



Electricals For You

اللجنة الأكاديمية لقسم الهندسة الكهربائية - HU

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لجنة كلية الهندسة - الجامعة الهاشمية

Chapter 1

1.1

$$n_i = BT^{3/2} e^{-E_g/2kT}$$

(a) Silicon

$$(i) \quad n_i = (5.23 \times 10^{15})(250)^{3/2} \exp\left[\frac{-1.1}{2(86 \times 10^{-6})(250)}\right] \\ = 2.067 \times 10^{19} \exp[-25.58] \\ \underline{n_i = 1.61 \times 10^8 \text{ cm}^{-3}}$$

$$(ii) \quad n_i = (5.23 \times 10^{15})(350)^{3/2} \exp\left[\frac{-1.1}{2(86 \times 10^{-6})(350)}\right] \\ = 3.425 \times 10^{19} \exp[-18.27] \\ \underline{n_i = 3.97 \times 10^{11} \text{ cm}^{-3}}$$

(b) GaAs

$$(i) \quad n_i = (2.10 \times 10^{14})(250)^{3/2} \exp\left[\frac{-1.4}{2(86 \times 10^{-6})(250)}\right] \\ = (8.301 \times 10^{17}) \exp[-32.56] \\ \underline{n_i = 6.02 \times 10^3 \text{ cm}^{-3}}$$

$$(ii) \quad n_i = (2.10 \times 10^{14})(350)^{3/2} \exp\left[\frac{-1.4}{2(86 \times 10^{-6})(350)}\right] \\ = (1.375 \times 10^{18}) \exp[-23.26] \\ \underline{n_i = 1.09 \times 10^8 \text{ cm}^{-3}}$$

1.2

a. $n_i = BT^{3/2} \exp\left(\frac{-Eg}{2kT}\right)$

$$10^{12} = 5.23 \times 10^{15} T^{3/2} \exp\left(\frac{-1.1}{2(86 \times 10^{-6})(T)}\right)$$

$$1.91 \times 10^{-4} = T^{3/2} \exp\left(-\frac{6.40 \times 10^3}{T}\right)$$

By trial and error, $T \approx 368$ K

b. $n_i = 10^9 \text{ cm}^{-3}$

$$10^9 = 5.23 \times 10^{15} T^{3/2} \exp\left(\frac{-1.1}{2(86 \times 10^{-6})(T)}\right)$$

$$1.91 \times 10^{-7} = T^{3/2} \exp\left(-\frac{6.40 \times 10^3}{T}\right)$$

By trial and error, $T \approx 268^\circ$ K

1.3

Silicon

$$(a) \quad n_i = (5.23 \times 10^{15}) (100)^{3/2} \exp\left[\frac{-1.1}{2(86 \times 10^{-6})(100)}\right] \\ = (5.23 \times 10^{18}) \exp[-63.95] \\ \underline{n_i = 8.79 \times 10^{-10} \text{ cm}^{-3}}$$

$$(b) \quad n_i = (5.23 \times 10^{15}) (300)^{3/2} \exp\left[\frac{-1.1}{2(86 \times 10^{-6})(300)}\right] \\ = (2.718 \times 10^{19}) \exp[-21.32] \\ \underline{n_i = 1.5 \times 10^{10} \text{ cm}^{-3}}$$

$$(c) \quad n_i = (5.23 \times 10^{15}) (500)^{3/2} \exp\left[\frac{-1.1}{2(86 \times 10^{-6})(500)}\right] \\ = (5.847 \times 10^{19}) \exp[-12.79] \\ \underline{n_i = 1.63 \times 10^{14} \text{ cm}^{-3}}$$

Germanium.

$$(a) \quad n_i = (1.66 \times 10^{15}) (100)^{3/2} \exp\left[\frac{-0.66}{2(86 \times 10^{-6})(100)}\right] = (1.66 \times 10^{18}) \exp[-38.37] \\ \underline{n_i = 35.9 \text{ cm}^{-3}}$$

$$(b) \quad n_i = (1.66 \times 10^{15}) (300)^{3/2} \exp\left[\frac{-0.66}{2(86 \times 10^{-6})(300)}\right] = (8.626 \times 10^{18}) \exp[-12.79] \\ \underline{n_i = 2.40 \times 10^{13} \text{ cm}^{-3}}$$

$$(c) \quad n_i = (1.66 \times 10^{15}) (500)^{3/2} \exp\left[\frac{-0.66}{2(86 \times 10^{-6})(500)}\right] = (1.856 \times 10^{19}) \exp[-7.674] \\ \underline{n_i = 8.62 \times 10^{15} \text{ cm}^{-3}}$$

1.4

$$(a) \quad \text{n-type; } n_o = 10^{15} \text{ cm}^{-3}; \quad p_o = \frac{n_i^2}{n_o} = \frac{(2.4 \times 10^{13})^2}{10^{15}} = 5.76 \times 10^{11} \text{ cm}^{-3}$$

$$(b) \quad \text{n-type; } n_o = 10^{15} \text{ cm}^{-3}; \quad p_o = \frac{n_i^2}{n_o} = \frac{(1.5 \times 10^{10})^2}{10^{15}} = 2.25 \times 10^5 \text{ cm}^{-3}$$

1.5

(a) p-type; $p_o = 10^{16} \text{ cm}^{-3}$; $n_o = \frac{n_i^2}{p_o} = \frac{(1.8 \times 10^6)^2}{10^{16}} = 3.24 \times 10^{-4} \text{ cm}^{-3}$

(b) p-type; $p_o = 10^{16} \text{ cm}^{-3}$; $n_o = \frac{n_i^2}{p_o} = \frac{(2.4 \times 10^{13})^2}{10^{16}} = 5.76 \times 10^{10} \text{ cm}^{-3}$

1.6

(a) n-type

(b) $n_o = N_d = 5 \times 10^{16} \text{ cm}^{-3}$

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{16}} = 4.5 \times 10^3 \text{ cm}^{-3}$$

(c) $n_o = N_d = 5 \times 10^{16} \text{ cm}^{-3}$

From Problem 1.1(a)(ii) $n_i = 3.97 \times 10^{11} \text{ cm}^{-3}$

$$p_o = \frac{(3.97 \times 10^{11})^2}{5 \times 10^{16}} = 3.15 \times 10^6 \text{ cm}^{-3}$$

1.7

(a) p-type; $p_o = 5 \times 10^{16} \text{ cm}^{-3}$; $n_o = \frac{n_i^2}{p_o} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{16}} = 4.5 \times 10^3 \text{ cm}^{-3}$

(b) p-type; $p_o = 5 \times 10^{16} \text{ cm}^{-3}$; $n_o = \frac{n_i^2}{p_o} = \frac{(1.8 \times 10^6)^2}{5 \times 10^{16}} = 6.48 \times 10^{-5} \text{ cm}^{-3}$

1.8

(a) Add boron atoms

(b) $N_a = p_o = 2 \times 10^{17} \text{ cm}^{-3}$

(c) $n_o = \frac{n_i^2}{p_o} = \frac{(1.5 \times 10^{10})^2}{2 \times 10^{17}} = 1.125 \times 10^3 \text{ cm}^{-3}$

1.9

(a) $n_o = 5 \times 10^{15} \text{ cm}^{-3}$

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{15}} \Rightarrow p_o = 4.5 \times 10^4 \text{ cm}^{-3}$$

(b) $n_o > p_o \Rightarrow$ n-type

(c) $n_o \cong N_d = 5 \times 10^{15} \text{ cm}^{-3}$

1.10

a. Add Donors

$$N_d = 7 \times 10^{15} \text{ cm}^{-3}$$

$$\text{b. Want } p_o = 10^6 \text{ cm}^{-3} = n_i^2 / N_d$$

$$\text{So } n_i^2 = (10^6)(7 \times 10^{15}) = 7 \times 10^{21}$$

$$= B^2 T^3 \exp\left(\frac{-Eg}{kT}\right)$$

$$7 \times 10^{21} = (5.23 \times 10^{15})^2 T^3 \exp\left(\frac{-1.1}{(86 \times 10^{-6})(T)}\right)$$

By trial and error, $T \approx 324^\circ \text{ K}$

1.11

$$(a) I = A\sigma E = (10^{-5})(1.5)(10) \Rightarrow I = 0.15 \text{ mA}$$

$$(b) I = \frac{AE}{\rho} \Rightarrow E = \frac{I\rho}{A} = \frac{(1.2 \times 10^{-3})(0.4)}{(2 \times 10^{-4})} = 2.4 \text{ V/cm}$$

1.12

$$J = \sigma E \Rightarrow \sigma = \frac{J}{E} = \frac{120}{18} = 6.67 (\Omega - \text{cm})^{-1}$$

$$\sigma \approx e\mu_n N_d \Rightarrow N_d = \frac{\sigma}{e\mu_n} = \frac{(6.67)}{(1.6 \times 10^{-19})(1250)} = 3.33 \times 10^{16} \text{ cm}^{-3}$$

1.13

$$(a) \rho \approx \frac{1}{e\mu_n N_d} \Rightarrow N_d = \frac{1}{e\mu_n \rho} = \frac{1}{(1.6 \times 10^{-19})(1250)(0.65)} = 7.69 \times 10^{15} \text{ cm}^{-3}$$

$$(b) J = \frac{E}{\rho} \Rightarrow E = \rho J = (0.65)(160) = 104 \text{ V/cm}$$

1.14

$$(a) \sigma \approx e\mu_n N_d \Rightarrow N_d = \frac{\sigma}{e\mu_n} = \frac{1.5}{(1.6 \times 10^{-19})(1000)} = 9.375 \times 10^{15} \text{ cm}^{-3}$$

$$(b) N_a = \frac{\sigma}{e\mu_p} = \frac{0.8}{(1.6 \times 10^{-19})(400)} = 1.25 \times 10^{16} \text{ cm}^{-3}$$

1.15

$$(a) \text{For n-type, } \sigma \approx e\mu_n N_d = (1.6 \times 10^{-19})(8500)N_d$$

$$\text{For } 10^{15} \leq N_d \leq 10^{19} \text{ cm}^{-3} \Rightarrow 1.36 \leq \sigma \leq 1.36 \times 10^4 (\Omega - \text{cm})^{-1}$$

$$(b) J = \sigma E = \sigma(0.1) \Rightarrow 0.136 \leq J \leq 1.36 \times 10^3 \text{ A/cm}^2$$

1.16

$$D_n = (0.026)(1250) = 32.5 \text{ cm}^2/\text{s}; \quad D_p = (0.026)(450) = 11.7 \text{ cm}^2/\text{s}$$

$$J_n = eD_n \frac{dn}{dx} = (1.6 \times 10^{-19})(32.5) \left(\frac{10^{16} - 10^{12}}{0 - 0.001} \right) = -52 \text{ A/cm}^2$$

$$J_p = -eD_p \frac{dp}{dx} = -(1.6 \times 10^{-19})(11.7) \left(\frac{10^{12} - 10^{16}}{0 - 0.001} \right) = -18.72 \text{ A/cm}^2$$

Total diffusion current density

$$J = -52 - 18.72 = -70.7 \text{ A/cm}^2$$

1.17

$$\begin{aligned} J_p &= -eD_p \frac{dp}{dx} \\ &= -eD_p (10^{15}) \left(\frac{-1}{L_p} \right) \exp\left(\frac{-x}{L_p}\right) \\ J_p &= \frac{(1.6 \times 10^{-19})(15)(10^{15})}{10 \times 10^{-4}} \exp\left(\frac{-x}{L_p}\right) \end{aligned}$$

$$J_p = 2.4 e^{-x/L_p}$$

(a) $x = 0 \quad J_p = 2.4 \text{ A/cm}^2$

(b) $x = 10 \mu\text{m} \quad J_p = 2.4 e^{-1} = 0.883 \text{ A/cm}^2$

(c) $x = 30 \mu\text{m} \quad J_p = 2.4 e^{-3} = 0.119 \text{ A/cm}^2$

1.18

a. $N_a = 10^{17} \text{ cm}^{-3} \Rightarrow p_o = 10^{17} \text{ cm}^{-3}$

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.8 \times 10^6)^2}{10^{17}} \Rightarrow n_o = 3.24 \times 10^{-5} \text{ cm}^{-3}$$

b. $n = n_o + \delta n = 3.24 \times 10^{-5} + 10^{15} \Rightarrow n = 10^{15} \text{ cm}^{-3}$
 $p = p_o + \delta p = 10^{17} + 10^{15} \Rightarrow p = 1.01 \times 10^{17} \text{ cm}^{-3}$

1.19 $V_{bi} = V_T \ln\left(\frac{N_a N_d}{n_i^2}\right)$

(a) (i) $V_{bi} = (0.026) \ln \left[\frac{(5 \times 10^{15})(5 \times 10^{15})}{(1.5 \times 10^{10})^2} \right] = 0.661 \text{ V}$

(ii) $V_{bi} = (0.026) \ln \left[\frac{(5 \times 10^{17})(10^{15})}{(1.5 \times 10^{10})^2} \right] = 0.739 \text{ V}$

(iii) $V_{bi} = (0.026) \ln \left[\frac{(10^{18})(10^{18})}{(1.5 \times 10^{10})^2} \right] = 0.937 \text{ V}$

$$(b) \quad (i) \quad V_{bi} = (0.026) \ln \left[\frac{(5 \times 10^{15})(5 \times 10^{15})}{(1.8 \times 10^6)^2} \right] = 1.13 \text{ V}$$

$$(ii) \quad V_{bi} = (0.026) \ln \left[\frac{(5 \times 10^{17})(10^{15})}{(1.8 \times 10^6)^2} \right] = 1.21 \text{ V}$$

$$(iii) \quad V_{bi} = (0.026) \ln \left[\frac{(10^{18})(10^{18})}{(1.8 \times 10^6)^2} \right] = 1.41 \text{ V}$$

1.20

$$V_{bi} = V_T \ln \left(\frac{N_a N_d}{n_i^2} \right)$$

or

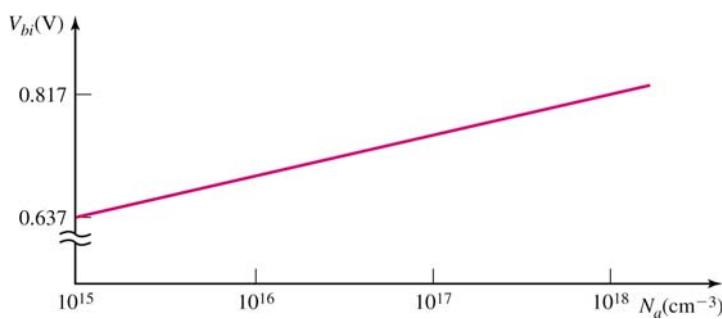
$$N_a = \frac{n_i^2}{N_d} \exp \left(\frac{V_{bi}}{V_T} \right) = \frac{(1.5 \times 10)^2}{10^{16}} \exp \left(\frac{0.712}{0.026} \right) = 1.76 \times 10^{16} \text{ cm}^{-3}$$

1.21

$$V_{bi} = V_T \ln \left(\frac{N_a N_d}{n_i^2} \right) = (0.026) \ln \left[\frac{N_a (10^{16})}{(1.5 \times 10^{10})^2} \right]$$

For $N_a = 10^{15} \text{ cm}^{-3}$, $V_{bi} = 0.637 \text{ V}$

For $N_a = 10^{18} \text{ cm}^{-3}$, $V_{bi} = 0.817 \text{ V}$



1.22

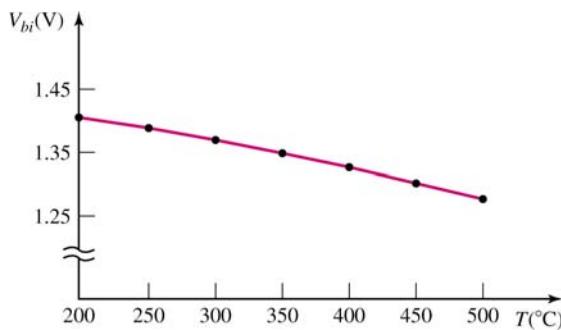
$$kT = (0.026) \left(\frac{T}{300} \right)$$

	kT	$(T)^{3/2}$
200	0.01733	2828.4
250	0.02167	3952.8
300	0.026	5196.2
350	0.03033	6547.9
400	0.03467	8000.0
450	0.0390	9545.9
500	0.04333	11,180.3

$$n_i = (2.1 \times 10^{14})(T^{3/2}) \exp\left(\frac{-1.4}{2(86 \times 10^{-6})(T)}\right)$$

$$V_{bi} = V_T \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

T	n _i	V _{bi}
200	1.256	1.405
250	6.02 × 10 ³	1.389
300	1.80 × 10 ⁶	1.370
350	1.09 × 10 ⁸	1.349
400	2.44 × 10 ⁹	1.327
450	2.80 × 10 ¹⁰	1.302
500	2.00 × 10 ¹¹	1.277



1.23

$$C_j = C_{jo} \left(1 + \frac{V_R}{V_{bi}}\right)^{-1/2}$$

$$V_{bi} = (0.026) \ln \left[\frac{(1.5 \times 10^{16})(4 \times 10^{15})}{(1.5 \times 10^{10})^2} \right] = 0.684 \text{ V}$$

$$(a) \quad C_j = (0.4) \left(1 + \frac{1}{0.684}\right)^{-1/2} = 0.255 \text{ pF}$$

$$(b) \quad C_j = (0.4) \left(1 + \frac{3}{0.684}\right)^{-1/2} = 0.172 \text{ pF}$$

$$(c) \quad C_j = (0.4) \left(1 + \frac{5}{0.684}\right)^{-1/2} = 0.139 \text{ pF}$$

1.24

$$(a) \quad C_j = C_{jo} \left(1 + \frac{V_R}{V_{bi}}\right)^{-1/2}$$

$$\text{For } V_R = 5 \text{ V, } C_j = (0.02) \left(1 + \frac{5}{0.8}\right)^{-1/2} = 0.00743 \text{ pF}$$

$$\text{For } V_R = 1.5 \text{ V, } C_j = (0.02) \left(1 + \frac{1.5}{0.8}\right)^{-1/2} = 0.0118 \text{ pF}$$

$$C_j(\text{avg}) = \frac{0.00743 + 0.0118}{2} = 0.00962 \text{ pF}$$

$$v_C(t) = v_C(\text{final}) + (v_C(\text{initial}) - v_C(\text{final}))e^{-t/\tau}$$

where

$$\tau = RC = RC_j(\text{avg}) = (47 \times 10^3)(0.00962 \times 10^{-12})$$

or

$$\tau = 4.52 \times 10^{-10} \text{ s}$$

$$\text{Then } v_C(t) = 1.5 = 0 + (5 - 0)e^{-t_1/\tau}$$

$$\frac{5}{1.5} = e^{+r_1/\tau} \Rightarrow t_1 = \tau \ln\left(\frac{5}{1.5}\right)$$

$$t_1 = 5.44 \times 10^{-10} \text{ s}$$

$$(b) \quad \text{For } V_R = 0 \text{ V, } C_j = C_{jo} = 0.02 \text{ pF}$$

$$\text{For } V_R = 3.5 \text{ V, } C_j = (0.02) \left(1 + \frac{3.5}{0.8}\right)^{-1/2} = 0.00863 \text{ pF}$$

$$C_j(\text{avg}) = \frac{0.02 + 0.00863}{2} = 0.0143 \text{ pF}$$

$$\tau = RC_j(\text{avg}) = 6.72 \times 10^{-10} \text{ s}$$

$$v_C(t) = v_C(\text{final}) + (v_C(\text{initial}) - v_C(\text{final}))e^{-t/\tau}$$

$$3.5 = 5 + (0 - 5)e^{-t_2/\tau} = 5(1 - e^{-t_2/\tau})$$

$$\text{so that } t_2 = 8.09 \times 10^{-10} \text{ s}$$

1.25

$$C_j = C_{jo} \left(1 + \frac{V_R}{V_{bi}}\right)^{-1/2}; \quad V_{bi} = (0.026) \ln \left[\frac{(5 \times 10^{15})(10^{17})}{(1.5 \times 10^{10})^2} \right] = 0.739 \text{ V}$$

For $V_R = 1 \text{ V}$,

$$C_j = \frac{0.60}{\sqrt{1 + \frac{1}{0.739}}} = 0.391 \text{ pF}$$

For $V_R = 3 \text{ V}$,

$$C_j = \frac{0.60}{\sqrt{1 + \frac{3}{0.739}}} = 0.267 \text{ pF}$$

For $V_R = 5 \text{ V}$,

$$C_j = \frac{0.60}{\sqrt{1 + \frac{5}{0.739}}} = 0.215 \text{ pF}$$

$$(a) \quad f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1.5 \times 10^{-3})(0.391 \times 10^{-12})}} \Rightarrow f_o = 6.57 \text{ MHz}$$

$$(b) \quad f_o = \frac{1}{2\pi\sqrt{(1.5 \times 10^{-3})(0.267 \times 10^{-12})}} \Rightarrow f_o = 7.95 \text{ MHz}$$

$$(c) \quad f_o = \frac{1}{2\pi\sqrt{(1.5 \times 10^{-3})(0.215 \times 10^{-12})}} \Rightarrow f_o = 8.86 \text{ MHz}$$

1.26

$$a. \quad I = I_s \left[\exp\left(\frac{V_D}{V_T}\right) - 1 \right] - 0.90 = \exp\left(\frac{V_D}{V_T}\right) - 1$$

$$\exp\left(\frac{V_D}{V_T}\right) = 1 - 0.90 = 0.10$$

$$V_D = V_T \ln(0.10) \Rightarrow \underline{V_D = -0.0599 \text{ V}}$$

b.

$$\begin{aligned} \left| \frac{I_F}{I_R} \right| &= \frac{I_s}{I_s} \cdot \frac{\left[\exp\left(\frac{V_F}{V_T}\right) - 1 \right]}{\left[\exp\left(\frac{V_R}{V_T}\right) - 1 \right]} = \left| \frac{\exp\left(\frac{0.2}{0.026}\right) - 1}{\exp\left(\frac{-0.2}{0.026}\right) - 1} \right| \\ &= \left| \frac{2190}{-1} \right| \end{aligned}$$

$$\underline{\frac{I_F}{I_R} = 2190}$$

1.27 $I_D = I_s \left[\exp\left(\frac{V_D}{V_T}\right) - 1 \right]$

$$(a) \quad (i) \quad I_D = (10^{-11}) \exp\left(\frac{0.3}{0.026}\right) \Rightarrow 1.03 \mu \text{A}$$

$$(ii) \quad I_D = (10^{-11}) \exp\left(\frac{0.5}{0.026}\right) \Rightarrow 2.25 \text{ mA}$$

$$(iii) \quad I_D = (10^{-11}) \exp\left(\frac{0.7}{0.026}\right) \Rightarrow 4.93 \text{ A}$$

$$(iv) \quad I_D = (10^{-11}) \left[\exp\left(\frac{-0.02}{0.026}\right) - 1 \right] = -5.37 \times 10^{-12} \text{ A}$$

$$(v) \quad I_D = (10^{-11}) \left[\exp\left(\frac{-0.20}{0.026}\right) - 1 \right] \cong -10^{-11} \text{ A}$$

$$(vi) \quad I_D = -(10^{-11}) \text{ A}$$

$$(b) \quad (i) \quad I_D = (10^{-13}) \exp\left(\frac{0.3}{0.026}\right) \Rightarrow 0.0103 \mu \text{A}$$

$$(ii) \quad I_D = (10^{-13}) \exp\left(\frac{0.5}{0.026}\right) \Rightarrow 22.5 \mu \text{A}$$

$$(iii) \quad I_D = (10^{-13}) \exp\left(\frac{0.7}{0.026}\right) \Rightarrow 49.3 \text{ mA}$$

$$(iv) \quad I_D = \left(10^{-13}\right) \left[\exp\left(\frac{-0.02}{0.026}\right) - 1 \right] = -5.37 \times 10^{-14} \text{ A}$$

$$(v) \quad I_D \approx -10^{-13} \text{ A}$$

$$(vi) \quad I_D \approx -10^{-13} \text{ A}$$

$$\mathbf{1.28} \quad V_D = V_T \ln\left(\frac{I_D}{I_S}\right)$$

$$(a) \quad (i) \quad V_D = (0.026) \ln\left(\frac{10 \times 10^{-6}}{10^{-11}}\right) = 0.359 \text{ V}$$

$$V_D = (0.026) \ln\left(\frac{100 \times 10^{-6}}{10^{-11}}\right) = 0.419 \text{ V}$$

$$V_D = (0.026) \ln\left(\frac{10^{-3}}{10^{-11}}\right) = 0.479 \text{ V}$$

$$(ii) \quad -5 \times 10^{-12} = 10^{-11} \left[\exp\left(\frac{V_D}{0.026}\right) - 1 \right] \Rightarrow V_D = -0.018 \text{ V}$$

$$(b) \quad (i) \quad V_D = (0.026) \ln\left(\frac{10 \times 10^{-6}}{10^{-13}}\right) = 0.479 \text{ V}$$

$$V_D = (0.026) \ln\left(\frac{100 \times 10^{-6}}{10^{-13}}\right) = 0.539 \text{ V}$$

$$V_D = (0.026) \ln\left(\frac{10^{-3}}{10^{-13}}\right) = 0.599 \text{ V}$$

$$(ii) \quad -10^{-14} = 10^{-13} \left[\exp\left(\frac{V_D}{0.026}\right) - 1 \right] \Rightarrow V_D = -0.00274 \text{ V}$$

1.29

$$(a) \quad 10^{-3} = I_S \exp\left(\frac{0.7}{0.026}\right)$$

$$I_S = 2.03 \times 10^{-15} \text{ A}$$

(b)

V_D	$I_D(A)(n=1)$	$I_D(A)(n=2)$
0.1	9.50×10^{-14}	1.39×10^{-14}
0.2	4.45×10^{-12}	9.50×10^{-14}
0.3	2.08×10^{-10}	6.50×10^{-13}
0.4	9.75×10^{-9}	4.45×10^{-12}
0.5	4.56×10^{-7}	3.04×10^{-11}
0.6	2.14×10^{-5}	2.08×10^{-10}
0.7	10^{-3}	1.42×10^{-9}

1.30

(a)

$$I_s = 10^{-12} \text{ A}$$

$V_D(\text{v})$	$I_D(\text{A})$	$\log_{10} I_D$
0.10	4.68×10^{-11}	-10.3
0.20	2.19×10^{-9}	-8.66
0.30	1.03×10^{-7}	-6.99
0.40	4.80×10^{-6}	-5.32
0.50	2.25×10^{-4}	-3.65
0.60	1.05×10^{-2}	-1.98
0.70	4.93×10^{-1}	-0.307

(b)

$$I_s = 10^{-14} \text{ A}$$

$V_D(\text{v})$	$I_D(\text{A})$	$\log_{10} I_D$
0.10	4.68×10^{-13}	-12.3
0.20	2.19×10^{-11}	-10.66
0.30	1.03×10^{-9}	-8.99
0.40	4.80×10^{-8}	-7.32
0.50	2.25×10^{-6}	-5.65
0.60	1.05×10^{-4}	-3.98
0.70	4.93×10^{-3}	-2.31

1.31

a.

$$\frac{I_{D2}}{I_{D1}} = 10 = \exp\left(\frac{V_{D2} - V_{D1}}{V_T}\right)$$

$$\Delta V_D = V_T \ln(10) \Rightarrow \underline{\Delta V_D = 59.9 \text{ mV} \approx 60 \text{ mV}}$$

$$\text{b. } \Delta V_D = V_T \ln(100) \Rightarrow \underline{\Delta V_D = 119.7 \text{ mV} \approx 120 \text{ mV}}$$

1.32

$$\text{(a) (i) } V_D = (0.026) \ln\left(\frac{2}{2 \times 10^{-9}}\right) = 0.539 \text{ V}$$

$$\text{(ii) } V_D = (0.026) \ln\left(\frac{20}{2 \times 10^{-9}}\right) = 0.599 \text{ V}$$

$$\text{(b) (i) } I_D = (2 \times 10^{-9}) \exp\left(\frac{0.4}{0.026}\right) \Rightarrow 9.60 \text{ mA}$$

$$\text{(ii) } I_D = (2 \times 10^{-9}) \exp\left(\frac{0.65}{0.026}\right) \Rightarrow 144 \text{ A}$$

1.33

$$V_D = V_t \ln\left(\frac{I_D}{I_S}\right) = (0.026) \ln\left(\frac{2 \times 10^{-3}}{5 \times 10^{-14}}\right) = 0.6347 \text{ V}$$

$$V_D = (0.026) \ln\left(\frac{2 \times 10^{-3}}{5 \times 10^{-12}}\right) = 0.5150 \text{ V}$$

$$\underline{0.5150 \leq V_D \leq 0.6347 \text{ V}}$$

1.34

$$(a) \quad 1.5 \times 10^{-3} = I_S \exp\left(\frac{0.30}{0.026}\right) \Rightarrow I_S = 1.46 \times 10^{-8} \text{ A}$$

$$(b) \quad (i) \quad I_D = (1.462 \times 10^{-8}) \exp\left(\frac{0.35}{0.026}\right) \Rightarrow I_D = 10.3 \text{ mA}$$

$$(ii) \quad I_D = (1.462 \times 10^{-8}) \exp\left(\frac{0.25}{0.026}\right) \Rightarrow I_D = 0.219 \text{ mA}$$

1.35

$$(a) \quad I_D = (10^{-22}) \exp\left(\frac{0.8}{0.026}\right) \Rightarrow 2.31 \text{ nA}$$

$$I_D = (10^{-22}) \exp\left(\frac{1.0}{0.026}\right) \Rightarrow 5.05 \mu\text{A}$$

$$I_D = (10^{-22}) \exp\left(\frac{1.2}{0.026}\right) \Rightarrow 11.1 \text{ mA}$$

$$I_D = (10^{-22}) \left[\exp\left(\frac{-0.02}{0.026}\right) - 1 \right] = -5.37 \times 10^{-23} \text{ A}$$

For $V_D = -0.20 \text{ V}$, $I_D = -10^{-22} \text{ A}$

For $V_D = -2 \text{ V}$, $I_D = -10^{-22} \text{ A}$

(b)

$$I_D = (5 \times 10^{-24}) \exp\left(\frac{0.8}{0.026}\right) \Rightarrow 115 \text{ pA}$$

$$I_D = (5 \times 10^{-24}) \exp\left(\frac{1.0}{0.026}\right) \Rightarrow 0.253 \mu\text{A}$$

$$I_D = (5 \times 10^{-24}) \exp\left(\frac{1.2}{0.026}\right) \Rightarrow 0.554 \text{ mA}$$

$$I_D = (5 \times 10^{-24}) \left[\exp\left(\frac{-0.02}{0.026}\right) - 1 \right] = -2.68 \times 10^{-24} \text{ A}$$

For $V_D = -0.20 \text{ V}$, $I_D = -5 \times 10^{-24} \text{ A}$

For $V_D = -2 \text{ V}$, $I_D = -5 \times 10^{-24} \text{ A}$

1.36

I_s doubles for every 5C increase in temperature.

$$I_s = 10^{-12} \text{ A at } T = 300\text{K}$$

$$\text{For } I_s = 0.5 \times 10^{-12} \text{ A } \Rightarrow T = 295 \text{ K}$$

$$\text{For } I_s = 50 \times 10^{-12} \text{ A, } (2)^n = 50 \Rightarrow n = 5.64$$

Where n equals number of 5C increases.

$$\text{Then } \Delta T = (5.64)(5) = 28.2 \text{ K}$$

$$\text{So } \underline{295 \leq T \leq 328.2 \text{ K}}$$

1.37

$$\frac{I_s(T)}{I_s(-55)} = 2^{\Delta T/5}, \quad \Delta T = 155^\circ \text{ C}$$

$$\frac{I_s(100)}{I_s(-55)} = 2^{155/5} = 2.147 \times 10^9$$

$$V_T @ 100^\circ \text{C} \Rightarrow 373^\circ \text{K} \Rightarrow V_T = 0.03220$$

$$V_T @ -55^\circ \text{C} \Rightarrow 216^\circ \text{K} \Rightarrow V_T = 0.01865$$

$$\begin{aligned} \frac{I_D(100)}{I_D(-55)} &= (2.147 \times 10^9) \times \frac{\exp\left(\frac{0.6}{0.0322}\right)}{\exp\left(\frac{0.6}{0.01865}\right)} \\ &= \frac{(2.147 \times 10^9)(1.237 \times 10^8)}{(9.374 \times 10^{13})} \end{aligned}$$

$$\underline{\frac{I_D(100)}{I_D(-55)} = 2.83 \times 10^3}$$

1.38

$$(a) \quad V_{PS} = I_D R + V_D$$

$$2.8 = I_D(10^6) + V_D; \quad I_D = (5 \times 10^{-11}) \exp\left(\frac{V_D}{0.026}\right)$$

By trial and error,

$$V_D = 0.282 \text{ V, } I_D = 2.52 \mu \text{A}$$

(b)

$$I_D \cong -5 \times 10^{-11} \text{ A, } V_D = -2.8 \text{ V}$$

1.39

$$10 = I_D \left(2 \times 10^4 \right) + V_D \text{ and } V_D = (0.026) \ln \left(\frac{I_D}{10^{-12}} \right)$$

Trial and error.

$V_D(v)$	$I_D(A)$	$V_D(v)$
0.50	4.75×10^{-4}	0.5194
0.517	4.7415×10^{-4}	0.5194
0.5194	4.740×10^{-4}	0.5194

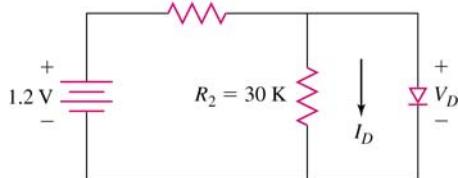
$$\underline{V_D = 0.5194 \text{ V}}$$

$$\underline{I_D = 0.4740 \text{ mA}}$$

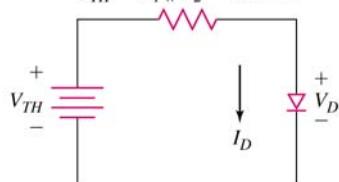
1.40

$$I_s = 5 \times 10^{-13} \text{ A}$$

$$R_1 = 50 \text{ K}$$



$$R_{TH} = R_1 \parallel R_2 = 18.75 \text{ K}$$



$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (1.2) = \left(\frac{30}{80} \right) (1.2) = 0.45 \text{ V}$$

$$0.45 = I_D R_{TH} + V_D, \quad V_D = V_T \ln \left(\frac{I_D}{I_s} \right)$$

By trial and error:

$$\underline{I_D = 2.56 \mu\text{A}}, \quad \underline{V_D = 0.402 \text{ V}}$$

1.41

(a) $I_{D1} = I_{D2} = 1 \text{ mA}$

$$(i) V_{D1} = V_{D2} = (0.026) \ln\left(\frac{10^{-3}}{10^{-13}}\right) = 0.599 \text{ V}$$

$$(ii) V_{D1} = (0.026) \ln\left(\frac{10^{-3}}{5 \times 10^{-14}}\right) = 0.617 \text{ V}$$

$$V_{D2} = (0.026) \ln\left(\frac{10^{-3}}{5 \times 10^{-13}}\right) = 0.557 \text{ V}$$

(b) $V_{D1} = V_{D2}$

$$(i) I_{D1} = I_{D2} = \frac{I_i}{2} = 0.5 \text{ mA}$$

$$V_{D1} = V_{D2} = (0.026) \ln\left(\frac{0.5 \times 10^{-3}}{10^{-13}}\right) = 0.581 \text{ V}$$

$$(ii) \frac{I_{D1}}{I_{D2}} = \frac{I_{S1}}{I_{S2}} = \frac{5 \times 10^{-14}}{5 \times 10^{-13}} = 0.10$$

So $I_{D1} = 0.10I_{D2}$

$$I_{D1} + I_{D2} = 1.1I_{D2} = 1 \text{ mA}$$

So $I_{D2} = 0.909 \text{ mA}$, $I_{D1} = 0.0909 \text{ mA}$

Now

$$V_{D1} = (0.026) \ln\left(\frac{0.0909 \times 10^{-3}}{5 \times 10^{-14}}\right) = 0.554 \text{ V}$$

$$V_{D2} = (0.026) \ln\left(\frac{0.909 \times 10^{-3}}{5 \times 10^{-13}}\right) = 0.554 \text{ V}$$

1.42

(a) $I_{D3} = (6 \times 10^{-14}) \exp\left(\frac{0.635}{0.026}\right) \Rightarrow 2.426 \text{ mA}$

$$I_R = \frac{0.635}{1} = 0.635 \text{ mA}$$

$$I_{D1} = I_{D2} = 2.426 + 0.635 = 3.061 \text{ mA}$$

$$V_{D1} = V_{D2} = (0.026) \ln\left(\frac{3.061 \times 10^{-3}}{6 \times 10^{-14}}\right) = 0.641 \text{ V}$$

$$V_I = 2(0.641) + 0.635 = 1.917 \text{ V}$$

(b) $I_{D3} = 2.426 \text{ mA}$

$$I_R = \frac{0.635}{0.5} = 1.27 \text{ mA}$$

$$I_{D1} = I_{D2} = 2.426 + 1.27 = 3.696 \text{ mA}$$

$$V_{D1} = V_{D2} = (0.026) \ln\left(\frac{3.696 \times 10^{-3}}{6 \times 10^{-14}}\right) = 0.6459 \text{ V}$$

$$V_I = 2(0.6459) + 0.635 = 1.927 \text{ V}$$

1.43

(a) Assume diode is conducting.

Then, $V_D = V_\gamma = 0.7 \text{ V}$

$$\text{So that } I_{R2} = \frac{0.7}{30} \Rightarrow 23.3 \mu\text{A}$$

$$I_{R1} = \frac{1.2 - 0.7}{10} \Rightarrow 50 \mu\text{A}$$

$$\text{Then } I_D = I_{R1} - I_{R2} = 50 - 23.3$$

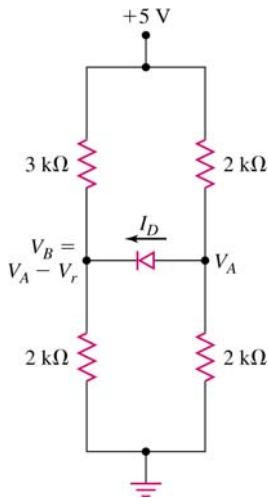
$$\text{Or } I_D = 26.7 \mu\text{A}$$

(b) Let $R_i = 50 \text{ k}\Omega$ Diode is cutoff.

$$V_D = \frac{30}{30 + 50} \cdot (1.2) = 0.45 \text{ V}$$

Since $V_D < V_\gamma$, $I_D = 0$

1.44



At node V_A :

$$(1) \quad \frac{5 - V_A}{2} = I_D + \frac{V_A}{2}$$

At node $V_B = V_A - V_r$

$$(2) \quad \frac{5 - (V_A - V_r)}{2} + I_D = \frac{(V_A - V_r)}{2}$$

$$\text{So } \frac{5 - (V_A - V_r)}{3} + \left[\frac{5 - V_A}{2} - \frac{V_A}{2} \right] = \frac{V_A - V_r}{2}$$

Multiply by 6:

$$10 - 2(V_A - V_r) + 15 - 6V_A = 3(V_A - V_r)$$

$$25 + 2V_r + 3V_r = 11V_A$$

$$(a) \quad V_r = 0.6 \text{ V}$$

$$11V_A = 25 + 5(0.6) = 28 \Rightarrow V_A = 2.545 \text{ V}$$

From (1) $I_D = \frac{5-V_A}{2} - \frac{V_A}{2} = 2.5 - V_A \Rightarrow I_D \text{ Neg.} \Rightarrow \underline{I_D = 0}$

Both (a), (b) $\underline{I_D = 0}$

$$V_A = 2.5, V_B = \frac{2}{5} \cdot 5 = 2 \text{ V} \Rightarrow \underline{V_D = 0.50 \text{ V}}$$

1.45

(a) $V_o = I_i(1); I_D = 0; \text{ for } 0 \leq I_i \leq 0.7 \text{ mA}$

$$V_o = 0.7 \text{ V}; I_D = (I_i - 0.7) \text{ mA; for } I_i \geq 0.7 \text{ mA}$$

(b) $V_o = I_i(1); I_D = 0; \text{ for } 0 \leq I_i \leq 1.7 \text{ mA}$

$$V_o = 1.7 \text{ V}; I_D = (I_i - 1.7) \text{ mA; for } I_i \geq 1.7 \text{ mA}$$

(c) $V_o = 0.7 \text{ V}; I_{D1} = I_i; I_{D2} = 0; \text{ for } 0 \leq I_i \leq 2 \text{ mA}$

1.46

Minimum diode current for V_{PS} (min)

$$I_D(\text{min}) = 2 \text{ mA}, V_D = 0.7 \text{ V}$$

$$I_2 = \frac{0.7}{R_2}, I_1 = \frac{5-0.7}{R_1} = \frac{4.3}{R_1}$$

We have $I_1 = I_2 + I_D$

$$\text{so (1)} \frac{4.3}{R_1} = \frac{0.7}{R_2} + 2$$

Maximum diode current for V_{PS} (max)

$$P = I_D V_D - 10 = I_D (0.7) \Rightarrow I_D = 14.3 \text{ mA}$$

$$I_1 = I_2 + I_D$$

or

$$(2) \quad \frac{9.3}{R_1} = \frac{0.7}{R_2} + 14.3$$

$$\text{Using Eq. (1), } \frac{9.3}{R_1} = \frac{4.3}{R_1} - 2 + 14.3 \Rightarrow \underline{R_1 = 0.41 \text{ k}\Omega}$$

Then $\underline{R_2 = 82.5\Omega} \quad 82.5\Omega$

1.47

(a) (i) $I = \frac{5-0.7}{20} = 0.215 \text{ mA}, V_o = 0.7 \text{ V}$

(ii) $I = \frac{5-0.6}{20} = 0.220 \text{ mA}, V_o = 0.6 \text{ V}$

(b) (i) $I = \frac{5-0.7-(-5)}{40} = 0.2325 \text{ mA}, V_o = (0.2325)(20) - 5 = -0.35 \text{ V}$

(iii) $I = \frac{5-0.6-(-5)}{40} = 0.235 \text{ mA}, V_o = (0.235)(20) - 5 = -0.30 \text{ V}$

- (c) (i) $I = \frac{2 - 0.7 - (-8)}{25} = 0.372 \text{ mA}$, $V_o = 2 - (0.372)(5) = 0.14 \text{ V}$
(ii) $I = \frac{2 - 0.6 - (-8)}{25} = 0.376 \text{ mA}$, $V_o = 2 - (0.376)(5) = 0.12 \text{ V}$
- (d) (i) $I = 0$, $V_o = -5 \text{ V}$
(ii) $I = 0$, $V_o = -5 \text{ V}$
-

1.48

- (a) $I = \frac{5 - V_o}{20}$, $I = (5 \times 10^{-14}) \exp\left(\frac{V_D}{0.026}\right)$
By trial and error, $V_D = V_o = 0.5775 \text{ V}$, $I = 0.221 \text{ mA}$
- (b) $I = \frac{10 - V_D}{40}$, $V_o = 5 - I(20) - V_D$
 $I = 0.2355 \text{ mA}$, $V_D = 0.579 \text{ V}$, $V_o = -0.289$
- (c) $I = \frac{10 - V_D}{25}$, $V_o = 2 - I(5)$
 $I = 0.3763 \text{ mA}$, $V_D = 0.5913 \text{ V}$, $V_o = 0.1185$
- (d) $I = -5 \times 10^{-14} \text{ A}$, $V_o \approx -5 \text{ V}$
-

1.49

- (a) Diode forward biased $V_D = 0.7 \text{ V}$
 $5 = (0.4)(4.7) + 0.7 + V \Rightarrow V = 2.42 \text{ V}$
- (b) $P = I \cdot V_D = (0.4)(0.7) \Rightarrow P = 0.28 \text{ mW}$
-

1.50

- (a) $I_{R2} = I_{D1} = \frac{0.65}{1} = 0.65 \text{ mA} = I_{D1}$
 $I_{D2} = 2(0.65) = 1.30 \text{ mA}$
 $I_{D2} = \frac{V_I - 2V_r - V_0}{R_l} = \frac{5 - 3(0.65)}{R_l} = 1.30 \Rightarrow R_l = 2.35 \text{ K}$
- (b) $I_{R2} = \frac{0.65}{1} = 0.65 \text{ mA}$
 $I_{D2} = \frac{8 - 3(0.65)}{2} \Rightarrow I_{D2} = 3.025 \text{ mA}$
 $I_{D1} = I_{D2} - I_{R2} = 3.025 - 0.65$
 $I_{D1} = 2.375 \text{ mA}$
-

1.51

a. $\tau_d = \frac{V_T}{I_{DQ}} = \frac{(0.026)}{1} = 0.026 \text{ k}\Omega = 26\Omega$
 $i_d = 0.05I_{DQ} = 50 \mu\text{A} \text{ peak-to-peak}$
 $v_d = i_d \tau_d = (26)(50) \mu\text{A} \Rightarrow v_d = 1.30 \text{ mV peak-to-peak}$

b. For $I_{DQ} = 0.1 \text{ mA} \Rightarrow \tau_d = \frac{(0.026)}{0.1} = 260\Omega$
 $i_d = 0.05I_{DQ} = 5 \mu\text{A} \text{ peak-to-peak}$
 $v_d = i_d \tau_d = (260)(5) \mu\text{V} \Rightarrow v_d = 1.30 \text{ mV peak-to-peak}$

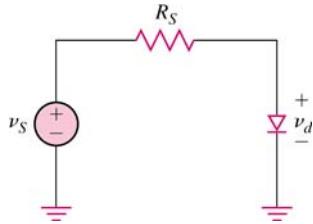
1.52

(a) $r_d = \frac{V_T}{I_{DQ}} = \frac{0.026}{0.026} = 1 \text{ k}\Omega$

(b) $r_d = \frac{0.026}{0.26} \Rightarrow 100\Omega$

(c) $r_d = \frac{0.026}{2.6} \Rightarrow 10\Omega$

1.53



a. diode resistance $r_d = V_T/I$

$$v_d = \left(\frac{r_d}{r_d + R_s} \right) v_s = \left(\frac{V_T/I}{V_T/I + R_s} \right) v_s$$

$$v_d = \left(\frac{V_T}{V_T + IR_s} \right) v_s = v_o$$

b. $R_s = 260\Omega$

$$I = 1 \text{ mA}, \frac{v_0}{v_s} = \left(\frac{V_T}{V_T + IR_s} \right) = \frac{0.026}{0.026 + (1)(0.26)} \Rightarrow \frac{v_0}{v_s} = 0.0909$$

$$I = 0.1 \text{ mA}, \frac{v_0}{v_s} = \frac{0.026}{0.026 + (0.1)(0.26)} \Rightarrow \frac{v_0}{v_s} = 0.50$$

$$I = 0.01 \text{ mA}, \frac{v_0}{v_s} = \frac{0.026}{0.026 + (0.01)(0.26)} \Rightarrow \frac{v_0}{v_s} = 0.909$$

1.54

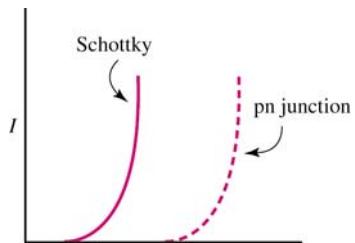
pn junction diode

$$V_D = (0.026) \ln \left(\frac{0.72 \times 10^{-3}}{5 \times 10^{-13}} \right) = 0.548 \text{ V}$$

Schottky diode

$$V_D = (0.026) \ln \left(\frac{0.72 \times 10^{-3}}{5 \times 10^{-8}} \right) = 0.249 \text{ V}$$

1.55



$$\text{Schottky: } I \cong I_s \exp \left(\frac{V_a}{V_T} \right)$$

$$V_a = V_T \ln \left(\frac{I}{I_s} \right) = (0.026) \ln \left(\frac{0.5 \times 10^{-3}}{5 \times 10^{-7}} \right) = 0.1796 \text{ V}$$

Then

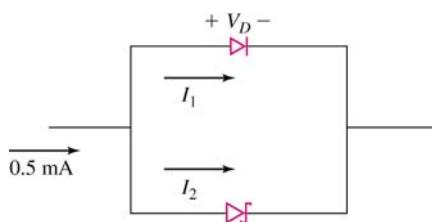
$$V_a \text{ of pn junction} = 0.1796 + 0.30 = 0.4796$$

$$I_s = \frac{I}{\exp \left(\frac{V_a}{V_T} \right)} = \frac{0.5 \times 10^{-3}}{\exp \left(\frac{0.4796}{0.026} \right)}$$

$$\underline{I_s = 4.87 \times 10^{-12} \text{ A}}$$

1.56

(a)



$$I_1 + I_2 = 0.5 \times 10^{-3}$$

$$5 \times 10^{-8} \exp\left(\frac{V_D}{V_T}\right) + 10^{-12} \exp\left(\frac{V_D}{V_T}\right) = 0.5 \times 10^{-3}$$

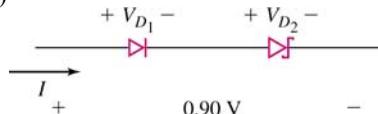
$$5.0001 \times 10^{-8} \exp\left(\frac{V_D}{V_T}\right) = 0.5 \times 10^{-3}$$

$$V_D = (0.026) \ln\left(\frac{0.5 \times 10^{-3}}{5.0001 \times 10^{-8}}\right) \Rightarrow V_D = 0.2395$$

Schottky diode, $I_2 = 0.49999 \text{ mA}$

pn junction, $I_1 = 0.00001 \text{ mA}$

(b)



$$I = 10^{-12} \exp\left(\frac{V_{D1}}{V_T}\right) = 5 \times 10^{-8} \exp\left(\frac{V_{D2}}{V_T}\right)$$

$$V_{D1} + V_{D2} = 0.9$$

$$10^{-12} \exp\left(\frac{V_{D1}}{V_T}\right) = 5 \times 10^{-8} \exp\left(\frac{0.9 - V_{D1}}{V_T}\right)$$

$$= 5 \times 10^{-8} \exp\left(\frac{0.9}{V_T}\right) \exp\left(-\frac{V_{D1}}{V_T}\right)$$

$$\exp\left(\frac{2V_{D1}}{V_T}\right) = \left(\frac{5 \times 10^{-8}}{10^{-12}}\right) \exp\left(\frac{0.9}{0.026}\right)$$

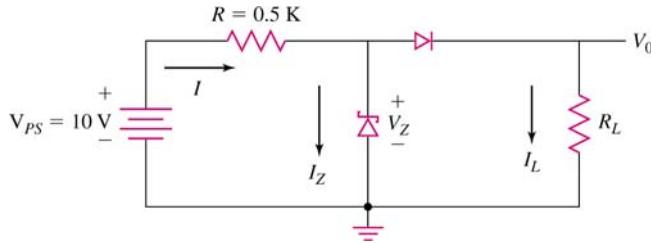
$$2V_{D1} = V_T \ln\left(\frac{5 \times 10^{-8}}{10^{-12}}\right) + 0.9 = 1.1813$$

$V_{D1} = 0.5907$ pn junction

$V_{D2} = 0.3093$ Schottky diode

$$I = 10^{-12} \exp\left(\frac{0.5907}{0.026}\right) \Rightarrow I = 7.35 \text{ mA}$$

1.57



$$V_Z = V_{Z0} = 5.6 \text{ V at } I_Z = 0.1 \text{ mA}$$

$$r_Z = 10 \Omega$$

$$I_z r_z = (0.1)(10) = 1 \text{ mV}$$

$$V_{z0} = 5.599$$

a. $R_L \rightarrow \infty \Rightarrow$

$$I_z = \frac{10 - 5.599}{R + r_z} = \frac{4.401}{0.50 + 0.01} = 8.63 \text{ mA}$$

$$V_z = V_{z0} + I_z r_z = 5.599 + (0.00863)(10)$$

$$\underline{V_z = V_0 = 5.685 \text{ V}}$$

b. $V_{ps} = 11 \text{ V} \Rightarrow I_z = \frac{11 - 5.599}{0.51} = 10.59 \text{ mA}$

$$V_z = V_0 = 5.599 + (0.01059)(10) = 5.7049 \text{ V}$$

$$V_{ps} = 9 \text{ V} \Rightarrow I_z = \frac{9 - 5.599}{0.51} = 6.669 \text{ mA}$$

$$V_z = V_0 = 5.599 + (0.006669)(10) = 5.66569 \text{ V}$$

$$\Delta V_0 = 5.7049 - 5.66569 \Rightarrow \underline{\Delta V_0 = 0.0392 \text{ V}}$$

c. $I = I_z + I_L$

$$I_L = \frac{V_0}{R_L}, \quad I = \frac{V_{ps} - V_0}{R}, \quad I_z = \frac{V_0 - V_{z0}}{r_z}$$

$$\frac{10 - V_0}{0.50} = \frac{V_0 - 5.599}{0.010} + \frac{V_0}{2}$$

$$\frac{10}{0.50} + \frac{5.599}{0.010} = V_0 \left[\frac{1}{0.50} + \frac{1}{0.010} + \frac{1}{2} \right]$$

$$20.0 + 559.9 = V_0 (102.5)$$

$$\underline{V_0 = 5.658 \text{ V}}$$

1.58

(a) $I_z = \frac{10 - 6.8}{0.5} = 6.4 \text{ mA}$

$$P = I_z V_z = (6.4)(6.8) = 43.5 \text{ mW}$$

(b) $I_z = (0.1)(6.4) = 0.64 \text{ mA}$

$$I_L = 6.4 - 0.64 = 5.76 \text{ mA}$$

$$I_L = \frac{V_z}{R_L} \Rightarrow R_L = \frac{V_z}{I_z} = \frac{6.8}{5.76} = 1.18 \text{ k}\Omega$$

1.59

$$I_z r_z = (0.1)(20) = 2 \text{ mV}$$

$$V_{z0} = 6.8 - 0.002 = 6.798 \text{ V}$$

a. $R_L = \infty$

$$I_z = \frac{10 - 6.798}{0.5 + 0.02} \Rightarrow I_z = 6.158 \text{ mA}$$

$$V_0 = V_Z = V_{Z0} + I_Z r_Z = 6.798 + (0.006158)(20)$$

$$\underline{V_0 = 6.921 \text{ V}}$$

b. $I = I_Z + I_L$

$$\frac{10 - V_0}{0.50} = \frac{V_0 - 6.798}{0.020} + \frac{V_0}{1}$$

$$\frac{10}{0.50} + \frac{6.798}{0.020} = V_0 \left[\frac{1}{0.50} + \frac{1}{0.020} + \frac{1}{1} \right]$$

$$359.9 = V_0 \quad (53)$$

$$V_0 = 6.791 \text{ V}$$

$$\Delta V_0 = 6.791 - 6.921$$

$$\underline{\Delta V_0 = -0.13 \text{ V}}$$

1.60

For $V_D = 0$, $\underline{I_{SC} = 0.1 \text{ A}}$

$$\text{For } I_D = 0 \quad V_D = V_T \ln \left(\frac{0.2}{5 \times 10^{-14}} + 1 \right)$$

$$\underline{V_D = V_{DC} = 0.754 \text{ V}}$$

1.61 $V_D = 0, I_D = 0.2 \text{ A}$

$$V_D = 0.60 \text{ V}, \quad I_D = 0.1995 \text{ A}$$

$$V_D = 0.65 \text{ V}, \quad I_D = 0.1964 \text{ A}$$

$$V_D = 0.70 \text{ V}, \quad I_D = 0.1754 \text{ A}$$

$$V_D = 0.72 \text{ V}, \quad I_D = 0.1468 \text{ A}$$

$$V_D = 0.74 \text{ V}, \quad I_D = 0.0853 \text{ A}$$

$$V_D = 0.7545 \text{ V}, \quad I_D = 0$$

1.62

$$(a) \quad 0.16 = 0.20 - \left(5 \times 10^{-14} \left[\exp \left(\frac{V_D}{0.026} \right) - 1 \right] \right) \Rightarrow V_D = 0.7126 \text{ V}$$

$$(b) \quad P = (0.16)(0.7126) = 0.114 \text{ W}$$

Chapter 2

2.1

(a) For $v_I > 0.6$ V, $v_o = \left(\frac{1000}{1020}\right)(v_I - 0.6)$

For $v_I < 0.6$ V, $v_o = 0$

(b) (ii) $v_o = 0 = \left(\frac{1000}{1020}\right)[10 \sin(\omega t)_1 - 0.6]$

Then $\sin(\omega t)_1 = \frac{0.6}{10} = 0.06 \Rightarrow (\omega t)_1 = 3.44^\circ \Rightarrow 0.01911\pi$ rad

Also $(\omega t)_2 = 180 - 3.44 = 176.56^\circ \Rightarrow 0.9809\pi$ rad

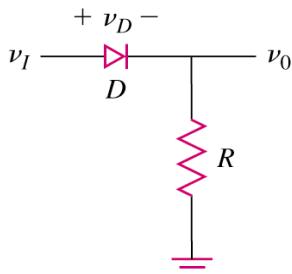
Now

$$\begin{aligned} v_o(\text{avg}) &= \frac{1}{T} \int_0^T v_o(t) dt = \frac{1}{2\pi} \int_0^{2\pi} [10 \sin x - 0.6] dx \\ &= \frac{1}{2\pi} \left[-10 \cos x \Big|_{0.01911\pi}^{0.9809\pi} - 0.6x \Big|_{0.01911\pi}^{0.9809\pi} \right] \\ &= \frac{1}{2\pi} [(-10)(-0.9982 - 0.9982) - 0.6(0.9809\pi - 0.01911\pi)] \\ v_o(\text{avg}) &= 2.89 \text{ V} \end{aligned}$$

(iii) $v_o(\text{peak}) = \left(\frac{1000}{1020}\right) \left[10 \sin\left(\frac{\pi}{2}\right) - 0.6 \right] = 9.2157 \text{ V}; i_d(\text{max}) = 9.2157 \text{ mA}$

(iv) $PIV = 10 \text{ V}$

2.2



$$v_0 = v_I - v_D$$

$$v_D = V_T \ln\left(\frac{i_D}{I_S}\right) \text{ and } i_D = \frac{v_0}{R}$$

$$v_0 = v_I - V_T \ln\left(\frac{v_0}{I_S R}\right)$$

2.3

$$(a) v_s = 120\sqrt{2} \left(\frac{1}{10} \right) = 16.97 \text{ V (peak)}$$

$$v_o(\text{peak}) = 16.27 \text{ V}$$

$$(b) i_D(\text{peak}) = \frac{16.27}{2} = 8.14 \text{ mA}$$

$$(c) v_o = 16.97 \sin \omega t - 0.7$$

$$\sin(\omega t)_1 = \frac{0.7}{16.97} = 0.04125 \Rightarrow (\omega t)_1 = 2.364^\circ$$

$$(\omega t)_2 = 180 - 2.364 = 177.64^\circ$$

$$\% = \left(\frac{177.64 - 2.364}{360} \right) \times 100\% = 48.7\%$$

(d)

$$\begin{aligned} v_o(\text{avg}) &= \frac{1}{2\pi} \int_{0.01313\pi}^{0.9869\pi} [16.97 \sin x - 0.7] dx \\ &= \frac{1}{2\pi} \left[(-16.97) \cos x \Big|_{0.01313\pi}^{0.9869\pi} - 0.7x \Big|_{0.01313\pi}^{0.9869\pi} \right] \\ &= \frac{1}{2\pi} [(-16.97)(-0.99915 - 0.99915) - 0.7(0.9738\pi)] \\ v_o(\text{avg}) &= 5.06 \text{ V} \end{aligned}$$

$$(e) i_D(\text{avg}) = \frac{v_o(\text{avg})}{2} = \frac{5.06}{2} = 2.53 \text{ mA}$$

2.4

$$(a) v_R(t) = 15 \sin \omega t - 0.7 - 9 = 15 \sin \omega t - 9.7$$

$$(\omega t)_1 = \sin^{-1} \left(\frac{9.7}{15} \right) = 40.29^\circ \Rightarrow 0.2238\pi \text{ rad}$$

$$(\omega t)_2 = 180 - 40.29 = 139.71^\circ \Rightarrow 0.7762\pi \text{ rad}$$

$$\begin{aligned} v_R(\text{avg}) &= \frac{1}{2\pi} \int_{0.2238\pi}^{0.7762\pi} [15 \sin x - 9.7] dx \\ &= \frac{1}{2\pi} \left[(-15) \cos x \Big|_{0.2238\pi}^{0.7762\pi} - 9.7x \Big|_{0.2238\pi}^{0.7762\pi} \right] = \frac{1}{2\pi} [(-15)(-0.7628 - 0.7628) - 9.7(0.5523\pi)] \\ v_R(\text{avg}) &= 0.9628 \text{ V} \end{aligned}$$

$$i_D(\text{avg}) = 0.8 = \frac{0.9628}{R} \Rightarrow R = 1.20 \Omega$$

(b)

$$\% = \left(\frac{139.71 - 40.29}{360} \right) \times 100\% = 27.6\%$$

2.5

(a) $i(\text{peak}) = \frac{v_R(\text{peak})}{R} \Rightarrow R = \frac{15 - 9.7}{1.2} = 4.417 \Omega$

(b) $v_R(t) = 15 \sin \omega t - 9.7$

$(\omega t)_1 = 0.2238\pi ; (\omega t)_2 = 0.7762\pi$

$$v_R(\text{avg}) = \frac{1}{\pi} \int_{0.2238\pi}^{0.7762\pi} [15 \sin x - 9.7] dx$$

Or from Problem 2.4, $v_R(\text{avg}) = 2(0.9628) = 1.9256 \text{ V}$

$$i_D(\text{avg}) = \frac{v_R(\text{avg})}{R} = \frac{1.9256}{4.417} = 0.436 \text{ A}$$

(c)

$$\% = \left(\frac{139.71 - 40.29}{360} \right) \times 100\% = 27.6\%$$

2.6

(a) $v_s(\text{peak}) = 12 + 0.7 = 12.7 \text{ V}$

$$\frac{N_1}{N_2} = \frac{120\sqrt{2}}{12.7} = 13.4$$

(b) $R = \frac{12}{0.2} = 60 \Omega$

$$C = \frac{V_M}{2fRV_r} = \frac{12}{2(60)(60)(0.25)} \Rightarrow 6667 \mu \text{ F}$$

(c) $PIV = 2v_s(\text{max}) - V_\gamma = 2(12.7) - 0.7 = 24.7 \text{ V}$

2.7

$v_0 = v_s - 2V_\gamma \Rightarrow v_s(\text{max}) = v_0(\text{max}) + 2V_\gamma$

a. For $v_0(\text{max}) = 25 \text{ V} \Rightarrow v_s(\text{max}) = 25 + 2(0.7) = 26.4 \text{ V}$

$$\frac{N_1}{N_2} = \frac{160}{26.4} \Rightarrow \frac{N_1}{N_2} = 6.06$$

b. For $v_0(\text{max}) = 100 \text{ V} \Rightarrow v_s(\text{max}) = 101.4 \text{ V}$

$$\frac{N_1}{N_2} = \frac{160}{101.4} \Rightarrow \frac{N_1}{N_2} = 1.58$$

From part (a) $PIV = 2v_s(\text{max}) - V_\gamma = 2(26.4) - 0.7$

or $\underline{PIV = 52.1 \text{ V}}$ or, from part (b) $\underline{PIV = 2(101.4) - 0.7}$ or $\underline{PIV = 202.1 \text{ V}}$

2.8

(a)

$v_s(\text{max}) = 12 + 2(0.7) = 13.4 \text{ V}$

$$v_s(\text{rms}) = \frac{13.4}{\sqrt{2}} \Rightarrow \underline{v_s(\text{rms}) = 9.48 \text{ V}}$$

(b)

$$V_r = \frac{V_M}{2f R_C} \Rightarrow C = \frac{V_M}{2f V_r R}$$

$$C = \frac{12}{2(60)(0.3)(150)} \Rightarrow C = 2222 \mu\text{F}$$

(c)

$$i_d, \text{ peak} = \frac{V_M}{R} \left[1 + \pi \sqrt{\frac{2V_M}{V_r}} \right]$$

$$= \frac{12}{150} \left[1 + \pi \sqrt{\frac{2(12)}{0.3}} \right]$$

$$\underline{i_d, \text{ peak} = 2.33 \text{ A}}$$

2.9

(a)

$$v_s(\text{max}) = 12 + 0.7 = 12.7 \text{ V}$$

$$v_s(\text{rms}) = \frac{v_s(\text{max})}{\sqrt{2}} \Rightarrow v_s(\text{rms}) = 8.98 \text{ V}$$

(b)

$$V_r = \frac{V_M}{fRC} \Rightarrow C = \frac{V_M}{fRV_r} = \frac{12}{(60)(150)(0.3)} \text{ or } C = 4444 \mu\text{F}$$

(c) For the half-wave rectifier

$$i_{D, \text{ max}} = \frac{V_M}{R} \left(1 + 4\pi \sqrt{\frac{V_M}{2V_r}} \right) = \frac{12}{150} \left(1 + 4\pi \sqrt{\frac{12}{2(0.3)}} \right) \text{ or } i_{D, \text{ max}} = 4.58 \text{ A}$$

2.10

$$(a) v_o(\text{peak}) = 10 - 0.7 = 9.3 \text{ V}$$

$$(b) C = \frac{V_M}{fRV_r} = \frac{9.3}{(60)(500)(0.5)} \Rightarrow 620 \mu\text{F}$$

$$(c) PIV = 10 + 9.3 = 19.3 \text{ V}$$

2.11

$$(a) 10.3 \leq v_o \leq 12.3 \text{ V}$$

$$(b) V_r = \frac{V_M}{fRC} = \frac{12.3}{(60)(1000)(350 \times 10^{-6})} = 0.586 \text{ V}$$

$$V_r = \frac{10.3}{(60)(1000)(350 \times 10^{-6})} = 0.490 \text{ V}$$

$$\text{So } 0.490 \leq V_r \leq 0.586 \text{ V}$$

$$(c) C = \frac{V_M}{fRV_r} = \frac{12.3}{(60)(1000)(0.4)} \Rightarrow 513 \mu\text{F}$$

2.12

$$(a) v_s(\text{peak}) = 8.5(\sqrt{2}) = 12.02 \text{ V}$$

$$|V_o|_{\max} = 12.02 - 0.7 = 11.32 \text{ V}$$

$$(b) C = \frac{V_M}{2fRV_r} = \frac{11.32}{2(60)(10)(0.25)} = 0.03773 \text{ F}$$

$$(c) \text{PIV} = 2v_s(\text{peak}) - V_\gamma = 2(12.02) - 0.7 = 23.34 \text{ V}$$

2.13

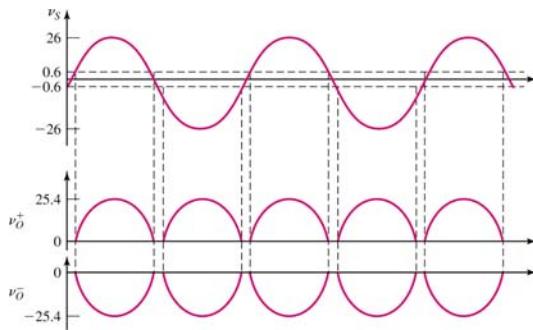
(a)

$$v_s(\text{peak}) = 15 + 2(0.7) = 16.4 \text{ V}$$

$$v_s(\text{rms}) = \frac{16.4}{\sqrt{2}} = 11.6 \text{ V}$$

$$(b) C = \frac{V_M}{2fRV_r} = \frac{15}{2(60)(125)(0.35)} = 2857 \mu\text{F}$$

2.14



2.15

$$(a) v_s = 12.8 \text{ V}$$

$$\frac{N_1}{N_2} = \frac{120\sqrt{2}}{12.8} = 13.3$$

$$(b) R = \frac{12}{0.5} = 24 \Omega$$

$$V_r = 3\% \Rightarrow V_r = (0.03)(12) = 0.36 \text{ V}$$

$$C = \frac{V_M}{2fRV_r} = \frac{12}{2(60)(24)(0.36)} = 0.0116 \text{ F}$$

$$(c) i_D(\text{peak}) = \frac{V_M}{R} \left(1 + \pi \sqrt{\frac{2V_M}{V_r}} \right) = \frac{12}{24} \left(1 + \pi \sqrt{\frac{2(12)}{0.36}} \right)$$

$$i_D(\text{peak}) = 13.3 \text{ A}$$

$$(d) i_D(\text{avg}) = \frac{1}{\pi} \sqrt{\frac{2V_r}{V_M}} \cdot \frac{V_M}{R} \left(1 + \frac{\pi}{2} \sqrt{\frac{2V_M}{V_r}} \right) = \frac{1}{\pi} \sqrt{\frac{2(0.36)}{12}} \left(\frac{12}{24} \right) \left(1 + \frac{\pi}{2} \sqrt{\frac{2(12)}{0.36}} \right)$$

$$i_D(\text{avg}) = 0.539 \text{ A}$$

$$(e) PIV = 12.8 + 12 = 24.8 \text{ V}$$

2.16

$$(a) v_s = 9 + 2(0.8) = 10.6 \text{ V}$$

$$\frac{N_1}{N_2} = \frac{120\sqrt{2}}{10.6} = 16$$

$$(b) R = \frac{9}{0.1} = 90 \Omega$$

$$C = \frac{V_M}{2fRV_r} = \frac{9}{2(60)(90)(0.2)} \Rightarrow 4167 \mu\text{F}$$

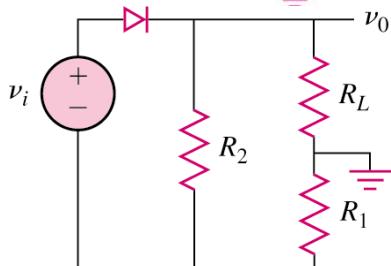
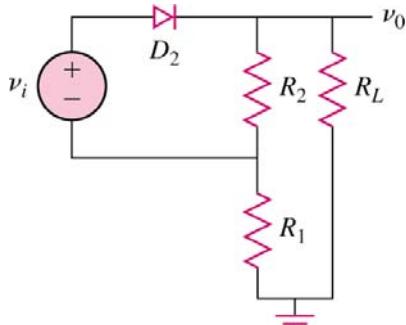
$$(c) i_D(\text{peak}) = \frac{V_M}{R} \left(1 + \pi \sqrt{\frac{2V_M}{V_r}} \right) = \frac{9}{90} \left(1 + \pi \sqrt{\frac{2(9)}{0.2}} \right) = 3.08 \text{ A}$$

$$(d) i_D(\text{avg}) = \frac{1}{\pi} \sqrt{\frac{2V_r}{V_M}} \cdot \frac{V_M}{R} \left(1 + \frac{\pi}{2} \sqrt{\frac{2V_M}{V_r}} \right) = \frac{1}{\pi} \sqrt{\frac{2(0.2)}{9}} \left(\frac{9}{90} \right) \left(1 + \frac{\pi}{2} \sqrt{\frac{2(9)}{0.2}} \right)$$

$$i_D(\text{avg}) = 0.1067 \text{ A}$$

$$(e) \text{PIV} = v_s(\text{max}) - V_\gamma = 10.6 - 0.8 = 9.8 \text{ V}$$

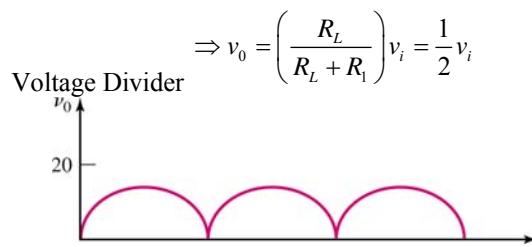
2.17



For $v_i > 0$

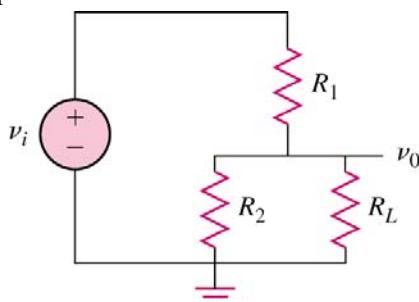
$$V_\gamma = 0$$

$$\text{Voltage across } R_L + R_1 = v_i$$



2.18

For $v_i > 0$, ($V_g = 0$)

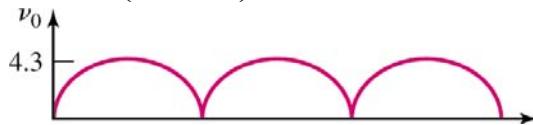


a.

$$v_0 = \left(\frac{R_2 \parallel R_L}{R_2 \parallel R_L + R_1} \right) |v_i|$$

$$R_2 \parallel R_L = 2.2 \parallel 6.8 = 1.66 \text{ k}\Omega$$

$$v_0 = \left(\frac{1.66}{1.66 + 2.2} \right) v_i = 0.43 |v_i|$$



b.

$$v_0 (\text{rms}) = \frac{v_0 (\text{max})}{\sqrt{2}} \Rightarrow v_0 (\text{rms}) = 3.04 \text{ V}$$

2.19

$$(a) I_L = \frac{3.9}{4} = 0.975 \text{ mA}$$

$$I_I = \frac{20 - 3.9}{12} = 1.342 \text{ mA}$$

$$I_Z = I_I - I_L = 1.342 - 0.975 = 0.367 \text{ mA}$$

$$P_Z = I_Z V_Z = (0.367)(3.9) = 1.43 \text{ mW}$$

$$(b) I_L = \frac{3.9}{10} = 0.39 \text{ mA}$$

$$I_Z = 1.342 - 0.39 = 0.952 \text{ mA}$$

$$P_Z = (0.952)(3.9) = 3.71 \text{ mW}$$

2.20

(a)

$$I_Z = \frac{40 - 12}{120} = 0.233 \text{ A}$$

$$P = (0.233)(12) = 2.8 \text{ W}$$

(b) $I_R = 0.233 \text{ A}$, $I_L = (0.9)(0.233) = 0.21 \text{ A}$

$$\text{So } 0.21 = \frac{12}{R_L} \Rightarrow R_L = 57.1 \Omega$$

$$(c) P = (0.1)(0.233)(12) \Rightarrow P = 0.28 \text{ W}$$

2.21

(a) $P_Z = I_Z V_Z$

$$4 = I_z(15.4) \Rightarrow I_z(\max) = 259.74 \text{ mA}$$

$$\text{So } 15 \leq I_z \leq 259.74 \text{ mA}$$

(b) $I_I = \frac{60 - 15.4}{0.15} = 297.33 \text{ mA}$

$$\text{So } I_L(\max) = 297.33 - 15 = 282.33 \text{ mA}$$

$$I_L(\min) = 297.33 - 259.74 = 37.59 \text{ mA}$$

$$\text{Then } R_L(\min) = \frac{15.4}{0.28233} = 54.55 \Omega$$

$$R_L(\max) = \frac{15.4}{0.03759} = 410 \Omega$$

$$\text{So } 54.55 \leq R_L \leq 410 \Omega$$

2.22

a.

$$I_I = \frac{20 - 10}{222} \Rightarrow I_I = 45.0 \text{ mA}$$

$$I_L = \frac{10}{380} \Rightarrow I_L = 26.3 \text{ mA}$$

$$I_Z = I_I - I_L \Rightarrow I_Z = 18.7 \text{ mA}$$

b.

