of the

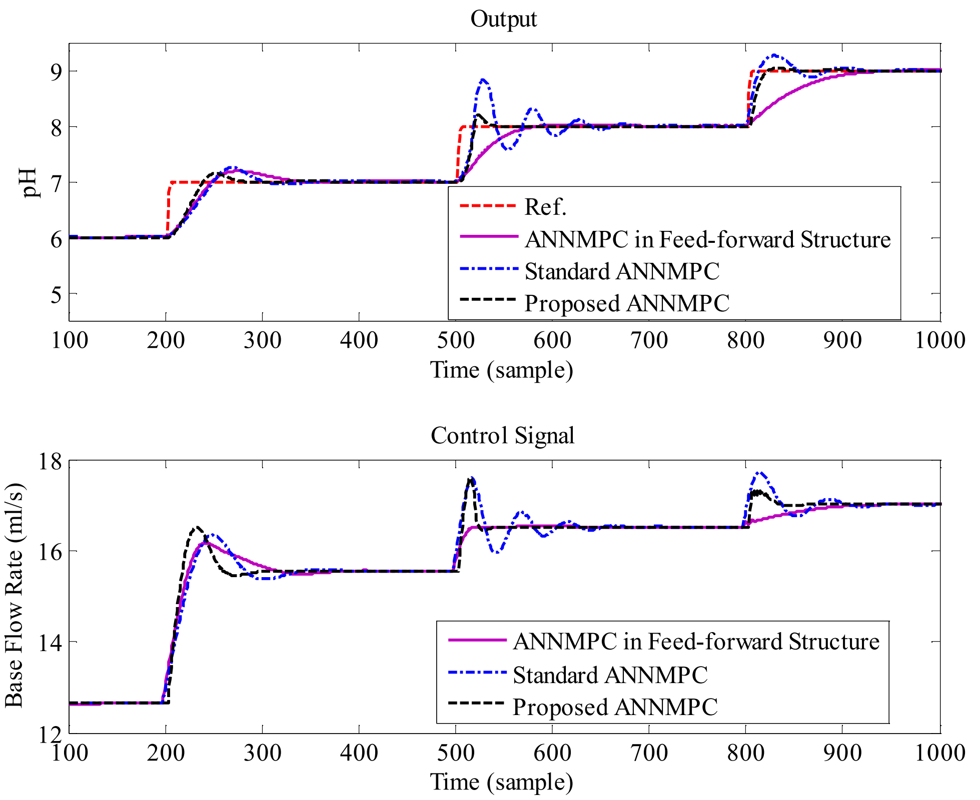

Fig. 3. Reference tracking responses.

model and/or the controller are updated in each sampling time. In these adaptive strategies, the effects of the parameter estimation error are minimized by a robust control method such as min–max or Lipschitz-based methods [17–20]. Feed-forward controllers can also reject the disturbances. They are widely used due to their simple structure. However, they need the disturbance to be measurable [21–24]. To solve this problem, disturbance observer-based (DOB) control strategies are utilized [25–27]. In most of these methods, system disturbances can be estimated using the inverse model of the system. This structure has been widely used in different industries, e.g., robotics [28], flight systems [29], and process control systems [30]. Nevertheless, utilizing the inverse model limits the DOB applications to the minimum-phase systems. Although there have been some researches on the DOB for the non-minimum-phase systems as well [9,31].

Several researchers have employed the MPC along with the DOB in feed-forward structure [26–31]. The main drawback of these methods is that the control signal is not optimal because the output of the feed-forward controller is not produced by an optimization problem and hence, the constraints on the manipulated and controlled signals may not be satisfied. In this paper, to improve performance of the MPC in the feed-forward control structure, the disturbance that is estimated by the DOB is utilized in the predictor model. Hence, the disturbance effect is introduced to the optimization problem of the MPC via the predictor

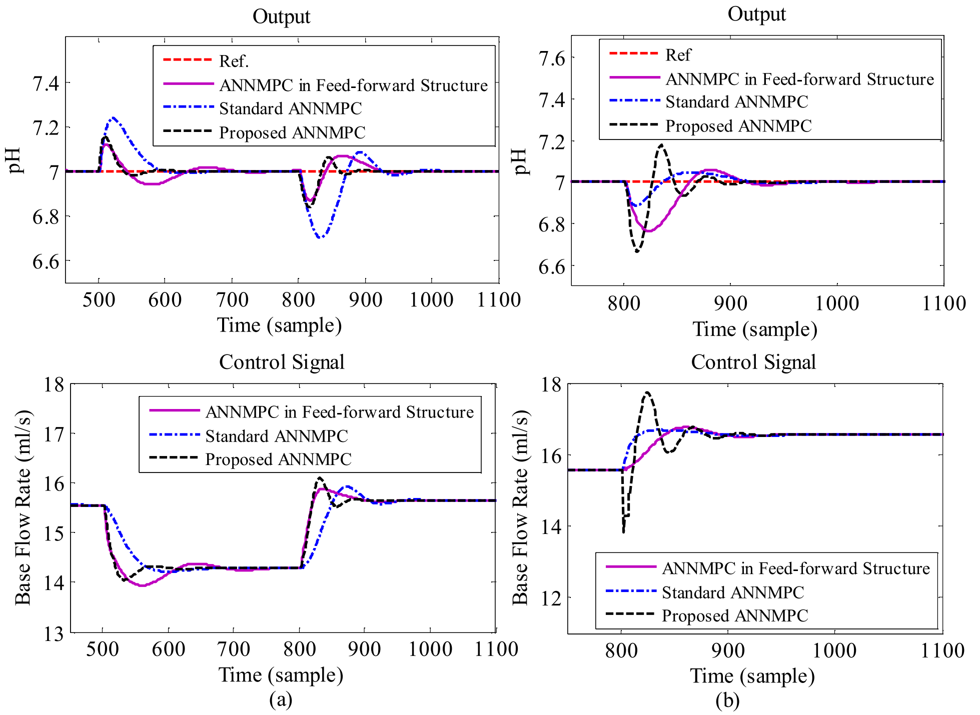

Fig. 4. Response of the closed-loop system to (a) parameter variations and (b) step disturbance.

model and an optimal control signal is generated to attenuate the influence of the disturbances (internal and external) on the closed-loop system.

Based on the aforementioned explanations, in this paper, a predictor NN model with adaptation ability is proposed to realize an Adaptive NN MPC (ANNMPC) to cope with the parameter variations and external disturbances. In addition, a NN is employed in the DOB structure for producing the inverse model of the system. Moreover, another NN with on-line training is designed to provide the estimation of the disturbance, which is required in the predictor NN model. The convergence and stability of the closed-loop system is investigated based on the Input-to-State Stability (ISS) method.

The advantages of the proposed method can be summarized as follows:

- Increasing the precision of the predictor model by using estimated disturbance.
- Predicting the future disturbances using an adaptive NN structure.
- Satisfaction of the system constraints.
- Optimality of the control signal in comparison with the MPC in feed-forward control structure.

The proposed control method is simulated on the pH neutralization and CSTR processes. These systems exhibit severe nonlinearities and have various disturbances, which make them appropriate for evaluation of advanced controllers.

This paper is organized as follows: Section 2 provides the problem statement. The structure of the proposed MPC strategy and the adaptive NN predictor along with the nonlinear disturbance observer and the disturbance predictor model are explained in this section. The analytical study and stability of the closed-loop system is discussed in Section 3. Simulation results are given in Section 4. Section 5 concludes the paper.

2. Problem formulation

Consider the nonlinear system described by the following discrete-time nonlinear equation:

$$y(k) = f\big(y(k-1), \ldots, \; y(k-n_y), \; u(k-1), \; \ldots, \; u(k-n_u), d(k)\big), \tag{1}$$

where k is the sampling instant, f denotes the unknown nonlinear function, $y(\cdot)$ and $u(\cdot)$ are the output and input of the system, respectively, $d(k)$ represents the unknown but bounded disturbance, and n_y and n_u refer to the maximum lags in the system output and input, respectively.

