Attenuation Requirements and Effective Noise Temperature for Qubit Drive Lines in a Dilution Refrigerator

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1 Motivation and model

Superconducting qubits and resonators are very sensitive to thermal photons in their drive lines. At frequency f the relevant scale is $\hbar\omega/k_B$ with $\omega = 2\pi f$. For a typical qubit at $f \approx 6\,\mathrm{GHz}$ this corresponds to $\hbar\omega/k_B \approx 0.288\,\mathrm{K}$, so even tens of millikelvin of effective noise temperature produce a non-negligible thermal population.

The mean thermal photon number of a bosonic mode at temperature T is

$$\bar{n}(T) = \frac{1}{e^{\hbar\omega/k_B T} - 1}. \tag{1}$$

A passive attenuator with linear attenuation $L > 1$ (for example $L = 100$ for $20\,\mathrm{dB}$) at physical temperature T_{att} can be modeled as a beam splitter that mixes the incoming field with Johnson-Nyquist noise at T_{att} [1]. The output occupation is

$$\bar{n}_{\mathrm{out}} = \frac{\bar{n}_{\mathrm{in}}}{L} + \left(1 - \frac{1}{L}\right)\bar{n}(T_{\mathrm{att}}). \tag{2}$$

This is the standard model used in practical wiring guides for cQED experiments [2, 3].

We consider a simple three-stage attenuation chain between room temperature and the mixing chamber (MXC):

$$\begin{aligned}
T_0 &= 300\,\mathrm{K} && \text{(room)},\\
T_1 &= 4\,\mathrm{K} && \text{(4 K plate)},\\
T_2 &= 0.10\,\mathrm{K} && \text{(cold plate or still)},\\
T_3 &= 0.020\,\mathrm{K} && \text{(mixing chamber)},
\end{aligned}$$

with attenuations L_1, L_2, L_3 at T_1, T_2, T_3 respectively, fed from T_0.

Repeated application of Eq. (2) gives the effective occupation at the MXC output:

$$\bar{n}_{\mathrm{eff}} = \bar{n}(T_3) + \frac{\bar{n}(T_2)}{L_3} + \frac{\bar{n}(T_1)}{L_2 L_3} + \frac{\bar{n}(T_0)}{L_1 L_2 L_3}, \tag{3}$$

with total attenuation

$$L_{\mathrm{tot}} = L_1 L_2 L_3, \qquad A_{\mathrm{tot}}^{(\mathrm{dB})} = 10\log_{10} L_{\mathrm{tot}}. \tag{4}$$

Assumptions. Throughout we assume:

- Narrowband consideration around a single frequency f (for example the qubit drive frequency).
- Each attenuator is well thermalized to the refrigerator stage where it is mounted and can be treated as a lumped element at that temperature.
- The line is well matched (no strong reflections), so we can ignore standing-wave effects.
- Cable losses between stages are either small or can be modeled as additional attenuation at the temperature of the stage where most of the dissipation occurs.

2 Standard 60 dB attenuation chain

We first evaluate the thermal population for a typical configuration with three 20 dB attenuators, so that

$$A_1 = A_2 = A_3 = 20\,\text{dB}, \qquad L_1 = L_2 = L_3 = 10^2, \qquad A_{\text{tot}}^{(\text{dB})} = 60\,\text{dB}. \tag{5}$$

At $f = 6\,\text{GHz}$ we have $\hbar\omega/k_B \approx 0.288\,\text{K}$. The corresponding thermal occupations are

$$\begin{aligned}
\bar{n}(300\,\text{K}) &\approx 1.04 \times 10^3, \\
\bar{n}(4\,\text{K}) &\approx 1.34 \times 10^1, \\
\bar{n}(0.10\,\text{K}) &\approx 5.96 \times 10^{-2}, \\
\bar{n}(0.020\,\text{K}) &\approx 5.65 \times 10^{-7}.
\end{aligned}$$

Inserting these values into Eq. (3) gives

$$\begin{aligned}
\bar{n}_{\text{eff}} &= \bar{n}(T_3) + \frac{\bar{n}(T_2)}{L_3} + \frac{\bar{n}(T_1)}{L_2 L_3} + \frac{\bar{n}(T_0)}{L_1 L_2 L_3} \\
&\approx 5.65 \times 10^{-7} + \frac{5.96 \times 10^{-2}}{10^2} + \frac{1.34 \times 10^1}{10^4} + \frac{1.04 \times 10^3}{10^6} \\
&\approx 2.98 \times 10^{-3}.
\end{aligned}$$

The effective temperature T_{eff} is defined by $\bar{n}(T_{\text{eff}}) = \bar{n}_{\text{eff}}$, which yields

$$T_{\text{eff}} \approx 50\,\text{mK} \quad \text{at } f = 6\,\text{GHz}. \tag{6}$$

So a symmetric 20/20/20 dB attenuation chain (60 dB total) produces an effective microwave bath around 50 mK at the qubit drive frequency, under the assumptions above.