$$P_Z(\max) = 400 \text{ mW} \Rightarrow I_Z(\max) = \frac{400}{10} = 40 \text{ mA}$$

$$\Rightarrow I_L(\min) = I_I - I_Z(\max) = 45 - 40$$

$$\Rightarrow I_L(\min) = 5 \text{ mA} = \frac{10}{R_L}$$

$$\Rightarrow R_L = 2 \text{ k}\Omega$$

(c) For $R_i = 175 \Omega$ $I_I = 57.1 \text{ mA}$ $I_L = 26.3 \text{ mA}$ $I_Z = 30.8 \text{ mA}$

$$I_Z(\max) = 40 \text{ mA} \Rightarrow I_L(\min) = 57.1 - 40 = 17.1 \text{ mA}$$

$$R_L = \frac{10}{17.1} \Rightarrow R_L = 585 \Omega$$

2.23

- a. From Eq. (2.30)

$$I_Z(\max) = \frac{500[20-10] - 50[15-10]}{15 - (0.9)(10) - (0.1)(20)} \\ = \frac{5000 - 250}{4}$$

$$I_Z(\max) = 1.1875 \text{ A}$$

$$I_Z(\min) = 0.11875 \text{ A}$$

$$\text{From Eq. (2.28(b)) } R_i = \frac{20-10}{1187.5+50} \Rightarrow R_i = 8.08\Omega$$

b.

$$P_Z = (1.1875)(10) \Rightarrow P_Z = 11.9 \text{ W}$$

$$P_L = I_L(\max)V_0 = (0.5)(10) \Rightarrow P_L = 5 \text{ W}$$

2.24

- (a) $I_L = 0$

$$I_Z = \frac{10-5.6}{50+3} \Rightarrow 83.0 \text{ mA}$$

$$V_Z = 5.6 + (0.083)(3) = 5.85 \text{ V} = V_L$$

$$P_Z = I_Z V_Z = (0.083)(5.85) = 0.486 \text{ W}$$

$$(b) \frac{10-V_L}{50} = \frac{V_L-5.6}{3} + \frac{V_L}{200}$$

$$0.20 + 1.867 = V_L(0.02 + 0.3333 + 0.005)$$

$$\text{So } V_L = 5.769 \text{ V}$$

$$\text{Then } I_L = \frac{5.769}{0.2} = 28.84 \text{ mA}$$

$$I_I = \frac{10-5.769}{0.050} = 84.62 \text{ mA}$$

$$\text{And } I_Z = I_I - I_L = 55.8 \text{ mA}$$

$$P_Z = (0.0558)(5.769) = 0.322 \text{ W}$$

- (c) $I_L = 0$

$$I_Z = \frac{12-5.6}{50+3} \Rightarrow 120.8 \text{ mA}$$

$$V_Z = V_L = 5.6 + (0.1208)(3) = 5.962 \text{ V}$$

$$P_Z = (0.1208)(5.962) = 0.72 \text{ W}$$

$$(d) \frac{12-V_L}{50} = \frac{V_L-5.6}{3} + \frac{V_L}{200}$$

$$0.24 + 1.867 = V_L(0.02 + 0.333 + 0.005)$$

$$\text{So } V_L = 5.88 \text{ V}$$

$$\text{Then } I_L = \frac{5.88}{0.20} = 29.4 \text{ mA}; \quad I_I = \frac{12-5.88}{0.05} = 122.4 \text{ mA}$$

$$I_Z = 122.4 - 29.4 = 93 \text{ mA}$$

$$P_Z = (0.093)(5.88) = 0.547 \text{ W}$$

2.25

(a) Set $I_Z = 10 \text{ mA}$; $I_L = \frac{V_L}{R_L} = \frac{7.5}{1} = 7.5 \text{ mA}$

$$I_I = 10 + 7.5 = 17.5 \text{ mA}$$

$$I_I = 17.5 = \frac{12 - 7.5}{R_i} \Rightarrow R_i = 257 \Omega$$

(b) $7.5 = V_{ZO} + (0.01)(12) \Rightarrow V_{ZO} = 7.38 \text{ V}$

For $V_I = (1.1)(12) = 13.2 \text{ V}$

$$\frac{13.2 - V_L}{257} = \frac{V_L - 7.38}{12} + \frac{V_L}{1000}$$

$$0.05136 + 0.615 = V_L (0.00389 + 0.0833 + 0.001) \Rightarrow V_L = 7.556 \text{ V}$$

For $V_I = (0.9)(12) = 10.8 \text{ V}$

$$\frac{10.8 - V_L}{257} = \frac{V_L - 7.38}{12} + \frac{V_L}{1000}$$

$$0.04202 + 0.615 = V_L (0.08819) \Rightarrow V_L = 7.450 \text{ V}$$

Then, Source Reg = $\left(\frac{7.556 - 7.450}{13.2 - 10.8} \right) \times 100\% = 4.42\%$

(c) For $R_L = 1 \text{ k}\Omega$, $V_L = 7.50 \text{ V}$

For $R_L = \infty$, $I_Z = \frac{12 - 7.38}{257 + 12} = 17.17 \text{ mA}$

$$V_L = 7.38 + (0.01717)(12) = 7.586 \text{ V}$$

Then, Load Reg = $\left(\frac{7.586 - 7.50}{7.50} \right) \times 100\% = 1.15\%$

2.26

$$\begin{aligned} \% \text{ Reg} &= \frac{V_L(\max) - V_L(\min)}{V_L(\text{nom})} \times 100\% \\ &= \frac{V_L(\text{nom}) + I_Z(\max)r_z - (V_L(\text{nom}) + I_Z(\min)r_z)}{V_L(\text{nom})} \\ &= \frac{[I_Z(\max) - I_Z(\min)](3)}{6} = 0.05 \end{aligned}$$

So $I_Z(\max) - I_Z(\min) = 0.1 \text{ A}$

Now $I_L(\max) = \frac{6}{500} = 0.012 \text{ A}$, $I_L(\min) = \frac{6}{1000} = 0.006 \text{ A}$

Now $R_i = \frac{V_{PS}(\min) - V_z}{I_z(\min) + I_L(\max)}$

or $280 = \frac{15 - 6}{I_z(\min) + 0.012} \Rightarrow I_z(\min) = 0.020 \text{ A}$

Then $I_Z(\max) = 0.1 + 0.02 = 0.12 \text{ A}$ and $R_i = \frac{V_{PS}(\max) - V_Z}{I_Z(\max) + I_L(\min)}$
 or $280 = \frac{V_{PS}(\max) - 6}{0.12 + 0.006} \Rightarrow V_{PS}(\max) = 41.3 \text{ V}$

2.27

Using Figure 2.19

a. $V_{PS} = 20 \pm 25\% \Rightarrow 15 \leq V_{PS} \leq 25 \text{ V}$

For $V_{PS}(\min)$:

$$I_I = I_Z(\min) + I_L(\max) = 5 + 20 = 25 \text{ mA}$$

$$R_i = \frac{V_{PS}(\min) - V_Z}{I_I} = \frac{15 - 10}{25} \Rightarrow R_i = 200\Omega$$

b. For $V_{PS}(\max) \Rightarrow I_I(\max) = \frac{25 - 10}{R_i} \Rightarrow I_I(\max) = 75 \text{ mA}$

For $I_L(\min) = 0 \Rightarrow I_Z(\max) = 75 \text{ mA}$

$$V_{Z_0} = V_Z - I_Z r_Z = 10 - (0.025)(5) = 9.875 \text{ V}$$

$$V_0(\max) = 9.875 + (0.075)(5) = 10.25$$

$$V_0(\min) = 9.875 + (0.005)(5) = 9.90$$

$$\Delta V_0 = 0.35 \text{ V}$$

c. $\% \text{ Reg} = \frac{\Delta V_0}{V_0(\text{nom})} \times 100\% \Rightarrow \% \text{ Reg} = 3.5\%$

2.28

From Equation (2.28(a))

$$R_i = \frac{V_{PS}(\min) - V_Z}{I_Z(\min) + I_L(\max)} = \frac{24 - 16}{40 + 400} \text{ or } R_i = 18.2\Omega$$

$$\text{Also } V_r = \frac{V_M}{2fRC} \Rightarrow C = \frac{V_M}{2fRV_r}$$

$$R \cong R_i + r_z = 18.2 + 2 = 20.2\Omega$$

Then

$$C = \frac{24}{2(60)(1)(20.2)} \Rightarrow C = 9901 \mu F$$

2.29

$$V_Z = V_{Z_0} + I_Z r_Z \quad V_Z(\text{nom}) = 8 \text{ V}$$

$$8 = V_{Z_0} + (0.1)(0.5) \Rightarrow V_{Z_0} = 7.95 \text{ V}$$

$$I_i = \frac{V_s(\max) - V_Z(\text{nom})}{R_i} = \frac{12 - 8}{3} = 1.333 \text{ A}$$

For $I_L = 0.2 \text{ A} \Rightarrow I_Z = 1.133 \text{ A}$

For $I_L = 1 \text{ A} \Rightarrow I_Z = 0.333 \text{ A}$

$$\begin{aligned}
 V_L(\max) &= V_{Z_0} + I_Z(\max)r_Z \\
 &= 7.95 + (1.133)(0.5) = 8.5165 \\
 V_L(\min) &= V_{Z_0} + I_Z(\min)r_Z \\
 &= 7.95 + (0.333)(0.5) = 8.1165 \\
 \Delta V_L &= 0.4 \text{ V} \\
 \% \text{ Reg} &= \frac{\Delta V_L}{V_0(\text{nom})} = \frac{0.4}{8} \Rightarrow \% \text{ Reg} = 5.0\%
 \end{aligned}$$

$$V_r = \frac{V_m}{2fRC} \Rightarrow C = \frac{V_m}{2fRV_r}$$

$$R = R_i + r_z = 3 + 0.5 = 3.5\Omega$$

$$C = \frac{12}{2(60)(3.5)(0.8)} \Rightarrow C = 0.0357 \text{ F}$$

Then

2.30

For $-6.3 \leq v_I \leq 3 \text{ V}$, $v_o = v_I$

For $v_I > 3 \text{ V}$, $I = \frac{v_I - 3}{1.5}$ and $v_o = v_I - I(0.5)$

$$v_o = v_I - (0.5) \left(\frac{v_I - 3}{1.5} \right) = 0.667v_I + 1.0$$

For $v_I < -6.3 \text{ V}$, $I = \frac{v_I + 6.3}{2.5}$ and $v_o = v_I - I(0.5)$

$$v_o = v_I - (0.5) \left(\frac{v_I + 6.3}{2.5} \right) = 0.8v_I - 1.26$$

2.31

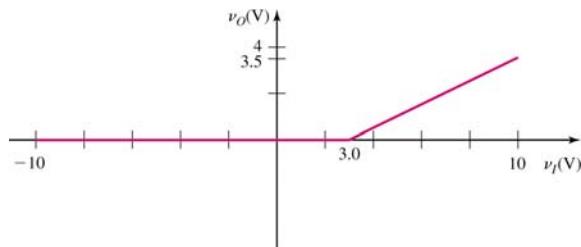
(a) For $-10 \leq v_I \leq 0$, both diodes are conducting $\Rightarrow v_o = 0$

For $0 \leq v_I \leq 3$, Zener not in breakdown, so $i_l = 0, v_o = 0$

$$\text{For } v_I > 3 \quad i_l = \frac{v_I - 3}{20} \text{ mA}$$

$$v_o = \left(\frac{v_I - 3}{20} \right)(10) = \frac{1}{2}v_I - 1.5$$

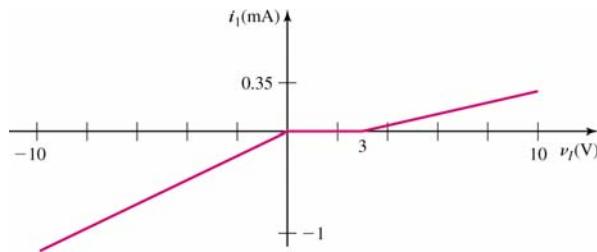
At $v_I = 10 \text{ V}$, $v_o = 3.5 \text{ V}$, $i_l = 0.35 \text{ mA}$



(b) For $v_I < 0$, both diodes forward biased

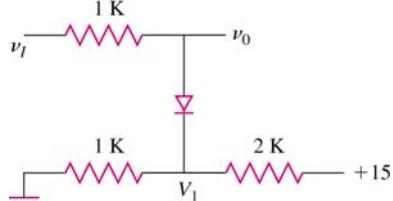
$$-i_1 = \frac{0 - v_I}{10} \text{. At } v_I = -10 \text{ V, } i_1 = -1 \text{ mA}$$

$$\text{For } v_I > 3, i_1 = \frac{v_I - 3}{20} \text{. At } v_I = 10 \text{ V, } i_1 = 0.35 \text{ mA}$$



2.32

(a)



$$V_i = \frac{1}{3} \times 15 = 5 \text{ V} \Rightarrow \text{for } v_I \leq 5.7, v_0 = v_I$$

For $v_I > 5.7 \text{ V}$

$$\frac{v_I - (V_i + 0.7)}{1} + \frac{15 - V_i}{2} = \frac{V_i}{1}, v_0 = V_i + 0.7$$

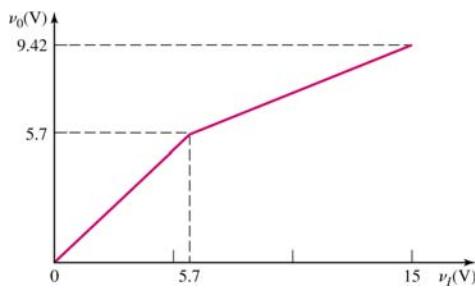
$$\frac{v_I - v_0}{1} + \frac{15 - (v_0 - 0.7)}{2} = \frac{v_0 - 0.7}{1}$$

$$\frac{v_I}{1} + \frac{15.7}{2} + \frac{0.7}{1} = v_0 \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{1} \right) = v_0 (2.5)$$

$$v_I + 8.55 = v_0 (2.5) \Rightarrow v_0 = \frac{1}{2.5} v_I + 3.42$$

$$v_I = 5.7 \Rightarrow v_0 = 5.7$$

$$v_I = 15 \Rightarrow v_0 = 9.42$$

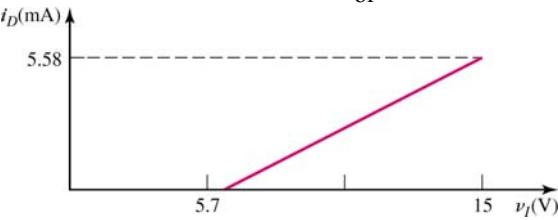


(b) $i_D = 0$ for $0 \leq v_I \leq 5.7$

Then for $v_I > 5.7$ V

$$i_D = \frac{v_I - v_O}{1} = \frac{v_I - \left(\frac{v_I}{2.5} + 3.42 \right)}{1} \quad \text{or} \quad i_D = \frac{0.6v_I - 3.42}{1}$$

For $v_I = 15$, $i_D = 5.58$ mA



2.33

(a) (i) $V_B = 1.8$ V

For $v_I \geq 1.1$ V, $v_O = v_I$

For $v_I \leq 1.1$ V, $v_O = 1.1$ V

(ii) $V_B = -1.8$ V

For $v_I \geq -2.5$ V, $v_O = v_I$

For $v_I \leq -2.5$ V, $v_O = -2.5$ V

(b) (i) $V_B = 1.8$ V

For $v_I \geq 2.5$ V, $v_O = 2.5$ V

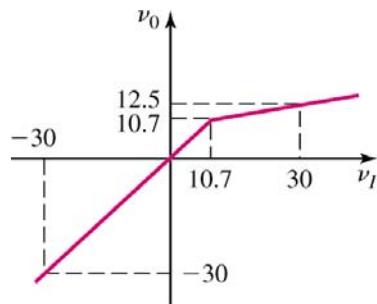
For $v_I \leq 2.5$ V, $v_O = v_I$

(ii) $V_B = -1.8$ V

For $v_I \geq -1.1$ V, $v_O = -1.1$ V

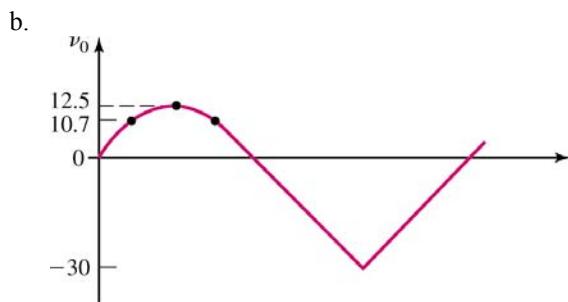
For $v_I \leq -1.1$ V, $v_O = v_I$

2.34



For $v_I = 30$ V, $i = \frac{30 - 10.7}{100 + 10} = 0.175$ A

$v_O = i(10) + 10.7 = 12.5$ V



2.35

(a) (i) $V_B = 5 \text{ V}$

For $v_I \geq 5.7 \text{ V}$, $v_o = v_I - 5.7$

For $v_I \leq 5.7 \text{ V}$, $v_o = 0$

(ii) $V_B = -5 \text{ V}$

For $v_I \geq -4.3 \text{ V}$, $v_o = v_I + 4.3$

For $v_I \leq -4.3 \text{ V}$, $v_o = 0$

(b) (i) $V_B = 5 \text{ V}$

For $v_I \geq 4.3 \text{ V}$, $v_o = 0$

For $v_I \leq 4.3 \text{ V}$, $v_o = v_I - 4.3$

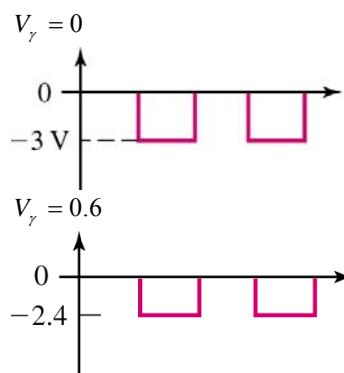
(ii) $V_B = -5 \text{ V}$

For $v_I \geq -5.7 \text{ V}$, $v_o = 0$

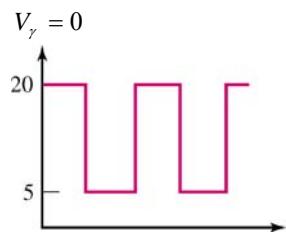
For $v_I \leq -5.7 \text{ V}$, $v_o = v_I + 5.7$

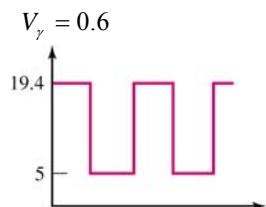
2.36

a.

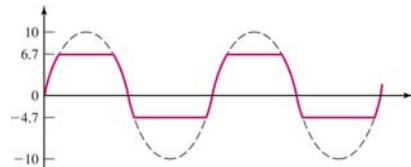


b.



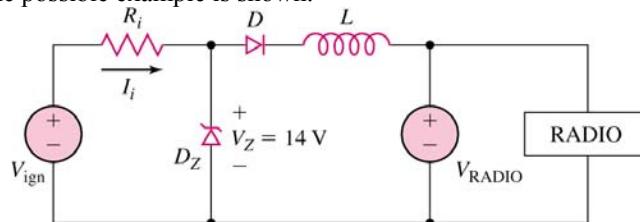


2.37



2.38

One possible example is shown.



L will tend to block the transient signals

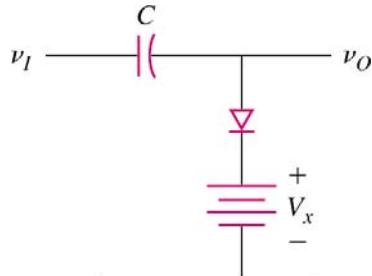
D_z will limit the voltage to +14 V and -0.7 V.

Power ratings depends on number of pulses per second and duration of pulse.

2.39

- (a) Square wave between +40 V and 0.
- (b) Square wave between +35 V and -5 V.
- (c) Square wave between +5 V and -35 V.

2.40



- a. For $V_\gamma = 0 \Rightarrow V_x = 2.7 \text{ V}$
- b. For $V_\gamma = 0.7 \text{ V} \Rightarrow V_x = 2.0 \text{ V}$

2.41

Circuit similar to Figure 2.31(a) with $V_B = -10$ V.

2.42

In steady-state, $v_o = (10 \sin \omega t + 5)$ V

2.43

(i) $V_B = 5$ V, In steady-state,

$$v_o = (10 \sin \omega t - 5) \text{ V}$$

(ii) $V_B = -5$ V, In steady-state,

$$v_o = (10 \sin \omega t - 15) \text{ V}$$

2.44

a.

$$I_{D1} = \frac{10 - 0.6}{9.5 + 0.5} \Rightarrow I_{D1} = 0.94 \text{ mA} \quad I_{D2} = 0$$

$$V_0 = I_{D1}(9.5) \Rightarrow V_0 = 8.93 \text{ V}$$

b.

$$I_{D1} = \frac{5 - 0.6}{9.5 + 0.5} \Rightarrow I_{D1} = 0.44 \text{ mA} \quad I_{D2} = 0$$

$$V_0 = I_{D1}(9.5) \Rightarrow V_0 = 4.18 \text{ V}$$

c. Same as (a)

d.

$$10 = \frac{(I)}{2}(0.5) + 0.6 + I(9.5) \Rightarrow I = 0.964 \text{ mA}$$

$$V_0 = I(9.5) \Rightarrow V_0 = 9.16 \text{ V}$$

$$I_{D1} = I_{D2} = \frac{I}{2} \Rightarrow I_{D1} = I_{D2} = 0.482 \text{ mA}$$

2.45

a.

$$I = I_{D1} = I_{D2} = 0 \quad V_0 = 10$$

b.

$$10 = I(9.5) + 0.6 + I(0.5) \Rightarrow I = I_{D2} = 0.94 \text{ mA} \quad I_{D1} = 0$$

$$V_0 = 10 - I(9.5) \Rightarrow V_0 = 1.07 \text{ V}$$

c.

$$10 = I(9.5) + 0.6 + I(0.5) + 5 \Rightarrow I = I_{D2} = 0.44 \text{ mA} \quad I_{D1} = 0$$

$$V_0 = 10 - I(9.5) \Rightarrow V_0 = 5.82 \text{ V}$$

d.

$$10 = I(9.5) + 0.6 + \frac{I}{2}(0.5) \Rightarrow I = 0.964 \text{ mA}$$

$$I_{D1} = I_{D2} = \frac{I}{2} \Rightarrow I_{D1} = I_{D2} = 0.482 \text{ mA}$$

$$V_0 = 10 - I(9.5) \Rightarrow V_0 = 0.842 \text{ V}$$

2.46

a.

$$V_1 = V_2 = 0 \Rightarrow D_1, D_2, D_3, \text{ on } V_0 = 4.4 \text{ V}$$

$$I = \frac{10 - 4.4}{9.5} \Rightarrow I = 0.589 \text{ mA}$$

$$I_{D1} = I_{D2} = \frac{4.4 - 0.6}{0.5} \Rightarrow I_{D1} = I_{D2} = 7.6 \text{ mA}$$

$$I_{D3} = I_{D1} + I_{D2} - I = 2(7.6) - 0.589 \Rightarrow I_{D3} = 14.6 \text{ mA}$$

b.

$$V_1 = V_2 = 5 \text{ V } D_1 \text{ and } D_2 \text{ on, } D_3 \text{ off}$$

$$10 = I(9.5) + 0.6 + \frac{I}{2}(0.5) + 5 \Rightarrow I = 0.451 \text{ mA}$$

$$I_{D1} = I_{D2} = \frac{I}{2} \Rightarrow I_{D1} = I_{D2} = 0.226 \text{ mA}$$

$$\underline{I_{D3} = 0}$$

$$V_0 = 10 - I(9.5) = 10 - (0.451)(9.5) \Rightarrow V_0 = 5.72 \text{ V}$$

c. $V_1 = 5 \text{ V}, V_2 = 0$ D_1 off, D_2, D_3 on $\underline{V_0 = 4.4 \text{ V}}$

$$I = \frac{10 - 4.4}{9.5} \Rightarrow I = 0.589 \text{ mA}$$

$$I_{D2} = \frac{4.4 - 0.6}{0.5} \Rightarrow \underline{I_{D2} = 7.6 \text{ mA}}$$

$$\underline{I_{D1} = 0}$$

$$I_{D3} = I_{D2} - I = 7.6 - 0.589 \Rightarrow \underline{I_{D3} = 7.01 \text{ mA}}$$

d. $V_1 = 5 \text{ V}, V_2 = 2 \text{ V}$ D_1 off, D_2, D_3 on $\underline{V_0 = 4.4 \text{ V}}$

$$I = \frac{10 - 4.4}{9.5} \Rightarrow I = 0.589 \text{ mA}$$

$$I_{D2} = \frac{4.4 - 0.6 - 2}{0.5} \Rightarrow \underline{I_{D2} = 3.6 \text{ mA}}$$

$$\underline{I_{D1} = 0}$$

$$I_{D3} = I_{D2} - I = 3.6 - 0.589 \Rightarrow \underline{I_{D3} = 3.01 \text{ mA}}$$

2.47

$$(a) \quad V_1 = 4.4 \text{ V}, \quad I_{D1} = 0.2 = \frac{10 - 0.6 - 4.4}{R_1} \Rightarrow R_1 = 25 \text{ k}\Omega$$

$$I_{R2} = 0.2 + 0.3 = 0.5 \text{ mA}$$

$$V_2 = -0.6 \text{ V}, \quad I_{R2} = 0.5 = \frac{4.4 - (-0.6)}{R_2} \Rightarrow R_2 = 10 \text{ k}\Omega$$

$$I_{R_3} = 0.5 + 0.5 = 1.0 \text{ mA}$$

$$R_3 = \frac{-0.6 - (-5)}{1.0} = 4.4 \text{ k}\Omega$$

(b) Assume all diodes conducting

$$V_1 = 4.4 \text{ V}, I_{D1} = \frac{10 - 0.6 - 4.4}{10} = 0.5 \text{ mA}$$

$$V_2 = -0.6 \text{ V}, I_{R2} = \frac{4.4 - (-0.6)}{4} = 1.25 \text{ mA}$$

Then $I_{D2} = 1.25 - 0.5 = 0.75 \text{ mA}$

$$I_{R3} = \frac{-0.6 - (-5)}{2.2} = 2 \text{ mA}$$

Then $I_{D3} = 2 - 1.25 = 0.75 \text{ mA}$

(c) Diode D_2 cutoff $\Rightarrow I_{D2} = 0$

$$V_2 = -0.6 \text{ V}, I_{D1} = \frac{10 - 0.6 - (-0.6)}{R_1 + R_2} = \frac{10}{9} = 1.11 \text{ mA}$$

$$V_1 = 10 - 0.6 - (1.11)(3) = 6.07 \text{ V}$$

$$I_{R3} = \frac{-0.6 - (-5)}{2.5} = 1.76 \text{ mA}$$

Then $I_{D3} = 1.76 - 1.11 = 0.65 \text{ mA}$

(d) Diode D_3 cutoff $\Rightarrow I_{D3} = 0$

$$V_1 = 4.4 \text{ V}, I_{D1} = \frac{10 - 0.6 - 4.4}{6} = 0.833 \text{ mA}$$

$$I_{R2} = \frac{4.4 - (-5)}{R_2 + R_3} = \frac{9.4}{9} = 1.044 \text{ mA}$$

$$V_2 = (1.044)(6) - 5 = 1.27 \text{ V}$$

Then $I_{D2} = 1.044 - 0.833 = 0.211 \text{ mA}$

2.48

(a) $I_{D1} = I_{D2} = 2.5 \text{ mA}$

$$I_{D2} = 2.5 = \frac{0.7 - 0.7 - (-2)}{R} \Rightarrow R = 0.8 \text{ k}\Omega$$

(b) $I_{D1} = 0.2I_{D2}, I_{D1} + I_{D2} = 5$

$$0.2I_{D2} + I_{D2} = 5 \Rightarrow I_{D2} = 4.167 \text{ mA}$$

$$I_{D2} = 4.167 = \frac{2}{R} \Rightarrow R = 0.48 \text{ k}\Omega$$

(c) $I_{D1} = 5I_{D2}, I_{D1} + I_{D2} = 5$

$$5I_{D2} + I_{D2} = 5 \Rightarrow I_{D2} = 0.833 \text{ mA}$$

$$I_{D2} = 0.833 = \frac{2}{R} \Rightarrow R = 2.4 \text{ k}\Omega$$

2.49

(a) D_1 and D_2 on

$$5 + \frac{5 - 0.7 - V_A}{2} = \frac{V_A - 0.7 - (-5)}{1.1}$$

$$5 + 2.15 - 3.909 = V_A \left(\frac{1}{2} + \frac{1}{1.1} \right) \Rightarrow V_A = 2.30 \text{ V}$$

$$\text{Then } I_{D1} = \frac{5 - 0.7 - 2.3}{2} = 1.0 \text{ mA}$$

$$I_{D2} = \frac{2.3 - 0.7 - (-5)}{1.1} = 6.0 \text{ mA}$$

(b) D_1 cutoff, $I_{D1} = 0$

$$I_{D2} = 5 \text{ mA}, \quad V_A = 0.7 + (5)(2.5) - 5 = 8.2 \text{ V}$$

$$(c) \quad V_A = 0, \quad I_{D1} = \frac{5 - 0.7 - 0}{2} = 2.15 \text{ mA}$$

$$\text{Then } I_{D2} = 5 + 2.15 = 7.15 \text{ mA}$$

$$I_{D2} = 7.15 = \frac{0 - 0.7 - (-5)}{R_2} \Rightarrow R_2 = 0.60 \text{ k}\Omega$$

2.50

(a) (i) $v_I = 5 \text{ V}$, D_1 and D_2 on

$$\frac{5 - (v_o + 0.6)}{5} + \frac{5 - v_o}{5} = \frac{v_o}{0.5} + \frac{v_o - 0.6}{0.5}$$

$$0.88 + 1.0 + 1.2 = v_o (0.20 + 0.20 + 2.0 + 2.0) \Rightarrow v_o = 0.7 \text{ V}$$

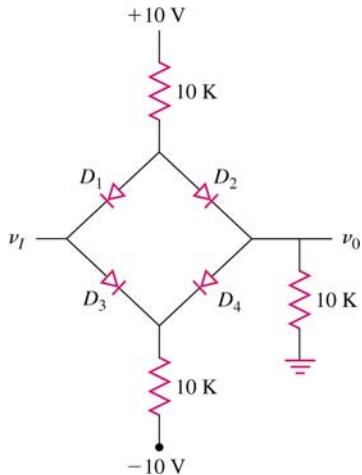
(ii) $v_I = -5 \text{ V}$

$$v_o = \left(\frac{0.5}{0.5 + 5} \right) v_I = -0.455 \text{ V}$$

(b) (i) $v_I = 5 \text{ V}$, $v_o = 4.4 \text{ V}$

(ii) $v_I = -5 \text{ V}$, $v_o = -0.6 \text{ V}$

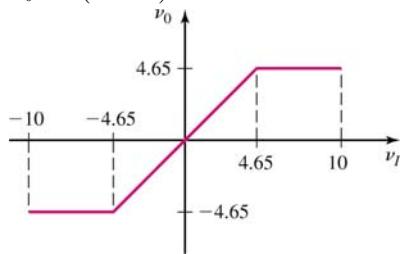
2.51



For $v_I > 0$, when D_1 and D_4 turn off

$$I = \frac{10 - 0.7}{20} = 0.465 \text{ mA}$$

$$v_0 = I(10 \text{ k}\Omega) = 4.65 \text{ V}$$



$$\underline{v_0 = v_I \text{ for } -4.65 \leq v_I \leq 4.65}$$

2.52

(a) All diodes on

$$\frac{15 - V_A}{6.15} = \frac{V_A - 0.7}{2} + \frac{V_A - 0.7 - (-5)}{14} + \frac{V_A - 0.7 - (-10)}{24}$$

$$2.439 + 0.35 - 0.307 - 0.3875 = V_A(0.1626 + 0.50 + 0.0714 + 0.0417) \Rightarrow V_A = 2.70 \text{ V}$$

$$\text{Then } I_{D1} = \frac{2.70 - 0.7}{2} = 1.0 \text{ mA}$$

$$I_{D2} = \frac{2.70 - 0.7 - (-5)}{14} = 0.50 \text{ mA}$$

$$I_{D3} = \frac{2.70 - 0.7 - (-10)}{24} = 0.50 \text{ mA}$$

(b) D_1 cutoff, $I_{D1} = 0$

$$\frac{15 - V_A}{6.15} = \frac{V_A - 0.7 - (-5)}{3.3} + \frac{V_A - 0.7 - (-10)}{5.2}$$

$$2.439 - 1.303 - 1.788 = V_A(0.1626 + 0.303 + 0.1923) \Rightarrow V_A = -0.991 \text{ V}$$

$$\text{Then } I_{D2} = \frac{-0.991 - 0.7 - (-5)}{3.3} = 1.0 \text{ mA}$$

$$I_{D3} = \frac{-0.991 - 0.7 - (-10)}{5.2} = 1.60 \text{ mA}$$

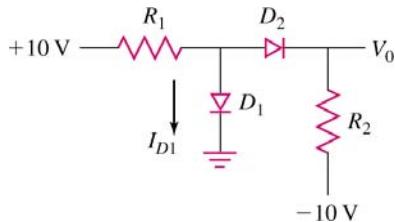
(c) D_1 and D_2 cutoff, $I_{D1} = I_{D2} = 0$

$$I_{D3} = \frac{15 - 0.7 - (-10)}{R_1 + R_4} = \frac{24.3}{6.15 + 1.32} = 3.25 \text{ mA}$$

$$V_A = 15 - (3.25)(6.15) = -5 \text{ V}$$

2.53

a.



$$R_1 = 5 \text{ k}\Omega, R_2 = 10 \text{ k}\Omega$$

$$D_1 \text{ and } D_2 \text{ on} \Rightarrow V_0 = 0$$

$$I_{D1} = \frac{10 - 0.7}{5} - \frac{0 - (-10)}{10} = 1.86 - 1.0$$

$$\underline{I_{D1} = 0.86 \text{ mA}}$$

b.

$$R_1 = 10 \text{ k}\Omega, R_2 = 5 \text{ k}\Omega, D_1 \text{ off}, D_2 \text{ on} \quad \underline{I_{D1} = 0}$$

$$I = \frac{10 - 0.7 - (-10)}{15} = 1.287$$

$$V_0 = IR_2 - 10 \Rightarrow \underline{V_0 = -3.57 \text{ V}}$$

2.54

If both diodes on

(a)

$$V_A = -0.7 \text{ V}, \underline{V_O = -1.4 \text{ V}}$$

$$I_{R1} = \frac{10 - (-0.7)}{10} = 1.07 \text{ mA}$$

$$I_{R2} = \frac{-1.4 - (-15)}{5} = 2.72 \text{ mA}$$

$$I_{R1} + I_{D1} = I_{R2} \Rightarrow I_{D1} = 2.72 - 1.07$$

$$\underline{I_{D1} = 1.65 \text{ mA}}$$

(b) D_1 off, D_2 on

$$I_{R1} = I_{R2} = \frac{10 - 0.7 - (-15)}{5 + 10} = 1.62 \text{ mA}$$

$$V_O = I_{R2}R_2 - 15 = (1.62)(10) - 15 \Rightarrow V_O = 1.2 \text{ V}$$

$$V_A = 1.2 + 0.7 = 1.9 \text{ V} \Rightarrow D_1 \text{ off},$$

$$I_{D1} = 0$$

2.55

(a) D_1 on, D_2 off

$$I_{D1} = \frac{10 - 0.7}{10} = 0.93 \text{ mA}$$

$$V_O = -15 \text{ V}$$

(b) D_1 on, D_2 off

$$I_{D1} = \frac{10 - 0.7}{5} = 1.86 \text{ mA}$$

$$V_o = -15 \text{ V}$$

2.56

$$\frac{15 - (V_0 + 0.7)}{10} = \frac{V_0 + 0.7}{20} + \frac{V_0}{20}$$

$$\frac{15}{10} - \frac{0.7}{10} - \frac{0.7}{20} = V_0 \left(\frac{1}{10} + \frac{1}{20} + \frac{1}{20} \right) = V_0 \left(\frac{4.0}{20} \right)$$

$$V_0 = 6.975 \text{ V}$$

$$I_D = \frac{V_0}{20} \Rightarrow I_D = 0.349 \text{ mA}$$

2.57

(a) Diode is cutoff, $I_D = 0$, $V_D = 0$

$$V_A = V_B = 3 \text{ V}$$

(b) Diode is conducting, $V_D = 0.7 \text{ V}$

$$\frac{5 - V_B}{10} = \frac{V_B}{10} + \frac{V_B - 0.7}{10} + \frac{V_B - 0.7 - 2}{10}$$

$$0.50 + 0.07 + 0.27 = V_B (0.10 + 0.10 + 0.10 + 0.10) \Rightarrow V_B = 2.1 \text{ V} \text{ and } V_A = 1.4 \text{ V}$$

$$\frac{5 - V_B}{10} = \frac{V_B}{10} + I_D$$

$$\text{So } I_D = \frac{5 - 2.1}{10} - \frac{2.1}{10} = 0.08 \text{ mA}$$

(c) Diode is cutoff, $I_D = 0$

$$V_A = \frac{1}{2}(5) = 2.5 \text{ V}, \quad V_B = \frac{1}{2}(4) = 2.0 \text{ V}$$

$$V_D = 2 - 2.5 = -0.5 \text{ V}$$

(d) Diode is conducting, $V_D = 0.7 \text{ V}$

$$\frac{8 - V_B}{10} = \frac{V_B}{10} + \frac{V_B - 0.7}{10} + \frac{V_B - 0.7 - 2}{10}$$

$$0.80 + 0.07 + 0.27 = V_B (0.40) \Rightarrow V_B = 2.85 \text{ V} \text{ and } V_A = 2.15 \text{ V}$$

$$\text{Then } I_D = \frac{8 - V_B}{10} - \frac{V_B}{10} = \frac{1}{10} [8 - 2(2.85)] = 0.23 \text{ mA}$$

2.58

$v_I = 0$, D_1 off, D_2 on

$$I = \frac{10 - 2.5}{15} 0.5 \text{ mA}$$

$$v_o = 10 - (0.5)(5) \Rightarrow v_o = 7.5 \text{ V} \quad \text{for } 0 \leq v_I \leq 7.5 \text{ V}$$

For $v_I > 7.5 \text{ V}$, Both D_1 and D_2 on

$$\frac{v_I - v_o}{15} = \frac{v_o - 2.5}{10} + \frac{v_o - 10}{5} \quad \text{or} \quad \frac{v_I = v_o(5.5) - 33.75}{}$$

When $v_o = 10 \text{ V}$, D_2 turns off

$$v_I = (10)(5.5) - 33.75 = 21.25 \text{ V}$$

For $v_I > 21.25 \text{ V}$, $v_o = 10 \text{ V}$

2.59

(a) For $v_I = 0.5 \text{ V}$, $I_{D1} = I_{D2} = I_{D3} = 0$, $v_o = 0.5 \text{ V}$

(b) For $v_I = 1.5 \text{ V}$, D_1 on; $I_{D2} = I_{D3} = 0$

$$I_{D1} = \frac{1.5 - 0.7}{4 + 8} = 0.0667 \text{ mA}$$

$$v_o = 0.7 + (0.0667)(8) = 1.23 \text{ V}$$

(c) For $v_I = 3 \text{ V}$, D_1 and D_2 conducting, $I_{D3} = 0$

$$\frac{3 - v_o}{4} = \frac{v_o - 0.7}{8} + \frac{v_o - 1.7}{6}$$

$$0.75 + 0.0875 + 0.2833 = v_o(0.25 + 0.125 + 0.1667) \Rightarrow v_o = 2.069 \text{ V}$$

$$\text{Then } I_{D1} = \frac{2.069 - 0.7}{8} = 0.171 \text{ mA}$$

$$I_{D2} = \frac{2.069 - 1.7}{6} = 0.0615 \text{ mA}$$

(d) For $v_I = 5 \text{ V}$, all diodes conducting

$$\frac{5 - v_o}{4} = \frac{v_o - 0.7}{8} + \frac{v_o - 1.7}{6} + \frac{v_o - 2.7}{4}$$

$$1.25 + 0.0875 + 0.2833 + 0.675 = v_o(0.25 + 0.125 + 0.1667 + 0.25)$$

$$\text{So } v_o = 2.90 \text{ V}$$

$$\text{Then } I_{D1} = \frac{2.90 - 0.7}{8} = 0.275 \text{ mA}$$

$$I_{D2} = \frac{2.90 - 1.7}{6} = 0.20 \text{ mA}$$

$$I_{D3} = \frac{2.90 - 2.7}{4} = 0.05 \text{ mA}$$

2.60

(a) $I_{D2} = 0$ for $v_I < 4.5 \text{ V}$

$$I_{D2} = 100 \text{ mA} \text{ for } v_I > 4.5 \text{ V}$$

(b) $I_{D2} = 0$ for $v_I < 9 \text{ V}$

$$I_{D2} = 100 \text{ mA} \text{ for } v_I > 9 \text{ V}$$

2.61

- a. $\underline{V_{01} = V_{02} = 0}$
b. $\underline{V_{01} = 4.4 \text{ V}, \quad V_{02} = 3.8 \text{ V}}$
c. $\underline{V_{01} = 4.4 \text{ V}, \quad V_{02} = 3.8 \text{ V}}$

Logic “1” level degrades as it goes through additional logic gates.

2.62

- a. $\underline{V_{01} = V_{02} = 5 \text{ V}}$
b. $\underline{V_{01} = 0.6 \text{ V}, \quad V_{02} = 1.2 \text{ V}}$
c. $\underline{V_{01} = 0.6 \text{ V}, \quad V_{02} = 1.2 \text{ V}}$

Logic “0” signal degrades as it goes through additional logic gates.

2.63

$$(V_1 \text{ AND } V_2) \text{ OR } (V_3 \text{ AND } V_4)$$

2.64

$$I = \frac{10 - 1.5 - 0.2}{R + 10} = 12 \text{ mA} = 0.012$$
$$R + 10 = \frac{8.3}{0.012} = 691.7\Omega$$
$$\underline{R = 681.7\Omega}$$

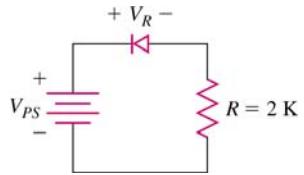
2.65

$$I = \frac{10 - 1.7 - V_I}{0.75} = 8$$
$$V_I = 10 - 1.7 - 8(0.75) \Rightarrow \underline{V_I = 2.3 \text{ V}}$$

2.66

$$R + r_f = \frac{5 - 1.7}{15} = 0.220 \text{ k}\Omega$$
$$r_f = 20 \Omega \Rightarrow R = 200 \Omega$$

2.67



$$V_R = 1 \text{ V}, I = 0.8 \text{ mA}$$

$$V_{PS} = 1 + (0.8)(2)$$

$$\underline{V_{PS} = 2.6 \text{ V}}$$

2.68

$$I_{Ph} = \eta e \Phi A$$

$$0.6 \times 10^{-3} = (1)(1.6 \times 10^{-19})(10^{17}) A$$

$$\underline{A = 3.75 \times 10^{-2} \text{ cm}^2}$$

Chapter 3

3.1

$$K_n = \frac{k'_n}{2} \cdot \frac{W}{L} = \frac{120}{2} \left(\frac{10}{0.8} \right) \Rightarrow 0.75 \text{ mA/V}^2$$

- (a) (i) $I_D = 0$
(ii) $I_D = (0.75)[2(1-0.4)(0.1)-(0.1)^2] \Rightarrow 82.5 \mu\text{A}$
(iii) $I_D = (0.75)[2(2-0.4)(0.1)-(0.1)^2] = 0.2325 \text{ mA}$
(iv) $I_D = (0.75)[2(3-0.4)(0.1)-(0.1)^2] = 0.3825 \text{ mA}$
- (b) (i) $I_D = 0$
(ii) $I_D = (0.75)(1-0.4)^2 = 0.27 \text{ mA}$
(iii) $I_D = (0.75)(2-0.4)^2 = 1.92 \text{ mA}$
(iv) $I_D = (0.75)(3-0.4)^2 = 5.07 \text{ mA}$
-

3.2

$$\begin{aligned} I_D &= K_n [2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2] \\ 0.5 &= K_n [2(0.6)V_{DS} - V_{DS}^2] \\ 1.0 &= K_n [2(1.0)V_{DS} - V_{DS}^2] \end{aligned}$$

Take ratio

$$0.5 = \frac{1.2V_{DS} - V_{DS}^2}{2V_{DS} - V_{DS}^2} \Rightarrow V_{DS} - 0.5V_{DS}^2 = 1.2V_{DS} - V_{DS}^2$$

$$\text{or } 1 - 0.5V_{DS} = 1.2 - V_{DS}$$

$$\text{which yields } V_{DS} = 0.4 \text{ V}$$

$$\text{Then } 0.5 = K_n [(1.2)(0.4) - (0.4)^2] \Rightarrow K_n = 1.56 \text{ mA/V}^2$$

3.3

- (a) Enhancement-mode
(b) From Graph $V_T = 1.5 \text{ V}$

Now

$$0.03 = K_n (2 - 1.5)^2 = 0.25 K_n \Rightarrow K_n = 0.12$$

$$0.15 = K_n (3 - 1.5)^2 = 2.25 K_n \quad K_n = 0.0666$$

$$0.39 = K_n (4 - 1.5)^2 = 6.25 K_n \quad K_n = 0.0624$$

$$0.77 = K_n (5 - 1.5)^2 = 12.25 K_n \quad K_n = 0.0629$$

From last three, $K_n(\text{Avg}) = 0.0640 \text{ mA/V}^2$

$$i_D(\text{sat}) = 0.0640(3.5 - 1.5)^2 \Rightarrow i_D(\text{sat}) = 0.256 \text{ mA for } V_{GS} = 3.5 \text{ V}$$

$$(c) \quad i_D(\text{sat}) = 0.0640(4.5 - 1.5)^2 \Rightarrow i_D(\text{sat}) = 0.576 \text{ mA for } V_{GS} = 4.5 \text{ V}$$

3.4

- $V_{GS} = 0$
- a. $V_{DS}(\text{sat}) = V_{GS} - V_{TN} = 0 - (-2.5) = 2.5 \text{ V}$
 $V_{DS} = 0.5 \text{ V} \Rightarrow \text{Biased in nonsaturation}$
- i. $I_D = (1.1) \left[2(0 - (-2.5))(0.5) - (0.5)^2 \right] \Rightarrow I_D = 2.48 \text{ mA}$
 $V_{DS} = 2.5 \text{ V} \Rightarrow \text{Biased in saturation}$
- ii. $I_D = (1.1)(0 - (-2.5))^2 \Rightarrow I_D = 6.88 \text{ mA}$
- iii. $V_{DS} = 5 \text{ V}$ Same as (ii) $\Rightarrow I_D = 6.88 \text{ mA}$
- b. $V_{GS} = 2 \text{ V}$
 $V_{DS}(\text{sat}) = 2 - (-2.5) = 4.5 \text{ V}$
 $V_{DS} = 0.5 \text{ V} \Rightarrow \text{Nonsaturation}$
- i. $I_D = (1.1) \left[2(2 - (-2.5))(0.5) - (0.5)^2 \right] \Rightarrow I_D = 4.68 \text{ mA}$
 $V_{DS} = 2.5 \text{ V} \Rightarrow \text{Nonsaturation}$
- ii. $I_D = (1.1) \left[2(2 - (-2.5))(2.5) - (2.5)^2 \right] \Rightarrow I_D = 17.9 \text{ mA}$
 $V_{DS} = 5 \text{ V} \Rightarrow \text{Saturation}$
- iii. $I_D = (1.1)(2 - (-2.5))^2 \Rightarrow I_D = 22.3 \text{ mA}$
-

3.5

- (a) $V_{DS}(\text{sat}) = V_{GS} - V_{TN} = 2.2 - 0.4 = 1.8 \text{ V}$
 $2.2 = V_{DS} > V_{DS}(\text{sat}) = 1.8 \Rightarrow \text{Saturation}$
- (b) $V_{DS}(\text{sat}) = V_{GS} - V_{TN} = 1 - 0.4 = 0.6 \text{ V}$
 $V_{DS} = -0.6 - (-1) = 0.4 \text{ V} < V_{DS}(\text{sat}) = 0.6 \text{ V} \Rightarrow \text{Nonsaturation}$
- (c) $V_{GS} = 1 - 1 = 0 \Rightarrow \text{Cutoff}$
-

3.6

- (a) $V_{SG} = 2.2 - 2.2 = 0 \Rightarrow \text{Cutoff}$
- (b) $V_{SG} = 2 \text{ V}, V_{SD} = 2 - (-1) = 3 \text{ V}$
 $V_{SD}(\text{sat}) = V_{SG} + V_{TP} = 2 + (-0.4) = 1.6 \text{ V}$
 So $V_{SD} = 3 > V_{SD}(\text{sat}) = 1.6 \Rightarrow \text{Saturation}$
- (c) $V_{SG} = 2 \text{ V}, V_{SD} = 2 - 1 = 1 \text{ V}$
 $V_{SD}(\text{sat}) = V_{SG} + V_{TP} = 2 + (-0.4) = 1.6 \text{ V}$
 So $V_{SD} = 1 < V_{SD}(\text{sat}) = 1.6 \Rightarrow \text{Nonsaturation}$
-

3.7

$$I_D = \frac{k'_n}{2} \left(\frac{W}{L} \right) (V_{GS} - V_{TN})^2$$

$$0.5 = \frac{0.12}{2} \left(\frac{W}{L} \right) [0 - (-1.2)]^2 \Rightarrow \left(\frac{W}{L} \right) = 5.79$$

3.8

$$k'_n = \mu_n C_{ox} = \frac{\mu_n \epsilon_{ox}}{t_{ox}} = \frac{(600)(3.9)(8.85 \times 10^{-14})}{200 \times 10^{-8}} = \frac{2.071 \times 10^{-10}}{t_{ox}}$$

- (a) 500 A $k'_n = 41.4 \text{ } \mu\text{A/V}^2$
 (b) 250 $k'_n = 82.8 \text{ } \mu\text{A/V}^2$
 (c) 100 $k'_n = 207 \text{ } \mu\text{A/V}^2$
 (d) 50 $k'_n = 414 \text{ } \mu\text{A/V}^2$
 (e) 25 $k'_n = 828 \text{ } \mu\text{A/V}^2$
-

3.9

$$(a) K_n = \frac{W \mu_n \epsilon_{ox}}{2L t_{ox}} = \frac{(20 \times 10^{-4})(650)(3.9)(8.85 \times 10^{-14})}{2(0.8 \times 10^{-4})(200 \times 10^{-8})} = 1.40 \text{ mA/V}^2$$

$$(b) I_D = K_n (V_{GS} - V_{TN})^2 = (1.40)(2 - 0.4)^2$$

Or $I_D = 3.58 \text{ mA}$

$$(c) V_{DS}(\text{sat}) = V_{GS} - V_{TN} = 2 - 0.4 = 1.6 \text{ V}$$

3.10

$$(a) I_D = \frac{k'_n}{2} \left(\frac{W}{L} \right) (V_{GS} - V_{TN})^2$$

$$0.6 = \left(\frac{0.12}{2} \right) \left(\frac{W}{L} \right) (1.4 - 0.8)^2 \Rightarrow \left(\frac{W}{L} \right) = 27.8$$

Or $W = (27.8)(0.8) = 22.2 \mu\text{m}$

$$(b) I_D = \left(\frac{0.12}{2} \right) (27.8) [2(1.4 - 0.8)(0.4) - (0.4)^2] = 0.534 \text{ mA}$$

$$(c) V_{DS}(\text{sat}) = V_{GS} - V_{TN} = 1.4 - 0.8 = 0.6 \text{ V}$$

3.11

$$k'_n = \mu_n C_{ox} = \frac{\mu_n \epsilon_{ox}}{t_{ox}} = \frac{(600)(3.9)(8.85 \times 10^{-14})}{200 \times 10^{-8}}$$

$$k'_n = 0.1035 \text{ mA/V}^2$$

$$I_D = \frac{k'_n}{2} \left(\frac{W}{L} \right) (V_{GS} - V_{TN})^2$$

$$1.2 = \left(\frac{0.1035}{2} \right) \left(\frac{W}{L} \right) (3 - 0.6)^2 \Rightarrow \left(\frac{W}{L} \right) = 4.026$$

Then $W = (4.026)(0.8) = 3.22 \mu\text{m}$

3.12

$$I_D = WC_{ox}(V_{GS} - V_{TN})v_{sat}$$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{200 \times 10^{-8}} = 1.726 \times 10^{-7} \text{ F/cm}^2$$

$$I_D = (3.22 \times 10^{-4})(1.726 \times 10^{-7})(3 - 0.6)(2 \times 10^7)$$

$$I_D = 2.67 \text{ mA}$$

3.13

$$I_D = \frac{k'_p}{2} \left(\frac{W}{L} \right) (V_{SG} + V_{TP})^2$$

$$0.225 = \left(\frac{0.05}{2} \right) \left(\frac{W}{L} \right) (2 + V_{TP})^2$$

$$0.65 = \left(\frac{0.05}{2} \right) \left(\frac{W}{L} \right) (3 + V_{TP})^2$$

Then $\sqrt{\frac{0.65}{0.225}} = \frac{3 + V_{TP}}{2 + V_{TP}} = 1.70 \Rightarrow V_{TP} = -0.571 \text{ V}$

And $0.225 = \left(\frac{0.05}{2} \right) \left(\frac{W}{L} \right) (2 - 0.571)^2 \Rightarrow \left(\frac{W}{L} \right) = 4.41$

3.14

- $V_S = 5 \text{ V}, V_G = 0 \Rightarrow V_{SG} = 5 \text{ V}$
 $V_{TP} = -0.5 \text{ V} \Rightarrow V_{SD}(\text{sat}) = V_{SG} + V_{TP} = 5 - 0.5 = 4.5 \text{ V}$
 $V_D = 0 \Rightarrow V_{SD} = 5 \text{ V} \Rightarrow \text{Biased in saturation}$
- a. $I_D = 2(5 - 0.5)^2 \Rightarrow I_D = 40.5 \text{ mA}$
 $V_D = 2 \text{ V} \Rightarrow V_{SD} = 3 \text{ V} \Rightarrow \text{Nonsaturation}$
b. $I_D = 2[2(5 - 0.5)(3) - (3)^2] \Rightarrow I_D = 36 \text{ mA}$
 $V_D = 4 \text{ V} \Rightarrow V_{SD} = 1 \text{ V} \Rightarrow \text{Nonsaturation}$
c. $I_D = 2[2(5 - 0.5)(1) - (1)^2] \Rightarrow I_D = 16 \text{ mA}$
d. $V_D = 5 \text{ V} \Rightarrow V_{SD} = 0 \Rightarrow I_D = 0$
-

3.15

- (a) Enhancement-mode
(b) From Graph $V_{TP} = +0.5 \text{ V}$
- $$0.45 = k_p (2 - 0.5)^2 = 2.25 K_p \Rightarrow K_p = 0.20$$
- $$1.25 = k_p (3 - 0.5)^2 = 6.25 K_p \qquad \qquad \qquad 0.20$$
- $$2.45 = k_p (4 - 0.5)^2 = 12.25 K_p \qquad \qquad \qquad 0.20$$
- $$4.10 = k_p (5 - 0.5)^2 = 20.25 K_p \qquad \qquad \qquad 0.202$$
- $$\text{Avg } K_p = 0.20 \text{ mA/V}^2$$

$$i_D(\text{sat}) = 0.20 (3.5 - 0.5)^2 = 1.8 \text{ mA}$$

(c) $i_D(\text{sat}) = 0.20 (4.5 - 0.5)^2 = 3.2 \text{ mA}$

3.16

$$V_{SD}(\text{sat}) = V_{SG} + V_{TP}$$

(a) $V_{SD}(\text{sat}) = -1 + 2 \Rightarrow V_{SD}(\text{sat}) = 1 \text{ V}$

(b) $V_{SD}(\text{sat}) = 0 + 2 \Rightarrow V_{SD}(\text{sat}) = 2 \text{ V}$

(c) $V_{SD}(\text{sat}) = 1 + 2 \Rightarrow V_{SD}(\text{sat}) = 3 \text{ V}$

$$I_D = \frac{k'_p}{2} \cdot \frac{W}{L} (V_{SG} + V_{TP})^2 = \frac{k'_p}{2} \cdot \frac{W}{L} \cdot [V_{SD}(\text{sat})]^2$$

(a) $I_D = \left(\frac{0.040}{2} \right) (6)(1)^2 \Rightarrow I_D = 0.12 \text{ mA}$

(b) $I_D = \left(\frac{0.040}{2} \right) (6)(2)^2 \Rightarrow I_D = 0.48 \text{ mA}$

(c) $I_D = \left(\frac{0.040}{2} \right) (6)(3)^2 \Rightarrow I_D = 1.08 \text{ mA}$

3.17

$$K_p = \frac{k'_p}{2} \left(\frac{W}{L} \right) = \frac{50}{2} \left(\frac{12}{0.8} \right) \Rightarrow K_p = 0.375 \text{ mA/V}^2$$

(a) Nonsaturation
 $I_D = (0.375) [2(2 - 0.5)(0.2) - (0.2)^2] = 0.21 \text{ mA}$

(b) Nonsaturation
 $I_D = (0.375) [2(2 - 0.5)(0.8) - (0.8)^2] = 0.66 \text{ mA}$

(c) Nonsaturation
 $I_D = (0.375) [2(2 - 0.5)(1.2) - (1.2)^2] = 0.81 \text{ mA}$

(d) Saturation
 $I_D = (0.375)(2 - 0.5)^2 = 0.844 \text{ mA}$

(e) Saturation
 $I_D = (0.375)(2 - 0.5)^2 = 0.844 \text{ mA}$

3.18

$$k'_p = \mu_p C_{ox} = \frac{\mu_p \epsilon_{ox}}{t_{0x}} = \frac{(250)(3.9)(8.85 \times 10^{-14})}{t_{0x}} = \frac{8.629 \times 10^{-11}}{t_{0x}}$$

(a) $t_{ox} = 500 \text{ \AA} \Rightarrow k'_p = 17.3 \text{ } \mu\text{A/V}^2$

(b) $250 \text{ \AA} \Rightarrow k'_p = 34.5 \text{ } \mu\text{A/V}^2$

(c) $100 \text{ \AA} \Rightarrow k'_p = 86.3 \text{ } \mu\text{A/V}^2$

(d) $50 \text{ \AA} \Rightarrow k'_p = 173 \text{ } \mu\text{A/V}^2$

(e) $25 \text{ \AA} \Rightarrow k'_p = 345 \text{ } \mu\text{A/V}^2$

3.19

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{500 \times 10^{-8}} = 6.90 \times 10^{-8} \text{ F/cm}^2$$

$$k'_n = (\mu_n C_{ox}) = (675)(6.90 \times 10^{-8}) \Rightarrow 46.6 \mu\text{A/V}^2$$

$$k'_p = (\mu_p C_{ox}) = (375)(6.90 \times 10^{-8}) \Rightarrow 25.9 \mu\text{A/V}^2$$

PMOS:

$$I_D = \frac{k'_p}{2} \left(\frac{W}{L} \right)_p (V_{SG} + V_{TP})^2$$

$$0.8 = \left(\frac{0.0259}{2} \right) \left(\frac{W}{L} \right)_p (5 - 0.6)^2 \Rightarrow \left(\frac{W}{L} \right)_p = 3.19$$

$$L = 4 \mu\text{m} \Rightarrow \underline{W_p = 12.8 \mu\text{m}}$$

$$K_p = \left(\frac{0.0259}{2} \right) (3.19) \Rightarrow \underline{K_p = 41.3 \mu\text{A/V}^2 = K_n}$$

Want $K_n = K_p$

$$\frac{k'_n}{2} \left(\frac{W}{L} \right)_N = \frac{k'_p}{2} \left(\frac{W}{L} \right)_p = 41.3$$

$$\left(\frac{46.6}{2} \right) \left(\frac{W}{L} \right)_N = 41.3 \Rightarrow \left(\frac{W}{L} \right)_N = 1.77$$

$$L = 4 \mu\text{m} \Rightarrow \underline{W_N = 7.09 \mu\text{m}}$$

3.20

$$V_{GS} = 2 \text{ V}, \quad I_D = (0.2)(2 - 1.2)^2 = 0.128 \text{ mA}$$

$$r_0 = \frac{1}{\lambda I_D} = \frac{1}{(0.01)(0.128)} \Rightarrow \underline{r_0 = 781 \text{ k}\Omega}$$

$$V_{GS} = 4 \text{ V}, \quad I_D = (0.2)(4 - 1.2)^2 = 1.57 \text{ mA}$$

$$r_0 = \frac{1}{(0.01)(1.57)} \Rightarrow \underline{r_0 = 63.7 \text{ k}\Omega}$$

$$V_A = \frac{1}{\lambda} = \frac{1}{(0.01)} \Rightarrow \underline{V_A = 100 \text{ V}}$$

3.21

$$I_D = \frac{k'_n}{2} \left(\frac{W}{L} \right) (V_{GS} - V_{TN})^2 = \left(\frac{0.12}{2} \right) (4)(2 - 0.5)^2 = 0.54 \text{ mA}$$

$$r_o = \frac{1}{\lambda I_{DQ}} \Rightarrow \lambda = \frac{1}{r_o I_{DQ}} = \frac{1}{(200 \times 10^3)(0.54 \times 10^{-3})} = 0.00926 \text{ V}^{-1}$$

$$\text{Then } V_A = \frac{1}{\lambda} = 108 \text{ V}$$

3.22

$$V_{TN} = V_{TNO} + \gamma \left[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f} \right]$$

$$\Delta V_{TN} = 2 = (0.8) \left[\sqrt{2\phi_f + V_{SB}} - \sqrt{2(0.35)} \right]$$

$$2.5 + 0.837 = \sqrt{2(0.35)} + V_{SB} \Rightarrow V_{SB} = 10.4 \text{ V}$$

3.23

$$V_{TN} = V_{TNo} + r \left[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f} \right]$$

$$= 0.75 + 0.6 \left[\sqrt{2(0.37) + 3} - \sqrt{2(0.37)} \right]$$

$$= 0.75 + 0.6 [1.934 - 0.860]$$

$$V_{TN} = 1.39 \text{ V}$$

$$V_{DS}(\text{sat}) = 2.5 - 1.39 = 1.11 \text{ V}$$

$$\text{Sat Region } I_D = (15) \left(\frac{0.08}{2} \right) (2.5 - 1.39)^2$$

$$(a) \quad I_D = 0.739 \text{ mA}$$

$$\text{Non-Sat } I_D = (15) \left(\frac{0.08}{2} \right) \left[2(2.5 - 1.39)(0.25) - (0.25)^2 \right]$$

$$(b) \quad I_D = 0.296 \text{ mA}$$

3.24

$$E_{ox} = 6 \times 10^6 \text{ V/cm}$$

$$(a) \quad (i) \quad V_G = E_{ox} t_{ox} = (6 \times 10^6) (120 \times 10^{-8}) = 7.2 \text{ V}$$

$$(ii) \quad V_G = \frac{7.2}{3} = 2.4 \text{ V}$$

$$(b) \quad (i) \quad V_G = (6 \times 10^6) (200 \times 10^{-8}) = 12 \text{ V}$$

$$(ii) \quad V_G = \frac{12}{3} = 4 \text{ V}$$

3.25

$$\text{Want } (3)(24) = E_{ox} t_{ox} = (6 \times 10^6) t_{ox}$$

$$\underline{t_{ox} = 1.2 \times 10^{-5} \text{ cm} = 1200 \text{ Angstroms}}$$

3.26

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \left(\frac{18}{18+32} \right) (10) = 3.6 \text{ V}$$

Assume transistor biased in saturation region

$$I_D = \frac{V_S}{R_S} = \frac{V_G - V_{GS}}{R_S} = K_n (V_{GS} - V_{TN})^2$$

$$3.6 - V_{GS} = (0.5)(2)(V_{GS} - 0.8)^2 \\ = V_{GS}^2 - 1.6V_{GS} + 0.64$$

$$V_{GS}^2 - 0.6V_{GS} - 2.96 = 0$$

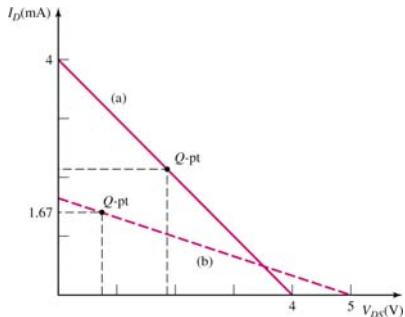
$$V_{GS} = \frac{0.6 \pm \sqrt{(0.6)^2 + 4(2.96)}}{2} \Rightarrow V_{GS} = 2.046 \text{ V}$$

$$I_D = \frac{V_G - V_{GS}}{R_S} = \frac{3.6 - 2.046}{2} \Rightarrow I_D = 0.777 \text{ mA}$$

$$V_{DS} = V_{DD} - I_D (R_D + R_S) \\ = 10 - (0.777)(4 + 2) \Rightarrow V_{DS} = 5.34 \text{ V}$$

$$\underline{V_{DS} > V_{DS}(\text{sat})}$$

3.27



$$(a) \quad V_{GS} = 4 \text{ V} \quad V_{DS}(\text{sat}) = 4 - 0.8 = 3.2 \text{ V}$$

$$\text{If Sat } I_D = 0.25(4 - 0.8)^2 = 2.56$$

$$V_{DS} = 1.44 \quad \times$$

Non-Sat

$$4 = I_D R_D + V_{DS} = K_n R_D [2(V_{GS} - V_T) V_{DS} - V_{DS}^2] + V_{DS}$$

$$4 = (0.25)(1)[2(4 - 0.8)V_{DS} - V_{DS}^2] + V_{DS}$$

$$4 = 2.6V_{DS} - 0.25V_{DS}^2$$

$$0.25V_{DS}^2 - 2.6V_{DS} + 4 = 0$$

$$V_{DS} = \frac{2.6 \pm \sqrt{6.76 - 4}}{2(0.25)} = 1.88 \text{ V}$$

$$I_D = \frac{4 - 1.88}{1} = 2.12 \text{ mA}$$

(b) Non-Sat region

$$5 = I_D R_D + V_{DS} = K_n R_D \left[2(V_{GS} - V_T) V_{DS} - V_{DS}^2 \right] + V_{DS}$$

$$5 = (0.25)(3) \left[2(5 - 0.8) V_{DS} - V_{DS}^2 \right] + V_{DS}$$

$$5 = 7.3 V_{DS} - 0.75 V_{DS}^2$$

$$0.75 V_{DS}^2 - 7.3 V_{DS} + 5 = 0$$

$$V_{DS} = \frac{7.3 \pm \sqrt{53.29 - 15}}{2(0.75)}$$

$$V_{DS} = 0.741 \text{ V}$$

$$I_D = \frac{5 - 0.741}{3} = 1.42 \text{ mA}$$

3.28

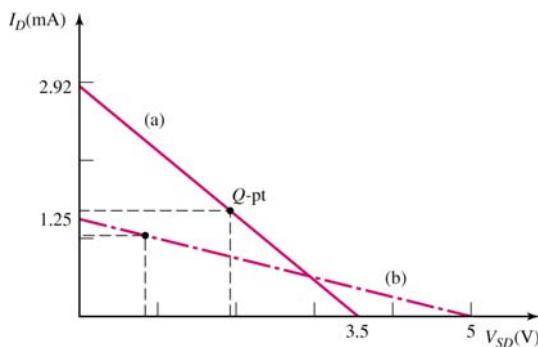
$$0.8 = \left(\frac{0.12}{2} \right) (80) (V_{GS} - 0.4)^2 \Rightarrow V_{GS} = 0.808 \text{ V}$$

$$V_{GS} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$0.80825 = \frac{1}{R_1} (200) (1.8) \Rightarrow R_1 = 445 \text{ k}\Omega$$

$$R_1 \| R_2 = R_{in} = 200 \text{ k}\Omega \Rightarrow R_2 = 363 \text{ k}\Omega$$

3.29



$$(a) \quad V_{GS} = V_{DD} = 3.5 \quad V_{SD} (\text{sat}) = 3.5 - 0.8 = 2.7 \text{ V}$$

$$\begin{aligned} \text{If biased in Sat region, } I_D &= (0.2)(3.5 - 0.8)^2 \\ &= 1.46 \text{ mA} \end{aligned}$$

$$V_{SD} = 3.5 - (1.46)(1.2) = 1.75 \text{ V} \quad \times$$

Biased in Non-Sat Region.

$$\begin{aligned}
 3.5 &= V_{SD} + I_D R_D = V_{SD} + K_p R_D [2(V_{SG} + V_{TP})V_{SD} - V_{SD}^2] \\
 3.5 &= V_{SD} + (0.2)(1.2)[2(3.5 - 0.8)V_{SD} - V_{SD}^2] \\
 3.5 &= V_{SD} + 1.296 V_{SD} - 0.24 V_{SD}^2 \\
 0.24 V_{SD}^2 - 2.296 V_{SD} + 3.5 &= 0 \\
 V_{SD} &= \frac{+2.296 \pm \sqrt{5.272 - 3.36}}{2(0.24)} \text{ use - sign } \underline{V_{SD} = 1.90 \text{ V}} \\
 I_D &= (0.2)[2(3.5 - 0.8)(1.9) - (1.9)^2] = 0.2[10.26 - 3.61] \\
 I_D &= \frac{3.5 - 1.90}{1.2} = 1.33 \text{ mA} \\
 \underline{I_D = 1.33 \text{ mA}}
 \end{aligned}$$

(b)

$$V_{SG} = V_{DD} = 5 \text{ V} \quad V_{SD}(\text{sat}) = 5 - 0.8 = 4.2 \text{ V}$$

If Sat Region $I_D = (0.2)(5 - 0.8)^2 = 3.53 \text{ mA}$, $V_{SD} < 0$

Non-Sat Region.

$$\begin{aligned}
 5 &= V_{SD} + K_p R_D [2(V_{SG} + V_{TP})V_{SD} - V_{SD}^2] \\
 5 &= V_{SD} + (0.2)(4)[2(5 - 0.8)V_{SD} - V_{SD}^2] \\
 5 &= V_{SD} + 6.72 V_{SD} - 0.8 V_{SD}^2 \\
 0.8 V_{SD}^2 - 7.72 V_{SD} + 5 &= 0 \\
 V_{SD} &= \frac{7.72 \pm \sqrt{59.598 - 16}}{2(0.8)} \text{ use - sign } \underline{V_{SD} = 0.698 \text{ V}} \\
 I_D &= \frac{5 - 0.698}{4} \Rightarrow \underline{I_D = 1.08 \text{ mA}}
 \end{aligned}$$

3.30

$$\begin{aligned}
 V_G &= \left(\frac{22}{22+8} \right)(6) - 3 = 1.40 \text{ V} \\
 3 &= K_p R_S (V_{SG} + V_{TP})^2 + V_{SG} + V_G \\
 3 &= (0.5)(0.5)(V_{SG}^2 - 1.6V_{SG} + 0.64) + V_{SG} + 1.40 \\
 0.25V_{SG}^2 + 0.6V_{SG} - 1.44 &= 0 \Rightarrow V_{SG} = 1.483 \text{ V} \\
 I_D &= (0.5)(1.483 - 0.8)^2 = 0.2332 \text{ mA} \\
 V_{SD} &= 6 - (0.2332)(0.5 + 5) = 4.72 \text{ V}
 \end{aligned}$$

3.31

$$V_G = 0, \quad V_{SG} = V_S$$

Assume saturation region

$$I_D = 0.4 = K_p (V_{SG} + V_{TP})^2$$

$$0.4 = (0.2)(V_S - 0.8)^2$$

$$V_S = \sqrt{\frac{0.4}{0.2}} + 0.8 \Rightarrow V_S = 2.21 \text{ V}$$

$$V_D = I_D R_D - 5 = (0.4)(5) - 5 = -3 \text{ V}$$

$$V_{SD} = V_S - V_D = 2.21 - (-3) \Rightarrow V_{SD} = 5.21 \text{ V}$$

$$V_{SD} > V_{SD} (\text{sat})$$

3.32

$$I_D = \frac{V_{DD} - V_{DSQ} - V_{RS}}{R_D} = \frac{3.3 - 1.6 - 0.8}{5} = 0.18 \text{ mA}$$

$$0.18 = \left(\frac{0.12}{2} \right) \left(\frac{W}{L} \right) (0.8 - 0.4)^2 \Rightarrow \left(\frac{W}{L} \right) = 18.75$$

$$R_S = \frac{0.8}{0.18} = 4.44 \text{ k}\Omega$$

$$V_G = 0.8 + 0.8 = 1.6 = \left(\frac{R_2}{R_1 + R_2} \right) (3.3)$$

$$\text{Now } I_R = (0.05)(0.18) \Rightarrow 9 \mu\text{A}$$

$$\text{So } R_1 + R_2 = \frac{3.3}{0.009} = 367 \text{ k}\Omega$$

$$\text{Then } 1.6 = \left(\frac{R_2}{367} \right) (3.3) \Rightarrow R_2 = 178 \text{ k}\Omega \text{ and } R_1 = 189 \text{ k}\Omega$$

3.33

$$I_D = \frac{0.2}{1} = 0.2 \text{ mA}$$

$$V_{DS} = 1.8 - (0.2)(4+1) = 0.8 \text{ V}$$

$$\text{Now } V_{DS} (\text{sat}) = 0.8 - 0.4 = 0.4 \text{ V}$$

$$V_{DS} (\text{sat}) = V_{GS} - V_{TN} \Rightarrow 0.4 = V_{GS} - 0.4 \Rightarrow V_{GS} = 0.8 \text{ V}$$

$$I_D = \frac{k'_n}{2} \left(\frac{W}{L} \right) (V_{GS} - V_{TN})^2$$

$$0.2 = \left(\frac{0.12}{2} \right) \left(\frac{W}{L} \right) (0.8 - 0.4)^2 \Rightarrow \left(\frac{W}{L} \right) = 20.8$$

$$\text{Now } V_G = V_{GS} + I_D R_S = 0.8 + (0.2)(1) = 1.0 \text{ V}$$

$$V_G = 1 = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD} = \frac{1}{R_1} (200)(1.8) \Rightarrow R_1 = 360 \text{ k}\Omega$$

$$R_1 \| R_2 = R_{in} = 200 \text{ k}\Omega \Rightarrow R_2 = 450 \text{ k}\Omega$$

