FUZZY LOGIC CONTROLLER DESIGN FOR PH-CONTROL IN A CSTR

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Abstract: In this paper two different approaches for the design of fuzzy logic controllers for the control of pH in a laboratory-scale CSTR are shown and compared. First a fuzzy controller was designed based on pure heuristics; the design process was fairly demanding due to the complexity of the system. Second, a fuzzy controller was identified based on a time-optimal controller derived from a coarse mathematical model. After addition of some heuristic rules to the rulebase which increase the robustness, this fuzzy controller shows an excellent performance.

Keywords: pH-control, fuzzy control, fuzzy modelling, time-optimal control.

1. INTRODUCTION

In the chemical industry one often encounters processes with considerable nonlinearity and unknown interactions. An example of such a process is the titration in a continuous stirred tank reactor (CSTR) which has strong gain nonlinearity. Hence common linear control laws (PID) are not able to achieve acceptable control performance. Fuzzy controllers, on the other hand, are known for their inherent nonlinearity which might enable them to handle nonlinear systems better.

The starting point of our research was an existing CSTR in laboratory scale. Our goal in the controller design was to get short regulation times for set point changes as well as good performance in rejecting disturbances. A coarse mathematical model was used to simulate the behaviour of the system. For the design of the fuzzy logic controllers we compared two different approaches. First we used our engineering common sense in order to develop a fuzzy rule base heuristically. Because of the complex system dynamics, this approach - which is the most commonly used in the design of fuzzy logic controllers - turned out to be more difficult than expected. Fuzzy controllers offer large flexibility to build nonlinear relationships, but clear systematics how to compose a rulebase based purely on heuristics are not available. Therefore the performance of such a controller is strongly dependent on the capability of the designer and the CAD-tool.

Our second approach was to make use of a coarse mathematical model of the plant. In order to build a fuzzy logic controller automatically, we computed a time-optimal controller first which is used as a reference to identify the kernel of the rulebase. By adding further rules to the automatically generated basic fuzzy controller in order to cope with modelling errors and measurement noise, it was possible to obtain excellent controller performance in simulations as well as at the actual plant.

This paper is divided into six sections. In section 2 we will describe the plant and its model. Then in section 3 we illustrate the problems encountered in the design of a fuzzy controller based on heuristics. Section 4 will briefly describe the identification of fuzzy controllers approximating a given optimal control law. The model based approach which includes the time-optimal controller is then discussed for the pH-control problem in section 5. Conclusions are given in section 6.

2. DESCRIPTION AND MODELLING OF THE PLANT

The CSTR which we want to control consists of a tank with two inlets and one outlet as shown in Fig. 1. The physical parameters are listed in Table 1.

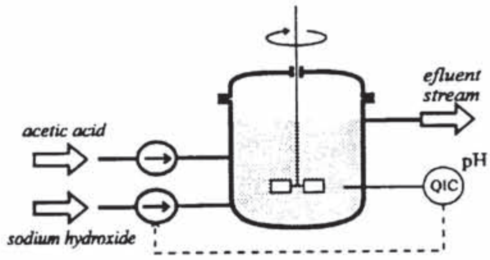

Fig 1: A CSTR for pH-control

Table 1: Physical parameters of the CSTR

Variable	Meaning	Setting
V	volume of tank	5.7 l
F_1	flow of acetic acid	20 l/h
F_2	flow of sodium hydroxide	[0, 45] l/h
$c_{1,e}$	concentration of acid in F_1	0.007 mol/l
$c_{2,e}$	concentration of hydroxide in F_2	0.01 mol/l
K_a	acid equilibrium constant	1.8×10^{-5}
K_w	water equilibrium constant	10^{-14}

The base flow rate F_2 is used to control the pH value of the solution. The acid flow is held constant or acts as a disturbance. We use diaphragm pumps to obtain the necessary precision of the flow rates. All control actions are performed by a commercial PC-based process control system which has a minimal sampling period of 1s. The sodium hydroxide flow is controlled by modulating an impulse frequency, which leads to a quantisation of the control amplitude because the frequency can assume only certain discrete values.

