LETTER

Bell violation using entangled photons without the fair–sampling assumption

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The violation of a Bell inequality is an experimental observation that forces the abandonment of a local realistic viewpoint—namely, one in which physical properties are (probabilistically) defined before and independently of measurement, and in which no physical influence can propagate faster than the speed of light[1,2]. All such experimental violations require additional assumptions depending on their specific construction, making them vulnerable to so-called loopholes. Here we use entangled photons to violate a Bell inequality while closing the fair-sampling loophole, that is, without assuming that the sample of measured photons accurately represents the entire ensemble[3]. To do this, we use the Eberhard form of Bell's inequality, which is not vulnerable to the fair-sampling assumption and which allows a lower collection efficiency than other forms[4]. Technical improvements of the photon source[5,6] and high-efficiency transition-edge sensors[7] were crucial for achieving a sufficiently high collection efficiency. Our experiment makes the photon the first physical system for which each of the main loopholes has been closed, albeit in different experiments.

In 1935, Einstein, Podolsky and Rosen[1] argued that quantum mechanics is incomplete when assuming that no physical influence can be faster than the speed of light and that the properties of physical systems are elements of reality. They considered measurements on spatially separated pairs of entangled particles. Measurement on one particle of an entangled pair instantly projects the other particle onto a well-defined state, independently of their spatial separation. In 1964, Bell[2] showed that no local realistic theory can reproduce all quantum mechanical predictions for entangled states. His renowned Bell inequality proved that there is an upper limit to the strength of the observed correlations predicted by local realistic theories. Quantum theory's predictions violate this limit.

In a Bell experiment, one prepares pairs of entangled particles and sends them to two observers, Alice and Bob, for measurement and detection. Alice and Bob observe correlations between their results that, for specific choices of their measurement settings, violate the Bell inequality and hence force abandonment of local realism.

It is common that in an experiment, some particles emitted by the source will not be detected[3,8]. In such a case, the subset of detected particles might display correlations that violate the Bell inequality although the entire ensemble can be described by a local realistic theory. To achieve a conclusive Bell violation without assuming that the detected particles are a 'fair' sample, a highly efficient experimental set-up is necessary. This efficiency need not be perfect[3].

Experimental limitations have made it necessary to assume fair sampling in nearly every Bell experiment performed to date, with a few exceptions[9–13]. In particular, owing to the lack of efficient sources and detectors, this assumption has always been unavoidable in Bell experiments on entangled photon pairs.

Since the first experimental Bell test[14], a satisfactory laboratory realization of the motivating *gedankenexperiment* has remained a challenge[15,16]. The two other main assumptions include "locality"[17,18] and "freedom of choice"[19]. Invoking any of these three assumptions renders an experiment vulnerable to explanation by a local realistic theory. The realization of an experiment that is free of all three assumptions—a loophole-free Bell test—remains an important goal for the physics community[20]. An important step has been the realization of quantum steering experiments that have also addressed the issue of loopholes[21–23]. Our experiment makes photons the first physical system for which all three assumptions have been successfully addressed in a Bell test, albeit in different experiments.

In our experiment, we employ Eberhard's inequality, a Bell inequality that inherently does not rely on the fair-sampling assumption[4]. Our scheme is characterized by a number of technical improvements over previous experiments. Each such improvement contributed crucially to reaching the high collection efficiency and visibility necessary for violating the inequality. Our source of photon pairs uses spontaneous parametric down-conversion in a Sagnac configuration, which has proved to be efficient[5,6]. For photon detection, we use superconducting transition-edge sensors (TESs), which not only have a high detection efficiency but are also intrinsically free of dark counts[7]. These two characteristics are essential for an experiment in which no correction of count rates can be tolerated.

Eberhard's inequality, which was proposed almost two decades ago[4], is a Clauser–Horne-type Bell inequality[24] that explicitly includes undetected (inconclusive) events. Therefore, its mere violation directly implies that the fair-sampling loophole is closed. Also, the derivation of Eberhard's inequality includes pairs not detected on either side (and can be generalized for those not even produced), which means that no post-selection on the created pairs is necessary to violate the inequality.

