

Model Reference Neural-Fuzzy Adaptive Control of the Concentration in a Chemical Reactor (CSTR)

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Abstract: In this work, we consider the application of a special type of neural networks based on radial basis functions (RBF) which belongs to a class of associative memory neural networks. The RBF neural network is used in a novel approach as a controller in a model reference adaptive control (MRAC) architecture. The global control scheme is based on a one step ahead fuzzy model and the objective is to control a Continuous Stared Tank Reactor (CSTR) complex nonlinear system.

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1. INTRODUCTION

To prevent the environment from pollution and preserve its stability, it will be better to look for new methods of control and modeling especially in chemical engineering domains. This can improve the leaving conditions and assure good health for the human being. The design of a controller which can change or modify the behavior and the response of an unknown system can be a delicate problem in the most control applications.

The control design task, is to choose the input such that the output response satisfies a given requested performances. Because, the process in most cases is complex or unknown, then the appropriate choice of the control input is not easy in general. This fact is due to the nonlinearity and complexity of these systems. One solution among many solutions is to model or identify these systems and then pass to the control design task. Fuzzy logic theory and artificial neural networks can be used for the identification and the control of this class of systems (Radu-Emil Precup *et al.*, 2011, Geng. F., 2006).

A special case of the classical neural networks is the so called associative memory networks (AMNs) (K.-L. Du *et al.*, 2014) which have as members the Cerebellar Model Articulation Controller (CMAC), B-spline and fuzzy networks, Kanerva's Sparse Distributed Memory Model (KSDMM) and Radial Basis Functions (RBF). These networks store information locally, and it is this feature which distinguishes them from other neural networks, such as Multi layered Perceptron (MLPs). The objective of the present work is to shown the control effectiveness of the RBF neural networks. This work is organized as follows: The second part gives a brief description of the RBF networks. The third part presents a fuzzy model reference adaptive control (MRAC) approach based on an RBF controller with a fuzzy model. This approach is tested in simulation on the control of the

concentration in a Continuous Stared Tank Reactor (CSTR). The last part concludes the paper.

2. RADIAL BASIS FUNCTION NETWORKS

2.1 Radial Basis function network : state of the art

RBF networks have been widely used for data modelling and control schemes from direct inverse modelling to nonlinear internal model control and predictive control. (Hunt and Sbarbaro-Hofer., 1991, Sbarbaro-Hofer *et al.*, 1993). They have also been used to compensate for the plant's nonlinearities (Sanner *et al.*, 1992, Tzirkel *et al.*, 1991) where a sliding mode controller is used to stabilise the system when the input lies outside the network's domain.

2.2 RBF Network Architecture

The RBF network can be considered as a two-layer network with only one hidden layer. The output depends linearly on the weights (Bahita *et al.*, 2012, 2014). More explicitly, the output of an RBF neural network system can be put in the following form

$$u = \underline{\theta}^T \cdot \underline{\xi}(x) = \sum_{i=1}^{nr} \xi_i \theta_i, \text{ with } \xi_i = \psi(\|x - c_i\|_2) \quad (1)$$

x is the input vector, ψ is a non linear function called radial basis function, $\underline{\theta}$ are connections weights to be adapted (parameters) between the hidden layer and the output layer, c_i are centres of basis functions and nr is the number of

basis functions. The most used basis function is the Gaussian function as will be described in section 4.

3. MODEL REFERENCE ADAPTIVE CONTROL BASED ON THE RBF CONTROLLER AND THE FUZZY MODEL

An adaptive controller is a controller with adjustable parameters and a mechanism for adjusting the parameters. The MRAC has been originally proposed to solve problems in which the desired performance is expressed in terms of a reference model, which gives the desired response to a command signal (Aström *et al.*, 1995). The general structure of the MRAC is shown in Fig.1

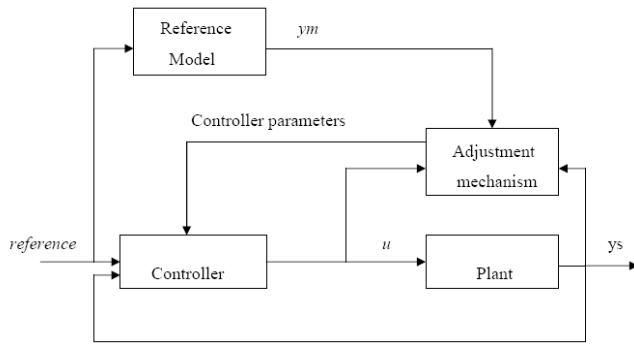


Fig. 1. General structure of the MRAC

The objective is to make the controlled output $y_s(t)$ follows the output of the reference model $y_m(t)$. This condition is satisfied by changing the controller parameters on the basis of feedback from the error $e(t) = y_m(t) - y_s(t)$ which is the difference between the output of the system (plant) and the output of the reference model. This operation is the role of the adaptation mechanism which can be obtained in two ways: by using a gradient method or by applying stability theory. The gradient method realizes this mechanism by varying the controller parameters θ in the direction of the negative gradient of a cost function $J(\theta) = \frac{1}{2} \cdot e^2$ (function of