Including cold cable insertion loss (10–15 dB)

In practice there is additional attenuation from coaxial cables and connectors between the cold plates and at the MXC stage. If these losses are well anchored and thermalized, to first approximation they can be treated as extra cold attenuation at T_3.

Let us model an additional cold loss of $A_{\text{cable}} = 10$ to $15\,\text{dB}$ as extra attenuation after the MXC attenuator. This is equivalent to increasing L_3 by a factor

$$L_{\text{cable}} = 10^{A_{\text{cable}}/10}, \qquad L_3 \to L_3' = L_3 L_{\text{cable}}. \tag{7}$$

Using Eq. (3) with $L_1 = L_2 = 10^2$ and $L_3' = 10^2 L_{\text{cable}}$ we obtain:

- For $A_{\text{cable}} = 10\,\text{dB}$ ($L_{\text{cable}} = 10$):

$$\bar{n}_{\text{eff}} \approx 3.0 \times 10^{-4}, \quad T_{\text{eff}} \approx 35\,\text{mK}.$$

- For $A_{\text{cable}} = 15\,\text{dB}$ ($L_{\text{cable}} \approx 31.6$):

$$\bar{n}_{\text{eff}} \approx 9.5 \times 10^{-5}, \quad T_{\text{eff}} \approx 31\,\text{mK}.$$

So a realistic line with 60 dB of discrete attenuators plus 10–15 dB of well-thermalized cable loss at the cold stages corresponds to an effective noise temperature of roughly

$$T_{\text{eff}} \sim 30\text{–}35\,\text{mK}, \qquad \bar{n}_{\text{eff}} \sim 10^{-4}\text{–}3 \times 10^{-4}, \tag{8}$$

as seen by a 6 GHz qubit drive line at the MXC.

Under these conditions the line is well within the single-photon regime and close to the ground state of the mode, which is consistent with standard engineering practice [2, 3].

3 Going colder: approaching 20–10 mK

To reduce the effective noise temperature further one can increase the total attenuation and, in particular, the attenuation mounted at the MXC stage.

Example: 80 dB total (20/20/40 dB)

Consider

$$\begin{aligned} A_1 &= 20\,\mathrm{dB}, & L_1 &= 10^2, \\ A_2 &= 20\,\mathrm{dB}, & L_2 &= 10^2, \\ A_3 &= 40\,\mathrm{dB}, & L_3 &= 10^4, \end{aligned}$$

so that $A_{\mathrm{tot}}^{(\mathrm{dB})} = 80\,\mathrm{dB}$. Inserting into Eq. (3) gives

$$\begin{aligned} \bar{n}_{\mathrm{eff}} &\approx 5.65 \times 10^{-7} + \frac{5.96 \times 10^{-2}}{10^4} + \frac{1.34 \times 10^1}{10^6} + \frac{1.04 \times 10^3}{10^8} \\ &\approx 3.0 \times 10^{-5}, \end{aligned}$$

which corresponds to

$$T_{\mathrm{eff}} \approx 28\,\mathrm{mK} \quad \text{at } 6\,\mathrm{GHz}. \tag{9}$$

So increasing the total attenuation from 60 to 80 dB, with 40 dB mounted at MXC, reduces T_{eff} from about 50 mK to below 30 mK.

Example: 100 dB total (20/20/60 dB)

In the limit of very large cold attenuation contributions from hotter stages vanish and $\bar{n}_{\mathrm{eff}}$ approaches $\bar{n}(T_3)$. As a concrete example take

$$\begin{aligned} A_1 &= 20\,\mathrm{dB}, & L_1 &= 10^2, \\ A_2 &= 20\,\mathrm{dB}, & L_2 &= 10^2, \\ A_3 &= 60\,\mathrm{dB}, & L_3 &= 10^6, \end{aligned}$$

so that $A_{\mathrm{tot}}^{(\mathrm{dB})} = 100\,\mathrm{dB}$. Then

$$\begin{aligned} \bar{n}_{\mathrm{eff}} &\approx 5.65 \times 10^{-7} + \frac{5.96 \times 10^{-2}}{10^6} + \frac{1.34 \times 10^1}{10^8} + \frac{1.04 \times 10^3}{10^{10}} \\ &\approx 8.6 \times 10^{-7}, \end{aligned}$$

which corresponds to

$$T_{\mathrm{eff}} \approx 21\,\mathrm{mK} \quad \text{at } 6\,\mathrm{GHz}. \tag{10}$$

This is already very close to the physical mixing chamber temperature $T_3 = 20\,\mathrm{mK}$. Increasing attenuation further produces only small changes in T_{eff}.

