Bayesian Filtering and Smoothing: Exercise Set 4

1. Modify the script `car_kf_demo` to use a stationary Kalman filter.

2. Modify the script `car_kf_demo` so that $\mathbf{P}_0^{-1} = \mathbf{0}$. Don't modify the data. (Hint: use an information filter with $\mathbf{J}_0 = \mathbf{0}$.)

3. The Langevin stochastic differential equation

$$\dot{x} = -\alpha x + w,$$

where w is Gaussian white noise with spectral power density Q_c and $\alpha \geq 0$, is used to model Brownian motion, price fluctuations, and electrical circuit thermal noise. Show that the coefficients of the discrete-time model for $x_k = x(t_k)$ are

$$A_k = \mathrm{e}^{-\alpha\Delta_k}, \quad Q_k = Q_c \frac{1 - \mathrm{e}^{-2\alpha\Delta_k}}{2\alpha},$$

and verify that $\lim\limits_{\Delta_k \to 0} \frac{1}{\Delta_k} Q_k = Q_c$.

4. (continued) Plot process paths $x(t)$ for $Q_c = 1$, $0 \leq t \leq 2$, and various α values. How does α affect the characteristics of the paths?

5. Consider an integrated Langevin path

$$\dot{u} = v, \quad \dot{v} = -\alpha v + w.$$

Show that the coefficients of the discrete-time model for $x_k = \left[\begin{smallmatrix} u(t_k) \\ v(t_k) \end{smallmatrix}\right]$ are

$$A_k = \begin{bmatrix} 1 & \frac{1-\mathrm{e}^{-\alpha\Delta_k}}{\alpha} \\ 0 & \mathrm{e}^{-\alpha\Delta_k} \end{bmatrix}, \quad Q_k = Q_c \begin{bmatrix} \frac{2\alpha\Delta_k - 3 + 4\mathrm{e}^{-\alpha\Delta_k} - \mathrm{e}^{-2\alpha\Delta_k}}{2\alpha^3} & \frac{1-2\mathrm{e}^{-\alpha\Delta_k} + \mathrm{e}^{-2\alpha\Delta_k}}{2\alpha^2} \\ \frac{1-2\mathrm{e}^{-\alpha\Delta_k} + \mathrm{e}^{-2\alpha\Delta_k}}{2\alpha^2} & \frac{1-\mathrm{e}^{-2\alpha\Delta_k}}{2\alpha} \end{bmatrix},$$

and verify that $\lim\limits_{\Delta \to 0} \frac{1}{\Delta} \mathbf{Q}_k = \left[\begin{smallmatrix} 0 & 0 \\ 0 & 1 \end{smallmatrix}\right] Q_c$.

6. (continued) Plot process paths $u(t)$ for $Q_c = 1$, $0 \leq t \leq 2$, and various α values. How does α affect the characteristics of the paths?

Answers

1. RMSE 0.4248