Information Sciences 397–398 (2017) 206–218

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Multiattribute decision making based on interval-valued intuitionistic fuzzy values and particle swarm optimization techniques

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ARTICLE INFO

Article history:
Received 17 December 2016
Revised 15 February 2017
Accepted 16 February 2017
Available online 20 February 2017

Keywords:
Accuracy function
IIFWGA operator
Interval-valued intuitionistic fuzzy sets
Interval-valued intuitionistic fuzzy values
Multiattribute decision making
Particle swarm optimization

ABSTRACT

In this paper, we propose a new multiattribute decision making (MADM) method based on the interval-valued intuitionistic fuzzy weighted geometric average (IIFWGA) operator, the accuracy function of interval-valued intuitionistic fuzzy values (IVIFVs) and particle swarm optimization (PSO) techniques, where the weights of attributes and the evaluating values of alternatives with respect to attributes are represented by IVIFVs. First, the proposed method uses an accuracy function to transform the decision matrix given by the decision maker and represented by IVIFVs into a transformed decision matrix represented by real values in $[-1, 1]$. Then, it produces the optimal weights of the attributes based on the obtained transformed decision matrix and PSO techniques. It determines the weighted evaluating IVIFV of each alternative based on the IIFWGA operator, the obtained optimal weights of the attributes and the decision matrix given by the decision maker represented by IVIFVs. Finally, it calculates the transformed value of the weighted evaluating IVIFV of each alternative based on the accuracy function to get the preference order of the alternatives. The main contribution of this paper is that we propose a new MADM method based on the IIFWGA operator of IVIFVs, the accuracy function of IVIFVs and PSO techniques, which can overcome the drawbacks of the existing MADM methods for MADM in interval-valued intuitionistic fuzzy (IVIF) environments.

1. Introduction

In [2], Atanassov and Gargov proposed the theory of interval-valued intuitionistic fuzzy sets (IVIFSs), where the degree of an element belonging to an IVIFS is represented by an interval-valued intuitionistic fuzzy value (IVIFV). IVIFSs are the extension of intuitionistic fuzzy sets (IFSs) [1], where the degree of an element belonging to an IFS is represented by an intuitionistic fuzzy value (IFV) [31]. In recent years, some multiattribute decision making (MADM) methods [5,6,13,16,21,25–27,33–36,39] based on IVIFSs have been presented and some multiattribute group decision making (MAGDM) methods [4,23,24,28,37,38] based on IVIFSs have been presented. In MADM problems, a decision maker evaluates alternatives based on a set of attributes, where the alternatives are ranked and the best alternative(s) are selected. A decision maker can use IVIFSs or IVIFVs to evaluate the attribute values of the alternatives for increasing the flexibility. In [5], Chen and Chiou presented a MADM method based on IVIFSs, particle swarm optimization (PSO) [18] techniques and the evidential reasoning

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http://dx.doi.org/10.1016/j.ins.2017.02.046
0020-0255/

methodology [32]. In [13], Li presented a TOPSIS-based nonlinear-programming methodology for MADM with IVIFSs based on the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method [11]. In [16], Nayagam et al. presented an accuracy function of IVIFSs for MCDM. In [21], Tsao and Chen presented a projection-based compromising method for multiple criteria decision analysis with IVIF information. In [25], Wang et al. presented a method for evaluating the risk of failure modes with a hybrid MCDM model under IVIF environments. In [26], Wei et al. presented an entropy measure of IVIFVs for pattern recognition, MCDM and medical diagnosis. In [27], Xu presented some aggregation operators of IVIFVs for MADM. In [33], Ye presented a MCDM method based on an accuracy function, the IIFWGA operator and the IIFWAA operator of IVIFVs. In [34], Ye presented a MCDM method using entropy weights-based correlation coefficients of IVIFSs. In [35], Zhang et al. presented some information measures for IVIFSs to deal with pattern recognition problems and MADM problems. In [36], Zhang and Yu presented a MADM model based on cross-entropies of IVIFSs and the extended TOPSIS method. In [39], Zhitao and Yingjun presented a MADM method using an accuracy function of IVIFVs.

However, the methods presented in [13] and [39] have the drawbacks that they have “the division by zero” problem which is mentioned in [5], such that they cannot get the preference order of alternatives in some particular situations. Moreover, the method presented in [5] has the drawback that it obtains incorrect combined IVIFVs in some particular situations, such that it obtains incorrect transformed values of the combined IVIFVs and obtains unreasonable preference orders of alternatives. Therefore, it is necessary to propose a new MADM method to overcome the drawbacks of the methods presented in [5,13,39]. The motivation of this paper is to develop a new method to overcome the drawbacks of the methods presented in [5,13,39] for MADM in IVIF environments.

In this paper, we propose a new MADM method based on the interval-valued intuitionistic fuzzy weighted geometric average (IIFWGA) operator [27] of IVIFVs, the accuracy function [33] of IVIFVs and PSO techniques [7], where the evaluating values of alternatives with respect to attributes and the weights of attributes are represented by IVIFVs. First, the proposed method uses an accuracy function to transform the decision matrix given by the decision maker represented by IVIFVs into a transformed decision matrix represented by real values in $[-1, 1]$. Then, based on the obtained transformed decision matrix and PSO techniques, it gets the optimal weights of the attributes. Then, based on the IIFWGA operator, the obtained optimal weights of the attributes and the decision matrix given by the decision maker represented by IVIFVs, it calculates the weighted evaluating IVIFV of each alternative. Finally, it calculates the transformed value of the obtained weighted evaluating IVIFV of each alternative based on the accuracy function to get the preference order of the alternatives. The main contribution of this paper is that we propose a new MADM method based on the IIFWGA operator of IVIFVs, the accuracy function of IVIFVs and PSO techniques, which can overcome the drawbacks of the MADM methods presented in [5,13,39] for MADM in IVIF environments.

The paper is organized as follows. In Section 2, we briefly review the concepts of IVIFSs [2], the accuracy function [33] of IVIFVs, the concept of the largest range [5] of an IVIFV and the interval-valued intuitionistic fuzzy weighted geometric average (IIFWGA) operator [27] of IVIFVs. In Section 3, we briefly review some concepts of particle swarm optimization techniques [7,10,12,22]. In Section 4, we propose a new MADM method based on the IIFWGA operator of IVIFVs, the accuracy function of IVIFVs and PSO techniques. In Section 5, we compare the experimental results of the proposed method with the ones of the existing methods [5,13,39]. The conclusions are discussed in Section 6.

2. Preliminaries

Let $P = \{\langle x_i, \ \mu_P(x_i), \upsilon_P(x_i)\rangle | \ x_i \in X\}$ be an IVIFS in the universe of discourse $X = \{x_1, x_2, \ldots, x_m\}$, where μ_P and υ_P are the membership function and the non-membership function of the IVIFS P, respectively, $\mu_P(x_i) = [\mu_P(x_i)^l, \ \mu_P(x_i)^u]$ and $\upsilon_P(x_i) = [\upsilon_P(x_i)^l, \ \upsilon_P(x_i)^u]$, $0 \le \mu_P(x_i)^l \le \mu_P(x_i)^u \le 1$, $0 \le \upsilon_P(x_i)^l \le \upsilon_P(x_i)^u \le 1$, $0 \le \mu_P(x_i)^u + \upsilon_P(x_i)^u \le 1$ and $1 \le i \le m$. According to [27], the IVIFV $(\mu_P(x_i), \ \nu_P(x_i))$ of element x_i belonging to the IVIFS P also can be represented as $([\mu_P(x_i)^l, \ \mu_P(x_i)^u], \ [\upsilon_P(x_i)^l, \ \upsilon_P(x_i)^u])$, where $0 \le \mu_P(x_i)^l \le \mu_P(x_i)^u \le 1$, $0 \le \upsilon_P(x_i)^l \le \upsilon_P(x_i)^u \le 1$, $0 \le \mu_P(x_i)^u + \upsilon_P(x_i)^u \le 1$ and $1 \le i \le m$.

In [33], Ye presented an accuracy function K of IVIFVs for MCDM. The accuracy value $K(\tilde{\alpha})$ of the IVIFV $\tilde{\alpha} = ([a, \ b], \ [c, \ d])$ is defined as follows:

$$K(\tilde{\alpha}) = \ a + b - 1 + \frac{c \ + \ d}{2}, \tag{1}$$

where $K(\tilde{\alpha}) \in [-1, \ 1]$, $0 \le a \le b \le 1$, $0 \le c \le d \le 1$ and $0 \le b + d \le 1$. The larger the accuracy value $K(\tilde{\alpha})$, the larger the IVIFV $\tilde{\alpha}$.

In [5], Chen and Chiou presented the concept of the largest range of an IVIFV for MADM. The largest range $\bar{\alpha} = [a, \ 1 - c]$ of the IVIFV $\tilde{\alpha} = ([a, \ b], \ [c, \ d])$ is shown in Fig. 1, where

$$\bar{\alpha} = [a, \ 1 - c], \tag{2}$$

$0 \le a \le b \le 1$, $0 \le c \le d \le 1$ and $0 \le b + d \le 1$. That is, the possible values of the IVIFV $\tilde{\alpha} = ([a, \ b], \ [c, \ d])$ are between a and $1 - c$, where $0 \le a \le 1 - c \le 1$.