Eberhard's inequality requires the lowest known symmetric arm efficiency for non-maximally entangled qubit states, namely $\eta = 2/3 \approx$
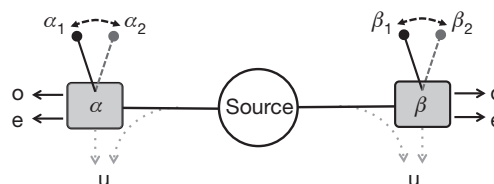

Figure 1 | Principle of the experiment. Violation of an Eberhard inequality involves a source of polarization-entangled pairs as well as polarization measurements. Each measurement device can rotate the photon's polarization according to one of two settings (α_1, α_2 and β_1, β_2) before projecting the photon into the 'ordinary' (o) or 'extraordinary' (e) output of a polarizing beam splitter and detecting it. All lost photons are also included in the derivation of the inequality as 'undetected' (u) events. The different terms of the inequality are photon counts recorded in the different settings.

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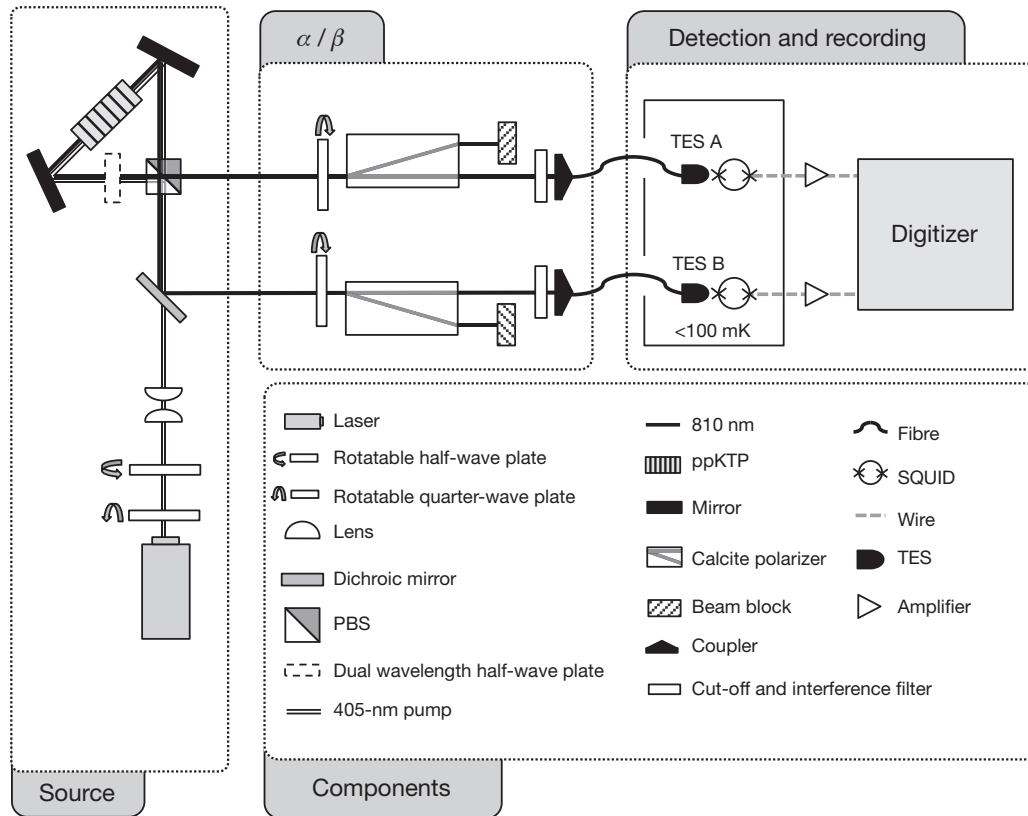*These authors contributed equally to this work.

Figure 2 | Measurement set-up. The source, based on spontaneous parametric down-conversion in ppKTP (periodically poled potassium titanyl phosphate) in a Sagnac configuration, produces polarization-entangled photons with a wavelength of 810 nm. A measurement setting is implemented in each arm by rotating a half-wave plate to the desired angle in front of a calcite polarizer. Photons transmitted through the calcite polarizer (ordinary output beam) are spectrally filtered and coupled into an optical fibre (SMF-28), which leads them to TESs for detection. The output signals from the detectors are amplified by SQUIDs and further electronics before being digitized and processed by an algorithm that identifies photons and time-correlated photon pairs.