the error) such that this cost function be minimized. This principle is known as the MIT rule which is the original approach to MRAC and the name is derived from the fact that it was developed at the Instrumentation Laboratory at Massachusetts Institute of Technology (MIT), that is,

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma \cdot e \cdot \frac{\partial e}{\partial \theta} \quad (2)$$

With γ is the adaptation gain. In this work, the RBF network is used as a controller, then the RBF input vector is $x = [y_s(t), ref(t)]$ and the RBF output is $u(t)$, with $y_s(t)$ is the system output, $ref(t)$ is the system desired output and $u(t)$ is the control input. Concerning the adaptation of the centers of the basis functions, the *k-means* algorithm (Darken *et al.*, 1990) is used and we can also refer to section: Training

and centers placement in an RBF network in (Bahita *et al.*, 2012). The adaptation of the RBF controller parameters θ_i is done using the MIT rule given in (2). As mentioned previously, one possibility is to adjust these parameters in such a way that the cost function $J(\theta) = \frac{1}{2} \cdot e^2$ is minimized. To make J small, it is reasonable to change the parameters in the negative gradient of J , that is,

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma \cdot e \cdot \frac{\partial e}{\partial \theta} \quad (3)$$

Using the chain rule in (3), we obtain:

$$\frac{d\theta_i}{dt} = -\gamma \frac{\partial J}{\partial \theta_i} = -\gamma \cdot e \cdot \frac{\partial e}{\partial \theta_i} = -\gamma \cdot e \cdot \frac{\partial e}{\partial u} \cdot \frac{\partial u}{\partial \theta_i} \quad (4)$$

The term $\frac{\partial u}{\partial \theta_i}$ is the derivative of the RBF controller output with respect of its weight number i . From (1), we have $u = \theta^T \cdot \xi(x) = \sum_{i=1}^{nr} \xi_i \theta_i$, with ξ_i is the basis function output number i , then $\frac{\partial u}{\partial \theta_i} = \xi_i$. It remains now to develop the partial derivative $\frac{\partial e}{\partial u}$, which is called the sensitivity derivative of the system. Because the error is $e = y_m - y_s$, and knowing that the reference model y_m is fixed by the designer and does not depend on u , then

$$\frac{\partial e}{\partial u} = \frac{\partial (y_m - y_s)}{\partial u} = -\frac{\partial y_s}{\partial u} \quad (5)$$

The problem here is how to compute the term $\frac{\partial y_s}{\partial u}$. As a solution, we replace the system by a model, i.e., in (5), the output y_s is replaced by a model output y_{mod} , then (5) became:

$$\frac{\partial e}{\partial u} = \frac{\partial (y_m - y_{mod})}{\partial u} = -\frac{\partial y_{mod}}{\partial u} \quad (6)$$

The term $\frac{\partial y_{mod}}{\partial u}$ can be computed using a neural network as a model of the system and make a back propagation via this model. But in order to avoid this and benefit from the computing time, we use instead a fuzzy or a neural-fuzzy model which give in their outputs a linear model (linear combination). In this work, we have used a fuzzy model of Takagi-Sugeno-type (Takagi *et al.*, 1985) with two inputs: the actual control signal $u(t)$ and the actual output of the real system $y_s(t)$. The output of the fuzzy model is a one step ahead prediction $y_{mod}(t+1)$ of the real system. Using this fuzzy identification, the output of the fuzzy model (Bahita *et al.*, 2015) is as follows:

$$y_{mod}(t+1) = p \cdot y_s(t) + q \cdot u(t) + r \quad (7)$$

From (6), it came immediately

$$\frac{\partial y \text{ mod}(t+1)}{\partial u(t)} = q \quad (8)$$

Using (4) and (8), the MIT rule can be written as:

$$\frac{d\theta_i}{dt} = -\gamma \cdot e \cdot \frac{\partial e}{\partial \theta_i} = \gamma \cdot e \cdot \frac{\partial y \text{ mod}}{\partial u} \cdot \frac{\partial u}{\partial \theta_i} = \gamma \cdot e \cdot q \cdot \frac{\partial u}{\partial \theta_i} \quad (9)$$

γ is the adaptation gain (set by an appropriate choice). The closed loop containing the RBF controller, the system model, the reference model and the adjustment mechanism is given by Fig. 2.