3.34

$$(a) 0.35 = \left(\frac{0.12}{2}\right)(50)(V_{GS} - 0.4)^2 \Rightarrow V_{GS} = 0.742 \text{ V}$$

$$(b) V_{DS} = 1.8 - (0.35)(2) = 1.1 \text{ V}$$

$$V_{DS}(\text{sat}) = V_{GS} - V_{TN} = 0.742 - 0.4 = 0.342 \text{ V}$$

$V_{DS} > V_{DS}(\text{sat}) \Rightarrow \text{Saturation}$

3.35

$$V_G = \left(\frac{6}{6+14}\right)(10) - 5 = -2 \text{ V}$$

$$V_G = V_{GS} + I_D R_S - 5 = V_{GS} + \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)R_S(V_{GS} - V_{TN})^2 - 5$$

$$5 - 2 = V_{GS} + \left(\frac{0.12}{2}\right)(25)(0.5)(V_{GS}^2 - 0.8V_{GS} + 0.16)$$

$$\text{Or } 0.75V_{GS}^2 + 0.4V_{GS} - 2.88 = 0 \Rightarrow V_{GS} = 1.71 \text{ V}$$

$$I_D = \left(\frac{0.12}{2}\right)(25)(1.71 - 0.4)^2 = 2.58 \text{ mA}$$

$$V_{DS} = 10 - (2.58)(1.2 + 0.5) = 5.62 \text{ V}$$

3.36

Let $\left(\frac{W}{L}\right) = 20$ for example, then

$$0.20 = \left(\frac{0.05}{2}\right)(20)(V_{SG} - 0.6)^2 \Rightarrow V_{SG} = 1.232 \text{ V}$$

$$\text{Then } V_{RS} = 1.232 = (0.2)R_S \Rightarrow R_S = 6.16 \text{ k}\Omega$$

$$R_D = \frac{6 - 1.232 - 3}{0.2} = 8.84 \text{ k}\Omega$$

$$V_G = 3 - 1.232 - 1.232 = 0.536 \text{ V}$$

$$I_R = \frac{6}{R_1 + R_2} = (0.1)(0.2) = 0.02 \text{ mA}, \Rightarrow R_1 + R_2 = 300 \text{ k}\Omega$$

$$V_G = 0.536 = \left(\frac{R_2}{R_1 + R_2}\right)(6) - 3 = \left(\frac{R_2}{300}\right)(6) - 3$$

$$\text{Or } R_2 = 176.8 \text{ k}\Omega \text{ and } R_1 = 123.2 \text{ k}\Omega$$

3.37

$$I_Q = 50 = 500(V_{GS} - 1.2)^2 \Rightarrow V_{GS} = 1.516 \text{ V}$$

$$(a) (i) V_{DS} = 5 - (-1.516) \Rightarrow V_{DS} = 6.516 \text{ V}$$

$$I_Q = 1 = (0.5)(V_{GS} - 1.2)^2 \Rightarrow V_{GS} = 2.61 \text{ V}$$

$$(ii) V_{DS} = 5 - (-2.61) \Rightarrow V_{DS} = 7.61 \text{ V}$$

(b) (i) Same as (a) $\underline{V_{GS} = V_{DS} = 1.516 \text{ V}}$

(ii) $\underline{V_{GS} = V_{DS} = 2.61 \text{ V}}$

3.38

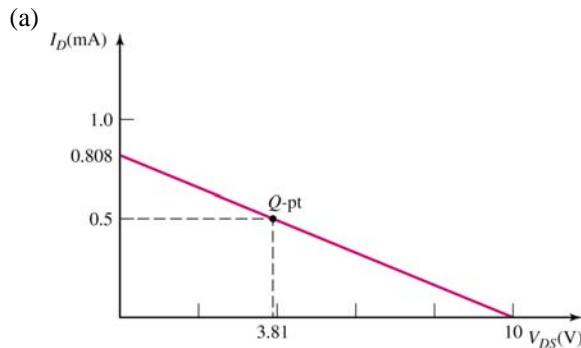
$$I_D = K_n (V_{GS} - V_{TN})^2$$

$$0.25 = (0.2)(V_{GS} - 0.6)^2$$

$$V_{GS} = \sqrt{\frac{0.25}{0.2}} + 0.6 \Rightarrow V_{GS} = 1.72 \text{ V} \Rightarrow \underline{V_s = -1.72 \text{ V}}$$

$$V_D = 9 - (0.25)(24) \Rightarrow \underline{V_D = 3 \text{ V}}$$

3.39



$$R_D = \frac{5-1}{0.5} \Rightarrow \underline{R_D = 8 \text{ K}}$$

$$I_{DQ} = 0.5 = 0.25(V_{GS} - 1.4)^2 \Rightarrow V_{GS} = 2.81 \text{ V}$$

$$R_S = \frac{-2.81 - (-5)}{0.5} \Rightarrow \underline{R_S = 4.38 \text{ K}}$$

(b) Let $R_D = 8.2 \text{ K}$, $R_S = 4.3 \text{ K}$

$$\text{Now } \frac{-V_{GS} - (-5)}{4.3} = I_D = 0.25(V_{GS} - 1.4)^2$$

$$5 - V_{GS} = 1.075(V_{GS}^2 - 2.8 V_{GS} + 1.96)$$

$$1.075 V_{GS}^2 - 2.01 V_{GS} - 2.89 = 0$$

$$V_{GS} = \frac{2.01 \pm \sqrt{4.04 + 12.427}}{2(1.075)} \Rightarrow V_{GS} = 2.82 \text{ V}$$

$$I_D = 0.25(2.82 - 1.4)^2 \Rightarrow I_D = 0.504 \text{ mA}$$

$$V_{DS} = 10 - (0.504)(8.2 + 4.3) \Rightarrow \underline{V_{DS} = 3.70 \text{ V}}$$

(c) If $R_s = 4.3 + 10\% = 4.73 \text{ K}$

$$5 - V_{GS} = 1.18(V_{GS}^2 - 2.8V_{GS} + 1.96)$$

$$1.18 V_{GS}^2 - 2.31 V_{GS} - 2.68 = 0$$

$$V_{GS} = \frac{2.31 \pm \sqrt{5.336 + 12.65}}{2(1.18)} = 2.78 \text{ V}$$

$$I_D = (0.25)(2.78 - 1.4)^2 \Rightarrow I_D = 0.476 \text{ mA}$$

If $R_s = 4.3 - 10\% = 3.87 \text{ K}$

$$5 - V_{GS} = (0.9675)(V_{GS}^2 - 2.8V_{GS} + 1.96)$$

$$0.9675V_{GS}^2 - 1.71V_{GS} - 3.10 = 0$$

$$V_{GS} = \frac{1.71 \pm \sqrt{2.924 + 12.0}}{2(0.9675)} = 2.88 \text{ V}$$

$$I_D = (0.25)(2.88 - 1.4)^2 = 0.548 \text{ mA}$$

3.40

$$I_D = 0.10 = \frac{9 - 2.5}{R} \Rightarrow R = 65 \text{ k}\Omega$$

$$I_D = \left(\frac{k'_p}{2} \right) \left(\frac{W}{L} \right) (V_{SG} + V_{TP})^2$$

Now $V_{SG} = V_{SD} = 2.5 \text{ V}$

$$0.10 = \left(\frac{0.05}{2} \right) \left(\frac{W}{L} \right) (2.5 - 0.7)^2 \Rightarrow \left(\frac{W}{L} \right) = 1.235$$

And $W = (1.235)(0.8) = 0.988 \mu\text{m}$

3.41

$$5 = I_{DQ}R_s + V_{SDQ} = I_{DQ}(2) + 2.5$$

$$\underline{I_{DQ} = 1.25 \text{ mA}}$$

$$I_R = \frac{10}{R_1 + R_2} = (1.25)(0.1) \Rightarrow R_1 + R_2 = 80 \text{ k}\Omega$$

$$I_{DQ} = K_p (V_{SG} + V_{TP})^2$$

$$1.25 = 0.5(V_{SG} + 1.5)^2 \Rightarrow \sqrt{\frac{1.25}{0.5}} - 1.5 = V_{SG}$$

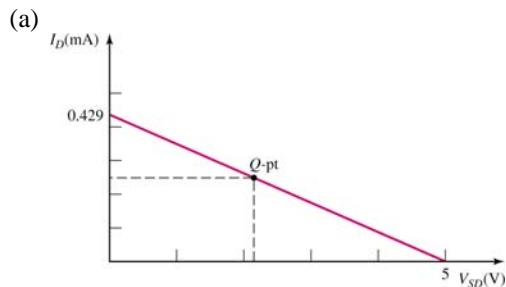
$$\underline{V_{SG} = 0.0811 \text{ V}}$$

$$V_G = V_S - V_{SG} = 2.5 - 0.0811 = 2.42 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5$$

$$2.42 = \left(\frac{R_2}{80} \right) (10) - 5 \Rightarrow R_2 = 59.4 \text{ k}\Omega, \quad R_1 = 20.6 \text{ k}\Omega$$

3.42



$$R_D = \frac{V_D - (-5)}{I_{DQ}} = \frac{5 - 2}{0.25} \Rightarrow R_D = 12 \text{ K}$$

$$I_D = \left(\frac{W}{L} \right) \left(\frac{k'_p}{2} \right) (V_{SG} + V_{TP})^2$$

$$0.25 = (15) \left(\frac{0.035}{2} \right) (V_{SG} - 1.2)^2 \Rightarrow V_{SG} = 2.18 \text{ V}$$

$$R_S = \frac{5 - 2.18}{0.25} \Rightarrow R_S = 11.3 \text{ K}$$

$$V_{SD} = 2.18 - (-2) = 4.18 \text{ V}$$

(b)

$$k'_p = 35 + 5\% = 36.75 \mu\text{A/V}^2$$

$$I_D = (15) \left(\frac{0.03675}{2} \right) (V_{SG} - 1.2)^2 = \frac{5 - V_{SG}}{11.3}$$

$$3.11(V_{SG}^2 - 2.4V_{SG} + 1.44) = 5 - V_{SG}$$

$$3.11V_{SG}^2 - 6.46V_{SG} - 0.522 = 0$$

$$V_{SG} = \frac{6.46 \pm \sqrt{41.73 + 6.49}}{2(3.11)} = 2.155 \text{ V}$$

$$I_D = \frac{5 - 2.155}{11.3} = 0.252 \text{ mA}$$

$$V_{SD} = 10 - (0.252)(12 + 11.3) = 4.13 \text{ V}$$

$$k'_p = 35 - 5\% = 33.25 \mu\text{A/V}^2$$

$$I_D = (15) \left(\frac{0.03325}{2} \right) (V_{SG} - 1.2)^2 = \frac{5 - V_{SG}}{11.3}$$

$$2.82(V_{SG}^2 - 2.4V_{SG} + 1.44) = 5 - V_{SG}$$

$$2.82V_{SG}^2 - 5.77V_{SG} - 0.939 = 0$$

$$V_{SG} = \frac{5.77 \pm \sqrt{33.29 + 10.59}}{2(2.82)} = 2.198 \text{ V}$$

$$I_D = \frac{5 - 2.198}{11.3} = 0.248 \text{ mA}$$

$$V_{SD} = 10 - (0.248)(12 + 11.3) = 4.22 \text{ V}$$

3.43

$$I_D = \frac{-V_{SD} - (-10)}{R_D} \Rightarrow 5 = \frac{-6 + 10}{R_D} \Rightarrow R_D = 0.8 \text{ k}\Omega$$

$$I_D = K_P (V_{SG} + V_{TP})^2 \Rightarrow 5 = 3(V_{SG} - 1.75)^2$$

$$V_{SG} = \sqrt{\frac{5}{3}} + 1.75 = 3.04 \text{ V} \Rightarrow V_G = -3.04$$

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5 = -3.04$$

$$R_m = R_1 \parallel R_2 = 80 \text{ k}\Omega$$

$$\frac{1}{R_1} \cdot (80)(10) = 5 - 3.04 \Rightarrow R_1 = 408 \text{ k}\Omega$$

$$\frac{408R_2}{408 + R_2} = 80 \Rightarrow R_2 = 99.5 \text{ k}\Omega$$

3.44

- (a) Both M_1 and M_2 in saturation

$$I_{D1} = I_{D2}$$

$$\frac{k'_n}{2}(4)(v_I - 0.4)^2 = \frac{k'_n}{2}(1)[0 - (-0.6)]^2 \Rightarrow v_I = 0.7 \text{ V}$$

- (b) (i) $v_I = 0.6 \text{ V}$; M_1 in saturation, M_2 in nonsaturation

$$I_{D1} = I_{D2}$$

$$\frac{k'_n}{2}(4)(0.6 - 0.4)^2 = \frac{k'_n}{2}(1)[2(0 - (-0.6))(5 - v_O) - (5 - v_O)^2]$$

$$\text{We find } (5 - v_O)^2 - 1.2(5 - v_O) + 0.16 = 0 \Rightarrow (5 - v_O) = 0.153$$

$$\text{So } v_O = 4.85 \text{ V}$$

- (ii) $v_I = 1.5 \text{ V}$; M_1 in nonsaturation, M_2 in saturation

$$I_{D1} = I_{D2}$$

$$\frac{k'_n}{2}(4)[2(1.5 - 0.4)v_O - v_O^2] = \frac{k'_n}{2}(1)(0 - (-0.6))^2$$

$$\text{We find } v_O^2 - 2.2v_O + 0.09 = 0 \Rightarrow v_O = 0.0417 \text{ V}$$

3.45

- M_1 in nonsaturation, M_2 in saturation

$$I_{D1} = I_{D2}$$

$$\frac{k'_n}{2} \left(\frac{W}{L} \right)_1 [2(3 - 0.4)(0.025) - (0.025)^2] = \frac{k'_n}{2}(1)[0 - (-0.6)]^2$$

$$\left(\frac{W}{L} \right)_1 (0.1294) = 0.36 \Rightarrow \left(\frac{W}{L} \right)_1 = 2.78$$

3.46

(a) Transistors matched

$$V_{GS1} = V_{GS2} = 2.5 \text{ V}, \quad V_o = 2.5 \text{ V}$$

$$I_D = \left(\frac{0.12}{2} \right) (30)(2.5 - 0.4)^2 = 7.938 \text{ mA}$$

(b) $I_{D1} = I_{D2}$

$$\frac{k'_n}{2} \left(\frac{W}{L} \right)_1 (V_{GS1} - V_{TN})^2 = \frac{k'_n}{2} \left(\frac{W}{L} \right)_2 (V_{GS2} - V_{TN})^2$$

$$V_{GS2} = 5 - V_{GS1}$$

$$\text{Then } \sqrt{\frac{30}{15}} (V_{GS1} - 0.4) = (5 - V_{GS1} - 0.4)$$

Which yields $V_{GS1} = 2.14 \text{ V}$, $V_{GS2} = V_o = 2.86 \text{ V}$

$$I_D = \left(\frac{0.12}{2} \right) (30)(2.14 - 0.4)^2 = 5.45 \text{ mA}$$

(c) $I_{D1} = I_{D2}$

$$(15)(V_{GS1} - 0.4)^2 = (30)(V_{GS2} - 0.4)^2$$

$$0.7071(V_{GS1} - 0.4) = 5 - V_{GS1} - 0.4$$

Which yields $V_{GS1} = 2.86 \text{ V}$, $V_{GS2} = V_o = 2.14 \text{ V}$

$$I_D = \left(\frac{0.12}{2} \right) (15)(2.86 - 0.4)^2 = 5.45 \text{ mA}$$

3.47

(a) $V_1 = V_{GS3} = 2.5 \text{ V}$

$$0.8 = \left(\frac{0.12}{2} \right) \left(\frac{W}{L} \right)_3 (2.5 - 0.6)^2 \Rightarrow \left(\frac{W}{L} \right)_3 = 3.69$$

$$V_2 = 6 = V_{GS2} + V_1 = V_{GS2} + 2.5 \Rightarrow V_{GS2} = 3.5 \text{ V}$$

$$0.8 = \left(\frac{0.12}{2} \right) \left(\frac{W}{L} \right)_2 (3.5 - 0.6)^2 \Rightarrow \left(\frac{W}{L} \right)_2 = 1.59$$

$$V_{GS1} = 9 - V_2 = 9 - 6 = 3 \text{ V}$$

$$0.8 = \left(\frac{0.12}{2} \right) \left(\frac{W}{L} \right)_1 (3 - 0.6)^2 \Rightarrow \left(\frac{W}{L} \right)_1 = 2.31$$

(b) (i) $k'_n = (120)(1.05) = 126 \mu \text{ A/V}^2$

No change; $V_1 = 2.5 \text{ V}$, $V_2 = 6 \text{ V}$

(ii) $k'_n = (120)(0.95) = 114 \mu \text{ A/V}^2$

No change; $V_1 = 2.5 \text{ V}$, $V_2 = 6 \text{ V}$

(c) $k'_{n1} = 114 \mu \text{ A/V}^2$, $k'_{n2} = k'_{n3} = 126 \mu \text{ A/V}^2$

$$I_{D2} = I_{D3}$$

$$\left(\frac{0.126}{2} \right) (1.59)(V_{GS2} - 0.6)^2 = \left(\frac{0.126}{2} \right) (3.69)(V_{GS3} - 0.6)^2$$

Now $V_{GS3} = V_1$, $V_{GS2} = V_2 - V_1$

So $(V_2 - V_1 - 0.6) = 1.523(V_1 - 0.6)$

And $V_2 = 2.523V_1 - 0.3138$

Also $I_{D1} = I_{D3}$

$$\left(\frac{0.114}{2}\right)(2.31)(V_{GS1} - 0.6)^2 = \left(\frac{0.126}{2}\right)(3.69)(V_{GS3} - 0.6)^2$$

We have $V_{GS1} = 9 - V_2 = 9 - (2.523V_1 - 0.3138) = 9.3138 - 2.523V_1$

Also $V_{GS3} = V_1$

Then $(0.13167)(9.3138 - 2.523V_1 - 0.6)^2 = (0.23247)(V_1 - 0.6)^2$

$8.7138 - 2.523V_1 = 1.3287(V_1 - 0.6)$

Which yields $V_1 = 2.469$ V and $V_2 = 5.916$ V

3.48

M_L in saturation, M_D in nonsaturation

$I_{DD} = I_{DL}$

$$\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_D [2(5 - 0.6)(0.15) - (0.15)^2] = \left(\frac{k'_n}{2}\right)(2)(5 - 0.15 - 0.6)^2$$

$$\left(\frac{W}{L}\right)_D (1.2975) = 36.125 \Rightarrow \left(\frac{W}{L}\right)_D = 27.8$$

3.49

M_L in saturation, M_D in nonsaturation

$I_{DD} = I_{DL}$

$$\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_D [2(5 - 0.6)(0.10) - (0.10)^2] = \left(\frac{k'_n}{2}\right)(2)[0 - (-1.2)]^2$$

$$\left(\frac{W}{L}\right)_D (0.87) = 2.88 \Rightarrow \left(\frac{W}{L}\right)_D = 3.31$$

3.50

(a) Transistor in nonsaturation

$$I_D = \frac{3 - 0.08}{30} = 0.09733 \text{ mA}$$

$$0.09733 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right) [2(2.6 - 0.4)(0.08) - (0.08)^2]$$

$$0.09733 = \left(\frac{W}{L}\right)(0.020736) \Rightarrow \left(\frac{W}{L}\right) = 4.69$$

(b) $0.09733 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right) [2(3 - 0.4)(0.08) - (0.08)^2]$

$$0.09733 = \left(\frac{W}{L}\right)(0.024576) \Rightarrow \left(\frac{W}{L}\right) = 3.96$$

3.51

$$5 = I_D R_D + V_\gamma + V_{DS}$$

$$5 = (12)R_D + 1.6 + 0.15 \Rightarrow R_D = 271\Omega$$

$$I_D = 12 = \left(\frac{0.08}{2}\right) \frac{W}{L} [2(5 - 0.6)(0.15) - (0.15)^2] \Rightarrow \left(\frac{W}{L}\right) = 231$$

3.52

$$5 = V_{SD} + I_D R_D + V_\gamma$$

$$5 = 0.20 + (15)R_D + 1.6 \Rightarrow R_D = 213\Omega$$

$$I_D = 15 = \left(\frac{0.04}{2}\right) \left(\frac{W}{L}\right) [2(5 - 0.6)(0.20) - (0.20)^2]$$

$$15 = \left(\frac{W}{L}\right)(0.0344) \Rightarrow \left(\frac{W}{L}\right) = 436$$

3.53

(a) $I_R = \frac{5 - 0.15}{50} = 0.097 \text{ mA}$

$$I_D = \frac{0.097}{2} = \left(\frac{0.12}{2}\right) \left(\frac{W}{L}\right) [2(5 - 0.6)(0.15) - (0.15)^2]$$

$$0.0485 = \left(\frac{W}{L}\right)(0.07785) \Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 0.623$$

(b) M_1 nonsaturation, M_2 cutoff

$$I_R = I_D = \frac{5 - V_o}{50} = \left(\frac{0.12}{2}\right)(0.623) [2(5 - 0.6)V_o - V_o^2]$$

$$\text{We find } 1.869V_o^2 - 17.45V_o + 5 = 0 \Rightarrow V_o = 0.297 \text{ V}$$

3.54

(a) $V_{DS2}(\text{sat}) = 0.5 = V_{GS2} - V_{TN} = V_{GS2} - 0.4 \Rightarrow V_{GS2} = 0.9 \text{ V}$

$$I_{Q1} = 125 = \left(\frac{120}{2}\right) \left(\frac{W}{L}\right)_2 (0.9 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 8.33$$

$$V_{GS3} = V_{GS2} = 0.9 \text{ V}$$

$$I_{REF1} = 225 = \left(\frac{120}{2}\right) \left(\frac{W}{L}\right)_3 (0.9 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 15$$

$$M_1 \text{ and } M_2 \text{ matched, so } \left(\frac{W}{L}\right)_1 = 8.33$$

(b) $V_{GS1} = 0.9 \text{ V} \Rightarrow V_{D1} = V_{DS1} - V_{GS1} = 2 - 0.9 = 1.1 \text{ V}$

$$R_D = \frac{2.5 - 1.1}{0.125} = 11.2 \text{ k}\Omega$$

3.55

$$(a) V_{SDB}(\text{sat}) = 0.8 = V_{SGB} + V_{TP} = V_{SGB} - 0.4 \Rightarrow V_{SGB} = 1.2 \text{ V}$$

$$I_{Q2} = 200 = \left(\frac{50}{2} \right) \left(\frac{W}{L} \right)_B (1.2 - 0.4)^2 \Rightarrow \left(\frac{W}{L} \right)_B = 12.5$$

$$V_{SGC} = V_{SGB} = 1.2 \text{ V}$$

$$I_{REF2} = 125 = \left(\frac{50}{2} \right) \left(\frac{W}{L} \right)_C (1.2 - 0.4)^2 \Rightarrow \left(\frac{W}{L} \right)_C = 7.81$$

$$M_A \text{ and } M_B \text{ matched, so } \left(\frac{W}{L} \right)_A = 12.5$$

$$(b) V_{SGA} = 1.2 \text{ V}$$

$$V_{DA} = V_{SGA} - V_{SDA} = 1.2 - 4 = -2.8 \text{ V}$$

$$R_D = \frac{-2.8 - (-5)}{0.20} = 11 \text{ k}\Omega$$

3.56

$$V_{DS2}(\text{sat}) = 0.5 = V_{GS2} - V_{TN} = V_{GS2} - 0.6 \Rightarrow V_{GS2} = 1.1 \text{ V}$$

$$I_Q = 0.35 = \left(\frac{0.12}{2} \right) \left(\frac{W}{L} \right)_2 (1.1 - 0.6)^2 \Rightarrow \left(\frac{W}{L} \right)_2 = 23.3$$

$$V_{GS3} = V_{GS2} = 1.1 \text{ V}$$

$$I_{REF} = 0.15 = \left(\frac{0.12}{2} \right) \left(\frac{W}{L} \right)_3 (1.1 - 0.6)^2 \Rightarrow \left(\frac{W}{L} \right)_3 = 10$$

$$V_{GS4} = 5 - 1.1 = 3.9 \text{ V}$$

$$I_{REF} = 0.15 = \left(\frac{0.12}{2} \right) \left(\frac{W}{L} \right)_4 (3.9 - 0.6)^2 \Rightarrow \left(\frac{W}{L} \right)_4 = 0.23$$

$$M_1 \text{ and } M_2 \text{ matched, so } \left(\frac{W}{L} \right)_1 = 23.3$$

$$V_{GS1} = 1.1 \text{ V}; V_{D1} = V_{DS1} - V_{GS1} = 3.5 - 1.1 = 2.4 \text{ V}$$

$$R_D = \frac{5 - 2.4}{0.35} = 7.43 \text{ k}\Omega$$

3.57

$$V_{DS}(\text{sat}) = V_{GS} - V_P$$

$$\text{So } \frac{V_{DS}}{V_{DS}(\text{sat})} > V_{DS}(\text{sat}) = -V_P, \underline{I_D = I_{DSS}}$$

3.58

$$V_{DS}(\text{sat}) = V_{GS} - V_P = \underline{V_{GS} + 3 = V_{DS}(\text{sat})}$$

$$\text{a. } V_{GS} = 0 \Rightarrow \underline{I_D = I_{DSS} = 6 \text{ mA}}$$

$$\text{b. } I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2 = 6 \left(1 - \frac{-1}{-3} \right)^2 \Rightarrow \underline{I_D = 2.67 \text{ mA}}$$

c. $I_D = 6 \left(1 - \frac{-2}{-3}\right)^2 \Rightarrow I_D = 0.667 \text{ mA}$

d. $\underline{I_D = 0}$

3.59

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_p}\right)^2$$

$$2.8 = I_{DSS} \left(1 - \frac{1}{V_p}\right)^2$$

$$0.30 = I_{DSS} \left(1 - \frac{3}{V_p}\right)^2$$

$$\frac{2.8}{0.30} = \frac{\left(1 - \frac{1}{V_p}\right)^2}{\left(1 - \frac{3}{V_p}\right)^2} = 9.33$$

$$\frac{\left(1 - \frac{1}{V_p}\right)}{\left(1 - \frac{3}{V_p}\right)} = 3.055$$

$$1 - \frac{1}{V_p} = 3.055 - \frac{9.165}{V_p}$$

$$\frac{8.165}{V_p} = 2.055 \Rightarrow V_p = 3.97 \text{ V}$$

$$2.8 = I_{DSS} \left(1 - \frac{1}{3.97}\right)^2 = I_{DSS} (0.560) \Rightarrow I_{DSS} = 5.0 \text{ mA}$$

3.60

$$V_S = -V_{GS}, V_{SD} = V_S - V_{DD}$$

Want $V_{SD} \geq V_{SD}(\text{sat}) = V_p - V_{GS}$

$$V_S - V_{DD} \geq V_p - V_{GS} - V_{GS} - V_{DD} \geq V_p - V_{GS} \Rightarrow V_{DD} \leq -V_p$$

So $\underline{V_{DD} \leq -2.5 \text{ V}}$

$$I_D = 2 = I_{DSS} \left(1 - \frac{V_{GS}}{V_p}\right)^2$$

$$2 = 6 \left(1 - \frac{V_{GS}}{2.5}\right)^2 \Rightarrow V_{GS} = 1.06 \text{ V} \Rightarrow \underline{V_S = -1.06 \text{ V}}$$

3.61

$$I_D = K_n (V_{GS} - V_{TN})^2$$

$$18.5 = K_n (0.35 - V_{TN})^2$$

$$86.2 = K_n (0.5 - V_{TN})^2$$

Then

$$\frac{18.5}{86.2} = 0.2146 = \frac{(0.35 - V_{TN})^2}{(0.50 - V_{TN})^2} \Rightarrow \underline{V_{TN} = 0.221 \text{ V}}$$

$$18.5 = K_n (0.35 - 0.221)^2 \Rightarrow \underline{K_n = 1.11 \text{ mA/V}^2}$$

3.62

$$I_D = K (V_{GS} - V_{TN})^2$$

$$250 = K (0.75 - 0.24)^2 \Rightarrow \underline{K = 0.961 \text{ mA/V}^2}$$

3.63

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_p} \right)^2 = \frac{V_s}{R_s} = -\frac{V_{GS}}{R_s}$$

$$10 \left(1 - \frac{V_{GS}}{-5} \right)^2 = -\frac{V_{GS}}{0.2}$$

$$2 \left(1 + \frac{2V_{GS}}{5} + \frac{V_{GS}^2}{25} \right) = -V_{GS}$$

$$\frac{2}{25} V_{GS}^2 + \frac{9}{5} V_{GS} + 2 = 0$$

$$2V_{GS}^2 + 45V_{GS} + 50 = 0$$

$$V_{GS} = \frac{-45 \pm \sqrt{(45)^2 - 4(2)(50)}}{2(2)} \Rightarrow \underline{V_{GS} = -1.17 \text{ V}}$$

$$I_D = -\frac{V_{GS}}{R_s} = \frac{1.17}{0.2} \Rightarrow \underline{I_D = 5.85 \text{ mA}}$$

$$V_D = 20 - (5.85)(2) = 8.3 \text{ V}$$

$$V_{DS} = V_D - V_s = 8.3 - 1.17 \Rightarrow \underline{V_{DS} = 7.13 \text{ V}}$$

3.64

$$V_{DS} = V_{DD} - V_s \\ 8 = 10 - V_s \Rightarrow V_s = 2 \text{ V} = I_D R_s = (5) R_s \Rightarrow \underline{R_s = 0.4 \text{ k}\Omega}$$

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_p} \right)^2$$

$$5 = I_{DSS} \left(1 - \frac{-1}{V_p} \right)^2 \text{ Let } \underline{I_{DSS} = 10 \text{ mA}}$$

$$5 = 10 \left(1 - \frac{-1}{V_p} \right)^2 \Rightarrow \underline{V_p = -3.41 \text{ V}}$$

$$V_G = V_{GS} + V_s = -1 + 2 = 1 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$1 = \frac{1}{R_1} (500)(10) \Rightarrow \underline{R_1 = 5 \text{ M}\Omega}$$

$$\frac{5R_2}{5 + R_2} = 0.5 \Rightarrow \underline{R_2 = 0.556 \text{ M}\Omega}$$

3.65

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_p} \right)^2$$

$$5 = 8 \left(1 - \frac{V_{GS}}{4} \right)^2 \Rightarrow \underline{V_{GS} = 0.838 \text{ V}}$$

$$V_{SD} = V_{DD} - I_D (R_s + R_D) \\ = 20 - (5)(0.5 + 2) \Rightarrow \underline{V_{SD} = 7.5 \text{ V}}$$

$$V_s = 20 - (5)(0.5) = 17.5 \text{ V}$$

$$V_G = V_s + V_{GS} = 17.5 + 0.838 = 18.3 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$18.3 = \frac{1}{R_1} (100)(20) \Rightarrow \underline{R_1 = 109 \text{ k}\Omega}$$

$$\frac{109R_2}{109 + R_2} = 100 \Rightarrow \underline{R_2 = 1.21 \text{ M}\Omega}$$

3.66

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2$$

$$5 = 7 \left(1 - \frac{V_{GS}}{3} \right)^2 \Rightarrow V_{GS} = 0.465 \text{ V}$$

$$V_{SD} = V_{DD} - I_D (R_S + R_D)$$

$$6 = 12 - (5)(0.3 + R_D) \Rightarrow R_D = 0.9 \text{ k}\Omega$$

$$V_S = 12 - (5)(0.3) = 10.5 \text{ V}$$

$$V_G = V_S + V_{GS} = 10.5 + 0.465 = 10.965 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD}$$

$$10.965 = \left(\frac{R_2}{100} \right) (12) \Rightarrow R_2 = 91.4 \text{ k}\Omega \Rightarrow R_1 = 8.6 \text{ k}\Omega$$

3.67

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \left(\frac{60}{140 + 60} \right) (20) \Rightarrow V_G = 6 \text{ V}$$

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2 = \frac{V_S}{R_S} = \frac{V_G - V_{GS}}{R_S}$$

$$(8)(2) \left(1 - \frac{V_{GS}}{(-4)} \right)^2 = 6 - V_{GS}$$

$$16 \left(1 + \frac{V_{GS}}{2} + \frac{V_{GS}^2}{16} \right) = 6 - V_{GS}$$

$$V_{GS}^2 + 9V_{GS} + 10 = 0$$

$$V_{GS} = \frac{-9 \pm \sqrt{(9)^2 - 4(10)}}{2} \Rightarrow V_{GS} = -1.30$$

$$I_D = 8 \left(1 - \frac{(-1.30)}{(-4)} \right)^2 \Rightarrow I_D = 3.65 \text{ mA}$$

$$V_{DS} = V_{DD} - I_D (R_S + R_D)$$

$$= 20 - (3.65)(2 + 2.7)$$

$$V_{DS} = 2.85 \text{ V}$$

$$V_{DS} > V_{DS} (\text{sat}) = V_{GS} - V_P$$

$$= -1.30 - (-4)$$

$$= 2.7 \text{ V (Yes)}$$

3.68

$$\begin{aligned} V_{DS} &= V_{DD} - I_D (R_S + R_D) \\ 5 &= 12 - I_D (0.5 + 1) \Rightarrow I_D = 4.67 \text{ mA} \\ V_S &= I_D R_S = (4.67)(0.5) \Rightarrow V_S = 2.33 \text{ V} \\ V_G &= \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \left(\frac{20}{450 + 20} \right) (12) \Rightarrow V_G = 0.511 \text{ V} \\ V_{GS} &= V_G - V_S = 0.511 - 2.33 \Rightarrow V_{GS} = -1.82 \text{ V} \\ I_D &= I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2 \\ 4.67 &= 10 \left(1 - \frac{(-1.82)}{V_P} \right)^2 \Rightarrow V_P = -5.75 \text{ V} \end{aligned}$$

3.69

$$\begin{aligned} I_D &= I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2, \quad V_{GS} = 0 \\ I_D &= I_{DSS} = 4 \text{ mA} \\ R_D &= \frac{V_{DD} - V_{DS}}{I_D} = \frac{10 - 3}{4} \Rightarrow R_D = 1.75 \text{ k}\Omega \end{aligned}$$

3.70

$$\begin{aligned} V_{SD} &= V_{DD} - I_D R_S \\ 10 &= 20 - (1) R_S \Rightarrow R_S = 10 \text{ k}\Omega \\ R_1 + R_2 &= \frac{V_{DD}}{I} = \frac{20}{0.1} = 200 \text{ k}\Omega \\ I_D &= I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2 \\ 1 &= 2 \left(1 - \frac{V_{GS}}{2} \right)^2 \Rightarrow V_{GS} = 0.586 \text{ V} \\ V_G &= V_S + V_{GS} = 10 + 0.586 = 10.586 \\ V_G &= \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} \\ 10.586 &= \left(\frac{R_2}{200} \right) (20) \Rightarrow R_2 = 106 \text{ k}\Omega \\ R_1 &= 94 \text{ k}\Omega \end{aligned}$$

3.71

$$V_{DS} = V_{DD} - I_D (R_S + R_D)$$

$$2 = 3 - (0.040)(10 + R_D) \Rightarrow \underline{R_D = 15 \text{ k}\Omega}$$

$$I_D = K(V_{GS} - V_{TN})^2$$

$$40 = 250(V_{GS} - 0.20)^2 \Rightarrow \underline{V_{GS} = 0.60 \text{ V}}$$

$$V_G = V_{GS} + V_S = 0.60 + (0.040)(10) = 1.0 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD}$$

$$1 = \left(\frac{R_2}{150} \right) (3) \Rightarrow \underline{R_2 = 50 \text{ k}\Omega}$$

$$\underline{R_1 = 100 \text{ k}\Omega}$$

3.72

$$V_O = 0.70 \text{ V} \Rightarrow V_{DS} = 0.70 > V_{DS} (\text{sat}) = V_{GS} - V_{TN}$$

For $0.75 - 0.15 = 0.6$

Biased in the saturation region

$$I_D = \frac{V_{DD} - V_{DS}}{R_D} = \frac{3 - 0.7}{50} \Rightarrow \underline{I_D = 46 \mu\text{A}}$$

$$I_D = K(V_{GS} - V_{TN})^2 \Rightarrow 46 = K(0.75 - 0.15)^2 \Rightarrow \underline{K = 128 \mu\text{A}/\text{V}^2}$$

Chapter 4

4.1

$$(a) \quad (i) \quad g_m = 2\sqrt{\frac{k'_n}{2} \left(\frac{W}{L} \right) I_{DQ}}$$

$$0.5 = 2\sqrt{\left(\frac{0.1}{2} \right) \left(\frac{W}{L} \right) (0.5)} \Rightarrow \left(\frac{W}{L} \right) = 2.5$$

$$(ii) \quad I_{DQ} = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) (V_{GSQ} - V_{TN})^2$$

$$0.5 = \left(\frac{0.1}{2} \right) (2.5) (V_{GSQ} - 0.4)^2 \Rightarrow V_{GSQ} = 2.4 \text{ V}$$

$$(b) \quad (i) \quad 0.5 = 2\sqrt{\left(\frac{0.1}{2} \right) \left(\frac{W}{L} \right) (0.15)} \Rightarrow \left(\frac{W}{L} \right) = 8.33$$

$$(ii) \quad 0.15 = \left(\frac{0.1}{2} \right) (8.33) (V_{GSQ} - 0.4)^2 \Rightarrow V_{GSQ} = 1.0 \text{ V}$$

4.2

$$(a) \quad (i) \quad g_m = 2\sqrt{\left(\frac{k'_p}{2} \right) \left(\frac{W}{L} \right) I_{DQ}}$$

$$1.2 = 2\sqrt{\left(\frac{0.04}{2} \right) \left(\frac{W}{L} \right) (0.15)} \Rightarrow \left(\frac{W}{L} \right) = 120$$

$$(ii) \quad I_{DQ} = \left(\frac{k'_p}{2} \right) \left(\frac{W}{L} \right) (V_{SGQ} + V_{TP})^2$$

$$0.15 = \left(\frac{0.04}{2} \right) (120) (V_{SGQ} - 0.6)^2 \Rightarrow V_{SGQ} = 0.85 \text{ V}$$

$$(b) \quad (i) \quad 1.2 = 2\sqrt{\left(\frac{0.04}{2} \right) \left(\frac{W}{L} \right) (0.50)} \Rightarrow \left(\frac{W}{L} \right) = 36$$

$$(ii) \quad 0.50 = \left(\frac{0.04}{2} \right) (36) (V_{SGQ} - 0.6)^2 \Rightarrow V_{SGQ} = 1.43 \text{ V}$$

4.3

$$I_D = K_n (V_{GS} - V_{TN})^2 (1 + \lambda V_{DS})$$

$$\frac{I_{D1}}{I_{D2}} = \frac{1 + \lambda V_{DS1}}{1 + \lambda V_{DS2}} \Rightarrow \frac{3.4}{3.0} = \frac{1 + \lambda(10)}{1 + \lambda(5)}$$

$$3.4[1 + 5\lambda] = 3.0[1 + 10\lambda]$$

$$3.4 - 3.0 = \lambda(3 \cdot 10 - (3.4) \cdot 5) \Rightarrow \underline{\lambda = 0.0308}$$

$$r_o = \frac{\Delta V_{DS}}{\Delta I_D} = \frac{5}{0.4} = 12.5 \text{ k}\Omega$$

4.4

$$r_o = \frac{1}{\lambda I_D}$$

$$I_D = \frac{1}{\lambda r_o} = \frac{1}{(0.012)(100)} \Rightarrow I_D = 0.833 \text{ mA}$$

4.5

$$(a) \quad I_D = K_n (V_{GS} - V_{TN})^2 (1 + \lambda V_{DS})$$

$$I_D = I_{DO} (1 + \lambda V_{DS})$$

$$\text{Then } \frac{0.258}{0.250} = \frac{(1 + 3.3\lambda)}{(1 + 1.5\lambda)}$$

$$\text{Or } 1.032(1 + 1.5\lambda) = 1 + 3.3\lambda \Rightarrow \lambda = 0.01826 \text{ V}^{-1}$$

$$0.250 = I_{DO} [1 + (0.01826)(1.5)] \Rightarrow I_{DO} = 0.2433 \text{ mA}$$

$$r_o = \frac{1}{\lambda I_{DO}} = \frac{1}{(0.01826)(0.2433)} = 225 \text{ k}\Omega$$

$$(b) \quad I_D = (0.2433)[1 + (0.01826)(5)] = 0.2655 \text{ mA}$$

4.6

(a)

$$(i) \quad r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.02)(0.05)} = 1000 \text{ K}$$

$$(ii) \quad r_o = \frac{1}{(0.02)(0.5)} = 100 \text{ K}$$

(b)

$$(i) \quad \Delta I_D = \frac{\Delta V_{DS}}{r_o} = \frac{1}{1000} = 0.001 \text{ mA} = 1.0 \mu\text{A}$$

$$\frac{\Delta I_D}{I_D} = \frac{1.0}{50} \Rightarrow 2\%$$

$$\Delta I_D = \frac{\Delta V_{DS}}{r_o} = \frac{1}{100} = 0.01 \Rightarrow 10 \mu\text{A}$$

$$(ii) \quad \frac{\Delta I_D}{I_D} = \frac{10}{500} \Rightarrow 2\%$$

4.7

$$I_D = 1.0 \text{ mA}$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.01)(1)} = 100 \text{ K}$$

4.8

(a) $V_{DD} = I_{DQ}R_D + V_{DSQ}$
 $3.3 = I_{DQ}(5) + 1.5 \Rightarrow I_{DQ} = 0.36 \text{ mA}$

$$I_{DQ} = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) (V_{GSQ} - V_{TN})^2$$

$$0.36 = \left(\frac{0.1}{2} \right) (40) (V_{GSQ} - 0.4)^2 \Rightarrow V_{GSQ} = 0.824 \text{ V}$$

(b) $K_n = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) = \left(\frac{0.1}{2} \right) (40) = 2 \text{ mA/V}^2$
 $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(2)(0.36)} = 1.697 \text{ mA/V}$
 $r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.025)(0.36)} = 111.1 \text{ k}\Omega$
 $A_v = -g_m(r_o \| R_D) = -(1.697)(111.1 \| 5) = -8.12$

4.9

(a) $A_v = -g_m R_D$
 $-3.8 = -g_m(10) \Rightarrow g_m = 0.38 \text{ mA/V}$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{\left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) I_{DQ}}$$

$$0.38 = 2\sqrt{\left(\frac{0.1}{2} \right) \left(\frac{W}{L} \right) (0.12)} \Rightarrow \left(\frac{W}{L} \right) = 6.02$$

(b) $-5 = -g_m(10) \Rightarrow g_m = 0.50 \text{ mA/V}$
 $0.50 = 2\sqrt{\left(\frac{0.1}{2} \right) \left(\frac{W}{L} \right) (0.12)} \Rightarrow \left(\frac{W}{L} \right) = 10.4$

4.10

(a) $V_{DD} = I_{DQ}R_D + V_{DSQ}$
 $5 = (0.5)R_D + 3 \Rightarrow R_D = 4 \text{ k}\Omega$
 $I_{DQ} = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) (V_{GSQ} - V_{TN})^2$
 $0.5 = \left(\frac{0.08}{2} \right) \left(\frac{W}{L} \right) (1.2 - 0.6)^2 \Rightarrow \left(\frac{W}{L} \right) = 34.7$

(b) $g_m = 2\sqrt{\left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) I_{DQ}} = 2\sqrt{\left(\frac{0.08}{2} \right) (34.7)(0.5)} = 1.666 \text{ mA/V}$
 $r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.015)(0.5)} = 133.3 \text{ k}\Omega$
(c) $A_v = -g_m(r_o \| R_D) = -(1.666)(133.3 \| 4) = -6.47$

4.11

$$K_n v_{gs}^2 = K_n [V_{gs} \sin \omega t]^2 = K_n V_{gs}^2 \sin^2 \omega t$$

$$\sin^2 \omega t = \frac{1}{2} [1 - \cos 2\omega t]$$

$$\text{So } K_n v_{gs}^2 = \frac{K_n V_{gs}^2}{2} [1 - \cos 2\omega t]$$

$$\text{Ratio of signal at } 2\omega \text{ to that at } \omega : \frac{\frac{K_n V_{gs}^2}{2} \cdot \cos 2\omega t}{2K_n (V_{GSQ} - V_{TN}) V_{gs} \cdot \sin \omega t}$$

$$\text{The coefficient of this expression is then: } \frac{V_{gs}}{4(V_{GSQ} - V_{TN})}$$

4.12

$$0.01 = \frac{V_{gs}}{4(V_{GSQ} - V_{TN})}$$

$$\text{So } V_{gs} = (0.01)(4)(3-1) \Rightarrow V_{gs} = 0.08 \text{ V}$$

4.13

$$(a) \quad V_{GS} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{DD} = \left(\frac{60}{60 + 240} \right) (3.3) = 0.66 \text{ V}$$

$$I_{DQ} = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) (V_{GS} - V_{TN})^2 = \left(\frac{0.1}{2} \right) (80) (0.66 - 0.4)^2 = 0.270 \text{ mA}$$

$$V_{DSQ} = V_{DD} - I_{DQ} R_D = 3.3 - (0.270)(8) = 1.14 \text{ V}$$

$$(b) \quad g_m = 2 \sqrt{\left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) I_{DQ}} = 2 \sqrt{\left(\frac{0.1}{2} \right) (80) (0.270)} = 2.078 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.27)} = 185 \text{ k}\Omega$$

$$(c) \quad A_v = -g_m (r_o \| R_D) \left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_{Si}} \right)$$

$$\text{We find } r_o \| R_D = 185 \| 8 = 7.668 \text{ k}\Omega$$

$$R_1 \| R_2 = 60 \| 240 = 48 \text{ k}\Omega$$

$$\text{So } A_v = -(2.078)(7.668) \left(\frac{48}{48 + 2} \right) = -15.3$$

4.14

$$A_v = -g_m (r_o \| R_D)$$

$$-10 = -g_m (100 \| 5) \Rightarrow g_m = 2.1 \text{ mA/V}$$

4.15

$$\begin{aligned}
 \text{(a)} \quad V_G &= \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{DD} = \left(\frac{175}{175 + 225} \right) (5) = 2.1875 \text{ V} \\
 2.1875 &= V_{GS} + I_D R_S = V_{GS} + K_n R_S (V_{GS} - V_{TN})^2 \\
 2.1875 &= V_{GS} + (1)(1)(V_{GS}^2 - 1.6V_{GS} + 0.64) \\
 \text{or } V_{GS}^2 - 0.6V_{GS} - 1.5475 &= 0 \Rightarrow V_{GS} = 1.58 \text{ V} \\
 I_{DQ} &= K_n (V_{GS} - V_{TN})^2 = (1)(1.58 - 0.8)^2 = 0.608 \text{ mA} \\
 V_{DSQ} &= V_{DD} - I_{DQ}(R_S + R_D) = 5 - (0.608)(1+4) = 1.96 \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad A_v &= \frac{-g_m R_D}{1 + g_m R_S} \\
 g_m &= 2\sqrt{(1)(0.608)} = 1.56 \text{ mA/V} \\
 A_v &= \frac{-(1.56)(4)}{1 + (1.56)(1)} = -2.44
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad A_v &= \frac{-g_m (R_D \| R_L)}{1 + g_m R_S} = \frac{-(1.56)(R_D \| R_L)}{1 + (1.56)(1)} = -0.6094(R_D \| R_L) \\
 -(0.75)(2.44) &= -0.6094(R_D \| R_L) \Rightarrow R_D \| R_L = 3.0 \text{ k}\Omega \\
 4 \| R_L &= 3 \Rightarrow R_L = 12 \text{ k}\Omega
 \end{aligned}$$

4.16

$$\begin{aligned}
 \text{(a)} \quad V_{DSQ} &= V_{DD} - I_{DQ}(R_S + R_D) \\
 5 &= 12 - (2)(R_S + R_D) \Rightarrow R_S + R_D = 3.5 \text{ k}\Omega \\
 R_S &= 0.5 \text{ k}\Omega, \text{ then } R_D = 3 \text{ k}\Omega \\
 I_{DQ} &= K_n (V_{GSQ} - V_{TN})^2 \\
 2 &= 1.5(V_{GSQ} - 1.2)^2 \Rightarrow V_{GSQ} = 2.355 \text{ V} \\
 V_G &= V_{GSQ} + I_{DQ} R_S = 2.355 + (2)(0.5) = 3.355 \text{ V} \\
 V_G &= \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD} \\
 3.355 &= \frac{1}{R_1} (250)(12) \Rightarrow R_1 = 894 \text{ k}\Omega \\
 R_1 \| R_2 &= 894 \| R_2 = 250 \Rightarrow R_2 = 347 \text{ k}\Omega
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad g_m &= 2\sqrt{(1.5)(2)} = 3.464 \text{ mA/V} \\
 A_v &= \frac{-g_m (R_D \| R_L)}{1 + g_m R_S} = \frac{-(3.464)(3 \| 10)}{1 + (3.464)(0.5)} = -2.93
 \end{aligned}$$

4.17

- From Problem 4.16;
 $R_S = 0.5 \text{ k}\Omega$, $R_D = 3 \text{ k}\Omega$,
 $R_1 = 894 \text{ k}\Omega$, $R_2 = 347 \text{ k}\Omega$

$$(b) \quad g_m = 2\sqrt{(1.5)(2)} = 3.464 \text{ mA/V}$$

$$A_v = -g_m(R_D \| R_L) = -(3.464)(3 \| 10) = -7.99$$

4.18

(a)

$$A_v = -g_m R_D$$

$$-15 = -2R_D \Rightarrow \underline{R_D = 7.5 \text{ K}}$$

(b)

$$A_v = \frac{-g_m R_D}{1 + g_m R_s}$$

$$-5 = \frac{-(2)(7.5)}{1 + (2)R_s} \Rightarrow \underline{R_s = 1 \text{ K}}$$

4.19

$$(a) \quad A_v = \frac{-g_m R_D}{1 + g_m R_s}$$

$$(1) \quad -8 = \frac{-g_m R_D}{1 + g_m (1)}$$

$$(2) \quad -16 = -g_m R_D$$

$$8 = \frac{16}{1 + g_m (1)} \Rightarrow \underline{g_m = 1 \text{ mA/V}}$$

$$\text{Then} \quad \underline{R_D = 16 \text{ K}}$$

$$A_v = -10 = \frac{-(1)(16)}{1 + (1)R_s}$$

$$(b) \quad \underline{R_s = 0.6 \text{ K}}$$

4.20

$$(a) \quad I_{DQ} = I_Q = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) (V_{GSQ} - V_{TN})^2$$

$$0.5 = \left(\frac{0.1}{2} \right) (50) (V_{GSQ} - 0.8)^2 \Rightarrow V_{GSQ} = 1.247 \text{ V}$$

$$V_{DSQ} = V^+ - I_{DQ} R_D - (-V_{GSQ}) = 5 - (0.5)(6) + 1.247 = 3.25 \text{ V}$$

$$(b) \quad g_m = 2 \sqrt{\left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) I_{DQ}} = 2 \sqrt{\left(\frac{0.1}{2} \right) (50)(0.5)} = 2.236 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.5)} = 100 \text{ k}\Omega$$

$$A_v = -g_m (r_o \| R_D) = -(2.236)(100 \| 6) = -12.7$$

$$(c) \quad A_v = -g_m (r_o \| R_D \| R_L) = -(2.236)(100 \| 6 \| 20) = -9.86$$

$$(d) \quad A_v = -g_m (r_o \| R_D \| R_L) = -(2.236)(100 \| 6 \| 6) = -6.51$$

4.21

$$(a) \quad V_{DSQ} = 5 - (-5) - I_{DQ}(R_s + R_D)$$

$$5.5 = 10 - (0.1)(R_s + R_D) \Rightarrow R_s + R_D = 45 \text{ k}\Omega$$

$$I_{DQ} = K_n (V_{GSQ} - V_{TN})^2$$

$$(0.1) = (0.85)(V_{GSQ} - 0.8)^2 \Rightarrow V_{GSQ} = 1.143 \text{ V}$$

$$V_{GS} + I_D R_s = 5$$

$$1.143 + (0.1)R_s = 5 \Rightarrow R_s = 38.6 \text{ k}\Omega$$

$$\Rightarrow R_D = 6.43 \text{ k}\Omega$$

$$(b) \quad g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.85)(0.1)} = 0.583 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$(c) \quad A_v = -g_m (R_D \| r_o \| R_L) = -(0.583)(6.43 \| 500 \| 40) = -3.19$$

4.22

$$(a) \quad V_{DSQ} = V_{DD} - I_{DQ}(R_s + R_D)$$

$$2 = 3.3 - (0.5)(R_s + R_D) \Rightarrow R_s + R_D = 2.6 \text{ k}\Omega$$

$$I_{DQ} = K_n (V_{GSQ} - V_{TN})^2$$

$$0.5 = 2(V_{GSQ} - (-0.8))^2 \Rightarrow V_{GSQ} = -0.3 \text{ V}$$

$$V_{GSQ} + I_{DQ} R_s = 0$$

$$-0.3 + (0.5)R_s = 0 \Rightarrow R_s = 0.6 \text{ k}\Omega$$

$$\Rightarrow R_D = 2 \text{ k}\Omega$$

$$(b) \quad g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(2)(0.5)} = 2 \text{ mA/V}$$

$$A_v = \frac{-g_m (R_D \| R_L)}{1 + g_m R_s} = \frac{-(2)(2 \| 10)}{1 + (2)(0.6)} = -1.52$$

4.23

$$(a) \quad V_{DD} = I_{DQ} R_D + V_{DSQ} + I_{DQ} R_s \quad \text{and} \quad V_{GS} + I_{DQ} R_s = 0$$

Then $V_{DD} = K_n R_D (V_{GS} - V_{TN})^2 + V_{DSQ} - V_{GS}$

$$5 = (2)(2)(V_{GS} + 0.8)^2 + 2.5 - V_{GS}$$

Which yields

$$4V_{GS}^2 + 5.4V_{GS} + 0.06 = 0, \Rightarrow V_{GS} = -0.0112 \text{ V}$$

and $I_{DQ} = 2(-0.0112 + 0.8)^2 = 1.244 \text{ mA}$

$$5 = (1.244)(2) + 2.5 + (1.244)R_s \Rightarrow R_s = 8.99 \Omega \approx 9 \Omega$$

$$(b) \ g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(2)(1.244)} = 3.155 \text{ mA/V}$$

$$A_v = \frac{-g_m(R_D \| R_L)}{1 + g_m R_S} = \frac{-(3.155)(2 \| 2)}{1 + (3.155)(0.009)} = -3.07$$

4.24

a.

$$5 = I_{DQ}R_s + V_{SDQ} + I_{DQ}R_D - 5$$

$$5 = I_{DQ}R_s + 6 + I_{DQ}(10) - 5$$

$$\underline{I_{DQ} = \frac{4}{R_s + 10}}$$

$$1. \quad V_s = V_{SDQ} + I_{DQ}R_b - 5 = V_{SGQ}$$

$$2. \quad 1 + I_{DQ}(10) = V_{SGQ}$$

$$3. \quad I_{DQ} = K_p(V_{SGQ} - 2)^2$$

$$\text{Choose } \underline{R_s = 10 \text{ k}\Omega} \Rightarrow I_{DQ} = \frac{4}{20} = 0.20 \text{ mA}$$

$$V_{SGQ} = 1 + (0.2)(10) = 3 \text{ V}$$

$$0.20 = K_p(3 - 2)^2 \Rightarrow \underline{K_p = 0.20 \text{ mA/V}^2}$$

b.

$$I_{DQ} = (0.20)(3 - 2)^2 = 0.20 \text{ mA}$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.2)(0.2)} = 0.4 \text{ mA/V}$$

$$A_v = -g_m(R_D \| R_L) = -(0.4)(10 \| 10) \Rightarrow \underline{A_v = -2.0}$$

c.

$$\text{Choose } \underline{R_s = 20 \text{ k}\Omega} \Rightarrow I_{DQ} = \frac{4}{30} = 0.133 \text{ mA}$$

$$V_{SGQ} = 1 + (0.133)(10) = 2.33 \text{ V}$$

$$0.133 = K_p(2.33 - 2)^2 \Rightarrow \underline{K_p = 1.22 \text{ mA/V}^2}$$

$$g_m = 2\sqrt{(1.22)(0.133)} = 0.806 \text{ mA/V}$$

$$A_v = -(0.806)(10 \| 10) \Rightarrow \underline{A_v = -4.03}$$

A larger gain can be achieved.

4.25

(a)

$$I_{DQ} = K_p (V_{SGQ} + V_{TP})^2$$

$$0.25 = 0.8(V_{SGQ} - 0.5)^2$$

$$V_{SGQ} = 1.059 \text{ V}$$

$$R_s = \frac{3 - 1.059}{0.25} \Rightarrow R_s = 7.76 \text{ K}$$

$$V_D = V_s - V_{SDQ} = 1.059 - 1.5 = -0.441 \text{ V}$$

$$R_D = \frac{-0.441 - (-3)}{0.25} \Rightarrow R_D = 10.2 \text{ K}$$

(b)

$$A_v = -g_m (R_D \| R_L)$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.8)(0.25)} = 0.8944 \text{ mA/V}$$

$$A_v = -(0.8944)(10.2 \| 2)$$

$$A_v = -1.50$$

(c)

$$\Delta V_o = \Delta I (R_D \| R_L) = 0.25 (10.2 \| 2) = 0.418$$

$$\text{So } \underline{\Delta V_o = 0.836 \text{ peak-to-peak}}$$

4.26

$$I_{DQ} = K_n (V_{GSQ} - V_{TN})^2$$

$$g_m = 2\sqrt{K_n I_{DQ}}$$

$$2.2 = 2\sqrt{K_n (6)} \Rightarrow K_n = 0.202 \text{ mA/V}^2$$

$$6 = 0.202(2.8 - V_{TN})^2 \Rightarrow V_{TN} = -2.65 \text{ V}$$

$$V_{DSQ} = 18 - I_{DQ} (R_S + R_D)$$

$$R_S + R_D = \frac{18 - 10}{6} = 1.33 \text{ k}\Omega \Rightarrow R_S = 1.33 - R_D$$

$$A_v = -\frac{g_m (R_D \| R_L)}{1 + g_m R_S}$$

$$-1 = \frac{-2.2 \left(\frac{R_D \cdot 1}{R_D + 1} \right)}{1 + (2.2)(1.33 - R_D)}$$

$$1 + 2.93 - 2.2R_D = \frac{2.2R_D}{1 + R_D}$$

$$(3.93 - 2.2R_D)(1 + R_D) = 2.2R_D$$

$$3.93 + 1.73R_D - 2.2R_D^2 = 2.2R_D$$

$$2.2R_D^2 + 0.47R_D - 3.93 = 0$$

$$R_D = \frac{-0.47 + \sqrt{(0.47)^2 + 4(2.2)(3.93)}}{2(2.2)} \Rightarrow R_D = 1.23 \text{ k}\Omega, \quad R_S = 0.10 \text{ k}\Omega$$

$$V_G = V_{GS} + V_S = 2.8 + (6)(0.1) = 3.4 \text{ V}$$

$$V_G = \frac{1}{R_1} \cdot R_m \cdot V_{DD} = \frac{1}{R_1}(100)(18) = 3.4 \Rightarrow R_1 = 529 \text{ k}\Omega$$

$$\frac{529R_2}{529 + R_2} = 100 \Rightarrow R_2 = 123 \text{ k}\Omega$$

4.27

$$(a) \quad V_S = V_{SGQ} = V_{SDQ} + I_{DQ} R_D - 9$$

$$V_{SGQ} = 5 + I_{DQ}(4) - 9 = K_p(4)(V_{SGQ} + V_{TP})^2 - 4$$

$$V_{SGQ} = 8(V_{SGQ}^2 - 2.4V_{SGQ} + 1.44) - 4$$

$$\text{Or } 8V_{SGQ}^2 - 20.2V_{SGQ} + 7.52 = 0 \Rightarrow V_{SGQ} = 2.071 \text{ V}$$

$$I_{DQ} = I_Q = K_p(V_{SGQ} + V_{TP})^2 = 2(2.071 - 1.2)^2 = 1.518 \text{ mA}$$

$$(b) \quad g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(2)(1.518)} = 3.485 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.03)(1.518)} = 22 \text{ k}\Omega$$

$$A_v = -g_m (R_D \| r_o) = -(3.485)(4 \| 22) = -11.8$$

$$(c) \quad A_v = -g_m (R_D \| r_o \| R_L) = -(3.485)(4 \| 22 \| 8) = -8.29$$

4.28

(a)

$$I_{DQ} = K_p (V_{SGQ} + V_{TP})^2$$

$$0.5 = 0.25(V_{SGQ} + 0.8)^2$$

$$V_{SGQ} = 0.614 \text{ V} = V_s$$

$$R_s = \frac{10 - 0.614}{0.5} \Rightarrow R_s = 18.8 \text{ k}\Omega$$

$$V_D = V_s - V_{SDQ} = 0.614 - 3 = -2.386 \text{ V}$$

$$R_D = \frac{-2.386 - (-10)}{0.5} \Rightarrow R_D = 15.2 \text{ k}\Omega$$

(b)

$$A_v = -g_m R_D$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.25)(0.5)} = 0.7071 \text{ mA/V}$$

$$A_v = -(0.7071)(15.2)$$

$$\underline{A_v = -10.7}$$

4.29

$$A_v = -g_m (R_D \| R_L)$$

$$V_{DSQ} = V_{DD} - I_{DQ} (R_s + R_D)$$

$$10 = 20 - (1)(R_s + R_D) \Rightarrow R_s + R_D = 10 \text{ k}\Omega$$

$$\text{Let } \underline{R_D = 8 \text{ k}\Omega}, \quad \underline{R_s = 2 \text{ k}\Omega}$$

$$A_v = -10 = -g_m (8 \| 20)$$

$$g_m = 1.75 \text{ mA/V} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{K_n (1)} \Rightarrow K_n = 0.766 \text{ mA/V}^2$$

$$V_s = I_{DQ} R_s = (1)(2) = 2 \text{ V}$$

$$I_{DQ} = K_n (V_{GS} - V_{TN})^2 \Rightarrow 1 = 0.766(V_{GS} - 2)^2 \Rightarrow V_{GS} = 3.14 \text{ V}$$

$$V_G = V_{GS} + V_s = 3.14 + 2 = 5.14$$

$$V_G = \frac{1}{R_i} \cdot R_{in} \cdot V_{DD} \Rightarrow \frac{1}{R_i}(200)(20) = 5.14 \Rightarrow \underline{R_i = 778 \text{ k}\Omega}$$

$$\frac{778 R_2}{778 + R_2} = 200 \Rightarrow \underline{R_2 = 269 \text{ k}\Omega}$$

4.30

$$(a) \quad A_v = \frac{g_m r_o}{1 + g_m r_o} = \frac{(5)(100)}{1 + (5)(100)} = 0.998$$

$$R_o = \frac{1}{g_m} \| r_o = \frac{1}{5} \| 100 = 0.2 \| 100 \Rightarrow R_o \cong 200 \Omega$$

$$(b) \quad A_v = \frac{g_m(r_o \| R_S)}{1 + g_m(r_o \| R_S)}$$

We have $r_o \| R_S = 100 \| 5 = 4.762 \text{ k}\Omega$

$$A_v = \frac{(5)(4.762)}{1 + (5)(4.762)} = 0.960$$

4.31

$$A_v = \frac{g_m(R_L \| r_o)}{1 + g_m(R_L \| r_o)}$$

$$0.98 = \frac{g_m r_o}{1 + g_m r_o} \Rightarrow g_m r_o = 49$$

$$\text{Also } 0.49 = \frac{g_m(R_L \| r_o)}{1 + g_m(R_L \| r_o)} = \frac{g_m\left(\frac{R_L r_o}{R_L + r_o}\right)}{1 + g_m\left(\frac{R_L r_o}{R_L + r_o}\right)}$$

$$0.49 = \frac{g_m(R_L r_o)}{R_L + r_o + g_m(R_L r_o)}$$

$$0.49 = \frac{(49)(1)}{1 + r_o + (49)(1)} = \frac{49}{50 + r_o}$$

$$\underline{r_o = 50 \text{ K}}$$

$$\underline{g_m = 0.98 \text{ mA/V}}$$

4.32

(a)

$$A_v = \frac{g_m r_o}{1 + g_m r_o} = \frac{(2)(25)}{1 + (2)(25)}$$

$$A_v = 0.98$$

$$R_o = \frac{1}{g_m} \| r_o = \frac{1}{2} \| 25 = 0.5 \| 25$$

$$\underline{R_o = 0.49 \text{ K}}$$

(b)

$$A_v = \frac{g_m(r_o \| R_L)}{1 + g_m(r_o \| R_L)} = \frac{2(25 \| 2)}{1 + 2(25 \| 2)} = \frac{2(1.852)}{1 + 2(1.852)}$$

$$\underline{A_v = 0.787}$$

4.33

$$(a) I_{DQ} = \frac{0 - (-1.5)}{10} = 0.15 \text{ mA}$$

$$I_{DQ} = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) (V_{GSQ} - V_{TN})^2$$

$$0.15 = \left(\frac{0.1}{2} \right) (80) (V_{GSQ} - 0.4)^2 \Rightarrow V_{GSQ} = 0.594 \text{ V}$$

$$(b) g_m = 2 \sqrt{\left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) I_{DQ}} = 2 \sqrt{\left(\frac{0.1}{2} \right) (80) (0.15)} = 1.549 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.15)} = 333 \text{ k}\Omega$$

We find $r_o \| R_S = 333 \| 10 = 9.708 \text{ k}\Omega$

$$\text{Then } A_v = \frac{g_m (r_o \| R_S)}{1 + g_m (r_o \| R_S)} = \frac{(1.549)(9.708)}{1 + (1.549)(9.708)} = 0.938$$

$$(c) R_o = \frac{1}{g_m} \| r_o \| R_S = \frac{1}{1.549} \| 333 \| 10 = 0.6456 \| 9.708$$

or $R_o = 605 \Omega$

4.34

$$(a) I_{DQ} = \frac{V_{DD} - V_{DSQ}}{R_S} = \frac{2.5 - 1.25}{0.5} = 2.5 \text{ mA}$$

$$(b) A_v = \frac{g_m R_S}{1 + g_m R_S}$$

$$0.85 = \frac{g_m (0.5)}{1 + g_m (0.5)} \Rightarrow g_m = 11.33 \text{ mA/V}$$

$$g_m = 2 \sqrt{\left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) I_{DQ}}$$

$$11.33 = 2 \sqrt{\left(\frac{0.1}{2} \right) \left(\frac{W}{L} \right) (2.5)} \Rightarrow \left(\frac{W}{L} \right) = 257$$

$$(c) I_{DQ} = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) (V_{GSQ} - V_{TN})^2$$

$$2.5 = \left(\frac{0.1}{2} \right) (257) (V_{GSQ} - 0.6)^2 \Rightarrow V_{GSQ} = 1.041 \text{ V}$$

$$V_{IQ} = V_{GSQ} + V_o = 1.041 + 1.25 = 2.291 \text{ V}$$

4.35

$$(a) P = I_Q V_{DD} \Rightarrow 2.5 = I_Q (2.5) \Rightarrow I_Q = 1 \text{ mA}$$

$$(b) R_o = \frac{1}{g_m} \| r_o$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(1)} = 50 \text{ k}\Omega$$

$$\text{So } 0.5 = \frac{1}{g_m} \| 50 \Rightarrow g_m = 1.98 \text{ mA/V}$$

$$g_m = 2 \sqrt{\left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right) I_{DQ}}$$

$$1.98 = 2 \sqrt{\left(\frac{0.1}{2}\right) \left(\frac{W}{L}\right) (1)} \Rightarrow \left(\frac{W}{L}\right) = 19.6$$

$$(c) A_v = \frac{g_m r_o}{1 + g_m r_o} = \frac{(1.98)(50)}{1 + (1.98)(50)} = 0.990$$

$$(d) g_m = 2 \sqrt{\left(\frac{0.1}{2}\right) (100) (1)} = 4.472 \text{ mA/V}$$

$$R_o = \frac{1}{g_m} \| r_o = \frac{1}{4.472} \| 50 = 0.2236 \| 50 \Rightarrow R_o = 223 \Omega$$

4.36

$$(a) V_G = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{DD} = \left(\frac{350}{350 + 850} \right) (10) = 2.917 \text{ V}$$

$$10 = I_{DQ} R_S + V_{SGQ} + V_G$$

$$\text{Now } K_p = \left(\frac{k'_p}{2} \right) \left(\frac{W}{L} \right) = \left(\frac{0.04}{2} \right) (80) = 1.6 \text{ mA/V}^2$$

$$\text{So } 10 = (1.6)(4)(V_{SGQ} - 1.2)^2 + V_{SGQ} + 2.917$$

$$\text{We find } 6.4V_{SGQ}^2 - 14.36V_{SGQ} + 2.133 = 0 \Rightarrow V_{SGQ} = 2.084 \text{ V}$$

$$I_{DQ} = 1.6(2.084 - 1.2)^2 = 1.25 \text{ mA}$$

$$V_{SDQ} = 10 - (1.25)(4) = 5 \text{ V}$$

$$(b) g_m = 2 \sqrt{K_p I_{DQ}} = 2 \sqrt{(1.6)(1.25)} = 2.828 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.05)(1.25)} = 16 \text{ k}\Omega$$

$$r_o \| R_S \| R_L = 16 \| 4 \| 4 = 1.778 \text{ k}\Omega$$

$$A_v = \frac{g_m (r_o \| R_S \| R_L)}{1 + g_m (r_o \| R_S \| R_L)} = \frac{(2.828)(1.778)}{1 + (2.828)(1.778)} = 0.834$$

$$(c) A_g = \frac{i_o}{v_i} = \frac{i_o}{v_o} \cdot \frac{v_o}{v_i} = \frac{1}{R_L} \cdot \frac{v_o}{v_i} = \frac{1}{4} (0.834) = 0.2085 \text{ mA/V}$$

$$(d) \quad R_o = \frac{1}{g_m} \| R_s \| r_o = \frac{1}{2.828} \| 4 \| 16 = 0.3536 \| 3.2 \\ R_o = 318 \Omega$$

4.37

$$(a) \quad (i) \quad K_n = \frac{k'_n}{2} \cdot \frac{W}{L} = \left(\frac{0.1}{2} \right) (20) = 1 \text{ mA/V}^2 \\ g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(1)(5)} = 4.472 \text{ mA/V} \\ r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(5)} = 10 \text{ k}\Omega \\ r_o \| R_L = 10 \| 4 = 2.857 \text{ k}\Omega \\ A_v = \frac{g_m (r_o \| R_L)}{1 + g_m (r_o \| R_L)} = \frac{(4.472)(2.857)}{1 + (4.472)(2.857)} = 0.927$$

$$(ii) \quad R_o = \frac{1}{g_m} \| r_o = \frac{1}{4.472} \| 10 \\ R_o = 219 \Omega$$

$$(b) \quad (i) \quad g_m = 2\sqrt{(1)(2)} = 2.828 \text{ mA/V}$$

$$r_o = \frac{1}{(0.02)(2)} = 25 \text{ k}\Omega \\ r_o \| R_L = 25 \| 4 = 3.448 \text{ k}\Omega \\ A_v = \frac{(2.828)(3.448)}{1 + (2.828)(3.448)} = 0.907$$

$$(ii) \quad R_o = \frac{1}{g_m} \| r_o = \frac{1}{2.828} \| 25 \\ R_o = 349 \Omega$$

4.38

a.

$$A_v = \frac{g_m R_L}{1 + g_m R_L} \Rightarrow 0.95 = \frac{g_m (4)}{1 + g_m (4)} \\ 0.95 = 4(1 - 0.95) g_m \Rightarrow g_m = 4.75 \text{ mA/V} \\ g_m = 2\sqrt{\left(\frac{1}{2} \mu_n C_{ox}\right) \left(\frac{W}{L}\right) I_Q} \\ 4.75 = 2\sqrt{(0.030) \left(\frac{W}{L}\right) (4)} \Rightarrow \underline{\underline{\frac{W}{L} = 47.0}} \\ g_m = 2\sqrt{\left(\frac{1}{2} \mu_n C_{ox}\right) \left(\frac{W}{L}\right) I_Q} \\ 4.75 = 2\sqrt{(0.030)(60)I_Q} \Rightarrow \underline{\underline{I_Q = 3.13 \text{ mA}}}$$

4.39

$$I_{DQ} = K_n (V_{GS} - V_{TN})^2$$

$$5 = 5(V_{GS} + 2)^2 \Rightarrow V_{GS} = -1 \text{ V} \Rightarrow V_s = -V_{GS} = 1 \text{ V}$$

$$I_{DQ} = \frac{V_s - (-5)}{R_s} \Rightarrow R_s = \frac{1+5}{5} \Rightarrow R_s = 1.2 \text{ k}\Omega$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(5)(5)} = 10 \text{ mA/V}$$

$$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(5)} = 20 \text{ k}\Omega$$

$$A_v = \frac{g_m (r_0 \| R_s \| R_L)}{1 + g_m (r_0 \| R_s \| R_L)}$$

$$= \frac{(10)(20 \| 1.2 \| 2)}{1 + (10)(20 \| 1.2 \| 2)} \Rightarrow A_v = 0.878$$

$$R_o = \frac{1}{g_m} \| r_0 \| R_s = \frac{1}{10} \| 20 \| 1.2 \Rightarrow R_o = 91.9 \text{ }\Omega$$

4.40

(a) $V_s = I_{DQ} R_s - 5 = (5)(1) - 5 = 0 \Rightarrow V_{GS} = 0$

$$I_{DQ} = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right) (V_{GSO} - V_{TN})^2$$

$$5 = \left(\frac{0.1}{2}\right) \left(\frac{W}{L}\right) [0 - (-2)]^2 \Rightarrow \left(\frac{W}{L}\right) = 25$$

(b) $g_m = 2\sqrt{\left(\frac{0.1}{2}\right)(25)(5)} = 5 \text{ mA/V}$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(5)} = 20 \text{ k}\Omega$$

$$r_o \| R_s = 20 \| 1 = 0.9524 \text{ k}\Omega$$

$$A_v = \frac{g_m (r_o \| R_s)}{1 + g_m (r_o \| R_s)} = \frac{(5)(0.9524)}{1 + (5)(0.9524)} = 0.826$$

(c) $R_o = \frac{1}{g_m} \| r_o \| R_s = \frac{1}{5} \| 20 \| 1 \Rightarrow R_o = 165 \text{ }\Omega$

(d) $r_o \| R_s \| R_L = 20 \| 1 \| 2 = 0.6452 \text{ k}\Omega$

$$A_v = \frac{g_m (r_o \| R_s \| R_L)}{1 + g_m (r_o \| R_s \| R_L)} = \frac{(5)(0.6452)}{1 + (5)(0.6452)} = 0.763$$

4.41

$$R_o = \frac{1}{g_m} \| R_s$$

Output resistance determined primarily by g_m

$$\begin{aligned} \text{Set } \frac{1}{g_m} &= 0.2 \text{ k}\Omega \Rightarrow g_m = 5 \text{ mA/V} \\ g_m &= 2\sqrt{K_n I_{DQ}} \Rightarrow 5 = 2\sqrt{(4)I_{DQ}} \Rightarrow I_{DQ} = 1.56 \text{ mA} \\ I_{DQ} &= K_n (V_{GS} - V_{TN})^2 \\ 1.56 &= 4(V_{GS} + 2)^2 \\ V_{GS} &= -1.38 \text{ V}, \quad V_S = -V_{GS} = 1.38 \text{ V} \\ R_s &= \frac{1.38 - (-5)}{1.56} \Rightarrow R_s = 4.09 \text{ k}\Omega \\ A_v &= \frac{g_m (R_s \| R_L)}{1 + g_m (R_s \| R_L)} = \frac{5(4.09 \| 2)}{1 + 5(4.09 \| 2)} \Rightarrow A_v = 0.870 \end{aligned}$$

