```
clear all

load AR.mat

phiT=@(theta) phi(theta,y);
R_MAP=fminsearch(phiT,0.5)
```

```
R_MAP = -0.2896
```

```
R_MAP1=fminunc(phiT,0.5)
```

```
Local minimum found.

Optimization completed because the size of the gradient is less than
the value of the optimality tolerance.

<stopping criteria details>
R_MAP1 = -0.2896
```

```
R_MAP2=fminbnd(phiT,-1,1)
```

```
R_MAP2 = -0.2896
```

```

%testing if this K gives same result as K_steady below
A=-0.2896; Q=0.2^2;  R=0.1^2; H=1;
X=dare(A',H',Q,R)
```

```
X = 0.0407
```

```
S=H*X*H'+R;
K=X*H'/S
```

```
K = 0.8027
```

```

A_minus = ((A-1)^2 + Q/R);
A_plus =  ((A+1)^2 + Q/R);
K_steady = (Q/R + A^2 -1 + sqrt(A_plus*A_minus))/(Q/R + A^2 + 1 + sqrt(A_plus*A_minus))
```

```
K_steady = 0.8027
```

Part b

```
%lets solve K symbolically using matlab
syms A H Q R X

XX = solve(X == Q + A*X*A -A^2*X^2/(X+R),X)
```

```
XX =
```

$$\begin{pmatrix} \frac{Q}{2} - \frac{R}{2} - \sigma_1 + \frac{A^2 R}{2} \\ \frac{Q}{2} - \frac{R}{2} + \sigma_1 + \frac{A^2 R}{2} \end{pmatrix}$$

where

$$\sigma_1 = \frac{\sqrt{(R A^2 - 2 R A + Q + R)\ (R A^2 + 2 R A + Q + R)}}{2}$$

```
X = XX(2) %lets pick this one. The other one would give negative answer
```

X =

$$\frac{Q}{2} - \frac{R}{2} + \frac{\sqrt{(R A^2 - 2 R A + Q + R)\ (R A^2 + 2 R A + Q + R)}}{2} + \frac{A^2 R}{2}$$

```
S = X + R
```

S =

$$\frac{Q}{2} + \frac{R}{2} + \frac{\sqrt{(R A^2 - 2 R A + Q + R)\ (R A^2 + 2 R A + Q + R)}}{2} + \frac{A^2 R}{2}$$

```
K = simplify(X/S)
```

K =

$$\frac{\frac{Q}{2} - \frac{R}{2} + \sigma_1 + \frac{A^2 R}{2}}{\frac{Q}{2} + \frac{R}{2} + \sigma_1 + \frac{A^2 R}{2}}$$

where

$$\sigma_1 = \frac{\sqrt{(R A^2 - 2 R A + Q + R)\ (R A^2 + 2 R A + Q + R)}}{2}$$

$$\frac{\frac{Q}{2} - \frac{R}{2} + \sigma_1 + \frac{A^2 R}{2}}{\frac{Q}{2} + \frac{R}{2} + \sigma_1 + \frac{A^2 R}{2}}$$

where

$$\sigma_1 = \frac{\sqrt{(R A^2 - 2 R A + Q + R)\ (R A^2 + 2 R A + Q + R)}}{2}$$

Let's do some simplification, we divide result by R/2. We get

$$\frac{\frac{Q}{R} - 1 + \sigma_1 + A^2}{\frac{Q}{R} + 1 + \sigma_1 + A^2}$$

where

$$\sigma_1 = \sqrt{\left((A-1)^2 + \frac{Q}{R}\right)\left((A+1)^2 + \frac{Q}{R}\right)}$$

which is given result in task 3b.

If we choose other solution, we get

$$\frac{\frac{Q}{R} - 1 - \sigma_1 + A^2}{\frac{Q}{R} + 1 - \sigma_1 + A^2}$$

where

$$\sigma_1 = \sqrt{\left((A-1)^2 + \frac{Q}{R}\right)\left((A+1)^2 + \frac{Q}{R}\right)}$$

```
eqn = Q/R-1+A^2 > sqrt(((A-1)^2 + Q/R)*((A-1)^2 + Q/R))
```

```
eqn =
```

$$\sqrt{\left(\frac{Q}{R} + (A-1)^2\right)^2} < \frac{Q}{R} + A^2 - 1$$

```
solve(eqn)
```

```
Warning: Unable to find explicit solution. For options, see help.

ans =

Empty sym: 0-by-1
```

```
%no solution found, so K would be always negative in this case which is not
%feasible solution. So we can drop this solution.
```

Functions

```
function val=phi(theta,Y)
%function taken from example12_1.m

A=theta; Q=0.2^2;  R=0.1^2; m=0;
H=1; %for AR model H = 1
%lets solve steady state kalman gain
X=dare(A',H',Q,R);
```

```matlab
S=H*X*H'+R;
K=X*H'/S;
val=0;  % flat prior; any constant will do
for k=1:size(Y,2)
    m=A*m; %KF prediction
    v=Y(:,k)-H*m;
    val=val+0.5*v'/S*v+sum(log(diag(chol(2*pi*S))));
    % KF update
    m=m+K*v;
end
end
```