It is assumed that the system is subject to the constraints on the control action, the system output, and the disturbance as

$$\begin{aligned} &u(k) \in U_c \\ &y(k) \in Y_c \\ &d(k) \in D_c, \end{aligned} \tag{2}$$

where Y_c is a closed set, and U_c and D_c are compact sets, all containing the origin.

2.1. Nonlinear model predictive control

The Nonlinear MPC (NMPC) is the same as the MPC but with a nonlinear prediction model and/or nonlinear constraints. The on-line optimization problem in this paper is defined as follows:

$$\begin{aligned} \min_{\boldsymbol{\Delta}\mathbf{u}(k|k)} \sum_{i=0}^{N-1} L(e_T(k+i), u(k+i)) \\ = \min_{\Delta u(k|k),\ldots,\Delta u(k+\mathrm{M}-1|k)} \Bigg[\sum_{i=0}^{N-1} \big(r(k+i) - \hat{y}_m(k+i\,|k)\big)^T Q_y \; \big(r(k+i) - \hat{y}_m(k+i\,|k)\big) \\ + \sum_{j=0}^{M-1} (\Delta u(k+j\,|k))^T R_u \; (\Delta u(k+j\,|k)) \Bigg] \end{aligned} \tag{3}$$

subject to

$$\begin{aligned} &\hat{y}_m(k+i\,|k) \in \mathrm{Y}_c \qquad 0 \le i \le N-1 \\ &u(k+i\,|k) \in \mathrm{U}_c \qquad 0 \le i \le M-1, \end{aligned} \tag{4}$$

where $e_T(k+i) = r(k+i) - \hat{y}_m(k+i\,|k)$ is the tracking error, $r(k+i)$ is the reference signal, $\hat{y}_m(k+i\,|k)$ is the predicted output, N and M are the prediction and control horizons, and Q_y and R_u are the weighting matrices on the predicted error and the control effort, respectively. Moreover, it is assumed that $\Delta u(k+j|k) = 0 \;\; \text{for}\, j \ge M-1$.

The output of system (1) is modeled using a Multi-Layer Perceptron (MLP) NN with NARMA structure as follows:

$$y_m(k) = f_{NN,k}\Big(y(k-1), \; \ldots, y(k-\hat{n}_y), u(k-1), \; \ldots, u(k-\hat{n}_u), \hat{d}(k-1), \; \ldots, \hat{d}(k-\hat{n}_d)\Big),$$

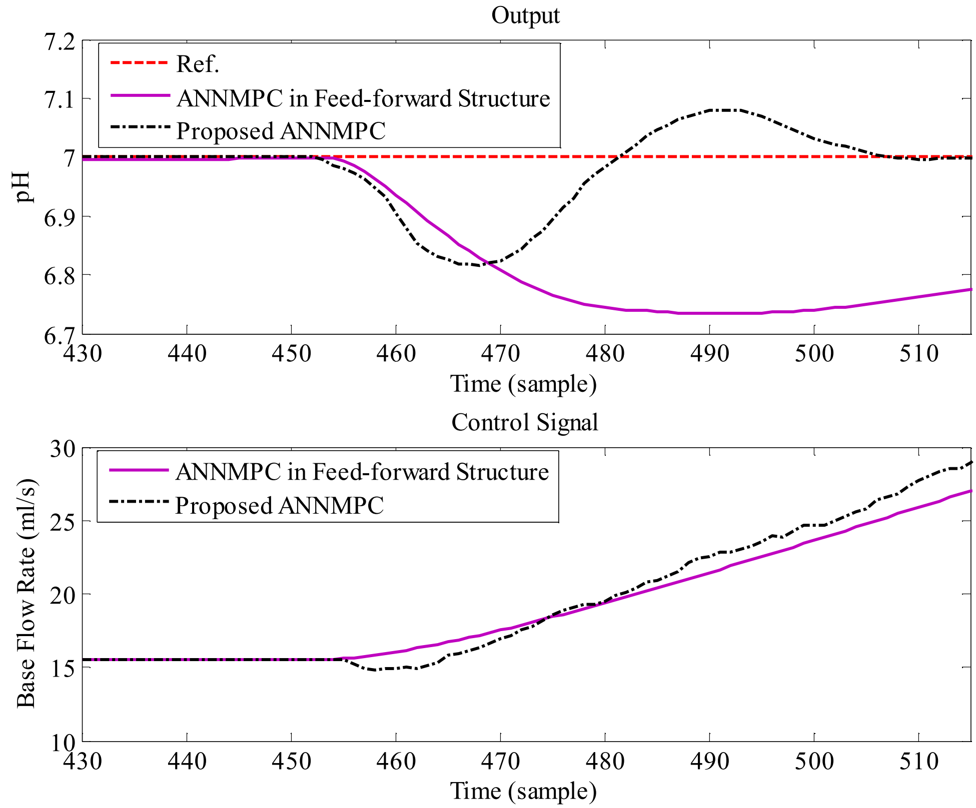

Fig. 5. Ramp disturbance rejection.

(5)

where $f_{NN,k}(.)$ represents the nonlinear mapping function at time step k, $\hat{d}(k)$ is the estimated disturbance, and $\hat{n}_y$, $\hat{n}_u$, and $\hat{n}_d$ are the maximum lags in the model output, input, and disturbance, respectively.

The NN model in Eq. (5) is trained on-line using the Levenberg–Marquardt (LM) algorithm. The objective function for the training is defined as

$$I(k) = \frac{1}{2}e_M^2(k), \tag{6}$$

where $e_M(k) = y(k) - y_m(k)$ is the modeling error. For predictions of the output over the prediction horizon in each sampling time, the same NN is used recursively. Hence, the weights of the NN remain fixed over the prediction horizon. At the next sampling time, the training of the NN is repeated and the weights are updated. The output predictions can be represented as follows:

$$\begin{cases} \hat{y}_m(k+i|k) = f_{NN,k}\big(\hat{y}_m(k+i-1|k),\ \ldots,\hat{y}_m(k+i-\hat{n}_y|k), u(k+i-1|k), \\ \qquad\qquad \ldots, u(k+i-\hat{n}_u|k), \hat{d}(k+i-1|k),\ \ldots,\hat{d}(k+i-\hat{n}_d|k)\big) \qquad 1 \le i \le N. \\ \hat{y}_m(k|k) = y(k) \end{cases} \tag{7}$$

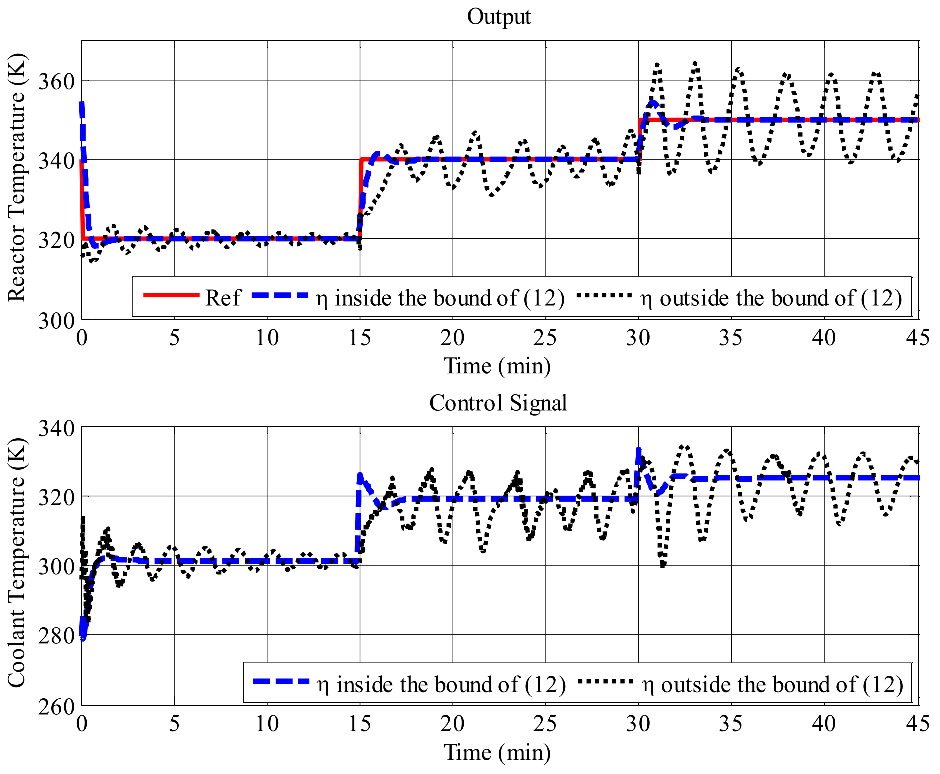

Fig. 6. Comparison of tracking response using different learning rates.