The mathematical equations of the CSTR can be derived as follows (McAvoy, 1972):

$$\frac{d(c_1)}{dt} = \frac{1}{V_R}\left[F_1 \cdot c_{1,e} - (F_1 + F_2)\cdot c_1\right] \tag{1}$$

$$\frac{d(c_2)}{dt} = \frac{1}{V_R}\left[F_2 \cdot c_{2,e} - (F_1 + F_2)\cdot c_2\right] \tag{2}$$

$$c_{H^+}^3 + c_{H^+}^2 \cdot (K_{HAc} + c_2) + c_{H^+} \cdot \left[K_{HAc}\cdot(c_2 - c_1) - K_W\right] - K_{HAc}\cdot K_W = 0 \tag{3}$$

$$pH = -\log[c_{H^+}] \ . \tag{4}$$

Due to the incomplete dissociation of acetic acid in water and the equilibrium reaction with sodium acetate, the system behaves like a buffer solution between pH 4 and pH 6.5. Consequently, the process gain varies greatly over the range of pH-values which can be controlled (see Fig. 2).

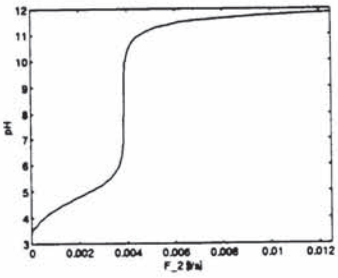

Fig. 2: The steady-state, nonlinear relationship of pH versus base flow rate.

The electrode which measures the pH in the tank exhibits a time delay with a time constant T_{el}=1.4s:

$$\frac{d(pH_{EL})}{dt} = \frac{-\log(c_{H^+}) - pH_{EL}}{T_{EL}} \tag{5}$$

Experimental step responses of the base flow rate showed immediately that the mixing in the tank is far from ideal. Even though the blade mixer is fairly powerful, different mixing zones in the tank exist which result in additional time delays and dead times. We decided not to try to build an accurate model of this very complex behaviour, but rather to lump the dynamical effects from the non-ideal mixing with the time delay of the electrode into one time constant, which was roughly estimated as $T_{El}^* = 10s$.

The difficulty of the control problem can be seen from the transients obtained with a fixed PI-controller tuned at pH 7, as shown in Fig. 3. This poor controller performance motivated us to design nonlinear fuzzy logic controllers.

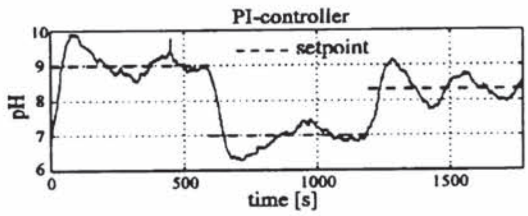

Fig. 3: Performance of PI-control (experiment)

3. FUZZY CONTROLLER DESIGN BASED ON HEURISTICS ALONE

The key to successful fuzzy controller design is the existence of heuristic knowledge which has been acquired by operators over an extensive period of manual control; this knowledge can then be modeled more or less easily by a fuzzy rulebase. For our plant we lacked such a source of knowledge for the design process. Therefore it was reasonable to implement the mathematical model in a fuzzy logic controller design tool to develop a fuzzy controller based on simulations instead of time-consuming on-line training.

First the regulation error, its derivative (here entitled dpH) and the change in the base flow rate were chosen as the controller inputs and outputs respectively. In the next step their range has to be partitioned into qualitative values which is known as fuzzification (see Fig. 4).