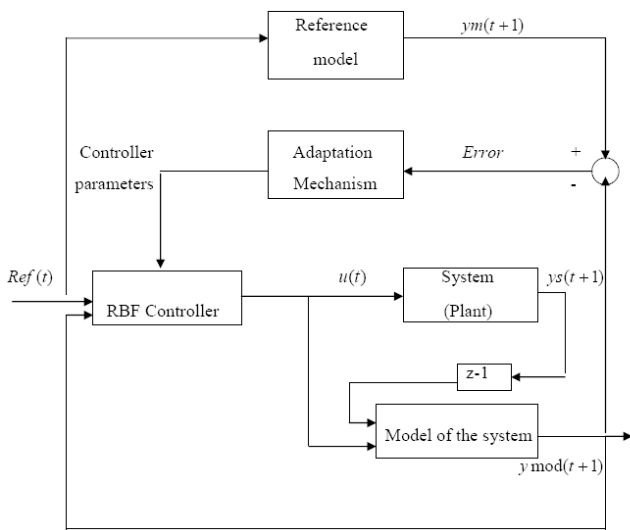


Fig. 2. Global structure of the MRAC- RBF controller with the fuzzy model

Remark1:

The key element in this work is the use of an off-line fuzzy model in order to can on-line compute the term in (5) and then we can update the RBF controller parameters. The resulting control input is first on-line tested on the fuzzy model and when the parameters are adjusted, the obtained new control input is then applied on the real plant in the future step sampling.

4. SIMULATION RESULTS

In this example, we consider the concentration control in a chemical reactor in order to test the effectiveness of the method MRAC-RBF fuzzy based model on a complex nonlinear system (CSTR). The continuously stirred tank reactor (CSTR) system (Ge *et al.*, 1998) consists of a constant volume reactor and it is cooled by circulation of water steam in the jacket. An irreversible, exothermic reaction, $A \rightarrow B$, occurs in the tank. The product concentration is represented by $C_a(t)$. This variable is

controlled by manipulating the coolant flow rate $q_c(t)$. The temperature of the mixture is represented by $T(t)$. The process is described by the following differential equations:

$$\dot{C}_a(t) = \frac{q}{V} \cdot (C_{a0} - C_a(t)) - k_0 \cdot C_a(t) \cdot \exp(-E/(R \cdot T(t))) \quad (10)$$

$$\begin{aligned} \dot{T}(t) = & \frac{q}{V} \cdot (T_0 - T(t)) - k_1 \cdot C_a(t) \cdot \exp(-E/(R \cdot T(t))) \\ & + k_2 \cdot q_c(t) \cdot (1 - \exp(-k_3/q_c(t))) \cdot (T_{c0} - T(t)) \end{aligned} \quad (11)$$

The coolant flow rate $q_c(t)$ is the control input $u(t)$ and the numerical variables of these parameters are given in Table 1. q is the products input flow, C_{a0} and T_0 represent respectively the concentration and the temperature of the process at the beginning; T_{c0} is the coolant temperature, all these values are supposed constants and nominal. The following variables k_0 , E/R , V , k_1 , k_2 , and k_3 are thermodynamical and chemical constants.

Table 1. Reactor parameters

Parameters	Description	Nominal value
q	Process flowrate	100 L/min
V	Reactor Volume	100 L
k_0	Reaction speed	$7.2 \times 10^{10} \text{ min}^{-1}$
E/R	Activation energy	$1 \times 10^4 \text{ K}$
T_0	Feed temperature of the product	350 K
T_{c0}	Coolant temperature	350 K
Δh	Heat of reaction	$-2 \times 10^5 \text{ cal/mol}$
C_p, C_{pc}	Heat capacities	1 cal/g/K
P, P_c	Liquid densities	$1 \times 10^3 \text{ g/l}$
ha	Heat transfert coefficient	$7 \times 10^5 \text{ cal/min/K}$
C_{a0}	Concentration of product at the beginning	1 mol/L

$$k_1 = (\Delta h \cdot k_0) / (p \cdot C_p) \quad (12)$$

$$k_2 = (p_c \cdot C_{pc}) / (p \cdot C_p \cdot V) \quad (13)$$

$$k_3 = ha / (p_c \cdot C_{pc}) \quad (14)$$

The nominal conditions for the following product concentration $C_a = 0.1 \text{ mol/L}$ are:

$$T=438.54 \text{ K}, q_c=103.41 \text{ L/min}. \quad (15)$$

For the control of the product concentration $C_a(t)$, an RBF network is used as a controller with the following components:

- 2 neurons in the input (first) layer.
- 5 neurons in the hidden layer (5 basis functions).
- 1 neuron in the output layer.