The structure of the proposed NN is depicted in Fig. 1.

2.2. Disturbance observer

As represented in Eqs. (5) and (7), the estimated disturbance is used as one of the inputs to the predictor model. For disturbance estimation, a DOB is utilized here [26]. In the process industries, disturbance on the process input is common.

As depicted in Fig. 2, the DOB uses the inverse model of the system and by comparing the output of the inverse model with the control input, the disturbance is estimated as

$$\hat{d}(k) = \hat{u}_p(k) - u(k) = g_{NN}\big(\mathbf{Y}(k), \mathbf{U}_{\mathrm{p}}(k)\big) - u(k) = g_{NN}(\mathbf{Y}(k), \mathbf{U}(k) + \mathbf{D}(k)) - u(k), \tag{8}$$

where $g_{NN}(\cdot)$ is the inverse model of the nonlinear system that is obtained using an MLP NN, which is trained off-line using the LM algorithm and

$$\begin{aligned} \mathbf{Y}(k) &= \Big[y(k-1),\ \ldots, y(k-\tilde{n}_y)\Big], \quad \mathbf{U}_{\mathrm{p}}(k) = \Big[u_p(k-1),\ \ldots, u_p(k-\tilde{n}_u)\Big] \\ \mathbf{U}(k) &= \Big[u(k-1),\ \ldots, u(k-\tilde{n}_u)\Big], \quad \mathbf{D}(k) = \Big[d(k-1),\ \ldots, d(k-\tilde{n}_d)\Big] \end{aligned} \tag{9}$$

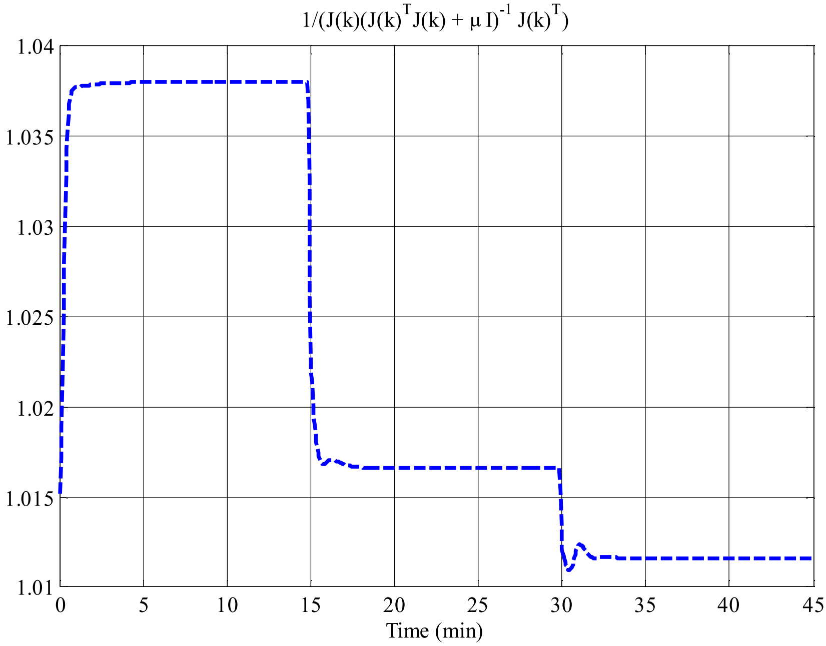

Fig. 7. The upper bound condition of Eq. (12) in Lemma 1.

in which $\tilde{n}_y$, $\tilde{n}_u$, and $\tilde{n}_d$ are the maximum lags in the model input, output, and disturbance, respectively, and $u_p(k)$ is the input of the system (Fig. 2). In order to make the proposed method more practical, only the input–output data of the system is utilized to train the NN.

2.3. Disturbance predictor

As Eq. (7) shows, predictions of the disturbance are needed in the predictor model. In this paper, a NN as a one-step-ahead predictor is utilized that is trained on-line with new data obtained from the system using the same structure over the prediction horizon as follows:

$$\hat{d}(k+i|k) = h_{NN,k}\left(\hat{d}(k+i-1|k),\ \ldots, \hat{d}(k+i-\tilde{n}_d|k)\right), \tag{10}$$

where $h_{NN,k}(\cdot)$ represents the nonlinear function at time step k. The structure of the disturbance predictor is depicted in the lower part of the block diagram in Fig. 1.

Remark 1. The DOB-based MPC methods in literature are mostly presented in feed-forward control structure [26], where the estimated disturbance is subtracted from the control signal to reject the disturbance influence in the system. Although this approach can reject the disturbances and has appropriate performance but the manipulated signal applied to the system is not optimal. Moreover, the constraints on the control signal may not be satisfied. Hence, in the proposed method in this paper, the estimated disturbance is used as the input to the predictor model and the effect of the disturbance can be eliminated through the optimization problem of the NMPC. Furthermore, the adaptive structure of the NN models helps to improve the precision of the predictor model of the system and better reject the disturbances. This fact will also be shown in the simulating examples.

3. Stability of the closed-loop system

In this section, the stability of the closed-loop system is investigated. Since the MPC is a model-based approach and the precision of the predictor model affects the performance of the closed-loop system, first the convergence of the proposed predictor model is studied. Then, the stability of the closed-loop system is investigates.

3.1. Convergence of the predictor model

To study the accuracy and convergence of the proposed predictor model depicted in Fig. 1, three parts of the proposed method should be studied: the NN that represents the inverse of the plant, the NN that predicts the future values of the disturbance, and the NN that provides a valid model prediction for the NMPC. All these NNs are of MLP type and their weights are trained using the LM algorithm. To study the convergence of the NN model with these properties, the following Lemma should be given first.

