

Bayesian Filtering and Smoothing: Exercise Set 7

1. Replicate the pendulum UKF and GHKF filtering results shown in the lecture.
2. Estimate the track from set 6 question 3 using UKF and GHKF.
3. Estimate the track from set 6 question 5 using UKF and GHKF and initial distribution $\mathbf{x}_0 \sim \mathcal{N}\left(\begin{bmatrix} 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}\right)$.
4. Experiment with different tracks generated by different rng values. Are UKF and GHKF as prone to diverge as EKF and EKF2?
5. As stated in the lecture, if $\mathbf{x} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ then $E(\mathbf{x}^\top \mathbf{x}) = n$ and $\text{var}(\mathbf{x}^\top \mathbf{x}) = 2n$, where $n = \text{length}(\mathbf{x})$. Verify that the fifth-order Gauss-Hermite cubature method give the correct mean and correct variance for $n \in \{1, 2, 3, 4, 5\}$.
6. Prove that the unscented transform gives the exact mean for the linear-affine mapping $\mathbf{g}(\mathbf{x}) = \mathbf{E}(\mathbf{x}) + \mathbf{A}(\mathbf{x} - \mathbf{E}(\mathbf{x}))$.