Can we reach 10 mK with passive attenuation?

The key point is that with passive elements one cannot make the effective noise temperature lower than the temperature of the coldest attenuator. In the limit of arbitrarily large L_3,

$$\bar{n}_{\mathrm{eff}} \to \bar{n}(T_3), \qquad T_{\mathrm{eff}} \to T_3. \tag{11}$$

If the MXC and the last attenuator are at $T_3 \approx 20\,\mathrm{mK}$, then T_{eff} is bounded below by this value. To reach $T_{\mathrm{eff}} \sim 10\,\mathrm{mK}$ one would need either a colder mixing chamber or a different type of engineered reservoir, not just more passive attenuation on a 20 mK stage.

4 Summary

Using a standard beam-splitter model for cryogenic attenuators, combined with typical fridge temperatures, one finds:

- A symmetric 20/20/20 dB attenuation chain (60 dB total) gives $T_{\mathrm{eff}} \approx 50\,\mathrm{mK}$ at 6 GHz.
- Including realistic cold cable insertion loss of 10–15 dB reduces this to $T_{\mathrm{eff}} \sim 30$–$35\,\mathrm{mK}$ with $\bar{n}_{\mathrm{eff}} \sim 10^{-4}$–$3 \times 10^{-4}$, which is already very close to the ground state of the qubit drive mode.
- Increasing the total attenuation to 80–100 dB, with a large fraction on the MXC plate, can bring T_{eff} down to the vicinity of the physical mixing chamber temperature, around 20–30 mK for the parameters considered here.

These estimates are consistent with experimental characterizations of cryogenic attenuators [1] and with the attenuation strategies recommended in modern cQED wiring guides [2, 3].

5 Noise Floor Along the Drive Line and "Single-Photon" Estimates

In addition to estimating the effective thermal occupation at the mixing chamber, it is useful to quantify how the noise floor evolves *along* the drive line. Following the standard thermal beam-splitter model [1–3], each attenuator at temperature T_i and attenuation L_i (linear) transforms the input thermal occupation according to

$$\bar{n}_{\mathrm{out}} = \frac{\bar{n}_{\mathrm{in}}}{L_i} + \left(1 - \frac{1}{L_i}\right) \bar{n}(T_i), \tag{12}$$

where $\bar{n}(T)$ is the Bose–Einstein thermal occupation at frequency $\omega = 2\pi f$. At $f = 6\,\mathrm{GHz}$, $\hbar\omega/k_B \simeq 0.288\,\mathrm{K}$, giving numerically

$$\bar{n}(300\,\mathrm{K}) \approx 1.0 \times 10^3, \quad \bar{n}(4\,\mathrm{K}) \approx 13.4, \quad \bar{n}(0.10\,\mathrm{K}) \approx 5.9 \times 10^{-2}, \quad \bar{n}(0.020\,\mathrm{K}) \approx 5.6 \times 10^{-7}.$$

For the standard 20/20/20 dB attenuation chain ($L_1 = L_2 = L_3 = 10^2$) the noise occupation *after each stage* is:

$$\bar{n}_{\mathrm{eff}}^{(1)} = \bar{n}(T_1) + \frac{\bar{n}(T_0)}{L_1} \approx 23.4, \tag{13}$$

$$\bar{n}_{\mathrm{eff}}^{(2)} = \bar{n}(T_2) + \frac{\bar{n}(T_1)}{L_2} + \frac{\bar{n}(T_0)}{L_1 L_2} \approx 0.21, \tag{14}$$

$$\bar{n}_{\mathrm{eff}}^{(3)} = \bar{n}(T_3) + \frac{\bar{n}(T_2)}{L_3} + \frac{\bar{n}(T_1)}{L_2 L_3} + \frac{\bar{n}(T_0)}{L_1 L_2 L_3} \approx 3 \times 10^{-3}. \tag{15}$$