Lemma 1. Assume that the weights of the NN model are trained using the LM method as

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \eta(k)\left(\mathbf{J}(k)^T\mathbf{J}(k) + \mu\mathbf{I}\right)^{-1}\mathbf{J}(k)^T e(k), \tag{11}$$

where $e(k)$ is the error between the desired output and the output of the NN model, $\mathbf{w}(k)$ is the weight vector, $\eta(k)$ is the learning rate, μ is the regularization parameter, and $\mathbf{J}(k) = \partial e(k)/\partial\mathbf{w}(k)$ is the Jacobian vector. The output of the NN model will converge to the desired output asymptotically if the learning rate satisfies the following condition:

$$0 < \eta(k) < \frac{1}{\mathbf{J}(k)\left(\mathbf{J}^T(k)\mathbf{J}(k) + \mu\mathbf{I}\right)^{-1}\mathbf{J}^T(k)}. \tag{12}$$

Proof. The proof procedure is similar to Theorem 1 in [32]. The candidate Lyapunov function is considered as $V(k) = e(k)^2$. The first difference of this Lyapunov function is

$$\Delta V(k) = V(k+1) - V(k) = 2e(k)\Delta e(k) + \Delta e(k)^2. \tag{13}$$

Using $\Delta e(k) = (\partial e(k)/\partial\mathbf{w}(k))^T\Delta\mathbf{w}(k)$, the definition of Jacobian vector, the training rule in Eq. (11), and after some manipulations, Eq. (13) can be written as

$$\begin{aligned}\Delta V(k) = &\left(2\eta(k)\mathbf{J}(k)\left(\mathbf{J}^T(k)\mathbf{J}(k) + \mu\mathbf{I}\right)^{-1}\mathbf{J}^T(k)e(k)^2\right)\\ &\left(-1 + \eta(k)\left[\mathbf{J}(k)\left(\mathbf{J}^T(k)\mathbf{J}(k) + \mu\mathbf{I}\right)^{-1}\mathbf{J}^T(k)\right]\right) < 0\ .\end{aligned} \tag{14}$$

Using the Sylvester criterion, it can be concluded that $\mathbf{J}(k)(\mathbf{J}^T(k)\mathbf{J}(k) + \mu\mathbf{I})^{-1}\mathbf{J}^T(k) > 0$. Therefore, the first term in Eq. (14) has a positive value and the second term must be negative. Hence, the stability bound for $\eta(k)$ is obtained as in Eq. (12). ❑

Next, the proposed predictor model is studied. First, the inverse model of the system is considered. Based on Lemma 1, the output of this off-line-trained NN converges to the desired value if the learning rate of the NN satisfies condition (12). Hence, Eq. (8) can be rewritten as

$$\begin{aligned}\lim_{k\to\infty}\hat{d}(k) &= \lim_{k\to\infty} g_{NN}(\mathbf{Y}(k), \mathbf{U}(k)+\mathbf{D}(k)) - u(k)\\ &= u(k)+d(k)-u(k)\\ &= d(k),\end{aligned} \tag{15}$$

which means that the estimated disturbance converges to its real value.

The second part of the proposed model is the disturbance predictor that is trained on-line. First, this NN is trained to learn the estimated disturbance. Then, as depicted in the box in Fig. 1, it is cascaded over the prediction horizon with fixed weights and updated inputs to predict the future disturbances. Based on Lemma 1, the output of this NN converges to the estimated value of the DOB if the learning rate satisfies condition (12).

Finally, the predicted disturbance is utilized in the NN as the predictor model of the system. This NN is trained on-line with learning rate that is provided in Lemma 1. Hence, the output of the proposed predictor model converges asymptotically to the actual value of the system output. Although Lemma 1 guarantees convergence of the NN's output to the desired values, but the stability of the closed-loop system cannot be guaranteed. This will be considered in Section 3.2.

3.2. Stability

In this section, stability of the closed-loop system is studied. A useful tool for studying the stability in the presence of disturbances and uncertainties is the Input-to-State Stability (ISS) approach. The definition of the ISS is given in the followings:

Definition. [33]**:** A function $V(\cdot)$ is said to be an ISS-Lyapunov function if there exist κ_∞-functions $\alpha_1(\cdot)$, $\alpha_2(\cdot)$, $\alpha_3(\cdot)$, and a κ-function $\delta(\cdot)$ satisfying

$$\begin{aligned}&\alpha_1(\|\mathbf{x}\|) \le V(\mathbf{x}) \le \alpha_2(\|\mathbf{x}\|)\\ &V(f(\mathbf{x},\mathbf{d})) - V(\mathbf{x}) \le -\alpha_3(\|\mathbf{x}\|)+\delta(\|\mathbf{d}\|).\end{aligned} \tag{16}$$

$\forall\, \mathbf{x}, \mathbf{d} \in \mathrm{R}^n$ and $V(\cdot): \mathrm{R}^n \to \mathrm{R}_+$ where $\mathrm{R}_+ = \{x \in \mathrm{R} | x > 0\}$. A system will be ISS if and only if it admits an ISS-Lyapunov function [34].

To analyze the ISS of the proposed control system, first the nonlinear system in Eq. (1) is rewritten in the state space form as follows:

$$\begin{cases}\mathbf{x}(k+1) = \begin{bmatrix} f\big(x_1(k),\ \ldots, x_{n_y}(k), x_{n_y+1}(k),\ \ldots, x_{n_y+n_u}(k), d(k)\big) \\ x_1(k) \\ \vdots \\ x_{n_y-1}(k) \\ 0 \\ x_{n_u-1}(k) \\ \vdots \\ x_{n_y+n_u-1}(k)\end{bmatrix} + \begin{bmatrix}0\\0\\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0\end{bmatrix}, \\ u(k) := \mathbf{F}(\mathbf{x}(k), u(k), d(k)) y(k) = f\big(x_1(k),\ \ldots, x_{n_y}(k), x_{n_y+1}(k),\ \ldots, x_{n_y+n_u}(k), d(k)\big)\end{cases} \tag{17}$$

where $\mathbf{x}(k) = [y(k-1)\ \ y(k-2)\ \ \cdots\ \ y(k-n_y)\ \ u(k-1)\ \ u(k-2)\ \ \cdots\ \ u(k-n_u)]^T$ is the state vector. Using Eq. (5), the predicted model in Eq. (7) can be written in the state-space form as

$$\begin{cases} \hat{\mathbf{x}}(k+i|k) = \begin{bmatrix} f_{NN,k}\Big(\hat{x}_1(k+i-1|k),\ \ldots, \hat{x}_{\hat{n}_y+\hat{n}_u}(k+i-1|k), \hat{d}(k+i-1|k)\ \ldots, \\ \hat{d}(k+i-\tilde{n}_d|k)\Big) \\ \hat{x}_1(k+i-1|k) \\ \vdots \\ \hat{x}_{\hat{n}_y+\hat{n}_u-1}(k+i-1|k) \end{bmatrix} \\ \quad +\begin{bmatrix}0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0\end{bmatrix}^T u(k+i-1|k) \\ \quad := \mathbf{F}_{NN,k}\Big(\hat{\mathbf{x}}(k+i-1|k), u(k+i-1|k), \hat{d}(k+i-1|k)\Big) \\ \hat{\mathbf{x}}(k|k) = \mathbf{x}(k) \end{cases}, \tag{18}$$

where $i = 1, \ldots, N$.

The optimization problem of the ANNMPC can be presented using the state variables as follows:

$$\begin{aligned} \mathbf{u}^* &= \min_{\boldsymbol{\Delta}\mathbf{u}(k|k)} \sum_{i=0}^{N-1} L(\mathbf{e}_x(k+i|k), u(k+i|k)) + V_F(\mathbf{e}_x(k+N|k)) \\ &= \min_{\Delta u(k|k),\ldots,\Delta u(k+\mathrm{M}-1|k)} \Bigg[\sum_{i=0}^{N-1} \big(\mathbf{r}_x(k+i) - \hat{\mathbf{x}}(k+i\,|k)\big)^T \mathbf{Q}_x \big(\mathbf{r}_x(k+i) - \hat{\mathbf{x}}(k+i\,|k)\big) \\ &\quad + \sum_{j=0}^{M-1} (\Delta u(k+j\,|k))^T R_u\ (\Delta u(k+j\,|k)) + V_F(\mathbf{e}_x(k+N|k))\Bigg] \end{aligned} \tag{19}$$

subject to

$$\begin{aligned} &\hat{\mathbf{x}}(k+1) = \mathbf{F}_{NN,k}\Big(\hat{\mathbf{x}}(k), u(k), \hat{d}(k)\Big) \\ &\qquad = \mathbf{F}_{NN,k}^{Nom}\big(\hat{\mathbf{x}}(k), u(k)\big) + \boldsymbol{\Delta}\mathbf{F}_{NN,k}\Big(\hat{\mathbf{x}}(k), u(k), \hat{d}(k)\Big) \\ &\hat{\mathbf{x}}(k+i\,|k) \in X_c \qquad 0 \le i \le N-1 \\ &u(k+i\,|k) \in \mathrm{U}_c \qquad 0 \le i \le M-1 \\ &\hat{\mathbf{x}}(k+N\,|k) \in X_f, \end{aligned} \tag{20}$$

where $\mathbf{e}_x(k+i) = \mathbf{r}_x(k+i) - \hat{\mathbf{x}}(k+i\,|k)$, $\mathbf{r}_x(k+i)$ is the reference vector of the states of the system, $V_F(\mathbf{e}_x(k+N|k))$ is the terminal cost function, $\mathbf{Q}_x$ is the weighting matrix on the predicted error, X_c is the constraint on the states of the system, and X_f is the terminal state set. The nonlinear function $\mathbf{F}_{NN,k}(.,.,.)$ is separated into two parts: $\mathbf{F}_{NN,k}^{Nom}(.,.)$ that is the nominal part and $\boldsymbol{\Delta}\mathbf{F}_{NN,k}(.,.,.)$, which is the effect of disturbances and uncertainties on the system.