In [27], Xu proposed the IIFWGA operator g_w of IVIFVs. Let $\tilde{\alpha}_1$, $\tilde{\alpha}_2$, ..., and $\tilde{\alpha}_n$ be IVIFVs, where $\tilde{\alpha}_i = ([a_i, \ b_i], \ [c_i, \ d_i])$, $0 \le a_i \le b_i \le 1$, $0 \le c_i \le d_i \le 1$, $0 \le b_i + d_i \le 1$ and $1 \le i \le n$. Let $W = \{w_1, \ w_2, \ \ldots, \ w_n\}^T$ be the weighting vector of the IIFWGA

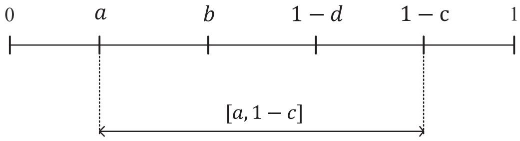

Fig. 1. The largest range $\bar{\alpha} = [a, 1 - c]$ of the IVIFV $\tilde{\alpha} = ([a, b],\ [c, d])$.

operator g_w, where w_i denotes the weight of $\tilde{\alpha}_i$, $w_i \in [0,\ 1]$, $1 \le i \le n$ and $\sum_{i=1}^{n} w_i = 1$. The IIFWGA operator g_w of the IVIFVs $\tilde{\alpha}_1$, $\tilde{\alpha}_2$, ..., and $\tilde{\alpha}_n$ is defined as follows:

$$g_w(\tilde{\alpha}_1,\ \tilde{\alpha}_2,\ \ldots,\ \tilde{\alpha}_n) = \left(\left[\prod_{i=1}^{n} {a_i}^{w_i},\ \ \prod_{i=1}^{n} {b_i}^{w_i} \right], \left[1 - \prod_{i=1}^{n} (1 - c_i)^{w_i},\ \ 1 - \prod_{i=1}^{n} (1 - d_i)^{w_i} \right] \right), \tag{3}$$

where $0 \le \prod_{i=1}^{n} {a_i}^{w_i} \le \prod_{i=1}^{n} {b_i}^{w_i} \le 1$, $0 \le (1 - \prod_{i=1}^{n} (1 - c_i)^{w_i}) \le (1 - \prod_{i=1}^{n} (1 - d_i)^{w_i}) \le 1$, $0 \le \prod_{i=1}^{n} {b_i}^{w_i} + (1 - \prod_{i=1}^{n} (1 - d_i)^{w_i}) \le 1$ and $1 \le i \le n$.

3. Particle swarm optimization techniques

In this section, we briefly review some concepts of PSO techniques [7,10,12,22]. In [12], Kennedy and Eberhant presented standard PSO techniques, where the position of particle *id* of dimensionality n is represented by the position vector $X_{id} = [x_{id,1}, x_{id,2}, \ldots, x_{id,n}]$; the velocity of particle *id* with dimension n is represented by the velocity vector $V_{id} = [v_{id,1}, v_{id,2}, \ldots, v_{id,n}]$. Each element $v_{id,j}$ in the velocity vector V_{id} and each element $x_{id,j}$ in the position vector X_{id} of particle *id* are calculated as follows [12]:

$$v_{id,j} = v_{id,j} + 2 \times rand() \times \left(p_{id,j} - x_{id,j}\right) + 2 \times Rand() \times \left(x_{gBest,j} - x_{id,j}\right), \tag{4}$$

$$x_{id,j} = x_{id,j} + v_{id,j}, \tag{5}$$

where $v_{id,j}$ is the jth element in the velocity vector V_{id} of particle *id*, $p_{id,j}$ is the jth element in the personal best position vector $P_{id} = [p_{id,1}, p_{id,2}, \ldots, p_{id,n}]$ of particle *id*, $x_{id,j}$ is the jth element in the position vector X_{id} of particle *id*, $x_{gBest,j}$ is the jth element in the position vector $X_{gBest} = [x_{gBest,1}, x_{gBest,2}, \ldots, x_{gBest,n}]$ of the best particle *gBest* among all particles, $rand()$ and $Rand()$ are random numbers uniformly distributed in [0, 1], respectively, and $1 \le j \le n$.

In [22], Umapathy et al. presented PSO techniques with the time-varying inertia weight ω, where

$$\omega = (\omega_{max} - \omega_{min}) \times \left(\frac{iter_{max}\ -\ iter}{iter_{max}} \right) +\ \omega_{min}, \tag{6}$$

ω_{max} and ω_{min} are the maximum value and the minimum value of the inertia weight, respectively, $iter$ is the current iteration number and $iter_{max}$ is the maximum number of iterations. Let the position of particle *id* with dimension n be represented by the position vector $X_{id} = [x_{id,1}, x_{id,2}, \ldots, x_{id,n}]$ and let the velocity of particle *id* with dimension n be represented by the velocity vector $V_{id} = [v_{id,1}, v_{id,2}, \ldots, v_{id,n}]$. Each element $v_{id,j}$ in the velocity vector V_{id} and each element $x_{id,j}$ in the position vector X_{id} of particle *id* are calculated as follows [22]:

$$v_{id,j} = \omega \times v_{id,j} + c_1 \times rand() \times \left(p_{id,j} - x_{id,j}\right) + c_2 \times Rand() \times \left(x_{gBest,j} - x_{id,j}\right), \tag{7}$$

$$x_{id,j} = x_{id,j} + v_{id,j}, \tag{8}$$

where ω is the inertia weight, where $\omega = (\omega_{max} - \omega_{min}) \times (\frac{iter_{max}\ -\ iter}{iter_{max}}) +\ \omega_{min}$, ω_{max} and ω_{min} are the maximum value and the minimum value of the inertia weight, respectively, $iter$ is the current iteration number and $iter_{max}$ is the maximum number of iterations, $v_{id,j}$ is the jth element in the velocity vector V_{id} of particle *id*, $p_{id,j}$ is the jth element in the personal best position vector $P_{id} = [p_{id,1}, p_{id,2}, \ldots, p_{id,n}]$ of particle *id*, $x_{id,j}$ is the jth element in the position vector X_{id} of particle *id*, $x_{gBest,j}$ is the jth element in the position vector $X_{gBest} = [x_{gBest,1}, x_{gBest,2}, \ldots, x_{gBest,n}]$ of the best particle *gBest* among all particles, c_1 and c_2 are the acceleration factors which are normally set as 2.0, respectively, $rand()$ are $Rand()$ are random numbers uniformly distributed in [0, 1] and $1 \le j \le n$. In [10], Eberhart and Shi pointed out that the inertia weight ω typically decreases linearly from about 0.9 to 0.4 during a run.

In [10], Eberhart and Shi presented PSO techniques with the constriction factor, where the position of particle *id* with dimension n is represented by the position vector $X_{id} = [x_{id,1}, x_{id,2}, \ldots, x_{id,n}]$; the velocity of particle *id* with dimension n is represented by the velocity vector $V_{id} = [v_{id,1}, v_{id,2}, \ldots, v_{id,n}]$. Each element $v_{id,j}$ in the velocity vector V_{id} and $x_{id,j}$ in the position vector X_{id} of particle *id* are calculated as follows:

$$v_{id,j} = k \times [v_{id,j} + c_1 \times rand() \times \left(p_{id,j} - x_{id,j}\right) + c_2 \times Rand() \times \left(x_{gBest,j} - x_{id,j}\right)], \tag{9}$$

$$x_{id,j} = x_{id,j} + v_{id,j}, \tag{10}$$

where $v_{id,j}$ is the jth element in the velocity vector V_{id} of particle id, k is the constriction factor, where $k = \frac{2}{|2-\varphi-\sqrt{\varphi^2-4\varphi}|}$, $\varphi = c_1 + c_2$ and $\varphi > 4$ (Note: In [10], Eberhart and Shi let $c_1 = 2.05$ and $c_2 = 2.05$, such that $\varphi = 4.1$ and the constriction factor $k = 0.7298$.), $p_{id,j}$ is the jth element in the personal best position vector $P_{id} = [p_{id,1}, p_{id,2}, \ldots, p_{id,n}]$ of the particle id, $x_{id,j}$ is the jth element in the position vector X_{id} of particle id, $x_{gBest,j}$ is the jth element in the position vector $X_{gBest} = [x_{gBest,1}, x_{gBest,2}, \ldots, x_{gBest,n}]$ of the best particle $gBest$ among all particles, $rand()$ and $Rand()$ are random numbers uniformly distributed in [0, 1], respectively, and $1 \le j \le n$.