Including wire and connector insertion loss

Cold coaxial wiring typically contributes an additional 10–15 dB of insertion loss between the 100 mK and 20 mK stages. If this loss is well thermalized, it can be modeled as extra attenuation L_{cable} at T_3. Replacing $L_3 \to L_3' = L_3\, L_{\mathrm{cable}}$ yields

$$A_{\mathrm{cable}} = 10\,\mathrm{dB}: \qquad \bar{n}_{\mathrm{eff}} \approx 3 \times 10^{-4}, \quad T_{\mathrm{eff}} \approx 35\,\mathrm{mK}, \tag{16}$$

$$A_{\mathrm{cable}} = 15\,\mathrm{dB}: \qquad \bar{n}_{\mathrm{eff}} \approx 1 \times 10^{-4}, \quad T_{\mathrm{eff}} \approx 31\,\mathrm{mK}. \tag{17}$$

Hence, the realistic noise floor at the sample is of order

$$\boxed{\bar{n}_{\mathrm{th}} \sim 10^{-4} \qquad \text{at } 6\,\mathrm{GHz},} \tag{18}$$

placing the drive line firmly in the sub-single-photon thermal regime.

Comparison with single-photon drive power

For a resonator or mode with linewidth κ and external coupling κ_{ext}, the steady-state photon number under a resonant drive is

$$\bar{n}_{\text{cav}} = \frac{4\kappa_{\text{ext}}}{\kappa^2}\frac{P_{\text{in}}}{\hbar\omega}. \tag{19}$$

For typical cQED parameters ($\kappa/2\pi \sim 1\,\text{MHz}$ and $\kappa_{\text{ext}} \sim \kappa/2$),

$$\bar{n}_{\text{cav}} = 1 \quad \Rightarrow \quad P_{\text{in}} \sim -139\,\text{dBm} \quad \text{(at the sample)}.$$

Accounting for $\sim 75\,\text{dB}$ total attenuation (60 dB fixed + 15 dB cold loss), the corresponding room-temperature power is

$$P_{\text{room}} \approx -64\,\text{dBm},$$

so a coherent drive tuned to $\bar{n} \sim 1$ exceeds the thermal background $\bar{n}_{\text{th}} \sim 10^{-4}$ by a factor of 10^4 in photon number.

This confirms that the "single-photon limit", in the conventional sense of coherent fields with $\bar{n} \lesssim 1$ and fixed phase, is quantitatively accessible given our attenuation and wiring.

Power-dependent dispersive shifts in the TLS experiment

In analogy with the candle-qubit calibration, we can use power-dependent frequency shifts to extract $\bar{n}(P)$ in the TLS experiment. A single TLS with detuning Δ_d and Rabi rate $\Omega \propto \sqrt{\bar{n}}$ acquires an AC Stark shift

$$\Delta\omega_{\text{Stark}} \approx \frac{\Omega^2}{2\Delta_d} \propto \bar{n}, \tag{20}$$

on top of a static Lamb-like shift from vacuum and thermal fields. In practice, TLS-mediated features should therefore move with drive power as

$$\Delta\omega_{\text{obs}}(P) \approx \Delta\omega_0 + C\,\bar{n}(P),$$

which can be used to calibrate the intracavity photon number and distinguish between vacuum/Lamb contributions and drive-induced Stark shifts—However, we do not have a cavity.

6 Broadband 3D Waveguide, Driven Dielectric Response, and Power Scales

In the input-line analysis above we implicitly treated a single mode coupled to a localized device (qubit or resonator). In the TLS experiment, the samples are placed inside a broadband 3D microwave waveguide, and we probe a 3–5 GHz band. Here the relevant object is a traveling-wave mode interacting with an extended dielectric volume, rather than a single high-Q cavity mode. We trade mode volume and resonant enhancement for a broadband, almost continuum-like spectrum, but the noise modelling per frequency remains the same: at each frequency ω the mean thermal occupation at the sample is still given by

$$\bar{n}_{\text{th}}(\omega) = \frac{1}{e^{\hbar\omega/k_B T_{\text{eff}}} - 1},$$

with T_{eff} set by the attenuation chain as discussed above. Over 3–5 GHz, $\hbar\omega/k_B$ varies from ~ 0.14 to $\sim 0.24\,\text{K}$, so the effective thermal occupation stays of order 10^{-4} at the mixing chamber for our wiring.

Driven dielectric response of a TLS ensemble in a waveguide

The dielectric sample inside the waveguide contains an ensemble of TLS defects with a distribution of resonance frequencies ω_j and dipole moments p_j. In the linear-response regime, the driven dielectric susceptibility at frequency ω can be written schematically as

$$\chi(\omega) \simeq \sum_j \frac{|p_j|^2}{\hbar\varepsilon_0 V_{\text{eff}}} \frac{1}{\omega_j - \omega - i\Gamma_{2,j}}, \tag{21}$$

where V_{eff} is an effective mode volume for the waveguide field in the sample region and $\Gamma_{2,j}$ are TLS decoherence rates. The imaginary part of $\chi(\omega)$ sets the dielectric loss tangent and absorption, while the real part modifies the phase of the transmitted wave.