Assumptions

1 Function $L(.,.)$ is Lipschitz and $\alpha_L(|\mathbf{e}_x|) \le L(\mathbf{e}_x, u) \le \beta_L(|\mathbf{e}_x, u|)$, where $\alpha_L(|\mathbf{e}_x|)$ and $\beta_L(|\mathbf{e}_x, u|)$ are κ-functions.
2 $X_f \subseteq X_c$ is a closed set and $0 \in X_f$.
3 Function $V_F(\cdot)$ is Lipschitz in X_f and $\alpha_{V_F}(|\mathbf{e}_x|) \le V_F(\mathbf{e}_x) \le \beta_{V_F}(|\mathbf{e}_x|)$, where $\alpha_{V_F}(|\mathbf{e}_x|)$ and $\beta_{V_F}(|\mathbf{e}_x|)$ are κ-functions.
4 $V_F((\mathbf{r}_x - f(\mathbf{x}, u, d))) - V_F(\mathbf{x}) \le -L(\mathbf{e}_x, u) \qquad \forall \mathbf{x} \in X_f$.

Theorem 1. *Let X^{ANNMPC} be the set of states where the ANNMPC problem is feasible and Assumptions 1–4 are satisfied. The closed-loop system is ISS in X^{ANNMPC} if*

$$\left|\Delta \mathbf{F}_{NN,k}\left(\hat{\mathbf{x}}(k), u^*(k), \hat{d}(k)\right)\right| \leq \frac{1}{L_V}\alpha_d\left(\left|\hat{d}(k)\right|\right), \tag{21}$$

where L_V is the Lipschitz constant of the optimal value of the ANNMPC objective function, and $\alpha_d(\cdot)$ is a κ-function.

Proof. The optimal value of the ANNMPC objective function is considered as the Lyapunov function candidate as

$$V(k) = \sum_{i=0}^{N-1} L\left(\mathbf{e}_x(k+i|k), u^*(k+i|k)\right) + V_F\left(\mathbf{e}_x(k+N|k)\right). \tag{22}$$

It is assumed that the Lyapunov function (22) is Lipschitz with constant L_V. Then, it can be written

$$\begin{aligned} V\left(\mathbf{F}_{NN,k}\left(\hat{\mathbf{x}}(k), u^*(k), \hat{d}(k)\right)\right) - V\left(\mathbf{F}^{Nom}_{NN,k}\left(\hat{\mathbf{x}}(k), u^*(k)\right)\right) \\ \leq L_V\left|\mathbf{F}_{NN,k}\left(\hat{\mathbf{x}}(k), u^*(k), \hat{d}(k)\right) - \mathbf{F}^{Nom}_{NN,k}\left(\hat{\mathbf{x}}(k), u^*(k)\right)\right| \\ = L_V\left|\Delta\mathbf{F}_{NN,k}\left(\hat{\mathbf{x}}(k), u^*(k), \hat{d}(k)\right)\right|. \end{aligned} \tag{23}$$

Next, the Lyapunov function for the nominal system $\mathbf{F}^{Nom}_{NN,k}(\hat{\mathbf{x}}^{Nom}(k), u^*(k))$ can be written as

$$\begin{aligned} V\left(\hat{\mathbf{x}}^{Nom}(k)\right) &= \sum_{i=0}^{N-1} L\left(\mathbf{e}_x^{Nom}(k+i|k), u^*(k+i|k)\right) + V_F\left(\mathbf{e}_x^{Nom}(k+N|k)\right) \\ &= L\left(\mathbf{e}_x^{Nom}(k), u^*(k)\right) + V_F\left(\mathbf{e}_x^{Nom}(k+N|k)\right) \\ &\quad + \sum_{i=1}^{N-1} L\left(\left(\mathbf{r}_x(k+i) - \mathbf{F}^{Nom}_{NN,k}\left(\hat{\mathbf{x}}^{Nom}(k+i-1), u^*(k+i-1)\right)\right), u^*(k+i|k)\right), \end{aligned} \tag{24}$$

where $\mathbf{e}_x^{Nom}(k+i|k) = \mathbf{r}_x(k+i|k) - \hat{\mathbf{x}}^{Nom}(k+i|k)$ and $\hat{\mathbf{x}}^{Nom}(k)$ is the nominal part of the system states.

Substituting the nominal part of the NN model and the nominal states and error, Assumption 4 can be written as

$$\begin{aligned} V_F\left(\mathbf{e}_x^{Nom}(k+N|k)\right) \geq V_F\left(\mathbf{r}_x(k+N+1) - \mathbf{F}^{Nom}_{NN,k}\left(\hat{\mathbf{x}}^{Nom}(k+N), u^*(k+N)\right)\right) \\ + L\left(\mathbf{e}_x^{Nom}(k+N|k), u^*(k+N|k)\right). \end{aligned} \tag{25}$$

Substituting Eq. (25) in Eq. (24) gives

$$\begin{aligned} V\left(\hat{\mathbf{x}}^{Nom}(k)\right) &\geq L\left(\mathbf{e}_x^{Nom}(k), u^*(k)\right) + V_F\left(\mathbf{e}_x^{Nom}(k+N+1|k)\right) + L\left(\mathbf{e}_x^{Nom}(k+N), u^*(k+N)\right) \\ &\quad + \sum_{i=0}^{N-2} L\left(\left(\mathbf{r}_\mathbf{x}(k+i+1) - \mathbf{F}^{Nom}_{NN,k}\left(\hat{\mathbf{x}}^{Nom}(k+i), u^*(k+i)\right)\right), u^*(k+i+1|k)\right) \\ &= L\left(\mathbf{e}_x^{Nom}(k), u^*(k)\right) + V_F\left(\mathbf{e}_x^{Nom}(k+N+1|k)\right) \\ &\quad + \sum_{i=0}^{N-1} L\left(\left(\mathbf{r}_\mathbf{x}(k+i+1) - \mathbf{F}^{Nom}_{NN,k}\left(\hat{\mathbf{x}}^{Nom}(k+i), u^*(k+i)\right)\right), u^*(k+i+1|k)\right) \\ &= L\left(\mathbf{e}_x^{Nom}(k), u^*(k)\right) + V\left(\mathbf{F}^{Nom}_{NN,k}\left(\hat{\mathbf{x}}^{Nom}(k), u^*(k)\right)\right). \end{aligned} \tag{26}$$

Using Assumption 1, Eq. (26) can be written

$$V\left(\hat{\mathbf{x}}^{Nom}(k)\right) \geq \alpha_L\left(\left|\mathbf{e}_x^{Nom}(k)\right|\right) + V\left(\mathbf{F}_{NN,k}^{Nom}\left(\hat{\mathbf{x}}^{Nom}(k), u^*(k)\right)\right). \tag{27}$$