66.7%. This arm efficiency (that is, the collection efficiency in one arm of the experiment) incorporates all losses, not least those in the source and the measurement set-up (including the detector). Thresholds lower than 2/3 have been reported for asymmetric efficiencies or higher-dimensionally entangled states[25,26]. For the most widely used Bell inequality, proposed by Clauser, Horne, Shimony and Holt[27], at least $\eta = 2\sqrt{2} - 2 \approx 82.8\%$ is necessary in the symmetric case. For polarization-entangled photon pairs, Eberhard's inequality considers three possible outcomes: o ('ordinary') and e ('extraordinary') for the two recorded outcomes of a polarization measurement, and u ('undetected') if no photon is detected (see Fig. 1). Two different measurement settings are used, (α_1, α_2) on Alice's side and (β_1, β_2) on Bob's side. Let $n_{kl}(\alpha_i, \beta_j)$ denote the number of pairs with the outcome k for Alice's photon and l for Bob's photon, where $k, l \in \{o, e, u\}$, when measured in settings α_i and β_j with $i, j \in \{1, 2\}$. Eberhard's inequality can then be written as:

$$J = -n_{oo}(\alpha_1, \beta_1) + n_{oe}(\alpha_1, \beta_2) + n_{ou}(\alpha_1, \beta_2) + \\ n_{eo}(\alpha_2, \beta_1) + n_{uo}(\alpha_2, \beta_1) + n_{oo}(\alpha_2, \beta_2) \geq 0 \quad (1)$$

Local realism allows J to take only non-negative values. Quantum mechanically, the maximal violation is given by $J/N = (1 - \sqrt{2})/2 \approx -0.207$ (ref. 15), where N denotes the number of entangled particle pairs produced per applied setting combination. This bound is attainable for a symmetric arm efficiency of $\eta = 1$ and maximally entangled states. For the largest possible violation of Eberhard's inequality with $\eta < 1$, non-maximally entangled states must be used. These have the form:

$$|\psi_r\rangle = \frac{1}{\sqrt{1 + r^2}} (|HV\rangle + r|VH\rangle) \quad (2)$$

where $0 < r < 1$ and H and V denote horizontal and vertical polarization of Alice's and Bob's photons. Depending on the background count rate,

efficiencies higher than $\eta = 2/3$ may be needed[4]. Interestingly, for $\eta < 82.8\%$, non-maximally entangled states are not only optimal but even necessary for a violation of Eberhard's inequality.

In an experiment, one records measurements of 'singles counts' S (number of detection events on one side) and 'coincidence counts' C (number of detected pairs) for the four combinations of settings (α_1, β_1), (α_1, β_2), (α_2, β_1) and (α_2, β_2). The number of events for which one of the outcomes is undetected follows directly from the measured rates. For a given measurement length, we denote the measured coincidence counts by $C_{kl}(\alpha_i, \beta_j)$ and the single counts by $S_k^A(\alpha_i)$ for Alice and $S_l^B(\beta_j)$ for Bob ($k, l \in \{o, e\}$). All the terms in Eberhard's inequality are then given by the following measured quantities:

$$n_{oo}(\alpha_1, \beta_1) = C_{oo}(\alpha_1, \beta_1)$$

$$n_{oe}(\alpha_1, \beta_2) = C_{oe}(\alpha_1, \beta_2)$$

$$n_{ou}(\alpha_1, \beta_2) = S_o^A(\alpha_1) - C_{oo}(\alpha_1, \beta_2) - C_{oe}(\alpha_1, \beta_2)$$

$$n_{eo}(\alpha_2, \beta_1) = C_{eo}(\alpha_2, \beta_1)$$

$$n_{uo}(\alpha_2, \beta_1) = S_o^B(\beta_1) - C_{oo}(\alpha_2, \beta_1) - C_{eo}(\alpha_2, \beta_1)$$

$$n_{oo}(\alpha_2, \beta_2) = C_{oo}(\alpha_2, \beta_2) \quad (3)$$

Inserting these expressions into Eberhard's inequality yields:

$$J = -C_{oo}(\alpha_1, \beta_1) + S_o^A(\alpha_1) - C_{oo}(\alpha_1, \beta_2) + \\ S_o^B(\beta_1) - C_{oo}(\alpha_2, \beta_1) + C_{oo}(\alpha_2, \beta_2) \geq 0 \quad (4)$$

where the coincidence counts $C_{oe}(\alpha_1, \beta_2)$ and $C_{eo}(\alpha_2, \beta_1)$ have dropped out. The resulting inequality, which is used in our experiment, now