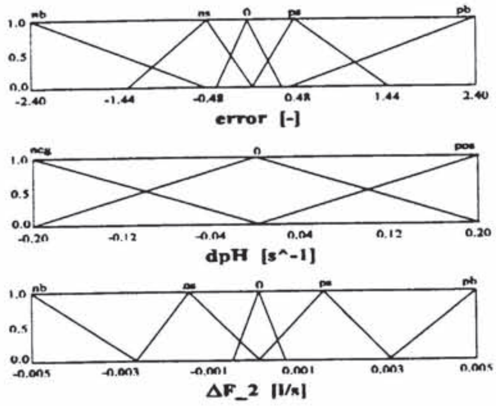

Fig. 4 Fuzzification of controller inputs and output

After an initial set of rules has been formulated, the behaviour of the system can be evaluated in simulations. Now the designer is challenged to change the rules and/or the fuzzification so that the system shows the desired behaviour. This procedure is characterized by a trial and error approach. Due to the numerous degrees of freedom in the fuzzy rulebase, this process can be very time-consuming. Fig. 5 shows typical behaviour of the system for a set of transients, illustrating the difficulties with such an approach for a complex system; even though the rules have been chosen symmetrically (see Fig. 6) the controller may show an excellent response for an initial set point change while for other set point changes the controller performs poorly. The designer has to change the fuzzification and the rules iteratively in order to obtain an acceptable behaviour over the whole range of operation. The success of the controller design is strongly dependent on the capability of the designer and the support provided by the tool which he is using. For example, the possibility to analyze which rules are triggered in every time step of the simulation is indispensable.

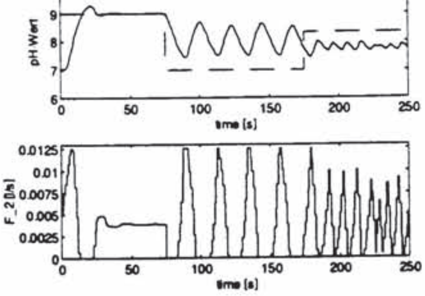

Fig. 5: Responses with fuzzy controller with 9 rules (simulation)

If *pH-Wert any* and ... then *ΔF_2*

↑ *error*	neg	0	pos ← *dpH*
nb	*ns*		*nb*
ns	*pb*		*ns*
0		*0*	
ps	*ps*		*nb*
pb	*pb*		*ps*

Fig. 6: Rulebase for fuzzy controller (9 rules)

To improve the transient behaviour of the fuzzy controller, it is necessary to use the pH value itself as an input as well. Three fuzzy values (*acidic*, *neutral* and *basic)* were chosen to characterize high and low gain regions of the plant. After a long time spent manipulating the rules and the fuzzification, a rulebase resulted with 29 rules which shows acceptable responses for set point changes (Chen, 1994) (see Fig. 7).

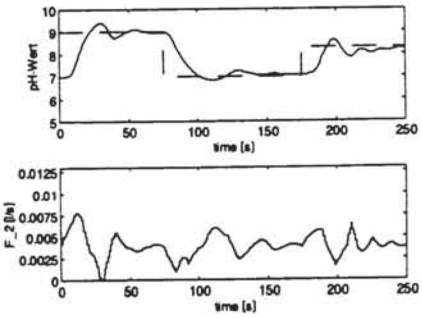

Fig 7: Responses with fuzzy controller with 29 rules (simulation)

4. APPROXIMATION OF OPTIMAL CONTROLLERS BY A FUZZY SYSTEM

The above section has shown that the purely heuristic approach is restricted to plants where either extensive operator knowledge is available or the dynamics of the system are not too complex. Since a coarse mathematical model is often available, the question arises how this information can be used directly for fuzzy logic controller design. The main idea is to compute a model-based optimal controller which can be used to identify the kernel of the fuzzy rulebase. In order to efficiently approximate an analytical control law, it is advantageous to choose a special format of the fuzzy rules which was first introduced by Takagi and Sugeno (1985):

Rule i: IF x_1 is A_1^i, x_2 is A_2^i, ..., x_m is A_m^i

THEN $y_j = p_0^i + p_1^i \cdot x_1 + p_2^i \cdot x_2 + ... + p_m^i \cdot x_m$

Here the output is determined by a linear function of the controller inputs x_j and not expressed by fuzzy values. If the fuzzy sets of the input variables have sufficient overlap, a soft interpolation of linear control laws results. The final output y^* is computed easily as the weighted mean of the individual outputs y_i of the rules:

$$y^* = \frac{\sum_{i=1}^{n} \min_j \mu A_j^i(x_j) \cdot y_i}{\sum_{i=1}^{n} \min_j \mu A_j^i(x_j)} \quad . \tag{6}$$

$\mu A_m^i(x_m)$ is the degree of membership of the (crisp) input x_m to the linguistic variable A_m^i. The parameters p_j^i, which determine the consequents of the rules, can be optimized by a least squares approximation of a priori given optimal trajectories. The interested reader is referred to (Heckenthaler and Engell, 1994) for details. The goal of the optimization is to approximate the model based controller characteristic with a minimum number of rules with sufficient accuracy. Therefore it is helpful to optimize the fuzzification as well, i.e. the coordinates of the membership functions. This can be accomplished using multidimensional minimization strategies like the downhill simplex method. To avoid getting trapped in local minima it is advisable to connect them with genetic algorithms.

