Bayesian Filtering and Smoothing: Exercise Set 2

1. A sequence of position measurements of a moving target is

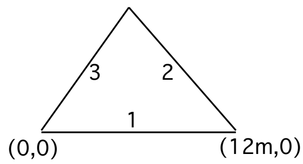
t_k (s)	1	2	3	4	5	6	7	8	9	10
y_k (m)	-0.083	0.028	0.285	0.780	0.757	1.076	1.173	1.409	1.521	1.773

Assuming the measurement model

$$y_k = \theta_1 + \theta_2 t_k + \varepsilon_k \qquad (k \in 1{:}10)$$

where $\varepsilon_k \sim \mathrm{N}(0, 0.01)$, and using the prior $\boldsymbol{\theta} \sim \mathrm{N}(\left[\begin{smallmatrix}0\\0\end{smallmatrix}\right], 100\mathbf{I})$, find the posterior distribution $\boldsymbol{\theta} \,|\, \mathbf{y}_{1:10}$.

2. A room's floor plan is an equilateral triangle:



Let r_k be the measured distance from the kth wall to a sensor that is located inside the room at $\boldsymbol{\theta} \in \mathbb{R}^2$. The measurements have mutually independent identically distributed zero-mean Gaussian errors. Show that the measurement model is $\mathbf{r} \,|\, \boldsymbol{\theta} \sim \mathrm{N}(\mathbf{H}\boldsymbol{\theta} + \mathbf{b}, \sigma^2\mathbf{I})$ with

$$\mathbf{H} = \begin{bmatrix} 0 & 1 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 6\sqrt{3} \\ 0 \end{bmatrix}$$

Consequently, the model for $\mathbf{y} = \mathbf{r} - \mathbf{b}$ is $\mathbf{y} \,|\, \boldsymbol{\theta} \sim \mathrm{N}(\mathbf{H}\boldsymbol{\theta}, \sigma^2\mathbf{I})$.

3. (continued) The measurement is $\mathbf{r} = \left[\begin{smallmatrix}2\\4\\6\end{smallmatrix}\right]$. Where is the sensor? You can assume a flat prior $p(\boldsymbol{\theta}) \propto 1$.

4. (continued) A new measurement of the sensor's distance to wall 3 is 5.5 m. Update your estimate of sensor location.

   Hint: use the answer of the previous question as your prior, and the measurement model $r_4 \,|\, \boldsymbol{\theta} \sim \mathrm{N}([\frac{\sqrt{3}}{2}, -\frac{1}{2}]\boldsymbol{\theta}, \sigma^2)$.

5. (continued) Assume $\sigma = 0.5$ m. Draw the 50% confidence ellipse.

   Hint: see next question.

6. Show that the PDF contour curve containing α of the probability of $\mathbf{x} \sim \mathrm{N}(\mathbf{m}, \mathbf{P})$ is given by $\{\mathbf{m} + \mathbf{L}\left[\begin{smallmatrix} r\cos(t) \\ r\sin(t)\end{smallmatrix}\right] : t \in [0, 2\pi)\}$, where $\mathbf{L} = \texttt{chol}(\mathbf{P}, \texttt{'lower'})$ and $r^2 = \texttt{chi2inv}(\alpha, 2)$.

Answers

1. $\mathrm{N}(\left[\begin{smallmatrix}-0.2705\\0.2077\end{smallmatrix}\right], 10^{-6}\left[\begin{smallmatrix}4666 & -667\\-667 & 121\end{smallmatrix}\right])$ 3. $\mathrm{N}(\left[\begin{smallmatrix}7.1547\\1.4641\end{smallmatrix}\right], 0.6667\sigma^2\mathbf{I})$ 4. $\mathrm{N}(\left[\begin{smallmatrix}7.1671\\1.4569\end{smallmatrix}\right], \sigma^2\left[\begin{smallmatrix}0.4667 & 0.1155\\0.1155 & 0.6000\end{smallmatrix}\right])$