In [7], Chen and Phuong presented PSO techniques combining with the inertia weight [22] and the constriction factor [10]. Let the position of particle id with dimension n be represented by the position vector $X_{id} = [x_{id,1}, x_{id,2}, \ldots, x_{id,n}]$ and let the velocity of particle id with dimension n be represented by the velocity vector $V_{id} = [v_{id,1}, v_{id,2}, \ldots, v_{id,n}]$. Each element $v_{id,j}$ in the velocity vector V_{id} and each element $x_{id,j}$ in the position vector X_{id} of particle id are calculated as follows [7]:

$$v_{id,j} = k \times [\omega \times v_{id,j} + c_1 \times rand() \times \left(p_{id,j} - x_{id,j}\right) + c_2 \times Rand() \times \left(x_{gBest,j} - x_{id,j}\right)], \tag{11}$$

$$x_{id,j} = x_{id,j} + v_{id,j}, \tag{12}$$

where $v_{id,j}$ is the jth element in the velocity vector V_{id} of particle id, ω is the inertia weight, $\omega = (\omega_{max} - \omega_{min}) \times \left(\frac{iter_{max} - iter}{iter_{max}}\right) + \omega_{min}$, ω_{max} and ω_{min} are the maximum value and the minimum value of the inertia weight, respectively, the inertia weight ω decreases linearly from 0.9 to 0.4 during a run, $iter$ is the current iteration number and $iter_{max}$ is the total number of iterations, k is the constriction factor, where $k = \frac{2}{|2-\varphi-\sqrt{\varphi^2-4\varphi}|}$, $\varphi = c_1 + c_2$, c_1 is the self-confidence coefficient, c_2 is the social confidence coefficient and $4.1 \le \varphi \le 4.2$ (Note: In [7], Chen and Phuong let $c_1 = 2.05$ and $c_2 = 2.05$, such that $\varphi = 4.1$ and the constriction factor $k = 0.7298$.), $p_{id,j}$ is the jth element in the personal best position vector $P_{id} = [p_{id,1}, p_{id,2}, \ldots, p_{id,n}]$ of particle id, $x_{id,j}$ is the jth element in the position vector X_{id} of particle id, $x_{gBest,j}$ is the jth element in the position vector $X_{gBest} = [x_{gBest,1}, x_{gBest,2}, \ldots, x_{gBest,n}]$ of the best particle $gBest$ among all particles, $rand()$ and $Rand()$ are random numbers uniformly distributed in [0, 1], respectively, and $1 \le j \le n$. In summary, Chen and Phuong's PSO techniques [7] are reviewed as follows:

Step 1: Initialize all particles with random position vectors and random velocity vectors.
Step 2: Calculate the objective value of each particle id.
Step 3: For each particle id, if the objective value of its current position vector is better than the objective value of its personal best position vector P_{id}, then update its personal best position vector P_{id} with its current position vector.
Step 4: Find the best particle $gBest$ which has the best objective value among the personal best position vectors of all particles to get the position vector P_{gBest} of the best particle $gBest$ among all particles.
Step 5: Update the elements in the velocity vector V_{id} and the elements in the position vector X_{id} of each particle id based on Eqs. (11) and (12), respectively.
Step 6: Repeatedly perform **Step 2** to **Step 5** until the number of iterations arrives at a predefined maximal number of iterations.

4. A new MADM method based on the IIFWGA operator, the accuracy function of IVIFVs and PSO techniques

In this section, we propose a new MADM method based on the IIFWGA operator [27] of IVIFVs, the accuracy function [33] of IVIFVs and Chen and Phuong's PSO techniques [7] to overcome the drawbacks of the methods presented in [5,13,39]. Let E_1, E_2, …, and E_m be alternatives, let A_1, A_2, …, and A_n be attributes, and let $\tilde{D} = (\tilde{\alpha}_{ij})_{m\times n} = (([\mu_{ij}^-, \mu_{ij}^+], [\nu_{ij}^-, \nu_{ij}^+]))_{m\times n}$ be a decision matrix given by the decision maker represented by IVIFVs, shown as follows:

$$\tilde{D} = \left(\tilde{\alpha}_{ij}\right)_{m\times n}$$

$$= \begin{array}{c} \\ E_1 \\ E_2 \\ \vdots \\ E_m \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_n \end{array} \\ \begin{pmatrix} ([\mu_{11}^-, \mu_{11}^+], [\nu_{11}^-, \nu_{11}^+]) & ([\mu_{12}^-, \mu_{12}^+], [\nu_{12}^-, \nu_{12}^+]) & \cdots & ([\mu_{1n}^-, \mu_{1n}^+], [\nu_{1n}^-, \nu_{1n}^+]) \\ ([\mu_{21}^-, \mu_{21}^+], [\nu_{21}^-, \nu_{21}^+]) & ([\mu_{22}^-, \mu_{22}^+], [\nu_{22}^-, \nu_{22}^+]) & \cdots & ([\mu_{2n}^-, \mu_{2n}^+], [\nu_{2n}^-, \nu_{2n}^+]) \\ \vdots & \vdots & \ddots & \vdots \\ ([\mu_{m1}^-, \mu_{m1}^+], [\nu_{m1}^-, \nu_{m1}^+]) & ([\mu_{m2}^-, \mu_{m2}^+], [\nu_{m2}^-, \nu_{m2}^+]) & \cdots & ([\mu_{mn}^-, \mu_{mn}^+], [\nu_{mn}^-, \nu_{mn}^+]) \end{pmatrix} \end{array},$$

where $\tilde{\alpha}_{ij} = ([\mu_{ij}^-, \mu_{ij}^+], [\nu_{ij}^-, \nu_{ij}^+])$ denotes the evaluating IVIFV of alternative E_i with respect to attribute A_j given by the decision maker, $0 \le \mu_{ij}^- \le \mu_{ij}^+ \le 1$, $0 \le \nu_{ij}^- \le \nu_{ij}^+ \le 1$, $0 \le \mu_{ij}^+ + \nu_{ij}^+ \le 1$, $1 \le i \le m$, m is the number of alternatives, $1 \le j \le n$ and n is the number of attributes. Let $\tilde{w}_j$ be the IVIF weight of attribute A_j given by the decision maker, where $\tilde{w}_j = ([a_j, b_j], [c_j, d_j])$ is an IVIFV, $0 \le a_j \le b_j \le 1$, $0 \le c_j \le d_j \le 1$, $0 \le b_j + d_j \le 1$, and $1 \le j \le n$. The proposed MADM method is now presented as follows:

$w_{k,1}$	$w_{k,2}$	...	$w_{k,n}$

Fig. 2. The position vector X_k of the kth particle.

$v_{k,1}$	$v_{k,2}$	...	$v_{k,n}$

Fig. 3. The velocity vector V_k of the kth particle.

Step 1: Based on Eq. (2), get the largest range $\bar{w}_j$ of the IVIF weight $\tilde{w}_j = ([a_j, b_j], [c_j, d_j])$ of attribute A_j given by the decision maker, where $\bar{w}_j = [a_j, 1 - c_j]$, $0 \le a_j \le 1 - c_j \le 1$ and $1 \le j \le n$.

Step 2: Based on the accuracy function shown in Eq. (1) and the decision matrix $\tilde{D} = (\tilde{\alpha}_{ij})_{m\times n} = ([\mu_{ij}^-, \mu_{ij}^+], [\nu_{ij}^-, \nu_{ij}^+])_{m\times n}$ given by the decision maker, calculate the accuracy value d_{ij} of the evaluating IVIFV $\tilde{\alpha}_{ij}$ to get the transformed decision matrix $D = (d_{ij})_{m\times n}$, where

$$d_{ij} = \mu_{ij}^- + \mu_{ij}^+ - 1 + \frac{\nu_{ij}^- + \nu_{ij}^+}{2}, \tag{13}$$

$d_{ij} \in [-1, 1]$, $1 \le i \le m$ and $1 \le j \le n$. Based on the obtained largest range $\bar{w}_j = [a_j, 1 - c_j]$ of the IVIF weight $\tilde{w}_j = ([a_j, b_j], [c_j, d_j])$ of attribute A_j, where $1 \le j \le n$, let the objective function F be the linear programming model defined by the optimal weight $w_j{}^*$ of attribute A_j and the accuracy value d_{ij} of the evaluating IVIFV $\tilde{\alpha}_{ij}$ of alternative E_i with respect to attribute A_j in the transformed decision matrix $D = (d_{ij})_{m\times n}$, shown as follows:

$$\max F = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(w_j{}^* \times d_{ij}\right), \tag{14}$$

$$s.t.\begin{cases} a_j \le w_j{}^* \le 1 - c_j \\ \sum\limits_{j=1}^{n} w_j{}^* = 1 \\ 0 \le w_j{}^* \le 1 \end{cases}$$

where $w_j{}^*$ is the optimal weight of attribute A_j , $w_j{}^* \in [0, 1]$, $d_{ij} \in [-1, 1]$, $1 \le i \le m$ and $1 \le j \le n$.

Step 3: Based on [7], use the PSO techniques to get the optimal weights $w_1{}^*$, $w_2{}^*$, $\cdots$, and $w_n{}^*$ of the attributes A_1, A_2, $\cdots$, and A_n, respectively, such that the value of the objective function F shown in Eq. (14) is maximal, described as follows. Initially, the system randomly generates s particles with dimension n (Note: In this paper, we let $s = 20$), where the position vector $X_k = [w_{k,1}, w_{k,2}, \ldots, w_{k,n}]$ of the kth particle is shown in Fig. 2, where $a_j \le w_{k,j} \le (1 - c_j)$, and the velocity vector $V_k = [v_{k,1}, v_{k,2}, \ldots, v_{k,n}]$ of the kth particle is shown in Fig. 3, where $\frac{-0.2\times[(1-c_j)-a_j]}{2} \le v_{k,j} \le \frac{0.2\times[(1-c_j)-a_j]}{2}$, $1 \le j \le n$ and $1 \le k \le s$. Let

$$w_{k,n} = \frac{w_{k,n}}{w_{k,1} + w_{k,2} + \cdots + w_{k,n}}, \tag{15}$$

where $a_j \le w_{k,j} \le (1 - c_j)$, $1 \le j \le n$ and $\sum\limits_{j=1}^{n} w_{k,j} = 1$. Let the personal best position vector $P_{Best,k} = [p_{k,1}, p_{k,2}, \ldots, p_{k,n}]$ of the kth particle be the same as its initial position vector $W_k = [w_{k,1}, w_{k,2}, \ldots, w_{k,n}]$, where the personal best position vector $P_{Best,k} = [p_{k,1}, p_{k,2}, \ldots, p_{k,n}]$ denotes the best position of the kth particle found so far.