5. MODEL BASED FUZZY CONTROLLER DESIGN

First we had to choose an optimal controller for this system. Observing the control actions of an experienced operator controlling a plant manually, it is often found that he or she is using the available range of the control variables to obtain quick responses for large deviations. For small ones, on the other hand, he operates more carefully to avoid "bang-bang"-type control. Thus the time-optimal controller seemed to be the right choice for large deviations, and a robust linear controller was used for small deviations.

To compute the time-optimal controller we took the following approach: since two state variables (c_1 and c_2) are not observable, the concentration c_2 is assumed to have a fixed value (5mmol/l) at the beginning of the sampling time. Equation (3) then yields along with the measured pH value an estimate of c_1. Now the state equations (1), (2) and (4) can be integrated for the next sampling interval with a maximal control action F_2, so that the pH value in the tank reaches the set point as fast as possible without overshoot (see Fig. 8). This controller is of course only time-optimal with respect to the assumed pH value in the tank and not with respect to the measured one.

Using this algorithm, 660 trajectories were computed such that the ranges of controller input space (pH, error, dpH) were covered well. Now a fuzzy controller was identified which has the format described in section 4.

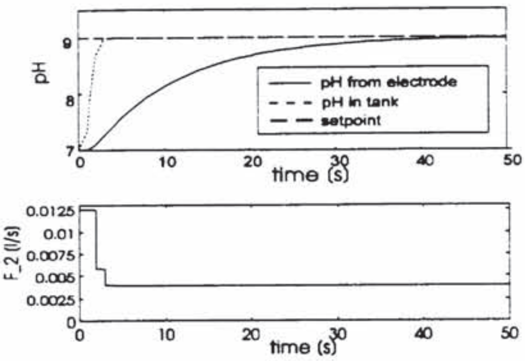

Fig. 8: Time-optimal controller for nominal model

An example of a rule is:

Rule i: If pH is *acidic* and error is *pb* and dpH is *ns*, then $F_2 = p_0^i + p_1^i \cdot pH + p_2^i \cdot error + p_3^i \cdot dpH$

The optimization of the rulebase showed that a fuzzy controller with 16 rules already yields a sufficient approximation of the control law (see Fig 9). A more exact approximation is possible with an increase in the number of rules, but this is not recommended because the model itself is inaccurate. In Fig. 10 the resulting fuzzy values are depicted with solid lines.

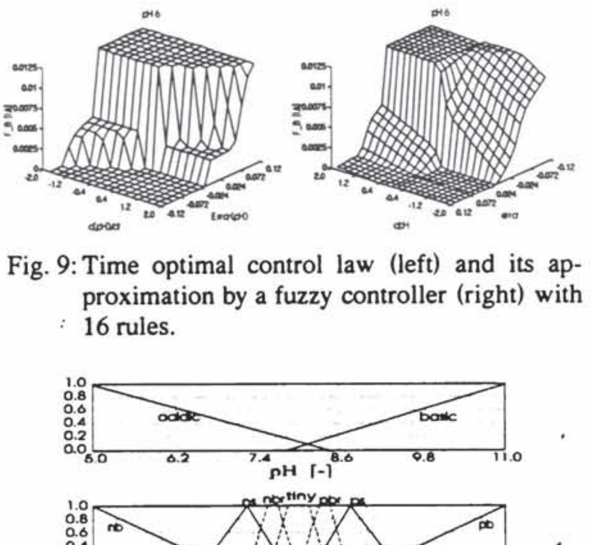

Fig. 9: Time optimal control law (left) and its approximation by a fuzzy controller (right) with 16 rules.

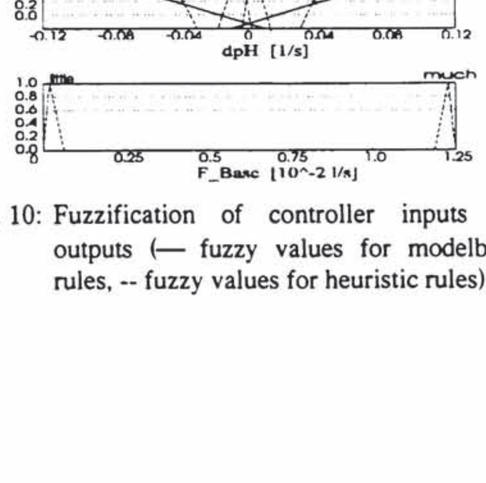

Fig. 10: Fuzzification of controller inputs and outputs (— fuzzy values for modelbased rules, -- fuzzy values for heuristic rules).