The basis function are Gaussian, $\xi_i(x) = \exp(-\frac{(c_i - x)^2}{2\sigma^2})$ with width $\sigma = 0.04$, and initialized centers c_i ($i = 1 : 5$) to:

$$c_1 = [0.06 ; 0.06]$$

$$c_2 = [0.08 ; 0.08]$$

$$c_3 = [0.10 ; 0.10]$$

$$c_4 = [0.12 ; 0.12]$$

$$c_5 = [0.14 ; 0.14]$$

These centers are adapted using the *k-means* algorithm with an adaptation gain $\alpha(t)$ initialized at $\alpha(1) = 0.9$, and for more detail concerning the centers adaptation, please refer to (Bahita et al., 2012).

The input vector of the RBF controller is $x = [y \bmod(t), Ref(t)]$. The RBF parameters θ_i are initialized with zero values (or random values near zero). They are adapted with an adaptation gain $\gamma = 7$ using the MIT rule given in (9). This operation is repeated with every presentation of the data couple $[x, ym(t+1)]$, with always

$x = [y \bmod(t), ref(t)]$, and $ym(t+1)$ is the model reference output which contains the desired performance of the controlled system. The reference $Ref(t)$ is changed every 200 samples (the sampling period is 6 seconds). The reference model is chosen as:

$$ym(t+1) = ym(t) + 0.02.(Ref(t) - ym(t))/0.2 \quad (16)$$

The model of the system is a Tagagi-Sugeno fuzzy model with nine (9) IF-THEN fuzzy rules. The consequent part of the fuzzy model gives a linear combination under the following form: $y \bmod(t+1) = p.y_s(t) + q.u(t)$, where the term r in (7) is not considered here, i.e., $r = 0$. Then we have 18 parameters to adjust by off-line training of this model.

The MRAC-RBF with the obtained fuzzy model is applied on simulation to the CSTR. As shown by the following Figs, the RBF controller was able to control the concentration $C_a(t)$ and making it following the reference model. Fig. 3 shows the evolution of the system output (concentration $C_a(t)$) with the reference model signal. The control input provided by the RBF controller is also shown in Fig. 4.

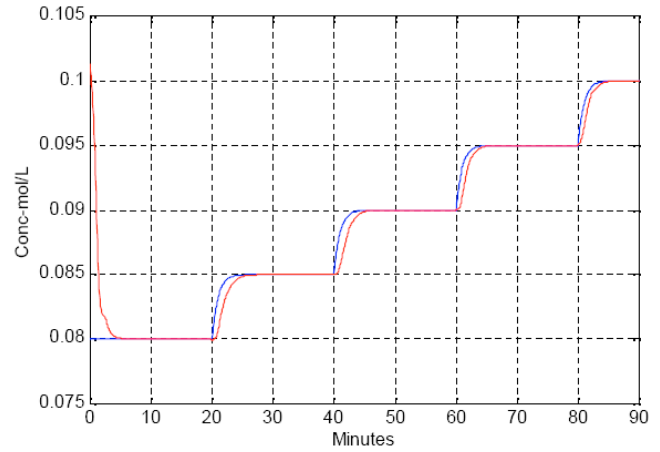


Fig. 3. Evolution of the system output and reference model

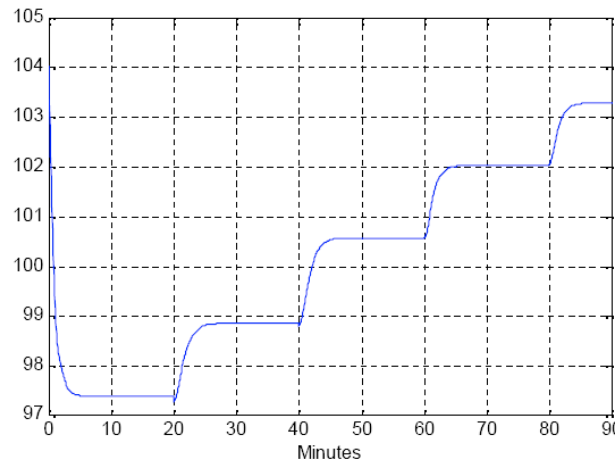


Fig. 4. Control signal : Coolant flow rate L/min

In order to test the robustness of the MRAC-RBF system, we introduce a perturbation with amplitude of 0,005 mol/L in the middle of every interval of reference variation as shown in Fig. 5.