By comparison, the Lyapunov function of the nominal system and the disturbed system in the steady state can be given as

$$V\left(\hat{\mathbf{x}}^{Nom}(k)\right) \leq V\left(\hat{\mathbf{x}}(k)\right). \tag{28}$$

Utilizing Eqs. (27) and (28), Eq. (23) can be written as

$$V\left(\mathbf{F}_{NN,k}\left(\hat{\mathbf{x}}(k), u^*(k), \hat{d}(k)\right)\right) \leq -\alpha_L\left(\left|\mathbf{e}_x^{Nom}(k)\right|\right) + V\left(\hat{\mathbf{x}}(k)\right) + L_V\left|\Delta\mathbf{F}_{NN,k}\left(\hat{\mathbf{x}}(k), u^*(k), \hat{d}(k)\right)\right|. \tag{29}$$

Therefore, the first difference of the Lyapunov function can be written as follows:

$$\Delta V\left(\hat{\mathbf{x}}(k)\right) \leq -\alpha_L\left(\left|\mathbf{e}_x^{Nom}(k)\right|\right) + L_V\left|\Delta\mathbf{F}_{NN,k}\left(\hat{\mathbf{x}}(k), u^*(k), \hat{d}(k)\right)\right|. \tag{30}$$

Based on the definition of ISS, the closed-loop system will be ISS in X^{ANNMPC} if the following condition is satisfied:

$$\left|\Delta\mathbf{F}_{NN,k}\left(\hat{\mathbf{x}}(k), u^*(k), \hat{d}(k)\right)\right| \leq \frac{1}{L_V}\alpha_d\left(\left|\hat{d}(k)\right|\right). \tag{31}$$

Hence, Eq. (30) can be written as

$$\Delta V\left(\hat{\mathbf{x}}(k)\right) \leq -\alpha_L\left(\left|\mathbf{e}_x^{Nom}(k)\right|\right) + \alpha_d\left(\left|\hat{d}(k)\right|\right). \tag{32}$$

This completes the proof. ❑

4. Simulation results

In this section, the ability of the proposed method in the reference tracking and disturbance rejection is evaluated and is compared with the existing NMPC methods. Control of a pH neutralization process and continuously stirred tank reactor (CSTR) as the benchmark processes are considered here.

4.1. pH neutralization process

The pH process has been widely used in different industries such as chemical industries and wastewater treatment. It has severe nonlinear behavior and consists of acid, buffer and base streams. The base flow rate is considered as the control input to regulate the pH value. The buffer and acid streams are considered as disturbances that any changes in their values can deflect the pH from the desired value. The static and dynamic equations of the pH process are as follows [35]:

$$\begin{cases} \begin{cases} \frac{dw_a(t)}{dt} = \frac{1}{V}\left((W_{aa} - w_a(t))F_a(t) + (W_{ba} - w_a(t))F_b(t) + \left(W_{bfa} - w_a(t)\right)F_{bf}(t)\right) \\ \frac{dw_b(t)}{dt} = \frac{1}{V}\left((W_{ab} - w_b(t))F_a(t) + (W_{bb} - w_b(t))F_b(t) + \left(W_{bfb} - w_b(t)\right)F_{bf}(t)\right) \end{cases} \\ H^{+4} + H^{+3}(K_{a1} - w_a(t)) + H^{+2}(K_{a1}(K_{a2} - w_a(t) - w_b(t)) - K_w) \\ \qquad + H^{+}(-K_{a1}(K_w + K_{a2}(w_a(t) - 2w_b(t)))) - K_{a1}K_{a2}K_w = 0 \\ pH = -\log_{10}H^{+}, \end{cases} \tag{33}$$

Table 1
pH process parameters.

Parameter	Value	Parameter	Value
V	$2900\,\mathrm{cm}^2$	W_{bb}	0.00005
W_{aa}	0.003	W_{bfb}	0.03
W_{ba}	-3.5×10^{-3}	K_{a1}	4.47×10^{-7}
W_{bfa}	-0.03	K_{a2}	5.62×10^{-11}
W_{ab}	0	K_w	1×10^{-14}

Table 2
Parameters of ANNMPC controllers.

Method	NN parameters			R_u	Q_y
ANNMPC in feed-forward control structure	Model predictor	$\hat{n}_u=3,\ \hat{n}_y=4$	$n_h=8$	0.08	1
Standard ANNMPC	Model predictor	$\hat{n}_u=3,\ \hat{n}_y=4$	$n_h=8$	0.2	1
Proposed ANNMPC	Inverse model in DOB	$\tilde{n}_u=3,\ \tilde{n}_y=4$	$n_h=8$	1	1.2
	Disturbance predictor	$\tilde{n}_d=3$	$n_h=2$		
	Model predictor	$\hat{n}_u=2,\ \hat{n}_y=3,\ \hat{n}_d=2$	$n_h=4$		

where $F_a(t)$, $F_b(t)$, and $F_{bf}(t)$ are acid, base, and buffer streams, respectively, and $w_a(t)$ and $w_b(t)$ are the reaction invariants. The nominal values of the parameters are given in Table 1.

The pH process is highly nonlinear near pH = 8 and its control around this point is more difficult. The reference tracking responses of the system by applying the proposed controller is depicted in Fig. 3. The sampling time is 5 s. The prediction horizon (N) is equal to 5 and the control horizon (M) is equal to 2. The reference tracking response of the proposed ANNMPC is compared with the standard ANNMPC (i.e., no disturbance is included in the NN predictor model) and with the standard ANNMPC in feed-forward control structure [32], which is the adaptive and nonlinear version of the controller proposed in [30]. All the NNs are of the MLP type with one hidden layer with tangent hyperbolic activation functions and a linear output layer. The parameters of the NNs and the controller are given in Table 2. These parameters are selected for the best response for each controller.

In Table 2, n_h shows the number of neurons in the hidden layer of NNs. The constraints on the base flow rate (as the input to the system) are set to $u_{\min}=0$ and $u_{\max}=30$. As shown in Fig. 3, the performance of the proposed controller is more appropriate than the other controllers.

One of the advantages of the proposed method is its ability to reject disturbances. As mentioned in the previous sections, the drawback of the feed-forward control method is that the constraint of the system signals especially the control input may not be satisfied and hence, the optimality of the control signal is lost. On the other hand, the proposed controller overcomes this problem. Fig. 4 demonstrates the disturbance rejection behavior of the proposed controller and the ANNMPC with DOB in the feed-forward structure. First, the value of the buffer flow rate $F_{bf}(t)$ is changed from 0.55 to 1.25 at sample 500 and back to 0.55 at sample 800 (Fig. 4(a)). Next, a step disturbance is added to the input with amplitude –1 at sample 800 (Fig. 4(b)). All controllers have appropriate responses. However, in rejection of parameter variations, the proposed controller has the fastest transient response. Nevertheless, for the step disturbance rejection, the performance of the standard ANNMPC is better.

As indicated in Table 2, the two adaptive NN models have simple structure with a small number of parameters to train. Hence, training of these NN models is simple and do not

Table 3
Nominal operating conditions for the CSTR.

Variable	Value	Variable	Value
F	100 L/min	E/R	8750 K
$(-\Delta H)$	5×10^4 J/mol	k_0	$7.2\times10^{10}\,\mathrm{min}^{-1}$
T_F	350 K	UA	5×10^4 J/min K
V	100 L	T_c	300 K
ρ	1000 g/L	C_A	0.5 mol/L
C_p	0.239 J/gK	T	350 K

incur considerable computational time. Moreover, a standard optimization problem is solved in the NMPC problem. Therefore, the computational complexity of the proposed strategy is such that it can be easily applied to practical systems with very good efficiency.