Table 1 | Measurement results and *J* value for a total measurement time of 300 s per setting.

$C_{oo}(\alpha_1, \beta_1)$	$S_o^A(\alpha_1)$	$C_{oo}(\alpha_1, \beta_2)$	$S_o^B(\beta_1)$	$C_{oo}(\alpha_2, \beta_1)$	$C_{oo}(\alpha_2, \beta_2)$	J
1,069,306 (−)	1,522,865 (+)	1,152,595 (−)	1,693,718 (+)	1,191,146 (−)	69,749 (+)	−126,715

Without background subtraction, the Eberhard *J* value can be calculated from the measured data according to equation (4). (−) values contribute beneficially to a negative *J* value (making it more negative). (+) values contribute detrimentally to a negative *J* value (making it less negative).

contains only directly available detection events related to the ordinary beams of Alice and Bob. Remarkably, this implies that Alice and Bob each need only one detector to test Eberhard's inequality, whereas they each require two detectors for testing a Clauser–Horne–Shimony–Holt inequality. This characteristic can be intuitively understood: consider detectors that monitor 'e' outcomes and whose detection efficiencies decrease gradually to zero. This will just move events from 'e' to 'u': that is, from $n_{oe}(\alpha_1, \beta_2)$ to $n_{ou}(\alpha_1, \beta_2)$ and from $n_{eo}(\alpha_2, \beta_1)$ to $n_{uo}(\alpha_2, \beta_1)$. Only their sum appears in Eberhard's inequality, so the value of *J* does not change.

The entangled photon pairs at 810 nm are produced in a Sagnac source[5,28] pumped by a 405-nm-wavelength laser. The source is based on type-II spontaneous parametric down-conversion using a non-linear crystal (periodically poled potassium titanyl phosphate). In each arm, a cut-off filter and a 3-nm interference filter with near 99% transmission are used to suppress counts from the pump laser and reduce the background counts. The source can be tuned to produce non-maximally entangled states with the form expressed by equation (2) for any *r* by setting the polarization of the pump light with half- and quarter-wave plates.

The measurement set-up (see Fig. 2), containing a rotatable half-wave plate in a high-precision rotation mount and a calcite polarizer, is positioned in front of the fibre coupler on both Alice's and Bob's sides to facilitate measurement of the desired polarization (α and β). The measurements require only one output of the polarizer, so only the transmitted ordinary beam of the polarizer is coupled into the fibre; the extraordinary beam is blocked after transmission. We couple the 810-nm photons into an optical fibre (SMF-28), which guides the photons to the sensitive areas of the detectors. To achieve a high coupling efficiency in both arms, we optimized the focusing of the pump laser and the fibre couplers.

To achieve highly efficient photon detection, we used TES calorimetric detectors that owe their sensitivity to operation at the superconducting transition, a regime characterized by a steep dependence of resistance on temperature[7]. By exploiting a wavelength-optimized optical structure, these detectors have been reported to demonstrate detection efficiencies of up to 98% (including losses from packaging and fibre coupling)[6,7]. Superconducting quantum interference devices (SQUIDs)[29] amplify the nanoampere-level TES current signal, which is subsequently digitized and stored for later analysis. Without requiring any additional information about the data, algorithms identify photon signatures in the analogue output signal, determine an arrival time for each event, and count two-photon coincidences.

As a guide for the experimental settings needed to observe a violation of local realism, we used numerical simulations and optimization to determine an optimal non-maximally entangled state. For input, the model used the estimated background rate, the observed visibility, and the overall efficiencies η_A and η_B on Alice's and Bob's sides. The model estimated a value for *r* but also appropriate measurement settings α_1, α_2 on Alice's side and β_1, β_2 on Bob's side.

