Bayesian Filtering and Smoothing: Exercise Set 3

1. Modify the script `linreg_demo` to solve exercise 1 of set 2.

2. Write Matlab functions that compute the Kalman filter prediction and update steps. The functions' first lines are

   ```
   function [m,P] = kf_predict(m,P,A,Q)

   function [m,P] = kf_update(m,P,y,H,R)
   ```

   Use these functions and the script `sine_demo` to replicate the lecture results.

3. (continued) Experiment with different values of the discrete random walk velocity model's variance parameter q, including $q = 1000$ and $q = 0.1$. How does the value affect the prior trajectories? Why? How does it affect the estimate? Why?

4. Generate and plot a sequence of 100 measurements using the discrete random walk velocity model

   $$\begin{aligned} u_k &= u_{k-1} + \Delta v_{k-1} \\ v_k &= v_{k-1} + e_{k-1} \\ y_k &= u_k + \varepsilon_k, \end{aligned}$$

   where $\Delta = 1$, $u_0 \sim \mathrm{N}(0,1)$, $v_0 \sim \mathrm{N}(0,1)$, $e_{k-1} \sim \mathrm{N}(0,1)$, and $\varepsilon_k \sim \mathrm{N}(0,100)$ are jointly Gaussian and uncorrelated.

5. (continued) Plot (as functions of k) the measurements y_k and the estimates $\hat{u}_k = \mathsf{E}(u_k|y_{1:k})$ on one figure, and plot the measurement errors ε_k and the estimation errors $u_k - \hat{u}_k$ on another figure. Compute the estimate's root mean square error

   $$\mathrm{RMSE} = \sqrt{\frac{1}{100}\sum_{k=1}^{100}(\hat{u}_k - u_k)^2}$$

   Compute also the RMSE of using y_k as the estimate.

6. (continued) Plot the variances of $u_k|y_{1:k}$ as a function of k. What is the radius of the 95% confidence interval of $u_{17}|y_{1:17}$?

   Hint: if $\mathbf{x}_k = \left[\begin{smallmatrix} u_k \\ v_k \end{smallmatrix}\right]$ and $\mathbf{x}_k|y_{1:k} \sim \mathrm{N}(\mathbf{m}_k, \mathbf{P}_k)$, then $\mathsf{var}(u_k|y_{1:k}) = \mathbf{P}_k(1,1)$.

Answers

6. 11.78