4.42

$$\begin{aligned} \text{(a)} \quad g_m &= 2\sqrt{K_p I_{DQ}} = 2\sqrt{(5)(10)} = 14.14 \text{ mA/V} \\ r_o &= \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(10)} = 10 \text{ k}\Omega \\ A_v &= \frac{g_m r_o}{1 + g_m r_o} = \frac{(14.14)(10)}{1 + (14.14)(10)} = 0.993 \\ \text{(b)} \quad R_o &= \frac{1}{g_m} \| r_o = \frac{1}{14.14} \| 10 = 0.07072 \| 10 \Rightarrow R_o = 70.2 \Omega \\ \text{(c)} \quad A_v &= \frac{g_m (r_o \| R_L)}{1 + g_m (r_o \| R_L)} \\ 0.90 &= \frac{(14.14)(r_o \| R_L)}{1 + (14.14)(r_o \| R_L)} \Rightarrow (r_o \| R_L) = 0.6365 \text{ k}\Omega \\ 10 \| R_L &= 0.6365 \Rightarrow R_L = 680 \Omega \end{aligned}$$

4.43

$$\begin{aligned} |\Delta i_D| &= I_{DQ} = \frac{-1}{R_s \| R_L} \cdot \Delta v_0 \\ \Delta v_0 &= -I_{DQ} \cdot R_s \| R_L = -\frac{I_{DQ} \cdot R_s R_L}{R_s + R_L} \\ v_0(\min) &= -\frac{I_{DQ} R_s}{1 + \frac{R_s}{R_L}} \\ A_v &= \frac{g_m (R_s \| R_L)}{1 + g_m (R_s \| R_L)} = \frac{v_0}{v_i} \\ v_i &= \frac{-I_{DQ} (R_s \| R_L) [1 + g_m (R_s \| R_L)]}{g_m (R_s \| R_L)} \\ v_i(\min) &= -\frac{I_{DQ}}{g_m} [1 + g_m (R_s \| R_L)] \end{aligned}$$

4.44

(a)

$$V_{DD} = V_{DSQ} + I_{DQ}R_s$$

$$3 = 1.5 + (0.25)R_s \Rightarrow R_s = 6 \text{ K}$$

$$V_s = 1.5 \text{ V}$$

$$I_{DQ} = K_n (V_{GSQ} - V_{TN})^2$$

$$0.25 = 0.5(V_{GSQ} - 0.4)^2$$

$$V_{GSQ} = 1.107 \text{ V}$$

$$V_G = V_{GSQ} + V_s + 1.107 + 1.5 = 2.607 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_i + R_2} \right) V_{DD} = \frac{1}{R_i} - R_L - V_{DD}$$

$$2.607 = \frac{1}{R_i} (300)(3) \Rightarrow R_i = 345.2 \text{ K} \Rightarrow R_2 = 2291 \text{ K}$$

(b)

$$A_v = \frac{g_m R_s}{1 + g_m R_s} \quad g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.25)} = 0.7071 \text{ mA/V}$$

$$A_v = \frac{(0.7071)(6)}{1 + (0.7071)(6)} \Rightarrow A_v = 0.809$$

$$R_o = \frac{1}{g_m} \left\| R_s = \frac{1}{(0.7071)} \right\| 6 = 1.414 \parallel 6$$

$$\underline{R_o = 1.14 \text{ K}}$$

4.45

$$R_i = \frac{1}{g_m} = 0.5 \Rightarrow g_m = \frac{1}{0.5} = 2 \text{ mA/V}$$

$$g_m = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)I_{DQ}}$$

$$2 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(0.25)} \Rightarrow \left(\frac{W}{L}\right) = 80$$

$$A_v = g_m R_D$$

$$20 = (2)R_D \Rightarrow R_D = 10 \text{ k}\Omega$$

4.46

(a) $R_o = R_D = 500 \text{ } \Omega$

(b) $V_{GSQ} = 1.2 \text{ V}$

$$I_{DQ} = \frac{V_{DD} - V_{DS}}{R_D} = \frac{2.2 - (V_{DS}(sat) + 0.3)}{0.5} = \frac{2.2 - (1.2 - 0.4 + 0.3)}{0.5}$$

$$I_{DQ} = 2.2 \text{ mA}$$

$$\begin{aligned}
 \text{(c)} \quad & I_{DQ} = K_n (V_{GSQ} - V_{TN})^2 \\
 & 2.2 = K_n (1.2 - 0.4)^2 \Rightarrow K_n = 3.438 \text{ mA/V}^2 \\
 & g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(3.438)(2.2)} = 5.5 \text{ mA/V} \\
 & R_i = \frac{1}{g_m} = \frac{1}{5.5} \Rightarrow R_i = 182 \Omega \\
 \text{(d)} \quad & A_v = g_m R_D = (5.5)(0.5) = 2.75
 \end{aligned}$$

4.47

$$\begin{aligned}
 K_n &= \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) = \left(\frac{0.1}{2} \right) (80) = 4 \text{ mA/V}^2 \\
 g_m &= 2\sqrt{K_n I_{DQ}} = 2\sqrt{(4)(0.5)} = 2.828 \text{ mA/V} \\
 A_v &= g_m R_D = (2.828)(4) = 11.3 \\
 R_i &= 10 \left\| \frac{1}{g_m} = 10 \left\| \frac{1}{2.828} = 10 \right\| 0.3536 \Rightarrow R_i = 342 \Omega \right.
 \end{aligned}$$

4.48

a.

$$\begin{aligned}
 V_{GS} + I_{DQ} R_S &= 5 \\
 I_{DQ} &= \frac{5 - V_{GS}}{R_S} = K_n (V_{GS} - V_{TN})^2 \\
 5 - V_{GS} &= (10)(3)(V_{GS}^2 - 2V_{GS} + 1) \\
 30V_{GS}^2 - 59V_{GS} + 25 &= 0 \\
 V_{GS} &= \frac{59 \pm \sqrt{(59)^2 - 4(30)(25)}}{2(30)} \Rightarrow \underline{V_{GS} = 1.35 \text{ V}} \\
 I_{DQ} &= (3)(1.35 - 1)^2 \Rightarrow \underline{I_{DQ} = 0.365 \text{ mA}} \\
 V_{DSQ} &= 10 - (0.365)(5 + 10) \Rightarrow \underline{V_{DSQ} = 4.53 \text{ V}}
 \end{aligned}$$

b.

$$\begin{aligned}
 g_m &= 2\sqrt{K_n I_{DQ}} = 2\sqrt{(3)(0.365)} \Rightarrow \underline{g_m = 2.093 \text{ mA/V}} \\
 r_0 &= \frac{1}{\lambda I_{DQ}} = \frac{1}{(0)(0.365)} \Rightarrow \underline{r_0 = \infty} \\
 \text{c. } & A_v = g_m (R_D \| R_L) = (2.093)(5 \| 4) \Rightarrow \underline{A_v = 4.65}
 \end{aligned}$$

4.49

a.

$$I_{DQ} = K_p (V_{SG} + V_{TP})^2$$

$$0.75 = (0.5)(V_{SG} - 1)^2 \Rightarrow V_{SG} = 2.225 \text{ V}$$

$$5 = I_{DQ}R_s + V_{SG} \Rightarrow R_s = \frac{5 - 2.225}{0.75} \Rightarrow R_s = 3.70 \text{ k}\Omega$$

$$V_{SDQ} = 10 - I_{DQ}(R_s + R_D)$$

$$6 = 10 - (0.75)(3.70 + R_D) \Rightarrow R_D = 1.63 \text{ k}\Omega$$

b.

$$R_i = \frac{1}{g_m}$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.5)(0.75)} = 1.225 \text{ mA/V}$$

$$R_i = \frac{1}{1.225} \Rightarrow R_i = 0.816 \text{ k}\Omega$$

$$R_o = R_D \Rightarrow R_o = 1.63 \text{ k}\Omega$$

c.

$$i_0 = \left(\frac{R_D}{R_D + R_L} \right) \left(\frac{R_s}{R_s + [1/g_m]} \right) \cdot i_i$$

$$i_0 = \left(\frac{1.63}{1.63 + 2} \right) \left(\frac{3.70}{3.70 + 0.816} \right) i_i$$

$$i_0 = 0.368 i_i = i_0 = 1.84 \sin \omega t (\mu\text{A})$$

$$v_0 = i_0 R_L = (1.84)(2) \sin \omega t \Rightarrow v_0 = 3.68 \sin \omega t (\text{mV})$$

4.50

(a) $V_O = (V_{DS(sat)} + 0.25) - V_{GS}$

$$V_O = V_{GS} - V_{TN} + 0.25 - V_{GS} = -0.4 + 0.25 = -0.15 \text{ V}$$

$$R_D = \frac{1.8 - (-0.15)}{2} = 0.975 \text{ k}\Omega$$

(b) $A_v = g_m R_D$

$$6 = g_m (0.975) \Rightarrow g_m = 6.154 \text{ mA/V}$$

$$g_m = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)I_{DQ}}$$

$$6.154 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(2)} \Rightarrow \left(\frac{W}{L}\right) = 94.7$$

(c) $I_{DQ} = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)(V_{GSQ} - V_{TN})^2$

$$2 = \left(\frac{0.1}{2}\right)(94.7)(V_{GSQ} - 0.4)^2 \Rightarrow V_{GSQ} = 1.05 \text{ V}$$

4.51

- (a) $I_{DQ} = I_Q = K_n (V_{GSQ} - V_{TN})^2$
 $2 = 4(V_{GSQ} - 0.6)^2 \Rightarrow V_{GSQ} = 1.307 \text{ V}$
 $V_D = V_{DSQ} - V_{GSQ} = 3.5 - 1.307 = 2.193 \text{ V}$
 $R_D = \frac{3.3 - 2.193}{2} = 0.554 \text{ k}\Omega$
- (b) $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(4)(2)} = 5.657 \text{ mA/V}$
 $R_i = \frac{1}{g_m} = \frac{1}{5.657} \Rightarrow R_i = 177 \Omega$
- (c) $A_v = g_m (R_D \| R_L) = (5.657)(0.554 \| 4) = 2.75$

4.52

- (a) $I_{DQ} = K_p (V_{SGQ} + V_{TP})^2$
 $1.2 = 2.5(V_{SGQ} - 0.8)^2 \Rightarrow V_{SGQ} = 1.493 \text{ V}$
 $R_S = \frac{V^+ - V_{SGQ}}{I_{DQ}} = \frac{3.3 - 1.493}{1.2} = 1.51 \text{ k}\Omega$
 $V_{SDQ} = 6.6 - I_{DQ}(R_S + R_D) \Rightarrow 3 = 6.6 - 1.2(1.51 + R_D) \Rightarrow R_D = 1.49 \text{ k}\Omega$
- (b) $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(2.5)(1.2)} = 3.464 \text{ mA/V}$
 $A_v = g_m (R_D \| R_L) = (3.464)(1.49 \| 4) = 3.76$

4.53

(a) $|A_v| = \sqrt{\frac{K_D}{K_L}} = \sqrt{\frac{(W/L)_D}{(W/L)_L}} = 5$
So $\left(\frac{W}{L}\right)_D = 25$

From Example 4.11,

$$v_{GSDt} = \frac{(3.3 - 0.6) + (0.6)(1 + 5)}{1 + 5} = 1.05 \text{ V}$$

$$v_{GSDQ} = \frac{1.05 - 0.6}{2} + 0.6 = 0.825 \text{ V}$$

(b) $I_{DQ} = \left(\frac{k'_n}{2}\right)_D \left(\frac{W}{L}\right)_D (V_{GSDQ} - V_{TN})^2 = \left(\frac{0.1}{2}\right)(25)(0.825 - 0.6)^2$
 $I_{DQ} = 0.0633 \text{ mA}$

Now $I_{DD} = I_{DL}$

$$\left(\frac{W}{L}\right)_D (V_{GSDQ} - V_{TN})^2 = \left(\frac{W}{L}\right)_L (V_{GSLQ} - V_{TN})^2$$

$$\sqrt{\frac{25}{1}} (V_{GSDQ} - V_{TN}) = V_{DD} - V_O - V_{TN}$$

$$5(0.825 - 0.6) = 3.3 - V_O - 0.6$$

Or $V_{DSQ} = V_O = 1.575 \text{ V}$

4.54

- (a) Neglect λ in dc analysis.

Transition points:

For point B,

$$V_{oB} = V_{DD} - |V_{TNL}| = 5 - 0.8 = 4.2 \text{ V}$$

For point A,

$$I_{DD} = I_{DL}$$

$$K_{nD}(V_{GSDQ} - V_{TND})^2 = K_{nL}(V_{GSLQ} - V_{TNL})^2$$

$$(1.2)(V_{GSDQ} - 0.6)^2 = (0.2)[0 - (-0.8)]^2$$

$$\text{So } V_{GSDQ} = \sqrt{\frac{0.2}{1.2}}(0.8) + 0.6 = 0.9266 \text{ V}$$

$$\text{Then } V_{oA} = V_{GSDQ} - V_{TND} = 0.9266 - 0.6 = 0.3266 \text{ V}$$

$$\text{For point A: } V_{oA} = 0.3266 \text{ V}, V_{GSDQ} = 0.9266 \text{ V}$$

$$\text{For point B: } V_{oB} = 4.2 \text{ V}, V_{GSDQ} = 0.9266 \text{ V}$$

- (b) $V_{GSDQ} = 0.9266 \text{ V}$,

$$V_{DSDQ} = \frac{4.2 - 0.3266}{2} + 0.3266 = 2.2633 \text{ V}$$

$$(c) I_{DQ} = K_{nD}(V_{GSDQ} - V_{TND})^2 = (1.2)(0.9266 - 0.6)^2 = 0.128 \text{ mA}$$

$$(d) A_v = -g_{mD}(r_{oD} \| r_{oL})$$

$$r_{oD} = r_{oL} = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.128)} = 390.6 \text{ k}\Omega$$

$$g_{mD} = 2\sqrt{K_{nD} I_{DQ}} = 2\sqrt{(1.2)(0.128)} = 0.7838 \text{ mA/V}$$

$$A_v = -(0.7838)(390.6 \| 390.6) = -153$$

4.55

- (a) $V_{TN} = 0.6 \text{ V}$

$$I_D = K_n(V_{DS} - V_{TN})^2$$

$$0.5 = K_n(1.5 - 0.6)^2 \Rightarrow K_n = 0.6173 \text{ mA/V}^2$$

$$\frac{1}{r} = \frac{dI_D}{dV_{DS}} = 2K_n(V_{DS} - V_{TN})$$

$$\text{Then } r = \frac{1}{2K_n(V_{DS} - V_{TN})} = \frac{1}{2(0.6173)(1.5 - 0.6)} \Rightarrow r = 900 \Omega$$

- (b) $I_D = (0.6173)(3 - 0.6)^2 = 3.56 \text{ mA}$

$$r = \frac{1}{2(0.6173)(3 - 0.6)} \Rightarrow r = 337 \Omega$$

4.56

a.

$$I_{DQ} = K_{nD} (V_{GS} - V_{TND})^2 = (0.5)(0 - (-1))^2$$

$$\underline{I_{DQ} = 0.5 \text{ mA}}$$

$$I_{DQ} = K_{nL} (V_{GSL} - V_{TNL})^2 = K_{nL} (V_{DD} - V_o - V_{TNL})^2$$

$$0.5 = 0.030(10 - V_0 - 1)^2$$

$$\sqrt{\frac{0.5}{0.030}} = 9 - V_0 \Rightarrow \underline{V_0 = 4.92 \text{ V}}$$

b.

$$I_{DD} = I_{DL}$$

$$K_{nD} (V_i - V_{TND})^2 = K_{nL} (V_{DD} - V_o - V_{TNL})^2$$

$$\sqrt{\frac{K_{nD}}{K_{nL}}} (V_i - V_{TND}) = V_{DD} - V_o - V_{TNL}$$

$$V_o = V_{DD} - V_{TNL} - \sqrt{\frac{K_{nD}}{K_{nL}}} (V_i - V_{TND})$$

$$A_v = \frac{dV_o}{dV_i} = -\sqrt{\frac{K_{nD}}{K_{nL}}} = -\sqrt{\frac{(W/L)_D}{(W/L)_L}}$$

$$A_v = -\sqrt{\frac{500}{30}} \Rightarrow \underline{A_v = -4.08}$$

4.57

(a)

$$I_{DQ} = K_L (V_{GSL} - V_{TNL})^2 = K_L (V_{DSL} - V_{TNL})^2$$

$$I_D = (0.1)(4 - 1)^2 = 0.9 \text{ mA}$$

$$I_{DQ} = K_D (V_{GSD} - V_{TND})^2$$

$$0.9 = (1)(V_{GSD} - 1)^2 \Rightarrow V_{GSD} = 1.95 \text{ V}$$

$$V_{GG} = V_{GSD} + V_{DSL} = 1.95 + 4 \Rightarrow \underline{V_{GG} = 5.95 \text{ V}}$$

b.

$$I_{DD} = I_{DL}$$

$$K_D (V_{GSD} - V_{TND})^2 = K_L (V_{GSL} - V_{TNL})^2$$

$$\sqrt{\frac{K_D}{K_L}} (V_{GG} + V_i - V_o - V_{TND}) = V_o - V_{TNL}$$

$$V_o \left(1 + \sqrt{\frac{K_D}{K_L}} \right) = \sqrt{\frac{K_D}{K_L}} (V_{GG} + V_i - V_{TND}) + V_{TNL}$$

$$A_v = \frac{dV_o}{dV_i} = \frac{\sqrt{K_D / K_L}}{1 + \sqrt{K_D / K_L}} \Rightarrow \underline{A_v = \frac{1}{1 + \sqrt{K_L / K_D}}}$$

(c) From Problem 4.55.

$$\begin{aligned}
 R_{LD} &= \frac{1}{2K_L(V_{DSL} - V_{TNL})} \\
 &= \frac{1}{2(0.1)(4-1)} = 1.67 \text{ k}\Omega \\
 g_m &= 2\sqrt{K_D I_{DQ}} = 2\sqrt{(1)(0.9)} = 1.90 \text{ mA/V} \\
 A_v &= \frac{g_m(R_{LD} \parallel R_L)}{1 + g_m(R_{LD} \parallel R_L)} = \frac{(1.90)(1.67 \parallel 4)}{1 + (1.90)(1.67 \parallel 4)} \Rightarrow \underline{A_v = 0.691}
 \end{aligned}$$

4.58

a. From Problem 4.57.

$$\begin{aligned}
 A_v &= \frac{g_m(R_{LD} \parallel R_L)}{1 + g_m(R_{LD} \parallel R_L)} = \frac{(1.90)(1.67 \parallel 10)}{1 + (1.90)(1.67 \parallel 10)} \\
 \underline{A_v = 0.731}
 \end{aligned}$$

b.

$$\begin{aligned}
 R_0 &= \frac{1}{g_m} \left\| R_{LD} \right\| = \frac{1}{1.90} \left\| 1.67 \right\| = 0.526 \parallel 1.67 \\
 \underline{R_0 = 0.40 \text{ k}\Omega}
 \end{aligned}$$

4.59

(a) $A_v = -g_{mD}(r_{oD} \parallel r_{oL})$

$$r_{oD} = \frac{1}{\lambda_D I_{DQ}} = \frac{1}{(0.02)(0.5)} = 100 \text{ k}\Omega$$

$$r_{oL} = \frac{1}{\lambda_L I_{DQ}} = \frac{1}{(0.04)(0.5)} = 50 \text{ k}\Omega$$

$$r_{oD} \parallel r_{oL} = 100 \parallel 50 = 33.33 \text{ k}\Omega$$

Then $-40 = -g_{mD}(33.33) \Rightarrow g_{mD} = 1.20 \text{ mA/V}$

$$\begin{aligned}
 g_{mD} &= 2 \sqrt{\left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_D (I_{DQ})} \\
 1.20 &= 2 \sqrt{\left(\frac{0.1}{2}\right) \left(\frac{W}{L}\right)_D (0.5)} \Rightarrow \left(\frac{W}{L}\right)_D = 14.4
 \end{aligned}$$

$$(b) I_{DQ} = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_L (V_{SGQ} + V_{TP})^2$$

$$0.5 = \left(\frac{0.04}{2}\right) (50) (V_{SGQ} - 0.4)^2 \Rightarrow V_{SGQ} = 1.107 \text{ V}$$

$$V_{SGQ} = V^+ - V_B$$

$$1.107 = 2.5 - V_B \Rightarrow V_B = 1.393 \text{ V}$$

$$(c) \quad I_{DQ} = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right)_D (V_{GSDQ} - V_{TN})^2$$

$$0.5 = \left(\frac{0.1}{2} \right) (14.4) (V_{GSDQ} - 0.4)^2 \Rightarrow V_{GSDQ} = 1.233 \text{ V}$$

4.60

$$(a) \quad (i) \quad r_{oD} = \frac{1}{\lambda_D I_{DQ}} = \frac{1}{(0.04)(0.25)} = 100 \text{ k}\Omega$$

$$r_{oL} = \frac{1}{\lambda_L I_{DQ}} = \frac{1}{(0.02)(0.25)} = 200 \text{ k}\Omega$$

$$r_{oD} \| r_{oL} = 100 \| 200 = 66.67 \text{ k}\Omega$$

$$A_v = -g_{mD} (r_{oD} \| r_{oL})$$

$$-25 = -g_{mD} (66.67) \Rightarrow g_{mD} = 0.375 \text{ mA/V}$$

$$g_{mD} = 2 \sqrt{\left(\frac{k'_p}{2} \right) \left(\frac{W}{L} \right)_D I_{DQ}}$$

$$0.375 = 2 \sqrt{\left(\frac{0.04}{2} \right) \left(\frac{W}{L} \right)_D (0.25)} \Rightarrow \left(\frac{W}{L} \right)_D = 7.03$$

$$(ii) \quad I_{DQ} = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right)_L (V_B - V_{TN})^2$$

$$0.25 = \left(\frac{0.1}{2} \right) (10) (V_B - 0.4)^2 \Rightarrow V_B = 1.107 \text{ V}$$

$$(iii) \quad I_{DQ} = \left(\frac{k'_p}{2} \right) \left(\frac{W}{L} \right)_D (V_{SGDQ} + V_{TP})^2$$

$$0.25 = \left(\frac{0.04}{2} \right) (7.03) (V_{SGDQ} - 0.6)^2 \Rightarrow V_{SGDQ} = 1.933 \text{ V}$$

$$(b) \quad (i) \quad r_{oD} = \frac{1}{\lambda_D I_{DQ}} = \frac{1}{(0.04)(0.1)} = 250 \text{ k}\Omega$$

$$r_{oL} = \frac{1}{\lambda_L I_{DQ}} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$r_{oD} \| r_{oL} = 250 \| 500 = 166.7 \text{ k}\Omega$$

$$A_v = -g_{mD} (r_{oD} \| r_{oL})$$

$$-25 = -g_{mD} (166.7) \Rightarrow g_{mD} = 0.15 \text{ mA/V}$$

$$g_{mD} = 2 \sqrt{\left(\frac{k'_p}{2} \right) \left(\frac{W}{L} \right)_D I_{DQ}}$$

$$0.15 = 2 \sqrt{\left(\frac{0.04}{2} \right) \left(\frac{W}{L} \right)_D (0.1)} \Rightarrow \left(\frac{W}{L} \right)_D = 2.81$$

$$\text{(ii)} \quad I_{DQ} = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right)_L (V_B - V_{TN})^2$$

$$0.1 = \left(\frac{0.1}{2} \right) (10) (V_B - 0.4)^2 \Rightarrow V_B = 0.874 \text{ V}$$

$$\text{(iii)} \quad I_{DQ} = \left(\frac{k'_p}{2} \right) \left(\frac{W}{L} \right)_D (V_{SGDQ} + V_{TP})^2$$

$$0.1 = \left(\frac{0.04}{2} \right) (2.81) (V_{SGDQ} - 0.6)^2 \Rightarrow V_{SGDQ} = 1.934 \text{ V}$$

4.61

$$K_{n1} = \left(\frac{85}{2} \right) (50) \Rightarrow 2.125 \text{ mA/V}^2$$

$$g_{m1} = 2\sqrt{K_{n1}I_{D1}} = 2\sqrt{(2.125)(0.1)} = 0.9220$$

$$r_{o1} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.05)(0.1)} = 200 \text{ K}$$

$$r_{o2} = \frac{1}{\lambda_2 I_{D2}} = \frac{1}{(0.075)(0.1)} = 133.3 \text{ K}$$

$$A_v = -g_{m1} (r_{o1} \parallel r_{o2}) = -(0.922)(200 \parallel 133.3)$$

$$\underline{A_v = -73.7}$$

4.62

$$K_{p1} = \left(\frac{k'_p}{2} \right) \left(\frac{w}{L} \right) = \left(\frac{40}{2} \right) (50) \Rightarrow 1.0 \text{ mA/V}^2$$

$$g_{m1} = 2\sqrt{K_{p1}I_{D1}} = 2\sqrt{(1)(0.1)} = 0.6325 \text{ mA/V}$$

$$r_{o1} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.075)(0.1)} = 133.3 \text{ K}$$

$$r_{o2} = \frac{1}{\lambda_2 I_{D2}} = \frac{1}{(0.05)(0.1)} = 200 \text{ K}$$

$$A_v = -g_{m1} (r_{o1} \parallel r_{o2}) = -(0.6325)(133.3 \parallel 200)$$

$$\underline{A_v = -50.6}$$

4.63

(a) $I_{DD} = I_{DL}$

$$K_{nD} (V_{GSD} - V_{TND})^2 = K_{nL} (V_{GSL} - V_{TNL})^2$$

$$\sqrt{\frac{2}{0.5}} (V_I - V_o - 0.4) = V_o - 0.4$$

$$2V_I - 0.8 = 3V_o - 0.4$$

$$V_o = \left(\frac{2}{3} \right) V_I - \left(\frac{1}{3} \right) (0.4)$$

For $V_I = 0.8 \text{ V}$, $V_O = 0.4 \text{ V}$
For $V_I = 2.5 \text{ V}$, $V_O = 1.533 \text{ V}$

(b) $I_D = K_n L (V_{GSL} - V_{TNL})^2 = (0.5)(V_O - 0.4)^2$

$$I_D = (0.5) \left[\left(\frac{2}{3} \right) V_I - \left(\frac{1}{3} \right) (0.4) - 0.4 \right]^2 = (0.5) \left[\left(\frac{2}{3} \right) V_I - 0.533 \right]^2$$

For $V_I = 0.8 \text{ V}$, $I_D = 0$; For $V_I = 2.5 \text{ V}$, $I_D = 0.642 \text{ mA}$

(c) From (a), voltage gain = constant = $2/3 = 0.667$

4.64

(a) $V_{SD}(\text{sat}) = V_{SG} + V_{TP} = (2.5 - 1) - 0.4 = 1.1 \text{ V}$
 $V_{SD} = V^+ - V_O \Rightarrow V_O(\text{max}) = V^+ - V_{SD}(\text{sat}) = 2.5 - 1.1 = 1.4 \text{ V}$

(b) $I_D = \left(\frac{k'_p}{2} \right) \left(\frac{W}{L} \right)_L (V_{SGL} + V_{TP})^2 = \left(\frac{0.04}{2} \right) (5)(2.5 - 1 - 0.4)^2 = 0.121 \text{ mA}$

(c) $I_D = \left(\frac{k'_p}{2} \right) \left(\frac{W}{L} \right)_D (V_{SGD} + V_{TP})^2$
 $0.121 = \left(\frac{0.04}{2} \right) (50)(V_{SGD} - 0.4)^2 \Rightarrow V_{SGD} = 0.748 \text{ V}$

(d) $g_{mD} = 2 \sqrt{\left(\frac{k'_p}{2} \right) \left(\frac{W}{L} \right)_D I_D} = 2 \sqrt{\left(\frac{0.04}{2} \right) (50)(0.121)} = 0.6957 \text{ mA/V}$

 $r_{oD} = r_{oL} = \frac{1}{\lambda I_D} = \frac{1}{(0.025)(0.121)} = 330.6 \text{ k}\Omega$
 $A_v = \frac{g_{mD} (r_{oD} \| r_{oL})}{1 + g_{mD} (r_{oD} \| r_{oL})} = \frac{(0.6957)(165.3)}{1 + (0.6957)(165.3)} = 0.9914$

4.65

(a) $I_{DQ} = K_n (V_{GSDQ} - V_{TN})^2$
 $1 = 2(V_{GSDQ} - 0.6)^2 \Rightarrow V_{GSDQ} = 1.307 \text{ V}$
 $I_{DQ} = K_p (V_{SGLQ} + V_{TP})^2$
 $1 = 0.5(V_{SGLQ} - 0.6)^2 \Rightarrow V_{SGLQ} = 2.014 \text{ V}$
 $V_O = 3.3 - 2.014 = 1.286 \text{ V}$
 $V_{DSDQ} = V_O - V_S = 1.286 - (-1.307) = 2.593 \text{ V}$

(b) $I_d = I_l$
 $g_{mD} V_i = g_{mL} V_o$
 $A_v = \frac{V_o}{V_i} = \frac{g_{mD}}{g_{mL}} = \sqrt{\frac{K_n}{K_p}}$

(c) $A_v = \sqrt{\frac{2}{0.5}} = 2$

4.66

(a)

$$K_{n1} = \left(\frac{85}{2} \right) (50) \Rightarrow 2.125 \text{ mA/V}^2$$

$$g_{m1} = 2\sqrt{K_{n1}I_{D1}} = 2\sqrt{(2.125)(0.1)} = 0.922 \text{ mA/V}$$

$$r_{o1} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.05)(0.1)} = 200 \text{ K}$$

$$r_{o2} = \frac{1}{\lambda_2 I_{D2}} = \frac{1}{(0.075)(0.1)} = 133.3 \text{ K}$$

(b)

$$R_{i1} = \frac{1}{g_{m1}} = \frac{1}{0.922} = 1.085 \text{ K}$$

$$V_{gs1} = -\left(\frac{R_{i1}}{R_{i1} + 0.050} \right) V_i = -\left(\frac{1.085}{1.085 + 0.050} \right) V_i = -0.956 V_i$$

$$A_v = -g_{m1} (r_{o1} \| r_{o2}) \cdot \frac{V_{gs1}}{V_i} = + (0.956)(0.922)(200)(133.3)$$

$$\underline{A_v = 70.5}$$

(c)

$$R_i = 0.05 + \frac{1}{g_{m1}} = 0.05 + \frac{1}{0.922} \Rightarrow R_i = 1.135 \text{ K}$$

(d)

$$R_o \approx r_{o1} \| r_{o2} = 200 \| 133.7 \Rightarrow R_o \approx 80 \text{ K}$$

4.67

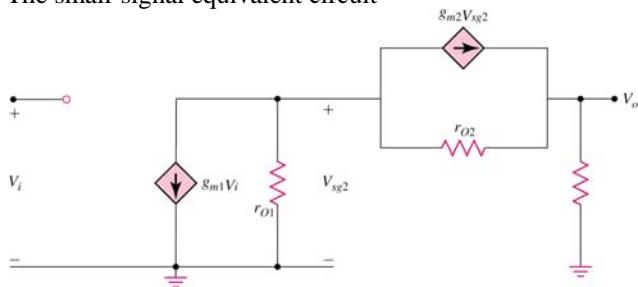
(a)

$$g_{m1} = 2\sqrt{K_n I_{D1}} = 2\sqrt{(2)(0.1)} = 0.8944 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{K_p I_{D2}} = 2\sqrt{(2)(0.1)} = 0.8944 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(0.1)} = 100 \text{ K}$$

(b) The small-signal equivalent circuit



$$(1) \quad g_{m1}V_i + \frac{V_{sg2}}{r_{o1}} + g_{m2}V_{sg2} + \frac{V_{sg2} - V_o}{r_{o2}} = 0$$

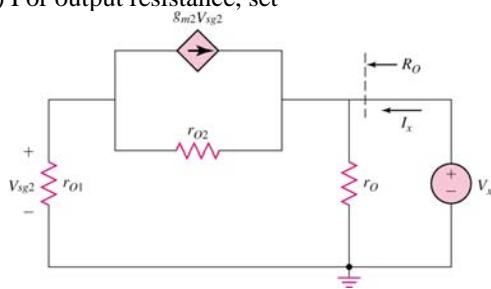
(2)

$$\begin{aligned}\frac{V_o}{r_o} + \frac{V_o - V_{sg2}}{r_{o2}} &= g_{m2}V_{sg2} \\ \frac{V_o}{r_o} \left(\frac{1}{r_o} + \frac{1}{r_{o2}} \right) &= V_{sg2} \left(\frac{1}{r_{o2}} + g_{m2} \right) \\ V_o \left(\frac{1}{50} + \frac{1}{100} \right) &= V_{sg2} \left(\frac{1}{100} + 0.8944 \right) \Rightarrow V_{sg2} = V_o (0.03317)\end{aligned}$$

(1)

$$\begin{aligned}g_{m1}V_i + V_{sg2} \left(\frac{1}{r_{o1}} + g_{m2} + \frac{1}{r_{o2}} \right) &= \frac{V_o}{r_{o2}} \\ 0.8944 V_i + V_o (0.03317) \left(\frac{1}{100} + 0.8944 + \frac{1}{100} \right) &= \frac{V_o}{100} \\ 0.8944 V_i &= V_o (0.01 - 0.03033) \\ \frac{V_o}{V_i} &= -44\end{aligned}$$

(c) For output resistance, set $V_i = 0$.



$$(1) \quad g_{m2}V_{sg2} + I_x = \frac{V_x}{r_o} + \frac{V_x - V_{sg2}}{r_{o2}}$$

$$(2) \quad \frac{V_{sg2}}{r_{o1}} + g_{m2}V_{sg2} + \frac{V_{sg2} - V_x}{r_{o2}} = 0$$

$$(2) \quad V_{sg2} \left(\frac{1}{r_{o1}} + g_{m2} + \frac{1}{r_{o2}} \right) = \frac{V_x}{r_{o2}}$$

$$V_{sg2} \left(\frac{1}{100} + 0.8944 + \frac{1}{100} \right) = \frac{V_x}{100}$$

$$V_{sg2} = V_x (0.010936)$$

(1)

$$I_x = V_x \left(\frac{1}{r_o} + \frac{1}{r_{o2}} \right) - V_{sg2} \left(\frac{1}{r_{o2}} + g_{m2} \right)$$

$$I_x = V_x \left(\frac{1}{50} + \frac{1}{100} \right) - V_x (0.010936) \left(\frac{1}{100} + 0.8944 \right)$$

$$I_x = V_x (0.03 - 0.0098905)$$

$$R_o = \frac{V_x}{I_x} = 49.7 \text{ K}$$

4.68

$$(a) I_{DQ1} = K_{n1} (V_{GSQ1} - V_{TN1})^2$$

$$0.2 = 0.2(V_{GSQ1} - 0.6)^2 \Rightarrow V_{GSQ1} = 1.6 \text{ V}$$

$$I_{DQ2} = K_{p2} (V_{SGQ2} + V_{TP2})^2$$

$$0.5 = 1.0(V_{SGQ2} - 0.6)^2 \Rightarrow V_{SGQ2} = 1.307 \text{ V}$$

$$R_{S1} = \frac{0.6}{0.2} = 3 \text{ k}\Omega$$

$$V_{D1} = 0.6 + 2 = 2.6 \text{ V}$$

$$R_{D1} = \frac{5 - 2.6}{0.2} = 12 \text{ k}\Omega$$

$$V_{G1} = V_{GSQ1} + 0.6 = 1.6 + 0.6 = 2.2 \text{ V}$$

$$V_{G1} = 2.2 = \left(\frac{R_2}{R_1 + R_2} \right) (5) = \frac{1}{R_1} \cdot R_{in} \cdot (5) = \frac{1}{R_1} (400)(5)$$

Or $R_1 = 909 \text{ k}\Omega$ and $R_1 \| R_2 = R_{in} = 400 \text{ k}\Omega \Rightarrow R_2 = 714 \text{ k}\Omega$

$$V_{S2} = V_{D1} + V_{SGQ2} = 2.6 + 1.307 = 3.907 \text{ V}$$

$$R_{S2} = \frac{5 - 3.907}{0.5} = 2.19 \text{ k}\Omega$$

$$V_{D2} = V_{S2} - 3 = 3.907 - 3 = 0.907 \text{ V}$$

$$R_{D2} = \frac{0.907}{0.5} = 1.81 \text{ k}\Omega$$

$$(b) g_{m1} = 2\sqrt{K_{n1} I_{DQ1}} = 2\sqrt{(0.2)(0.2)} = 0.4 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{K_{p2} I_{DQ2}} = 2\sqrt{(1)(0.5)} = 1.414 \text{ mA/V}$$

$$A_v = (-g_{m1} R_{D1})(-g_{m2} R_{D2}) = g_{m1} g_{m2} R_{D1} R_{D2}$$

$$A_v = (0.4)(1.414)(12)(1.81) = 12.3$$

4.69

$$(a) I_{DQ1} = K_{n1} (V_{GS1} - V_{TN1})^2$$

$$0.1 = 0.2(V_{GS1} - 0.6)^2 \Rightarrow V_{GSQ1} = 1.307 \text{ V}$$

$$I_{DQ2} = K_{p2} (V_{SGQ2} + V_{TP})^2$$

$$0.25 = 1.0(V_{SGQ2} - 0.6)^2 \Rightarrow V_{SGQ2} = 1.10 \text{ V}$$

$$V_{G1} = V_{GSQ1} + I_{DQ1} R_{S1} = 1.307 + (0.1)(1) = 1.407 \text{ V}$$

$$V_{G1} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$1.407 = \frac{1}{R_1} (250)(3.3) \Rightarrow R_1 = 586 \text{ k}\Omega$$

$$R_1 \| R_2 = R_{in} = 250 \text{ k}\Omega \Rightarrow R_2 = 436 \text{ k}\Omega$$

$$V_{D1} = I_{DQ1} R_{S1} + V_{DSQ1} = (0.1)(1) + 1.2 = 1.3 \text{ V}$$

$$R_{D1} = \frac{3.3 - 1.3}{0.1} = 20 \text{ k}\Omega$$

$$V_{S2} = V_{D1} + V_{SGQ2} = 1.3 + 1.1 = 2.4 \text{ V}$$

$$R_{S2} = \frac{3.3 - 2.4}{0.25} = 3.6 \text{ k}\Omega$$

$$V_{D2} = V_{S2} - V_{SDQ2} = 2.4 - 1.8 = 0.6 \text{ V}$$

$$R_{D2} = \frac{0.6}{0.25} = 2.4 \text{ k}\Omega$$

$$(b) \quad g_{m1} = 2\sqrt{K_{p1}I_{DQ1}} = 2\sqrt{(0.2)(0.1)} = 0.2828 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{K_{n2}I_{DQ2}} = 2\sqrt{(1)(0.25)} = 1.0 \text{ mA/V}$$

$$A_v = g_{m1}g_{m2}R_{D1}R_{D2} = (0.2828)(1.0)(20)(2.4) = 13.6$$

4.70

$$K_{p1} = \left(\frac{0.04}{2}\right)(20) = 0.4 \text{ mA/V}^2$$

$$K_{n2} = \left(\frac{0.1}{2}\right)(80) = 4.0 \text{ mA/V}^2$$

$$(a) \quad R_{S1} = \frac{0.6}{0.1} = 6 \text{ k}\Omega$$

$$V_{D1} = 1.8 - 0.6 - 1 = 0.2 \text{ V}$$

$$R_{D1} = \frac{0.2 - (-1.8)}{0.1} = 20 \text{ k}\Omega$$

$$I_{DQ1} = K_{p1}(V_{SGQ1} + V_{TP})^2$$

$$0.1 = 0.4(V_{SGQ1} - 0.4)^2 \Rightarrow V_{SGQ1} = 0.90 \text{ V}$$

$$I_{DQ2} = K_{n2}(V_{GSQ2} - V_{TN})^2$$

$$0.3 = 4(V_{GSQ2} - 0.4)^2 \Rightarrow V_{GSQ2} = 0.6739 \text{ V}$$

$$V_{G1} = 1.8 - 0.6 - V_{SGQ1} = 1.8 - 0.6 - 0.9 = 0.3 \text{ V}$$

$$V_{G1} = \left(\frac{R_2}{R_1 + R_2}\right)(3.6) - 1.8$$

$$0.3 = \frac{1}{R_1}(200)(3.6) - 1.8 \Rightarrow R_1 = 343 \text{ k}\Omega$$

$$R_1 \parallel R_2 = 200 \text{ k}\Omega \Rightarrow R_2 = 480 \text{ k}\Omega$$

$$V_{D1} = 1.8 - 0.6 - 1.0 = 0.2 \text{ V}$$

$$V_{S2} = V_{D1} - V_{GSQ2} = 0.2 - 0.6739 = -0.4739 \text{ V}$$

$$R_{S2} = \frac{-0.4739 - (-1.8)}{0.3} = 4.42 \text{ k}\Omega$$

$$(b) \quad A_v = \left(\frac{-g_{m1}R_{D1}}{1 + g_{m1}R_{S1}} \right) \left(\frac{g_{m2}R_{S2}}{1 + g_{m2}R_{S2}} \right)$$

$$g_{m1} = 2\sqrt{K_{p1}I_{DQ1}} = 2\sqrt{(0.4)(0.1)} = 0.4 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{K_{n2}I_{DQ2}} = 2\sqrt{(1)(0.3)} = 2.191 \text{ mA/V}$$

$$A_v = \frac{-(0.4)(20)}{1 + (0.4)(6)} \cdot \frac{(2.191)(4.42)}{1 + (2.191)(4.42)} = -2.13$$

$$(c) \quad R_o = \frac{1}{g_{m2}} \| R_{S2} = \frac{1}{2.191} \| 4.42 = 0.4564 \| 4.42 \Rightarrow R_o = 414 \Omega$$

4.71

(a)

$$I_{DQ1} = \frac{10 - V_{GS1}}{R_{S2}} = K_{n1} (V_{GS1} - V_{TN1})^2$$

$$10 - V_{GS1} = (4)(10)(V_{GS1}^2 - 4V_{GS1} + 4)$$

$$40V_{GS1}^2 - 159V_{GS1} + 150 = 0$$

$$V_{GS1} = \frac{159 \pm \sqrt{(159)^2 - 4(40)(150)}}{2(40)} \Rightarrow V_{GS1} = 2.435 \text{ V}$$

$$I_{DQ1} = (4)(2.435 - 2)^2 \Rightarrow I_{DQ1} = 0.757 \text{ mA}$$

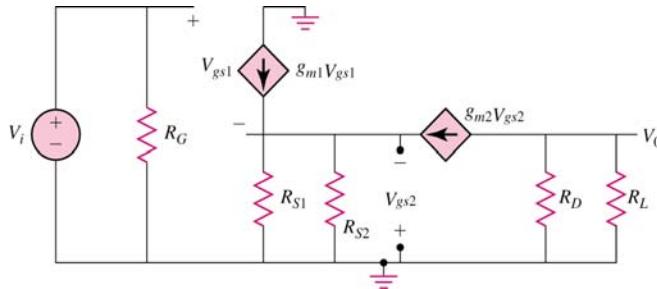
$$V_{DSQ1} = 20 - (0.757)(10) \Rightarrow V_{DSQ1} = 12.4 \text{ V}$$

$$\text{Also } I_{DQ2} = 0.757 \text{ mA}$$

$$V_{DSQ2} = 20 - (0.757)(10 + 5) \Rightarrow V_{DSQ2} = 8.65 \text{ V}$$

$$(b) \quad g_{m1} = g_{m2} = 2\sqrt{KI_{DQ}} = 2\sqrt{(4)(0.757)} \Rightarrow g_{m1} = g_{m2} = 3.48 \text{ mA/V}$$

c.



$$V_0 = -(g_{m2}V_{gs2})(R_D \| R_L)$$

$$V_{gs2} = (-g_{m1}V_{gs1} - g_{m2}V_{gs2})(R_{S1} \| R_{S2})$$

$$V_i = V_{gs1} - V_{gs2} \Rightarrow V_{gs1} = V_i + V_{gs2}$$

$$V_{gs2} + g_{m2}V_{gs2}(R_{S1} \| R_{S2}) = -g_{m1}(V_i + V_{gs2})(R_{S1} \| R_{S2})$$

$$V_{gs2} + g_{m2}V_{gs2}(R_{S1} \| R_{S2}) + g_{m1}V_{gs2}(R_{S1} \| R_{S2}) = -g_{m1}V_i(R_{S1} \| R_{S2})$$

$$V_{gs2} = \frac{-g_{m1}V_i(R_{S1} \| R_{S2})}{1 + g_{m2}(R_{S1} \| R_{S2}) + g_{m1}(R_{S1} \| R_{S2})}$$

$$A_v = \frac{V_0}{V_i} = \frac{g_{m1}g_{m2}(R_{S1} \| R_{S2})(R_D \| R_L)}{1 + (g_{m1} + g_{m2})(R_{S1} \| R_{S2})}$$

$$A_v = \frac{(3.48)^2 (10 \| 10) (5 \| 2)}{1 + (3.48 + 3.48)(10 \| 10)} \Rightarrow A_v = 2.42$$

4.72

a.

$$I_{DQ} = 3 \text{ mA}$$

$$V_{S1} = I_{DQ}R_s - 5 = (3)(1.2) - 5 = -1.4 \text{ V}$$

$$I_{DQ} = K_1(V_{GS} - V_{TN})^2$$

$$3 = 2(V_{GS} - 1)^2 \Rightarrow V_{GS} = 2.225 \text{ V}$$

$$V_{G1} = V_{GS} + V_{S1} = 2.225 - 1.4 = 0.825 \text{ V}$$

$$V_{G1} = \left(\frac{R_3}{R_1 + R_2 + R_3} \right)(5) \Rightarrow 0.825 = \left(\frac{R_3}{500} \right)(5) \Rightarrow R_3 = 82.5 \text{ k}\Omega$$

$$V_{D1} = V_{S1} + V_{DSQ1} = -1.4 + 2.5 = 1.1 \text{ V}$$

$$V_{G2} = V_{D1} + V_{GS} = 1.1 + 2.225 = 3.325 \text{ V}$$

$$V_{G2} = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3} \right)(5) \Rightarrow 3.325 = \left(\frac{R_2 + R_3}{500} \right)(5)$$

$$R_2 + R_3 = 332.5 \Rightarrow R_2 = 250 \text{ k}\Omega$$

$$R_l = 500 - 250 - 82.5 \Rightarrow R_l = 167.5 \text{ k}\Omega$$

$$V_{D2} = V_{D1} + V_{DSQ2} = 1.1 + 2.5 = 3.6 \text{ V}$$

$$R_D = \frac{5 - 3.6}{3} \Rightarrow R_D = 0.467 \text{ k}\Omega$$

b.

$$A_v = -g_m R_D$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(2)(3)} = 4.90 \text{ mA/V}$$

$$A_v = -(4.90)(0.467) \Rightarrow A_v = -2.29$$

4.73

a.

$$V_{S1} = I_{DQ}R_s - 10 = (5)(2) - 10 \Rightarrow V_{S1} = 0$$

$$I_{DQ} = K_1(V_{GS1} - V_{TN})^2$$

$$5 = 4(V_{GS1} - 1.5)^2 \Rightarrow V_{GS1} = 2.618 \text{ V}$$

$$V_{G1} = V_{GS1} + V_{S1} = 2.618 \text{ V} = IR_3 = (0.1)R_3 \Rightarrow R_3 = 26.2 \text{ k}\Omega$$

$$V_{D1} = V_{S1} + V_{DSQ1} = 0 + 3.5 = 3.5 \text{ V}$$

$$V_{G2} = V_{D1} + V_{GS} = 3.5 + 2.62 = 6.12 \text{ V}$$

$$= (0.1)(R_2 + R_3)$$

$$R_2 + R_3 = 61.2 \text{ k}\Omega \Rightarrow R_2 = 35 \text{ k}\Omega$$

$$V_{D2} = V_{D1} + V_{DSQ2} = 3.5 + 3.5 = 7.0 \text{ V}$$

$$R_D = \frac{10 - 7}{5} \Rightarrow R_D = 0.6 \text{ k}\Omega$$

$$R_l = \frac{10 - 6.12}{0.1} \Rightarrow R_l = 38.8 \text{ k}\Omega$$

b.

$$A_v = -g_m R_D$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(4)(5)} = 8.944 \text{ mA/V}$$

$$A_v = -(8.944)(0.6) \Rightarrow A_v = -5.37$$

4.74

a.

$$I_{DQ} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$4 = 6 \left(1 - \frac{V_{GS}}{(-3)}\right)^2$$

$$V_{GS} = (-3) \left[1 - \sqrt{\frac{4}{6}}\right] \Rightarrow V_{GS} = -0.551 \text{ V}$$

$$V_{DSQ} = V_{DD} - I_{DQ} R_D$$

$$6 = 10 - (4)R_D \Rightarrow R_D = 1 \text{ k}\Omega$$

b.

$$g_m = \frac{2I_{DSS}}{(-V_P)} \left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(6)}{3} \left(1 - \frac{-0.551}{-3}\right) \Rightarrow g_m = 3.265 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(4)} \Rightarrow r_o = 25 \text{ k}\Omega$$

c. $A_v = -g_m (r_o \| R_D) = -(3.265)(25 \| 1) \Rightarrow A_v = -3.14$

4.75

$$V_{GS} + I_{DQ} (R_{S1} + R_{S2}) = 0$$

$$I_{DQ} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2$$

$$V_{GS} + I_{DSS} (R_{S1} + R_{S2}) \left(1 - \frac{V_{GS}}{V_P} \right)^2 = 0$$

$$V_{GS} + (2)(0.1 + 0.25) \left(1 - \frac{V_{GS}}{V_P} \right)^2 = 0$$

$$V_{GS} + 0.7 \left(1 - \frac{2_{VGS}}{(-2)} + \frac{V_{GS}^2}{(-2)^2} \right) = 0$$

$$0.175V_{GS}^2 + 1.7V_{GS} + 0.7 = 0$$

$$V_{GS} = \frac{-1.7 \pm \sqrt{(1.7)^2 - 4(0.175)(0.7)}}{2(0.175)} \Rightarrow V_{GS} = -0.4314 \text{ V}$$

$$g_m = \frac{2I_{DSS}}{-V_P} \left(1 - \frac{V_{GS}}{V_P} \right) = \frac{2(2)}{2} \left(1 - \frac{-0.431}{-2} \right) \Rightarrow g_m = 1.569 \text{ mA/V}$$

$$A_v = \frac{-g_m (R_D \| R_L)}{1 + g_m R_{S1}} = \frac{-(1.569)(8\|4)}{1 + (1.569)(0.1)} \Rightarrow A_v = -3.62$$

$$A_i = \frac{i_0}{i_i} = \frac{(v_0 / R_L)}{(v_i / R_G)} = \frac{v_0}{v_i} \cdot \frac{R_G}{R_L} = (-3.62) \left(\frac{50}{4} \right) \Rightarrow A_i = -45.2$$

4.76

$$I_{DQ} = \frac{I_{DSS}}{2} = 4 \text{ mA}$$

$$V_{DSQ} = \frac{V_{DD}}{2} = 10 \text{ V}$$

$$V_{DSQ} = V_{DD} - I_{DQ} (R_S + R_D)$$

$$10 = 20 - (4)(R_S + R_D) \Rightarrow R_S + R_D = 2.5 \text{ k}\Omega$$

$$V_s = 2 \text{ V} = I_{DQ} R_S = 4R_S \Rightarrow R_S = 0.5 \text{ k}\Omega, \quad R_D = 2.0 \text{ k}\Omega$$

$$I_{DQ} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2$$

$$4 = 8 \left(1 - \frac{V_{GS}}{(-4.2)} \right)^2 \Rightarrow V_{GS} = (-4.2) \left(1 - \sqrt{\frac{4}{8}} \right) \Rightarrow V_{GS} = -1.23 \text{ V}$$

$$V_G = V_s + V_{GS} = 2 - 1.23$$

$$V_G = 0.77 \text{ V} = \left(\frac{R_2}{R_l + R_2} \right) (20) = \left(\frac{R_2}{100} \right) (20) \Rightarrow R_2 = 3.85 \text{ k}\Omega, \quad R_l = 96.2 \text{ K}\Omega$$

4.77

a.

$$I_{DQ} = \frac{I_{DSS}}{2} = 5 \text{ mA}$$

$$V_{DSQ} = \frac{V_{DD}}{2} = \frac{12}{2} = 6 \text{ V}$$

$$R_s = \frac{12 - 6}{5} \Rightarrow R_s = 1.2 \text{ k}\Omega$$

$$I_{DQ} = I_{DSS} \left(1 - \frac{V_{GS}}{V_p} \right)^2$$

$$5 = 10 \left(1 - \frac{V_{GS}}{(-5)} \right)^2 \Rightarrow V_{GS} = (-5) \left(1 - \sqrt{\frac{5}{10}} \right) \Rightarrow V_{GS} = -1.464 \text{ V}$$

$$V_G = V_S + V_{GS} = 6 - 1.464 = 4.536 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$4.536 = \frac{1}{R_1} (100)(12) \Rightarrow R_1 = 265 \text{ k}\Omega$$

$$\frac{265R_2}{265 + R_2} = 100 \Rightarrow R_2 = 161 \text{ k}\Omega$$

b.

$$g_m = \frac{2I_{DSS}}{(-V_p)} \left(1 - \frac{V_{GS}}{V_p} \right) = \frac{2(10)}{5} \left(1 - \frac{-1.46}{-5} \right) \Rightarrow g_m = 2.83 \text{ mA/V}$$

$$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(5)} = 20 \text{ k}\Omega$$

$$A_v = \frac{g_m (r_0 \| R_s \| R_L)}{1 + g_m (r_0 \| R_s \| R_L)}$$

$$A_v = \frac{(2.83)(20 \| 1.2 \| 0.5)}{1 + (2.83)(20 \| 1.2 \| 0.5)} \Rightarrow A_v = 0.495$$

$$R_0 = \frac{1}{g_m} \| R_s = \frac{1}{2.83} \| 1.2 = 0.353 \| 1.2 \Rightarrow R_0 = 0.273 \text{ k}\Omega$$

4.78

a.

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \left(\frac{110}{110+90} \right) (10) = 5.5 \text{ V}$$

$$I_{DQ} = \frac{10 - (V_G - V_{GS})}{R_S} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2$$

$$10 - 5.5 + V_{GS} = (2)(5) \left(1 - \frac{V_{GS}}{1.75} \right)^2$$

$$4.5 + V_{GS} = 10 \left(1 - 1.143V_{GS} + 0.3265V_{GS}^2 \right)$$

$$3.265V_{GS}^2 - 12.43V_{GS} + 5.5 = 0$$

$$V_{GS} = \frac{12.43 \pm \sqrt{(12.43)^2 - 4(3.265)(5.5)}}{2(3.265)} \Rightarrow V_{GS} = 0.511 \text{ V}$$

$$I_{DQ} = (2) \left(1 - \frac{0.511}{1.75} \right)^2 \Rightarrow I_{DQ} = 1.00 \text{ mA}$$

$$V_{SDQ} = 10 - (1.00)(5) \Rightarrow V_{SDQ} = 5.0 \text{ V}$$

b.

$$g_m = \frac{2I_{DSS}}{V_P} \left(1 - \frac{V_{GS}}{V_P} \right) = \frac{2(2)}{1.75} \left(1 - \frac{0.511}{1.75} \right) \Rightarrow g_m = 1.618 \text{ mA/V}$$

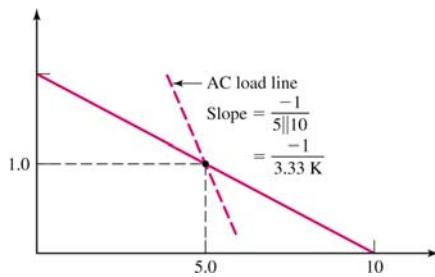
$$A_v = \frac{g_m (R_S \| R_L)}{1 + g_m (R_S \| R_L)} = \frac{(1.618)(5\|10)}{1 + (1.618)(5\|10)} \Rightarrow A_v = 0.844$$

$$A_i = \frac{i_o}{i_i} = \frac{(v_o / R_L)}{(v_i / R_i)} = A_v \cdot \left(\frac{R_i}{R_L} \right)$$

$$R_i = R_1 \| R_2 = 90 \| 110 = 49.5 \text{ k}\Omega$$

$$A_i = (0.844) \left(\frac{49.5}{10} \right) \Rightarrow A_i = 4.18$$

c.



$$\Delta i_d = 1.0 \text{ mA}$$

$$v_{sd} = (3.33)(1.0) = 3.33 \text{ V}$$

Maximum swing in output voltage = 6.66 V peak-to-peak

4.79

$$I_{DQ} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2$$

$$4 = 8 \left(1 - \frac{V_{GS}}{4} \right)^2 \Rightarrow V_{GS} = 4 \left(1 - \sqrt{\frac{4}{8}} \right) \Rightarrow V_{GS} = 1.17 \text{ V}$$

$$V_{SDQ} = V_{DD} - I_{DQ} (R_S + R_D)$$

$$7.5 = 20 - 4(R_S + R_D) \Rightarrow R_S + R_D = 3.125 \text{ k}\Omega$$

$$g_m = \frac{2I_{DSS}}{V_P} \left(1 - \frac{V_{GS}}{V_P} \right) = \frac{2(8)}{4} \left(1 - \frac{1.17}{4} \right) \Rightarrow g_m = 2.83 \text{ mA/V}$$

$$R_S = 3.125 - R_D$$

$$A_v = \frac{-g_m R_D}{1 + g_m R_S}$$

$$-3(1 + g_m R_S) = -g_m R_D$$

$$3[1 + (2.83)(3.125 - R_D)] = (2.83)R_D$$

$$9.844 - 2.83R_D = 0.9433R_D \Rightarrow R_D = 2.61 \text{ k}\Omega \quad R_S = 0.516 \text{ k}\Omega$$

$$V_S = 20 - (4)(0.516) \Rightarrow V_S = 17.94 \text{ V}$$

$$V_G = V_S - V_{GS} = 17.94 - 1.17 = 16.77 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \left(\frac{R_2}{400} \right) (20) \Rightarrow R_2 = 335 \text{ k}\Omega. \quad R_1 = 65 \text{ k}\Omega$$

Chapter 5

5.1

$$(a) \quad i_E = (1 + \beta)i_B \Rightarrow 1 + \beta = \frac{325}{2.8} = 116 \Rightarrow \beta = 115$$

$$\alpha = \frac{\beta}{1 + \beta} = \frac{115}{116} = 0.9914$$

$$i_C = i_E - i_B = 325 - 2.8 = 322 \mu A$$

$$(b) \quad 1 + \beta = \frac{1.80}{0.020} = 90 \Rightarrow \beta = 89$$

$$\alpha = \frac{89}{90} = 0.9889$$

$$i_C = 1.80 - 0.02 = 1.78 mA$$

5.2

$$(a) \quad \alpha = \frac{i_C}{i_E} = \frac{726}{732} = 0.9918$$

$$\beta = \frac{\alpha}{1 - \alpha} = \frac{0.9918}{1 - 0.9918} = 121$$

$$i_B = i_E - i_C = 732 - 726 = 6 \mu A$$

$$(b) \quad \alpha = \frac{2.902}{2.961} = 0.9801$$

$$\beta = \frac{\alpha}{1 - \alpha} = \frac{0.980074}{1 - 0.980074} = 49.19$$

$$i_B = 2.961 - 2.902 \Rightarrow i_B = 59 \mu A$$

5.3

$$(a) \quad \text{For } \beta = 110: \quad \alpha = \frac{\beta}{1 + \beta} = \frac{110}{111} = 0.99099$$

$$\text{For } \beta = 180: \quad \alpha = \frac{180}{181} = 0.99448$$

$$\underline{0.99099 \leq \alpha \leq 0.99448}$$

$$(b) \quad I_C = \beta I_B = 110(50 \mu A) \Rightarrow I_C = 5.50 mA$$

$$\text{or } I_C = 180(50 \mu A) \Rightarrow I_C = 9.00 mA$$

$$\text{so } \underline{5.50 \leq I_C \leq 9.0 mA}$$

5.4

$$(a) \quad i_B = \frac{i_E}{1+\beta} = \frac{1.25}{151} \Rightarrow i_B = 8.28 \mu A$$

$$i_C = \left(\frac{\beta}{1+\beta} \right) \cdot i_E = \left(\frac{150}{151} \right) (1.25) = 1.242 \text{ mA}$$

$$\alpha = \frac{150}{151} = 0.9934$$

$$(b) \quad i_B = \frac{4.52}{81} \Rightarrow i_B = 55.8 \mu A$$

$$i_C = \left(\frac{80}{81} \right) (4.52) = 4.46 \text{ mA}$$

$$\alpha = \frac{80}{81} = 0.9877$$

5.5

(a)

α	$\beta = \frac{\alpha}{1-\alpha}$
0.9	9
0.95	19
0.98	49
0.99	99
0.995	199
0.999	999

(b)

β	$\alpha = \frac{\beta}{1+\beta}$
20	0.9524
50	0.9804
100	0.9901
150	0.9934
220	0.9955
400	0.9975

5.6

$$(a) \quad I_B = \frac{I_E}{1+\beta} = \frac{1.2}{81} \Rightarrow I_B = 14.8 \mu A$$

$$I_C = \left(\frac{\beta}{1+\beta} \right) \cdot I_E = \left(\frac{80}{81} \right) (1.2) = 1.185 \text{ mA}$$

$$\alpha = \frac{80}{81} = 0.9877$$

$$V_C = 5 - (1.185)(2) = 2.63 \text{ V}$$

$$(b) \quad I_B = \frac{0.8}{81} \Rightarrow I_B = 9.88 \mu A$$

$$I_C = \left(\frac{80}{81} \right) (0.8) = 0.790 \text{ mA}$$

$$\alpha = 0.9877$$

$$V_C = 5 - (0.790)(2) = 3.42 \text{ V}$$

$$(c) \quad (i) \quad I_B = \frac{1.2}{121} \Rightarrow I_B = 9.92 \mu A$$

$$I_C = \left(\frac{120}{121} \right) (1.2) = 1.19 \text{ mA}$$

$$\alpha = \frac{120}{121} = 0.9917$$

$$V_C = 5 - (1.19)(2) = 2.62 \text{ V}$$

$$(ii) \quad I_B = \frac{0.8}{121} \Rightarrow I_B = 6.61 \mu A$$

$$I_C = \left(\frac{120}{121} \right) (0.8) = 0.7934 \text{ mA}$$

$$\alpha = 0.9917$$

$$V_C = 5 - (0.7934)(2) = 3.41 \text{ V}$$

5.7

$$I_E = I_{Eo} \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$0.80 \times 10^{-3} = (5 \times 10^{-14}) \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$\text{Then } V_{BE} = (0.026) \ln\left(\frac{0.80 \times 10^{-3}}{5 \times 10^{-14}}\right) = 0.6109 \text{ V}$$

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.9910}{1-0.9910} = 110$$

$$I_C = \alpha I_E = (0.9910)(0.80) = 0.7928 \text{ mA}$$

$$I_B = \frac{I_E}{1+\beta} = \frac{0.80}{111} \Rightarrow I_B = 7.21 \mu A$$

$$V_C = 5 - I_C R_C = 5 - (0.7928)(2) = 3.41 \text{ V}$$

5.8

$$I_B = \frac{0.75}{61} \Rightarrow 12.3 \mu\text{A}$$

$$I_C = (0.75) \left(\frac{60}{61} \right) = 0.738 \text{ mA}$$

$$\alpha = \frac{60}{61} = 0.9836$$

$$V_C = I_C R_C - 10 = (0.738)(5) - 10$$

(a) $V_C = -6.31 \text{ V}$

$$I_B = \frac{1.5}{61} \Rightarrow 24.6 \mu\text{A}$$

$$I_C = (1.5) \left(\frac{60}{61} \right) = 1.475 \text{ mA}$$

$$\alpha = \left(\frac{60}{61} \right) = 0.9836$$

(b) $V_C = (1.475)(5) - 10 \Rightarrow V_C = -2.625 \text{ V}$

(c) Yes, $V_C < 0$ in both cases so that B-C junction is reverse biased.

5.9

(a) $V_C = I_C(5) - 10$

$$-1.2 = I_C(5) - 10 \Rightarrow I_C = 1.76 \text{ mA}$$

$$I_E = \frac{I_C}{\alpha} = \frac{1.76}{0.986} = 1.785 \text{ mA}$$

$$I_B = I_E - I_C = 1.785 - 1.76 \Rightarrow I_B = 25 \mu\text{A}$$

(b) $I_E = I_{Eo} \exp\left(\frac{V_{EB}}{V_T}\right)$

$$1.785 \times 10^{-3} = 2 \times 10^{-15} \exp\left(\frac{V_{EB}}{V_T}\right)$$

$$V_{EB} = (0.026) \ln\left(\frac{1.785 \times 10^{-3}}{2 \times 10^{-15}}\right) = 0.7154 \text{ V}$$

5.10

$$i_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right) = (5 \times 10^{-15}) \exp\left(\frac{0.615}{0.026}\right)$$

$$i_C = 93.7 \mu\text{A}$$

$$i_B = \frac{93.7}{125} = 0.7495 \mu\text{A}$$

$$i_E = (126)(0.7495) = 94.44 \mu\text{A}$$

5.11

$$\text{Device 1: } i_E = I_{Eo1} e^{v_{EB}/V_T} \Rightarrow 0.5 \times 10^{-3} = I_{Eo1} e^{0.650/0.026}$$

So that

$$\underline{I_{Eo1} = 6.94 \times 10^{-15} \text{ A}}$$

$$\text{Device 2: } 12.2 \times 10^{-3} = I_{Eo2} e^{0.650/0.026}$$

Or

$$\underline{I_{Eo2} = 1.69 \times 10^{-13} \text{ A}}$$

$$\text{Ratio of areas} = \frac{I_{Eo2}}{I_{Eo1}} = \frac{1.69 \times 10^{-13}}{6.94 \times 10^{-15}} \Rightarrow \underline{\text{Ratio} = 24.4}$$

5.12

For transistor A:

$$v_{BE}(A) = V_T \ln\left(\frac{I_C}{I_{SA}}\right) = (0.026) \ln\left(\frac{275 \times 10^{-6}}{8 \times 10^{-16}}\right) = 0.6906 \text{ V}$$

For transistor B:

$$I_{SB} = 4I_{SA} = 4(8 \times 10^{-16}) = 3.2 \times 10^{-15} \text{ A}$$

$$v_{BE}(B) = (0.026) \ln\left(\frac{275 \times 10^{-6}}{3.2 \times 10^{-15}}\right) = 0.6546 \text{ V}$$

5.13

$$(a) \quad i_C = I_{Co} \left(1 + \frac{v_{CE}}{V_A} \right)$$

$$0.6 = I_{Co} \left(1 + \frac{2}{80} \right) \Rightarrow I_{Co} = 0.58537 \text{ mA}$$

At $v_{CE} = 5 \text{ V}$

$$i_C = (0.58537) \left(1 + \frac{5}{80} \right) = 0.622 \text{ mA}$$

$$(b) \quad r_o = \frac{\Delta v_{CE}}{\Delta i_C} = \frac{5 - 2}{0.621956 - 0.60} \Rightarrow r_o = 137 \text{ k}\Omega$$

5.14

$$BV_{CE0} = \frac{BV_{CB0}}{\sqrt[3]{\beta}} = \frac{60}{\sqrt[3]{100}}$$

$$\underline{BV_{CE0} = 12.9 \text{ V}}$$

5.15

$$BV_{CE0} = \frac{BV_{CB0}}{\sqrt[3]{\beta}}$$

$$56 = \frac{220}{\sqrt[3]{\beta}} \Rightarrow \sqrt[3]{\beta} = \frac{220}{56} = 3.93$$

$$\underline{\beta = 60.6}$$

5.16

$$BV_{CE0} = \frac{BV_{CB0}}{\sqrt[3]{\beta}}$$

$$BV_{CB0} = (BV_{CE0})\sqrt[3]{\beta} = (50)\sqrt[3]{50}$$

$$\underline{BV_{CB0} = 184 \text{ V}}$$

5.17

$$I_E = \frac{-0.7 - (-10)}{5} = 1.86 \text{ mA}$$

$$I_C = (1.86) \left(\frac{75}{76} \right) = 1.836 \text{ mA}$$

$$V_C = -0.7 + 4 = 3.3 \text{ V}$$

$$(a) R_C = \frac{10 - 3.3}{1.836} \Rightarrow R_C = 3.65 \text{ K}$$

$$I_B = \frac{0.5}{76} = 0.00658 \text{ mA}$$

$$V_B = I_B R_B = (0.00658)(25) \Rightarrow V_B = 0.164 \text{ V}$$

$$I_C = (0.5) \left(\frac{75}{76} \right) = \underline{0.493 \text{ mA}}$$

$$(b) R_C = \frac{-1 - (-5)}{0.493} \Rightarrow R_C = 8.11 \text{ K}$$

$$O = \frac{I_E}{76} (10) + 0.7 + I_E (4) - 8$$

$$7.3 = I_E (4 + 0.132) \Rightarrow I_E = 1.767 \text{ mA}$$

$$I_C = (1.767) \left(\frac{75}{76} \right) = \underline{1.744 \text{ mA}}$$

$$V_{CE} = 8 - (1.744)(4) - [(1.767)(4) - 8]$$

$$(c) = 16 - 6.972 - 7.068 \Rightarrow V_{CE} = 1.96 \text{ V}$$

$$5 = I_E (10) + \left(\frac{I_E}{76} \right) (20) + 0.7 + I_E (2) = I_E (10 + 0.263 + 2) + 0.7$$

$$I_E = 0.3506 \text{ mA} \Rightarrow \underline{I_B = 4.61 \mu\text{A}} \quad V_C = 5 - (0.3506)(10)$$

$$(d) \quad \underline{V_C = 1.49 \text{ V}}$$

5.18

For Fig. P5.17(a), $R_E = 5 + 5\% = 5.25 \text{ k}\Omega$

$$I_E = \frac{-0.7 - (-10)}{5.25} = 1.77 \text{ mA}$$

$$I_C = 1.75 \text{ mA}$$

$$R_C = \frac{10 - 3.3}{1.75} = 3.83 \text{ K}$$

$$R_E = 5 - 5\% = 4.75 \text{ K}$$

$$I_E = \frac{-0.7 - (-10)}{4.75} = 1.96 \text{ mA}$$

$$I_C = 1.93 \text{ mA}$$

$$R_C = \frac{10 - 3.3}{1.93} = 3.47 \text{ K}$$

So $1.75 \leq I_C \leq 1.93 \text{ mA}$ $3.47 \leq R_C \leq 3.83 \text{ K}$

For Fig. P5.17(c), $R_E = 4 + 5\% = 4.2 \text{ k}\Omega$

$$I_B = \frac{8 - 0.7}{10 + (76)(4.2)} = 0.0222 \text{ mA} \quad \underline{I_C = 1.66 \text{ mA}}$$

$$I_E = 1.69 \text{ mA}$$

$$\begin{aligned} V_{CE} &= 16 - (1.66)(4) - (1.69)(4.2) \\ &= 16 - 6.64 - 7.098 \Rightarrow \underline{V_{CE} = 2.26 \text{ V}} \end{aligned}$$

$$R_E = 4 - 5\% = 3.8 \text{ K}$$

$$I_B = \frac{8 - 0.7}{10 + (76)(3.8)} = 0.0244 \quad I_C = 1.83 \text{ mA}$$

$$I_E = 1.86 \text{ mA}$$

$$\begin{aligned} V_{CE} &= 16 - (1.83)(4) - (1.86)(3.8) \\ &= 16 - 7.32 - 7.068 \end{aligned}$$

$$V_{CE} = 1.61 \text{ V}$$

So $1.66 \leq I_C \leq 1.83 \text{ mA}$ $1.61 \leq V_{CE} \leq 2.26 \text{ V}$

5.19

(a) $V_{CC} = I_C R_C + V_{CE}$

$$I_C = \frac{2.5 - 1.1}{4} = 0.35 \text{ mA}$$

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$V_{BE} = V_{BB} = (0.026) \ln\left(\frac{0.35 \times 10^{-3}}{5 \times 10^{-16}}\right) = 0.7091 \text{ V}$$

$$(b) I_E = \frac{V_{CC} - V_{CE}}{R_E} = \frac{2.5 - 1.1}{2} = 0.7 \text{ mA}$$

$$I_C = \left(\frac{\beta}{1 + \beta} \right) \cdot I_E = \left(\frac{90}{91} \right) (0.70) = 0.6923 \text{ mA}$$

$$V_{BE} = (0.026) \ln \left(\frac{0.6923 \times 10^{-3}}{5 \times 10^{-16}} \right) = 0.7269 \text{ V}$$

$$V_{BB} = V_{BE} + I_E R_E = 0.7269 + (0.7)(2) = 2.127 \text{ V}$$

5.20

$$(a) I_C = 0, V_{CE} = 2 \text{ V}$$

$$(b) I_C = \beta I_B = (120)(2) \Rightarrow I_C = 0.24 \text{ mA}$$

$$V_{CE} = 2 - (0.24)(4) = 1.04 \text{ V}$$

$$(c) I_E = \frac{1.4 - 0.7}{2} = 0.35 \text{ mA}$$

$$I_C = \left(\frac{120}{121} \right) (0.35) = 0.3471 \text{ mA}$$

$$V_{CE} = 2 - (0.3471)(4) - (0.35)(2) = -0.088 \text{ V} - \text{Not possible}$$

Transistor in Saturation

$$V_{CE} = 0.2 \text{ V}$$

$$V_E = 0.7 \text{ V} \Rightarrow V_C = 0.9 \text{ V}$$

$$I_C = \frac{2 - 0.9}{4} = 0.275 \text{ mA}$$

5.21

$$(a) I_E = \frac{2 - (0.7 + 0.2)}{1.5} = 0.7333 \text{ mA}$$

$$I_C = \left(\frac{\beta}{1 + \beta} \right) \cdot I_E = \left(\frac{120}{121} \right) (0.7333) = 0.7273 \text{ mA}$$

$$V_{EC} = V_E = 0.9 \text{ V}$$

$$(b) I_C = \beta I_B = (120)(15) \Rightarrow I_C = 1.8 \text{ mA} - \text{Not possible}$$

Transistor in saturation

$$V_{EC} = 0.2 \text{ V}$$

$$I_E = \frac{2 - 0.2}{1.5} = 1.2 \text{ mA}$$

$$I_C = I_E - I_B = 1.2 - 0.015 = 1.185 \text{ mA}$$

$$(c) \text{ Transistor cutoff}$$

$$I_C = 0, V_{EC} = 2 \text{ V}$$

5.22

$$(a) I_{BQ} = \frac{V_{BB} - V_{BE}(\text{on})}{R_B} \Rightarrow R_B = \frac{1.3 - 0.7}{0.005} = 120 \text{ k}\Omega$$

$$I_{CQ} = \beta I_{BQ} = (100)(0.005) = 0.5 \text{ mA}$$

$$R_C = \frac{3 - 1.5}{0.5} = 3 \text{ k}\Omega$$

$$(b) \text{ For } \beta = 75, I_{CQ} = (75)(0.005) = 0.375 \text{ mA}$$

$$V_{CE} = 3 - (0.375)(3) = 1.875 \text{ V}$$

$$\text{For } \beta = 125, I_{CQ} = (125)(0.005) = 0.625 \text{ mA}$$

$$V_{CE} = 3 - (0.625)(3) = 1.125 \text{ V}$$

$$\text{So } 1.125 \leq V_{CE} \leq 1.875 \text{ V}$$

5.23

(a)

$$V_B = -I_B R_B \Rightarrow I_B = \frac{-V_B}{R_B} = \frac{-(-1)}{500}$$

$$I_B = 2.0 \mu\text{A}$$

$$V_E = -1 - 0.7 = -1.7 \text{ V}$$

$$I_E = \frac{V_E - (-3)}{R_E} = \frac{-1.7 + 3}{4.8} = 0.2708 \text{ mA}$$

$$\frac{I_E}{I_B} = (1 + \beta) = \frac{0.2708}{0.002} = 135.4 \Rightarrow \underline{\beta = 134.4}$$

$$\alpha = \frac{\beta}{1 + \beta} \Rightarrow \underline{\alpha = 0.9926}$$

$$I_C = \beta I_B \Rightarrow \underline{I_C = 0.269 \text{ mA}}$$

$$V_{CE} = 3 - V_E = 3 - (-1.7) \Rightarrow \underline{V_{CE} = 4.7 \text{ V}}$$

(b)

$$I_E = \frac{5 - 4}{2} \Rightarrow I_E = 0.5 \text{ mA}$$

$$4 = 0.7 + I_B R_B + (I_B + I_C) R_C - 5$$

$$I_B + I_C = I_E$$

$$I_B + I_C = I_E$$

$$4 = 0.7 + I_B (100) + (0.5)(8) - 5$$

$$I_B = 0.043 \Rightarrow \frac{I_E}{I_B} = (1 + \beta) = \frac{0.5}{0.043} = 11.63$$

$$\underline{\beta = 10.63, \alpha = \frac{\beta}{1 + \beta} \Rightarrow \alpha = 0.9140}$$

5.24

$$(a) \quad I_C = \frac{5 - V_B}{10}, \quad I_E = \frac{(V_B - 0.7) + 5}{3}, \quad I_C = \left(\frac{\beta}{1 + \beta} \right) \cdot I_E = \left(\frac{90}{91} \right) \cdot I_E$$

$$\text{Then } \frac{5 - V_B}{10} = \left(\frac{90}{91} \right) \left(\frac{V_B + 4.3}{3} \right) \Rightarrow V_B = -2.136 \text{ V}$$

$$I_E = \frac{-2.136 - 0.7 + 5}{3} = 0.721 \text{ mA}$$

$$(b) \quad V_{CE} = 10 - I_C(10) - I_E(3)$$

$$2 = 10 - I_E \left[3 + \left(\frac{90}{91} \right)(10) \right] = 10 - I_E(12.89)$$

$$\text{Then } I_E = 0.6206 \text{ mA}$$

$$\text{And } V_B = 0.7 + (0.6206)(3) - 5 = -2.438 \text{ V}$$

5.25

$$(a) \quad I_E = \frac{3.3 - 0.85}{10} = 0.245 \text{ mA}$$

$$I_B = \frac{0.85 - 0.7}{50} \Rightarrow I_B = 3 \mu \text{A}$$

$$I_C = I_E - I_B = 0.245 - 0.003 = 0.242 \text{ mA}$$

$$\beta = \frac{I_C}{I_B} = \frac{0.242}{0.003} = 80.67$$

$$\alpha = \frac{\beta}{1 + \beta} = \frac{80.67}{81.67} = 0.9878$$

$$V_C = (0.242)(10) - 3.3 = -0.88 \text{ V}$$

$$V_{EC} = 0.85 - (-0.88) = 1.73 \text{ V}$$

$$(b) \quad \beta = (80.67)(1.10) = 88.73$$

$$I_B = \frac{V_E - 0.7}{50}, \quad I_E = \frac{3.3 - V_E}{10} = (88.73) \left(\frac{V_E - 0.7}{50} \right), \quad \Rightarrow V_E = 0.8371 \text{ V}$$

$$I_E = \frac{3.3 - 0.8371}{10} = 0.2463 \text{ mA}$$

$$I_C = \left(\frac{88.73}{89.73} \right)(0.2463) = 0.2435 \text{ mA}$$

$$V_C = (0.2435)(10) - 3.3 = -0.8645 \text{ V}$$

$$V_{EC} = 0.8371 - (-0.8645) = 1.70 \text{ V}$$

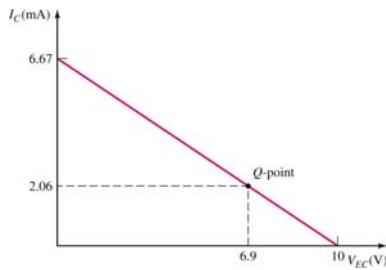
5.26

$$I_B = \frac{5 - 0.7}{250} \Rightarrow 17.2 \mu \text{A}$$

$$I_C = (120)(0.0172) = 2.064 \text{ mA}$$

$$V_C = (2.064)(1.5) - 5 = -1.90 \text{ V}$$

$$V_{EC} = 5 - (-1.90) \Rightarrow \underline{V_{EC} = 6.90 \text{ V}}$$



5.27

$$I_C = \left(\frac{50}{51} \right)(1) = 0.98 \text{ mA}$$

$$V_C = I_C R_C - 9 = (0.98)(4.7) - 9 \text{ or } V_C = -4.39 \text{ V}$$

$$I_B = \frac{1}{51} = 0.0196 \text{ mA}$$

$$V_E = I_B R_B + V_{EB} (\text{on}) = (0.0196)(50) + 0.7 \text{ or } V_E = 1.68 \text{ V}$$

5.28

$$I_C = \left(\frac{50}{51} \right)(0.5) = 0.49 \text{ mA}, \quad I_B = \frac{0.5}{51} = 0.0098 \text{ mA}$$

$$V_E = I_B R_B + V_{EB} (\text{on}) = (0.0098)(50) + 0.7 \text{ or } V_E = 1.19 \text{ V}$$

$$V_C = I_C R_C - 9 = (0.49)(4.7) - 9 = -6.70 \text{ V}$$

$$\text{Then } V_{EC} = V_E - V_C = 1.19 - (-6.7) = 7.89 \text{ V}$$

$$P_Q = I_C V_{EC} + I_B V_{EB} = (0.49)(7.89) + (0.0098)(0.7) \text{ or } P_Q = 3.87 \text{ mW}$$

$$\text{Power Dissipated} = P_S = I_Q (9 - V_E) = (0.5)(9 - 1.19)$$

$$\text{Or } P_S = 3.91 \text{ mW}$$

5.29

$$I_{E1} = I_{E2} = \frac{I}{2} \Rightarrow I_{E1} = I_{E2} = 0.5 \text{ mA}$$

$$I_{C1} = I_{C2} \approx 0.5 \text{ mA}$$

$$V_{C1} = V_{C2} = 5 - (0.5)(4) \Rightarrow V_{C1} = V_{C2} = 3 \text{ V}$$

5.30

$$R_E = 0 \quad I_B = \frac{2 - 0.7}{R_B} = \frac{1.3}{R_B}$$

$$I_C = (80) \left(\frac{1.3}{R_B} \right) = \frac{5 - 2}{R_C} = 0.8 \Rightarrow R_C = 3.75 \text{ K}$$

$$(a) \quad R_B = 130 \text{ K}$$

$$R_E = 1 \text{ K} \quad I_B = \frac{0.8}{80} = 0.010 \text{ mA} \quad I_E = 0.8 \left(\frac{81}{80} \right) = 0.81 \text{ mA}$$

$$2 = (0.010)(R_B) + 0.7 + (0.81)(1) \Rightarrow R_B = 49 \text{ K}$$

$$(b) \quad 5 = (0.8)R_C + 2 + (0.81)(1) \Rightarrow R_C = 2.74 \text{ K}$$

$$(c) \quad \text{For part (a)} \quad I_B = \frac{2 - 0.7}{130} = 0.01 \text{ mA}$$

$$I_C = (120)(0.01) \Rightarrow I_C = 1.20 \text{ mA}$$

$$V_{CE} = 5 - (1.2)(3.75) \Rightarrow V_{CE} = 0.5 \text{ V}$$

$$\text{For part (b)} \quad 2 = I_B (49) + 0.7 + (121)I_B (1)$$

$$I_B = 0.00765 \text{ mA}, I_E = 0.925 \text{ mA}, I_C = 0.918 \text{ mA}$$

$$V_{CE} = 5 - (0.918)(2.74) - (0.925)(1) \Rightarrow V_{CE} = 1.56 \text{ V}$$

Including R_E result in smaller changes in Q-point values.