We set the state with a value of ~0.3 for *r* and measured for a total of 300 s per setting at each of the four settings $\alpha_1\beta_1$, $\alpha_1\beta_2$, $\alpha_2\beta_1$ and $\alpha_2\beta_2$, where $\alpha_1 = 85.6°$, $\alpha_2 = 118.0°$, $\beta_1 = -5.4°$ and $\beta_2 = 25.9°$. The relevant single and coincidence counts obtained appear in Table 1 and yield $J = -126,715$.

After recording for a total of 300 s per setting we divided our data into 10-s blocks and calculated the standard deviation of the resulting 30 different *J* values. This yielded $\sigma = 1,837$ for our aggregate *J* value of $J = -126,715$, a 69-σ violation (see Fig. 3). Note that this calculation does not assume Poisson counting statistics or any error propagation rules. We estimate the number of produced pairs to $N = 24.2 \times 10^6$ per applied setting, yielding a normalized violation of $J/N = -0.00524$ (± 0.00008).

Under the assumptions of locality and freedom of choice, a negative *J* value refutes local realism without the fair-sampling assumption or post-selection on created pairs, regardless of the states and angles used for the measurement or any error in their implementation. Nonetheless, additional measurements can provide further insight into the obtained value. The directly measured arm efficiencies (each a ratio of observed coincidence and singles counts without any correction) measured in the *HV*-basis were $\eta_A = 73.77\%$ ($\pm 0.07\%$) in Alice's arm and $\eta_B = 78.59\%$ ($\pm 0.08\%$) in Bob's arm. We attribute these imperfect coupling efficiencies to various possibly arm-dependent effects including optical losses in the source, coupling, fibre splices, and detectors. We estimate our *r* value and visibility to be about 0.297 and 97.5%, respectively. Using these values, our numerical model (used for the abovementioned optimization) agrees very well with our measured *J* value.

Using photons, we have demonstrated an experimental Bell inequality violation that closes the fair-sampling loophole. Without relying on any assumed error distribution, we statistically verify a violation of Eberhard's inequality by nearly 70 standard deviations and thus clearly demonstrate the necessity of abandoning all local realistic theories that take advantage of unfair sampling to explain the observed values. Moreover, because the derivation of Eberhard's Bell inequality even includes events not detected on either side, no post-selection is necessary to violate the inequality. To achieve a loophole-free Bell test as described above, it will be necessary to introduce space-like separation sufficient to prohibit unwanted communication between Alice, Bob, the measurement decisions, and the photon emission event. This will require fast random-number generators, precise timing, and efficiency gains to offset the propagation losses introduced by the increased distance. We do not find this unreasonable.

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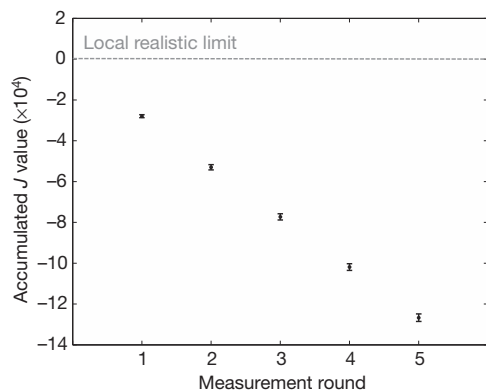
Figure 3 | Eberhard *J* value computed from up to five measurements of recorded data. Any negative *J* value violates the inequality and refutes all local realistic models that exploit the fair-sampling loophole. Error bars represent ±1 standard deviation calculated from the binned raw data.

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Author Contributions M.G. designed and carried out the experiment, and analysed data. A.M. designed and carried out the experiment. S.R. provided theoretical analysis, designed and carried out the experiment, and analysed data. B.W. designed and carried out the experiment. J.K. provided theoretical analysis, and analysed data. J.B., A.L., B.C., T.G. and S.W.N. provided experimental and conceptual assistance. R.U. designed the experiment and provided experimental, organizational and conceptual assistance. A.Z. conceived the research and guided the experiment. All authors wrote the manuscript.

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