Step 3.1: Let the number of iteration p be zero (i.e., let $p = 0$). Perform the following sub-steps:

Step 3.1.1: According to Eq. (14), calculate the value of the objective function $F_k = \sum_{i=1}^{m}\sum_{j=1}^{n}(w_j^* \times d_{ij})$ of the kth particle at the pth iteration, where $1 \le k \le s$.

Step 3.1.2: If the value of the objective function $F_k = \sum_{i=1}^{m}\sum_{j=1}^{n}(w_j^* \times d_{ij})$ of the kth particle with its position vector $X_k = [w_{k,1}, w_{k,2}, \ldots, w_{k,n}]$ at the pth iteration is larger than that of the personal best position vector $P_{Best,k} = [p_{k,1}, p_{k,2}, \ldots, p_{k,n}]$ of the kth particle, where $1 \le k \le s$, then assign the values of the elements in the position vector $X_k = [w_{k,1}, w_{k,2}, \ldots, w_{k,n}]$ at the pth iteration to the elements in the personal best position vector $P_{Best,k} = [p_{k,1}, p_{k,2}, \ldots, p_{k,n}]$, such that $P_{Best,k} = X_k$, i.e., let $p_{k,j} = w_{k,j}$, where $1 \le j \le n$ and $1 \le k \le s$.

Step 3.1.3: If the value of the objective function F_k of the personal best position vector $P_{best,k} = [p_{k,1}, p_{k,2}, \ldots, p_{k,n}]$ at the pth iteration is larger than the value of the objective function F_k of the global position vector $X_{gbest} = [x_{gBest,1}, x_{gBest,2}, \ldots, x_{gBest,n}]$ of the best particle $gBest$ among all particles, then let $X_{gBest} = P_{Best,k}$, i.e., let $x_{gBest,j} = w_{k,j}$, where $1 \le j \le n$ and $1 \le k \le s$.

Step 3.2: Calculate the inertial weight ω_p at the pth iteration, shown as follows [22]:

$$\omega_p = (\omega_{max} - \omega_{min}) \times \left(\frac{p_{max} - p}{p_{max}}\right) + \omega_{min}, \tag{16}$$

where ω_{max} and ω_{min} are the maximum value and the minimum value of the inertia weight, respectively (Note: In this paper, we let $\omega_{max} = 0.9$ and $\omega_{min} = 0.4$.), and p_{max} is the predefined maximum number of iterations.

Step 3.3: Update the element $v_{k,j}$ in the velocity vector $V_k = [v_{k,1}, v_{k,2}, \ldots, v_{k,n}]$ at the pth iteration and update the element $w_{k,j}$ in the position vector $W_k = [w_{k,1}, w_{k,2}, \ldots, w_{k,n}]$ of the kth particle at the pth iteration, respectively, where $1 \le k \le s$ and $1 \le j \le n$, shown as follows [7]:

$$v_{k,j} = k\left(\omega_p \times v_{k,j} + c_1 \times rand() \times \left(p_{k,j} - w_{k,j}\right) + h_2 \times Rand() \times \left(x_{gBest,j} - w_{k,j}\right)\right), \tag{17}$$

$$w_{k,j} = w_{k,j} + v_{k,j}, \tag{18}$$

where $1 \le k \le s$, $1 \le j \le n$, c_1 is the self-confidence coefficient, c_2 is the social confidence coefficient, ω_p is the inertial weight at the pth iteration, $0 \le p \le p_{max}$, p_{max} is a predefined maximum number of iterations, $rand()$ and $Rand()$ are two independent random numbers uniformly distributed in [0, 1] (Note: In this paper, we let $c_1 = 2.05$ and $c_2 = 2.05$, such that $k = 0.7298$). Based on [8], if $v_{k,j} > \frac{0.2 \times [(1 - c_j) - a_j]}{2}$, then let $v_{k,j} = \frac{0.2 \times [(1 - c_j) - a_j]}{2}$; if $v_{k,j} < \frac{-0.2 \times [(1 - c_j) - a_j]}{2}$, then let $v_{k,j} = \frac{-0.2 \times [(1 - c_j) - a_j]}{2}$, where $1 \le j \le n$. Normalized each element $w_{k,j}$ in the position vector $X_k = [w_{k,1}, w_{k,2}, \ldots, w_{k,n}]$ of the kth particle at the pth iteration, where $1 \le k \le s$ and $1 \le j \le n$, shown as follows [8]:

$$w_{k,j} = \begin{cases} \left(1 - c_j\right) - \frac{1}{2} \times rand() \times \left[\left(1 - c_j\right) - a_j\right], & \text{if } w_{k,j} > \left(1 - c_j\right) \\ a_j + \frac{1}{2} \times rand() \times \left[\left(1 - c_j\right) - a_j\right], & \text{if } w_{k,j} < a_j \end{cases} \tag{19}$$

where the largest range $\bar{w}_j$ of the IVIF weight $\tilde{w}_j = ([a_j, b_j], [c_j, d_j])$ of attribute A_j given by the decision maker is $[a_j, 1 - c_j]$, i.e., $\bar{w}_j = [a_j, 1 - c_j]$, $0 \le a_j \le 1 - c_j \le 1$, $1 \le j \le n$ and $rand()$ is a random number uniformly distributed in [0, 1]. Let

$$w_{k,j} = \frac{w_{k,j}}{w_{k,1} + w_{k,2} + \cdots + w_{k,n}}, \tag{20}$$

where $w_{k,j}$ is the jth element in the position vector $X_k = [w_{k,1}, w_{k,2}, \ldots, w_{k,n}]$ of the kth particle at the pth iteration, $1 \le k \le s$ and $1 \le j \le n$.

Step 3.4: Let $p = p + 1$. If p is larger than the predefined number of iterations p_{max}, then let the best position vector $X_{gbest} = [x_{gBest,1}, x_{gBest,2}, \ldots, x_{gBest,n}]$ obtained in **Step 3.1.3** be the optimal weighting vector $W^* = [w_1{}^*, w_2{}^*, \ldots, w_n{}^*]$, i.e., let $w_j{}^* = x_{gBest,j}$, where $w_j{}^*$ is the optimal weight of the attribute A_j, $w_j{}^* \in [0, 1]$, $1 \le j \le n$ and $\sum_{j=1}^{n} w_j{}^* = 1$; **go to Step 4**. Otherwise, go to **Step 3.1.1**.

Step 4: Based on the IIFWGA operator shown in Eq. (3), the obtained optimal weights $w_1{}^*, w_2{}^*, \cdots$, and $w_n{}^*$ of the attributes $A_1, A_2, \cdots$, and A_n, respectively, and the decision matrix $\tilde{D} = (\tilde{\alpha}_{ij})_{m\times n} = ([\mu_{ij}^-, \mu_{ij}^+], [\nu_{ij}^-, \nu_{ij}^+])_{m\times n}$ represented by IVIFVs given by the decision maker, calculate the weighted evaluating IVIFV $\tilde{t}_i = ([\rho_i^-, \rho_i^+], [\tau_i^-, \tau_i^+])$ of alternative E_i, where

$$\rho_i^- = \prod_{j=1}^{n} \mu_{ij}^{-w_j^*}, \tag{21}$$

$$\rho_i^+ = \prod_{j=1}^{n} \mu_{ij}^{+w_j^*}, \tag{22}$$

$$\tau_i^- = 1 - \prod_{j=1}^{n} \left(1 - \nu_{ij}^-\right)^{w_j^*}, \tag{23}$$

$$\tau_i^+ = 1 - \prod_{j=1}^{n} \left(1 - \nu_{ij}^+\right)^{w_j^*}, \tag{24}$$

$0 \le \boldsymbol{\rho}_i^- \le \boldsymbol{\rho}_i^+ \le 1, 0 \le \boldsymbol{\tau}_i^- \le \boldsymbol{\tau}_i^+ \le 1, 0 \le \boldsymbol{\rho}_i^+ + \boldsymbol{\tau}_i^+ \le 1, 1 \le i \le m$, w_j^* is the obtained optimal weight of attributes A_j, $w_j^* \in [0, 1]$, $1 \le j \le n$ and $\sum_{j=1}^{n} w_j^* = 1$.

Step 5: Based on Eq. (1), calculate the transformed value t_i of the weighted evaluating IVIFV $\tilde{\boldsymbol{t}}_{\boldsymbol{i}} = ([\boldsymbol{\rho}_i^-, \boldsymbol{\rho}_i^+], [\boldsymbol{\tau}_i^-, \boldsymbol{\tau}_i^+])$ of alternative E_i, shown as follows:

$$t_i = \rho_i^- + \rho_i^+ - 1 + \frac{\tau_i^- + \tau_i^+}{2}, \tag{25}$$

where $t_i \in [-1, 1]$ and $1 \le \boldsymbol{i} \le \boldsymbol{m}$. The larger the transformed value t_i, the better the preference order of alternative E_i, where $1 \le i \le m$.

In summary, Fig. 4 shows the flowchart of the proposed MADM method.