5.31

$$(a) \quad R_C = \frac{V_{CC} - V_{CEQ}}{I_{CQ}} = \frac{9 - 4.5}{0.25} = 18 \text{ k}\Omega$$

$$I_{BQ} = \frac{0.25}{80} \Rightarrow I_{BQ} = 3.125 \mu\text{A}$$

$$R_B = \frac{9 - 0.7}{0.003125} \Rightarrow R_B = 2.656 \text{ M}\Omega$$

$$(b) \quad I_{CQ} = (120)(0.003125) = 0.375 \text{ mA}$$

$$V_{CEQ} = 9 - (0.375)(18) = 2.25 \text{ V}$$

5.32

$$(a) \quad I_C = I_E = 0, \quad V_C = 6 \text{ V}$$

$$(b) \quad I_E = \frac{0.9 - 0.7}{1} = 0.2 \text{ mA}, \quad I_C = \left(\frac{150}{151} \right)(0.2) = 0.1987 \text{ mA}$$

$$V_C = 6 - (0.1987)(10) = 4.013 \text{ V}$$

$$(c) \quad I_E = \frac{1.5 - 0.7}{1} = 0.8 \text{ mA}$$

Transistor in saturation

$$V_C = 1.5 - 0.7 + 0.2 = 1 \text{ V}$$

$$I_C = \frac{6 - 1}{10} = 0.5 \text{ mA}$$

$$(d) \quad I_E = \frac{2.2 - 0.7}{1} = 1.5 \text{ mA}$$

$$V_C = 2.2 - 0.7 + 0.2 = 1.7 \text{ V}$$

$$I_C = \frac{6 - 1.7}{10} = 0.43 \text{ mA}$$

5.33

$$V_{BB} = 0.$$

$$\text{Cutoff } V_0 = \left(\frac{R_L}{R_C + R_L} \right) V_{CC} = \left(\frac{10}{10+5} \right) (5)$$

a. $\underline{V_0 = 3.33 \text{ V}}$

$$V_{BB} = 1 \text{ V}$$

$$I_B = \frac{1 - 0.7}{50} \Rightarrow 6 \mu\text{A}$$

$$I_C = \beta I_B = (75)(6) \Rightarrow I_C = 0.45 \text{ mA}$$

$$\frac{5 - V_0}{5} = I_C + \frac{V_0}{10}$$

b. $1 - 0.45 = V_0 \left(\frac{1}{5} + \frac{1}{10} \right) \Rightarrow \underline{V_0 = 1.83 \text{ V}}$

c. Transistor in saturation $\underline{V_0 = V_{CE}(\text{sat}) = 0.2 \text{ V}}$

5.34

(a) $\beta = 100$

$$I_Q = 0.1 \text{ mA} \quad I_C = \left(\frac{100}{101} \right) (0.1) = 0.0990 \text{ mA}$$

(i) $V_o = 5 - (0.099)(5) \Rightarrow \underline{V_o = 4.505 \text{ V}}$

$$I_Q = 0.5 \text{ mA} \quad I_C = \left(\frac{100}{101} \right) (0.5) = 0.495 \text{ mA}$$

(ii) $V_o = 5 - (0.495)(5) \Rightarrow \underline{V_o = 2.525 \text{ V}}$

$I_Q = 2 \text{ mA}$ Transistor is in saturation

(iii) $V_o = -V_{BE}(\text{sat}) + V_{CE}(\text{sat}) = -0.7 + 0.2 \Rightarrow \underline{V_o = -0.5 \text{ V}}$

(b) $\beta = 150$

$$I_Q = 0.1 \text{ mA} \quad I_C = \left(\frac{150}{151} \right) (0.1) = 0.09934 \text{ mA}$$

$$V_o = 5 - (0.09934)(5) \Rightarrow \underline{V_o = 4.503 \text{ V}}$$

(i) $\% \text{ change} = \frac{4.503 - 4.505}{4.503} \times 100\% = \underline{-0.044\%}$

$$I_Q = 0.5 \text{ mA} \quad I_C = \left(\frac{150}{151} \right) (0.5) = 0.4967 \text{ mA}$$

$$V_o = 5 - (0.4967)(5) \Rightarrow \underline{V_o = 2.517 \text{ V}}$$

(ii) $\% \text{ change} = \frac{2.517 - 2.525}{2.525} \times 100\% = \underline{-0.32\%}$

$I_Q = 2 \text{ mA}$ Transistor in saturation

(iii) $\underline{V_o = -8.5 \text{ V}} \quad \underline{\text{No change}}$

5.35

$$(a) I_C = \frac{5-4}{5} = 0.20 \text{ mA}$$

$$I_Q = I_E = \left(\frac{121}{120} \right) (0.20) = 0.2017 \text{ mA}$$

$$(b) I_C = \frac{5-2}{5} = 0.60 \text{ mA}$$

$$I_Q = I_E = \left(\frac{121}{120} \right) (0.60) = 0.605 \text{ mA}$$

$$(c) I_C = \frac{5-0}{5} = 1.0 \text{ mA}$$

$$I_Q = I_E = \left(\frac{121}{120} \right) (1.0) = 1.008 \text{ mA}$$

5.36

For $I_Q = 0$, then $P_Q = 0$

$$\text{For } I_Q = 0.5 \text{ mA}, \quad I_C = \left(\frac{50}{51} \right) (0.5) = 0.49 \text{ mA}$$

$$I_B = \frac{0.5}{51} = 0.0098 \text{ mA}, \quad V_B = 0.490 \text{ V}, \quad V_E = 1.19 \text{ V}$$

$$V_C = (0.49)(4.7) - 9 = -6.70 \text{ V} \Rightarrow V_{EC} = 7.89 \text{ V}$$

$$P \equiv I_C V_{EC} = (0.49)(7.89) \Rightarrow P = 3.87 \text{ mW}$$

For $I_Q = 1.0 \text{ mA}$, Using the same calculations as above, we find $P = 5.95 \text{ mW}$

For $I_Q = 1.5 \text{ mA}$, $P = 6.26 \text{ mW}$

For $I_Q = 2 \text{ mA}$, $P = 4.80 \text{ mW}$

For $I_Q = 2.5 \text{ mA}$, $P = 1.57 \text{ mW}$

For $I_Q = 3 \text{ mA}$, Transistor is in saturation.

$$0.7 + I_B (50) = 0.2 + I_C (4.7) - 9$$

$$I_E = I_Q = I_B + I_C \Rightarrow I_B = 3 - I_C$$

$$\text{Then, } 0.7 + (3 - I_C)(50) = 0.2 + I_C (4.7) - 9$$

Which yields $I_C = 2.916 \text{ mA}$ and $I_B = 0.084 \text{ mA}$

$$P = I_B V_{EB} + I_C V_{EC} = (0.084)(0.7) + (2.916)(0.2) \quad \text{or} \quad P = 0.642 \text{ mW}$$

5.37

$$I_E = \frac{V_{EE} - V_{EB} (\text{on})}{R_E} = \frac{9 - 0.7}{4} \Rightarrow I_E = 2.075 \text{ mA}$$

$$I_C = \alpha I_E = (0.9920)(2.075) \Rightarrow I_C = 2.06 \text{ mA}$$

$$V_{BC} + I_C R_C = V_{CC}$$

$$V_{BC} = 9 - (2.06)(2.2) \Rightarrow V_{BC} = 4.47 \text{ V}$$

5.38

$$(a) \quad I_C = \frac{12 - 6}{2.2} = 2.727 \text{ mA}, \quad I_B = \frac{2.727}{80} = 0.03409 \text{ mA}$$

$$I_{R2} = \frac{0.7 - (-12)}{100} = 0.127 \text{ mA}$$

$$I_{R1} = I_B + I_{R2} = 0.1611 \text{ mA}$$

$$V_I = I_{R1} R_1 + 0.7 = (0.1611)(15) + 0.7 = 3.12 \text{ V}$$

$$(b) \quad \text{For } V_{CEQ} = 9 \text{ V}, \quad I_C = \frac{12 - 9}{2.2} = 1.364 \text{ mA}, \quad I_B = 0.01705 \text{ mA}$$

$$I_{R1} = 0.01705 + 0.127 = 0.14405 \text{ mA}$$

$$V_I = (0.14405)(15) + 0.7 = 2.86 \text{ V}$$

$$\text{For } V_{CEQ} = 3 \text{ V}, \quad I_C = \frac{12 - 3}{2.2} = 4.0909 \text{ mA}, \quad I_B = 0.05114 \text{ mA}$$

$$I_{R1} = 0.05114 + 0.127 = 0.1781 \text{ mA}$$

$$V_I = (0.1781)(15) + 0.7 = 3.37 \text{ V}$$

So $2.86 \leq V_I \leq 3.37 \text{ V}$

5.39

For $V_{CE} = 4.5$

$$I_{CQ} = \frac{5 - 4.5}{1} = 0.5 \text{ mA}$$

$$I_{BQ} = \frac{0.5}{25} = 0.02 \text{ mA}$$

$$I_{R2} = \frac{0.7 - (-5)}{100} = 0.057 \text{ mA}$$

$$I_{R1} = I_{R2} + I_{BQ} = 0.057 + 0.02 = 0.077 \text{ mA}$$

$$V_I = I_{R1} R_1 + V_{BE} (\text{on}) = (0.077)(15) + 0.7 = 1.86 \text{ V}$$

For $V_{CE} = 1.0$

$$I_{CQ} = \frac{5 - 1}{1} = 4 \text{ mA}$$

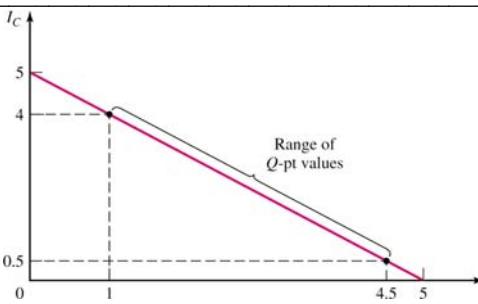
$$I_{BQ} = \frac{4}{25} = 0.16 \text{ mA}$$

$$I_{R2} = 0.057 \text{ mA}$$

$$I_{R1} = I_{R2} + I_{BQ} = 0.057 + 0.16 = 0.217 \text{ mA}$$

$$V_I = (0.217)(15) + 0.7 \Rightarrow 3.96 \text{ V}$$

So $\underline{1.86 \leq V_I \leq 3.96 \text{ V}}$

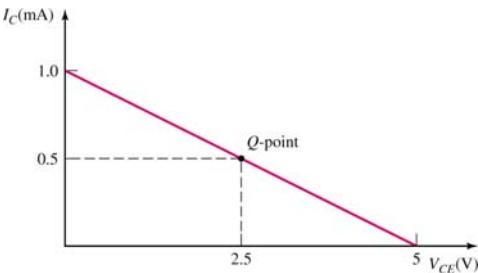


5.40

$$R_C = \frac{5 - 2.5}{0.5} = 5 \text{ K}$$

$$I_B = \frac{0.5}{120} = 0.00417 \text{ mA}$$

$$(a) R_B = \frac{5 - 0.7}{0.00417} = 1032 \text{ K}$$



Choose $R_C = 5.1 \text{ K}$

$$(b) R_B = 1 \text{ M}\Omega$$

For $R_B = 1 \text{ M}\Omega + 10\% = 1.1 \text{ M}$, $R_C = 5.1 \text{ k} + 10\% = 5.61 \text{ K}$

$$I_{BQ} = \frac{5 - 0.7}{1.1} = 3.91 \mu\text{A} \Rightarrow I_{CQ} = 0.469 \text{ mA}$$

$$V_{CEQ} = 2.37 \text{ V}$$

$$R_B = 1 \text{ M}\Omega + 10\% = 1.1 \text{ M}, R_C = 5.1 \text{ k} - 10\% = 4.59 \text{ K}$$

$$I_{BQ} = 3.91 \mu\text{A} \Rightarrow I_{CQ} = 0.469 \text{ mA}$$

$$V_{CEQ} = 2.85 \text{ V}$$

$$R_B = 1 \text{ M}\Omega - 10\% = 0.90 \text{ M}\Omega \quad R_C = 5.1 \text{ k} + 10\% = 5.61 \text{ K}$$

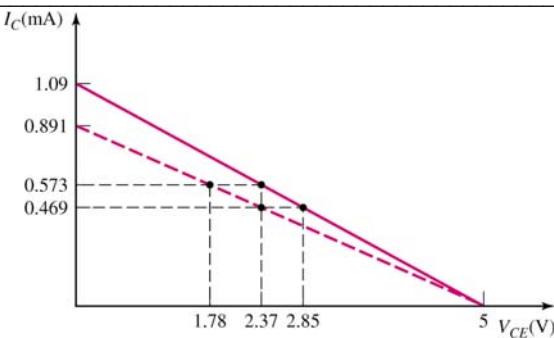
$$I_{BQ} = \frac{5 - 0.7}{0.90} = 4.78 \mu\text{A} \Rightarrow I_C = 0.573 \text{ mA}$$

$$V_{CEQ} = 1.78 \text{ V}$$

$$R_B = 1 \text{ M}\Omega - 10\% = 0.90 \text{ M}\Omega \quad R_C = 5.1 \text{ k} - 10\% = 4.59 \text{ K}$$

$$I_{BQ} = 4.78 \mu\text{A} \Rightarrow I_C = 0.573 \text{ mA}$$

$$V_{CEQ} = 2.37 \text{ V}$$



5.41

$$\begin{aligned} V_{E2} &= 5 - V_{BE2} & V_{E1} &= 5 - V_{BE1} \\ V_O &= V_{E2} - V_{E1} = (5 - V_{BE2}) - (5 - V_{BE1}) \\ V_O &= V_{BE1} - V_{BE2} \end{aligned}$$

We have $V_{BE1} = V_T \ln\left(\frac{I_{E1}}{I_{EO}}\right)$

$$V_{BE2} = V_T \ln\left(\frac{I_{E2}}{I_{EO}}\right)$$

$$V_O = V_T \left[\ln\left(\frac{I_{E1}}{I_{EO}}\right) - \ln\left(\frac{I_{E2}}{I_{EO}}\right) \right]$$

$$V_O = V_T \ln\left(\frac{I_{E1}}{I_{E2}}\right) = V_T \ln\left(\frac{10}{I}\right)$$

$$V_O = \frac{kT}{e} \ln(10)$$

5.42

(a) (i) $I_C = \frac{5-4}{4} = 0.25 \text{ mA}$, $I_B = \frac{0.25}{120} = 0.002083 \text{ mA}$

$$V_I = (0.002083)(200) + 0.7 = 1.117 \text{ V}$$

(ii) $I_C = 0.25 \text{ mA}$, $I_E = \left(\frac{121}{120}\right)(0.25) = 0.252 \text{ mA}$

$$V_I = (0.002083)(200) + 0.7 + (0.252)(1) = 1.369 \text{ V}$$

(b) (i) $I_C = \frac{5-2.5}{4} = 0.625 \text{ mA}$, $I_B = \frac{0.625}{120} = 0.005208 \text{ mA}$

$$V_I = (0.005208)(200) + 0.7 = 1.742 \text{ V}$$

(ii) $I_E = \left(\frac{121}{120}\right)(0.625) = 0.6302 \text{ mA}$

$$V_I = 1.742 + (0.6302)(1) = 2.372 \text{ V}$$

(c) Transistor biased in saturation

$$3.5 = I_B(200) + 0.7 + I_E(1)$$

$$I_C = \frac{5 - V_O}{4}, \quad I_E = \frac{V_O - 0.2}{1}$$

$$I_B = I_E - I_C = \frac{V_o - 0.2}{1} - \left(\frac{5 - V_o}{4} \right) = V_o(1.25) - 1.45$$

$$\text{Then } 3.5 = [V_o(1.25) - 1.45](200) + 0.7 + (V_o - 0.2)$$

$$V_o = 1.167 \text{ V}$$

5.43

For $4.3 \leq V_I \leq 5$ Q is cutoff $I_C = 0$

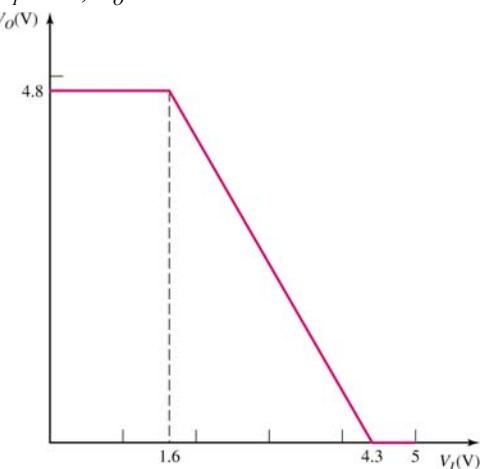
$$V_o = 0$$

If Q reaches saturation, $V_o = 4.8$

$$I_C = \frac{4.8}{4} = 1.2 \text{ mA}$$

$$I_B = \frac{1.2}{80} = 0.015 = \frac{5 - 0.7 - V_I}{180} \Rightarrow V_I = 1.6$$

So $V_I \leq 1.6$, $V_o = 4.8$



5.44

(a) For $V_I \geq 4.3$, Q is off and $V_o = 0$

$$5 = \left(\frac{101}{100}\right)I_c(1) + 0.2 + I_c(4) \Rightarrow I_c = 0.958 \text{ mA}$$

When transistor enters saturation,

$$V_o = 3.832 \text{ V}$$

$$I_B = 0.00958 \text{ mA}$$

$$5 = \left(\frac{101}{100}\right)(0.958)(1) + 0.7 + (0.00958)(180) + V_I$$

$$V_I = 5 - 0.7 - 0.9676 - 1.7244 \Rightarrow V_I = 1.61 \text{ V}$$

For $V_I = 0$, transistor in saturation

$$5 = I_E(1) + 0.2 + I_c(4) \Rightarrow 5 = I_c(1) + I_B(1) + 0.2 + I_c(4)$$

$$5 = I_E(1) + 0.7 + I_B(180) \quad 5 = I_c(1) + I_B(1) + 0.7 + I_B(180)$$

$$I_E = I_c + I_B$$

$$4.8 = 5I_c + I_B(1)$$

$$4.3 = 1I_c + 181I_B$$

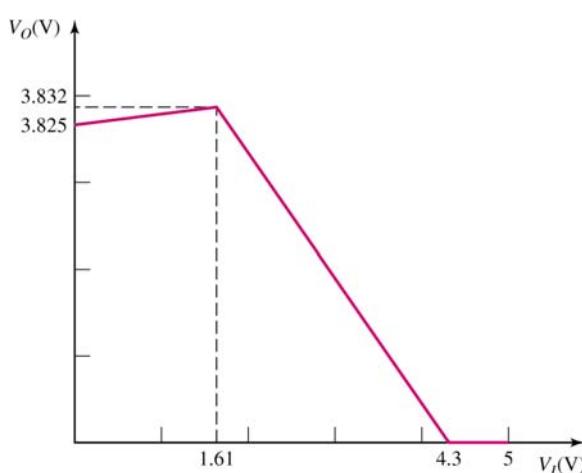
$$I_B = 4.8 - 5I_c$$

$$4.3 = I_c + (181)(4.8 - 5I_c)$$

$$904I_c = 864.5$$

$$I_c = 0.956 \text{ mA}$$

$$V_o = 3.825 \text{ V}$$



5.45

$$I_c = \frac{V_{CC} - V_{CE}(\text{sat})}{R_C} = \frac{5 - 0.2}{0.2} = 24 \text{ mA}$$

$$\frac{I_c}{I_B} = 20 \Rightarrow I_B = \frac{24}{20} = 1.2 \text{ mA}$$

$$I_B = \frac{V_I - V_{BE}(\text{on})}{R_B} \Rightarrow R_B = \frac{5 - 0.7}{1.2} = 3.58 \text{ k}\Omega$$

5.46

(a) $V_I = 0, \Rightarrow I_B = I_C = I_E = 0, V_O = 0$

(b) $V_I = 2.5 \text{ V}$,

$$V_I = I_B R_B + V_{BE}(\text{on}) + I_E R_E, \text{ and } I_E = (1 + \beta)I_B$$

$$\text{Then } I_B = \frac{V_I - V_{BE}(\text{on})}{R_B + (1 + \beta)R_E} = \frac{2.5 - 0.7}{10 + (51)(0.5)} \Rightarrow I_B = 50.7 \mu\text{A}$$

$$I_C = (50)(0.0507) = 2.535 \text{ mA}, \quad I_E = (51)(0.0507) = 2.586 \text{ mA}$$

$$V_O = I_E R_E = (2.586)(0.5) = 1.293 \text{ V}$$

(c) $V_I = 5 \text{ V}$, Transistor in saturation

$$V_O = 2.8 \text{ V}, \quad I_E = \frac{2.8}{0.5} = 5.6 \text{ mA}$$

$$I_B = \frac{V_I - V_{BE}(\text{on}) - V_O}{R_B} = \frac{5 - 0.7 - 2.8}{10} = 0.15 \text{ mA}$$

$$I_C = I_E - I_B = 5.6 - 0.15 = 5.45 \text{ mA}$$

5.47

$$I_C = \frac{V_O}{R_C} = \frac{8.8}{0.5} = 17.6 \text{ mA}$$

$$\frac{I_C}{I_B} = 25 \Rightarrow I_B = \frac{17.6}{25} = 0.704 \text{ mA}$$

$$I_B = \frac{9 - V_{EB}(\text{on}) - V_I}{R_B} \Rightarrow R_B = \frac{9 - 0.7 - 5}{0.704} = 4.69 \text{ k}\Omega$$

5.48

(a) $I_{CQ} = \frac{3 - 1.6}{2} = 0.7 \text{ mA}, \quad I_{BQ} = \frac{0.7}{120} \Rightarrow I_{BQ} = 5.833 \mu\text{A}$

$$R_B = \frac{V_{BB} - V_{BE}(\text{on})}{I_{BQ}} = \frac{1 - 0.7}{0.005833} = 51.4 \text{ k}\Omega$$

(b) $\Delta V_O = 3.3 - 0.2 = 2.8 \text{ V}$, peak-to-peak

(c) $\frac{\Delta V_O}{\Delta V_I} = \frac{3 - 1.6}{0.7 - 1.0} = A_v = -4.67$

(d) $\Delta V_I(\text{max}) = \frac{\Delta V_O(\text{max})}{|A_v|} = \frac{2.8}{4.667} = 0.6 \text{ V}$

So $v_i = 0.6 \text{ V}$, peak-to-peak

5.49

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.15}{120} \Rightarrow I_{BQ} = 1.25 \mu\text{A}$$

$$I_{EQ} = I_{CQ} \left(\frac{1 + \beta}{\beta} \right) = (0.15) \left(\frac{121}{120} \right) = 0.15125 \text{ mA}$$

We have $R_{TH} = 200 \text{ k}\Omega$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + I_{EQ}R_E = (0.00125)(200) + 0.7 + (0.15125)(2) = 1.2525 \text{ V}$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC}$$

$$\text{So } 1.2525 = \frac{1}{R_1} (200)(2.5)$$

$$\Rightarrow R_1 = 399 \text{ k}\Omega \text{ and } R_2 = 401 \text{ k}\Omega$$

$$V_{CEQ} = 2.5 - I_{CQ}R_C - I_{EQ}R_E = 2.5 - (0.15)(6) - (0.15125)(2) = 1.30 \text{ V}$$

5.50

$$I_{EQ} = \left(\frac{1+\beta}{\beta} \right) \cdot I_{CQ} = \left(\frac{151}{150} \right) (0.20) = 0.2013 \text{ mA}, \quad I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.20}{150} \Rightarrow I_{BQ} = 1.33 \mu \text{A}$$

$$V_{CC} = I_{CQ}R_C + V_{CEQ} + I_{EQ}R_E$$

$$2.5 = (0.20)R_C + 1.5 + (0.2013)(1) \Rightarrow R_C = 4 \text{ k}\Omega$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + I_{EQ}R_E = (0.00133)(120) + 0.7 + (0.2013)(1) = 1.061 \text{ V}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} \Rightarrow 1.061 = \frac{1}{R_1} (120)(2.5)$$

$$\text{So } R_1 = 283 \text{ k}\Omega \text{ and } R_2 = 208 \text{ k}\Omega$$

5.51

$$R_{TH} = R_1 \| R_2 = 20 \| 15 = 8.57 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left(\frac{15}{15+20} \right) (10) = 4.29 \text{ V}$$

$$V_{CC} = I_{EQ}R_E + V_{EB}(on) + \frac{I_{EQ}}{1+\beta} \cdot R_{TH} + V_{TH}$$

$$10 = I_{EQ}(1) + 0.7 + I_{EQ} \left(\frac{8.57}{101} \right) + 4.29$$

$$\text{Then } I_{EQ} = \frac{10 - 0.7 - 4.29}{1 + \frac{8.57}{101}} = \frac{5.01}{1.085} \Rightarrow I_{EQ} = 4.62 \text{ mA}$$

$$V_B = \frac{I_{EQ}}{1+\beta} \cdot R_{TH} + V_{TH} = \left(\frac{4.62}{101} \right) (8.57) + 4.29 \Rightarrow V_B = 4.68 \text{ V}$$

5.52

(a)

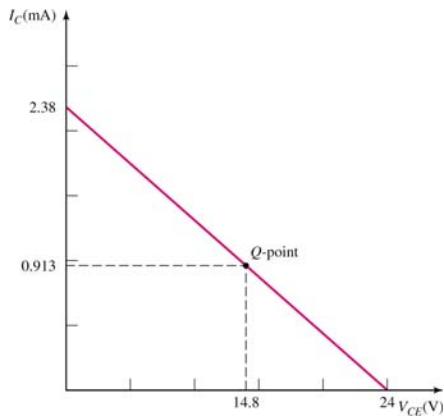
$$R_{TH} = 42 \| 58 = 24.36 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{42}{100} \right) (24) = 10.08 \text{ V}$$

$$I_{BQ} = \frac{10.08 - 0.7}{24.36 + (126)(10)} = \frac{9.38}{1284.36} \Rightarrow I_{BQ} = 7.30 \mu \text{A}$$

$$I_{CQ} = 0.913 \text{ mA}, \quad I_{EQ} = 0.9202 \text{ mA}$$

$$V_{CEQ} = 14.8 \text{ V}$$



(b)

$$R_1 + 5\% = 60.9, R_2 + 5\% = 44.1 \quad R_{TH} = 25.58 \text{ K} \quad V_{TH} = 10.08$$

$$I_{BQ} = \frac{10.08 - 0.7}{25.58 + 126(10)} = \frac{9.38}{1285.58} \Rightarrow 7.30 \mu\text{A}$$

$$I_{CQ} = 0.912 \text{ mA} \quad I_{EQ} = 0.919$$

$$V_{CEQ} = 14.81$$

$$R_1 + 5\% = 60.9, R_2 - 5\% = 39.90 \quad R_{TH} = 24.11 \text{ K} \quad V_{TH} = 9.50$$

$$I_{BQ} = \frac{9.50 - 0.7}{24.11 + (126)(10)} = \frac{8.8}{1284.11} = 6.85 \mu\text{A}$$

$$I_{CQ} = 0.857 \text{ mA} \quad I_{EQ} = 0.8635 \text{ mA}$$

$$V_{CEQ} = 15.37 \text{ V}$$

$$R_1 - 5\% = 55.1 \text{ K} \quad R_2 + 5\% = 44.1 \text{ K} \quad R_{TH} = 24.50 \text{ K} \quad V_{TH} = 10.67 \text{ V}$$

$$I_{BQ} = \frac{10.67 - 0.7}{24.50 + 1260} = \frac{9.97}{1284.5} = 7.76 \mu\text{A}$$

$$I_{CQ} = 0.970 \text{ mA} \quad I_{EQ} = 0.978 \text{ mA}$$

$$V_{CEQ} = 14.22 \text{ V}$$

$$R_1 - 5\% = 55.1 \text{ K} \quad R_2 - 5\% = 39.90 \quad R_{TH} = 23.14 \text{ K} \quad V_{TH} = 10.08$$

$$I_{BQ} = \frac{10.08 - 0.7}{23.14 + 1260} = \frac{9.38}{1283.14} = 7.31 \mu\text{A}$$

$$I_{CQ} = 0.914 \text{ mA} \quad I_{EQ} = 0.9211 \text{ mA}$$

$$V_{CEQ} = 14.79 \text{ V}$$

So we have $0.857 \leq I_{CQ} \leq 0.970 \text{ mA}$

$$14.22 \leq V_{CEQ} \leq 15.37 \text{ V}$$

5.53

$$(a) \quad R_{TH} = R_1 \| R_2 = 96 \| 24 = 19.2 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left(\frac{24}{24 + 96} \right) (9) = 1.80 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1 + \beta)R_E} = \frac{1.80 - 0.7}{19.2 + (81)(1)} \Rightarrow I_{BQ} = 10.98 \mu\text{A}$$

$$I_{CQ} = \beta I_{BQ} = (80)(0.01098) = 0.8782 \text{ mA}, \quad I_{EQ} = (81)(0.01098) = 0.8892 \text{ mA}$$

$$V_{CEQ} = 9 - (0.8782)(5.25) - (0.8892)(1) = 3.50 \text{ V}$$

$$(b) \quad I_{BQ} = \frac{1.80 - 0.7}{19.2 + (121)(1)} \Rightarrow I_{BQ} = 7.846 \mu\text{A}$$

$$I_{CQ} = (120)(0.007846) = 0.9415 \text{ mA}, \quad I_{EQ} = (121)(0.007846) = 0.9494 \text{ mA}$$

$$V_{CEQ} = 9 - (0.9415)(5.25) - (0.9494)(1) = 3.108 \text{ V}$$

$$\text{For } I_{CQ}: \left(\frac{0.9415 - 0.8782}{0.8782} \right) \times 100\% = 7.21\%$$

$$\text{For } V_{CEQ}: \left(\frac{3.108 - 3.500}{3.500} \right) \times 100\% = -11.2\%$$

5.54

(a)

$$I_{CQ} \cong I_{EQ} = 0.4 \text{ mA}$$

$$R_C = \frac{3}{0.4} \Rightarrow R_C = 7.5 \text{ k}\Omega; \quad R_E = \frac{3}{0.4} \Rightarrow R_E = 7.5 \text{ k}\Omega$$

$$R_1 + R_2 \cong \frac{9}{(0.2)(0.4)} = 112.5 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (V_{CC}) = I_{BQ} R_{TH} + V_{BE} (\text{on}) + (1 + \beta) I_{BQ} R_E$$

$$R_{TH} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(112.5 - R_2) R_2}{112.5}, \quad I_{BQ} = \frac{0.4}{100} = 0.004 \text{ mA}$$

$$R_2 \left(\frac{9}{112.5} \right) = (0.004) \left[\frac{(112.5 - R_2) R_2}{112.5} \right] + 0.7 + (101)(0.004)(7.5)$$

$$\text{We obtain } R_2 (0.08) = 0.004 R_2 - 3.56 \times 10^{-5} R_2^2 + 3.73$$

From this quadratic, we find $R_2 = 48 \text{ k}\Omega \Rightarrow R_1 = 64.5 \text{ k}\Omega$

(b) Standard resistor values:

Set $R_E = R_C = 7.5 \text{ k}\Omega$ and $R_1 = 62 \text{ k}\Omega, R_2 = 47 \text{ k}\Omega$

$$\text{Now } R_{TH} = R_1 \| R_2 = 62 \| 47 = 26.7 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (V_{CC}) = \left(\frac{47}{47 + 62} \right) (9) = 3.88 \text{ V}$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE} (\text{on}) + (1 + \beta) I_{BQ} R_E$$

$$\text{So } I_{BQ} = \frac{3.88 - 0.7}{26.7 + (101)(7.5)} = 0.00406 \text{ mA}$$

$$\text{Then } I_{CQ} = 0.406 \text{ mA}$$

$$V_{RC} = V_{RE} = (0.406)(7.5) = 3.05 \text{ V}$$

5.55

(a)

$$R_{TH} = R_1 \parallel R_2 = 12 \parallel 2 = 1.714 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5 = \left(\frac{2}{14} \right) (10) - 5 \Rightarrow V_{TH} = -3.571 \text{ V}$$

(b)

$$V_{TH} = I_{BQ} R_{TH} + V_{BE}(\text{on}) + (1 + \beta) I_{BQ} R_E - 5$$

$$-3.57 = I_{BQ} (1.714) + 0.7 + (101) I_{BQ} (0.5) - 5$$

$$I_{BQ} = \frac{5 - 0.7 - 3.571}{1.714 + (101)(0.5)} = \frac{0.729}{52.21} \Rightarrow 13.96 \mu\text{A}$$

$$I_{CQ} = 1.396 \text{ mA}, I_{EQ} = 1.410 \text{ mA}$$

$$V_{CEQ} = 10 - (1.396)(5) - (1.41)(0.5) \Rightarrow V_{CEQ} = 2.32 \text{ V}$$

(d)

$$R_E = 0.5 + 5\% = 0.525 \text{ K} \quad R_C = 5 + 5\% = 5.25 \text{ K}$$

$$I_{BQ} = \frac{0.729}{1.714 + (101)(0.525)} \Rightarrow 13.32 \mu\text{A}$$

$$\underline{I_{CQ} = 1.332 \text{ mA}} \quad I_{EQ} = 1.345 \text{ mA}$$

$$V_{CEQ} = 10 - (1.332)(5.25) - (1.345)(0.525)$$

$$= 10 - 6.993 - 0.7061 \Rightarrow \underline{V_{CEQ} = 2.30 \text{ V}}$$

$$R_E = 0.5 + 5\% = 0.525 \text{ K} \quad R_C = 5 - 5\% = 4.75 \text{ K}$$

$$\underline{I_{CQ} = 1.332 \text{ mA}} \quad I_{EQ} = 1.345 \text{ mA}$$

$$V_{CEQ} = 10 - (1.332)(4.75) - (1.345)(0.525)$$

$$= 10 - 6.327 - 0.7061 \Rightarrow \underline{V_{CEQ} = 2.97 \text{ V}}$$

$$R_E = 0.5 - 5\% = 0.475 \text{ K} \quad R_C = 5 + 5\% = 5.25 \text{ K}$$

$$I_{BQ} = \frac{0.729}{1.714 + (101)(0.475)} \Rightarrow 14.67 \mu\text{A}$$

$$\underline{I_{CQ} = 1.467 \text{ mA}} \quad I_{EQ} = 1.482 \text{ mA}$$

$$V_{CEQ} = 10 - (1.467)(5.25) - (1.482)(0.475)$$

$$= 10 - 7.70175 - 0.70395 \Rightarrow \underline{V_{CEQ} = 1.59 \text{ V}}$$

$$R_E = 0.5 - 5\% = 0.475 \text{ K} \quad R_C = 5 - 5\% = 4.75 \text{ K}$$

$$\underline{I_{CQ} = 1.467 \text{ mA}} \quad I_{EQ} = 1.482 \text{ mA}$$

$$V_{CEQ} = 10 - (1.467)(4.75) - (1.482)(0.475)$$

$$= 10 - 6.96825 - 0.70395 \Rightarrow \underline{V_{CEQ} = 2.33 \text{ V}}$$

5.56

(a) $R_{TH} = R_1 \parallel R_2 = 40 \parallel 40 = 20 \text{ k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V^+ = \left(\frac{40}{40 + 40} \right) (2.5) = 1.25 \text{ V}$$

$$V^+ = I_{EQ} R_E + V_{EB}(\text{on}) + I_{BQ} R_{TH} + V_{TH}$$

$$\text{So } I_{BQ} = \frac{V^+ - V_{EB}(on) - V_{TH}}{R_{TH} + (1+\beta)R_E} = \frac{2.5 - 0.7 - 1.25}{20 + (91)(0.7)} \Rightarrow I_{BQ} = 6.57 \mu\text{A}$$

$$I_{CQ} = 0.5914 \text{ mA}, \quad I_{EQ} = 0.5980 \text{ mA}$$

$$V_{ECQ} = 2.5 - (0.5914)(1.6) - (0.5980)(0.7) = 1.135 \text{ V}$$

$$(b) \quad I_{BQ} = \frac{2.5 - 0.7 - 1.25}{20 + (151)(0.7)} \Rightarrow I_{BQ} = 4.375 \mu\text{A}$$

$$I_{CQ} = 0.6563 \text{ mA}, \quad I_{EQ} = 0.6607 \text{ mA}$$

$$V_{ECQ} = 2.5 - (0.6563)(1.6) - (0.6607)(0.7) = 0.9874 \text{ V}$$

$$\text{For } I_{CQ}: \left(\frac{0.6563 - 0.5914}{0.5914} \right) \times 100\% = 10.97\%$$

$$\text{For } V_{ECQ}: \left(\frac{0.9874 - 1.135}{1.135} \right) \times 100\% = -13.0\%$$

5.57

(a)

$$R_{TH} = 36 \parallel 68 = 23.5 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{36}{36+68} \right)(10) = 3.46 \text{ V}$$

$$I_{BQ} = \frac{3.46 - 0.7}{23.5 + (51)(30)} = 0.00178 \text{ mA}$$

$$I_{CQ} = 0.0888 \text{ mA}, \quad I_{EQ} = 0.0906 \text{ mA}$$

$$V_{CEQ} = 10 - (0.0888)(42) - (0.0906)(30) \Rightarrow V_{CEQ} = 3.55 \text{ V}$$

(b)

$$R_1 = 22.7, R_2 = 12 \text{ K}, R_C = 14 \text{ K}, R_E = 10 \text{ K}$$

$$R_{TH} = 7.85 \text{ k} \quad V_{TH} = 3.46$$

$$I_{BQ} = \frac{3.46 - 0.7}{7.85 + (51)(10)} = 0.00533 \text{ mA}$$

$$I_{CQ} = 0.266 \text{ mA} \quad I_{EQ} = 0.272 \text{ mA}$$

$$V_{CE} = 10 - (0.266)(14) - (0.272)(10)$$

$$\underline{V_{CE} = 3.56 \text{ V}}$$

5.58

(a)

$$R_{TH} = 36 \parallel 68 = 23.5 \text{ k}\Omega ; V_{TH} = \left(\frac{68}{36+68} \right) (10) - 5 = 1.54 \text{ V}$$

$$5 = (51)I_{BQ}(30) + 0.7 + I_{BQ}(23.5) + 1.54$$

$$I_{BQ} = \frac{2.76}{1553.5} = 1.78 \mu\text{A} \Rightarrow I_{CQ} = 0.0888 \text{ mA}$$

$$I_{EQ} = 0.0906 \text{ mA}$$

$$\begin{aligned} V_{ECQ} &= 10 - (0.0906)(30) - (0.0888)(42) \\ &= 10 - 2.718 - 3.7296 \Rightarrow \underline{V_{ECQ} = 3.55 \text{ V}} \end{aligned}$$

(b)

$$R_{TH} = 12 \parallel 22.7 = 7.85 \text{ k}\Omega$$

$$V_{TH} = 1.54 \text{ V}, R_E = 10 \text{ k}\Omega, R_C = 14 \text{ k}\Omega$$

$$5 = (51)I_{BQ}(10) + 0.7 + I_{BQ}(7.85) + 1.54$$

$$I_{BQ} = \frac{2.76}{517.85} \Rightarrow 5.33 \mu\text{A}, I_{CQ} = 0.266 \text{ mA}, I_{EQ} = 0.272 \text{ mA}$$

$$V_{ECQ} = 10 - (0.272)(10) - (0.266)(14) = 3.56 \text{ V}$$

5.59

(a)

$$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(101)(0.5) = 5.05 \text{ k}\Omega$$

$$V_{TH} = \frac{1}{R_i} \cdot R_{TH} \cdot V_{CC} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1+\beta)I_{BQ}R_E$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.8}{100} = 0.008 \text{ mA}$$

$$\text{Then } \frac{1}{R_i}(5.05)(10) = (0.008)(5.05) + 0.7 + (101)(0.008)(0.5)$$

$$\text{or } R_i = 44.1 \text{ k}\Omega, \frac{44.1R_2}{44.1+R_2} = 5.05 \Rightarrow \underline{R_2 = 5.70 \text{ k}\Omega}$$

$$\text{Now } I_{EQ} = \left(\frac{101}{100} \right)(0.8) = 0.808 \text{ mA}$$

$$V_{CC} = I_{CQ}R_C + V_{CEQ} + I_{EQ}R_E$$

$$10 = (0.8)R_C + 5 + (0.808)(0.5)$$

$$\underline{R_C = 5.75 \text{ k}\Omega}$$

(b) For $75 \leq \beta \leq 150$

$$V_{TH} = \left(\frac{R_2}{R_i + R_2} \right) (V_{CC}) = \left(\frac{5.7}{5.7 + 44.1} \right) (10) = 1.145 \text{ V}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1+\beta)I_{BQ}R_E$$

For $\beta = 75$, $I_{BQ} = \frac{1.145 - 0.7}{5.05 + (76)(0.5)} = 0.0103 \text{ mA}$

Then $I_{CQ} = (75)(0.0103) = 0.775 \text{ mA}$

For $\beta = 150$, $I_{BQ} = \frac{1.145 - 0.7}{5.05 + (151)(0.5)} = 0.00552 \text{ mA}$

Then $I_{CQ} = 0.829 \text{ mA}$

$$\% \text{ Change} = \frac{\Delta I_{CQ}}{I_{CQ}} = \frac{0.829 - 0.775}{0.80} \times 100\% \Rightarrow \% \text{ Change} = 6.75\%$$

(c) For $R_E = 1 \text{ k}\Omega$

$$R_{TH} = (0.1)(101)(1) = 10.1 \text{ k}\Omega$$

$$V_{TH} = \frac{1}{R_i} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_i}(10.1)(10) = (0.008)(10.1) + 0.7 + (101)(0.008)(1)$$

which yields $R_i = 63.6 \text{ k}\Omega$

$$\text{And } \frac{63.6R_2}{63.6 + R_2} = 10.1 \Rightarrow R_2 = 12.0 \text{ k}\Omega$$

Now $V_{TH} = \left(\frac{R_2}{R_i + R_2} \right) (V_{CC}) = \left(\frac{12}{12 + 63.6} \right) (10) = 1.587 \text{ V}$

For $\beta = 75$, $I_{BQ} = \frac{1.587 - 0.7}{10.1 + (76)(1)} = 0.0103 \text{ mA}$

So $I_{CQ} = 0.773 \text{ mA}$

For $\beta = 150$, $I_{BQ} = \frac{1.587 - 0.7}{10.1 + (151)(1)} = 0.00551 \text{ mA}$

Then $I_{CQ} = 0.826 \text{ mA}$

$$\% \text{ Change} = \frac{\Delta I_{CQ}}{I_{CQ}} = \frac{0.826 - 0.773}{0.8} \times 100\% \Rightarrow \% \text{ Change} = 6.63\%$$

5.60

$$V_{CC} \approx I_{CQ} (R_C + R_E) + V_{CEO}$$

$$10 = (0.8)(R_C + R_E) + 5 \Rightarrow R_C + R_E = 6.25 \text{ k}\Omega$$

Let $R_E = 0.875 \text{ k}\Omega$

Then, for bias stable $R_{TH} = (0.1)(121)(0.875) = 10.6 \text{ k}\Omega$

$$I_{BQ} = \frac{0.8}{120} = 0.00667 \text{ mA}$$

$$\frac{1}{R_i}(10.6)(10) = (0.00667)(10.6) + 0.7 + (121)(0.00667)(0.875)$$

$$\text{So } R_i = 71.8 \text{ k}\Omega \text{ and } \frac{71.8R_2}{71.8 + R_2} = 10.6 \Rightarrow R_2 = 12.4 \text{ k}\Omega$$

Then $I_R \cong \frac{10}{71.8+12.4} = 0.119 \text{ mA}$

This is close to the design specification.

5.61

$$I_{CQ} \approx I_{EQ} \Rightarrow V_{CEQ} = V_{CC} - I_{CQ}(R_C + R_E)$$

$$6 = 12 - I_{CQ}(2 + 0.2)$$

$$\underline{I_{CQ} = 2.73 \text{ mA}}, \quad \underline{I_{BQ} = 0.0218 \text{ mA}}$$

$$V_{CEQ} = 6 \text{ V}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E - 6$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right)(12) - 6, \quad R_{TH} = R_1 \| R_2$$

$$\text{Bias stable} \Rightarrow R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(126)(0.2) = 2.52 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{1}{R_1} \right)(R_{TH})(12) - 6$$

$$\frac{1}{R_1}(2.52)(12) - 6 = (0.0218)(2.52) + 0.7 + (126)(0.0218)(0.2) - 6$$

$$\frac{1}{R_1}(30.24) = 0.7549 + 0.5494$$

$$R_1 = 23.2 \text{ k}\Omega, \quad \frac{23.2R_2}{23.2 + R_2} = 2.52$$

$$\underline{R_2 = 2.83 \text{ k}\Omega}$$

5.62

(a) $R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(121)(0.2) = 2.42 \text{ k}\Omega$

$$V_{CEQ} = 6 - I_{CQ} \left[R_C + \left(\frac{1 + \beta}{\beta} \right) R_E \right] = 6 - I_{CQ}(2.202)$$

$$2.8 = 6 - I_{CQ}(2.202) \Rightarrow I_{CQ} = 1.453 \text{ mA}$$

Then $I_{EQ} = 1.465 \text{ mA}, \quad I_{BQ} = 12.11 \mu\text{A}$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + I_{EQ}R_E - 3 = (0.01211)(2.42) + 0.7 + (1.465)(0.2) - 3$$

$$\text{Then } V_{TH} = -1.978 \text{ V} = \frac{1}{R_1} \cdot R_{TH}(6) - 3 = \frac{1}{R_1}(2.42)(6) - 3$$

Which yields $R_1 = 14.2 \text{ k}\Omega$ and $R_2 = 2.92 \text{ k}\Omega$

(b) For $R_1 = (1.05)(14.2) = 14.91 \text{ k}\Omega$

$$R_2 = (0.95)(2.92) = 2.774 \text{ k}\Omega$$

$$R_{TH} = R_1 \| R_2 = 2.34 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{2.774}{2.774 + 14.91} \right)(6) - 3 = -2.059 \text{ V}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E - 3$$

So $I_{BQ} = \frac{3 - 2.059 - 0.7}{2.34 + (121)(0.2)} \Rightarrow I_{BQ} = 9.08 \mu A$

$I_{CQ} = 1.090 \text{ mA}, I_{EQ} = 1.099 \text{ mA}$

$V_{CEQ} = 6 - (1.09)(2) - (1.099)(0.2) = 3.60 \text{ V}$

For $R_1 = (0.95)(14.2) = 13.49 \text{ k}\Omega$

$R_2 = (1.05)(2.92) = 3.066 \text{ k}\Omega$

$R_{TH} = R_1 \parallel R_2 = 2.50 \text{ k}\Omega$

$V_{TH} = \left(\frac{3.066}{3.066 + 13.49} \right)(6) - 3 = -1.889 \text{ V}$

$I_{BQ} = \frac{3 - 1.889 - 0.7}{2.50 + (121)(0.2)} \Rightarrow I_{BQ} = 15.39 \mu A$

$I_{CQ} = 1.847 \text{ mA}, I_{EQ} = 1.863 \text{ mA}$

$V_{CEQ} = 6 - (1.847)(2) - (1.863)(0.2) = 1.933 \text{ V}$

So $1.09 \leq I_{CQ} \leq 1.847 \text{ mA}$

$1.933 \leq V_{CEQ} \leq 3.60 \text{ V}$

5.63

$V_{CEQ} \cong V_{CC} - I_{CQ}(R_C + R_E)$

$5 = 12 - 3(R_C + R_E) \Rightarrow R_C + R_E = 2.33 \text{ k}\Omega$

Let $\underline{R_E = 0.333 \text{ k}\Omega}$ and $\underline{R_C = 2 \text{ k}\Omega}$

Nominal value of $\beta = 100$

$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(101)(0.333) = 3.36 \text{ k}\Omega$

$I_{BQ} = \frac{3}{100} = 0.03 \text{ mA}$

$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot (12) - 6 = \frac{1}{R_1}(3.36)(12) - 6$

Then $V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E - 6$

$\frac{1}{R_1}(3.36)(12) - 6 = (0.03)(3.36) + 0.7 + (101)(0.03)(0.333) - 6$

which yields $\underline{R_1 = 22.3 \text{ k}\Omega}$ and $\underline{R_2 = 3.96 \text{ k}\Omega}$

Now $V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right)(12) - 6 = \left(\frac{3.96}{3.96 + 22.3} \right)(12) - 6$ or $V_{TH} = -4.19 \text{ V}$

For $\beta = 75$, $V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E - 6$

$I_{BQ} = \frac{V_{TH} + 6 - 0.7}{R_{TH} + (1 + \beta)R_E} = \frac{-4.19 + 6 - 0.7}{3.36 + (76)(0.333)} = 0.0387 \text{ mA} \Rightarrow \underline{I_C = 2.90 \text{ mA}}$

$\beta = 150$, $I_{BQ} = \frac{-4.19 + 6 - 0.7}{3.36 + (151)(0.333)} = 0.0207 \text{ mA}$

For

Then $\underline{I_C = 3.10 \text{ mA}}$

Specifications are met.

5.64

$$(a) V^+ = V_{ECQ} + I_{CQ} \left[R_C + \left(\frac{1+\beta}{\beta} \right) R_E \right]$$

$$3.3 = 1.5 + I_{CQ} (3.022) \Rightarrow I_{CQ} = 0.5956 \text{ mA}$$

$$I_{BQ} = 6.618 \mu \text{A}, \quad I_{EQ} = 0.6022 \text{ mA}$$

$$V^+ = I_{EQ} R_E + V_{EB} (\text{on}) + I_{BQ} R_{TH} + V_{TH}$$

$$3.3 = (0.6022)(2) + 0.7 + (0.006618)(2.4) + V_{TH}$$

$$\text{So } V_{TH} = 1.380 \text{ V} = \frac{1}{R_1} \cdot R_{TH} \cdot V^+ = \frac{1}{R_1} (2.4)(3.3)$$

Which yields $R_1 = 5.74 \text{ k}\Omega$ and $R_2 = 4.12 \text{ k}\Omega$

$$(b) I_{BQ} = \frac{V^+ - V_{EB} (\text{on}) - V_{TH}}{R_{TH} + (1+\beta)R_E} = \frac{3.3 - 0.7 - 1.38}{2.4 + (131)(2)} \Rightarrow I_{BQ} = 4.61 \mu \text{ A}$$

$$I_{CQ} = 0.60 \text{ mA}, \quad I_{EQ} = 0.6045 \text{ mA}$$

$$V_{ECQ} = 3.3 - (0.60)(1) - (0.6045)(2) = 1.49 \text{ V}$$

5.65

$$I_{CQ} = 4.8 \text{ mA} \rightarrow I_{EQ} = 4.84 \text{ mA}$$

$$V_{CEQ} = V_{CC} - I_{CQ} R_C - I_{EQ} R_E$$

$$6 = 18 - (4.8)(2) - (4.84)R_E \Rightarrow R_E = 0.496 \text{ k}\Omega$$

$$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(121)(0.496) = 6.0 \text{ k}\Omega$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE} (\text{on}) + (1+\beta)I_{BQ} R_E$$

$$I_{BQ} = 0.040 \text{ mA}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (6.0)(18)$$

$$\frac{1}{R_1} (6.0)(18) = (0.04)(6.0) + 0.70 + (121)(0.04)(0.496)$$

$$\frac{1}{R_1} (108) = 3.34$$

$$\underline{R_1 = 32.3 \text{ k}\Omega}, \quad \frac{32.3 R_2}{32.3 + R_2} = 6.0$$

$$\underline{R_2 = 7.37 \text{ k}\Omega}$$

5.66

$$\text{For } I_{EQ} \cong I_{CQ}, \quad R_C + R_E = \frac{V_{CC} - V_{CEQ}}{I_{CQ}} = \frac{2.5 - 1.6}{0.2} = 4.5 \text{ k}\Omega$$

$$\text{So } R_E = 0.5 \text{ k}\Omega$$

$$\text{For } \beta = 100$$

$$R_{TH} = (0.1)(101)(0.5) = 5.05 \text{ k}\Omega$$

$$I_{BQ} = \frac{0.2}{100} \Rightarrow I_{BQ} = 2 \mu \text{A}, \quad I_{EQ} = 0.202 \text{ mA}$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE} (\text{on}) + I_{EQ} R_E = (0.002)(5.05) + 0.7 + (0.202)(0.5) = 0.8111 \text{ V}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} \Rightarrow 0.8111 = \frac{1}{R_1} (5.05)(2.5)$$

So $R_1 = 15.6 \text{ k}\Omega$ and $R_2 = 7.47 \text{ k}\Omega$

For $\beta = 80$,

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1+\beta)R_E} = \frac{0.8111 - 0.7}{5.05 + (81)(0.5)} \Rightarrow I_{BQ} = 2.439 \mu\text{A}, \quad I_{CQ} = 0.1951 \text{ mA}$$

For $\beta = 120$,

$$I_{BQ} = \frac{0.8111 - 0.7}{5.05 + (121)(0.5)} \Rightarrow I_{BQ} = 1.695 \mu\text{A}, \quad I_{CQ} = 0.2034 \text{ mA}$$

Design is valid

5.67

$$I_{CQ} = 1 \text{ mA} \rightarrow I_{EQ} = 1.017 \text{ mA}$$

$$V_{CEQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E$$

$$5 = 15 - (1)(5) - (1.017)R_E \Rightarrow R_E = 4.92 \text{ k}\Omega$$

$$\text{Bias stable: } R_{TH} = (0.1)(1+\beta)R_E = (0.1)(61)(4.92) = 30.0 \text{ k}\Omega$$

$$I_{BQ} = \frac{1}{60} = 0.0167 \text{ mA}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + I_{EQ}R_E$$

$$\frac{1}{R_1}(30.0)(15) = (0.0167)(30.0) + 0.70 + (1.017)(4.92)$$

$$\frac{1}{R_1}(448.5) = 6.197$$

$$R_1 = 72.5 \text{ k}\Omega, \quad \frac{72.5R_2}{72.5 + R_2} = 30.0$$

$$\underline{R_2 = 51.2 \text{ k}\Omega}$$

Check: For $\beta = 45$

$$V_{TH} = \left(\frac{51.2}{51.2 + 72.5} \right)(15) = 6.21 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1+\beta)R_E} = \frac{6.21 - 0.7}{30 + (46)(4.92)} = 0.0215 \text{ mA}$$

$$\underline{I_{CQ} = 0.967 \text{ mA}, \quad \frac{\Delta I_C}{I_C} = 3.27\%}$$

Check: For $\beta = 75$

$$I_{BQ} = \frac{6.21 - 0.7}{30.0 + (76)(4.92)} = 0.0136 \text{ mA}$$

$$\underline{I_{CQ} = 1.023 \text{ mA}, \quad \frac{\Delta I_C}{I_C} = 2.31\%}$$

Design criterion is satisfied.

5.68

(a)

$$\begin{aligned} V_{CC} &\equiv I_{CQ}(R_C + R_E) + V_{CEO} \\ 3 &= (0.1)(5R_E + R_E) + 1.4 \Rightarrow R_E = 2.67 \text{ k}\Omega \\ R_C &= 13.3 \text{ k}\Omega, \quad I_{BQ} = \frac{100}{120} = 0.833 \mu\text{A} \\ R_{TH} &= (0.1)(1 + \beta)R_E = (0.1)(121)(2.67) = 32.3 \text{ k}\Omega \\ V_{TH} &= \frac{1}{R_l} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_l}(32.3)(3) \\ &= I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E \\ &= (0.000833)(32.3) + 0.7 + (121)(0.000833)(2.67) \end{aligned}$$

which gives $R_l = 97.3 \text{ k}\Omega$, and $R_2 = 48.4 \text{ k}\Omega$

(b)

$$\begin{aligned} I_R &\equiv \frac{3}{R_l + R_2} = \frac{3}{97.3 + 48.4} \Rightarrow 20.6 \mu\text{A} \\ I_{CQ} &= 100 \mu\text{A} \\ P &= (I_{CQ} + I_R)V_{CC} = (100 + 20.6)(3) \\ \text{or } P &= 362 \mu\text{W} \end{aligned}$$

5.69

$$\begin{aligned} I_E &= \frac{5 - V_E}{R_E} = \frac{5}{3} = 1.67 \text{ mA} \\ R_{TH} &= R_l \parallel R_2 = (0.1)(1 + \beta)R_E = (0.1)(101)(3) = 30.3 \text{ k}\Omega \\ V_{TH} &= \left(\frac{R_2}{R_l + R_2} \right)(4) - 2 = \frac{1}{R_l} \cdot R_{TH} \cdot (4) - 2 \\ I_{BQ} &= \frac{I_{EQ}}{1 + \beta} = 0.0165 \text{ mA} \\ 5 &= I_{EQ}R_E + V_{EB}(\text{on}) + I_BR_{TH} + V_{TH} \\ 5 &= (1.67)(3) + 0.7 + (0.0165)(30.3) + \frac{1}{R_l}(30.3)(4) - 2 \\ 0.80 &= \frac{1}{R_l}(30.3)(4) \Rightarrow R_l = 152 \text{ k}\Omega \\ \frac{152R_2}{152 + R_2} &= 30.3 \Rightarrow R_2 = 37.8 \text{ k}\Omega \end{aligned}$$

5.70

a. $R_{TH} = R_1 \parallel R_2 = 10 \parallel 20 = 6.67 \text{ k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5 = \left(\frac{20}{20+10} \right) (10) - 5 = 1.67 \text{ V}$$

b.

$$10 = (1 + \beta) I_{BQ} R_E + V_{EB} (\text{on}) + I_{BQ} R_{TH} + V_{TH}$$

$$I_{BQ} = \frac{10 - 0.7 - 1.67}{6.67 + (61)(2)} = \frac{7.63}{128.7} \Rightarrow I_{BQ} = 0.0593 \text{ mA}$$

$$I_{CQ} = 3.56 \text{ mA}, \quad I_{EQ} = 3.62 \text{ mA}$$

$$V_E = 10 - I_{EQ} R_E = 10 - (3.62)(2)$$

$$V_E = 2.76 \text{ V}$$

$$V_C = I_{CQ} R_C - 10 = (3.56)(2.2) - 10$$

$$V_C = -2.17 \text{ V}$$

5.71

$$V^+ - V^- \cong I_{CQ} (R_C + R_E) + V_{ECQ}$$

$$20 = (0.5)(R_C + R_E) + 8 \Rightarrow (R_C + R_E) = 24 \text{ k}\Omega$$

Let $\frac{R_E = 10 \text{ k}\Omega}{R_E = 10 \text{ k}\Omega}$ then $\frac{R_C = 14 \text{ k}\Omega}{R_C = 14 \text{ k}\Omega}$

Let $\beta = 60$ from previous problem.

$$R_{TH} = (0.1)(1 + \beta) R_E = (0.1)(61)(10)$$

Or $R_{TH} = 61 \text{ k}\Omega$

$$I_{BQ} = \frac{0.5}{60} = 0.00833 \text{ mA}$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5 = \frac{1}{R_1} \cdot R_{TH} \cdot 10 - 5$$

Now $10 = (1 + \beta) I_{BQ} R_E + V_{EB} (\text{on}) + I_{BQ} R_{TH} + V_{TH}$

$$10 = (61)(0.00833)(10) + 0.7 + (0.00833)(61) + \frac{1}{R_1}(61)(10) - 5$$

Then $\frac{R_1 = 70.0 \text{ k}\Omega}{R_1 = 70.0 \text{ k}\Omega}$ and $\frac{R_2 = 474 \text{ k}\Omega}{R_2 = 474 \text{ k}\Omega}$

$$I_R \cong \frac{10}{R_1 + R_2} = \frac{10}{70 + 474} \Rightarrow 18.4 \mu\text{A}$$

So the $40 \mu\text{A}$ current limit is met.

5.72

(a) $V_{ECQ} = (V^+ - V^-) - I_{CQ} \left[R_C + \left(\frac{1+\beta}{\beta} \right) \cdot R_E \right]$

$$2.7 = 5 - (0.15) \left[R_C + \left(\frac{81}{80} \right) (2) \right] \Rightarrow R_C = 13.3 \text{ k}\Omega$$

$$I_{EQ} = 0.1519 \text{ mA}, \quad I_{BQ} = 1.875 \mu\text{A}$$

$$R_{TH} = (0.1)(81)(2) = 16.2 \text{ k}\Omega$$

$$V^+ = I_{EQ}R_E + V_{EB}(on) + I_{BQ}R_{TH} + V_{TH}$$

$$2.5 = (0.1519)(2) + 0.7 + (0.001875)(16.2) + V_{TH} \Rightarrow V_{TH} = 1.466 \text{ V}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} (5) - 2.5 \Rightarrow 1.466 = \frac{1}{R_1} (16.2)(5) - 2.5$$

So $R_1 = 20.4 \text{ k}\Omega$ and $R_2 = 78.7 \text{ k}\Omega$

(b) For $\beta = 60$

$$I_{BQ} = \frac{V^+ - V_{TH} - V_{EB}(on)}{R_{TH} + (1+\beta)R_E} = \frac{2.5 - 1.466 - 0.7}{16.2 + (61)(2)} \Rightarrow I_{BQ} = 2.417 \mu\text{A}$$

$$I_{CQ} = 0.145 \text{ mA}, \quad I_{EQ} = 0.1474 \text{ mA}$$

$$V_{ECQ} = 5 - (0.145)(13.3) - (0.1474)(2) = 2.777 \text{ V}$$

For $\beta = 100$

$$I_{BQ} = \frac{2.5 - 1.466 - 0.7}{16.2 + (101)(2)} \Rightarrow I_{BQ} = 1.531 \mu\text{A}$$

$$I_{CQ} = 0.1531 \text{ mA}, \quad I_{EQ} = 0.1546 \text{ mA}$$

$$V_{ECQ} = 5 - (0.1531)(13.3) - (0.1546)(2) = 2.655 \text{ V}$$

$$\text{For } I_{CQ}: \left(\frac{0.1531 - 0.145}{0.15} \right) \times 100\% = 5.4\%$$

$$\text{For } V_{ECQ}: \left(\frac{2.655 - 2.777}{2.70} \right) \times 100\% = -4.52\%$$

5.73

a.

$$R_{TH} = 500 \parallel 500 \parallel 70 = 250 \parallel 70 = 54.7 \text{ k}\Omega$$

$$\frac{5 - V_{TH}}{500} + \frac{3 - V_{TH}}{500} = \frac{V_{TH} - (-5)}{70}$$

$$\frac{5}{500} + \frac{3}{500} - \frac{5}{70} = V_{TH} \left(\frac{1}{500} + \frac{1}{500} + \frac{1}{70} \right) \Rightarrow -0.0554 = V_{TH} (0.0183)$$

$$V_{TH} = -3.03 \text{ V}$$

b.