Another disturbance that can be considered in the pH process is the ramp disturbance. This phenomenon can happen when the base flow sediments in the inlet streams with time. As Fig. 5 shows, the shortcoming of the ANNMPC in feed-forward structure is considerable in ramp rejection. For relatively large slopes and long-lasting ramp disturbances, the control signal of this controller may exceed the constraints because the output of ANNMPC is subtracted from the estimated disturbance. Hence, this controller may not reject strong ramp disturbances. However, the proposed method considers these limitations in the optimization problem and an optimal manipulated signal is produced within the system constraints.

4.2. Continuously stirred tank reactor

The CSTR process is a highly nonlinear system that is based on an energy balance and a component balance for a reactant. The dynamic model of CSTR can be represented by the following nonlinear differential equations [36]:

$$\begin{aligned}\dot{C}_\mathrm{A} &= \tfrac{F(C_\mathrm{AF}-C_\mathrm{A})}{V} - C_\mathrm{A}k_0\exp\left(-\tfrac{E}{RT}\right),\\ \dot{T} &= \tfrac{F(T_\mathrm{f}-T)}{V} + \tfrac{(-\Delta H)C_\mathrm{A}}{\rho C_p}k_0\exp\left(-\tfrac{E}{RT}\right) + \tfrac{UA(T_c-T)}{V\rho C_p},\end{aligned} \tag{34}$$

where C_A is the concentration of reactant A in the reactor, T (the reactor temperature) is output of the process, and T_c (the temperature of the coolant stream) is the input of the process. Table 3 contains the nominal operating conditions of the CSTR plant corresponding to an unstable steady state. The sampling time is 0.05 min. The prediction horizon (N) is equal to 10 and the control horizon (M) is equal to 2. The weighting factors in the objective function (3) are $Q_y = 0.3$ and $R_u = 0.5$. The parameters of the NNs are as the same as in Table 2. The input constraints are set to $u_{\min} = 280$ and $u_{\max} = 350$. The response of the CSTR plant using the proposed control strategy is shown in Fig. 6.

The bound of the NN learning rate in Eq. (12) is shown in Fig. 7 for the predictor model with $\mu = 1$. In order to show validity of the upper bound in Eq. (12), different values of η are considered for each reference signal in Fig. 6. In particular, $\eta_1 = 0.7$, $\eta_2 = 0.6$, and $\eta_3 = 0.5$ are considered as the learning rates that are smaller than the upper bound of Eq. (12) for the first, the second, and the third reference signals in Fig. 6, respectively. Moreover, $\eta_1 = 1.4$, $\eta_2 = 1.3$, and $\eta_3 = 1.2$ are considered as the learning rates that are larger than the upper bound of Eq. (12) for the first, the second, and the third reference signals, respectively. As Fig. 6 shows, exceeding the upper bound of Eq. (12) makes the response of the closed-loop system oscillatory or unstable, which are in agreement with Lemma 1.

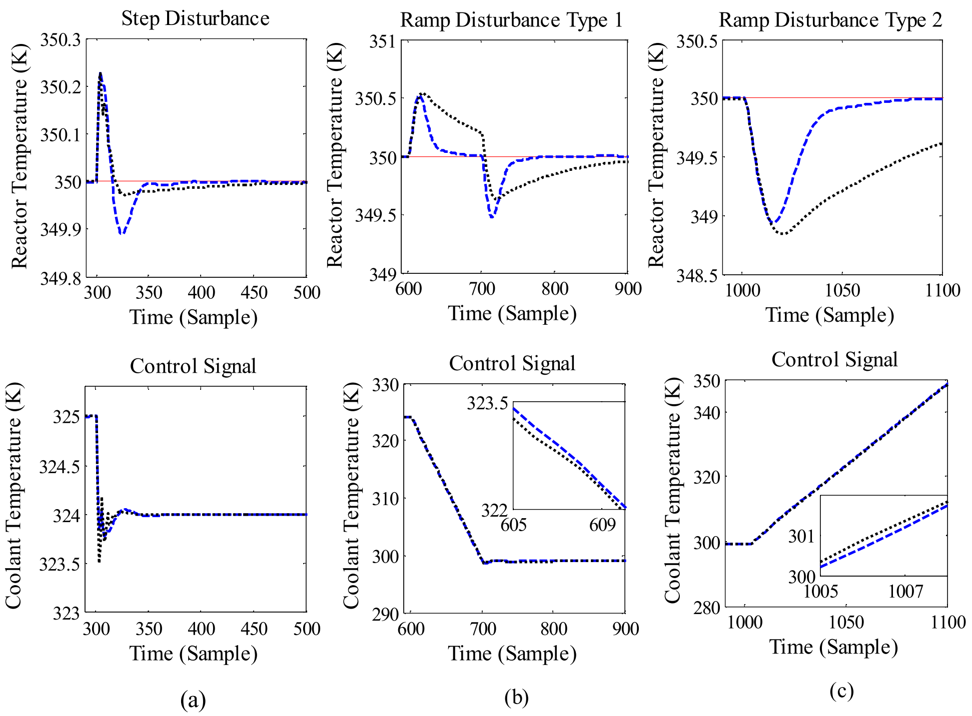

Fig. 8. Response of the closed-loop system to (a) step disturbance, (b) ramp disturbance type 1, and (c) ramp disturbance type 2; solid line: reference signal, dashed line: proposed ANNMPC, and dotted line: ANNMPC in feed-forward structure.

The response of the CSTR process using the proposed ANNMPC controller in disturbance rejection is depicted in Fig. 8. Around the unstable operating point of the CSTR process (i.e., $T = 350\,\text{K}$), a step disturbance with amplitude 1 at sample 300 and a ramp disturbance with slope 5 at sample 600 are added to the input. The ramp disturbance is removed at sample 700. This disturbance is called a ramp disturbance of type 1. Another ramp disturbance with slope 10 (called as ramp disturbance of type 2) is added to the input at sample 1000.

The performance of the proposed controller is compared with the ANNMPC in the feed-forward structure. The weighting factors in the objective function (3) are $Q_y = 0.3$ and $R_u = 0.5$ for both controllers. The parameters of the NNs are as the same as in Table 2.

Both controllers can reject the external input disturbances but in the ramp disturbance, the ANNMPC controller in the feed-forward structure cannot satisfy the constraint of the process input because the feed-forward signal is subtracted from the ANNMPC control signal in this structure. On the other hand, the control signals of the proposed controller are generated via an optimization problem and the constraints, especially on the control signal, are satisfied. To show the optimality of the proposed controller, the index $J(k) = (r(k) - y(k))^2 + \Delta u(k)^2$ is depicted in Fig. 9. As this figure shows, the proposed controller has lower index values than the ANNMPC with feed-forward structure.

Finally, the stability condition in Eq. (21) for the external input disturbances is shown in Figs. 8 and 9. Function $\alpha_d(|\hat{d}(k)|)$ is considered as $\|\hat{d}(k)\|^2$ and the Lipschitz constant of the Lyapunov function in Eq. (22) is computed based on the maximum derivative of Eq. (22) with respect to $\mathbf{e}_x(k)$.

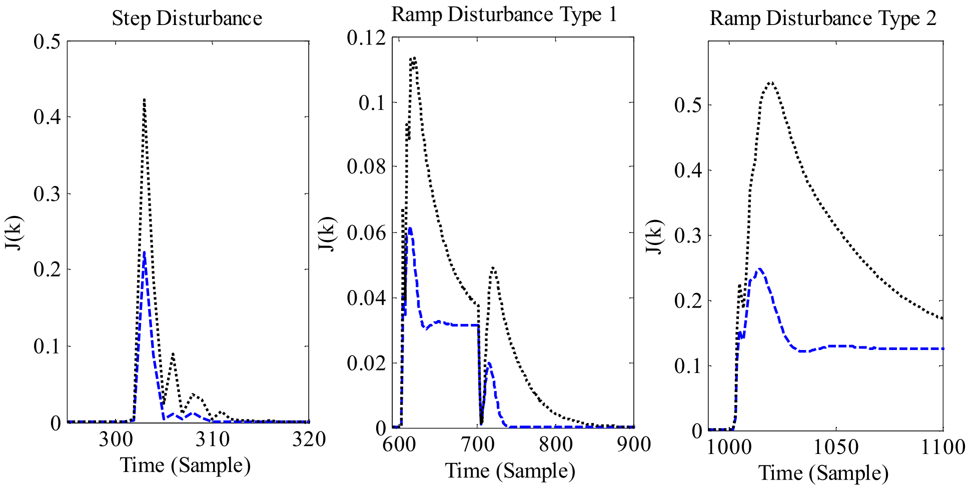

Fig. 9. Optimality index, dashed line: proposed ANNMPC and dotted line: ANNMPC in feed-forward structure.