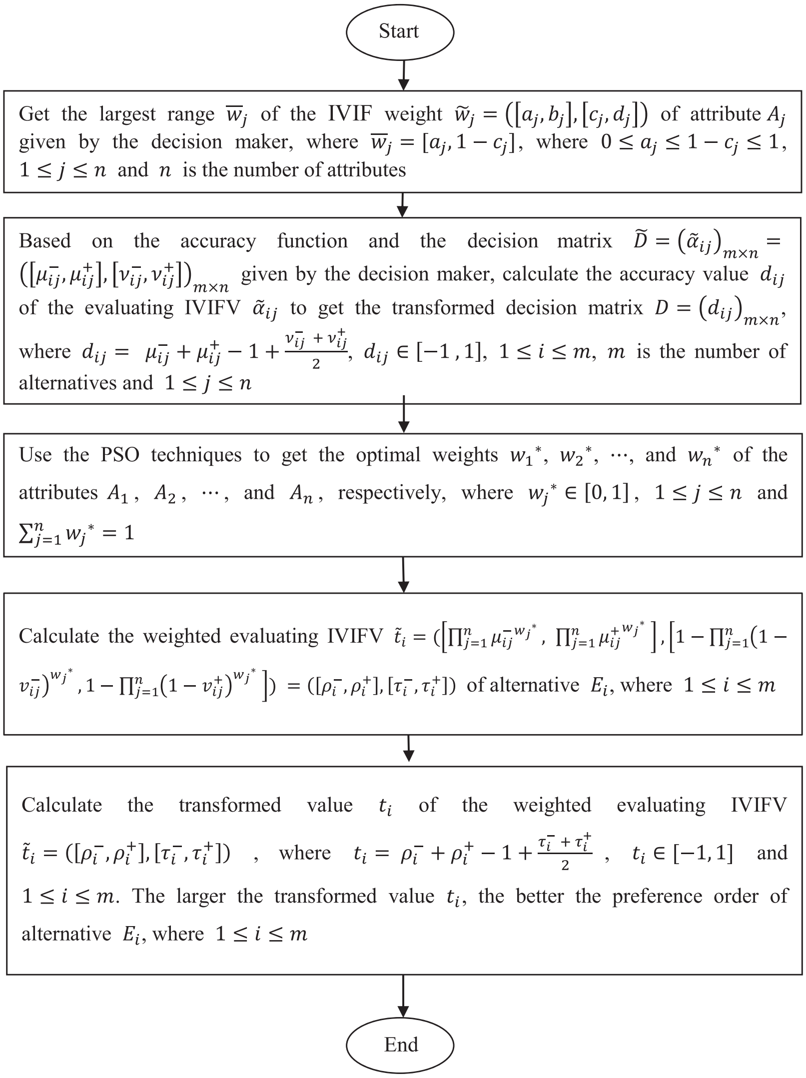

Fig. 4. Flowchart of the proposed MADM method.

5. Illustrative examples

We have implemented the proposed MADM method useing Visual Studio 2013 using C++ language on a Core i5×86–64 personal computer. In this section, we use some examples to compare the experimental results of the proposed method with the ones of Chen and Chiou's method [5], Li's method [13] and Zhitao and Yingjun's method [39].

Example 5.1 [13]. Let E_1, E_2, E_3 and E_4 be four alternatives and let A_1, A_2 and A_3 be three attributes. Assume that the IVIF weights $\tilde{w}_1$, $\tilde{w}_2$ and $\tilde{w}_3$ of the attributes A_1, A_2 and A_3 given by the decision maker represented by IVIFVs are ([0.10, 0.40], [0.20, 0.55]), ([0.20, 0.50], [0.15, 0.45]) and ([0.25, 0.60], [0.15, 0.38]), respectively, i.e.,

$$\begin{aligned} \tilde{w}_1 &= ([a_1, b_1], [c_1, d_1]) = ([0.10, 0.40], [0.20, 0.55]), \\ \tilde{w}_2 &= ([a_2, b_2], [c_2, d_2]) = ([0.20, 0.50], [0.15, 0.45]), \\ \tilde{w}_3 &= ([a_3, b_3], [c_3, d_3]) = ([0.25, 0.60], [0.15, 0.38]), \end{aligned}$$

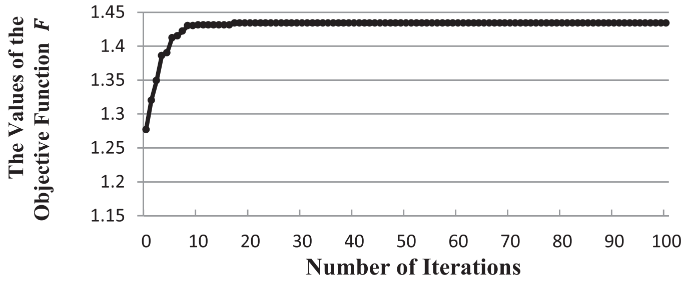

Fig. 5. The convergence process of the values of the objective function F of Example 5.1.

and assume that the decision matrix $\tilde{D}$ given by the decision maker represented by IVIFVs is shown as follows:

$$\tilde{D} = \left(\tilde{\alpha}_{ij}\right)_{4\times 3} = \left(\left[\mu_{ij}^-,\ \mu_{ij}^+\right],\ \left[\nu_{ij}^-,\ \nu_{ij}^+\right]\right)_{4\times 3}$$

$$= \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \\ E_4 \end{array} \begin{array}{c} \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \\ \begin{pmatrix} ([0.40,\ 0.50],\ [0.30,\ 0.40]) & ([0.40,\ 0.60],\ [0.20,\ 0.40]) & ([0.10,\ 0.30], [0.50,\ 0.60]) \\ ([0.60,\ 0.70],\ [0.20,\ 0.30]) & ([0.60,\ 0.70],\ [0.20,\ 0.30]) & ([0.40,\ 0.70], [0.10,\ 0.20]) \\ ([0.30,\ 0.60],\ [0.30,\ 0.40]) & ([0.50,\ 0.60],\ [0.30,\ 0.40]) & ([0.50,\ 0.60], [0.10,\ 0.30]) \\ ([0.70,\ 0.80],\ [0.10,\ 0.20]) & ([0.60,\ 0.70],\ [0.10,\ 0.30]) & ([0.30,\ 0.40], [0.10,\ 0.20]) \end{pmatrix} \end{array}.$$

[Step 1] Based on Eq. (2), we can get the largest range $\bar{w}_j$ of the IVIF weight $\tilde{w}_j = ([a_j,\ b_j],\ [c_j,\ d_j])$ of attribute A_j given by the decision maker, where $\bar{w}_j = [a_j,\ 1 - c_j],\ 0 \le a_j \le (1 - c_j) \le 1$ and $1 \le j \le 3$. Because $\tilde{w}_1 = ([a_1, b_1], [c_1, d_1]) = ([0.10,\ 0.40],\ [0.20,\ 0.55])$, $\tilde{w}_2 = ([a_2,\ b_2],\ [c_2,\ d_2]) = ([0.20,\ 0.50],\ [0.15,\ 0.45])$ and $\tilde{w}_3 = ([a_3,\ b_3],\ [c_3,\ d_3]) = ([0.25,\ 0.60],\ [0.15,\ 0.38])$, we can get the largest ranges $\bar{w}_1$, $\bar{w}_2$ and $\bar{w}_3$ of the IVIF weights $\tilde{w}_1 = ([0.10,\ 0.40], [0.20,\ 0.55])$, $\tilde{w}_2 = ([0.20,\ 0.50],\ [0.15,\ 0.45])$ and $\tilde{w}_3 = ([0.25,\ 0.60],\ [0.15,\ 0.38])$ of the attributes A_1, A_2 and A_3, respectively, where

$$\bar{w}_1 = [0.10,\ 0.80],\ \bar{w}_2 = [0.20,\ 0.85],\ \text{and}\ \bar{w}_3 = [0.25,\ 0.85].$$

[Step 2] Based on Eq. (13) and the decision matrix $\tilde{D} = (\tilde{\alpha}_{ij})_{4\times 3} = ([\mu_{ij}^-,\ \mu_{ij}^+],\ [\nu_{ij}^-,\ \nu_{ij}^+])_{4\times 3}$ given by the decision maker, we can get the transformed decision matrix $D = (d_{ij})_{4\times 3}$, where $d_{11} = 0.2500$, $d_{12} = 0.3000$,$d_{13} = -0.0500$, $d_{21} = 0.5500$, $d_{22} = 0.5500$, $d_{23} = 0.2500$, $d_{31} = 0.2500$, $d_{32} = 0.4500$, $d_{33} = 0.3000$, $d_{41} = 0.6500$, $d_{42} = 0.5000$ and $d_{43} = -0.1500$. Based on Eq. (14), the obtained largest ranges $\bar{w}_1 = [0.10,\ 0.80]$, $\bar{w}_2 = [0.20,\ 0.85]$ and $\bar{w}_3 = [0.25,\ 0.85]$ of the IVIF weights $\tilde{w}_1 = ([0.10,\ 0.40],\ [0.20,\ 0.55])$, $\tilde{w}_2 = ([0.20,\ 0.50],\ [0.15,\ 0.45])$ and $\tilde{w}_3 = ([0.25,\ 0.60],\ [0.15,\ 0.38])$, respectively, and the obtained transformed decision matrix $D = (d_{ij})_{4\times 3}$, we can get the following linear programming model:

$$\max F = \sum\nolimits_{i=1}^{4} \sum\nolimits_{j=1}^{3} \left(w_j{}^* \times d_{ij}\right),$$

$$s.t. \begin{cases} 0.10 \le w_1{}^* \le 0.80 \\ 0.20 \le w_2{}^* \le 0.85 \\ 0.25 \le w_3{}^* \le 0.85 \\ w_1{}^* +\ w_2{}^* +\ w_3{}^* = 1 \end{cases}$$

where $w_1{}^*$, $w_2{}^*$ and $w_3{}^*$ are the optimal weights of the attributes A_1, A_2 and A_3, respectively.