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on}) - (-5)}{R_{TH} + (1+\beta)R_E}$$

$$= \frac{-3.03 - 0.7 + 5}{54.7 + (101)(5)}$$

$$I_{BQ} = 0.00227 \text{ mA}$$

$$I_{CQ} = 0.227 \text{ mA}, \quad I_{EQ} = 0.229$$

$$V_{CEQ} = 20 - (0.227)(50) - (0.229)(5)$$

$$V_{CEQ} = 7.51 \text{ V}$$

5.74

$$R_E \approx \frac{V_E}{I_{CQ}} = \frac{1.5}{0.8} = 1.87 \text{ k}\Omega$$

$$10 = (0.8)R_C + 4 + 1.5 \Rightarrow R_C = 5.63 \text{ k}\Omega$$

$$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(121)(1.87) = 22.6 \text{ k}\Omega$$

$$I_{BQ} = \frac{0.8}{120} \Rightarrow I_{BQ} = 6.67 \mu\text{A}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + I_{EQ}R_E = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC}$$

$$(0.00667)(22.6) + 0.7 + (0.807)(1.87) = \frac{1}{R_1} (22.6)(10)$$

which yields $R_1 = 95.8 \text{ k}\Omega$ and $R_2 = 29.6 \text{ k}\Omega$

5.75

$$I_{CQ} = 50 \mu\text{A}, I_{BQ} = 0.625 \mu\text{A}, I_{EQ} = 50.6 \mu\text{A}$$

(a)

$$R_E = \frac{1}{0.0506} = 19.8 \text{ K}$$

$$5 = (0.050)R_C + 5 + (0.0506)(19.8) - 5$$

$$R_C = 80 \text{ K}$$

$R_{TH} = R_1 \| R_2$, Design bias stable circuit

$$R_{TH} = (0.1)(51)(19.8) = 101 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right)(10) - 5 = \frac{1}{R_1} \cdot R_{TH} \cdot (10) - 5$$

$$\text{So } \frac{1}{R_1}(101)(10) - 5 = I_{BQ}(101) + 0.7 + (0.0506)(19.8) - 5$$

$$\frac{1}{R_1}(1010) = 0.0631 + 0.7 + 1$$

$$R_1 = 573 \text{ K} \quad \frac{573 R_2}{573 + R_2} = 101$$

$$R_2 = 123 \text{ K}$$

(b)

$$R_{TH} = 101 \text{ K}, V_{TH} = -3.23 \text{ V}$$

$$V_{TH} = I_{BQ}R_{TH} + 0.7 + (121)(19.8)I_{BQ} - 5$$

$$1.07 = I_{BQ}(101 + 2395.8) \Rightarrow I_{BQ} = 0.429 \mu\text{A}$$

$$I_{CQ} = 0.0514 \text{ mA}, I_{EQ} = 0.0519 \text{ mA}$$

$$V_{CEQ} = 10 - (0.0514)(80) - (0.0519)(19.8) \\ = 10 - 4.11 - 1.03 \Rightarrow \underline{V_{CEQ} = 4.86 \text{ V}}$$

5.76

$$(a) R_E \approx \frac{0.7}{0.5} = 1.4 \text{ k}\Omega, I_{BQ} = \frac{0.5}{120} \Rightarrow I_{BQ} = 4.167 \mu\text{A}$$

$$V_{CEQ} = 6 - I_{CQ}R_C - 0.7$$

$$2.5 = 6 - (0.5)R_C - 0.7 \Rightarrow R_C = 5.6 \text{ k}\Omega$$

$$R_{TH} = (0.1)(121)(1.4) = 16.9 \text{ k}\Omega$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E - 3$$

$$V_{TH} = (0.004167)(16.9) + 0.7 + (121)(0.004167)(1.4) - 3 = -1.5237 \text{ V}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH}(6) - 3 \Rightarrow -1.5237 = \frac{1}{R_1}(16.9)(6) - 3$$

which yields $R_1 = 68.7 \text{ k}\Omega$ and $R_2 = 22.4 \text{ k}\Omega$

(b) For standard resistor values:

$$\text{Let } R_E = 1.5 \text{ k}\Omega, R_C = 5.6 \text{ k}\Omega, R_1 = 68 \text{ k}\Omega, R_2 = 22 \text{ k}\Omega$$

$$R_{TH} = R_1 \parallel R_2 = 68 \parallel 22 = 16.62 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right)(6) - 3 = \left(\frac{22}{22 + 68} \right)(6) - 3 = -1.533 \text{ V}$$

$$I_{BQ} = \frac{3 + V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1 + \beta)R_E} = \frac{3 - 1.533 - 0.7}{16.62 + (121)(1.5)} \Rightarrow I_{BQ} = 3.87 \mu\text{A}$$

$$I_{CQ} = 0.4646 \text{ mA}, I_{EQ} = 0.4684 \text{ mA}$$

$$V_{CEQ} = 6 - (0.4646)(5.6) - (0.4684)(1.5) = 2.70 \text{ V}$$

5.77

$$(a) R_E \approx \frac{0.7}{0.1} = 7 \text{ k}\Omega, R_C + R_E = \frac{6-3}{0.1} = 30 \text{ k}\Omega, \Rightarrow R_C = 23 \text{ k}\Omega$$

$$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(111)(7) = 77.7 \text{ k}\Omega$$

$$I_{BQ} = \frac{0.1}{110} \Rightarrow I_{BQ} = 0.909 \mu\text{A}, I_{EQ} = 0.1009 \text{ mA}$$

$$V^+ = I_{EQ}R_E + V_{EB}(\text{on}) + I_{BQ}R_{TH} + V_{TH}$$

$$3 = (0.1009)(7) + 0.7 + (0.000909)(77.7) + V_{TH}, \Rightarrow V_{TH} = 1.523 \text{ V}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH}(6) - 3 \Rightarrow 1.523 = \frac{1}{R_1}(77.7)(6) - 3$$

which yields $R_1 = 103 \text{ k}\Omega$ and $R_2 = 316 \text{ k}\Omega$

$$I_{BQ} = \frac{V^+ - V_{EB}(\text{on}) - V_{TH}}{R_{TH} + (1 + \beta)R_E} = \frac{3 - 0.7 - 1.523}{77.7 + (151)(7)} \Rightarrow I_{BQ} = 0.685 \mu\text{A}$$

$$I_{CQ} = 0.1027 \text{ mA}, I_{EQ} = 0.1034 \text{ mA}$$

$$V_{ECQ} = 6 - (0.1027)(23) - (0.1034)(7) = 2.914 \text{ V}$$

5.78

(a) $V_{ECQ} = 18 - I_{CQ}R_C - V_{RE}$
 $6 = 18 - (1.2)R_C - 1.5 \Rightarrow R_C = 8.75 \text{ k}\Omega$
 $I_{EQ} = \left(\frac{76}{75} \right)(1.2) = 1.216 \text{ mA}, \quad I_{BQ} = \frac{1.2}{75} \Rightarrow I_{BQ} = 16 \mu\text{A}$
 $R_E = \frac{1.5}{1.216} = 1.234 \text{ k}\Omega$
 $R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(76)(1.234) = 9.375 \text{ k}\Omega$
 $V^+ = I_{EQ}R_E + V_{EB}(\text{on}) + I_{BQ}R_{TH} + V_{TH}$
 $9 = (1.216)(1.234) + 0.7 + (0.016)(9.375) + V_{TH} \Rightarrow V_{TH} = 6.65 \text{ V}$
 $V_{TH} = \frac{1}{R_1}(R_{TH})(18) - 9 \Rightarrow 6.65 = \frac{1}{R_1}(9.375)(18) - 9$
which yields $R_1 = 10.78 \text{ k}\Omega$ and $R_2 = 71.8 \text{ k}\Omega$
Set $R_C = 9.1 \text{ k}\Omega$, $R_E = 1.2 \text{ k}\Omega$, $R_1 = 11 \text{ k}\Omega$, $R_2 = 68 \text{ k}\Omega$
 $R_{TH} = R_1 \parallel R_2 = 11 \parallel 68 = 9.47 \text{ k}\Omega$
 $V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right)(18) - 9 = \left(\frac{68}{68 + 11} \right)(18) - 9 = 6.494 \text{ V}$
 $I_{BQ} = \frac{V^+ - V_{EB}(\text{on}) - V_{TH}}{R_{TH} + (1 + \beta)R_E} = \frac{9 - 0.7 - 6.494}{9.47 + (76)(1.2)} \Rightarrow I_{BQ} = 17.94 \mu\text{A}$
 $I_{CQ} = 1.345 \text{ mA}, \quad I_{EQ} = 1.363 \text{ mA}$
 $V_{ECQ} = 18 - (1.345)(9.1) - (1.363)(1.2) = 4.12 \text{ V}$

5.79

$$R_{TH} = R_1 \parallel R_2 = 100 \parallel 40 = 28.6 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right)(10) = \left(\frac{40}{40 + 100} \right)(10) = 2.86 \text{ V}$$

$$I_{B1} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1 + \beta)R_{E1}} = \frac{2.86 - 0.7}{28.6 + (121)(1)}$$

$$I_{B1} = 0.0144 \text{ mA}, \quad I_{C1} = 1.73 \text{ mA}, \quad I_{E1} = 1.75 \text{ mA}$$

$$\frac{10 - V_{B2}}{3} = I_{C1} + I_{B2}$$

$$I_{E2} = \frac{V_{B2} - V_{BE}(\text{on}) - (-10)}{5}$$

$$\frac{10 - V_{B2}}{3} = I_{C1} + \frac{V_{B2} - 0.7 + 10}{(121)(5)}$$

$$\frac{10}{3} - 1.73 - \frac{9.3}{605} = V_{B2} \left(\frac{1}{3} + \frac{1}{(121)(5)} \right)$$

$$1.588 = V_{B2} (0.335) \Rightarrow V_{B2} = 4.74 \text{ V}$$

$$I_{E2} = \frac{4.74 - 0.7 - (-10)}{5} \Rightarrow I_{E2} = 2.808 \text{ mA}$$

$$\underline{I_{B2} = 0.0232 \text{ mA}}$$

$$\underline{I_{C2} = 2.785 \text{ mA}}$$

$$V_{CEQ1} = 4.74 - (1.75)(1) \Rightarrow V_{CEQ1} = 2.99 \text{ V}$$

$$V_{CEQ2} = 10 - (4.74 - 0.7) \Rightarrow V_{CEQ2} = 5.96 \text{ V}$$

5.80

$$V_{E1} = -0.7$$

$$I_{R1} = \frac{-0.7 - (-5)}{20} = 0.215 \text{ mA}$$

$$V_{E2} = -0.7 - 0.7 = -1.4$$

$$I_{E2} = \frac{-1.4 - (-5)}{1} \Rightarrow I_{E2} = 3.6 \text{ mA}$$

$$\underline{I_{B2} = 0.0444 \text{ mA}}$$

$$\underline{I_{C2} = 3.56 \text{ mA}}$$

$$I_{E1} = I_{R1} + I_{B2} = 0.215 + 0.0444$$

$$\underline{I_{E1} = 0.259 \text{ mA}}$$

$$\underline{I_{B1} = 0.00320 \text{ mA}}$$

$$\underline{I_{C1} = 0.256 \text{ mA}}$$

5.81

$$V_{B1} = V_{RE} + V_{BE} (\text{on}) = 0.5 + 0.7 = 1.2 \text{ V}$$

$$R_3 = \frac{1.2}{0.020} = 60 \text{ k}\Omega$$

$$R_E = \frac{0.5}{0.2} = 2.5 \text{ k}\Omega$$

$$V_{B2} = V_{BE} (\text{on}) + V_{CE} + V_{RE} = 0.7 + 1.2 + 0.5 = 2.4 \text{ V}$$

$$R_2 = \frac{V_{B2} - V_{B1}}{I_{R2}} = \frac{2.4 - 1.2}{0.020} = 60 \text{ k}\Omega$$

$$R_1 = \frac{V^+ - V_{B2}}{I_{R1}} = \frac{5 - 2.4}{0.020} = 130 \text{ k}\Omega$$

$$V_{C2} = 2V_{CE} + V_{RE} = 2(1.2) + 0.5 = 2.9 \text{ V}$$

$$R_C = \frac{V^+ - V_{C2}}{I_{C2}} = \frac{5 - 2.9}{0.20} = 10.5 \text{ k}\Omega$$

5.82

$$R_{TH} = 40\parallel 80 = 26.67 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{40}{40+80} \right) (9) = 3 \text{ V}$$

$$I_{B1} = \frac{V_{TH} - V_{BE}(on)}{R_{TH} + (1 + \beta_n)R_{E1}} = \frac{3 - 0.7}{26.67 + (121)(2)} \Rightarrow I_{B1} = 8.56 \mu\text{A}$$

$$I_{C1} = 1.027 \text{ mA}, \quad I_{E1} = 1.036 \text{ mA}$$

$$\frac{9 - V_{C1}}{2} + I_{B2} = I_{C1}$$

$$9 = I_{E2}(0.1) + V_{EB}(on) + V_{C1} \Rightarrow I_{B2} = \frac{9 - 0.7 - V_{C1}}{(1 + \beta_p)(0.1)} = \frac{8.3 - V_{C1}}{8.1}$$

$$\text{So } \frac{9 - V_{C1}}{2} + \frac{8.3 - V_{C1}}{8.1} = 1.027 \Rightarrow V_{C1} = 7.214 \text{ V}$$

$$I_{B2} = \frac{8.3 - 7.214}{8.1} = 0.134 \text{ mA}$$

$$I_{C2} = 10.73 \text{ mA}, \quad I_{E2} = 10.86 \text{ mA}$$

$$V_{E1} = I_{E1}R_{E1} = (1.036)(2) = 2.072 \text{ V}$$

$$V_{CE1} = V_{C1} - V_{E1} = 7.214 - 2.072 = 5.14 \text{ V}$$

$$V_{EC2} = 9 - (10.86)(0.1) - (10.73)(0.2) = 5.77 \text{ V}$$

5.83

$$R_{TH} = R_1\parallel R_2 = 50\parallel 100 = 33.3 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5 = \left(\frac{100}{100 + 50} \right) (10) - 5 = 1.67 \text{ V}$$

$$5 = I_{E1}R_{E1} + V_{EB}(on) + I_{B1}R_{TH} + V_{TH}$$

$$I_{E1} = \left(\frac{101}{100} \right) (0.8) = 0.808 \text{ mA}$$

$$I_{B1} = 0.008 \text{ mA}$$

$$5 = (0.808)R_{E1} + 0.7 + (0.008)(33.3) + 1.67$$

$$R_{E1} = 2.93 \text{ k}\Omega$$

$$V_{E1} = 5 - (0.808)(2.93) = 2.63 \text{ V}$$

$$V_{C1} = V_{E1} - V_{ECQ1} = 2.63 - 3.5 = -0.87 \text{ V}$$

$$V_{E2} = -0.87 - 0.70 = -1.57 \text{ V}$$

$$I_{E2} = \frac{-1.57 - (-5)}{R_{E2}} = 0.808 \Rightarrow \underline{R_{E2} = 4.25 \text{ k}\Omega}$$

$$V_{CEQ2} = 4 \Rightarrow V_{C2} = -1.57 + 4 = 2.43 \text{ V}$$

$$R_{C2} = \frac{5 - 2.43}{0.8} \Rightarrow \underline{R_{C2} = 3.21 \text{ k}\Omega}$$

$$I_{RC1} = I_{C1} - I_{B2} = 0.8 - 0.008 = 0.792 \text{ mA}$$

$$R_{C1} = \frac{-0.87 - (-5)}{0.792} \Rightarrow R_{C1} = 5.21 \text{ k}\Omega$$

Chapter 6

6.1

$$(a) \text{ (i)} \quad g_m = \frac{I_{CQ}}{V_T} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(180)(0.026)}{0.5} = 9.36 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{150}{0.5} = 300 \text{ k}\Omega$$

$$\text{(ii)} \quad g_m = \frac{2}{0.026} = 76.92 \text{ mA/V}$$

$$r_\pi = \frac{(180)(0.026)}{2} = 2.34 \text{ k}\Omega$$

$$r_o = \frac{150}{2} = 75 \text{ k}\Omega$$

$$(b) \text{ (i)} \quad g_m = \frac{0.25}{0.026} = 9.615 \text{ mA/V}$$

$$r_\pi = \frac{(80)(0.026)}{0.25} = 8.32 \text{ k}\Omega$$

$$r_o = \frac{100}{0.25} = 400 \text{ k}\Omega$$

$$\text{(ii)} \quad g_m = \frac{0.08}{0.026} = 3.077 \text{ mA/V}$$

$$r_\pi = \frac{(80)(0.026)}{0.08} = 26 \text{ k}\Omega$$

$$r_o = \frac{100}{0.08} = 1250 \text{ k}\Omega$$

6.2

$$(a) \quad g_m = \frac{I_{CQ}}{V_T} \Rightarrow I_{CQ} = (95)(0.026) = 2.47 \text{ mA}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(125)(0.026)}{2.47} = 1.32 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{200}{2.47} = 81 \text{ k}\Omega$$

$$(b) \quad I_{CQ} = g_m V_T = (120)(0.026) = 3.12 \text{ mA}$$

$$\beta = g_m r_\pi = (120)(1.2) = 144$$

6.3

$$g_m = \frac{0.8}{0.026} = 30.77 \text{ mA/V}; \quad g_m = \frac{1.2}{0.026} = 46.15 \text{ mA/V}$$

So $30.77 \leq g_m \leq 46.15 \text{ mA/V}$

$$r_\pi = \frac{(180)(0.026)}{0.8} = 5.85 \text{ k}\Omega ; \quad r_\pi = \frac{(90)(0.026)}{1.2} = 1.95 \text{ k}\Omega$$

So $1.95 \leq r_\pi \leq 5.85 \text{ k}\Omega$

6.4

$$(a) \quad i_C = I_{CQ} + g_m v_{be} = 0.12 + \left(\frac{0.12}{0.026} \right) (5 \times 10^{-3} \sin \omega t)$$

$$i_C = 0.12 + 0.0231 \sin \omega t \text{ (mA)}$$

$$v_{CE} = V_{CEQ} - i_C R_C = [3.3 - (0.12)(15)] - (15)(0.0231 \sin \omega t)$$

$$v_{CE} = 1.5 - 0.346 \sin \omega t \text{ (V)}$$

$$(b) \quad A_v = \frac{v_{ce}}{v_{be}} = \frac{-0.346}{0.005} = -69.2$$

6.5

$$(a) \quad I_{CQ} = \frac{\beta(V_{BB} - V_{BE}(\text{on}))}{R_B} = \frac{(120)(1.10 - 0.7)}{110} = 0.436 \text{ mA}$$

$$g_m = \frac{0.436}{0.026} = 16.78 \text{ mA/V}$$

$$r_\pi = \frac{(120)(0.026)}{0.436} = 7.15 \text{ k}\Omega$$

$$r_o = \frac{80}{0.436} = 183 \text{ k}\Omega$$

$$(b) \quad A_v = -g_m (R_C \| r_o) \left(\frac{r_\pi}{r_\pi + R_B} \right) = -(16.78)(4 \| 183) \left(\frac{7.15}{7.15 + 110} \right)$$

$$A_v = -4.0$$

$$(c) \quad v_s = \frac{v_o}{A_v} = \frac{0.5 \sin(100t)}{-4} = -0.125 \sin(100t) \text{ (V)}$$

6.6

a.

$$r_\pi = 5.4 = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{I_{CQ}} \Rightarrow I_{CQ} = 0.578 \text{ mA}$$

$$V_{CEQ} = \frac{1}{2} V_{CC} = \frac{1}{2}(5) = 2.5 \text{ V}$$

$$V_{CEQ} = V_{CC} - I_{CQ} R_C \Rightarrow 2.5 = 5.0 - (0.578) R_C \Rightarrow R_C = 4.33 \text{ k}\Omega$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.578}{120} = 0.00482 \text{ mA}$$

$$\begin{aligned} V_{BB} &= I_{BQ} R_B + V_{BE}(\text{on}) \\ &= (0.00482)(25) + 0.70 \Rightarrow V_{BB} = 0.820 \text{ V} \end{aligned}$$

b.

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{0.578} = 5.40 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.578}{0.026} = 22.2 \text{ mA/V}$$

$$r_0 = \frac{V_A}{I_{CQ}} = \frac{100}{0.578} = 173 \text{ k}\Omega$$

$$V_0 = -g_m (r_0 \| R_C) V_\pi, \quad V_\pi = \left(\frac{r_\pi}{r_\pi + R_B} \right) V_s$$

$$A_v = -g_m \left(\frac{r_\pi}{r_\pi + R_B} \right) (r_0 \| R_C) = -\frac{\beta (r_0 \| R_C)}{r_\pi + R_B}$$

$$A_v = -\frac{(120)[173 \| 4.33]}{5.40 + 25} = -\frac{(120)(4.22)}{30.4} \Rightarrow A_v = -16.7$$

6.7

$$r_\pi = \frac{(120)(0.026)}{0.5} = 6.24 \text{ k}\Omega$$

(a) $R_i = R_B + r_\pi = 50 + 6.24 = 56.24 \text{ k}\Omega$

(b) $R_i = R_B \| r_\pi = 100 \| 6.24 = 5.87 \text{ k}\Omega$

(c) $R_i = r_\pi = 6.24 \text{ k}\Omega$

6.8

$$r_o = \frac{V_A}{I_{CQ}} = \frac{80}{0.2} = 400 \text{ k}\Omega$$

(a) $R_o = R_C \| r_o = 4 \| 400 = 3.96 \text{ k}\Omega$

(b) $R_o = R_C \| r_o = 10 \| 400 = 9.76 \text{ k}\Omega$

(c) $R_o = R_C \| R_L \| r_o = 10 \| 5 \| 400 = 3.333 \| 400 = 3.31 \text{ k}\Omega$

6.9

$$I_{CQ} = \frac{10 - 4}{4} = 1.5 \text{ mA}$$

$$I_{BQ} = \frac{1.5}{100} = 0.015 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{1.5} = 1.73 \text{ K}$$

$$i_b = \frac{v_{be}}{r_\pi} = \frac{5 \sin \omega t (\text{mV})}{1.73 \text{ k}\Omega} = 2.89 \sin \omega t (\mu\text{A})$$

So

$$\begin{aligned}
 i_B(t) &= I_{BQ} + i_b(t) = 15 + 2.89 \sin \omega t \text{ } (\mu\text{A}) \\
 i_{C1}(t) &= \beta i_B(t) = 1.5 + 0.289 \sin \omega t \text{ (mA)} \\
 v_{C1}(t) &= 10 - i_{C1}(t)R_C = 10 - [1.5 + 0.289 \sin \omega t]4 \text{ (V)} \\
 v_{C1}(t) &= 4 - 1.156 \sin \omega t \text{ (V)} \\
 A_v &= \frac{v_c(t)}{v_{be}(t)} = \frac{-1.156}{0.005} \Rightarrow A_v = -231
 \end{aligned}$$

6.10

a.

$$V_{ECQ} = \frac{1}{2}V_{CC} = 5 \text{ V}$$

$$V_{ECQ} = 10 - I_{CQ}R_C \Rightarrow 5 = 10 - (0.5)R_C \Rightarrow R_C = 10 \text{ k}\Omega$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.5}{100} = 0.005$$

$$V_{EB} \text{ (on)} + I_{BQ}R_B = V_{BB} = (0.70) + (0.005)(50) \Rightarrow V_{BB} = 0.95 \text{ V}$$

b.

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.5}{0.026} \Rightarrow g_m = 19.2 \text{ mA/V}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.5} \Rightarrow r_\pi = 5.2 \text{ k}\Omega$$

$$r_0 = \frac{V_A}{I_{CQ}} = \frac{\infty}{0.5} \Rightarrow r_0 = \infty$$

$$A_v = -\frac{\beta R_C}{r_\pi + R_B} = -\frac{(100)(10)}{5.2 + 50} \Rightarrow A_v = -18.1$$

c.

6.11

$$v_o = 1.2 \sin \omega t \text{ (V)}$$

$$i_C(t)R_C + v_o = 0 \Rightarrow i_C(t) = \frac{-1.2 \sin \omega t}{2}$$

$$i_C(t) = -0.60 \sin \omega t \text{ (mA)}$$

$$i_b(t) = \frac{i_C(t)}{\beta} = -6 \sin \omega t \text{ } (\mu\text{A})$$

$$v_{be}(t) = i_b(t) \cdot r_\pi \quad g_m r_\pi = \beta$$

$$r_\pi = \frac{100}{50} = 2 \text{ K}$$

$$v_{be}(t) = -12 \sin \omega t \text{ (mV)}$$

6.12

a.

$$I_{CQ} \approx I_{EQ}$$

$$\begin{aligned} V_{CEQ} &= 5 = 10 - I_{CQ}(R_C + R_E) \\ &= 10 - I_{CQ}(1.2 + 0.2) \end{aligned}$$

$$I_{CQ} = 3.57 \text{ mA}$$

$$I_{BQ} = \frac{3.57}{150} = 0.0238 \text{ mA}$$

$$\begin{aligned} R_1 \| R_2 &= R_{TH} = (0.1)(1 + \beta)R_E \\ &= (0.1)(151)(0.2) = 3.02 \text{ k}\Omega \end{aligned}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot (10) - 5$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E - 5$$

$$\frac{1}{R_1}(3.02)(10) - 5 = (0.0238)(3.02) + 0.7 + (151)(0.0238)(0.2) - 5$$

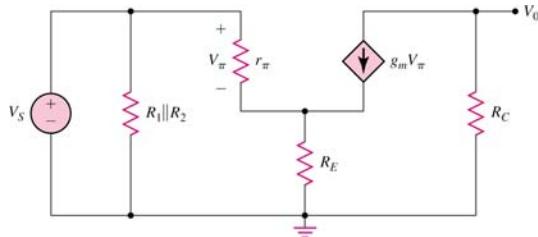
$$\frac{1}{R_1}(30.2) = 1.50 \Rightarrow R_1 = 20.1 \text{ k}\Omega$$

$$\frac{20.1R_2}{20.1 + R_2} = 3.02 \Rightarrow R_2 = 3.55 \text{ k}\Omega$$

b.

$$r_\pi = \frac{(150)(0.026)}{3.57} = 1.09 \text{ k}\Omega$$

$$g_m = \frac{3.57}{0.026} = 137 \text{ mA/V}$$



$$A_v = \frac{-\beta R_C}{r_\pi + (1 + \beta)R_E} = \frac{-(150)(1.2)}{1.09 + (151)(0.2)} \Rightarrow A_v = -5.75$$

6.13

(a) $R_{TH} = R_1 \| R_2 = 33 \| 50 = 19.88 \text{ k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left(\frac{50}{50 + 33} \right) (3.3) = 1.988 \text{ V}$$

$$V_{CC} = I_{BQ}(1 + \beta)R_E + V_{EB}(\text{on}) + I_{BQ}R_{TH} + V_{TH}$$

$$\text{Then } I_{BQ} = \frac{3.3 - 0.7 - 1.988}{19.88 + (101)(1)} = 0.005063 \text{ mA}$$

$$I_{CQ} = 0.506 \text{ mA}; \quad I_{EQ} = 0.511 \text{ mA}$$

$$V_{EQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E = 3.3 - (0.506)(2) - (0.511)(1)$$

$$V_{EQ} = 1.78 \text{ V}$$

$$(b) \quad r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.506} = 5.14 \text{ k}\Omega$$

$$A_v = \frac{-\beta R_C}{r_\pi + (1+\beta)R_E} = \frac{-(100)(2)}{5.14 + (101)(1)} = -1.884$$

$$(c) \quad R_1 = (1.05)(33) = 34.65 \text{ k}\Omega$$

$$R_2 = (0.95)(50) = 47.5 \text{ k}\Omega$$

$$R_{TH} = R_1 \parallel R_2 = 34.65 \parallel 47.5 = 20.03 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{47.5}{47.5 + 34.65} \right) (3.3) = 1.908 \text{ V}$$

$$I_{CQ} = (100) \left(\frac{3.3 - 0.7 - 1.908}{20.03 + (101)(1)} \right) = 0.5718 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{0.5718} = 4.547 \text{ k}\Omega$$

$$A_v = \frac{-(100)(2)}{4.547 + (101)(1)} = -1.895$$

$$\text{Also } R_1 = (0.95)(33) = 31.35 \text{ k}\Omega$$

$$R_2 = (1.05)(50) = 52.5 \text{ k}\Omega$$

$$R_{TH} = 31.35 \parallel 52.5 = 19.63 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{52.5}{52.5 + 31.35} \right) (3.3) = 2.066 \text{ V}$$

$$I_{CQ} = (100) \left(\frac{3.3 - 0.7 - 2.066}{19.63 + (101)(1)} \right) = 0.4427 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{0.4427} = 5.873 \text{ k}\Omega$$

$$A_v = \frac{-(100)(2)}{5.873 + (101)(1)} = -1.871$$

$$\text{So } 1.871 \leq |A_v| \leq 1.895$$

6.14

(a)

$$V_{CC} = \left(\frac{1+\beta}{\beta} \right) I_{CQ} R_E + V_{ECQ} + I_{CQ} R_C$$

$$12 = \left(\frac{101}{100} \right) I_{CQ} (1) + 6 + I_{CQ} (2)$$

so that $I_{CQ} = 1.99 \text{ mA}$

$$I_{BQ} = \frac{1.99}{100} = 0.0199 \text{ mA}$$

$$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(101)(1) = 10.1 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (10.1)(12)$$

$$V_{CC} = (1+\beta)I_{BQ}R_E + V_{EB} (\text{on}) + I_{BQ}R_{TH} + V_{TH}$$

$$12 = (101)(0.0199)(1) + 0.7 + (0.0199)(10.1) + \frac{121.2}{R_1}$$

which yields $R_1 = 13.3 \text{ k}\Omega$ and $R_2 = 41.6 \text{ k}\Omega$

$$(b) \quad A_v = \frac{-\beta R_C}{r_\pi + (1+\beta)R_E} = \frac{-(100)(2)}{1.31 + (101)(1)} \Rightarrow A_v = -1.95$$

6.15

$$I_{CQ} = 0.25 \text{ mA}, I_{EQ} = 0.2525 \text{ mA}$$

$$I_{BQ} = 0.0025 \text{ mA}$$

$$I_{BQ}R_B + V_{BE} (\text{on}) + I_{EQ}(R_S + R_E) - 5 = 0$$

$$(0.0025)(50) + 0.7 + (0.2525)(0.1 + R_E) = 5$$

$$R_E = 16.4 \text{ k}\Omega$$

$$V_E = -(0.0025)(50) - 0.7 = -0.825 \text{ V}$$

$$V_C = V_{CEO} + V_E = 3 - 0.825 = 2.175 \text{ V}$$

$$R_C = \frac{5 - 2.175}{0.25} \Rightarrow R_C = 11.3 \text{ k}\Omega$$

$$A_v = \frac{-\beta R_C}{r_\pi + (1+\beta)R_S}$$

$$r_\pi = \frac{(100)(0.026)}{0.25} = 10.4 \text{ k}\Omega$$

$$A_v = \frac{-(100)(11.3)}{10.4 + (101)(0.1)} \Rightarrow A_v = -55.1$$

$$R_i = R_B \left[r_\pi + (1+\beta)R_S \right]$$

$$= 50 \left[10.4 + (101)(0.1) \right]$$

$$R_i = 50 \parallel 20.5 \Rightarrow R_i = 14.5 \text{ k}\Omega$$

6.16

$$(a) I_{CQ} \approx \frac{V_{CC} - V_{CEO}}{R_C + R_E} = \frac{9 - 5.20}{2.2 + 2} = 0.905 \text{ mA}$$

$$I_{BQ} \approx 0.00754 \text{ mA}; I_{EQ} \approx 0.9123 \text{ mA}$$

$$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(121)(2) = 24.2 \text{ k}\Omega$$

$$\begin{aligned} V_{TH} &= I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E \\ &= (0.00754)(24.2) + 0.7 + (0.9123)(2) = 2.707 \text{ V} \end{aligned}$$

$$\text{Now } V_{TH} = 2.707 = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (24.2)(9)$$

Or $R_1 = 80.5 \text{ k}\Omega$ and then $R_2 = 34.6 \text{ k}\Omega$

$$(b) r_\pi = \frac{(120)(0.026)}{0.905} = 3.448 \text{ k}\Omega$$

$$g_m = \frac{0.905}{0.026} = 34.81 \text{ mA/V}$$

$$r_o = \frac{100}{0.905} = 110 \text{ k}\Omega$$

$$V_\pi = i_s(R_1 \parallel R_2 \parallel r_\pi) = i_s(80.5 \parallel 34.6 \parallel 3.448)$$

$$V_\pi = i_s(3.02)$$

$$v_o = -g_m V_\pi (r_o \parallel R_C \parallel R_L)$$

$$\text{Then } R_m = \frac{v_o}{i_s} = -g_m (3.02) (110 \parallel 2.2 \parallel 1) = -(34.81)(3.02)(110 \parallel 0.6875)$$

Or $R_m = -71.8 \text{ V/mA}$

(c) For $\beta = 100$,

$$I_{BQ} = \frac{2.707 - 0.7}{24.2 + (101)(2)} = 0.008873 \text{ mA}$$

$$I_{CQ} = 0.8873 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{0.8873} = 2.93 \text{ k}\Omega, g_m = \frac{0.8873}{0.026} = 34.13 \text{ mA/V}, r_o = \frac{100}{0.8873} = 113 \text{ k}\Omega$$

$$V_\pi = i_s(R_{TH} \parallel r_\pi) = i_s(24.2 \parallel 2.93) = i_s(2.614)$$

$$R_m = \frac{v_o}{i_s} = -(34.13)(2.614)(113 \parallel 0.6875) = -61.0 \text{ V/mA}$$

For $\beta = 150$,

$$I_{CQ} = (150) \left(\frac{2.707 - 0.7}{24.2 + (151)(2)} \right) = 0.923 \text{ mA}$$

$$r_\pi = 4.225 \text{ k}\Omega, g_m = 35.5 \text{ mA/V}, r_o = 108 \text{ k}\Omega$$

$$V_\pi = i_s(24.2 \parallel 4.225) = i_s(3.597)$$

$$R_m = \frac{v_o}{i_s} = -(35.5)(3.597)(108 \parallel 0.6875) = -87.2 \text{ V/mA}$$

So $61.0 \leq |R_m| \leq 87.2 \text{ V/mA}$

6.17

$$(a) \text{ (i)} I_{BQ} = \frac{0.8}{81} = 0.009877 \text{ mA}, \quad I_{CQ} = 0.790 \text{ mA}$$

$$R_B = \frac{0.2}{0.009877} = 20.25 \text{ k}\Omega$$

$$R_C = \frac{-3 - (-5)}{0.79} = 2.53 \text{ k}\Omega$$

$$\text{(ii)} \ g_m = \frac{0.79}{0.026} = 30.38 \text{ mA/V}, \quad r_o = \frac{100}{0.79} = 127 \text{ k}\Omega$$

$$r_o \| R_C = 127 \| 2.53 = 2.48 \text{ k}\Omega$$

$$i_o = -g_m v_s \left(\frac{2.48}{2.48 + 4} \right)$$

$$G_f = \frac{i_o}{v_s} = -(30.38) \left(\frac{2.48}{2.48 + 4} \right) = -11.63 \text{ mA/V}$$

$$(b) \text{ (i)} I_{BQ} = \frac{0.8}{121} = 0.00661 \text{ mA}, \quad I_{CQ} = 0.7934 \text{ mA}$$

$$R_B = \frac{0.2}{0.00661} = 30.3 \text{ k}\Omega$$

$$R_C = \frac{-3 - (-5)}{0.7934} = 2.52 \text{ k}\Omega$$

$$\text{(ii)} \ g_m = \frac{0.7934}{0.026} = 30.52 \text{ mA/V}, \quad r_o = \frac{80}{0.7934} = 101 \text{ k}\Omega$$

$$r_o \| R_C = 101 \| 2.52 = 2.459 \text{ k}\Omega$$

$$G_f = -(30.52) \left(\frac{2.459}{2.459 + 4} \right) = -11.62 \text{ mA/V}$$

6.18

$$(a) \text{ (i)} I_{BQ} = \frac{0.25}{120} = 0.00208 \text{ mA}, \quad I_{EQ} = 0.252 \text{ mA}$$

$$5 = (0.00208)(2.5) + 0.7 + (0.252)R_E \Rightarrow R_E = 17.0 \text{ k}\Omega$$

$$V_{CEQ} = 3 = 10 - (0.252)(17) - (0.25)R_C \Rightarrow R_C = 10.9 \text{ k}\Omega$$

$$\text{(ii)} \ g_m = \frac{0.25}{0.026} = 9.615 \text{ mA/V}, \quad r_\pi = \frac{(120)(0.026)}{0.25} = 12.48 \text{ k}\Omega$$

$$A_v = -g_m (R_C \| R_L) \left(\frac{r_\pi}{r_\pi + R_S} \right) = -(9.615)(10.9 \| 5) \left(\frac{12.48}{12.48 + 2.5} \right) = -27.5$$

$$\text{(iii)} \ v_o = -(27.49)(5 \times 10^{-3} \sin \omega t) = -0.137 \sin \omega t \text{ (V)}$$

$$(b) \text{ (i)} 5 = 0.7 + (0.252)R_E \Rightarrow R_E = 17.1 \text{ k}\Omega$$

$$V_{CEQ} = 3 = 10 - (0.252)(17.06) - (0.25)R_C \Rightarrow R_C = 10.8 \text{ k}\Omega$$

$$\text{(ii)} \ g_m = 9.615 \text{ mA/V}, \quad r_\pi = 12.48 \text{ k}\Omega$$

$$A_v = -g_m (R_C \| R_L) = -(9.615)(10.8 \| 5) = -32.9$$

$$\text{(iii)} \ v_o = -(32.86)(5 \times 10^{-3} \ sin \omega t) = -0.164 \sin \omega t \text{ (V)}$$

6.19

$$(a) \text{ (i)} I_{BQ} = \frac{5 - 0.7}{2.5 + (81)(10)} = 0.005292 \text{ mA}, \Rightarrow I_{CQ} = 0.4234 \text{ mA}$$

$$g_m = \frac{0.4234}{0.026} = 16.28 \text{ mA/V}, r_\pi = \frac{(80)(0.026)}{0.4234} = 4.91 \text{ k}\Omega$$

$$A_v = -g_m (R_C \parallel R_L) \left(\frac{r_\pi}{r_\pi + R_S} \right) = -(16.28)(5 \parallel 5) \left(\frac{4.91}{4.91 + 2.5} \right) = -26.97$$

$$i_o = \frac{v_o}{R_L} = \frac{A_v v_s}{R_L}$$

$$\text{or } G_f = \frac{i_o}{v_s} = \frac{-26.97}{5} = -5.39 \text{ mA/V}$$

$$\text{(ii)} v_o = -(26.97)(4 \times 10^{-3} \sin \omega t) = -0.108 \sin \omega t \text{ (V)}$$

$$i_o = (-5.39 \times 10^{-3})(4 \times 10^{-3} \sin \omega t) \Rightarrow -21.6 \sin \omega t \text{ (\mu A)}$$

$$(b) \text{ (i)} I_{CQ} = (120) \left(\frac{5 - 0.7}{2.5 + (121)(10)} \right) = 0.4256 \text{ mA}$$

$$g_m = \frac{0.4256}{0.026} = 16.37 \text{ mA/V}, r_\pi = \frac{(120)(0.026)}{0.4256} = 7.33 \text{ k}\Omega$$

$$A_v = -(16.37)(5 \parallel 5) \left(\frac{7.33}{7.33 + 2.5} \right) = -30.5$$

$$G_f = \frac{-30.5}{5} = -6.1 \text{ mA/V}$$

$$\text{(ii)} v_o = -(30.5)(4 \times 10^{-3} \sin \omega t) = -0.122 \sin \omega t \text{ (V)}$$

$$i_o = -24.4 \sin \omega t \text{ (\mu A)}$$

6.20

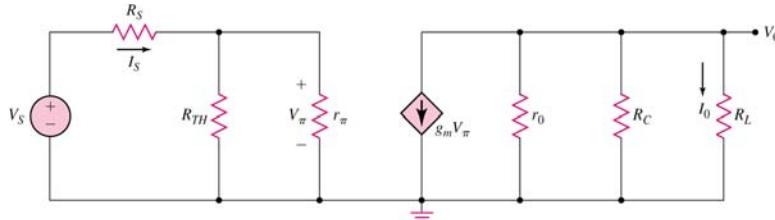
$$R_{TH} = R_1 \parallel R_2 = 27 \parallel 15 = 9.64 \text{ K}$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{15}{15 + 27} \right)(9) = 3.214 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1 + \beta)R_E} = \frac{3.214 - 0.7}{9.64 + (101)(1.2)} = \frac{2.514}{130.84}$$

$$I_{BQ} = 0.0192 \text{ mA} \quad I_{CQ} = 1.9214 \text{ mA}$$

$$g_m = \frac{1.92}{0.026} = 73.9 \text{ mA/V} \quad r_\pi = \frac{(100)(0.026)}{1.92} = 1.35 \text{ K}$$



$$r_o = \frac{100}{1.92} = 52.1 \text{ K}$$

$$V_o = -g_m V_\pi \left(r_0 \| R_C \| R_L \right) \quad V_\pi = \left(\frac{r_\pi \| R_{TH} }{r_\pi \| R_{TH} + R_S} \right) V_s$$

$$r_\pi \| R_{TH} = 1.35 \| 9.64 = 1.184 \text{ K}$$

$$V_\pi = \left(\frac{1.184}{1.184 + 10} \right) V_s$$

$$= 0.1059 V_s$$

$$A_v = -(73.9)(0.1059)(52.1 \| 2.2 \| 2)$$

$$= -(73.9)(0.1059)(52.1 \| 1.0476)$$

$$= -(73.9)(0.1059)(1.027)$$

$$A_v = -8.04$$

$$A_I = \frac{I_o}{I_s} = \frac{-g_m V_\pi \left(\frac{r_o \| R_C}{r_o \| R_C + R_L} \right)}{\frac{V_\pi}{R_{TH} \| r_\pi}}$$

$$A_I = -g_m (R_{TH} \| r_\pi) \left(\frac{r_o \| R_C}{r_o \| R_C + R_L} \right)$$

$$r_o \| R_C = 52.1 \| 2.2 = 2.11 \text{ K}$$

$$R_{TH} \| r_\pi = 9.64 \| 1.35 = 1.184 \text{ K}$$

$$A_I = -(73.9)(1.184) \left(\frac{2.11}{2.11 + 2} \right)$$

$$\underline{A_I = -44.9}$$

$$R_i = R_{TH} \| r_\pi = 9.64 \| 1.35$$

$$\underline{R_i = 1.184 \text{ K}}$$

6.21

a.

$$I_E = 0.35 \text{ mA}, \quad I_B = \frac{0.35}{101} = 0.00347 \text{ mA}$$

$$V_B = -I_B R_B = -(0.00347)(10) \Rightarrow V_B = -0.0347 \text{ V}$$

$$V_E = V_B - V_{BE} (\text{on}) \Rightarrow V_E = -0.735 \text{ V}$$

b.

$$V_C = V_{CEQ} + V_E = 3.5 - 0.735 = 2.77 \text{ V}$$

$$I_C = \left(\frac{\beta}{1 + \beta} \right) \cdot I_E = \left(\frac{100}{101} \right) (0.35) = 0.347 \text{ mA}$$

$$R_C = \frac{V^+ - V_C}{I_C} = \frac{5 - 2.77}{0.347} \Rightarrow R_C = 6.43 \text{ k}\Omega$$

(c)

$$A_v = -g_m \left(\frac{R_B \| r_\pi }{R_B \| r_\pi + R_S} \right) (R_C \| r_o)$$

$$g_m = \frac{0.347}{0.026} = 13.3 \text{ mA/V}, \quad r_o = \frac{100}{0.347} = 288 \text{ k}\Omega$$

$$r_\pi = \frac{(100)(0.026)}{0.347} = 7.49 \text{ k}\Omega$$

$$R_B \| r_\pi = 10 \| 7.49 = 4.28 \text{ k}\Omega$$

$$A_v = -(13.3) \left(\frac{4.28}{4.28 + 0.1} \right) (6.43 \| 288) \Rightarrow A_v = -81.7$$

d.

$$A_v = -g_m \left(\frac{R_B \| r_\pi }{R_B \| r_\pi + R_S} \right) (R_C \| r_o)$$

$$R_B \| r_\pi = 10 \| 7.49 = 4.28 \text{ k}\Omega$$

$$A_v = -(13.3) \left(\frac{4.28}{4.28 + 0.5} \right) (6.43 \| 288) \Rightarrow A_v = -74.9$$

6.22

a.

$$R_{TH} = R_1 \| R_2 = 6 \| 1.5 = 1.2 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V^+ = \left(\frac{1.5}{1.5 + 6} \right) (5) = 1.0 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE} (\text{on})}{R_{TH} + (1 + \beta) R_E} = \frac{1.0 - 0.7}{1.2 + (181)(0.1)} = 0.0155 \text{ mA}$$

$$I_{CQ} = 2.80 \text{ mA}, \quad I_{EQ} = 2.81$$

$$V_{CEQ} = V^+ - I_{CQ} R_C - I_{EQ} R_E$$

$$= 5 - (2.8)(1) - (2.81)(0.1) \Rightarrow \underline{V_{CEQ} = 1.92 \text{ V}}$$

b.

$$r_\pi = \frac{(180)(0.026)}{2.80} = 1.67 \text{ k}\Omega$$

$$g_m = \frac{2.80}{0.026} = 108 \text{ mA/V}$$

(c)

$$A_v = g_m \left(\frac{R_1 \| R_2 \| r_\pi }{R_1 \| R_2 \| r_\pi + R_S} \right) (R_C \| R_L)$$

$$R_1 \| R_2 \| r_\pi = 6 \| 1.5 \| 1.67 = 0.698 \text{ k}\Omega$$

$$A_v = (108) \left(\frac{0.698}{0.698 + 0.2} \right) (1 \| 1.2) = 45.8$$

6.23

a.

$$9 = I_{EQ}R_E + V_{EB}(\text{on}) + I_{BQ}R_S$$

$$I_{EQ} = 0.75 \text{ mA}, I_{BQ} = \frac{0.75}{81} = 0.00926 \text{ mA}$$

$$I_{CQ} = 0.741 \text{ mA}$$

$$9 = (0.75)R_E + 0.7 + (0.00926)(2) \Rightarrow R_E = 11.0 \text{ k}\Omega$$

b.

$$V_E = 9 - (0.75)(11) = 0.75 \text{ V}$$

$$V_C = V_E - V_{ECQ} = 0.75 - 7 = -6.25 \text{ V}$$

$$R_C = \frac{V_C - (-9)}{I_{CQ}} = \frac{9 - 6.25}{0.741} \Rightarrow R_C = 3.71 \text{ k}\Omega$$

c.

$$A_v = -g_m \left(\frac{r_\pi}{r_\pi + R_S} \right) (R_C \| R_L \| r_o)$$

$$r_\pi = \frac{(80)(0.026)}{0.741} = 2.81 \text{ k}\Omega$$

$$r_o = \frac{80}{0.741} = 108 \text{ k}\Omega$$

$$A_v = \frac{-80}{2.81 + 2} (3.71 \| 10 \| 108)$$

$$A_v = -43.9$$

d.

$$R_i = R_S + r_\pi = 2 + 2.81 \Rightarrow R_i = 4.81 \text{ k}\Omega$$

6.24

(a) For $\beta = 80$, $I_{CQ} = (80) \left[\frac{4 - 0.7}{5 + (81)(5)} \right] = 0.6439 \text{ mA}$

$$g_m = \frac{0.6439}{0.026} = 24.77 \text{ mA/V}, \quad r_\pi = \frac{(80)(0.026)}{0.6439} = 3.23 \text{ k}\Omega, \quad r_o = \frac{50}{0.6439} = 77.7 \text{ k}\Omega$$

$$V_\pi = \frac{r_\pi \| R_B}{r_\pi \| R_B + R_S} \cdot v_s = \frac{(3.23 \| 5)}{(3.23 \| 5) + 1} \cdot v_s = (0.6624)v_s$$

$$A_v = -g_m \cdot \frac{V_\pi}{v_s} \cdot (r_o \| R_C \| R_L) = -(24.77)(0.6624)(77.7 \| 4 \| 4) = -32$$

For $\beta = 120$, $I_{CQ} = (120) \left[\frac{4 - 0.7}{5 + (121)(5)} \right] = 0.6492 \text{ mA}$

$$g_m = 24.97 \text{ mA/V}, \quad r_\pi = 4.806 \text{ k}\Omega, \quad r_o = 77 \text{ k}\Omega$$

$$V_\pi = \frac{(4.806 \| 5)}{(4.806 \| 5) + 1} \cdot v_s = (0.710)v_s$$

$$A_v = -(24.97)(0.710)(77 \| 4 \| 4) = -34.6$$

So $|A_v| \leq 34.6$

(b) $R_i = R_B \parallel r_\pi$

For $\beta = 80$, $R_i = 5 \parallel 3.23 = 1.96 \text{ k}\Omega$

For $\beta = 120$, $R_i = 5 \parallel 4.806 = 2.45 \text{ k}\Omega$

So $1.96 \leq R_i \leq 2.45 \text{ k}\Omega$

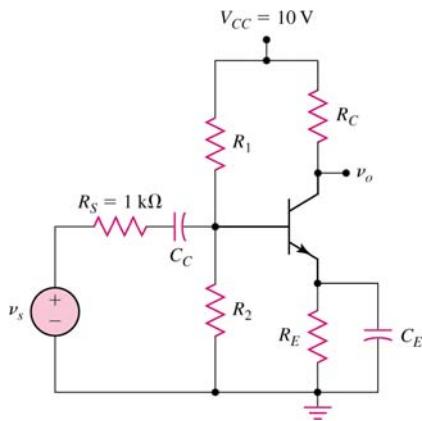
(c) $R_o = R_C \parallel r_o$

For $\beta = 80$, $R_o = 4 \parallel 77.7 = 3.804 \text{ k}\Omega$

For $\beta = 120$, $R_o = 4 \parallel 77 = 3.802 \text{ k}\Omega$

So $3.802 \leq R_o \leq 3.804 \text{ k}\Omega$

6.25



Assume an npn transistor with $\beta = 100$ and $V_A = \infty$. Let $V_{cc} = 10 \text{ V}$.

$$|A_v| = \frac{0.5}{0.01} = 50$$

Bias at $I_{CQ} = 1 \text{ mA}$ and let $R_E = 1 \text{ k}\Omega$

For a bias stable circuit

$$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(101)(1) = 10.1 \text{ k}\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (10.1)(10) = \frac{101}{R_1}$$

$$I_{BQ} = \frac{1}{100} = 0.01 \text{ mA}$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE} (\text{on}) + (1 + \beta) I_{BQ} R_E$$

$$\frac{101}{R_1} = (0.01)(10.1) + 0.7 + (101)(0.01)(1)$$

which yields $R_1 = 55.8 \text{ k}\Omega$, $R_2 = 12.3 \text{ k}\Omega$

Now

$$r_\pi = \frac{(100)(0.026)}{1} = 2.6 \text{ k}\Omega$$

$$g_m = \frac{1}{0.026} = 38.46 \text{ mA/V}$$

$$V_o = -g_m V_\pi R_C$$

$$\text{where } V_\pi = \left(\frac{R_1 \| R_2 \| r_\pi}{R_1 \| R_2 \| r_\pi + R_S} \right) \cdot V_s = \left(\frac{10.1 \| 2.6}{10.1 \| 2.6 + 1} \right) \cdot V_s = 0.674 V_s$$

$$\text{Then } A_v = \frac{V_o}{V_s} = -(0.674) g_m R_C = -(0.674)(38.46) R_C = -50$$

$$\text{which yields } R_C = 1.93 \text{ k}\Omega$$

With this R_C , the dc bias is OK.

Finish Design, Set $R_C = 2 \text{ k}\Omega$, $R_E = 1 \text{ k}\Omega$

$$R_1 = 56 \text{ k}\Omega, R_2 = 12 \text{ k}\Omega$$

$$R_{TH} = R_1 \| R_2 = 9.88 \text{ K}$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{12}{12 + 56} \right) (10) = 1.765 \text{ V}$$

$$I_{BQ} = \frac{1.765 - 0.7}{9.88 + (101)(1)} = 9.60 \mu\text{A}$$

$$I_{CQ} = 0.9605 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{0.9605} = 2.707 \text{ K} \quad g_m = \frac{0.9605}{0.026} = 36.94$$

$$R_{TH} \| r_\pi = 2.125 \text{ K}$$

$$V_\pi = \left(\frac{R_{TH} \| r_\pi}{R_{TH} \| r_\pi + R_S} \right) \cdot V_i = \left(\frac{2.125}{2.125 + 1} \right) \cdot V_i = (0.680) \cdot V_i$$

$$A_v = -(0.680) g_m R_C = -(0.680)(36.94)(2) = -50.2$$

Design specification met.

6.26

a.

$$I_{BQ} = \frac{6 - 0.7}{10 + (101)(3)} = 0.0169 \text{ mA}$$

$$I_{CQ} = 1.69 \text{ mA}, I_{EQ} = 1.71 \text{ mA}$$

$$V_{CEO} = (16 + 6) - (1.69)(6.8) - (1.71)(3)$$

$$V_{CEO} = 5.38 \text{ V}$$

b.

$$g_m = \frac{1.69}{0.026} = 65 \text{ mA/V}$$

$$r_\pi = \frac{(100)(0.026)}{1.69} = 1.54 \text{ k}\Omega, r_o = \infty$$

(c)

$$A_v = \frac{-\beta(R_C \| R_L)}{r_\pi + (1 + \beta)R_E} \cdot \frac{R_B \| R_{ib}}{R_B \| R_{ib} + R_S}$$

$$R_{ib} = r_\pi + (1 + \beta)R_E = 1.54 + (101)(3) = 304.5 \text{ k}\Omega$$

$$R_B \| R_{ib} = 10 \| 304.5 = 9.68 \text{ k}\Omega$$

Then

$$A_v = \frac{-(100)(6.8\parallel 6.8)}{1.54 + (101)(3)} \cdot \left(\frac{9.68}{9.68 + 0.5} \right) = -1.06$$

$$i_o = \left(\frac{R_C}{R_C + R_L} \right) (-\beta i_b)$$

$$i_b = \left(\frac{R_B}{R_B + r_\pi + (1+\beta)R_E} \right) \cdot i_s$$

$$\begin{aligned} A_i &= -(\beta) \left(\frac{R_C}{R_C + R_L} \right) \left(\frac{R_B}{R_B + r_\pi + (1+\beta)R_E} \right) \\ &= -(100) \left(\frac{6.8}{6.8 + 6.8} \right) \left(\frac{10}{10 + 1.54 + (101)(3)} \right) \Rightarrow A_i = -1.59 \end{aligned}$$

(d) $R_{is} = R_s + R_B \parallel R_{ib} = 0.5 + 10 \parallel 304.5 = 10.2 \text{ k}\Omega$

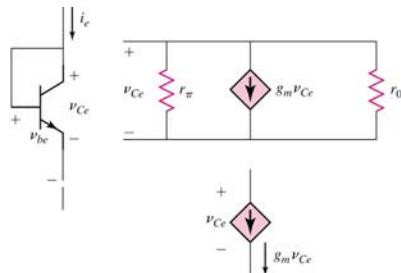
(e)

$$A_v = \frac{-\beta(R_C \parallel R_L)}{r_\pi + (1+\beta)R_E} = \frac{-(100)(6.8\parallel 6.8)}{1.54 + (101)(3)}$$

$$A_v = -1.12$$

$A_i = -1.59$, same as part (c)

6.27



$$r = \frac{v_{ce}}{g_m v_{ce}} = \frac{1}{g_m}$$

$$\text{So } r_e = r_\pi \left| \frac{1}{g_m} \right| r_o$$

6.28

(a) Set $\frac{R_C}{R_E} = 12.5 \Rightarrow R_C = 12.5R_E$

$$\text{Set } V_{CEQ} = 1.5 \equiv 3.3 - I_{CQ}(R_C + R_E) = 3.3 - I_{CQ}(13.5R_E)$$

$$\text{Set } I_{CQ} = 0.1 \text{ mA, } \Rightarrow R_E = 1.33 \text{ k}\Omega \text{ and } R_C = 16.7 \text{ k}\Omega$$

$$\text{We find } I_{BQ} = 0.833 \mu \text{ A}$$

$$\text{Set } R_{TH} = (0.1)(1+\beta)R_E = (0.1)(121)(1.33) = 16.1 \text{ k}\Omega$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE} (\text{on}) + (1+\beta) I_{BQ} R_E = (0.000833)(16.1) + 0.7 + (121)(0.000833)(1.33)$$

$$V_{TH} = 0.8475 = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (16.1)(3.3)$$

So $R_1 = 62.7 \text{ k}\Omega$, $R_2 = 21.7 \text{ k}\Omega$

Actual gain:

$$r_\pi = \frac{(120)(0.026)}{0.1} = 31.2 \text{ k}\Omega$$

$$R_{ib} = r_\pi + (1 + \beta)R_E = 31.2 + (121)(1.33) = 192.1 \text{ k}\Omega$$

$$R_i = R_{TH} \| R_{ib} = 16.1 \| 192.1 = 14.85 \text{ k}\Omega$$

$$\text{Then } A_v = \frac{-(120)(16.7)}{31.2 + (121)(1.33)} \cdot \left(\frac{14.85}{14.85 + 0.1} \right) = -10.4$$

$$(b) R_{is} = R_S + R_i = 0.1 + 14.85 = 14.95 \text{ k}\Omega$$

$$R_o = R_C = 16.7 \text{ k}\Omega$$

6.29

$$\frac{100}{5} = 20.$$

Need a voltage gain of $\frac{100}{5} = 20$.

Assume a sign inversion from a common-emitter is not important. Use the configuration for Figure 6.31.

Let $R_s = 0$. Need an input resistance of

$$R_i = \frac{5 \times 10^{-3}}{0.2 \times 10^{-6}} = 25 \times 10^3 \Rightarrow R_i = 25 \text{ k}\Omega$$

$R_i = R_{TH} \| R_{ib}$, Let $R_{TH} = 50 \text{ k}\Omega$, then $R_{ib} = 50 \text{ k}\Omega$

$$R_{ib} = r_\pi + (1 + \beta)R_E \approx (1 + \beta)R_E$$

$$\text{For } \beta = 100, R_E \approx \frac{R_{ib}}{1 + \beta} = \frac{50}{101} = 0.495 \text{ k}\Omega$$

Let $R_E = 0.5 \text{ k}\Omega$, $V_{CC} = 10 \text{ V}$, $I_{CQ} = 0.2 \text{ mA}$

$$\text{Then } I_{BQ} = \frac{0.2}{100} = 0.002 \text{ mA}$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE} (\text{on}) + (1 + \beta) I_{BQ} R_E$$

$$\frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (50)(10) = (0.002)(50) + 0.7 + (101)(0.002)(0.5)$$

which yields $R_1 = 555 \text{ k}\Omega$ and $R_2 = 55 \text{ k}\Omega$

$$\text{Now } A_v = \frac{-\beta R_C}{r_\pi + (1 + \beta)R_E}, \quad r_\pi = \frac{(100)(0.026)}{0.2} = 13 \text{ k}\Omega$$

So

$$-20 = \frac{-(100)R_C}{13 + (101)(0.5)} \Rightarrow R_C = 12.7 \text{ k}\Omega$$

[Note: $I_{CQ}R_C = (0.2)(12.7) = 2.54 \text{ V}$. So dc biasing is OK.]

6.30

$$\text{Set } \frac{R_C}{R_E} = 15 \Rightarrow R_C = 15R_E$$

Set $R_C = 5 \text{ k}\Omega$, then $R_E = 0.333 \text{ k}\Omega$

Set $I_{CQ} = 0.5 \text{ mA}$, then $V_{ECQ} \geq 5 - (0.5)(5.333) = 2.33 \text{ V}$

$r_\pi = 4.68 \text{ k}\Omega$, $R_{ib} = r_\pi + (1 + \beta)R_E = 4.68 + (91)(0.333) = 35 \text{ k}\Omega$

Set $R_i = 22 \text{ k}\Omega = R_{TH} \| R_{ib} = R_{TH} \| 35 \Rightarrow R_{TH} \geq 60 \text{ k}\Omega$

Now $I_{BQ} = 0.00556 \text{ mA}$, $I_{EQ} = 0.506 \text{ mA}$

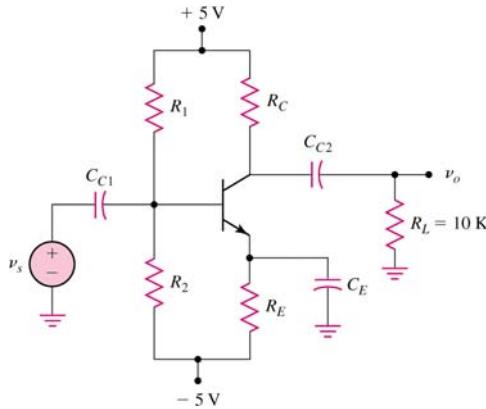
$$V_{CC} = I_{EQ}R_E + V_{EB}(\text{on}) + I_{BQ}R_{TH} + V_{TH}$$

$$5 = (0.506)(0.333) + 0.7 + (0.00556)(60) + V_{TH}$$

$$V_{TH} = 3.798 = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (60)(5)$$

So that $R_1 = 79 \text{ k}\Omega$ and $R_2 = 249 \text{ k}\Omega$

6.31



$$\beta = 120$$

$$\text{Let } I_{CQ} = 0.35 \text{ mA}, I_{EQ} = 0.353 \text{ mA}$$

$$I_{BQ} = 0.00292 \text{ mA}$$

$$\text{Let } R_E = 2 \text{ k}\Omega. \text{ For } V_{CEQ} = 4 \text{ V} \Rightarrow 10 = 4 + (0.35)R_C + (0.353)(2)$$

$$R_C = 15.1 \text{ k}\Omega, r_\pi = \frac{(120)(0.026)}{0.35} = 8.91 \text{ k}\Omega$$

$$A_v = \frac{-\beta(R_C \| R_L)}{r_\pi} = -\frac{(120)(15.1 \| 10)}{8.91}$$

$$A_v = -81.0$$

For bias stable circuit:

$$R_1 \| R_2 = R_{TH} = (0.1)(1 + \beta)R_E$$

$$= (0.1)(121)(2) = 24.2 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5 = \frac{1}{R_1} \cdot R_{TH} \cdot (10) - 5$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1 + \beta)I_{BQ}R_E - 5$$

$$\frac{1}{R_1}(24.2)(10) - 5 = (0.00292)(24.2) + 0.7 + (121)(0.00292)(2) - 5$$

$$\frac{1}{R_1}(242) = 1.477, \quad R_1 = 164 \text{ k}\Omega$$

$$\frac{164R_2}{164 + R_2} = 24.2 \Rightarrow R_2 = 28.4 \text{ k}\Omega$$

$$\frac{10}{164 + 28.4} = 0.052, \quad 0.35 + 0.052 = 0.402 \text{ mA}$$

So bias current specification is met.

6.32

$$R_{TH} = R_1 \parallel R_2 = 33 \parallel 50 = 19.88 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left(\frac{50}{50 + 33} \right) (3.3) = 1.988 \text{ V}$$

$$3.3 = (101)I_{BQ}(1) + 0.7 + I_{BQ}(19.88) + 1.988$$

$$\text{So } I_{BQ} = 0.005063 \text{ mA}, \quad I_{CQ} = 0.5063 \text{ mA}, \quad I_{EQ} = 0.5114 \text{ mA}$$

$$V_{ECQ} = 3.3 - (0.5114)(1) - (0.5063)(2) = 1.776 \text{ V}$$

$$\text{Then } \Delta V_{EC} = 1.776 - 0.5 = 1.276 \text{ V, or } \Delta V_{EC} = 3 - 1.776 = 1.224 \text{ V}$$

$$\text{So } \Delta V_{EC} = 2(1.224) = 2.448 \text{ V peak-to-peak}$$

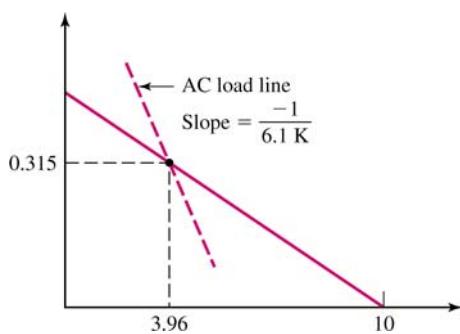
6.33

$$I_{BQ} = \frac{5 - 0.7}{50 + (101)(0.1 + 12.9)} = 0.00315 \text{ mA}$$

$$I_{CQ} = 0.315 \text{ mA}, \quad I_{EQ} = 0.319 \text{ mA}$$

$$V_{CEQ} = (5 + 5) - (0.315)(6) - (0.319)(13)$$

$$V_{CEQ} = 3.96 \text{ V}$$



$$\Delta i_C = -\frac{1}{6.1} \Delta v_{eC}$$

$$\text{For } \Delta i_C = 0.315 - 0.05 = 0.265 \Rightarrow |\Delta v_{eC}| = 1.62$$

$$v_{EC} (\text{min}) = 3.96 - 1.62 = 2.34$$

Output signal swing determined by current:

$$\text{Max. output swing} = \underline{3.24 \text{ V peak-to-peak}}$$

6.34

$$(a) 5 = (81)I_{BQ}(10) + 0.7 + I_{BQ}(2.5)$$

So that $I_{BQ} = 0.005292 \text{ mA}$, $I_{CQ} = 0.4234 \text{ mA}$, $I_{EQ} = 0.4287 \text{ mA}$

$$\text{Now } V_{ECQ} = 10 - (0.4234)(5) - (0.4287)(10) = 3.60 \text{ V}$$

$$\Delta V_{EC} = \Delta I_C (R_C \| R_L) = \Delta I_C (2.5)$$

$$\text{For } \Delta I_C = 0.4234 \Rightarrow \Delta V_{EC} = 1.06 \text{ V}$$

$$\text{So that } \Delta V_{EC} = 2(1.06) = 2.12 \text{ V peak-to-peak}$$

$$(b) \Delta I_C = 2(0.4234) = 0.847 \text{ mA peak-to-peak}$$

6.35

$$I_{EQ} = 0.8 \text{ mA}, I_{CQ} = 0.790 \text{ mA}, I_{BQ} = 0.009877 \text{ mA}$$

$$V_E = 0.7 + (0.009877)(20) = 0.898 \text{ V}$$

$$V_C = (0.79)(2.5) - 5 = -3.025 \text{ V}$$

$$\text{Then } V_{ECQ} = V_E - V_C = 3.923 \text{ V}$$

$$\Delta V_{EC} = \Delta I_C (R_C \| R_L) = \Delta I_C (2.5 \| 4) = \Delta I_C (1.538)$$

$$\text{For } \Delta I_C = 0.79 - 0.08 = 0.71 \text{ mA, then } \Delta V_{EC} = (0.71)(1.538) = 1.09 \text{ V}$$

$$\text{So, } \Delta I_C = 2(0.71) = 1.42 \text{ mA peak-to-peak,}$$

$$\Delta i_o = \left(\frac{2.5}{2.5 + 4} \right) \Delta I_C = 0.546 \text{ mA peak-to-peak}$$

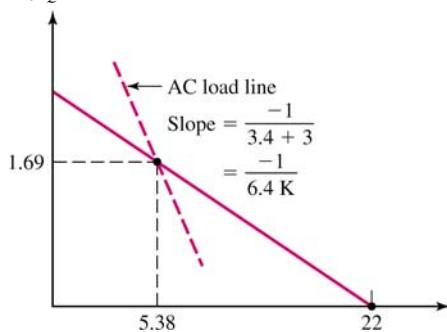
6.36

$$I_{BQ} = \frac{6 - 0.7}{10 + (101)(3)} = 0.0169 \text{ mA}$$

$$I_{CQ} = 1.69 \text{ mA}, I_{EQ} = 1.71 \text{ mA}$$

$$V_{CEQ} = (16 + 6) - (1.69)(6.8) - (1.71)(3)$$

$$V_{CEQ} = 5.38 \text{ V}$$



$$\Delta i_c = -\frac{1}{6.4} \Delta v_{ce}$$

For $v_{ce}(\min) = 1 \text{ V}$, $\Delta v_{ce} = 5.38 - 1 = 4.38 \text{ V} \Rightarrow |\Delta i_c| = \frac{4.38}{6.4} = 0.684 \text{ mA}$

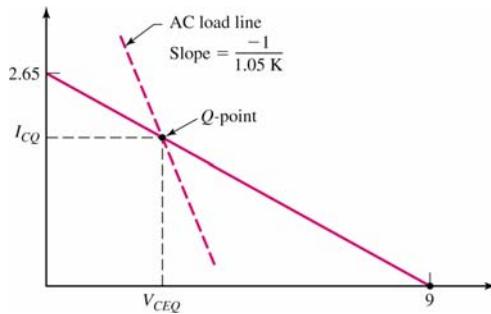
Output swing limited by voltage:

$$\begin{aligned}\Delta v_{ce} &= \text{Max. swing in output voltage} \\ &= \underline{8.76 \text{ V peak-to-peak}}\end{aligned}$$

$$\Delta i_0 = \frac{1}{2} \Delta i_c \Rightarrow \Delta i_0 = 0.342 \text{ mA}$$

or $\Delta i_0 = 0.684 \text{ mA}$ (peak-to-peak)

6.37



$$r_o = \frac{100}{I_{cQ}}$$

Neglect r_o as 1st approx.: dc load line $V_{CE} = 9 - I_C(3.4)$

$$\Delta I_C = I_{cQ} - 0.1$$

$$\Delta V_{CE} = V_{CEQ} - 1$$

Also $\Delta V_{CE} = \Delta I_C (R_C \parallel R_L) = \Delta I_C (1.05)$

$$\text{Or } V_{CEQ} - 1 = (I_{cQ} - 0.1)(1.05)$$

Substituting the expression for the dc load line.

$$[9 - I_{cQ}(3.4) - 1] = (I_{cQ} - 0.1)(1.05)$$

$$8.105 = I_{cQ}(4.45) \Rightarrow I_{cQ} = 1.821 \text{ mA}$$

$$V_{CEQ} = 2.81 \text{ V}$$

$$I_{BQ} = \frac{1.821}{100} = 0.01821$$

$$R_{TH} = (0.1)(101)(1.2) = 12.12 \text{ K}$$

$$V_{TH} = \frac{1}{R_i} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_i} (12.12)(9) = (0.01821)(12.12) + 0.7 + (101)(0.01821)(1.2)$$

$$= 0.2207 + 0.7 + 2.20705$$

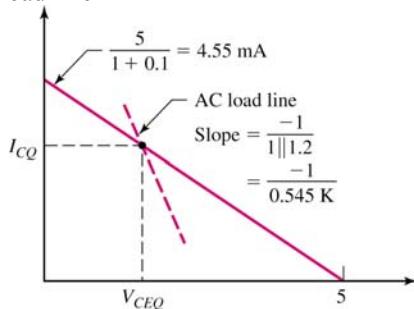
$$R_1 = 34.9 \text{ K}$$

$$R_2 = 18.6 \text{ K}$$

$$\frac{34.9R_2}{34.9 + R_2} = 12.12$$

6.38

dc load line



For maximum symmetrical swing

$$\Delta i_c = I_{CQ} - 0.25$$

$$\Delta v_{CE} = V_{CEQ} - 0.5 \text{ and } |\Delta i_c| = \frac{1}{0.545 \text{ k}\Omega} \cdot |\Delta v_{CE}|$$

$$I_{CQ} - 0.25 = \frac{V_{CEQ} - 0.5}{0.545}$$

$$V_{CEQ} = 5 - I_{CQ}(1.1)$$

$$0.545(I_{CQ} - 0.25) = [5 - I_{CQ}(1.1)] - 0.5$$

$$(0.545 + 1.1)I_{CQ} = 5 - 0.5 + 0.136$$

$$\underline{I_{CQ} = 2.82 \text{ mA}}, \quad I_{BQ} = 0.0157 \text{ mA}$$

$$R_{TH} = R_1 \parallel R_2 = (0.1)(1 + \beta)R_E \\ = (0.1)(181)(0.1) = 1.81 \text{ k}\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V^+ = I_{BQ} R_{TH} + V_{BE} (\text{on}) + (1 + \beta) I_{BQ} R_E$$

$$\frac{1}{R_1}(1.81)(5) = (0.0157)(1.81) + 0.7 + (181)(0.0157)(0.1)$$

$$\frac{1}{R_1}(9.05) = 1.013 \Rightarrow \underline{R_1 = 8.93 \text{ k}\Omega}$$

$$\frac{8.93R_2}{8.93 + R_2} = 1.81 \Rightarrow \underline{R_2 = 2.27 \text{ k}\Omega}$$

6.39

$$I_{CQ} = 0.647 \text{ mA}, \quad V_{CEQ} = 10 - (0.647)(9) = 4.18 \text{ V}$$

$$\Delta i_c = I_{CQ} = 0.647 \text{ mA}$$

$$\text{So } \Delta v_{ce} = \Delta i_c (4\parallel 4) = (0.647)(2) = 1.294 \text{ V}$$

Voltage swing is well within the voltage specification. Then $\Delta v_{ce} = 2(1.294) = 2.59 \text{ V peak-to-peak}$.