Fig. 11 shows the resulting performance for a set point change. As expected, the regulation time is very short without overshoot. The fuzzy controller shows the same deficiencies as its origin, the time-optimal controller, in correcting small deviations with hectic, ineffective control actions. It is therefore necessary to use a more cautious, linear controller for small control errors. This can be done conveniently by adding one rule to the fuzzy rulebase:

Rule 17: If pH is *any* and error is *tiny* and
dpH is *any*, then F_2=a+b·pH +c·error +d·dpH

The parameters a and b were found by approximating the titration curve (Fig. 2) in the manner described in section 4. Actually at least 2 rules for pH values *acidic* and *basic* are necessary to cope with the nonlinearity. The parameters c and d are the gains for the proportional and derivative action which were found experimentally. Fig. 12 shows the effect of these 2 additional rules. The controller actions for small deviations are now much smoother and a steady state is reached.

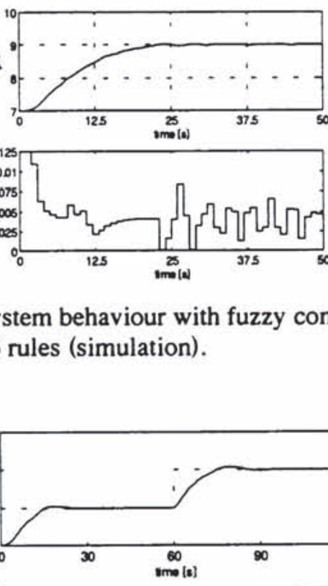

Fig. 11: System behaviour with fuzzy controller with 16 rules (simulation).

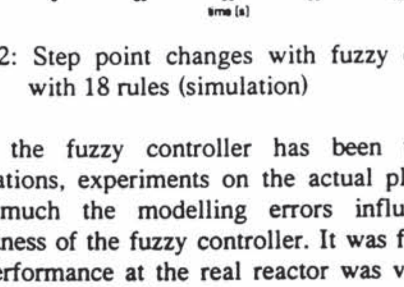

Fig 12: Step point changes with fuzzy controller with 18 rules (simulation)

After the fuzzy controller has been tested in simulations, experiments on the actual plant show how much the modelling errors influence the robustness of the fuzzy controller. It was found that the performance at the real reactor was very good, too. A small overshoot occurred for large set point changes. This observation led to the formulation of two more rules, which are triggered when the pH approaches the set point too quickly:

Rule 19: If pH *any* and error *p_br* and dpH *p_br*,
then F_2 *little*
Rule 20: If pH *any* and error *n_br* and dpH *n_br*,
then F_2 *much.*

Fig. 13 shows the performance at the real plant. The chattering of the controller output at steady state is caused by the measurement noise and the quantisation of the controller output.

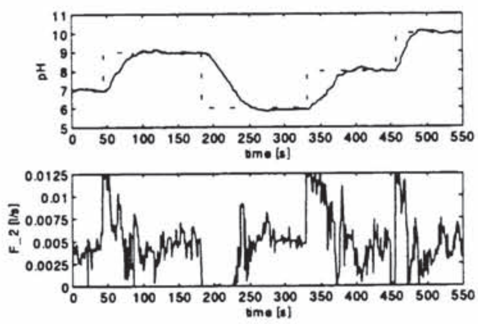

Fig 13: Step point changes with fuzzy controller with 20 rules (experiment)

The rejection of disturbances is very important in pH control. Therefore the fuzzy controller was tested while the concentration of the acetic acid was reduced to 50% from 0.007 to 0.0035 mol/*l* at t=0s (see Fig. 14). Compared to the strong disturbance, only a relatively small remaining regulating error occurs. This is because the time-optimal control characteristic is very steep.