[Step 3] By applying the PSO techniques to solve the programming model obtained in **Step 2**, we can obtain the optimal weights $w_1{}^*$, $w_2{}^*$ and $w_3{}^*$ of the attributes A_1, A_2 and A_3, respectively, where $w_1{}^* = 0.2620$, $w_2{}^* = 0.5040$ and $w_3{}^* = 0.2340$. (Note: In this paper, we let the maximum value of the inertia weight $\omega_{max} = 0.9$ and let the minimum value of the inertia weight $\omega_{min} = 0.4$, which are adopted from [7] and [10], let the maximum number of iterations p_{max} be 100, let $c_1 = 2.05$ and $c_2 = 2.05$, which are the same as the ones presented in [7] and [10]). The convergence process of the values of the objective function F for obtaining the optimal weights $w_1{}^*$, $w_2{}^*$ and $w_3{}^*$ of the attributes A_1, A_2 and A_3, respectively, is shown in Fig. 5, where $w_1{}^* = 0.2620$, $w_2{}^* = 0.5040$ and $w_3{}^* = 0.2340$.

[Step 4] Based on Eqs. (21)–(24), the decision matrix $\tilde{D} = (\tilde{\alpha}_{ij})_{4\times 3} = ([\mu_{ij}^-,\ \mu_{ij}^+],\ [\nu_{ij}^-,\ \nu_{ij}^+])_{4\times 3}$ given by the decision maker, and the obtained optimal weights $w_1{}^*$, $w_2{}^*$ and $w_3{}^*$ of the attributes A_1, A_2 and A_3, respectively, where $w_1{}^* = 0.2620$, $w_2{}^* = 0.5040$ and $w_3{}^* = 0.2340$, we can get $\rho_1^- = 0.2892$, $\rho_1^+ = 0.4864$, $\tau_1^- = 0.3080$, $\tau_1^+ = 0.4543$, $\rho_2^- = 0.5457$, $\rho_2^+ = 0.7000$,$\tau_2^- = 0.1776$, $\tau_2^+ = 0.2778$, $\rho_3^- = 0.4374$, $\rho_3^+ = 0.6000$, $\tau_3^- = 0.2576$, $\tau_3^+ = 0.3780$, $\rho_4^- = 0.4071$, $\rho_4^+ = 0.5749$, $\tau_4^- = 0.2659$ and $\tau_4^+ = 0.3971$. Therefore, we can get the weighted evaluating IVIFVs $\tilde{t}_1$, $\tilde{t}_2$, $\tilde{t}_3$ and $\tilde{t}_4$ of the alternatives E_1, E_2, E_3 and E_4, respectively, where

$$\tilde{t}_1 = \left(\left[\rho_1^-,\ \rho_1^+\right],\ \left[\tau_1^-,\ \tau_1^+\right]\right) = ([0.2892,\ 0.4864],\ [0.3080,\ 0.4543]),$$

Table 1
A comparison of the preference orders of the alternatives of **Example 5.1** for different methods.

Methods	Preference order
Chen and Chiou's method [5]	$E_2 > E_3 > E_4 > E_1$
Li's method [13]	$E_2 > E_3 > E_4 > E_1$
Zhitao and Yingjun's method [39]	$E_2 > E_4 > E_3 > E_1$
The proposed method	$E_2 > E_3 > E_4 > E_1$

$$\tilde{t}_2 = \left(\left[\rho_2^-,\ \rho_2^+\right],\ \left[\tau_2^-,\ \tau_2^+\right]\right) = ([0.5457,\ 0.7000],\ [0.1776,\ 0.2778]),$$
$$\tilde{t}_3 = \left(\left[\rho_3^-,\ \rho_3^+\right],\ \left[\tau_3^-,\ \tau_3^+\right]\right) = ([0.4374,\ 0.6000],\ [0.2576,\ 0.3780]),$$
$$\tilde{t}_4 = \left(\left[\rho_4^-,\ \rho_4^+\right],\ \left[\tau_4^-,\ \tau_4^+\right]\right) = ([0.4071,\ 0.5749],\ [0.2659,\ 0.3971]).$$

[Step 5] Based on Eq. (25) and the obtained weighted evaluating IVIFVs $\tilde{t}_1$, $\tilde{t}_2$, $\tilde{t}_3$ and $\tilde{t}_4$ of the alternatives E_1, E_2, E_3 and E_4, respectively, we can get the transformed values t_1, t_2, t_3 and t_4 of the weighted evaluating IVIFVs $\tilde{t}_1$, $\tilde{t}_2$, $\tilde{t}_3$ and $\tilde{t}_4$, respectively, where $t_1 =$ 0.1567, $t_2 =$ 0.4734, $t_3 =$ 0.3551 and $t_4 =$ 0.3135. Because $t_2 > t_3 > t_4 > t_1$, we can see that the preference order of the alternatives E_1, E_2, E_3 and E_4 is: $E_2 > E_3 > E_4 > E_1$.

Table 1 shows a comparison of the preference orders of the alternatives of **Example 5.1** for different methods. From Table 1, we can see that Chen and Chiou's method [5], Li's method [13] and the proposed method get the same preference order of the alternatives, i.e., $E_2 > E_3 > E_4 > E_1$. Moreover, we also can see that the best alternative obtained by Zhitao and Yingjun's method [39] is E_2, which coincides with the results of Chen and Chiou's method [5], Li's method [13] and the proposed method.

Example 5.2. Assume that there are three "table tennis players" E_1, E_2, E_3 and assume that there are three attributes "Reaction", "Explosive Strength" and "Hand-Eye Coordination", assume that the IVIF weights $\tilde{w}_1$, $\tilde{w}_2$ and $\tilde{w}_3$ of the attributes "Reaction", "Explosive Strength" and "Hand-Eye Coordination" given by the decision maker represented by IVIFVs are $\tilde{w}_1 = ([0.25,\ 0.25],\ [0.25,\ 0.25])$, $\tilde{w}_2 = ([0.35,\ 0.35],\ [0.40,\ 0.40])$ and $\tilde{w}_3 = ([0.30,\ 0.30],\ [0.65,\ 0.65])$, respectively, and assume that the decision matrix $\tilde{D}$ given by the decision maker represented by IVIFVs is shown as follows:

$$\tilde{D} = \left(\tilde{\alpha}_{ij}\right)_{3\times 3} = \left(\left[\mu_{ij}^-,\ \mu_{ij}^+\right],\ \left[\nu_{ij}^-,\ \nu_{ij}^+\right]\right)_{3\times 3}$$
$$= \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \end{array} \begin{array}{c} \begin{array}{ccc} \text{Reaction} & \text{Explosive Strength} & \text{Hand} - \text{Eye Coordination} \end{array} \\ \begin{pmatrix} ([0.75,\ 0.75],\ [0.10,\ 0.10]) & ([0.60,\ 0.60],\ [0.25,\ 0.25]) & ([0.80,\ 0.80],\ [0.20,\ 0.20]) \\ ([0.80,\ 0.80],\ [0.15,\ 0.15]) & ([0.68,\ 0.68],\ [0.20,\ 0.20]) & ([0.45,\ 0.45],\ [0.50,\ 0.50]) \\ ([0.30,\ 0.30],\ [0.45,\ 0.45]) & ([0.70,\ 0.70],\ [0.05,\ 0.05]) & ([0.60,\ 0.60],\ [0.30,\ 0.30]) \end{pmatrix} \end{array}.$$

[Step 1] Based on Eq. (2), we can get the largest range $\bar{w}_j$ of the IVIF weight $\tilde{w}_j =$ $([a_j,\ b_j],\ [c_j,\ d_j])$ of attribute A_j given by the decision maker, where $\bar{w}_j = [a_j,\ 1 - c_j]$, $0 \le a_j \le (1 - c_j) \le 1$ and $1 \le j \le 3$. Because $\tilde{w}_1 = ([a_1,\ b_1],\ [c_1,\ d_1]) = ([0.10,\ 0.40], [0.20,\ 0.55])$, $\tilde{w}_2 = ([a_2,\ b_2],\ [c_2,\ d_2]) = ([0.20,\ 0.50],\ [0.15,\ 0.45])$ and $\tilde{w}_3 = ([a_3,\ b_3],\ [c_3,\ d_3]) = ([0.25,\ 0.60], [0.15,\ 0.38])$, we can get the largest ranges $\bar{w}_1$, $\bar{w}_2$ and $\bar{w}_3$ of the IVIF weights $\tilde{w}_1 = ([0.25,\ 0.25],\ [0.25,\ 0.25])$, $\tilde{w}_2 = ([0.35,\ 0.35],\ [0.40,\ 0.40])$ and $\tilde{w}_3 = ([0.30,\ 0.30], [0.65,\ 0.65])$ of the attributes A_1, A_2 and A_3, respectively, where

$$\bar{w}_1 = [0.25,\ 0.75],\ \bar{w}_2 = [0.35,\ 0.60] \text{ and } \bar{w}_3 = [0.30,\ 0.35].$$