6.40

$$(a) \quad A_v = \frac{(1+\beta)R_E}{r_\pi + (1+\beta)R_E}$$

$$0.92 = \frac{(121)(0.5)}{r_\pi + (121)(0.5)} \Rightarrow r_\pi = 5.261 = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{I_{CQ}} \Rightarrow I_{CQ} = 0.593 \text{ mA}$$

$$(b) \quad r_o = \frac{V_A}{I_{CQ}} = \frac{20}{0.593} = 33.7 \text{ k}\Omega$$

$$A_v = \frac{(1+\beta)(r_o \| R_E)}{r_\pi + (1+\beta)(r_o \| R_E)}$$

$$r_o \| R_E = 33.7 \| 0.5 = 0.4927 \text{ k}\Omega$$

$$\text{Then } A_v = \frac{(121)(0.4927)}{5.261 + (121)(0.4927)} = 0.919$$

$$(c) \quad \text{For (a), } R_o = R_E \left\| \frac{r_\pi}{1+\beta} \right\| = 0.5 \left\| \frac{5.261}{121} \right\| = 0.5 \| 0.04348 \Rightarrow R_o = 40 \Omega$$

$$\text{For (b), } R_o = R_E \| r_o \left\| \frac{r_\pi}{1+\beta} \right\| = 0.04 \| 33.7 \Rightarrow R_o = 39.95 \Omega$$

6.41

$$(a) \quad R_{ib} = r_\pi + (1+\beta)R_E$$

$$A_v = \frac{(1+\beta)R_E}{r_\pi + (1+\beta)R_E} = \frac{(1+\beta)R_E}{R_{ib}}$$

$$R_E = \frac{(A_v)(R_{ib})}{1+\beta} = \frac{(0.95)(50)}{81} = 0.586 \text{ k}\Omega$$

$$50 = r_\pi + (81)(0.586) \Rightarrow r_\pi = 2.5 \text{ k}\Omega$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} \Rightarrow I_{CQ} = \frac{(80)(0.026)}{2.5} = 0.832 \text{ mA}$$

$$(b) \quad R_o = R_E \left\| \frac{r_\pi}{1+\beta} \right\| = 0.586 \left\| \frac{2.5}{81} \right\| = 0.586 \| 0.03086$$

$$R_o = 29.3 \Omega$$

6.42

$$(a) \quad g_m = 76.92 \text{ mA/V}, \quad r_\pi = 1.04 \text{ k}\Omega, \quad r_o = 25 \text{ k}\Omega$$

$$A_v = \frac{(1+\beta)r_o}{R_S + r_\pi + (1+\beta)r_o} = \frac{(81)(25)}{1+1.04+(81)(25)} = 0.999$$

$$R_i = R_S + r_\pi + (1+\beta)r_o = 1+1.04+(81)(25) \Rightarrow R_i = 2.027 \text{ M}\Omega$$

$$R_o = r_o \left\| \frac{r_\pi + R_S}{1+\beta} \right\| = 25 \left\| \frac{1.04+1}{81} \right\| = 25 \| 0.0252 \Rightarrow R_o = 25.2 \Omega$$

$$(b) \quad r_\pi = 10.4 \text{ k}\Omega, \quad r_o = 250 \text{ k}\Omega$$

$$A_v = \frac{(81)(250)}{1+10.4+(81)(250)} = 0.9994$$

$$R_i = 1 + 10.4 + (81)(250) \Rightarrow R_i = 20.26 \text{ M}\Omega$$

$$R_o = 250 \left\| \left(\frac{10.4+1}{81} \right) \right\| = 250 \left\| 0.1407 \right\| \Rightarrow R_o = 141 \Omega$$

6.43

$$(a) R_o = \frac{r_\pi + R_s}{1 + \beta}$$

$$0.015 = \frac{r_\pi + 0.5}{121} \Rightarrow r_\pi = 1.315 \text{ k}\Omega$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} \Rightarrow I_{CQ} = \frac{(120)(0.026)}{1.315} = 2.37 \text{ mA}$$

$$(b) r_o = \frac{50}{2.373} = 21.07 \text{ k}\Omega$$

$$A_v = \frac{(1 + \beta)r_o}{R_s + r_\pi + (1 + \beta)r_o} = \frac{(121)(21.07)}{0.5 + 1.315 + (121)(21.07)} = 0.9993$$

6.44

a.

$$R_{TH} = R_1 \parallel R_2 = 10 \parallel 10 = 5 \text{ k}\Omega$$

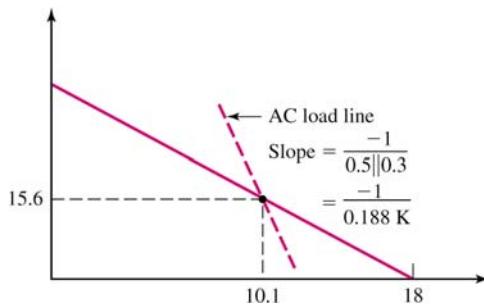
$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (18) - 9 = \left(\frac{10}{10 + 10} \right) (18) - 9 = 0$$

$$I_{BQ} = \frac{0 - 0.7 - (-9)}{5 + (181)(0.5)} = 0.0869 \text{ mA}$$

$$I_{CQ} = 15.6 \text{ mA}, \quad I_{EQ} = 15.7 \text{ mA}$$

$$V_{CEQ} = 18 - (15.7)(0.5) = 10.1 \text{ V}$$

b.



c.

$$r_\pi = \frac{(180)(0.026)}{15.6} = 0.30 \text{ k}\Omega$$

$$A_v = \frac{(1 + \beta)(R_E \parallel R_L)}{r_\pi + (1 + \beta)(R_E \parallel R_L)} \cdot \left(\frac{R_1 \parallel R_2 \parallel R_{ib}}{R_1 \parallel R_2 \parallel R_{ib} + R_s} \right)$$

$$R_{ib} = r_\pi + (1 + \beta)(R_E \| R_L) = 0.30 + (181)(0.5 \| 0.3) = 34.2 \text{ k}\Omega$$

$$R_1 \| R_2 \| R_{ib} = 5 \| 34.2 = 4.36 \text{ k}\Omega$$

$$A_v = \frac{(181)(0.5 \| 0.3)}{0.3 + (181)(0.5 \| 0.3)} \cdot \left(\frac{4.36}{4.36 + 1} \right) \Rightarrow A_v = 0.806$$

d.

$$R_{ib} = r_\pi + (1 + \beta)(R_E \| R_L)$$

$$R_{ib} = 0.30 + (181)(0.188) = 34.3 \text{ k}\Omega$$

$$R_o = R_E \left\| \frac{r_\pi + R_1 \| R_2 \| R_S}{1 + \beta} \right\| = 0.5 \left\| \frac{0.3 + 5 \| 1}{181} \right\| \Rightarrow R_o = 6.18 \Omega$$

6.45

a.

$$R_{TH} = R_1 \| R_2 = 10 \| 10 = 5 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (-10) = -5 \text{ V}$$

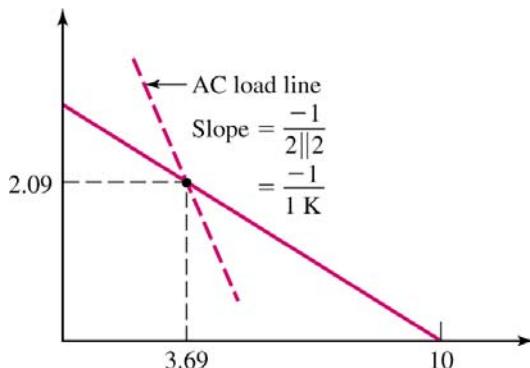
$$V_{TH} = I_{BQ} R_{TH} + V_{BE} (\text{on}) + (1 + \beta) I_{BQ} R_E - 10$$

$$I_{BQ} = \frac{-5 - 0.7 - (-10)}{5 + (121)(2)} = 0.0174 \text{ mA}$$

$$I_{CQ} = 2.09 \text{ mA}, \quad I_{EQ} = 2.11 \text{ mA}$$

$$V_{CEQ} = 10 - (2.09)(1) - (2.11)(2) = 3.69 \text{ V}$$

b.



c.

$$r_\pi = \frac{(120)(0.026)}{2.09} = 1.49 \text{ k}\Omega$$

$$A_v = \frac{(1 + \beta)(R_E \| R_L)}{r_\pi + (1 + \beta)(R_E \| R_L)} \cdot \left(\frac{R_1 \| R_2 \| R_{ib}}{R_1 \| R_2 \| R_{ib} + R_S} \right)$$

$$R_{ib} = r_\pi + (1 + \beta)(R_E \| R_L) = 1.49 + (121)(2 \| 2)$$

$$R_{ib} = 122.5 \text{ k}\Omega, \quad R_1 \| R_2 \| R_{ib} = 5 \| 122.5 = 4.80 \text{ k}\Omega$$

$$A_v = \frac{(121)(2 \| 2)}{1.49 + (121)(2 \| 2)} \cdot \left(\frac{4.80}{4.80 + 5} \right) \Rightarrow A_v = 0.484$$

d.

$$\begin{aligned} R_{ib} &= r_\pi + (1 + \beta)(R_E \parallel R_L) \\ R_{ib} &= 1.49 + (121)(2 \parallel 2) \Rightarrow R_{ib} = 122 \text{ k}\Omega \\ R_o &= R_E \left\| \frac{r_\pi + R_1 \parallel R_2 \parallel R_S}{1 + \beta} \right\| = 2 \left\| \frac{1.49 + 5 \parallel 5}{121} \right\| \Rightarrow R_o = 32.4 \Omega \end{aligned}$$

6.46

(a) $R_{TH} = R_1 \parallel R_2 = 585 \parallel 135 = 109.7 \text{ k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left(\frac{135}{135 + 585} \right) (3.3) = 0.61875 \text{ V}$$

$$3.3 = (1 + \beta)I_{BQ}R_E + V_{EB}(\text{on}) + I_{BQ}R_{TH} + V_{TH}$$

$$I_{BQ} = \frac{3.3 - 0.7 - 0.61875}{109.7 + (91)(12)} = 0.001649 \text{ mA}$$

Then $I_{CQ} = 0.1484 \text{ mA}$, $I_{EQ} = 0.150 \text{ mA}$

$$V_{ECQ} = 3.3 - (0.15)(12) = 1.5 \text{ V}$$

(c) $r_\pi = \frac{(90)(0.026)}{0.1484} = 15.77 \text{ k}\Omega$, $r_o = \frac{60}{0.1484} = 404 \text{ k}\Omega$

$$A_v = \frac{(1 + \beta)(r_o \parallel R_E \parallel R_L)}{r_\pi + (1 + \beta)(r_o \parallel R_E \parallel R_L)}$$

$$r_o \parallel R_E \parallel R_L = 404 \parallel 12 \parallel 4 = 2.978 \text{ k}\Omega$$

$$A_v = \frac{(91)(2.978)}{15.77 + (91)(2.978)} = 0.945$$

$$R_{ib} = r_\pi + (1 + \beta)(r_o \parallel R_E \parallel R_L) = 15.77 + (91)(2.978) = 286.8 \text{ k}\Omega$$

$$A_i = (1 + \beta) \left(\frac{R_{TH}}{R_{TH} + R_{ib}} \right) \left(\frac{r_o \parallel R_E}{r_o \parallel R_E + R_L} \right) = (91) \left(\frac{109.7}{109.7 + 286.8} \right) \left(\frac{404 \parallel 12}{404 \parallel 12 + 4} \right) = 18.7$$

(d) $R_{ib} = 286.8 \text{ k}\Omega$

$$R_o = R_E \left\| r_o \left\| \frac{r_\pi}{1 + \beta} \right\| \right\| = 12 \left\| 404 \left\| \frac{15.77}{91} \right\| \right\| \Rightarrow R_o = 171 \Omega$$

6.47

(a)

$$I_{BQ} = \frac{0.5}{81} = 0.00617 \text{ mA}$$

$$V_B = I_{BQ}R_B = (0.00617)(10) \Rightarrow \underline{V_B = 0.0617 \text{ V}}$$

$$V_E = V_B + 0.7 \Rightarrow \underline{V_E = 0.7617 \text{ V}}$$

(b)

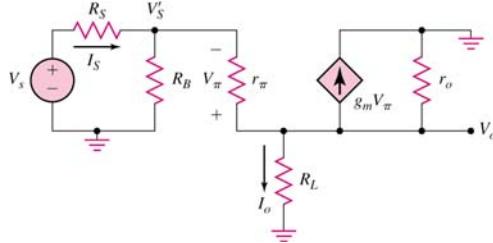
$$I_{CQ} = (0.5) \left(\frac{80}{81} \right) = 0.494 \text{ mA}$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.494}{0.026} \Rightarrow g_m = 19 \text{ mA/V}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(80)(0.026)}{0.494} \Rightarrow r_\pi = 4.21 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{150}{0.494} \Rightarrow r_o = 304 \text{ k}\Omega$$

(c)



For $R_S = 0$

$$V_o = -\left(\frac{V_\pi}{r_\pi} + g_m V_\pi \right) (R_L \| r_o)$$

$$\text{so that } V_\pi = \frac{-V_o}{\left(\frac{1+\beta}{r_\pi} \right) (R_L \| r_o)}$$

Now $V_s + V_\pi = V_o$

$$V_s = V_o - V_\pi = V_o + \frac{V_o}{\left(\frac{1+\beta}{r_\pi} \right) (R_L \| r_o)}$$

We find

$$A_v = \frac{V_o}{V_s} = \frac{(1+\beta)(R_L \| r_o)}{r_\pi + (1+\beta)(R_L \| r_o)} = \frac{(81)(0.5 \| 304)}{4.21 + (81)(0.5 \| 304)}$$

$$A_v = 0.906$$

$$R_{ib} = r_\pi + (1+\beta)(R_L \| r_o) \cong 4.21 + (81)(0.5) = 44.7 \text{ k}\Omega$$

$$I_b = \left(\frac{R_B}{R_B + R_{ib}} \right) I_s \quad \text{and} \quad I_o = \left(\frac{r_o}{r_o + R_L} \right) (1+\beta) I_b$$

Then

$$A_i = \frac{I_o}{I_s} = (1+\beta) \left(\frac{R_B}{R_B + R_{ib}} \right) \left(\frac{r_o}{r_o + R_L} \right)$$

$$A_i \cong (81) \left(\frac{10}{10 + 44.7} \right) (1) \Rightarrow A_i = 14.8$$

(d)

$$V'_s = \left(\frac{R_B \| R_{ib}}{R_B \| R_{ib} + R_S} \right) \cdot V_s = \left(\frac{10 \| 44.7}{10 \| 44.7 + 2} \right) \cdot V_s = (0.803) V_s$$

$$\text{Then } A_v = (0.803)(0.906) \Rightarrow A_v = 0.728$$

$$A_i = 14.8 \text{ (unchanged)}$$

6.48

(a)

$$I_{CQ} = 1.98 \text{ mA}, \quad r_\pi = \frac{(100)(0.026)}{1.98} = 1.313 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{1.98} = 50.5 \text{ k}\Omega$$

$$R_o = \frac{r_\pi + R_S}{1 + \beta} \| r_o = \frac{1.31 + 10}{101} \| 50.5 \Rightarrow R_o = 112 \Omega$$

(b) From Equation (6.68)

$$A_v = \frac{(1 + \beta)(r_o \| R_L)}{r_\pi + (1 + \beta)(r_o \| R_L)} \quad r_o = \frac{100}{1.98} = 50.5 \text{ K}$$

(i)

$$R_L = 0.5 \text{ K}$$

$$A_v = \frac{(101)(50.5 \| 0.5)}{1.31 + (101)(50.5 \| 0.5)}$$

$$A_v = \frac{(101)(0.4951)}{1.31 + (101)(0.4951)} \Rightarrow A_v = 0.974$$

(ii)

$$R_L = 5 \text{ K} \quad r_o \| R_L = 50.5 \| 5 = 4.5495$$

$$A_v = \frac{(101)(4.55)}{1.31 + (101)(4.55)} \Rightarrow A_v = 0.997$$

6.49

$$(a) \frac{5 - 0.7}{3.3} = I_{EQ} = 1.303 \text{ mA}, \quad I_{CQ} = 1.29 \text{ mA}$$

$$V_{ECQ} = 0.7 - (-5) = 5.7 \text{ V}$$

$$(b) \quad r_\pi = \frac{(110)(0.026)}{1.29} = 2.217 \text{ k}\Omega, \quad r_o = \frac{50}{1.29} = 38.76 \text{ k}\Omega$$

$$A_v = \frac{(1 + \beta)(r_o \| R_E \| R_L)}{r_\pi + (1 + \beta)(r_o \| R_E \| R_L)}$$

$$r_o \| R_E \| R_L = 38.76 \| 3.3 \| 1 = 0.7525 \text{ k}\Omega$$

$$A_v = \frac{(111)(0.7525)}{2.217 + (111)(0.7525)} = 0.974$$

$$R_{ib} = r_\pi + (1 + \beta)(r_o \| R_E \| R_L) = 2.217 + (111)(0.7525) = 85.7 \text{ k}\Omega$$

$$R_o = \frac{r_\pi}{1+\beta} \| R_E \| r_o = \frac{2.217}{111} \| 3.3 \| 38.76 \Rightarrow R_o = 19.8 \Omega$$

$$(c) \quad i_s(t) = \frac{v_s(t)}{R_{ib}} = \frac{2.8 \sin \omega t}{85.7} \Rightarrow i_s(t) = 32.7 \sin \omega t \text{ } (\mu \text{A})$$

$$i_o(t) = (1 + \beta) \left(\frac{R_E \| r_o}{R_E \| r_o + R_L} \right) \cdot i_s(t) = (111) \left(\frac{3.041}{3.041 + 1} \right) (32.7 \sin \omega t)$$

$$\text{or } i_o(t) = 2.73 \sin \omega t \text{ } (\text{mA})$$

$$v_o(t) = i_o(t) R_L = 2.73 \sin \omega t \text{ } (\text{V})$$

$$v_{eb}(t) = -i_s(t) r_\pi = -(32.7 \sin \omega t)(2.217)$$

$$v_{eb}(t) = -72.5 \sin \omega t \text{ } (\text{mV})$$

6.50

a.

$$I_{EQ} = 1 \text{ mA}, V_{CEO} = V_{CC} - I_{EQ} R_E$$

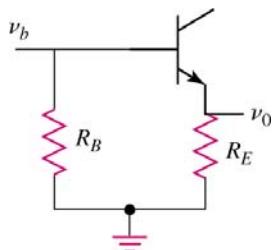
$$5 = 10 - (1)(R_E) \Rightarrow R_E = 5 \text{ k}\Omega$$

$$I_{BQ} = \frac{1}{101} = 0.0099 \text{ mA}$$

$$10 = I_{BQ} R_B + V_{BE} \text{ (on)} + I_{EQ} R_E$$

$$10 = (0.0099) R_B + 0.7 + (1)(5) \Rightarrow R_B = 434 \text{ k}\Omega$$

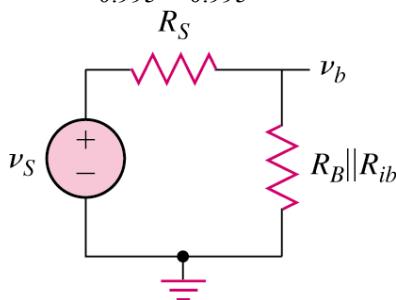
b.



$$r_\pi = \frac{(100)(0.026)}{0.99} = 2.63 \text{ k}\Omega$$

$$\frac{v_0}{v_b} = \frac{(1+\beta)R_E}{r_\pi + (1+\beta)R_E} = \frac{(101)(5)}{2.63 + (101)(5)} = 0.995$$

$$\Rightarrow v_b = \frac{v_0}{0.995} = \frac{4}{0.995} \Rightarrow v_b = 4.02 \text{ V peak-to-peak at base}$$



$$R_{ib} = r_\pi + (1 + \beta) R_E = 508 \text{ k}\Omega$$

$$R_B \| R_{ib} = 434 \| 508 = 234 \text{ k}\Omega$$

$$v_b = \frac{R_B \| R_{ib}}{R_B \| R_{ib} + R_S} \cdot v_s = \frac{234 v_s}{234 + 0.7} = \frac{234}{234.7} v_s$$

$$v_b = 0.997 v_s \Rightarrow v_s = \frac{4.02}{0.997} \Rightarrow v_s = 4.03 \text{ V peak-to-peak}$$

c.

$$R_{ib} = r_\pi + (1 + \beta)(R_E \| R_L)$$

$$R_{ib} = 2.63 + (101)(5 \| 1) = 86.8 \text{ k}\Omega$$

$$R_B \| R_{ib} = 434 \| 86.8 = 72.3 \text{ k}\Omega$$

$$v_b = \left(\frac{72.3}{72.3 + 0.7} \right) \cdot v_s = 0.99 v_s = (0.99)(4.03)$$

$$v_b = 3.99 \text{ V peak-to-peak}$$

$$v_o = \frac{(1 + \beta)(R_E \| R_L)}{r_\pi + (1 + \beta)(R_E \| R_L)} \cdot v_b$$

$$= \frac{(101)(0.833)}{2.63 + (101)(0.833)} \cdot (3.99)$$

$$v_o = 3.87 \text{ V peak-to-peak}$$

6.51

$$R_{TH} = R_1 \| R_2 = 40 \| 60 = 24 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{60}{60 + 40} \right) (10) = 6 \text{ V}$$

$$\text{For } \beta = 75, I_{BQ} = \frac{6 - 0.7}{24 + (76)(5)} = 0.0131 \text{ mA}$$

$$I_{CQ} = 0.984 \text{ mA}$$

$$\text{For } \beta = 150, I_{BQ} = \frac{6 - 0.7}{24 + (151)(5)} = 0.00680 \text{ mA}$$

$$I_{CQ} = 1.02 \text{ mA}$$

$$\text{For } \beta = 75, r_\pi = \frac{(75)(0.026)}{0.984} = 1.98 \text{ k}\Omega$$

$$\text{For } \beta = 150, r_\pi = \frac{(150)(0.026)}{1.02} = 3.82 \text{ k}\Omega$$

$$\text{For } \beta = 75, R_{ib} = r_\pi + (1 + \beta)(R_E \| R_L) = 65.3 \text{ k}\Omega$$

$$\text{For } \beta = 150, R_{ib} = 130 \text{ k}\Omega$$

$$A_v = \frac{(1 + \beta)(R_E \| R_L)}{r_\pi + (1 + \beta)(R_E \| R_L)} \cdot \frac{R_1 \| R_2 \| R_{ib}}{R_1 \| R_2 \| R_{ib} + R_S}$$

$$\text{For } \beta = 75, R_1 \| R_2 \| R_{ib} = 40 \| 60 \| 65.3 = 17.5 \text{ k}\Omega$$

$$A_v = \frac{(76)(0.833)}{1.98 + (76)(0.833)} \cdot \frac{17.5}{17.5 + 4} = 0.789$$

For $\beta = 150$, $R_1 \parallel R_2 \parallel R_{ib} = 40 \parallel 60 \parallel 130 = 20.3 \text{ k}\Omega$

$$A_v = \frac{(151)(0.833)}{3.82 + (151)(0.833)} \cdot \frac{20.3}{20.3 + 4} = 0.811$$

So $0.789 \leq A_v \leq 0.811$

$$A_i = (1 + \beta) \left(\frac{R_E}{R_E + R_L} \right) \left(\frac{R_{TH}}{R_{TH} + R_{ib}} \right)$$

$\beta = 75$

$$A_i = (76) \left(\frac{5}{5+1} \right) \left(\frac{24}{24+65.3} \right) \Rightarrow A_i = 17.0$$

$\beta = 150$

$$A_i = (151) \left(\frac{5}{6} \right) \left(\frac{24}{24+130} \right) \Rightarrow A_i = 19.6$$

$17.0 \leq A_i \leq 19.6$

6.52

(a)

$$9 = \left(\frac{I_E}{1 + \beta} \right) (100) + V_{BE} (\text{on}) + I_E R_E$$

$$I_E = \frac{9 - 0.7}{\left(\frac{100}{1 + \beta} \right) + R_E}$$

$$\beta = 50 \quad I_E = \frac{8.3}{\left(\frac{100}{51} \right) + 1} = 2.803 \text{ mA}$$

$$\beta = 200 \quad I_E = \frac{8.3}{\left(\frac{100}{201} \right) + 1} = 5.543 \text{ mA}$$

$2.80 \leq I_E \leq 5.54 \text{ mA}$

$V_E = I_E R_E$, $\beta = 50$, $V_E = 2.80 \text{ V}$

$\beta = 200$, $V_E = 5.54 \text{ V}$

(b) $\beta = 50$, $I_{CQ} = 2.748 \text{ mA}$, $r_\pi = 0.473 \text{ K}$

$\beta = 200$, $I_{CQ} = 5.515 \text{ mA}$, $r_\pi = 0.943 \text{ K}$

$$R_i = R_B \left[r_\pi + (1 + \beta) (R_E \parallel R_L) \right]$$

$$\beta = 50, \quad R_i = 100 \left[0.473 + (51)(1) \right] = 100 \parallel 25.97 = 20.6 \text{ k}\Omega$$

$$\beta = 200, \quad R_i = 100 \left[0.943 + (201)(1) \right] = 100 \parallel 101.4 = 50.3 \text{ k}\Omega$$

From Fig. (6.68)

$$\begin{aligned}
 A_v &= \frac{(1+\beta)(R_E \| R_L)}{r_\pi + (1+\beta)(R_E \| R_L)} \cdot \left(\frac{R_i}{R_i + R_S} \right) \\
 &= \frac{(51)(1\|1)}{0.473 + (51)(1\|1)} \cdot \left(\frac{20.6}{20.6+10} \right) \\
 \beta = 50 \Rightarrow A_v &= 0.661 \\
 \beta = 200 \Rightarrow A_v &= \frac{(201)(1\|1)}{0.943 + (201)(1\|1)} \left(\frac{50.3}{50.3+10} \right) \\
 A_v &= 0.826
 \end{aligned}$$

6.53

$$\begin{aligned}
 V_o &= (1+\beta) I_b R_L \\
 I_b &= \frac{V_s}{r_\pi + (1+\beta) R_L} \\
 \text{so } A_v &= \frac{(1+\beta) R_L}{r_\pi + (1+\beta) R_L}
 \end{aligned}$$

For $\beta = 100$, $R_L = 0.5 \text{ k}\Omega$

$$r_\pi = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$\text{Then } A_v(\min) = \frac{(101)(0.5)}{5.2 + (101)(0.5)} = 0.9066$$

Then $\beta = 180$, $R_L = 500 \text{ k}\Omega$

$$r_\pi = \frac{(180)(0.026)}{0.5} = 9.36 \text{ k}\Omega$$

$$\text{Then } A_v(\max) = \frac{(181)(500)}{9.36 + (181)(500)} = 0.9999$$

6.54

$$(a) V_{CEQ} = V_{CC} - I_{EQ} R_E$$

Let $V_{CEQ} = 2.5 \text{ V}$, then $2.5 = 5 - I_{EQ}(0.5) \Rightarrow I_{EQ} \cong I_{CQ} = 5 \text{ mA}$, $I_{BQ} = 0.04167 \text{ mA}$

$$r_\pi = \frac{(120)(0.026)}{5} = 0.624 \text{ k}\Omega$$

$$R_{ib} = r_\pi + (1+\beta)(R_E \| R_L) = 0.624 + (121)(0.5\|0.5) = 30.87 \text{ k}\Omega$$

$$A_i = (1+\beta) \left(\frac{R_{TH}}{R_{TH} + R_{ib}} \right) \left(\frac{R_E}{R_E + R_L} \right)$$

$$10 = (121) \left(\frac{R_{TH}}{R_{TH} + 30.87} \right) \left(\frac{0.5}{0.5 + 0.5} \right) \Rightarrow R_{TH} = 6.113 \text{ k}\Omega$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE(on)} + I_{EQ} R_E = (0.04167)(6.113) + 0.7 + (5)(0.5) = 3.455 \text{ V}$$

$$V_{TH} = 3.455 = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (6.113)(5)$$

So $R_1 = 8.85 \text{ k}\Omega$ and $R_2 = 19.8 \text{ k}\Omega$

$$R_o = R_E \left\| \frac{r_\pi}{1+\beta} = 0.5 \right\| \frac{0.624}{121} \Rightarrow R_o = 5.10 \Omega$$

$$(b) \quad A_i = (1 + \beta) \left(\frac{R_{TH}}{R_{TH} + R_{ib}} \right) \left(\frac{R_E}{R_E + R_L} \right)$$

$$R_{ib} = r_\pi + (1 + \beta)(R_E \| R_L) = 0.624 + (121)(0.5 \| 2) = 49.02 \text{ k}\Omega$$

$$A_i = (121) \left(\frac{6.113}{6.113 + 49.02} \right) \left(\frac{0.5}{0.5 + 2} \right) = 2.68$$

6.55

$$R_i = R_{TH} \| R_{ib} \text{ where } R_{ib} = r_\pi + (1 + \beta)R_E$$

$$V_{CEQ} = 3.5, \quad I_{CQ} \frac{5 - 3.5}{2} = 0.75 \text{ mA}$$

$$r_\pi = \frac{(120)(0.026)}{0.75} = 4.16 \text{ k}\Omega$$

$$R_{ib} = 4.16 + (121)(2) = 246 \text{ k}\Omega$$

$$\text{Then } R_i = 120 = R_{TH} \| 246 \Rightarrow R_{TH} = 234 \text{ k}\Omega$$

$$I_{BQ} = \frac{0.75}{120} = 0.00625 \text{ mA}$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE} (\text{on}) + (1 + \beta) I_{BQ} R_E$$

$$\frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (234)(5) = (0.00625)(234) + 0.7 + (121)(0.00625)(2)$$

which yields $R_1 = 318 \text{ k}\Omega$ and $R_2 = 886 \text{ k}\Omega$

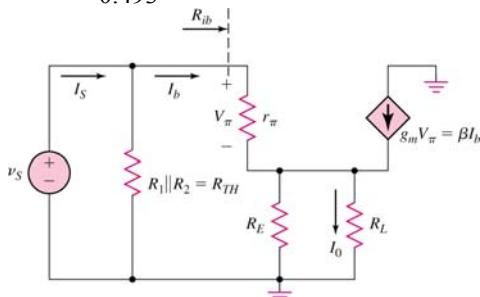
6.56

a.

$$\text{Let } R_E = 24 \Omega \text{ and } V_{CEQ} = \frac{1}{2} V_{CC} = 12 \text{ V} \Rightarrow I_{EQ} = \frac{12}{24} = 0.5 \text{ A}$$

$$I_{CQ} = 0.493 \text{ A}, \quad I_{BQ} = 6.58 \text{ mA}$$

$$r_\pi = \frac{(75)(0.026)}{0.493} = 3.96 \Omega$$



$$I_0 = (1 + \beta) I_b \left(\frac{R_E}{R_E + R_L} \right)$$

$$I_b = I_S \left(\frac{R_{TH}}{R_{TH} + R_{ib}} \right)$$

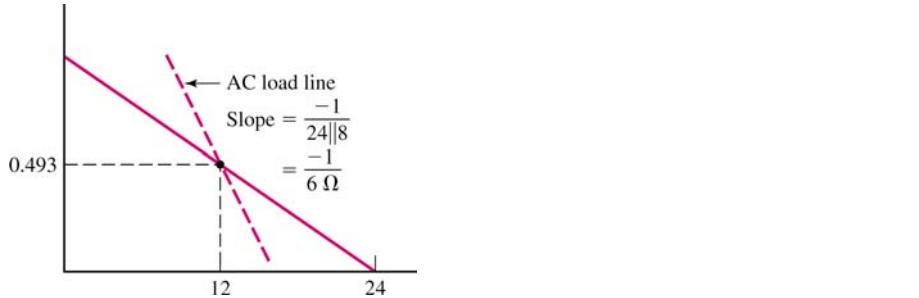
$$\begin{aligned}
 R_{ib} &= r_\pi + (1 + \beta)(R_E \parallel R_L) \\
 &= 3.96 + (76)(24 \parallel 8) \Rightarrow R_{ib} = 460 \Omega \\
 A_i &= \frac{I_0}{I_s} = (1 + \beta) \left(\frac{R_E}{R_E + R_L} \right) \left(\frac{R_{TH}}{R_{TH} + R_{ib}} \right) \\
 8 &= (76) \left(\frac{24}{24+8} \right) \left(\frac{R_{TH}}{R_{TH} + 460} \right) \\
 0.140 &= \frac{R_{TH}}{R_{TH} + 460} \Rightarrow R_{TH} = 74.9 \Omega \text{ (Minimum value)}
 \end{aligned}$$

dc analysis:

$$\begin{aligned}
 V_{TH} &= \frac{1}{R_i} \cdot R_{TH} \cdot V_{CC} \\
 &= I_{BQ} R_{TH} + V_{BE} \text{ (on)} + I_{EQ} R_E \\
 \frac{1}{R_i} (74.9)(24) &= (0.00658)(74.9) + 0.70 + (0.5)(24) \\
 &= 13.19
 \end{aligned}$$

$$R_i = 136 \Omega, \frac{136R_2}{136 + R_2} = 74.9 \Rightarrow R_2 = 167 \Omega$$

b.



$$\Delta i_C = -\frac{1}{6} \Delta v_{ce}$$

For $\Delta i_C = 0.493 \Rightarrow |\Delta v_{ce}| = (0.493)(6) \Rightarrow \text{Max. swing in output voltage for this design} = 5.92 \text{ V peak-to-peak}$

c.

$$R_0 = \frac{r_\pi}{1 + \beta} \parallel R_E = \frac{3.96}{76} \parallel 24 = 0.0521 \parallel 24 \Rightarrow R_0 = 52 \text{ m}\Omega$$

6.57

The output of the emitter follower is

$$v_o = \left(\frac{R_L}{R_L + R_o} \right) \cdot v_{TH}$$

For v_o to be within 5% for a range of R_L , we have

$$\frac{R_L(\min)}{R_L(\min) + R_o} = (0.95) \frac{R_L(\max)}{R_L(\max) + R_o}$$

$$\frac{4}{4 + R_o} = (0.95) \frac{10}{10 + R_o} \text{ which yields } R_o = 0.364 \text{ k}\Omega$$

We have $R_o = \left(\frac{r_\pi + R_1 \| R_2 \| R_S}{1 + \beta} \right) \| R_E \| r_o$

The first term dominates

Let $R_1 \| R_2 \| R_S \approx R_S$, then

$$R_o \approx \frac{r_\pi + R_S}{1 + \beta} \Rightarrow 0.364 = \frac{r_\pi + 4}{1 + \beta}$$

$$\text{or } 0.364 = \frac{r_\pi}{1 + \beta} + \frac{4}{1 + \beta} = \frac{\beta V_T}{I_{cQ}(1 + \beta)} + \frac{4}{1 + \beta}$$

$$0.364 \approx \frac{V_T}{I_{cQ}} + \frac{4}{1 + \beta}$$

The factor $\frac{4}{1 + \beta}$ is in the range of $\frac{4}{91} = 0.044$ to $\frac{4}{131} = 0.0305$. We can set $R_o \approx 0.32 = \frac{V_T}{I_{cQ}}$

Or $I_{cQ} = 0.08125 \text{ mA}$. To take into account other factors, set $I_{cQ} = 0.15 \text{ mA}$,

$$I_{BQ} = \frac{0.15}{110} = 0.00136 \text{ mA}$$

For $V_{CEQ} \approx 5 \text{ V}$, set $R_E = \frac{5}{0.15} = 33.3 \text{ k}\Omega$

Design a bias stable circuit.

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (10) - 5 = \frac{1}{R_1} (R_{TH}) (10) - 5$$

$$R_{TH} = (0.1)(1 + \beta) R_E = (0.1)(111)(33.3) = 370 \text{ k}\Omega$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE} (\text{on}) + (1 + \beta) I_{BQ} R_E - 5$$

$$\text{So } \frac{1}{R_1}(370)(10) - 5 = (0.00136)(370) + 0.7 + (111)(0.00136)(33.3) - 5$$

which yields $R_1 = 594 \text{ k}\Omega$ and $R_2 = 981 \text{ k}\Omega$

$$\text{Now } A_v = \frac{(1+\beta)(R_E \| R_L)}{r_\pi + (1+\beta)(R_E \| R_L)} \cdot \left(\frac{R_{TH} \| R_{ib}}{R_{TH} \| R_{ib} + R_s} \right)$$

$$R_{ib} = r_\pi + (1+\beta)(R_E \| R_L) \text{ and } r_\pi = \frac{\beta V_T}{I_{CQ}}$$

For $\beta = 90$, $R_L = 4 \text{ k}\Omega$,

$$r_\pi = 15.6 \text{ k}\Omega, \quad R_{ib} = 340.6 \text{ k}\Omega$$

$$A_v = \frac{(91)(33.3\|4)}{15.6 + (91)(33.3\|4)} \cdot \frac{370\|340.6}{370\|340.6 + 4} \Rightarrow A_v = 0.9332$$

For $\beta = 90$, $R_L = 10 \text{ k}\Omega$

$$R_{ib} = 715.4 \text{ k}\Omega$$

$$A_v = \frac{(91)(33.3\|10)}{15.6 + (91)(33.3\|10)} \cdot \frac{370\|715.4}{370\|715.4 + 4} \Rightarrow A_v = 0.9625$$

For $\beta = 130$, $R_L = 4 \text{ k}\Omega$

$$r_\pi = 22.5 \text{ k}\Omega, \quad R_{ib} = 490 \text{ k}\Omega$$

$$A_v = \frac{(131)(33.3\|4)}{22.5 + (131)(33.3\|4)} \cdot \frac{370\|490}{370\|490 + 4} \Rightarrow A_v = 0.9360$$

For $\beta = 130$, $R_L = 10 \text{ k}\Omega$

$$R_{ib} = 1030 \text{ k}\Omega$$

$$A_v = \frac{(131)(33.3\|10)}{22.5 + (131)(33.3\|10)} \cdot \frac{370\|1030}{370\|1030 + 4} \Rightarrow A_v = 0.9645$$

Now $v_o(\text{min}) = |A_v(\text{min})| \cdot v_s = 3.73 \sin \omega t$

$$v_o(\text{max}) = |A_v(\text{max})| \cdot v_s = 3.86 \sin \omega t$$

$$\frac{\Delta v_o}{v_o} = 3.5\%$$

6.58

$$P_{AVG} = i_L^2(rms)R_L \Rightarrow 1 = i_L^2(rms)(12)$$

$$\text{so } i_L(rms) = 0.289 \text{ A} \Rightarrow i_L(\text{peak}) = \sqrt{2}(0.289)$$

$$i_L(\text{peak}) = 0.409 \text{ A}$$

$$v_L(\text{peak}) = i_L(\text{peak}) \cdot R_L = (0.409)(12) = 4.91 \text{ V}$$

$$\text{Need a gain of } \frac{4.91}{5} = 0.982$$

With $R_s = 10 \text{ k}\Omega$, we will not be able to meet this voltage gain requirement. Need to insert a buffer or an op-amp voltage follower (see Chapter 9) between R_s and C_{c1} .

Set $I_{EQ} = 0.5A$, $V_{CEQ} = \frac{1}{3}(12 - (-12)) = 8 V$

$$24 = I_{EQ}R_E + V_{CEQ} = (0.5)R_E + 8 \Rightarrow R_E = 32 \Omega$$

Let $\beta = 50$, $I_{CQ} = \frac{50}{51}(0.5) = 0.49 A$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(50)(0.026)}{0.49} = 2.65 \Omega$$

$$R_{ib} = r_\pi + (1 + \beta)(R_E \| R_L) = 2.65 + (51)(32 \| 12)$$

$$R_{ib} = 448 \Omega$$

$$A_v = \frac{(1 + \beta)(R_E \| R_L)}{r_\pi + (1 + \beta)(R_E \| R_L)} = \frac{(51)(32 \| 12)}{2.65 + (51)(32 \| 12)} = 0.994$$

So gain requirement has been met.

$$I_{BQ} = \frac{0.49}{50} = 0.0098 A = 9.8 mA$$

$$\text{Let } I_R \cong \frac{24}{R_1 + R_2} \cong 10I_B = 98 mA$$

$$\text{So that } R_1 + R_2 = 245 \Omega$$

$$V_{TH} = \frac{R_2}{R_1 + R_2}(24) - 12 = I_{BQ}R_{TH} + V_{BE}(\text{on}) + I_{EQ}R_E - 12$$

$$\left(\frac{R_2}{245} \right)(24) = \frac{(0.0098)R_1R_2}{245} + 0.7 + (0.5)(32)$$

$$\text{Now } R_1 = 245 - R_2$$

So we obtain

$$4 \times 10^{-5} R_2^2 + 0.0882R_2 - 16.7 = 0 \text{ which yields } R_2 = 175 \Omega \text{ and } R_1 = 70 \Omega$$

6.59

(a) $g_m = \frac{1}{0.026} = 38.46 \text{ mA/V}$

$$A_v = g_m R_C = (38.46)(2) = 76.9$$

(b) $A_i = \frac{\beta}{1 + \beta} = \frac{120}{121} = 0.9917$

(c) $R_i = \frac{r_\pi}{1 + \beta}, \quad r_\pi = \frac{(120)(0.026)}{1} = 3.12 \text{ k}\Omega$

$$R_i = \frac{3.12}{121} \Rightarrow R_i = 25.8 \Omega$$

(d) $R_o = R_C = 2 \text{ k}\Omega$

6.60

(a) $I_{CQ} = \left(\frac{80}{81} \right)(2) = 1.975 \text{ mA}$

$$g_m = \frac{1.975}{0.026} = 75.97 \text{ mA/V}, \quad r_\pi = \frac{(80)(0.026)}{1.975} = 1.053 \text{ k}\Omega$$

$$A_v = g_m (R_C \| R_L) = (75.97)(1.5 \| 2.5) = 71.2$$

$$(b) \quad A_i = \left(\frac{\beta}{1+\beta} \right) \left(\frac{R_C}{R_C + R_L} \right) = \left(\frac{80}{81} \right) \left(\frac{1.5}{1.5+2.5} \right) = 0.370$$

$$(c) \quad R_i = \frac{r_\pi}{1+\beta} = \frac{1.053}{81} \Rightarrow R_i = 13 \Omega$$

$$(d) \quad r_o = \frac{80}{1.975} = 40.5 \text{ k}\Omega$$

$$(i) \quad R_{oc} = r_o = 40.5 \text{ k}\Omega$$

$$(ii) \quad R_o = r_o \| R_C \| R_L = 40.5 \| 1.5 \| 2.5 = 0.916 \text{ k}\Omega$$

6.61

$$(a) \quad I_{CQ} = \left(\frac{110}{111} \right) (0.5) = 0.4955 \text{ mA}$$

$$g_m = \frac{0.4955}{0.026} = 19.06 \text{ mA/V}, \quad r_\pi = \frac{(110)(0.026)}{0.4955} = 5.77 \text{ k}\Omega$$

$$A_v = g_m \left(\frac{R_C}{R_S} \right) \left[\frac{r_\pi}{1+\beta} \| R_S \right] = (19.06) \left(\frac{4}{1} \right) \left[\frac{5.77}{111} \| 1 \right] = (76.24) [0.05198 \| 1]$$

$$A_v = 3.77$$

$$(b) \quad A_i = \frac{\beta}{1+\beta} = \frac{110}{111} = 0.991$$

$$(c) \quad R_i = R_S + \frac{r_\pi}{1+\beta} = 1 + \frac{5.77}{111} = 1.052 \text{ k}\Omega$$

$$(d) \quad R_o = R_C = 4 \text{ k}\Omega$$

6.62

$$(a) \quad I_{R2} = \frac{0.7}{2.8} = 0.25 \text{ mA}$$

$$I_{CQ} \cong 1.50 - 0.25 = 1.25 \text{ mA}$$

$$V_C = (0.25)(5 + 2.8) = 1.95 \text{ V} = V_{CEO}$$

$$(b) \quad g_m = \frac{1.25}{0.026} = 48.08 \text{ mA/V}, \quad r_\pi = \frac{(120)(0.026)}{1.25} = 2.5 \text{ k}\Omega$$

$$A_v = g_m (R_1 \| R_L) = (48.08)(5 \| 10) = 160.3$$

$$(c) \quad A_i = \left(\frac{\beta}{1+\beta} \right) \left(\frac{R_1}{R_1 + R_L} \right) = \left(\frac{120}{121} \right) \left(\frac{5}{5+10} \right) = 0.3306$$

6.63

(a)

$$I_{EQ} = \frac{10 - 0.7}{10} = 0.93 \text{ mA}$$

$$\underline{I_{CQ} = 0.921 \text{ mA}}$$

$$V_{ECQ} = 20 - (0.93)(10) - (0.921)(5)$$

$$\underline{V_{ECQ} = 6.10 \text{ V}}$$

(b)

$$g_m = \frac{0.921}{0.026} = 35.42 \text{ mA/V}$$

$$A_v = g_m (R_C \| R_L) = (35.42)(5\|50)$$

$$\underline{A_v = 161}$$

6.64

(a) $I_{EQ} = 0.93 \text{ mA}, I_{CQ} = 0.921 \text{ mA}$

$$V_{ECQ} = 6.10 \text{ V}$$

(b) $g_m = \frac{0.921}{0.026} = 35.42 \text{ mA/V} \quad r_\pi = 2.82 \text{ K}$

From Eq. 6.90

$$A_v = g_m \frac{(R_C \| R_L)}{R_s} \left[\frac{r_\pi}{1 + \beta} \| R_E \| R_s \right]$$

$$= \frac{(35.42)(50\|5)}{0.1} \left[\frac{2.82}{101} \| 10 \| 0.1 \right]$$

$$A_v = \frac{(35.42)(4.545)}{0.1} [0.0218]$$

$$\underline{A_v = 35.1}$$

6.65

(a) $I_{EQ} = 1 \text{ mA}, I_{CQ} = 0.9917 \text{ mA}$

$$V_C = 5 - (0.9917)(2) = 3.017 \text{ V}$$

$$V_E = -0.7 \text{ V}$$

$$\underline{V_{CEO} = 3.72 \text{ V}}$$

(b)

$$A_v = g_m (R_C \| R_L)$$

$$g_m = \frac{0.9917}{0.026} = 38.14 \text{ mA/V}$$

$$A_v = (38.14)(2\|10) \Rightarrow \underline{A_v = 63.6}$$

6.66

a. Emitter current

$$I_{EQ} = I_{CC} = 0.5 \text{ mA}$$

$$I_{BQ} = \frac{0.5}{101} = 0.00495 \text{ mA}$$

$$V_E = I_{EQ}R_E = (0.5)(1) \Rightarrow V_E = 0.5 \text{ V}$$

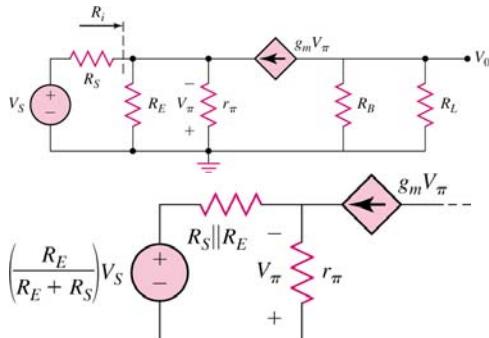
$$V_B = V_E + V_{BE}(\text{on}) = 0.5 + 0.7 \Rightarrow V_B = 1.20 \text{ V}$$

$$V_C = V_B + I_{BQ}R_B = 1.20 + (0.00495)(100) \Rightarrow V_C = 1.7 \text{ V}$$

b.

$$r_\pi = \frac{(100)(0.026)}{(100)(0.00495)} = 5.25 \text{ k}\Omega$$

$$g_m = \frac{(100)(0.00495)}{0.026} = 19.0 \text{ mA/V}$$



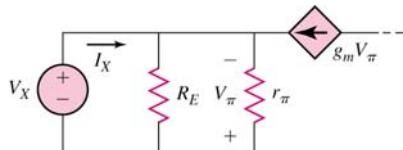
$$V_o = -g_m V_\pi (R_B \| R_L)$$

$$V_\pi = \frac{-(R_E \| R_{ie})}{R_E \| R_{ie} + R_S} \cdot V_s = -(0.4971)V_s$$

$$V_o = (19)(0.4971)V_s (100\|1)$$

$$A_v = 9.37$$

c.



$$I_X = \frac{V_X}{R_E} + \frac{V_X}{r_\pi} - g_m V_\pi, \quad V_\pi = -V_X$$

$$\frac{I_X}{V_X} = \frac{1}{R_i} = \frac{1}{R_E} + \frac{1}{r_\pi} + g_m$$

$$\text{or } R_i = R_E \| r_\pi \left\| \frac{1}{g_m} = 1 \| 5.253 \right\| \frac{1}{19} = 0.84 \| 0.05252$$

$$R_i = 49.4 \Omega$$

6.67

(a) $R_{TH} = R_1 \parallel R_2 = 150 \parallel 50 = 37.5 \text{ k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left(\frac{50}{50+150} \right)(9) = 2.25 \text{ V}$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE} (\text{on}) + (1+\beta) I_{BQ} R_E$$

$$\text{or } I_{BQ} = \frac{2.25 - 0.7}{37.5 + (126)(3)} = 0.00373 \text{ mA}, \quad I_{CQ} = 0.4663 \text{ mA}, \quad I_{EQ} = 0.470 \text{ mA}$$

$$V_{CEQ} = 9 - (0.4663)(6) - (0.470)(3) = 4.79 \text{ V}$$

(b) $g_m = \frac{0.4663}{0.026} = 17.93 \text{ mA/V}, \quad r_\pi = \frac{(125)(0.026)}{0.4663} = 6.97 \text{ k}\Omega$

$$R_m = \frac{v_o}{i_s} = \left(\frac{R_S \parallel R_E}{R_S \parallel R_E + R_{ie}} \right) \left(\frac{\beta}{1+\beta} \right) (R_C \parallel R_L)$$

We find $R_S \parallel R_E = 100 \parallel 3 = 2.913 \text{ k}\Omega$

$$R_{ie} = \frac{r_\pi + R_{TH}}{1+\beta} = \frac{6.97 + 37.5}{126} = 0.3529 \text{ k}\Omega$$

$$R_m = \left(\frac{2.913}{2.913 + 0.3529} \right) \left(\frac{125}{126} \right) (6 \parallel 4) = 2.12 \text{ V/mA}$$

(c) $v_s = i_s (R_S \parallel R_E \parallel R_{ie}) = i_s (100 \parallel 3 \parallel 0.3529) = i_s (0.3148)$

So $i_s = \frac{v_s}{0.3148}$

Then $A_v = \frac{v_o}{v_s} = \frac{v_o}{i_s} \cdot \frac{i_s}{v_s} = \frac{2.12}{0.3148} = 6.73$

6.68

(a) $V_{CEQ} \approx V_{CC} - I_{CQ} (R_C + R_E)$

Let $V_{CEQ} = 2.5 \text{ V}$ and $I_{CQ} = 0.25 \text{ mA}$

Then $2.5 = 5 - (0.25)(R_C + 0.5) \Rightarrow R_C = 9.5 \text{ k}\Omega$

$$v_o = i_c (R_C \parallel R_L) = \left(\frac{\beta}{1+\beta} \right) \cdot i_e \cdot (R_C \parallel R_L)$$

$$i_e = \frac{v_s}{R_{ie}} = \frac{v_s}{\left(\frac{r_\pi + R_{TH}}{1+\beta} \right)}$$

$$A_v = \frac{v_o}{v_s} = \left(\frac{\beta}{r_\pi + R_{TH}} \right) (R_C \parallel R_L)$$

Now $r_\pi = \frac{(100)(0.026)}{0.25} = 10.4 \text{ k}\Omega$

$$A_v = 25 = \left(\frac{100}{10.4 + R_{TH}} \right) (9.5 \parallel 12) = \frac{530.2}{10.4 + R_{TH}} \Rightarrow R_{TH} = 10.81 \text{ k}\Omega$$

Also $I_{BQ} = 0.0025 \text{ mA}$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE} (\text{on}) + (1+\beta) I_{BQ} R_E = (0.0025)(10.81) + 0.7 + (101)(0.0025)(0.5)$$

$$V_{TH} = 0.853275 = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} (10.81)(5)$$

We obtain $R_1 = 63.3 \text{ k}\Omega$ and $R_2 = 13 \text{ k}\Omega$

(b) From part (a), $I_{CQ} = 0.25 \text{ mA}$, $V_{CEQ} = 2.5 \text{ V}$

(c) $A_v = g_m (R_C \| R_L)$

$$g_m = \frac{0.25}{0.026} = 9.615 \text{ mA/V}$$

$$A_v = (9.615)(9.5 \| 12) = 51$$

6.69

(a)

$$I_{CQ} = \left(\frac{60}{61} \right) (1) \Rightarrow I_{CQ} = 0.984 \text{ mA}$$

$$V_{CEQ} = I_{BQ} R_B + V_{BE} (\text{on}) = \left(\frac{1}{61} \right) (100) + 0.7$$

$$\underline{V_{CEQ} = 2.34 \text{ V}}$$

(b)

$$A_v = g_m \frac{(R_B \| R_L)}{R_S} \left[\frac{r_\pi}{1 + \beta} \| R_S \right]$$

$$g_m = \frac{0.984}{0.026} = 37.85 \text{ mA/V}$$

$$r_\pi = 1.59 \text{ K}$$

$$A_v = \frac{(37.85)(100 \| 2)}{0.05} \left[\frac{1.59}{61} \| 0.05 \right] \\ = 1484 [0.0261 \| 0.05]$$

$$A_v = 25.4$$

6.70

$$i_s(\text{peak}) = 2.5 \mu\text{A}, V_o(\text{peak}) = 5 \text{mV}$$

$$\text{So we need } R_m = \frac{V_o}{i_s} = \frac{5 \times 10^{-3}}{2.5 \times 10^{-6}} = 2 \times 10^3 \Omega \Rightarrow R_m = 2 \text{k}\Omega$$

We have

$$\frac{V_o}{I_s} = \left(\frac{\beta}{1 + \beta} \right) (R_C \parallel R_L) \left(\frac{R_S \| R_E}{R_S \| R_E + R_{ie}} \right)$$

Let $R_C = 4 \text{k}\Omega$, $R_L = 5 \text{k}\Omega$, $R_E = 2 \text{k}\Omega$

Now $\beta = 120$, so we have

$$2 = \left(\frac{120}{121} \right) \left(4 \| 5 \right) \left(\frac{R_S \| R_E}{R_S \| R_E + R_{ie}} \right) = 2.204 \left(\frac{R_S \| R_E}{R_S \| R_E + R_{ie}} \right)$$

$$\text{Then } \frac{R_S \| R_E}{R_S \| R_E + R_{ie}} = 0.9075$$

$R_S \| R_E = 50 \| 2 = 1.923 \text{k}\Omega$, so that $R_{ie} = 0.196 \text{k}\Omega$

Assume $V_{CEQ} = 3 \text{V}$

$$V_{CC} \approx I_{CQ} (R_C + R_E) + V_{CEQ}$$

$$5 = I_{CQ} (4 + 2) + 3 \Rightarrow I_{CQ} = 0.333 \text{mA}$$

$$r_\pi = \frac{(120)(0.026)}{0.333} = 9.37 \text{k}\Omega$$

$$R_{ie} = \frac{r_\pi + R_{TH}}{1 + \beta} \Rightarrow 0.196 = \frac{9.37 + R_{TH}}{121}$$

which yields $R_{TH} = 14.35 \text{k}\Omega$

$$\text{Now } V_{TH} = I_{BQ} R_{TH} + V_{BE} (\text{on}) + I_{EQ} R_E$$

$$I_{BQ} = \frac{1}{120} = 0.00833 \text{mA}, I_{EQ} = \left(\frac{121}{120} \right) (1) = 1.008 \text{mA}$$

$$V_{TH} = \frac{1}{R_i} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_i} (14.35) (5) = (0.00833) (14.35) + 0.7 + (1.008) (2)$$

which yields $R_i = 25.3 \text{k}\Omega$

and $R_2 = 33.2 \text{k}\Omega$

6.71

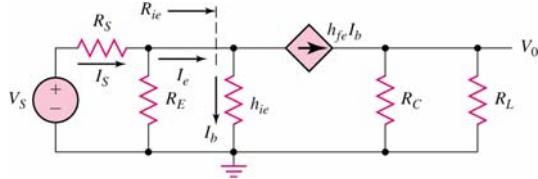
a.

$$I_{EQ} = \frac{20 - 0.7}{10} = 1.93 \text{mA}$$

$$\underline{I_{CQ} = 1.91 \text{mA}}$$

$$\begin{aligned} V_{ECQ} &= V_{CC} + V_{EB} (\text{on}) - I_C R_C \\ &= 25 + 0.7 - (1.91)(6.5) \Rightarrow \underline{V_{ECQ} = 13.3 \text{V}} \end{aligned}$$

b.



Neglect effect h_{oe}

Assume

$$2.45 \leq h_{ie} \leq 3.7 \text{ k}\Omega$$

$$80 \leq h_{fe} \leq 120$$

$$V_o = (h_{fe} I_b) (R_C \| R_L)$$

$$R_{ie} = \frac{h_{ie}}{1 + h_{fe}}, \quad I_e = \left(\frac{R_E}{R_E + R_{ie}} \right) \cdot I_s$$

$$I_b = \left(\frac{I_e}{1 + h_{fe}} \right), \quad I_s = \frac{V_s}{R_s + (R_E \| R_{ie})}$$

$$A_v = \left(\frac{h_{fe}}{1 + h_{fe}} \right) (R_C \| R_L) \left(\frac{R_E}{R_E + R_{ie}} \right) \left(\frac{1}{R_s + R_E \| R_{ie}} \right)$$

High gain device: $h_{ie} = 3.7 \text{ k}\Omega, h_{fe} = 120$

$$R_{ie} = \frac{3.7}{121} = 0.0306 \text{ k}\Omega$$

$$R_E \| R_{ie} = 10 \| 0.0306 = 0.0305 \text{ k}\Omega$$

$$A_v = \left(\frac{120}{121} \right) \left(6.5 \| 5 \right) \left(\frac{10}{10 + 0.0306} \right) \left(\frac{1}{1 + 0.0305} \right) \Rightarrow A_v = 2.711$$

Low gain device: $h_{ie} = 2.45 \text{ k}\Omega, h_{fe} = 80$

$$R_{ie} = \frac{2.45}{81} = 0.03025 \text{ k}\Omega$$

$$R_E \| R_{ie} = 10 \| 0.03025 = 0.0302 \text{ k}\Omega$$

$$A_v = \left(\frac{80}{81} \right) \left(6.5 \| 5 \right) \left(\frac{10}{10 + 0.03025} \right) \left(\frac{1}{1 + 0.0302} \right) \Rightarrow A_v = 2.70$$

$$\underline{2.70 \leq A_v \leq 2.71} \quad \text{So } A_v \cong \text{constant}$$

c.

$$R_i = R_E \| R_{ie}$$

We found $\underline{0.0302 \leq R_i \leq 0.0305 \text{ k}\Omega}$

Neglecting $h_{oe}, \underline{R_o = R_C = 6.5 \text{ k}\Omega}$

6.72

- a. Small-signal voltage gain

$$A_v = g_m (R_C \parallel R_L) \Rightarrow 25 = g_m (R_C \parallel 1)$$

For $V_{ECQ} = 3 \text{ V}$, $V_C = -V_{ECQ} + V_{EB} (\text{on}) = -3 + 0.7 = -2.3 \text{ V}$

$$V_{CC} - I_{CQ} R_C + V_C = 0 \Rightarrow I_{CQ} = \frac{5 - 2.3}{R_C} = \frac{2.7}{R_C}$$

For $I_{CQ} = 1 \text{ mA}$, $R_C = 2.7 \text{ k}\Omega$

$$g_m = \frac{1}{0.026} = 38.5 \text{ mA/V}$$

$$A_v = (38.5)(2.7 \parallel 1) = 28.1$$

Design criterion satisfied and V_{ECQ} satisfied.

$$I_E = \left(\frac{101}{100} \right) (1) = 1.01 \text{ mA}$$

$$V_{EE} = I_E R_E + V_{EB} (\text{on}) \Rightarrow R_E = \frac{5 - 0.7}{1.01} \Rightarrow R_E = 4.26 \text{ k}\Omega$$

b.

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1} \Rightarrow r_\pi = 2.6 \text{ k}\Omega, g_m = 38.5 \text{ mA/V}, r_o = \infty$$

6.73

$$(a) g_{m1} = g_{m2} = \frac{1}{0.026} = 38.46 \text{ mA/V}$$

$$r_{\pi 1} = \frac{(120)(0.026)}{1} = 3.12 \text{ k}\Omega, r_{\pi 2} = \frac{(80)(0.026)}{1} = 2.08 \text{ k}\Omega$$

$$A_{v1} = \frac{V_{o1}}{V_i} = \frac{\left(1 + \beta_1\right) \left(R_E \parallel \left| \frac{r_{\pi 2}}{1 + \beta_2} \right| \right)}{r_{\pi 1} + \left(1 + \beta_1\right) \left(R_E \parallel \left| \frac{r_{\pi 2}}{1 + \beta_2} \right| \right)}$$

$$\text{We find } R_E \parallel \left| \frac{r_{\pi 2}}{1 + \beta_2} \right| = 1 \parallel \frac{2.08}{81} = 0.02504 \text{ k}\Omega$$

$$\text{Then } A_{v1} = \frac{(121)(0.02504)}{3.12 + (121)(0.02504)} = 0.4927$$

$$(b) A_{v2} = \frac{V_{o2}}{V_{o1}} = g_{m2} R_C = (38.46)(4) = 153.8$$

$$(c) A_v = A_{v1} \cdot A_{v2} = (0.4927)(153.8) = 75.8$$

6.74

$$(a) g_{m1} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}, g_{m2} = \frac{2}{0.026} = 76.92 \text{ mA/V}$$

$$r_{\pi 1} = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega, r_{\pi 2} = \frac{(100)(0.026)}{2} = 1.3 \text{ k}\Omega$$

$$A_{v1} = \frac{V_{o1}}{V_i} = -g_{m1}(R_{C1} \| R_{ib2})$$

where $R_{ib2} = r_{\pi2} + (1 + \beta_2)R_{E2} = 1.3 + (101)(4) = 405.3 \text{ k}\Omega$

$$A_{v1} = -(19.23)(4 \| 405.3) = -76.17$$

$$(b) \quad A_{v2} = \frac{V_{o2}}{V_{o1}} = \frac{(1 + \beta_2)(R_{E2})}{r_{\pi2} + (1 + \beta_2)R_{E2}} = \frac{(101)(4)}{1.3 + (101)(4)} = 0.9968$$

$$(c) \quad A_v = A_{v1} \cdot A_{v2} = (-76.17)(0.9968) = -75.93$$

6.75

a.

$$V_{TH1} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left(\frac{20}{20 + 80} \right) (10) = 2.0 \text{ V}$$

$$R_{TH1} = R_1 \| R_2 = 20 \| 80 = 16 \text{ k}\Omega$$

$$I_{B1} = \frac{2 - 0.7}{16 + (101)(1)} = 0.0111 \text{ mA}$$

$$I_{C1} = 1.11 \text{ mA}; \quad g_{m1} = \frac{1.11}{0.026} = 42.74 \text{ mA/V}$$

$$r_{\pi1} = \frac{(100)(0.026)}{1.11} = 2.34 \text{ k}\Omega$$

$$r_{o1} = \frac{\infty}{1.11} = \infty$$

$$V_{TH2} = \left(\frac{R_4}{R_3 + R_4} \right) \cdot V_{CC} = \left(\frac{15}{15 + 85} \right) (10) = 1.50 \text{ V}$$

$$R_{TH2} = R_3 \| R_4 = 15 \| 85 = 12.75 \text{ k}\Omega$$

$$I_{B2} = \frac{1.50 - 0.70}{12.75 + (101)(0.5)} = 0.01265 \text{ mA}$$

$$I_{C2} = 1.265 \text{ mA} \Rightarrow g_{m2} = \frac{1.265}{0.026} \Rightarrow \underline{g_{m2} = 48.65 \text{ mA/V}}$$

$$r_{\pi2} = \frac{(100)(0.026)}{1.26} \Rightarrow \underline{r_{\pi2} = 2.06 \text{ k}\Omega}$$

$$\underline{r_{o2} = \infty}$$

b.

$$A_{v1} = -g_{m1} R_{C1} = -(42.7)(2) \Rightarrow \underline{A_{v1} = -85.48}$$

$$A_{v2} = -g_{m2} (R_{C2} \| R_L) = -(48.5)(4 \| 4) \Rightarrow \underline{A_{v2} = -97.3}$$

c. Input resistance of 2nd stage

$$R_{i2} = R_3 \| R_4 \| r_{\pi2} = 15 \| 85 \| 2.06 = 12.75 \| 2.06$$

$$R_{i2} = 1.773 \text{ k}\Omega$$

$$A'_{v1} = -g_{m1} (R_{C1} \| R_{i2}) = -(42.7)(2 \| 1.773)$$

$$A'_{v1} = -40.17$$

Overall gain: $A_v = (-40.17)(-97.3) \Rightarrow A_v = 3909$

If we had $A_{v1} \cdot A_{v2} = (-85.48)(-97.3) = 8317$

Loading effect reduces overall gain

6.76

a.

$$V_{TH1} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left(\frac{12.7}{12.7 + 67.3} \right) (12) = 1.905 \text{ V}$$

$$R_{TH1} = R_1 \| R_2 = 12.7 \| 67.3 = 10.68 \text{ k}\Omega$$

$$I_{B1} = \frac{1.905 - 0.7}{10.68 + (121)(2)} = 0.00477 \text{ mA}$$

$$I_{C1} = 0.572 \text{ mA}$$

$$g_{m1} = \frac{0.572}{0.026} = 22 \text{ mA/V}$$

$$r_{\pi 1} = \frac{(120)(0.026)}{0.572} = 5.45 \text{ k}\Omega$$

$$r_{o1} = \frac{\infty}{0.572} = \infty$$

$$V_{TH2} = \left(\frac{R_4}{R_3 + R_4} \right) \cdot V_{CC} = \left(\frac{45}{45 + 15} \right) (12) = 9.0 \text{ V}$$

$$R_{TH2} = R_3 \| R_4 = 15 \| 45 = 11.25 \text{ k}\Omega$$

$$I_{B2} = \frac{9.0 - 0.7}{11.25 + (121)(1.6)} = 0.0405 \text{ mA}$$

$$I_{C2} = 4.86 \text{ mA}$$

$$g_{m2} = \frac{4.86}{0.026} = 187 \text{ mA/V}$$

$$r_{\pi 2} = \frac{(120)(0.026)}{4.86} = 0.642 \text{ k}\Omega, \quad r_{o2} = \infty$$

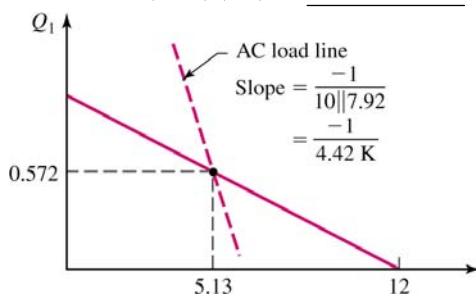
b.

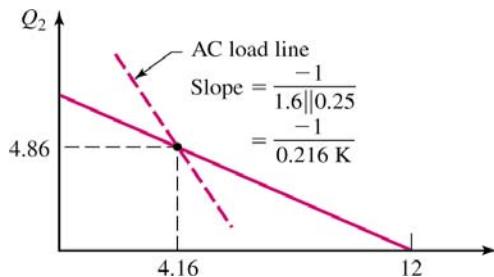
$$I_{E1} = 0.577 \text{ mA}$$

$$V_{CEQ1} = 12 - (0.572)(10) - (0.577)(2) \Rightarrow V_{CEQ1} = 5.13 \text{ V}$$

$$I_{E2} = 4.90$$

$$V_{CEQ2} = 12 - (4.90)(1.6) \Rightarrow V_{CEQ2} = 4.16 \text{ V}$$





$$R_{i2} = R_3 \parallel R_4 \parallel R_{ib}$$

$$R_{ib} = r_{\pi 2} + (1 + \beta)(R_{E2} \parallel R_L)$$

$$= 0.642 + (121)(1.6 \parallel 0.25) = 26.8 \text{ k}\Omega$$

$$R_{i2} = 15 \parallel 45 \parallel 26.8 = 7.92 \text{ k}\Omega$$

c.

$$A_{v1} = -g_m R_{C1} \parallel R_{i2} = -(22)(10 \parallel 7.92) \Rightarrow A_{v1} = -97.2$$

$$A_{v2} = \frac{(1 + \beta)(R_{E2} \parallel R_L)}{r_{\pi 2} + (1 + \beta)(R_{E2} \parallel R_L)} = \frac{(121)(0.216)}{0.642 + (121)(0.216)}$$

$$A_{v2} = 0.976$$

Overall gain, $A_v = -(97.2)(0.976) = -94.9$

d.

$$R_{iS} = R_1 \parallel R_2 \parallel r_{\pi 1} = 67.3 \parallel 12.7 \parallel 5.45 = 3.61 \text{ k}\Omega$$

$$R_o = \frac{r_{\pi 2} + R_S}{1 + \beta} \parallel R_{E2}$$

where $R_S = R_3 \parallel R_{C1} = 15 \parallel 45 \parallel 10 = 5.29 \text{ k}\Omega$

$$R_o = \frac{0.642 + 5.29}{121} \parallel 1.6 = 0.049 \parallel 1.6 \Rightarrow R_o = 47.6 \Omega$$

e.

$$\Delta i_C = \frac{-1}{0.216 \text{ k}\Omega} \cdot \Delta v_{ce}, \Delta i_C = 4.86$$

$$|\Delta v_{ce}| = (4.86)(0.216) = 1.05 \text{ V}$$

Max. output voltage swing = 2.10 V peak-to-peak

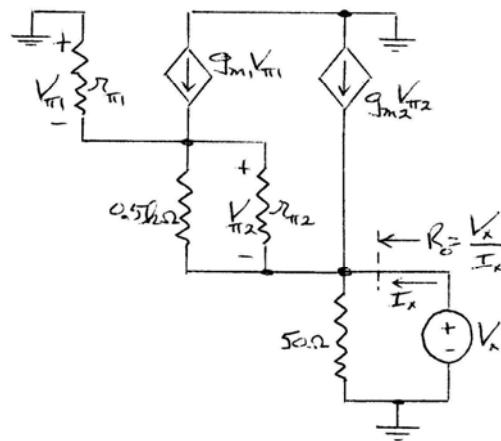
6.77

$$(a) I_{R2} = \frac{5 - 2(0.7)}{0.050} = 72 \text{ mA}, \quad I_{R1} = \frac{0.7}{0.5} = 1.4 \text{ mA}$$

$$I_{E2} = 72 - 1.4 = 70.6 \text{ mA}, \quad I_{C2} = \left(\frac{80}{81}\right)(70.6) = 69.73 \text{ mA}, \quad I_{B2} = \frac{70.6}{81} = 0.8716 \text{ mA}$$

$$I_{E1} = 1.4 + 0.8716 = 2.2716 \text{ mA}, \quad I_{C1} = \left(\frac{120}{121}\right)(2.2716) = 2.253 \text{ mA}$$

(b)



$$g_{m1} = \frac{2.253}{0.026} = 86.65 \text{ mA/V}, \quad g_{m2} = \frac{69.73}{0.026} = 2681.9 \text{ mA/V}$$

$$r_{\pi 1} = \frac{(120)(0.026)}{2.253} = 1.385 \text{ k}\Omega, \quad r_{\pi 2} = \frac{(80)(0.026)}{69.73} = 0.02983 \text{ k}\Omega$$

$$V_s = V_{\pi 1} + V_{\pi 2} + V_o$$

$$V_o = \left[\left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} \right) + g_{m2}V_{\pi 2} \right] (0.05)$$

$$V_{\pi 2} = \left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} \right) (0.5 \| r_{\pi 2}) = V_{\pi 1} \left(\frac{1}{1.385} + 86.65 \right) (0.5 \| 0.02983) = V_{\pi 1} (2.4595)$$

$$V_o = \left[V_{\pi 1} \left(\frac{1}{1.385} + 86.65 \right) + (2681.9)V_{\pi 1} (2.4595) \right] (0.05) = V_{\pi 1} (334.175)$$

$$V_s = V_{\pi 1} + V_{\pi 2} + V_o = V_{\pi 1} + V_{\pi 1} (2.4595) + V_o$$

$$\text{So } V_{\pi 1} = (V_s - V_o)(0.28906)$$

$$\text{And } V_o = (334.175)(V_s - V_o)(0.28906) = 96.596(V_s - V_o)$$

$$A_v = \frac{V_o}{V_s} = 0.990$$

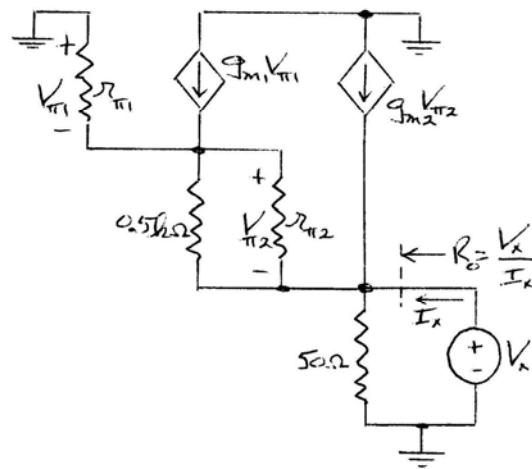
(c) For R_{ib} :

We have $V_o = (0.989754)V_s$

$$V_{\pi 1} = (V_s - V_o)(0.28906) = V_s (1 - 0.98975)(0.28906) = V_s (0.0029618)$$

$$R_{ib} = \frac{V_s}{I_s} = \frac{V_s}{\left(\frac{V_{\pi 1}}{r_{\pi 1}} \right)} = \frac{r_{\pi 1}}{0.0029618} \Rightarrow R_{ib} = 467.6 \text{ k}\Omega$$

For R_o :



$$(1) \quad V_{\pi 1} + V_{\pi 2} + V_x = 0$$

$$(2) \quad I_x + g_{m2}V_{\pi 2} + \left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} \right) = \frac{V_x}{0.05}$$

We had $V_{\pi 2} = V_{\pi 1}(2.4595)$

$$(1) \quad V_{\pi 1} + V_{\pi 1}(2.4595) + V_x = 0 \Rightarrow V_{\pi 1} = -V_x(0.28906)$$

$$(2) \quad I_x + g_{m2}V_{\pi 1}(2.4595) + V_{\pi 1}\left(\frac{1}{r_{\pi 1}} + g_{m1} \right) = \frac{V_x}{0.05}$$

$$I_x + V_{\pi 1} \left[(2681.9)(2.4595) + \left(\frac{1}{1.385} + 86.65 \right) \right] = \frac{V_x}{0.05}$$

$$I_x - V_x(0.28906)[6683.5] = \frac{V_x}{0.05}$$

$$R_o = \frac{V_x}{I_x} = 0.512 \Omega$$

6.78

a.

$$R_{TH} = R_1 \parallel R_2 = 335 \parallel 125 = 91.0 \text{ k}\Omega$$

$$\begin{aligned} V_{TH} &= \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} \\ &= \left(\frac{125}{125 + 335} \right) (10) = 2.717 \text{ V} \end{aligned}$$

$$V_{TH} = I_{B1}R_{TH} + V_{BE1} + V_{BE2} + I_{E2}R_{E2}$$

$$I_{E2} = (1 + \beta)I_{E1} = (1 + \beta)^2 I_{B1}$$

$$I_{B1} = \frac{2.717 - 1.40}{91.0 + (101)^2 (1)} \Rightarrow I_{B1} = 0.128 \mu\text{A}$$

$$I_{C1} = 12.8 \mu\text{A}$$

$$I_{C2} = \beta I_{E1} = \beta (1 + \beta) I_{B1} = (100)(101)(0.128 \mu\text{A})$$

$$I_{C2} = 1.29 \text{ mA}, I_{E2} = 1.31 \text{ mA}$$

$$I_{RC} = I_{C2} + I_{C1} = 1.29 + 0.0128 = 1.30 \text{ mA}$$

$$V_C = 10 - I_{RC} R_C = 10 - (1.30)(2.2) = 7.14 \text{ V}$$

$$V_E = I_{E2} R_{E2} = (1.30)(1) = 1.30 \text{ V}$$

$$V_{CE2} = 7.14 - 1.30 = 5.84 \text{ V}$$

$$V_{CE1} = V_{CE2} - V_{BE2} = 5.84 - 0.7$$

$$V_{CE1} = 5.14 \text{ V}$$

Summary:

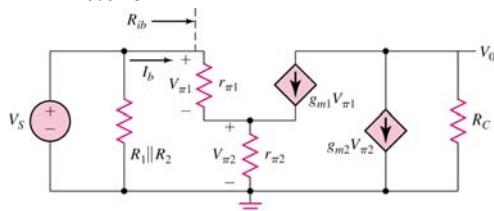
$$I_{C1} = 12.8 \mu\text{A} \quad I_{C2} = 1.29 \text{ mA}$$

$$V_{CE1} = 5.14 \text{ V} \quad V_{CE2} = 5.84 \text{ V}$$

b.

$$g_{m1} = \frac{0.0128}{0.026} = 0.492 \text{ mA/V}$$

$$g_{m2} = \frac{1.292}{0.026} = 49.7 \text{ mA/V}$$



$$V_0 = -(g_{m1}V_{\pi1} + g_{m2}V_{\pi2})R_C$$

$$V_S = V_{\pi1} + V_{\pi2}, \quad V_{\pi1} = V_S - V_{\pi2}$$

$$V_{\pi2} = \left(\frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} \right) r_{\pi2}$$

$$V_{\pi2} = V_{\pi1} \left(\frac{1 + \beta}{r_{\pi1}} \right) r_{\pi2}$$

$$V_0 = -[g_{m1}(V_S - V_{\pi2}) + g_{m2}V_{\pi2}]R_C$$

$$V_0 = -[g_{m1}V_S + (g_{m2} - g_{m1})V_{\pi2}]R_C$$

$$V_{\pi2} = (V_S - V_{\pi2})(1 + \beta) \left(\frac{r_{\pi2}}{r_{\pi1}} \right)$$

$$V_{\pi2} \left[1 + (1 + \beta) \left(\frac{r_{\pi2}}{r_{\pi1}} \right) \right] = V_S (1 + \beta) \left(\frac{r_{\pi2}}{r_{\pi1}} \right)$$

$$V_0 = - \left\{ g_{m1} V_s + (g_{m2} - g_{m1}) \cdot \frac{V_s (1 + \beta) \left(\frac{r_{\pi2}}{r_{\pi1}} \right)}{1 + (1 + \beta) \left(\frac{r_{\pi2}}{r_{\pi1}} \right)} \right\} R_C$$

$$A_v = \frac{V_0}{V_s}$$

$$= - \left\{ (0.492) + \frac{(49.7 - 0.492)(101) \left(\frac{2.01}{203} \right)}{1 + (101) \left(\frac{2.01}{203} \right)} \right\} 2.2$$

$$A_v = -55.2$$

c.

