Volume of interest (VOI) limited linear filtering

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In medical imaging, it is sometimes useful to limit the effect of filtering to a certain volume of interest (VOI) or region of interest (ROI). This note explains how to perform such filtering using convolution and masking operations, convenient for Matlab-based implementation.

Let $\Omega \subset D$ denote the set of voxels within the limiting VOI, where D is the image domain and let M be the indicator function of Ω, i.e., the mask defining the VOI:

$$\begin{aligned} M(\mathbf{x}) &= 1 \text{ if } \mathbf{x} \in \Omega \\ M(\mathbf{x}) &= 0 \text{ if } \mathbf{x} \notin \Omega. \end{aligned}$$

Let $I(\mathbf{p})$ be the image intensity at the voxel $\mathbf{p}$. The filtered image I^f can then be defined as

$$I^f(\mathbf{q}) = \frac{1}{W(\mathbf{q})} \sum_{\mathbf{p} \in \Omega} K(\mathbf{p} - \mathbf{q}) I(\mathbf{p}),$$

when $\mathbf{q} \in \Omega$, where

$$W(\mathbf{q}) = \sum_{\mathbf{p} \in \Omega} K(\mathbf{p} - \mathbf{q}),$$

and $K : \mathbb{R}^3 \rightarrow \mathbb{R}$ is the applied filtering kernel, typically a Gaussian. We are not interested on the values of I^f outside the VOI Ω and thus they need not to be (and are not) defined.

This can be conveniently implemented with (in practice) just two lines of Matlab code, since, for all $\mathbf{q} \in \Omega$,

$$\begin{aligned} W(\mathbf{q}) &= \sum_{\mathbf{p} \in \Omega} K(\mathbf{p} - \mathbf{q}) \\ &= \sum M(\mathbf{p}) K(\mathbf{p} - \mathbf{q}) \\ &= (M * K)(\mathbf{q}) \end{aligned}$$

where $*$ denotes 3-D convolution, and

$$\begin{aligned} & \sum_{\mathbf{p} \in \Omega} K(\mathbf{p} - \mathbf{q}) I(\mathbf{p}) \\ = & \sum M(\mathbf{p}) I(\mathbf{p}) K(\mathbf{p} - \mathbf{q}) \\ = & (K * (I \odot M))(\mathbf{q}), \end{aligned}$$

where $\odot$ denotes element-by-element product. Thus, in Matlab, because the convolution is a commutative operator, the following two lines give the desired filter:

```
W = conv(M,K,'same');
Ifilt = (1./W).*conv(M.*I,K,'same');
```

If one wants to ensure that the values outside the VOI are NaN's, then additional masking is required:

```
Ifilt = Ifilt.*M;
```

The Matlab-code for this VOI limited filtering can be downloaded at `http://www.cs.tut.fi/~jupeto/matlab_code/gaussian3dfil_roi.m`

The Matlab function uses Gaussian kernel $K(\mathbf{x}) = \frac{1}{Z} \exp(-\sigma^T \mathbf{x})$, where Z is a normalization constant and σ is a 3-component vector defined based on the full-width half maximum

$$FWHM = [FWHM_x, FWHM_y, FWHM_z]$$

of the kernel at each direction: $\sigma_x = \frac{8 \log(2)}{FWHM_x^2}$ and similarly for y and z directions.