[Step 2] Based on Eq. (13) and the decision matrix $\tilde{D} = (\tilde{\alpha}_{ij})_{3\times 3} =$ $([\mu_{ij}^-,\ \mu_{ij}^+],\ [\nu_{ij}^-,\ \nu_{ij}^+])_{3\times 3}$ given by the decision maker, we can get the transformed decision matrix $D = (d_{ij})_{3\times 3}$, where $d_{11} = 0.6000$, $d_{12} = 0.4500$, $d_{13} = 0.8000$, $d_{21} = 0.7500$, $d_{22} = 0.5600$, $d_{23} = 0.4000$, $d_{31} =$ 0.0500, $d_{32} =$ 0.4500 and $d_{33} =$ 0.5000. Based on Eq. (14), the obtained largest ranges $\bar{w}_1 = [0.25,\ 0.75]$, $\bar{w}_2 = [0.35,\ 0.60]$ and $\bar{w}_3 = [0.30,\ 0.35]$ of the IVIF weights $\tilde{w}_1 = ([0.25,\ 0.25],\ [0.25,\ 0.25]$, $\tilde{w}_2 = ([0.35,\ 0.35],\ [0.40,\ 0.40])$ and $\tilde{w}_3 = ([0.30,\ 0.30],\ [0.65,\ 0.65])$, respectively, and the obtained transformed decision matrix $D = (d_{ij})_{3\times 3}$, we can get the following linear programming model:

$$\max F = \sum_{i=1}^{3} \sum_{j=1}^{3} \left(w_j{}^* \times d_{ij}\right),$$
$$s.t. \begin{cases} 0.25 \le w_1{}^* \le 0.75 \\ 0.35 \le w_2{}^* \le 0.60 \\ 0.30 \le w_3{}^* \le 0.35 \\ w_1{}^* + w_2{}^* + w_3{}^* = 1 \end{cases}$$

where $w_1{}^*$, $w_2{}^*$ and $w_3{}^*$ are the optimal weights of the attributes "Reaction", "Explosive Strength" and "Hand-Eye Coordination", respectively.

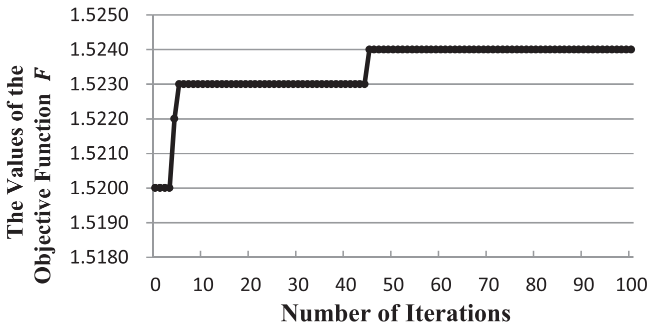

Fig. 6. The convergence process of the values of the objective function F of Example 5.2.

Table 2
A comparison of the preference orders of the "table tennis players" of ***Example 5.2*** for different methods.

Methods	Preference order
Chen and Chiou's method [5]	$E_2 > E_1 > E_3$
Li's method [13]	N/A
Zhitao and Yingjun's method [39]	$E_1 > E_2 > E_3$
The proposed method	$E_1 > E_2 > E_3$

Note: "N/A" denotes it cannot get the preference order of the "table tennis players" due to the fact that it has "the division by zero" problem.

[Step 3] By applying the PSO techniques to solve the programming model obtained in **Step 2**, we can obtain the optimal weights $w_1{}^*$, $w_2{}^*$ and $w_3{}^*$ of the attributes "Reaction", "Explosive Strength" and "Hand-Eye Coordination", respectively, where $w_1{}^* = 0.2507$, $w_2{}^* = 0.4184$ and $w_3{}^* = 0.3309$. The converge process of the values of the objective function F for obtaining the optimal weights $w_1{}^*$, $w_2{}^*$ and $w_3{}^*$ of the attributes "Reaction", "Explosive Strength" and "Hand-Eye Coordination", respectively, is shown in Fig. 6, where $w_1{}^* = 0.2507$, $w_2{}^* = 0.4184$ and $w_3{}^* = 0.3309$.

[Step 4] Based on Eqs. (21)–(24), the decision matrix $\tilde{D} = (\tilde{\alpha}_{ij})_{3\times3} = ([\mu_{ij}^-,\ \mu_{ij}^+],\ [\nu_{ij}^-,\ \nu_{ij}^+])_{3\times3}$ given by the decision maker, and the obtained optimal weights $w_1{}^*$, $w_2{}^*$ and $w_3{}^*$ of the attributes "Reaction", "Explosive Strength" and "Hand-Eye Coordination", respectively, where $w_1{}^* = 0.2507$, $w_2{}^* = 0.4184$ and $w_3{}^* = 0.3309$, we can get $\rho_1^- = 0.6979$, $\rho_1^+ = 0, 6979$, $\tau_1^- = 0.1980$, $\tau_1^+ = 0.1980$, $\rho_2^- = 0.6178$, $\rho_2^+ = 0.6178$, $\tau_2^- = 0.3047$, $\tau_2^+ = 0.3047$, $\rho_3^- = 0.5379$, $\rho_3^+ = 0.5379$, $\tau_3^- = 0.2513$ and $\tau_3^+ = 0.2513$. Therefore, we can get the weighted evaluating IVIFVs $\tilde{t}_1$, $\tilde{t}_2$ and $\tilde{t}_3$ of the "table tennis players" E_1, E_2 and E_3, respectively, where

$$\tilde{t}_1 = ([\rho_1^-,\ \rho_1^+],\ [\tau_1^-,\ \tau_1^+]) = ([0.6979,\ 0.6979],\ [0.1980,\ 0.1980]),$$

$$\tilde{t}_2 = ([\rho_2^-,\ \rho_2^+],\ [\tau_2^-,\ \tau_2^+]) = ([0.6178,\ 0.6178],\ [0.3047,\ 0.3047]),$$

$$\tilde{t}_3 = ([\rho_3^-,\ \rho_3^+],\ [\tau_3^-,\ \tau_3^+]) = ([0.5379,\ 0.5379],\ [0.2513,\ 0.2513]),$$

[Step 5] Based on Eq. (25) and the obtained weighted evaluating IVIFVs $\tilde{t}_1$, $\tilde{t}_2$ and $\tilde{t}_3$ of the "table tennis players" E_1, E_2 and E_3, respectively, we can get the transformed values t_1, t_2 and t_3 of weighted evaluating IVIFVs $\tilde{t}_1$, $\tilde{t}_2$ and $\tilde{t}_3$, respectively, where $t_1 = 0.5938$, $t_2 = 0.5403$ and $t_3 = 0.3271$. Because $t_1 > t_2 > t_3$, we can see that the preference order of the "table tennis players" E_1, E_2 and E_3 is: $E_1 > E_2 > E_3$.

Table 2 shows a comparison of the preference orders of the "table tennis players" of **Example 5.2** for different methods. From Table 2, we can see that Zhitao and Yingjun's method [39] and the proposed method get the same preference order of the "table tennis players", i.e., $E_1 > E_2 > E_3$, whereas Chen and Chiou's method [5] has the drawback that it gets an unreasonable preference order of the "table tennis players" in this situation; Li's method [13] has the drawback that it cannot get the preference order of the "table tennis players" in this situation due to the fact that it has "the division by zero" problem.

Example 5.3. Assume that there are three "table tennis players" E_1, E_2, E_3, assume that there are three attributes "Reaction", "Explosive Strength" and "Hand-Eye Coordination", and assume that the IVIF weights $\tilde{w}_1$, $\tilde{w}_2$ and $\tilde{w}_3$ of the attributes "Reaction", "Explosive Strength" and "Hand-Eye Coordination" given by the decision maker represented by IVIFVs are $\tilde{w}_1 = ([0,\ 0],\ [0,\ 0])$, $\tilde{w}_2 = ([0,\ 0],\ [0,\ 0])$ and $\tilde{w}_3 = ([0,\ 0],\ [0,\ 0])$, respectively, That is, the IVIF weights of the attributes "Reaction", "Explosive Strength" and "Hand-Eye Coordination" are unknown, respectively. Assume that the decision matrix $\tilde{D}$ given by the decision maker represented by IVIFVs is shown as follows:

$$\tilde{D} = \left(\tilde{\alpha}_{ij}\right)_{3\times 3} = \left(\left[\mu_{ij}^{-},\ \mu_{ij}^{+}\right],\ \left[\nu_{ij}^{-},\ \nu_{ij}^{+}\right]\right)_{3\times 3}$$

$$= \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \end{array} \begin{array}{c} \begin{array}{ccc} \text{Reaction} & \text{Explosive Strength} & \text{Hand – Eye Coordination} \end{array} \\ \begin{pmatrix} ([0.5,\ 0.5],\ [0.5,\ 0.5]) & ([0.5,\ 0.5],\ [0.5,\ 0.5]) & ([0.5,\ 0.5],\ [0.5,\ 0.5]) \\ ([0.5,\ 0.5],\ [0.5,\ 0.5]) & ([0.5,\ 0.5],\ [0.5,\ 0.5]) & ([0.5,\ 0.5],\ [0.5,\ 0.5]) \\ ([0.5,\ 0.5],\ [0.5,\ 0.5]) & ([0.5,\ 0.5],\ [0.5,\ 0.5]) & ([0.5,\ 0.5],\ [0.5,\ 0.5]) \end{pmatrix} \end{array}.$$