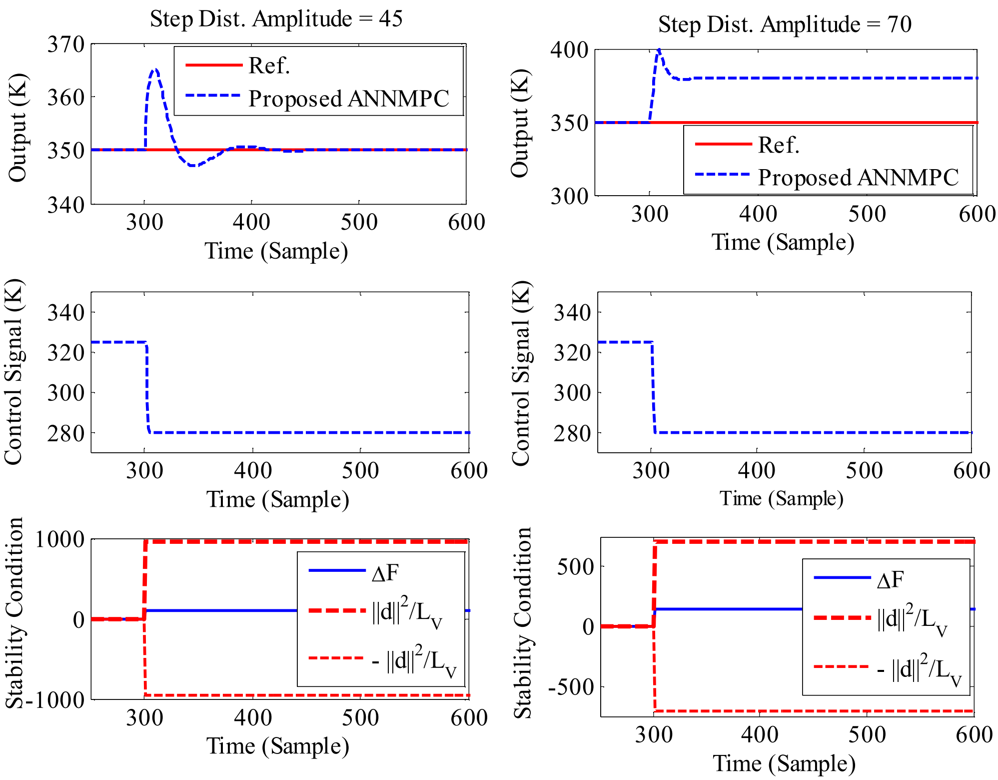

Fig. 10. Stability condition of the closed-loop system in presence of step disturbances.

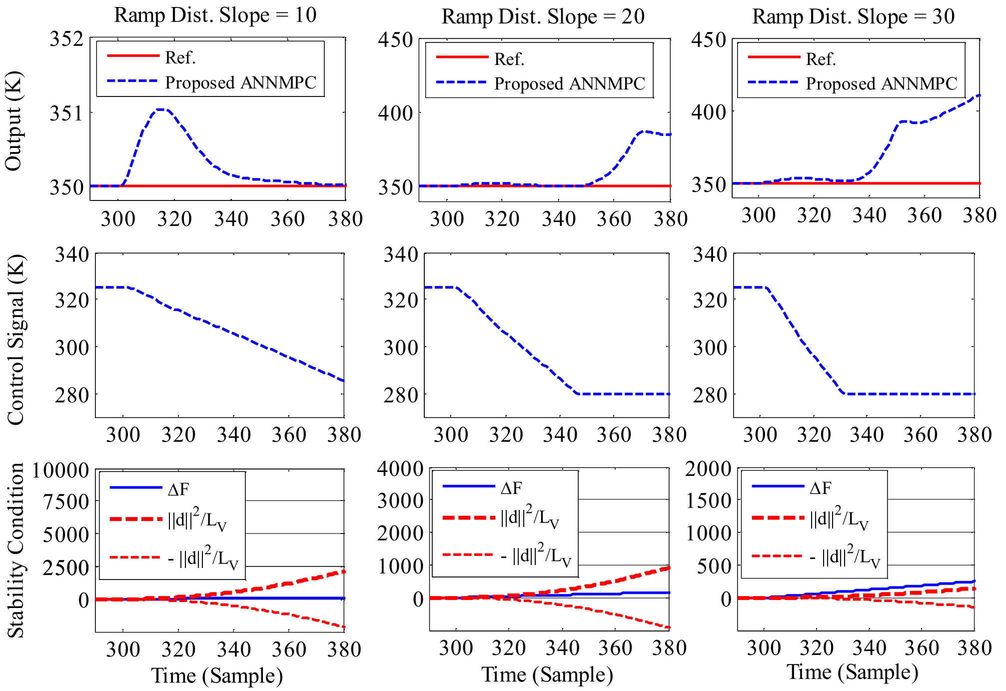

Fig. 11. Stability condition of the closed-loop system in presence of ramp disturbance.

As shown in Fig. 10, for the step disturbances with amplitude less than 45, the proposed controller can reject the disturbances and satisfy the constraint on the control signal as well as the stability condition. However, for the step disturbances with amplitude more than 45, the output response of the process has offset because of the control signal constraints. Nevertheless, the stability condition is satisfied. Simulation results confirm that no step disturbance on the input can make the output response to become unstable inside the acceptable region of the system output (i.e., $y_{\min} = 300$ and $y_{\max} = 400$).

The stability condition in the presence of ramp disturbances is demonstrated in Fig. 11. As this figure shows, for the ramp disturbances with slope less than 30, the proposed controller can reject the disturbance and satisfy the stability condition. However, for ramp disturbances with slope greater than 30, the constraints on the control signal does not allow it to become less than 280 K to reject the disturbance. As a result, since the disturbance still exists, the output becomes unstable and the stability condition cannot be satisfied.

5. Conclusions

In this paper, a nonlinear model predictive control method based on the adaptive neural network predictor was proposed and the reference tracking and disturbance rejection behavior of the closed-loop system was studied. Both the internal and external disturbances were considered. First, disturbance was estimated by the nonlinear DOB and then it was used as one of the inputs of the NN predictor model besides the input and output of the system. Moreover, for future disturbances that are used in the main NN predictor model, another

NN model was used to predict the disturbance over the prediction horizon. This structure imposes the disturbance to the optimization problem of the ANNMPC and the effect of the disturbance on the closed-loop system response was considered. Hence, the drawbacks of the feed-forward control, which is a common method in disturbance rejection, were solved. The output of the ANNMPC with proposed NN structure became optimal and the constraints on the system parameters especially on the control signal were satisfied. Simulating examples for the reference tracking and disturbance rejection (internal and external) were performed on the pH and CSTR processes and effectiveness of the new ANNMPC method was demonstrated. The main advantages of the new ANNMPC method over the ANNMPC with feed-forward control structure were the optimality of the control signal, satisfying the system constraints, and improving the prediction precision of the future disturbances using adaptive NNs. The future works include extending the proposed method to the non-minimum phase systems and obtaining offset-free response of the closed-loop system.

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