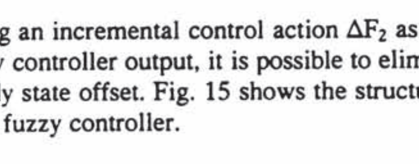

Fig. 14: Response to disturbance c_1(t=0s)=0.0035 mol/*l* with fuzzy controller (20 rules) (simulation)

Using an incremental control action ΔF_2 as a second fuzzy controller output, it is possible to eliminate the steady state offset. Fig. 15 shows the structure of the final fuzzy controller.

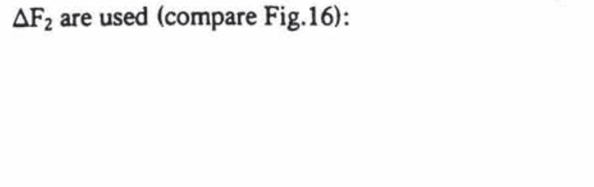

Fig. 15: Structure of control loop

To handle the disturbances, three rules for the output ΔF_2 are used (compare Fig.16):

Rule 21: If pH *any* and error *any* and dpH *any*, *then* ΔF_2 *zero*

Rule 22: If pH *any* and error *p_br* and dpH *nt*, *then* ΔF_2 *more*

Rule 23: If pH *any* and error *n_br* and dpH *pt*, then ΔF_2 *less*.

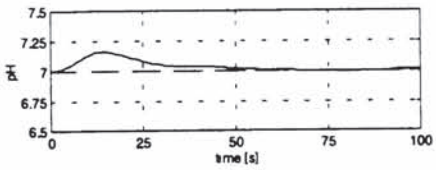

Fig. 16 Response to disturbance $c_1(t=0s)=0.0035$ mol/*l* with fuzzy controller (23 rules) (simulation)

The effectiveness of these rules can be evaluated from the transients depicted in Fig. 17, where the same disturbance occurred at t=0s. While the first set point change causes a small overshoot, the controller adapts to the disturbance in a comparatively short time.

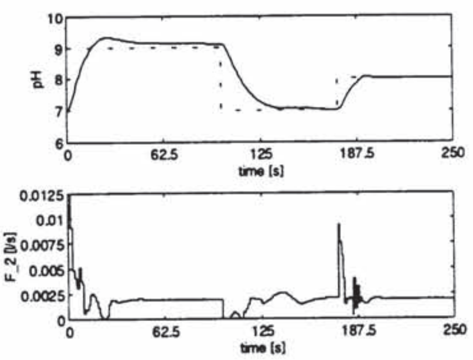

Fig. 17: Response with fuzzy controller (23 rules) with permanent disturbance $c_1(t=0s)=0.0035$ mol/*l* (simulation)

6. CONCLUSIONS

The design of two different fuzzy controllers has shown that fuzzy technology offers a powerful tool to design nonlinear controllers. The complexity of the neutralization plant showed the limits of a purely heuristic approach, which is strongly dependent on existing operator knowledge and the skill of the designer. Heuristic knowledge for complex systems takes a long time to acquire, and it is difficult to evaluate the quality of the control.

The automatic design of the fuzzy controller based on model information on the other hand offers a systematic design procedure. First the user has to formulate a coarse mathematical model and then an optimal controller is computed for this model. We decided to use time-optimal control which was in our case not very hard to compute. Often the computation of a time-optimal control law for nonlinear systems will be difficult to determine. A promising general approach to compute time-optimal trajectories for nonlinear systems is the cell state space method introduced by Hsu (1985). This optimization procedure requires large memory and computational effort, but the resulting identified time-optimal cell trajectories can be used directly to identify the kernel of the rulebase off-line. The on-line computational effort to determine the output of the fuzzy rulebase, on the other hand, is fairly small, so that on-line control is possible. We are currently investigating and refining this approach for nolinear systems in order to implement it into our design tool (FFUN-tool).

The pH-control of a laboratory CSTR with the model based fuzzy controller has shown that it is advantageous to use a coarse mathematical model as a starting point for the design of the rulebase. Adding heuristic knowledge to a set of model based fuzzy rules is according to our experience simple and effective to correct the deficiencies of the approximated time-optimal behaviour. This increases the robustness of the fast fuzzy controller significantly. The fuzzy rulebase provides a flexible platform to realize different control strategies, which can be integrated efficiently and smoothly.

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