[Step 1] Based on Eq. (2), we can get the largest range $\bar{w}_j$ of the IVIF weight $\tilde{w}_j = ([a_j,\ b_j],\ [c_j,\ d_j])$ of attribute A_j given by the decision maker, where $\bar{w}_j = [a_j,\ 1 - c_j]$, $0 \le a_j \le (1 - c_j) \le 1$ and $1 \le j \le 3$. Because $\tilde{w}_1 = ([a_1,\ b_1],\ [c_1,\ d_1]) = ([0,\ 0], [0,\ 0])$, $\tilde{w}_2 = ([a_2,\ b_2],\ [c_2,\ d_2]) = ([0,\ 0],\ [0,\ 0])$ and $\tilde{w}_3 = ([a_3,\ b_3],\ [c_3,\ d_3]) = ([0,\ 0],\ [0,\ 0])$, we can get the largest ranges $\bar{w}_1$, $\bar{w}_2$ and $\bar{w}_3$ of the IVIF weights $\tilde{w}_1 = ([0,\ 0],\ [0,\ 0])$, $\tilde{w}_2 = ([0,\ 0],\ [0,\ 0])$ and $\tilde{w}_3 = ([0,\ 0],\ [0,\ 0])$ of the attributes A_1, A_2 and A_3, respectively, where $\bar{w}_1 = [0,\ 1]$, $\bar{w}_2 = [0,\ 1]$ and $\bar{w}_3 = [0,\ 1]$.

[Step 2] Based on Eq. (13) and the decision matrix $\tilde{D} = (\tilde{\alpha}_{ij})_{3\times 3} = ([\mu_{ij}^{-},\ \mu_{ij}^{+}],\ [\nu_{ij}^{-},\ \nu_{ij}^{+}])_{3\times 3}$ given by the decision maker, we can get the transformed decision matrix $D = (d_{ij})_{3\times 3}$, where $d_{11} = 0.5000$, $d_{12} = 0.5000$, $d_{13} = 0.5000$, $d_{21} = 0.5000$, $d_{22} = 0.5000$, $d_{23} = 0.5000$, $d_{31} = 0.5000$, $d_{32} = 0.5000$ and $d_{33} = 0.5000$. Based on Eq. (14), the obtained largest ranges $\bar{w}_1 = [0,\ 1]$, $\bar{w}_2 = [0,\ 1]$ and $\bar{w}_3 = [0,\ 1]$ of the IVIF weights $\tilde{w}_1 = ([0,\ 0],\ [0,\ 0])$, $\tilde{w}_2 = ([0,\ 0],\ [0,\ 0])$ and $\tilde{w}_3 = ([0,\ 0],\ [0,\ 0])$, respectively, and the obtained transformed decision matrix $D = (d_{ij})_{3\times 3}$, we can get the following linear programming model:

$$\max F = \sum\nolimits_{i=1}^{3} \sum\nolimits_{j=1}^{3} \left(w_j{}^{*} \times d_{ij}\right),$$

$$s.t. \begin{cases} 0 \le w_1{}^{*} \le 1 \\ 0 \le w_2{}^{*} \le 1 \\ 0 \le w_3{}^{*} \le 1 \\ w_1{}^{*} + w_2{}^{*} + w_3{}^{*} = 1 \end{cases}$$

where $w_1{}^{*}$, $w_2{}^{*}$ and $w_3{}^{*}$ are the optimal weights of the attributes “Reaction”, “Explosive Strength” and “Hand-Eye Coordination”, respectively.

[Step 3] By applying the PSO techniques to solve the programming model obtained in **Step 2**, we can obtain the optimal weights $w_1{}^{*}$, $w_2{}^{*}$ and $w_3{}^{*}$ of the attributes “Reaction”, “Explosive Strength” and “Hand-Eye Coordination”, respectively, where $w_1{}^{*} = 0.6894$, $w_2{}^{*} = 0.2525$ and $w_3{}^{*} = 0.0581$, where the value of the objective function F for each iteration does not change.

[Step 4] Based on Eqs. (21)–(24), the decision matrix $\tilde{D} = (\tilde{\alpha}_{ij})_{3\times 3} = ([\mu_{ij}^{-},\ \mu_{ij}^{+}],\ [\nu_{ij}^{-},\ \nu_{ij}^{+}])_{3\times 3}$ given by the decision maker, and the obtained optimal weights $w_1{}^{*}$, $w_2{}^{*}$ and $w_3{}^{*}$ of the attributes “Reaction”, “Explosive Strength” and “Hand-Eye Coordination”, respectively, where $w_1{}^{*} = 0.6894$, $w_2{}^{*} = 0.2525$ and $w_3{}^{*} = 0.0581$, we can get $\rho_1^{-} = 0.5000$, $\rho_1^{+} = 0.5000$, $\tau_1^{-} = 0.5000$, $\tau_1^{+} = 0.5000$, $\rho_2^{-} = 0.5000$, $\rho_2^{+} = 0.5000$, $\tau_2^{-} = 0.5000$, $\tau_2^{+} = 0.5000$, $\rho_3^{-} = 0.5000$, $\rho_3^{+} = 0.5000$, $\tau_3^{-} = 0.5000$ and $\tau_3^{+} = 0.5000$. Therefore, we can get the weighted evaluating IVIFVs $\tilde{t}_1$, $\tilde{t}_2$ and $\tilde{t}_3$ of the “table tennis players” E_1, E_2 and E_3, respectively, where

$$\tilde{t}_1 = \left(\left[\rho_1^{-},\ \rho_1^{+}\right],\ \left[\tau_1^{-},\ \tau_1^{+}\right]\right) = ([0.5000,\ 0.5000],\ [0.5000,\ 0.5000]),$$

$$\tilde{t}_2 = \left(\left[\rho_2^{-},\ \rho_2^{+}\right],\ \left[\tau_2^{-},\ \tau_2^{+}\right]\right) = ([0.5000,\ 0.5000],\ [0.5000,\ 0.5000]),$$

$$\tilde{t}_3 = \left(\left[\rho_3^{-},\ \rho_3^{+}\right],\ \left[\tau_3^{-},\ \tau_3^{+}\right]\right) = ([0.5000,\ 0.5000],\ [0.5000,\ 0.5000]).$$

[Step 5] Based on Eq. (25) and the obtained the weighted evaluating IVIFVs $\tilde{t}_1$, $\tilde{t}_2$ and $\tilde{t}_3$ of the “table tennis players” E_1, E_2 and E_3, respectively, we can get the transformed values t_1, t_2 and t_3 of the weighted evaluating IVIFVs $\tilde{t}_1$, $\tilde{t}_2$ and $\tilde{t}_3$, respectively, where $t_1 = 0.5000$, $t_2 = 0.5000$ and $t_3 = 0.5000$. Because $t_1 = t_2 = t_3$, we can see that the preference order of the “table tennis players” x_1, x_2 and x_3 is: $E_1 = E_2 = E_3$.

Table 3 shows a comparison of the preference orders of the “table tennis players” of **Example 5.3** for different methods. From Table 3, we can see that Chen and Chiou’s method [5] and the proposed method obtain the same preference order of the “table tennis players”, i.e., $E_1 = E_2 = E_3$, whereas Zhitao and Yingjun’s method [39] and Li’s method [13] cannot obtain the preference order of the “table tennis players” in this situation due to the fact that they have “the division by zero” problem.

6. Conclusions

In this paper, we have proposed a new MADM method based on the IIFWGA operator of IVIFVs, the accuracy function of IVIFVs and PSO techniques. The proposed method uses PSO techniques to get the optimal weights of the attributes, calculates the weighted evaluating IVIFV of each alternative based on the IIFWGA operator, the obtained optimal weights

Table 3
A comparison of the preference orders of the “table tennis players” of ***Example 5.3*** for different methods.

Methods	Preference order
Chen and Chiou’s method [5]	$E_1 = E_2 = E_3$
Li’s method [13]	N/A
Zhitao and Yingjun’s method [39]	N/A
The proposed method	$E_1 = E_2 = E_3$

Note: “N/A” denotes it cannot get the preference order of the “table tennis players” due to the fact that it has “the division by zero” problem.

of the attributes and the decision matrix given by the decision maker, and then calculates the transformed value of the weighted evaluating IVIFV of each alternative based on the accuracy function of IVIFVs to obtain the preference order of the alternatives. From Tables 1, 2 and 3, we can see that the proposed method can overcome the drawbacks of the methods presented in [5,13,39]. The main contribution of this paper is that we propose a new MADM method based on the IIFWGA operator of IVIFVs, the accuracy function of IVIFVs and PSO techniques, which can overcome the drawbacks of the MADM methods presented in [5,13] and [39] for MADM in IVIF environments. In the future, we will apply granular computing techniques [3,9,14,15,17,19,20,29,30,40] to develop simpler MADM methods for dealing with MADM problems.

Acknowledgements

The authors would like to thank Professor Shou-Hsiung Cheng, Chienkuo Technology University, Changhua, Taiwan, for his helpful suggestions during this work. This work is supported by the Ministry of Science and Technology, Republic of China, under Grant MOST 104-2221-E-011-084-MY3.

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