Under a coherent drive with local field amplitude $E(\omega)$, TLSs experience a Rabi frequency $\Omega_j \propto p_j E/\hbar$. In the standard Bloch-equation picture, the steady-state inversion w_j is

$$w_j^{(\text{ss})} \approx -\frac{1}{1 + \Omega_j^2 T_{1,j} T_{2,j} + (\Delta_j T_{2,j})^2}, \qquad \Delta_j = \omega - \omega_j, \tag{22}$$

so the effective loss tangent is reduced (saturated) when $\Omega_j^2 T_{1,j} T_{2,j} \gtrsim 1$. In the TLS experiment we are interested in the regime where the drive is strong enough to produce a measurable change in dielectric response, but still close to the "single-photon" limit defined below, such that the response is sensitive to individual or small clusters of TLSs rather than a heavily saturated bath.

Relaxation dynamics after the pulse

In a pulsed experiment, the waveguide is driven for a finite time τ_{pulse}, which prepares a nonequilibrium distribution of TLS excitations throughout the sample. After the pulse is switched off, the TLS ensemble relaxes back toward equilibrium with characteristic times $T_{1,j}$. The time-dependent dielectric function can be written schematically as

$$\varepsilon(\omega, t) = \varepsilon_\infty + \delta\varepsilon'(\omega, t) - i\,\delta\varepsilon''(\omega, t), \tag{23}$$

where the changes $\delta\varepsilon', \delta\varepsilon''$ are determined by the evolving TLS inversions $w_j(t)$, which typically decay as $w_j(t) \sim w_j^{(\text{ss})} e^{-t/T_{1,j}}$ (with a distribution of $T_{1,j}$ in a real material).

The transmitted waveguide signal $S_{21}(\omega, t)$ therefore encodes both the driven dielectric response during the pulse and the relaxation dynamics afterward. By choosing drive powers in the single-photon regime (see below), one can probe the intrinsic TLS relaxation and spectral diffusion without overwhelming the system with a large classical field.

Single-photon power scale in a traveling-wave picture

In a broadband 3D waveguide the relevant "mode" is the spatiotemporal mode of the pulse that overlaps the sample. For a coherent pulse of duration τ_{int} at frequency ω, the energy in a single photon is $U = \hbar\omega$, so the average power at the *sample* for one photon in that mode is

$$P_{\text{single}}(\omega, \tau_{\text{int}}) = \frac{\hbar\omega}{\tau_{\text{int}}}. \tag{24}$$

For example, at $f = 4\,\text{GHz}$ ($\hbar\omega \approx 2.6 \times 10^{-24}\,\text{J}$) and an effective interaction time $\tau_{\text{int}} = 100\,\text{ns}$,

$$P_{\text{single}} \approx 2.6 \times 10^{-17}\,\text{W} \approx -136\,\text{dBm} \quad \text{(at the sample)}. \tag{25}$$

If the total attenuation from room temperature to the sample is $A_{\text{tot}} \approx 75\,\text{dB}$ (e.g. 60 dB of fixed attenuators plus 15 dB of cold cable loss), the corresponding room-temperature power is

$$P_{\text{room,single}} \approx -136\,\text{dBm} + 75\,\text{dB} \approx -61\,\text{dBm}. \tag{26}$$

Thus, for pulses of order 100 ns, driving in the range $P_{\text{room}} \sim -60$ to -70 dBm corresponds to an average of $\bar{n} \sim 1$ photon in the spatiotemporal mode that interacts with the sample, while the thermal background at the sample is $\bar{n}_{\text{th}} \sim 10^{-4}$ per frequency mode over 3–5 GHz.

More generally, the average photon number in the relevant mode is

$$\bar{n}_{\text{mode}} = \frac{P_{\text{sample}}\, \tau_{\text{int}}}{\hbar\omega}, \qquad P_{\text{sample}} = P_{\text{room}} \times 10^{-A_{\text{tot}}/10}, \tag{27}$$

so for fixed attenuation one can select P_{room} to achieve $\bar{n}_{\text{mode}} \lesssim 1$. This defines the "single-photon limit" for the broadband waveguide TLS experiment and sets the power window in which we can investigate the driven dielectric response and subsequent relaxation dynamics of the ensemble without leaving the few-photon regime.

References

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