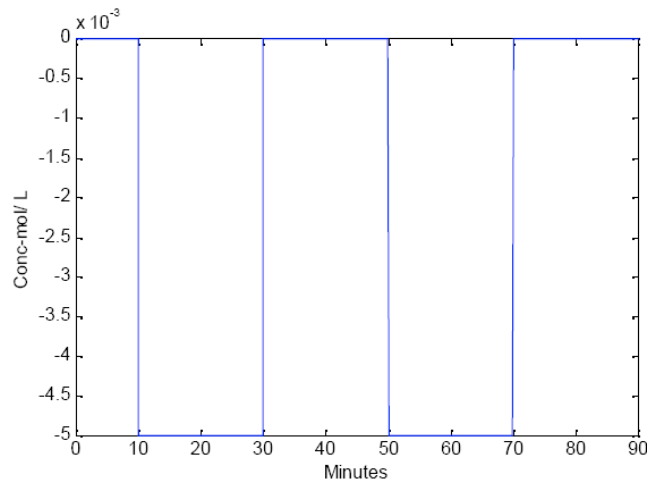


Fig. 5. Perturbation

As shown by Fig. 6, in spite of the perturbation effect, the controller MRAC-RBF was able to manipulate the output $C_a(t)$ as desired in the reference model and eliminates rapidly the introduced perturbation in the middle of every interval.

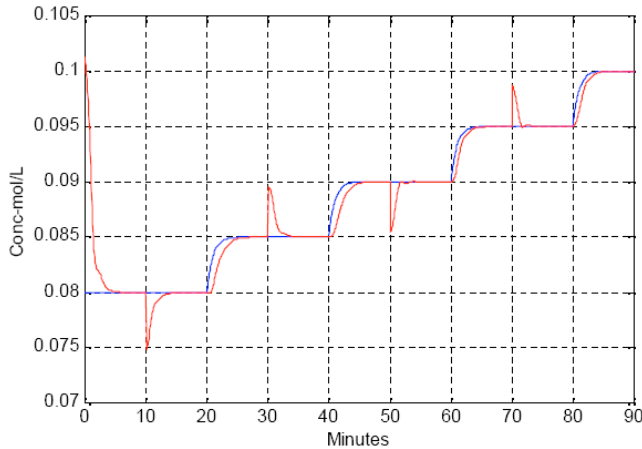


Fig. 6. Evolution of the perturbed system output and reference model

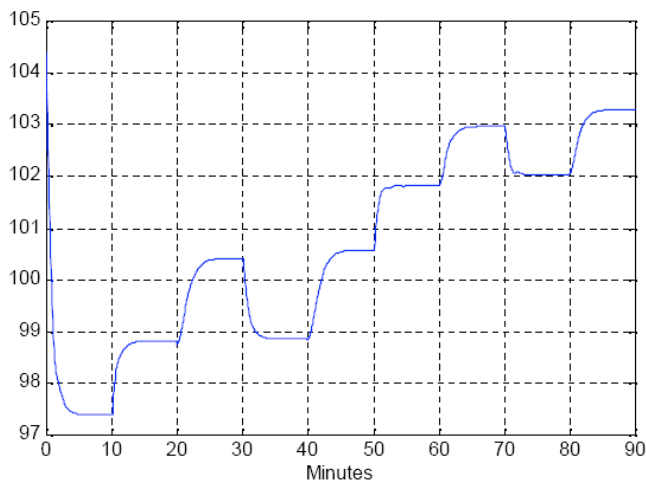


Fig. 7 Control signal in case of system perturbation: Coolant flow rate L/min

Remark2:

From the Figs above, we can confirm that simulation results have showed good performance, expressing good control which leads to good stability and as a consequence this leads to good preservation of the environment especially when using chemical reactors or other CSTR systems.

5. CONCLUSION

The Radial basis function (RBF) neural network was used on-line as a controller in a model reference adaptive control (MRAC) scheme. The controller parameters are changed via the MIT rule and based on the error which is the difference between the output of the system (plant) and the output of the

reference model. The adaptation of the centers of the basis functions are adjusted using the *k-means* algorithm. A one step ahead fuzzy identification with a linear model of the plant is also used in order to compute an unknown term in the MIT rule. This approach MRAC-RBF fuzzy based model is tested on controlling the concentration in a chemical reactor which makes part of a CSTR series. Simulation results applied on this chemical reactor have shown encouraging efforts leading to good stability and environment preservation.

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