$$R_{is} = R_l \| R_2 \| R_{ib}$$

$$R_{ib} = r_{\pi1} + (1 + \beta) r_{\pi2}$$

$$= 203 + (101)(2.01) = 406 \text{ k}\Omega$$

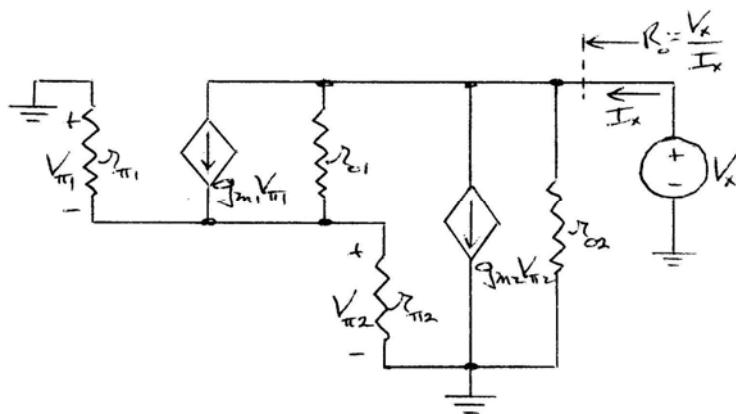
$$R_{is} = 91 \| 406 = \underline{74.3 \text{ k}\Omega = R_{is}}$$

$$R_0 = R_C = 2.2 \text{ k}\Omega$$

6.79

$$I_{E1} = I_{Bias} + I_{B2} = I_{Bias} + \frac{I_{C2}}{\beta_2}$$

$$I_{C1} = \left(\frac{\beta_1}{1 + \beta_1} \right) \left[I_{Bias} + \frac{I_{C2}}{\beta_2} \right]$$



$$(a) \quad I_{C2} = I_{Bias} = 1 \text{ mA}, \quad I_{C1} = \left(\frac{120}{121} \right) \left[1 + \frac{1}{80} \right] = 1.004 \text{ mA}$$

$$g_{m1} = 38.62 \text{ mA/V}, \quad r_{\pi1} = 3.108 \text{ k}\Omega, \quad r_{o1} = 79.68 \text{ k}\Omega$$

$$g_{m2} = 38.46 \text{ mA/V}, \quad r_{\pi2} = 2.08 \text{ k}\Omega, \quad r_{o2} = 50 \text{ k}\Omega$$

$$(1) \quad I_x = \frac{V_x}{r_{o2}} + g_{m2}V_{\pi2} + \frac{V_x - V_{\pi2}}{r_{o1}} + g_{m1}V_{\pi1}$$

$$(2) \quad V_{\pi2} = \left[\frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} + \frac{V_x - V_{\pi2}}{r_{o1}} \right] \cdot r_{\pi2}$$

$$(3) \quad V_{\pi1} + V_{\pi2} = 0 \Rightarrow V_{\pi1} = -V_{\pi2}$$

$$\text{Then } (1) \quad I_x = \frac{V_x}{r_{o2}} + g_{m2}V_{\pi2} + \frac{V_x - V_{\pi2}}{r_{o1}} - g_{m1}V_{\pi2}$$

$$I_x = V_x \left(\frac{1}{r_{o2}} + \frac{1}{r_{o1}} \right) + V_{\pi2} \left(g_{m2} - \frac{1}{r_{o1}} - g_{m1} \right)$$

$$(2) \quad V_{\pi2} = -V_{\pi2} \left(\frac{1}{r_{\pi1}} + g_{m1} + \frac{1}{r_{o1}} \right) \cdot r_{\pi2} + V_x \left(\frac{r_{\pi2}}{r_{o1}} \right)$$

$$V_{\pi2} \left[1 + \left(\frac{1}{r_{\pi1}} + g_{m1} + \frac{1}{r_{o1}} \right) \cdot r_{\pi2} \right] = V_x \left(\frac{r_{\pi2}}{r_{o1}} \right)$$

$$\text{Now } (2) \quad V_{\pi2} \left[1 + \left(\frac{1}{3.108} + 38.62 + \frac{1}{79.68} \right) (2.08) \right] = V_x \left(\frac{2.08}{79.68} \right)$$

$$V_{\pi2} = V_x (0.00031825)$$

$$\text{Then } (1) \quad I_x = V_x \left(\frac{1}{50} + \frac{1}{79.68} \right) + V_x (0.00031825) \left(38.46 - \frac{1}{79.68} - 38.62 \right) = V_x (0.32495)$$

$$R_o = \frac{V_x}{I_x} = 30.77 \text{ k}\Omega$$

$$(b) \quad I_{C2} = 1 \text{ mA}, \quad I_{Bias} = 0.2 \text{ mA}, \quad I_{C1} = \left(\frac{120}{121} \right) \left(0.2 + \frac{1}{80} \right) = 0.2107 \text{ mA}$$

$$g_{m1} = 8.104 \text{ mA/V}, \quad r_{\pi1} = 14.81 \text{ k}\Omega, \quad r_{o1} = 379.7 \text{ k}\Omega$$

$$g_{m2} = 38.46 \text{ mA/V}, \quad r_{\pi2} = 2.08 \text{ k}\Omega, \quad r_{o2} = 50 \text{ k}\Omega$$

$$\text{Now } (2) \quad V_{\pi2} \left[1 + \left(\frac{1}{14.81} + 8.104 + \frac{1}{379.7} \right) (2.08) \right] = V_x \left(\frac{2.08}{379.7} \right)$$

$$V_{\pi2} = V_x (0.0003043)$$

$$(1) \quad I_x = V_x \left(\frac{1}{50} + \frac{1}{379.7} \right) + V_x (0.0003043) \left(38.46 - \frac{1}{379.7} - 8.104 \right) = V_x (0.031867)$$

$$R_o = \frac{V_x}{I_x} = 31.38 \text{ k}\Omega$$

$$(c) \quad I_{C2} = 2 \text{ mA}, \quad I_{Bias} = 0, \quad I_{C1} = \left(\frac{120}{121} \right) \left(\frac{2}{80} \right) = 0.02479 \text{ mA}$$

$$g_{m1} = 0.9536 \text{ mA/V}, \quad r_{\pi1} = 125.9 \text{ k}\Omega, \quad r_{o1} = 3327 \text{ k}\Omega$$

$$g_{m2} = 76.92 \text{ mA/V}, \quad r_{\pi2} = 1.04 \text{ k}\Omega, \quad r_{o2} = 25 \text{ k}\Omega$$

$$\text{Now } (2) \quad V_{\pi2} \left[1 + \left(\frac{1}{125.9} + 0.9536 + \frac{1}{3327} \right) (1.04) \right] = V_x \left(\frac{1.04}{3327} \right)$$

$$V_{\pi2} = V_x (0.00015627)$$

$$(1) \quad I_x = V_x \left(\frac{1}{25} + \frac{1}{3327} \right) + V_x (0.00015627) \left(76.92 - \frac{1}{3327} - 0.9536 \right) = V_x (0.05217)$$

$$R_o = \frac{V_x}{I_x} = 19.17 \text{ k}\Omega$$

6.80

$$(a) R_{TH} = R_1 \| R_2 = 250 \| 75 = 57.69 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left(\frac{75}{75 + 250} \right) (5) = 1.1538 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1+\beta)R_E} = \frac{1.1538 - 0.7}{57.69 + (121)(0.6)} = 0.003483 \text{ mA}$$

$$\Rightarrow I_{CQ} = 0.418 \text{ mA}, \quad I_{EQ} = 0.4214 \text{ mA}$$

$$V_{CEQ} = 5 - (0.418)(5.6) - (0.4214)(0.6) = 2.406 \text{ V}$$

$$\text{For } Q: P_Q = I_{CQ}V_{CEQ} = (0.418)(2.406) = 1.01 \text{ mW}$$

$$\text{For } R_C: P_{RC} = I_{CQ}^2 R_C = (0.418)^2 (5.6) = 0.978 \text{ mW}$$

$$\text{For } R_E: P_{RE} = I_{EQ}^2 R_E = (0.4214)^2 (0.6) = 0.107 \text{ mW}$$

$$(b) R_{ib} = r_\pi + (1+\beta)R_E$$

$$r_\pi = \frac{(120)(0.026)}{0.418} = 7.464 \text{ k}\Omega$$

$$R_{ib} = 7.464 + (121)(0.6) = 80.06 \text{ k}\Omega$$

$$R_i = R_{TH} \| R_{ib} = 57.69 \| 80.06 = 33.53 \text{ k}\Omega$$

$$R_{iS} = R_S + R_i = 0.5 + 33.53 = 34.03 \text{ k}\Omega$$

$$i_s = \frac{V_s}{R_{iS}}; \quad i_b = \left(\frac{R_{TH}}{R_{TH} + R_{ib}} \right) \cdot i_s = \left(\frac{R_{TH}}{R_{TH} + R_{ib}} \right) \left(\frac{V_s}{R_{iS}} \right)$$

$$i_c = \beta \left(\frac{R_{TH}}{R_{TH} + R_{ib}} \right) \left(\frac{V_s}{R_{iS}} \right); \quad i_e = (1+\beta) \left(\frac{R_{TH}}{R_{TH} + R_{ib}} \right) \left(\frac{V_s}{R_{iS}} \right)$$

$$\text{Now } i_c = (120) \left(\frac{57.69}{57.69 + 80.06} \right) \left(\frac{0.1 \sin \omega t}{34.03} \right) = 0.1477 \sin \omega t \text{ (mA)}$$

$$i_e = 0.1489 \sin \omega t \text{ (mA)}$$

$$\text{For } R_C: \bar{P}_{RC} = I_{CQ}^2 R_C + \frac{1}{2} I_c^2 R_C = 0.978 + \frac{1}{2} (0.1477)^2 (5.6) = 1.039 \text{ mW}$$

$$\text{For } R_E: \bar{P}_{RE} = I_{EQ}^2 R_E + \frac{1}{2} I_e^2 R_E = 0.107 + \frac{1}{2} (0.1489)^2 (0.6) = 0.1137 \text{ mW}$$

$$\text{For } Q: \bar{P}_Q = I_{CQ}V_{CEQ} - \frac{1}{2} I_c^2 R_C - \frac{1}{2} I_e^2 R_E = 1.01 - 0.0611 - 0.00665 = 0.942 \text{ mW}$$

6.81

$$(a) I_{BQ}R_B + V_{BE}(\text{on}) + (1+\beta)I_{BQ}R_E + V^- = 0$$

$$I_{BQ} = \frac{5 - 0.7}{100 + (121)(4)} = 0.007363 \text{ mA}; \quad I_{CQ} = 0.8836 \text{ mA}, \quad I_{EQ} = 0.8909 \text{ mA}$$

$$V_{CEQ} = 10 - (0.8836)(4) - (0.8909)(4) = 2.902 \text{ V}$$

$$\text{For } Q: P_Q = I_{CQ}V_{CEQ} = (0.8836)(2.902) = 2.564 \text{ mW}$$

For R_C : $P_{RC} = I_{CQ}^2 R_C = (0.8836)^2 (4) = 3.123 \text{ mW}$

For R_E : $P_{RE} = I_{EQ}^2 R_E = (0.8909)^2 (4) = 3.175 \text{ mW}$

(b) $\Delta V_{CE} = \Delta I_C (R_C) = (0.8836)(4) = 3.534 \text{ V}$ - Not possible

$$\Delta V_{CE} = 2.902 - 0.5 = 2.402 \text{ V}$$

So $\Delta I_C = \frac{2.402}{4} = 0.6005 \text{ mA}$

$$\bar{P}_{RC} = \frac{1}{2} (\Delta I_C)^2 R_C = \frac{1}{2} (0.6005)^2 (4) = 0.721 \text{ mW}$$

6.82

a.

$$I_{BQ} = \frac{10 - 0.7}{50 + (151)(10)} = 0.00596 \text{ mA}$$

$$I_{CQ} = 0.894 \text{ mA}, \quad I_{EQ} = 0.90 \text{ mA}$$

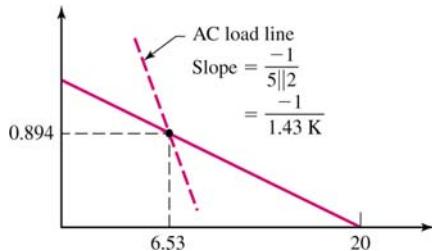
$$V_{ECQ} = 20 - (0.894)(5) - (0.90)(10) \Rightarrow V_{ECQ} = 6.53 \text{ V}$$

$$P_Q \cong I_{CQ} V_{ECQ} = (0.894)(6.53) \Rightarrow P_Q = 5.84 \text{ mW}$$

$$P_{RC} \cong I_{CQ}^2 R_C = (0.894)^2 (5) \Rightarrow P_{RC} = 4.0 \text{ mW}$$

$$P_{RE} \cong I_{EQ}^2 R_E = (0.90)^2 (10) \Rightarrow P_{RE} = 8.1 \text{ mW}$$

b.



$$\Delta i_C = \frac{-1}{1.43 \text{ k}\Omega} \cdot \Delta v_{ec}$$

$$\Delta i_C = 0.894 \Rightarrow |\Delta v_{ec}| = (0.894)(1.43) = 1.28 \text{ V}$$

$$\Delta i_0 = \left(\frac{5}{5+2} \right) \Delta i_C = 0.639 \text{ mA}$$

$$\overline{P_{RL}} = \frac{1}{2} (0.639)^2 (2) \Rightarrow \overline{P_{RL}} = 0.408 \text{ mW}$$

$$\overline{P_{RC}} = \frac{1}{2} \cdot (0.894 - 0.639)^2 (5) \Rightarrow \overline{P_{RC}} = 0.163 \text{ mW}$$

$$\overline{P_{RE}} = 0$$

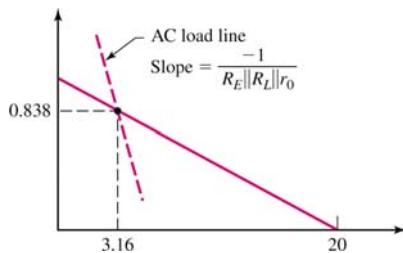
$$\overline{P_Q} = 5.84 - 0.408 - 0.163 \Rightarrow \overline{P_Q} = 5.27 \text{ mW}$$

6.83

$$I_{BQ} = \frac{10 - 0.7}{100 + (101)(10)} = 0.00838 \text{ mA}$$

$$I_{CQ} = 0.838 \text{ mA}, \quad I_{EQ} = 0.846 \text{ mA}$$

$$V_{CEQ} = 20 - (0.838)(10) - (0.846)(10) \Rightarrow V_{CEQ} = 3.16 \text{ V}$$



$$r_0 = \frac{100}{0.838} = 119 \text{ k}\Omega$$

Neglecting base currents:

a.

$$R_L = 1 \text{ k}\Omega$$

$$\text{slope} = \frac{-1}{10 \parallel 1 \parallel 119} = \frac{-1}{0.902 \text{ k}\Omega}$$

$$\Delta i_C = \frac{-1}{0.902 \text{ k}\Omega} \cdot \Delta V_{ce}$$

$$\Delta i_C = 0.838 \Rightarrow |\Delta V_{ce}| = (0.902)(0.838) = 0.756 \text{ V}$$

$$\overline{P_{RL}} = \frac{1}{2} \frac{(0.756)^2}{1} \Rightarrow \overline{P_{RL}} = 0.286 \text{ mW}$$

b.

$$R_L = 10 \text{ k}\Omega$$

$$\text{slope} = \frac{-1}{10 \parallel 10 \parallel 119} = \frac{-1}{4.80}$$

$$\text{For } \Delta i_C = 0.838 \Rightarrow |\Delta V_{ce}| = (0.838)(4.80) = 4.02$$

$$\text{Max. swing determined by voltage } \overline{P_{RL}} = \frac{1}{2} \frac{(3.16)^2}{10} \Rightarrow \overline{P_{RL}} = 0.499 \text{ mW}$$

6.84

a.

$$I_{BQ} = \frac{10 - 0.7}{100 + (101)(10)} = 0.00838 \text{ mA}$$

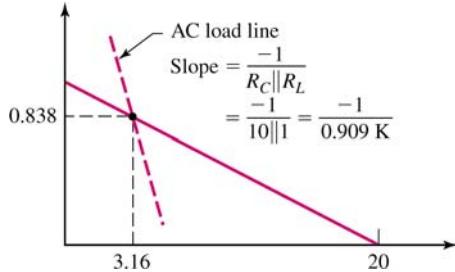
$$I_{CQ} = 0.838 \text{ mA}, \quad I_{EQ} = 0.846 \text{ mA}$$

$$V_{CEQ} = 20 - (0.838)(10) - (0.846)(10) \Rightarrow V_{CEQ} = 3.16 \text{ V}$$

$$P_Q \cong I_{CQ} V_{CEQ} = (0.838)(3.16) \Rightarrow P_Q = 2.65 \text{ mW}$$

$$P_{RC} \cong I_{CQ}^2 R_C = (0.838)^2 (10) \Rightarrow P_{RC} = 7.02 \text{ mW}$$

b.



$$\Delta i_C = \frac{-1}{0.909 \text{ k}\Omega} \cdot \Delta v_{ce}$$

$$\text{For } \Delta i_C = 0.838 \Rightarrow |\Delta v_{ce}| = (0.909)(0.838) = 0.762 \text{ V}$$

$$\Delta i_0 = \left(\frac{R_C}{R_C + R_L} \right) \Delta i_C = \left(\frac{10}{10+1} \right) \Delta i_C = 0.762 \text{ mA}$$

$$\overline{P_{RL}} = \frac{1}{2} (0.762)^2 (1) \Rightarrow \overline{P_{RL}} = 0.290 \text{ mW}$$

$$\overline{P_{RC}} = \frac{1}{2} \cdot (0.838 - 0.762)^2 (10) \Rightarrow \overline{P_{RC}} = 0.0289 \text{ mW}$$

$$\overline{P_Q} = 2.65 - 0.290 - 0.0289 \Rightarrow \overline{P_Q} = 2.33 \text{ mW}$$

Chapter 7

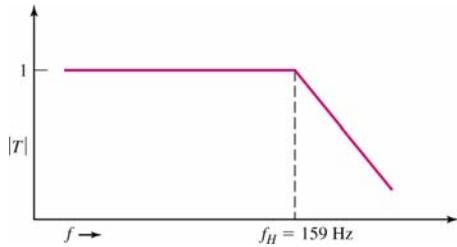
7.1

a.

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{1/(sC_1)}{[1/(sC_1)] + R_i}$$

$$T(s) = \frac{1}{1 + sR_i C_1}$$

b.



$$f_H = \frac{1}{2\pi R_i C_1} = \frac{1}{2\pi (10^3)(10^{-6})} \Rightarrow f_H = 159 \text{ Hz}$$

c.

$$V_o(s) = V_i(s) \cdot \frac{1}{1 + sR_i C_1}$$

$$V_i(s) = \frac{1}{s}$$

For a step function

$$\begin{aligned} V_o(s) &= \frac{1}{s} \cdot \frac{1}{1 + sR_i C_1} = \frac{K_1}{s} + \frac{K_2}{1 + sR_i C_1} \\ &= \frac{K_1(1 + sR_i C_1) + K_2 s}{s(1 + sR_i C_1)} \\ &= \frac{K_1 + s(K_1 R_i C_1 + K_2)}{s(1 + sR_i C_1)} \end{aligned}$$

$$K_2 = -K_1 R_i C_1 \text{ and } K_1 = 1$$

$$\begin{aligned} V_o(s) &= \frac{1}{s} + \frac{-R_i C_1}{1 + sR_i C_1} \\ &= \frac{1}{s} - \frac{\frac{1}{R_i C_1}}{\frac{1}{R_i C_1} + s} \end{aligned}$$

$$v_o(t) = 1 - e^{-t/R_i C_1}$$

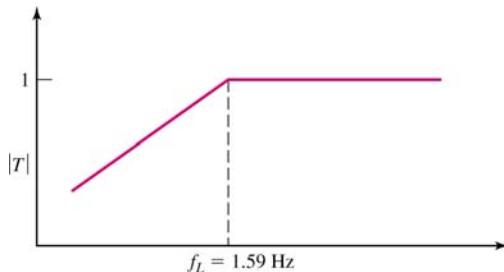
7.2

a.

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_2 + [1/(sC_2)]}$$

$$T(s) = \frac{sR_2 C_2}{1 + sR_2 C_2}$$

b.



$$f_L = \frac{1}{2\pi R_2 C_2} = \frac{1}{2\pi(10^4)(10 \times 10^{-6})} \Rightarrow f_L = 1.59 \text{ Hz}$$

c.

$$V_o(s) = V_i(s) \cdot \frac{sR_2C_2}{1 + sR_2C_2}$$

$$V_i(s) = \frac{1}{s}$$

$$V_o(s) = \frac{R_2C_2}{1 + sR_2C_2} = \frac{1}{s + \frac{1}{R_2C_2}}$$

$$v_o(t) = e^{-t/R_2C_2}$$

7.3

$$(a) T(s) = \frac{V_o}{V_i} = \frac{\left. \frac{R_2}{sC_2} \right|_1}{\left. \frac{1}{sC_2} + R_1 \right|_{sC_2}}$$

$$\text{Now } R_2 \left| \frac{1}{sC_2} \right. = \frac{R_2 \left(\frac{1}{sC_2} \right)}{R_2 + \frac{1}{sC_2}} = \frac{R_2}{1 + sR_2C_2}$$

$$\text{Then } T(s) = \frac{\frac{R_2}{1 + sR_2C_2}}{\frac{R_2}{1 + sR_2C_2} + R_1} = \frac{R_2}{R_1 + R_2 + sR_1R_2C_2}$$

$$T(s) = \left(\frac{R_2}{R_1 + R_2} \right) \cdot \frac{1}{1 + s(R_1 \| R_2)C_2}$$

$$(b) \tau = (R_1 \| R_2)C_2 = (10 \| 20) \times 10^3 \times 10 \times 10^{-6} \Rightarrow \tau = 66.7 \text{ ms}$$

$$(c) f = \frac{1}{2\pi\tau} = \frac{1}{2\pi(66.7 \times 10^{-3})} = 2.39 \text{ Hz}$$

7.4

a.

$$\begin{aligned}\tau_s &= (R_i + R_p)C_s = (30 + 10) \times 10^3 \times (10 \times 10^{-6}) \Rightarrow \tau_s = 0.40 \text{ s} \\ \tau_p &= (R_i \| R_p)C_p = (30 \| 10) \times 10^3 \times (50 \times 10^{-12}) \Rightarrow \tau_p = 0.375 \mu\text{s}\end{aligned}$$

b.

$$f_L = \frac{1}{2\pi\tau_s} = \frac{1}{2\pi(0.4)} \Rightarrow f_L = 0.398 \text{ Hz}$$

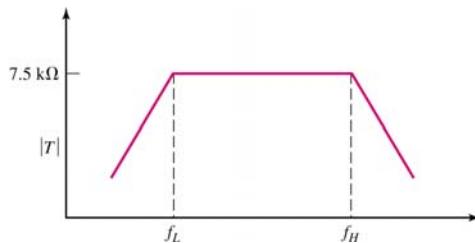
$$f_H = \frac{1}{2\pi\tau_p} = \frac{1}{2\pi(0.375 \times 10^{-6})} \Rightarrow f_H = 424 \text{ kHz}$$

At midband, $C_s \rightarrow$ short, $C_p \rightarrow$ open

$$V_o = I_i(R_i \| R_p)$$

$$T(s) = R_i \| R_p = 30 \| 10 \Rightarrow T(s) = 7.5 \text{ k}\Omega$$

c.



7.5

$$(a) \frac{V_o}{V_i} = \frac{R_2}{R_2 + R_1} = \frac{20}{20 + 10} = 0.667$$

$$(b) \frac{V_o}{V_i} = 1$$

$$(c) T(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_2 + R_1 \left\| \frac{1}{sC_1} \right\|} = \frac{R_2}{R_2 + \frac{R_1}{1 + sR_1 C_1}}$$

$$T(s) = \frac{R_2(1 + sR_1 C_1)}{R_1 + R_2 + sR_1 R_2 C_1} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot \frac{(1 + sR_1 C_1)}{[1 + s(R_1 \| R_2)C_1]}$$

$$\text{We have } K = \frac{R_2}{R_1 + R_2}, \quad \tau_A = R_1 C_1, \quad \tau_B = (R_1 \| R_2) C_1$$

7.6

a.

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_p \left| \frac{1}{sC_p} \right|}{R_p \left| \frac{1}{sC_p} + \left(R_s + \frac{1}{sC_s} \right) \right|}$$

$$R_p \left| \frac{1}{sC_p} \right| = \frac{R_p \cdot \frac{1}{sC_p}}{R_p + \frac{1}{sC_p}} = \frac{R_p}{1 + sR_p C_p}$$

Then

$$T(s) = \frac{R_p}{R_p + \left(R_s + \frac{1}{sC_s} \right) (1 + sR_p C_p)}$$

$$= \frac{R_p}{R_p + R_s + \frac{R_p C_p}{C_s} + \frac{1}{sC_s} + sR_s R_p C_p}$$

$$T(s) = \left(\frac{R_p}{R_p + R_s} \right) \times \left(1 / \left[1 + \frac{R_p}{R_p + R_s} \cdot \frac{C_p}{C_s} + \frac{1}{s(R_s + R_p)C_s} + \frac{sR_p R_s}{R_s + R_p} \cdot C_p \right] \right)$$

b.

$$T(s) = \left(\frac{10}{10+10} \right) \times \left(1 / \left[1 + \left[\frac{10}{20} \cdot \frac{10^{-11}}{10^{-6}} + \frac{1}{s(2 \times 10^4) \cdot 10^{-6}} + s(5 \times 10^3) \cdot 10^{-11} \right] \right] \right)$$

$$\cong \frac{1}{2} \cdot \frac{1}{1 + \frac{1}{s(0.02)} + s(5 \times 10^{-8})}$$

$$s = j\omega$$

$$T(j\omega) = \frac{1}{2} \cdot \frac{1}{1 + j \left[\omega(5 \times 10^{-8}) - \frac{1}{\omega(0.02)} \right]}$$

$$\text{For } \omega_L = \frac{1}{(R_s + R_k)C_s} = \frac{1}{(2 \times 10^4)(10^{-6})} = 50$$

$$T(j\omega) = \frac{1}{2} \cdot \frac{1}{1 + j \left[(50)(5 \times 10^{-8}) - \frac{1}{(50)(0.02)} \right]}$$

$$\approx \frac{1}{2} \cdot \frac{1}{1-j} \Rightarrow |T(j\omega)| = \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$

For

$$\omega_H = \frac{1}{(R_S \| R_P) C_P} = \frac{1}{(5 \times 10^3)(10^{-11})} = 2 \times 10^7$$

$$T(j\omega) = \frac{1}{2} \cdot \frac{1}{1 + j \left[(2 \times 10^7)(5 \times 10^{-8}) - \frac{1}{(2 \times 10^7)(0.02)} \right]}$$

$$T(j\omega) \approx \frac{1}{2} \cdot \frac{1}{1 + j} \Rightarrow |T(j\omega)| = \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$

$$\text{In each case, } |T(j\omega)| = \frac{1}{\sqrt{2}} \cdot \frac{R_p}{R_s + R_p}$$

c.

$$R_s = R_p = 10 \text{ k}\Omega, \quad C_s = C_p = 0.1 \mu\text{F}$$

$$T(s) = \frac{1}{2} \cdot \left(1 / \left[1 + \frac{1}{2} \cdot \frac{1}{1 + \frac{1}{s(2 \times 10^4(10^{-7}))}} + s(5 \times 10^3)(10^{-7}) \right] \right)$$

$$s = j\omega$$

$$T(j\omega) = \frac{1}{2} \cdot \frac{1}{1 + \frac{1}{2} + j \left[\omega(5 \times 10^{-4}) - \frac{1}{\omega(2 \times 10^{-3})} \right]}$$

$$\text{For } \omega = \frac{1}{(2 \times 10^4)(10^{-7})} = 500$$

$$\begin{aligned} T(j\omega) &= \frac{1}{2} \cdot \frac{1}{1.5 + j \left[(500)(5 \times 10^{-4}) - \frac{1}{(500)(2 \times 10^{-3})} \right]} \\ &= \frac{1}{2} \cdot \frac{1}{1.5 - j(0.75)} \Rightarrow |T(j\omega)| = 0.298 \end{aligned}$$

$$\text{For } \omega = \frac{1}{(5 \times 10^3)(10^{-7})} = 2 \times 10^3$$

$$\begin{aligned} T(j\omega) &= \frac{1}{2} \cdot \left\{ 1 / \left(1.5 + j \left[(2 \times 10^3)(5 \times 10^{-4}) - \frac{1}{(2 \times 10^3)(2 \times 10^{-3})} \right] \right) \right\} \\ &= \frac{1}{2} \cdot \frac{1}{1.5 + j(0.75)} \Rightarrow |T(j\omega)| = 0.298 \end{aligned}$$

$$\text{In each case, } |T(j\omega)| < \frac{1}{\sqrt{2}} \cdot \frac{R_p}{R_p + R_s}$$

7.7

$$(a) |T| = \frac{1}{\left[\sqrt{1 + \left(\frac{f}{f_T} \right)^2} \right]^3}$$

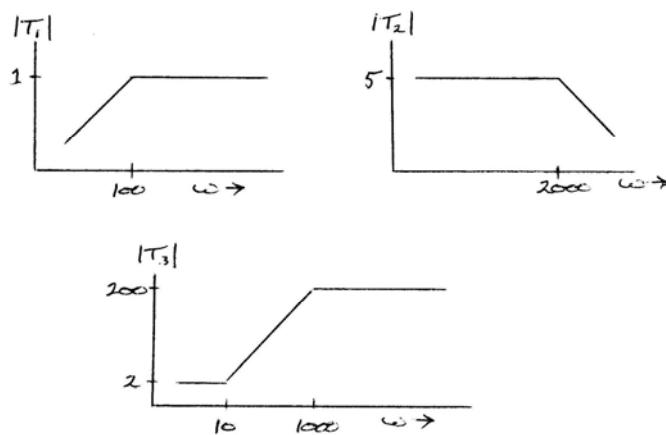
$$\text{At } f = f_T, |T| = \frac{1}{(\sqrt{2})^3} = 0.35355$$

Or $|T|_{dB} = 20 \log_{10}(0.35355) = -9.03 \text{ dB}$

$$\phi = -\tan^{-1}\left(\frac{f}{f_T}\right)^3 = -3 \tan^{-1}\left(\frac{f}{f_T}\right) = -3 \tan^{-1}(1) = -135^\circ$$

- (b) Slope = $3(-6) = -18 \text{ dB/octave} = -60 \text{ dB/decade}$
 $\phi = 3(-90) = -270^\circ$
-

7.8



7.9

- (a) (ii) $\omega_1 = 1 \text{ rad/s}$; $\omega_2 = 10 \text{ rad/s}$; $\omega_3 = 100 \text{ rad/s}$; $\omega_4 = 1000 \text{ rad/s}$
 (iii) $|T(0)| = 10$
 (iv) $|T(\infty)| = 10$
- (b) (ii) $\omega = 5 \text{ rad/s}$
 (iii) $|T(0)| = 0$
 (iv) $|T(\infty)| = \frac{8}{(0.2)^2} = 200$
-

7.10

$$(a) T(j\omega) = 5 \left(\frac{j \frac{\omega}{10^2}}{1 + j \frac{\omega}{10^2}} \right) \left(\frac{1}{1 + j \frac{\omega}{5 \times 10^4}} \right)$$

$$\text{or } T(j\omega) = 2.5 \times 10^5 \left(\frac{j\omega}{10^2 + j\omega} \right) \left(\frac{1}{5 \times 10^4 + j\omega} \right)$$

$$(b) |T| = \frac{5\left(\frac{\omega}{10^2}\right)}{\sqrt{1+\left(\frac{\omega}{10^2}\right)^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{\omega}{5\times 10^4}\right)^2}}$$

(i) At $\omega = 50 \text{ rad/s}$

$$|T| = \frac{5\left(\frac{50}{100}\right)}{\sqrt{1+\left(\frac{50}{100}\right)^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{50}{5\times 10^4}\right)^2}} = 2.236$$

(ii) At $\omega = 150 \text{ rad/s}$

$$|T| = \frac{5\left(\frac{150}{100}\right)}{\sqrt{1+\left(\frac{150}{100}\right)^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{150}{5\times 10^4}\right)^2}} = 4.16$$

(iii) At $\omega = 10^5$

$$|T| = \frac{5\left(\frac{10^5}{10^2}\right)}{\sqrt{1+\left(\frac{10^5}{10^2}\right)^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{10^5}{5\times 10^4}\right)^2}} = 2.236$$

7.11

a.

$$V_0 = -g_m V_\pi R_L \quad V_\pi = \left(\frac{r_\pi}{r_\pi + R_S} \right) V_i$$

$$|T| = g_m R_L \left(\frac{r_\pi}{r_\pi + R_S} \right) = (29)(6) \left(\frac{5.2}{5.2 + 0.5} \right)$$

$$\underline{|T_{\text{midband}}| = 159}$$

b.

$$\tau_s = (R_S + r_\pi) C_C$$

$$f_L = \frac{1}{2\pi\tau_s} \Rightarrow \tau_s = \frac{1}{2\pi f_L} = \frac{1}{2\pi(30)} \Rightarrow \tau_s = 5.31 \text{ ms}, \text{ Open-Circuit}$$

$$\tau_p = \frac{1}{2\pi f_H} = \frac{1}{2\pi(480 \times 10^3)} \Rightarrow \tau_p = 0.332 \mu \text{ s}, \text{ Short-Circuit}$$

c.

$$C_C = \frac{\tau_s}{(R_S + r_\pi)} = \frac{5.31 \times 10^{-3}}{(0.5 + 5.2) \times 10^3} \Rightarrow C_C = 0.932 \mu \text{ F}$$

$$\tau_p = R_L C_L$$

$$C_L = \frac{\tau_p}{R_L} = \frac{0.332 \times 10^{-6}}{6 \times 10^3} \Rightarrow C_L = 55.3 \text{ pF}$$

7.12

$$(a) \frac{V_o}{V_i} = \frac{R_2 + R_3}{R_1 + R_2 + R_3} = \frac{10 + 40}{10 + 10 + 40} = 0.833$$

$$(b) \frac{V_o}{V_i} = \frac{R_2}{R_1 + R_2} = \frac{10}{10 + 10} = 0.50$$

$$(c) R_3 \parallel \frac{1}{sC} = \frac{R_3 \left(\frac{1}{sC} \right)}{R_3 + \frac{1}{sC}} = \frac{R_3}{1 + sR_3 C}$$

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_2 + \frac{R_3}{1 + sR_3 C}}{R_1 + R_2 + \frac{R_3}{1 + sR_3 C}} = \frac{R_2 + R_3 + sR_2 R_3 C}{R_1 + R_2 + R_3 + s(R_1 + R_2)R_3 C}$$

$$\text{or } T(s) = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3} \right) \cdot \frac{[1 + s(R_2 \parallel R_3)C]}{[1 + s((R_1 + R_2) \parallel R_3)C]}$$

$$\text{where } K = \frac{R_2 + R_3}{R_1 + R_2 + R_3}, \quad \tau_A = (R_2 \parallel R_3)C, \quad \tau_B = ((R_1 + R_2) \parallel R_3)C$$

7.13 Computer Analysis

7.14

$$(a) |A_v|_{\max} = g_m R_D, \quad g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.4)(0.8)} = 1.131 \text{ mA/V}$$

$$|A_v|_{\max} = (1.131)(1) = 1.13$$

$$(b) f_H = \frac{1}{2\pi R_D C_L} = \frac{1}{2\pi(10^3)(10^{-12})}$$

$$f_H = BW = 159 \text{ MHz}$$

7.15

$$(a) f_H = \frac{1}{2\pi R_C C_L} \Rightarrow R_C = \frac{1}{2\pi f_H C_L} = \frac{1}{2\pi(800 \times 10^6)(0.08 \times 10^{-12})}$$

$$\text{or } R_C = 2.49 \text{ k}\Omega$$

$$(b) I_{CQ} = \frac{V_{CC} - V_{CEQ}}{R_C} = \frac{2.5 - 1.25}{2.487} = 0.503 \text{ mA}$$

$$(c) |A_v|_{\max} = g_m R_C, \quad g_m = \frac{0.5026}{0.026} = 19.33 \text{ mA/V}$$

$$|A_v|_{\max} = (19.33)(2.487) = 48.1$$

7.16

$$(a) \quad T(s) = \frac{V_o(s)}{V_i(s)} = -g_m \left(r_o \left\| \frac{1}{sC_L} \right\| \right) = -g_m \left(\frac{r_o \cdot \frac{1}{sC_L}}{r_o + \frac{1}{sC_L}} \right) = -g_m r_o \left(\frac{1}{1 + s r_o C_L} \right)$$

$$(b) \quad g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.05)(0.1)} = 0.1414 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(0.1)} = 1000 \text{ k}\Omega$$

$$|A_v|_{\max} = g_m r_o = (0.1414)(1000) = 141.4$$

$$(c) \quad f_H = BW = \frac{1}{2\pi r_o C_L} = \frac{1}{2\pi(10^6)(0.5 \times 10^{-12})} \Rightarrow f_H = 318 \text{ kHz}$$

7.17

a.

$$R_{TH} = R_1 \parallel R_2 = 10 \parallel 1.5 = 1.304 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{1.5}{1.5 + 10} \right) (12) = 1.565 \text{ V}$$

$$I_{BQ} = \frac{1.565 - 0.7}{1.30 + (10)(0.1)} = 0.0759 \text{ mA}$$

$$I_{CQ} = 7.585 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{7.59} = 0.343 \text{ k}\Omega$$

$$g_m = \frac{7.59}{0.026} = 292 \text{ mA/V}$$

$$R_i = R_1 \parallel [R_2 \parallel [r_\pi + (1 + \beta)R_E]]$$

$$= 10 \parallel 1.5 \parallel [0.343 + (10)(0.1)]$$

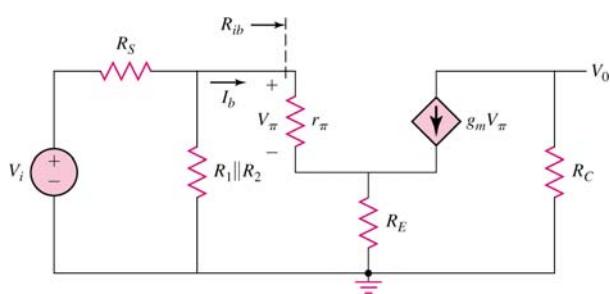
$$= 1.30 \parallel 10.44 \Rightarrow R_i = 1.159 \text{ k}\Omega$$

$$\tau = (R_S + R_i)C_C = (0.5 + 1.16) \times 10^3 \times (0.1 \times 10^{-6})$$

$$\tau = 1.659 \times 10^{-4} \text{ s}$$

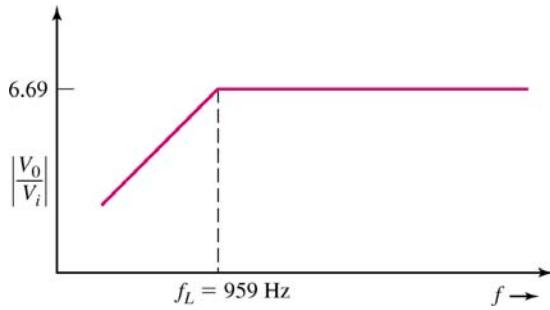
$$f_L = \frac{1}{2\pi\tau} = \frac{1}{2\pi(1.66 \times 10^{-4})} \Rightarrow f_L = 959 \text{ Hz}$$

b.



$$\begin{aligned}
 V_0 &= -(\beta I_b) R_C \\
 R_{ib} &= r_\pi + (1 + \beta) R_E \\
 &= 0.343 + (101)(0.1) = 10.44 \text{ k}\Omega \\
 I_b &= \left(\frac{R_1 \| R_2 }{R_1 \| R_2 + R_{ib}} \right) I_i \\
 &= \left(\frac{1.30}{1.30 + 10.4} \right) I_i = (0.111) I_i \\
 I_i &= \frac{V_i}{R_s + R_1 \| R_2 \| R_{ib}} \\
 &= \frac{V_i}{0.5 + (1.3) \| (10.44)} \\
 I_i &= \frac{V_i}{1.659} \\
 \left| \frac{V_0}{V_i} \right| &= \frac{\beta R_C (0.111)}{1.659} \Rightarrow \left| \frac{V_0}{V_i} \right|_{\text{midband}} = \frac{(100)(1)(0.111)}{1.659} \Rightarrow \left| \frac{V_0}{V_i} \right|_{\text{midband}} = 6.69
 \end{aligned}$$

c.



7.18

$$\begin{aligned}
 \text{(a)} \quad V_{DSQ} &= V_{DD} - I_{DQ}(R_D + R_S) \\
 3.2 &= 9 - (0.8)(R_D + 0.5) \Rightarrow R_D = 6.75 \text{ k}\Omega \\
 I_{DQ} &= K_n (V_{GSQ} - V_{TN})^2 \\
 0.8 &= 0.5(V_{GSQ} - 1.2)^2 \Rightarrow V_{GSQ} = 2.465 \text{ V} \\
 V_G &= (0.8)(0.5) + 2.465 = 2.865 \text{ V} \\
 V_G &= \frac{1}{R_1} \cdot R_{in} \cdot V_{DD} \Rightarrow 2.865 = \frac{1}{R_1} (160)(9)
 \end{aligned}$$

which yields $R_1 = 503 \text{ k}\Omega$ and $R_2 = 235 \text{ k}\Omega$

$$(b) \quad g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.8)} = 1.265 \text{ mA/V}$$

$$A_v = \frac{-g_m R_D}{1 + g_m R_S} = \frac{-(1.265)(6.75)}{1 + (1.265)(0.5)} = -5.23$$

$$\begin{aligned}
 \text{(c)} \quad f_L &= \frac{1}{2\pi R_{in} C_C} \Rightarrow C_C = \frac{1}{2\pi f_L R_{in}} = \frac{1}{2\pi(16)(160 \times 10^3)} \Rightarrow C_C = 0.06217 \mu\text{F} \\
 \tau_s &= R_{in} C_C = (160 \times 10^3)(0.06217 \times 10^{-6}) = 9.947 \times 10^{-3} \text{ s}
 \end{aligned}$$

$$|A_v| = 5.23 \left| \frac{s\tau_s}{1+s\tau_s} \right| = (5.23) \frac{\left(\frac{f}{f_L} \right)}{\sqrt{1+\left(\frac{f}{f_L} \right)^2}}$$

(i) For $f = 5$ Hz,

$$|A_v| = (5.23) \frac{\frac{5}{16}}{\sqrt{1+\left(\frac{5}{16} \right)^2}} = 1.56$$

(ii) For $f = 14$ Hz,

$$|A_v| = (5.23) \frac{\frac{14}{16}}{\sqrt{1+\left(\frac{14}{16} \right)^2}} = 3.44$$

(iii) For $f = 25$ Hz,

$$|A_v| = (5.23) \frac{\frac{25}{16}}{\sqrt{1+\left(\frac{25}{16} \right)^2}} = 4.405$$

7.19

$$I_{DQ} = K_n (V_{GS} - V_{TN})^2 \Rightarrow V_{GS} = \sqrt{\frac{I_{DQ}}{K_n}} + V_{TN} = \sqrt{\frac{1}{0.5}} + 1 = 2.414 \text{ V}$$

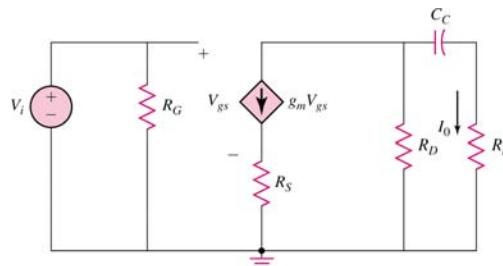
$$V_S = -2.414 \text{ V}$$

$$R_S = \frac{-2.414 - (-5)}{1} \Rightarrow R_S = 2.59 \text{ k}\Omega$$

$$V_D = V_{DSQ} + V_S = 3 - 2.414 = 0.586 \text{ V}$$

$$R_D = \frac{5 - 0.586}{1} \Rightarrow R_D = 4.41 \text{ k}\Omega$$

b.



$$I_0 = -\left(g_m V_{gs}\right) \begin{pmatrix} R_D \\ R_D + R_L + \frac{1}{sC_C} \end{pmatrix}$$

$$V_{gs} = \frac{V_i}{1 + g_m R_s}$$

$$\frac{I_0(s)}{V_i(s)} = \frac{-g_m}{1 + g_m R_s} \cdot R_D \left[\frac{sC_C}{1 + s(R_D + R_L)C_C} \right]$$

$$T(s) = \frac{I_0(s)}{V_i(s)}$$

$$= \frac{-g_m R_D}{1 + g_m R_s} \cdot \frac{1}{R_D + R_L} \cdot \frac{s(R_D + R_L)C_C}{1 + s(R_D + R_L)C_C}$$

c.

$$f_L = \frac{1}{2\pi\tau_L} \Rightarrow \tau_L = \frac{1}{2\pi f_L} = \frac{1}{2\pi(10)} \Rightarrow \tau_L = 15.92 \text{ ms}$$

$$\tau_L = (R_D + R_L)C_C \Rightarrow C_C = \frac{\tau_L}{R_D + R_L} = \frac{15.9 \times 10^{-3}}{(4.41 + 4) \times 10^3} \Rightarrow C_C = 1.89 \mu \text{F}$$

7.20

a.

$$\frac{9 - V_{SG}}{R_s} = I_D = K_p (V_{SG} + V_{TP})^2$$

$$9 - V_{SG} = (0.5)(12)(V_{SG}^2 - 4V_{SG} + 4)$$

$$6V_{SG}^2 - 23V_{SG} + 15 = 0$$

$$V_{SG} = \frac{23 \pm \sqrt{(23)^2 - 4(6)(15)}}{2(6)} \Rightarrow V_{SG} = 3 \text{ V}$$

$$g_m = 2K_p (V_{SG} + V_{TP}) = 2(0.5)(3 - 2) \Rightarrow g_m = 1 \text{ mA/V}$$

$$R_o = \frac{1}{g_m} \| R_s = 1 \| 12 \Rightarrow R_o = 0.923 \text{ k}\Omega$$

b. $\tau = (R_o + R_L)C_C$

$$f_L = \frac{1}{2\pi\tau} \Rightarrow \tau = \frac{1}{2\pi f_L} = \frac{1}{2\pi(20)} \Rightarrow \tau = 7.96 \text{ ms}$$

$$\text{c. } C_C = \frac{\tau}{R_o + R_L} = \frac{7.96 \times 10^{-3}}{(0.923 + 10) \times 10^3} \Rightarrow C_C = 0.729 \mu \text{F}$$

7.21

$$(a) R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(121)(4) = 48.4 \text{ k}\Omega$$

$$I_{BQ} = \frac{I_{EQ}}{1 + \beta} = \frac{1.5}{121} = 0.012397 \text{ mA}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + I_{EQ}R_E = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC}$$

$$\text{so } \frac{1}{R_1} (48.4)(12) = (0.012397)(48.4) + 0.7 + (1.5)(4)$$

which yields $R_1 = 79.6 \text{ k}\Omega$ and $R_2 = 124 \text{ k}\Omega$

$$(b) \quad I_{CQ} = \left(\frac{120}{121} \right) (1.5) = 1.488 \text{ mA}$$

$$r_\pi = \frac{(120)(0.026)}{1.488} = 2.097 \text{ k}\Omega, \quad r_o = \frac{50}{1.488} = 33.6 \text{ k}\Omega$$

$$A_v = \frac{(1+\beta)(r_o \| R_E \| R_L)}{r_\pi + (1+\beta)(r_o \| R_E \| R_L)}$$

$$\text{Now } r_o \| R_E \| R_L = 33.6 \| 4 \| 4 = 1.888 \text{ k}\Omega$$

$$A_v = \frac{(121)(1.888)}{2.097 + (121)(1.888)} = 0.991$$

$$(c) \quad R_o = R_E \| r_o \left| \frac{r_\pi}{1+\beta} \right. = 4 \| 33.6 \left| \frac{2.097}{121} \right. \Rightarrow R_o = 17.25 \Omega$$

$$(d) \quad f_L = \frac{1}{2\pi(R_o + R_L)C_{C2}} = \frac{1}{2\pi(17.25 + 4000)(2 \times 10^{-6})} \\ f_L = 19.8 \text{ Hz}$$

7.22

$$(a) \quad V_o(s) = -g_m \left(r_o \| R_D \left| \frac{1}{sC_L} \right. \right) \cdot V_{gs} = -g_m \left[\frac{\left(r_o \| R_D \right) \left(\frac{1}{sC_L} \right)}{r_o \| R_D + \frac{1}{sC_L}} \right] \cdot V_{gs} = -g_m \left[\frac{r_o \| R_D}{1 + s(r_o \| R_D)C_L} \right] \cdot V_{gs}$$

$$V_{gs} = \frac{\left(\frac{1}{sC_i} \right) (V_i(s))}{\frac{1}{sC_i} + R_{Si}} = \frac{V_i(s)}{1 + sR_{Si}C_i}$$

$$T(s) = \frac{V_o(s)}{V_i(s)} = -g_m \left(\frac{1}{1 + sR_{Si}C_i} \right) \left(\frac{r_o \| R_D}{1 + s(r_o \| R_D)C_L} \right)$$

$$(b) \quad \tau = R_{Si}C_i$$

$$(c) \quad \tau = (r_o \| R_D)C_L$$

7.23

$$(a) \quad \frac{V_{gs}}{V_i} = \frac{-\left(\frac{1}{g_m} \left| \frac{1}{sC_i} \right. \right)}{\left(\frac{1}{g_m} \left| \frac{1}{sC_i} \right. \right) + R_s}$$

$$\text{Now } \left(\frac{1}{g_m} \parallel \frac{1}{sC_i} \right) = \frac{\left(\frac{1}{g_m} \right) \left(\frac{1}{sC_i} \right)}{\frac{1}{g_m} + \frac{1}{sC_i}} = \frac{\frac{1}{g_m}}{1 + s \left(\frac{1}{g_m} \right) C_i}$$

$$\text{So } \frac{V_{gs}}{V_i} = \frac{-\frac{1}{g_m}}{\frac{1}{g_m} + R_s \left(1 + s \left(\frac{1}{g_m} \right) C_i \right)} = \left(\frac{-\frac{1}{g_m}}{\frac{1}{g_m} + R_s} \right) \cdot \frac{1}{1 + s \left(\frac{1}{g_m} \parallel R_s \right) C_i}$$

We have

$$V_o = -g_m V_{gs} \left[\frac{R_D}{R_D + R_L + \frac{1}{sC_C}} \right] \cdot R_L = -g_m V_{gs} \left[\frac{R_D R_L (sC_C)}{1 + s(R_D + R_L) C_C} \right]$$

$$V_o = -g_m V_{gs} \left(\frac{R_D R_L}{R_D + R_L} \right) \left[\frac{s(R_D + R_L) C_C}{1 + s(R_D + R_L) C_C} \right]$$

$$\text{Then } T(s) = \frac{V_o(s)}{V_i(s)} = \frac{+g_m (R_D \parallel R_L)}{1 + g_m R_s} \cdot \frac{1}{\left[1 + s \left(\frac{1}{g_m} \parallel R_s \right) C_i \right]} \cdot \left[\frac{s(R_D + R_L) C_C}{1 + s(R_D + R_L) C_C} \right]$$

- (b) $\tau = \left(\frac{1}{g_m} \parallel R_s \right) C_i$
 (c) $\tau = (R_D + R_L) C_C$

7.24

(a)

$$I_{EQ} = \frac{5 - 0.7}{4} = 1.075 \text{ mA} \quad I_{CQ} = 1.064 \text{ mA}$$

$$V_{CEQ} = 10 - (1.064)(2) - (1.075)(4)$$

$$V_{CEQ} = 3.57 \text{ V}$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1.064}{0.026} = 40.92 \text{ mA/V}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1.064} = 2.44 \text{ K}$$

(b)

$$\text{For } C_{C1}; R_{eq1} = R_s + R_E \left\| \frac{r_\pi}{1 + \beta} = 200 + 4000 \right\| \frac{2440}{101}$$

$$R_{eq1} = 224.0 \Omega; \quad \tau_1 = R_{eq1} C_{C1} = 1.053 \text{ ms}$$

$$\text{For } C_{C2}; R_{eq2} = R_C + R_L = 2 + 47 = 49 \text{ k}\Omega$$

$$\tau_2 = R_{eq2} C_{C2} = 49 \text{ ms}$$

$$(c) f_1 = \frac{1}{2\pi\tau_1} = \frac{1}{2\pi(1.053 \times 10^{-3})} \Rightarrow f_1 = 151 \text{ Hz}$$

7.25

(a)

$$\tau_H = (R_C \| R_L) C_L = (2 \| 47) \times 10^3 \times 10 \times 10^{-12} \\ = 1.918 \times 10^{-8} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi(1.918 \times 10^{-8})} \Rightarrow f_H = 8.30 \text{ MHz}$$

(b)

$$\frac{1}{\sqrt{1 + (2\pi\tau_H f)^2}} = 0.1$$

$$\left(\frac{1}{0.1}\right)^2 = 100 = 1 + (2\pi\tau_H f)^2$$

$$f = \frac{\sqrt{99}}{2\pi\tau_H} = \frac{\sqrt{99}}{2\pi(1.918 \times 10^{-8})}$$

$$f = 82.6 \text{ MHz}$$

7.26

(a)

$$\frac{5 - V_{SG}}{R_i} = K_p (V_{SG} + V_{TP})^2$$

$$5 - V_{SG} = (1)(1.2)(V_{SG} - 1.5)^2 = (1.2)(V_{SG}^2 - 3V_{SG} + 2.25)$$

$$1.2V_{SG}^2 - 2.6V_{SG} - 2.3 = 0 \Rightarrow V_{SG} = 2.84 \text{ V}$$

$$\underline{I_{DQ} = 1.8 \text{ mA}}$$

$$V_{SDQ} = 10 - (1.8)(1.2 + 1.2) \Rightarrow V_{SDQ} = 5.68 \text{ V}$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(1)(1.8)} = 2.683 \text{ mA/V}$$

$$r_o = \infty$$

(b)

$$R_{is} = \frac{1}{g_m} = \frac{1}{2.68} = 0.3727 \text{ k}\Omega$$

$$R_i = 1.2 \| 0.373 = 0.284 \text{ k}\Omega$$

$$\text{For } C_{C1}, \tau_{s1} = (284 + 200)(4.7 \times 10^{-6}) = 2.27 \text{ ms}$$

$$\text{For } C_{C2}, \tau_{s2} = (1.2 \times 10^3 + 50 \times 10^3)(10^{-6}) = 51.2 \text{ ms}$$

(c)

C_{C2} dominates,

$$f_{3-dB} = \frac{1}{2\pi\tau_{s2}} = \frac{1}{2\pi(51.2 \times 10^{-3})} = 3.1 \text{ Hz}$$

7.27

Assume $V_{TN} = 1V$, $k'_n = 80 \mu A/V^2$, $\lambda = 0$

Neglecting $R_{Si} = 200\Omega$, Midband gain is:

$$|A_v| = g_m R_D$$

Let $I_{DQ} = 0.2 \text{ mA}$, $V_{DSQ} = 5V$

$$R_D = \frac{9 - 5}{0.2} \Rightarrow R_D = 20 \text{ k}\Omega$$

Then

$$\text{We need } g_m = \frac{|A_v|}{R_D} = \frac{10}{20} = 0.5 \text{ mA/V and } g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)I_{DQ}}$$

$$\text{or } 0.5 = 2\sqrt{\left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)(0.2)} \Rightarrow \frac{W}{L} = 7.81$$

Let

$$R_1 + R_2 = \frac{9}{(0.2)I_{DQ}} = \frac{9}{(0.2)(0.2)} = 225 \text{ k}\Omega$$

$$I_{DQ} = 0.2 = \left(\frac{0.080}{2}\right)(7.81)(V_{GS} - 1)^2 \Rightarrow V_{GS} = 1.80 = \left(\frac{R_2}{R_1 + R_2}\right)(9) = \left(\frac{R_2}{225}\right)(9) \Rightarrow$$

$$\underline{R_2 = 45 \text{ k}\Omega, R_1 = 180 \text{ k}\Omega}$$

$$R_{TH} = R_1 \| R_2 = 180 \| 45 = 36 \text{ k}\Omega$$

$$\tau_1 = \frac{1}{2\pi f_1} = \frac{1}{2\pi(200)} = 7.958 \times 10^{-4} \text{ s} = (R_{Si} + R_{TH})C_C \text{ or } C_C = \frac{7.96 \times 10^{-4}}{(200 + 36 \times 10^3)} \Rightarrow$$

$$C_C = 0.022 \text{ }\mu\text{F}$$

$$\tau_2 = \frac{1}{2\pi f_2} = \frac{1}{2\pi(3 \times 10^3)} = 5.305 \times 10^{-5} \text{ s} = R_D C_L \text{ or } C_L = \frac{5.31 \times 10^{-5}}{20 \times 10^3} \Rightarrow \underline{C_L = 2.65 \text{ nF}}$$

7.28

$$I_{BQ} = \frac{10 - 0.7}{430 + (201)(2.5)} = 0.00997 \text{ mA}$$

$$I_{CQ} = (200)I_{BQ} = 1.995 \text{ mA}$$

$$r_\pi = \frac{(200)(0.026)}{1.99} = 2.61 \text{ k}\Omega$$

$$R_{ib} = 2.61 + (201)(2.5) = 505 \text{ k}\Omega$$

$$\tau_s = \frac{1}{2\pi f_L} = \frac{1}{2\pi(15)} = 0.0106 \text{ s}$$

$$= R_{eq} C_C = (0.5 + 505 \| 430) \times 10^3 C_C = 232.7 \times 10^3 C_C$$

Or $\frac{C_C = 4.55 \times 10^{-8} F}{C_C = 45.5 \text{ nF}}$

7.29

(a) $V^+ = V_{CEQ} + I_{EQ}R_E$

$$3.3 = 1.8 + (0.25)R_E \Rightarrow R_E = 6 \text{ k}\Omega$$

$$I_{BQ} = \frac{0.25}{121} = 0.002066 \text{ mA}$$

$$V^+ = I_{BQ}R_B + V_{BE}(\text{on}) + I_{EQ}R_E$$

$$3.3 = (0.002066)(R_B) + 0.7 + (0.25)(6) \Rightarrow R_B = 532 \text{ k}\Omega$$

(b) $I_{CQ} = \left(\frac{120}{121}\right)(0.25) = 0.2479 \text{ mA}, r_\pi = \frac{(120)(0.026)}{0.2479} = 12.59 \text{ k}\Omega$

$$R_{ib} = r_\pi + (1 + \beta)R_E = 12.59 + (121)(6) = 738.6 \text{ k}\Omega$$

$$R_i = R_B \| R_{ib} = 532 \| 738.6 = 309.25 \text{ k}\Omega$$

$$\tau_s = \frac{1}{2\pi f_L} = \frac{1}{2\pi(20)} = 0.007958 = (R_s + R_i)C_C$$

$$\text{so } C_C = \frac{0.007958}{(0.1 + 309.25) \times 10^3} \Rightarrow C_C = 0.0257 \mu\text{F}$$

(c) For $R_s \ll R_B$,

$$A_v \cong \frac{(1 + \beta)R_E}{r_\pi + (1 + \beta)R_E} = \frac{(121)(6)}{12.59 + (121)(6)} = 0.983$$

7.30

$$R_{TH} = R_1 \parallel R_2 = 1.2 \parallel 1.2 = 0.6 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (V_{CC}) = \left(\frac{1.2}{1.2 + 1.2} \right) (5) = 2.5 \text{ V}$$

$$I_{BO} = \frac{2.5 - 0.7}{0.6 + (101)(0.05)} = 0.319 \text{ mA}$$

$$I_{CQ} = 31.9 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{31.9} = 0.0815 \text{ k}\Omega$$

$$\tau_{C_{C1}} \gg \tau_{C_{C2}} \text{ and } f = \frac{1}{2\pi\tau} \text{ so that } f_{3-dB}(C_{C1}) \ll f_{3-dB}(C_{C2})$$

Then, for $f_{3-dB}(C_{C1}) \Rightarrow C_{C2}$ acts as an open and for $f_{3-dB}(C_{C2}) \Rightarrow C_{C1}$ acts as a short circuit.

$$f_{3-dB}(C_{C2}) = 25 \text{ Hz} = \frac{1}{2\pi\tau_2} \quad \tau_2 = \frac{1}{2\pi(25)} = 0.006366 \text{ s} = R_{eq}C_{C2}$$

where

$$R_{eq} = R_L + R_E \left| \left(\frac{r_\pi + R_1 \parallel R_2 \parallel R_S}{1 + \beta} \right) \right| \\ = 10 + 50 \left| \left(\frac{81.5 + 600 \parallel 300}{101} \right) \right| = 10 + 50 \parallel 2.787 \Rightarrow$$

$$R_{eq} = 12.64 \text{ }\Omega \Rightarrow C_{C2} = \frac{0.00637}{12.6} \Rightarrow C_{C2} = 504 \text{ }\mu\text{F}$$

$$R_{ib} = r_\pi + (1 + \beta)R_E \text{ Assume } C_{C2} \text{ an open}$$

$$R_{ib} = 81.5 + (101)(50) = 5132 \text{ }\Omega$$

$$\tau_1 = (100)\tau_2 = (100)(0.006366) = 0.6366 \text{ s} = R_{eq1}C_{C1}$$

$$R_{eq1} = R_S + R_{TH} \parallel R_{ib} = 300 + 600 \parallel 5132 = 837.2 \text{ }\Omega$$

$$\text{So } C_{C1} = \frac{0.6366}{837.2} \Rightarrow C_{C1} = 760 \text{ }\mu\text{F}$$

7.31

From Problem 7.30 $R_{TH} = 0.6 \text{ K}$, $I_{CQ} = 31.9 \text{ mA}$, $r_\pi = 81.5 \text{ }\Omega$

$$\tau_{C2} \gg \tau_{C1} \text{ and } f = \frac{1}{2\pi\tau} \text{ so } f_{3-dB}(C_{C2}) \ll f_{3-dB}(C_{C1})$$

Then $f_{3-dB}(C_{C2}) \Rightarrow C_{C1}$ acts as an open circuit and for $f_{3-dB}(C_{C1}) \Rightarrow C_{C2}$ acts as a short circuit.

$$f_{3-dB}(C_{C1}) = 20 \text{ Hz} = \frac{1}{2\pi\tau_{C1}} \Rightarrow \tau_{C1} = 0.007958 \text{ s}$$

$$R_{ib} = r_\pi + (1 + \beta)(R_E \| R_L) = 81.5 + (101)(50 \| 10) = 923.2 \Omega$$

$$\tau_{C1} \Rightarrow R_{eq1} = R_S + R_{TH} \| R_{ib} = 300 + 600 \| 923.2 = 663.7 \Omega$$

$$C_{C1} = \frac{0.007958}{663.7} \Rightarrow C_{C1} = 12 \mu\text{F}$$

$$\tau_{C2} = 100\tau_{C1} = 0.7958 \text{ s}$$

$$R_{eq2} = R_L + R_E \left\| \left(\frac{r_\pi + R_{TH}}{1 + \beta} \right) \right\| = 10 + 50 \left\| \left(\frac{81.5 + 600}{101} \right) \right\|$$

$$R_{eq2} = 10 + 50 \| 6.748 = 15.95 \Omega$$

$$C_{C2} = \frac{0.7958}{15.95} \Rightarrow C_{C2} = 0.050 \text{ F}$$

7.32

$$(a) I_{EQ} = 0.2 \text{ mA}, I_{BQ} = \frac{0.2}{121} = 0.001653 \text{ mA}, I_{CQ} = \left(\frac{120}{121} \right) (0.2) = 0.1983 \text{ mA}$$

$$V_E = -(I_{BQ} R_i + V_{BE}(\text{on})) = -[(0.001653)(10)] + 0.7 = -0.7165 \text{ V}$$

$$V_C = V_E + V_{CEQ} = -0.7165 + 2.2 = 1.483 \text{ V}$$

$$R_C = \frac{3 - 1.483}{0.1983} = 7.65 \text{ k}\Omega$$

$$(b) r_\pi = \frac{(120)(0.026)}{0.1983} = 15.73 \text{ k}\Omega, g_m = \frac{0.1983}{0.026} = 7.627 \text{ mA/V}$$

$$A_v = -g_m (R_C \| R_L) \left(\frac{r_\pi}{r_\pi + R_i} \right) = -(7.627)(7.65 \| 20) \left(\frac{15.73}{15.73 + 10} \right) = -25.8$$

$$(c) \text{ For } C_C: \tau_C = (R_C + R_L)C_C$$

$$f_C = \frac{1}{2\pi\tau_C} = \frac{1}{2\pi(R_C + R_L)C_C}$$

$$\text{For } C_E: \tau_E = \left(\frac{r_\pi + R_i}{1 + \beta} \right) C_E \Rightarrow f_E = \frac{1}{2\pi\tau_E}$$

$$(d) f_E = 10 = \frac{1}{2\pi\tau_E} \Rightarrow \tau_E = 0.015915 \text{ s}$$

$$0.015915 = \left(\frac{15.73 + 10}{121} \right) \times 10^3 \times C_E \Rightarrow C_E = 74.8 \mu\text{F}$$

$$f_C = 50 = \frac{1}{2\pi\tau_C} \Rightarrow \tau_C = 0.003183 \text{ s}$$

$$0.003183 = (7.65 + 20) \times 10^3 \times C_C \Rightarrow C_C = 0.115 \mu\text{F}$$

7.33

a.

$$I_D = K_n (V_{GS} - V_{TN})^2$$

$$V_{GS} = \sqrt{\frac{I_D}{K_n}} + V_{TN} = \sqrt{\frac{0.5}{0.5}} + 0.8 = 1.8 \text{ V}$$

$$R_s = \frac{-V_{GS} - (-5)}{0.5} = \frac{5 - 1.8}{0.5} \Rightarrow R_s = 6.4 \text{ k}\Omega$$

$$V_D = V_{DSQ} + V_S = 4 - 1.8 = 2.2 \text{ V}$$

$$R_D = \frac{5 - 2.2}{0.5} \Rightarrow R_D = 5.6 \text{ k}\Omega$$

(b)

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.5)} = 1 \text{ mA/V}$$

$$\tau_A = R_s C_s = (6.4 \times 10^3) (5 \times 10^{-6})$$

$$= 3.2 \times 10^{-2} \text{ s}$$

$$f_A = \frac{1}{2\pi\tau_A} = \frac{1}{2\pi(3.2 \times 10^{-2})} \Rightarrow f_A = 4.97 \text{ Hz}$$

$$\tau_B = \left(\frac{R_s}{1 + g_m R_s} \right) C_s = \left[\frac{6.4 \times 10^3}{1 + (1)(6.4)} \right] (5 \times 10^{-6})$$

$$= 4.32 \times 10^{-3} \text{ s}$$

$$f_B = \frac{1}{2\pi\tau_B} = \frac{1}{2\pi(4.32 \times 10^{-3})} \Rightarrow f_B = 36.8 \text{ Hz}$$

c.

$$|A_v| = \frac{g_m R_D (1 + s R_s C_s)}{(1 + g_m R_s) \left[1 + s \left(\frac{R_s}{1 + g_m R_s} \right) C_s \right]}$$

As R_s becomes large

$$|A_v| \rightarrow \frac{g_m R_D (s R_s C_s)}{(g_m R_s) \left[1 + s \left(\frac{R_s}{g_m R_s} \right) C_s \right]}$$

$$A_v = \frac{(g_m R_D) \left[s \left(\frac{1}{g_m} \right) C_s \right]}{1 + s \left(\frac{1}{g_m} \right) C_s}$$

The corner frequency $f_B = \frac{1}{2\pi(1/g_m)C_s}$ and the corresponding $f_A \rightarrow 0$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{(0.5)(0.5)} = 1 \text{ mA/V}$$

$$f_B = \frac{1}{2\pi \left(\frac{1}{10^{-3}} \right) (5 \times 10^{-6})} \Rightarrow f_B = 31.8 \text{ Hz}$$

7.34

$$(a) \text{ (i)} \quad T_1(s) = \frac{V_{o1}}{V_i} = -g_{m1} \left(r_o \parallel R_D \parallel \frac{1}{sC_L} \right) = -g_{m1} \left[\frac{\left(r_o \parallel R_D \right) \left(\frac{1}{sC_L} \right)}{\left(r_o \parallel R_D \right) + \left(\frac{1}{sC_L} \right)} \right]$$

$$T_1(s) = -g_{m1} \left(r_o \parallel R_D \right) \cdot \frac{1}{\left[1 + s(r_o \parallel R_D) C_L \right]}$$

$$\text{(ii)} \quad T_2(s) = \frac{V_o}{V_{o1}} = -g_{m2} \left(r_o \parallel R_D \right) \cdot \frac{1}{\left[1 + s(r_o \parallel R_D) C_L \right]}$$

$$\text{(iii)} \quad T(s) = \frac{V_o}{V_i} = g_{m1} g_{m2} \left(r_o \parallel R_D \right)^2 \cdot \frac{1}{\left[1 + s(r_o \parallel R_D) C_L \right]^2}$$

$$(b) \text{ (i)} \quad f_{3-dB} = \frac{1}{2\pi(r_o \parallel R_D) C_L}$$

$$\text{Now } r_o = \frac{1}{(0.02)(0.5)} = 100 \text{ k}\Omega, \quad r_o \parallel R_D = 100 \parallel 5 = 4.762 \text{ k}\Omega$$

$$f_{3-dB} = \frac{1}{2\pi(4.762 \times 10^3)(12 \times 10^{-12})} \Rightarrow f_{3-dB} = 2.785 \text{ MHz}$$

$$\text{(ii)} \quad f_{3-dB} = 2.785 \text{ MHz}$$

$$\text{(iii)} \quad \text{Want} \left\{ \frac{1}{\sqrt{1 + [(2\pi f)(r_o \parallel R_D) C_L]^2}} \right\}^2 = \frac{1}{\sqrt{2}}$$

$$\text{So} \quad \frac{1}{1 + \left(\frac{f}{2.785 \times 10^6} \right)^2} = \frac{1}{\sqrt{2}} = 0.7071$$

$$\left(\frac{f}{2.785 \times 10^6} \right)^2 = \frac{1}{0.7071} - 1 = 0.4142$$

$$\text{which yields } f = 1.792 \text{ MHz}$$

7.35

- a. Expression for the voltage gain is the same as Equation (7.59) with $R_s = 0$.
- b.

$$\tau_A = R_E C_E$$

$$\tau_B = \frac{R_E r_\pi C_E}{r_\pi + (1 + \beta) R_E}$$

7.36

$$(a) I_{EQ} = \left(\frac{91}{90} \right) (0.15) = 0.1517 \text{ mA}$$

$$R_E = \frac{3 - 0.7}{0.1517} = 15.16 \text{ k}\Omega$$

$$V_C = 0.7 - V_{ECQ} = 0.7 - 2.2 = -1.5 \text{ V}$$

$$R_C = \frac{-1.5 - (-3)}{0.15} = 10 \text{ k}\Omega$$

$$(b) g_m = \frac{0.15}{0.026} = 5.769 \text{ mA/V}$$

$$A_v = -g_m R_C = -(5.769)(10) = -57.7$$

$$(c) \tau_A = R_E C_E = \left(15.16 \times 10^3 \right) \left(3 \times 10^{-6} \right) = 4.548 \times 10^{-2} \text{ s}$$

$$f_A = \frac{1}{2\pi\tau_A} = \frac{1}{2\pi(4.548 \times 10^{-2})} = 3.5 \text{ Hz}$$

$$\tau_B = \frac{R_E r_\pi C_E}{r_\pi + (1 + \beta)R_E}, \text{ where } r_\pi = \frac{(90)(0.026)}{0.15} = 15.6 \text{ k}\Omega$$

$$\tau_B = \frac{\left(15.16 \times 10^3 \right) \left(15.6 \times 10^3 \right) \left(3 \times 10^{-6} \right)}{15.6 \times 10^3 + (91)(15.16 \times 10^3)} = 5.085 \times 10^{-4} \text{ s}$$

$$f_B = \frac{1}{2\pi\tau_B} = \frac{1}{2\pi(5.085 \times 10^{-4})} = 313 \text{ Hz}$$

7.37

$$(a) I_{EQ} = \frac{10 - 0.7}{10} = 0.93 \text{ mA}, I_{CQ} = \left(\frac{90}{91} \right) (0.93) = 0.9198 \text{ mA}$$

$$g_m = \frac{0.9198}{0.026} = 35.38 \text{ mA/V}$$

$$A_v = g_m (R_C \| R_L) = (35.38)(5 \| 10) = 118$$

$$(b) f = \frac{1}{2\pi\tau} = \frac{1}{2\pi(R_C \| R_L)C_L} = \frac{1}{2\pi(5 \| 10) \times 10^3 \times (3 \times 10^{-12})}$$

$$f = 15.9 \text{ MHz}$$

7.38

$$(a) I_{DQ} = K_p (V_{SGQ} + V_{TP})^2$$

$$0.2 = 0.1(V_{SGQ} - 0.6)^2 \Rightarrow V_{SGQ} = 2.014 \text{ V}$$

$$R_s = \frac{3 - 2.014}{0.2} = 4.93 \text{ k}\Omega$$

$$V_D = V_{SGQ} - V_{SDQ} = 2.014 - 1.9 = 0.114 \text{ V}$$

$$R_D = \frac{0.114 - (-3)}{0.2} = 15.6 \text{ k}\Omega$$

$$(b) f_H = \frac{1}{2\pi(R_D \| R_L)C_L}$$

or $C_L = \frac{1}{2\pi(15.6 \| 20) \times 10^3 \times 4 \times 10^6} \Rightarrow C_L = 4.54 \text{ pF}$

7.39

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD} = \left(\frac{166}{166 + 234} \right) (10) = 4.15 \text{ V}$$

$$I_D = \frac{V_G - V_{GS}}{R_S} = K_n (V_{GS} - V_{TN})^2$$

$$4.15 - V_{GS} = (0.5)(0.5)(V_{GS}^2 - 4V_{GS} + 4)$$

$$0.25V_{GS}^2 - 3.15 = 0 \Rightarrow V_{GS} = 3.55 \text{ V}$$

$$g_m = 2K_n (V_{GS} - V_{TN}) = 2(0.5)(3.55 - 2)$$

$$g_m = 1.55 \text{ mA/V}$$

$$R_0 = R_S \left\| \frac{1}{g_m} = 0.5 \left\| \frac{1}{1.55} = 0.5 \right\| 0.645 \right.$$

$$R_0 = 0.282 \text{ k}\Omega$$

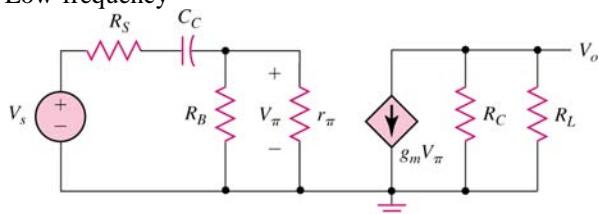
$$\tau = (R_o \| R_L) C_L \quad \text{and} \quad f_H = \frac{1}{2\pi\tau}$$

$$\text{BW} \cong f_H = 5 \text{ MHz} \Rightarrow \tau = \frac{1}{2\pi(5 \times 10^6)} = 3.18 \times 10^{-8} \text{ s}$$

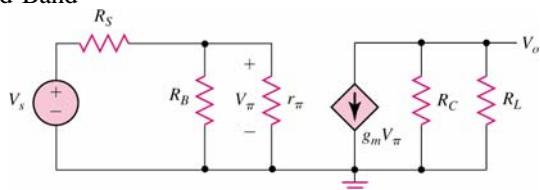
$$C_L = \frac{\tau}{R_o \| R_L} = \frac{3.18 \times 10^{-8}}{(0.282 \| 4) \times 10^3} \Rightarrow C_L = 121 \text{ pF}$$

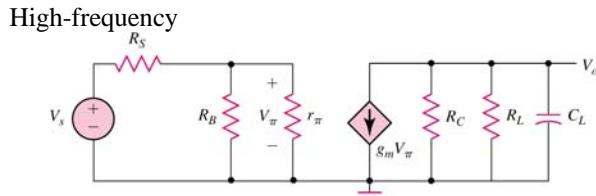
7.40

(a) Low-frequency

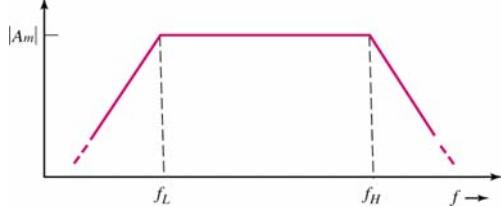


Mid-Band





(b)



(c)

$$I_{BQ} = \frac{12 - 0.7}{1 M\Omega} = 11.3 \mu A$$

$$I_{CQ} = 1.13 mA$$

$$r_\pi = \frac{(100)(0.026)}{1.13} = 2.3 k\Omega$$

$$g_m = \frac{1.13}{0.026} = 43.46 mA/V$$

$$\begin{aligned} A_m &= \frac{V_o}{V_s} (\text{midband}) = -g_m (R_C \| R_L) \left(\frac{R_B \| r_\pi}{R_B \| r_\pi + R_S} \right) \\ &= -(43.46)(5.1 \| 500) \left(\frac{1000 \| 2.3}{1000 \| 2.3 + 1} \right) \\ &= -(43.46)(5.05) \left(\frac{2.29}{2.29 + 1} \right) \Rightarrow |A_m| = 153 \end{aligned}$$

$$|A_m|_{dB} = 43.7 \text{ dB}$$

$$f_L = \frac{1}{2\pi\tau_L}, \quad \tau_L = (R_S + R_B \| r_\pi) C_C = (1 + 1000 \| 2.3) \times 10^3 \times (10 \times 10^{-6})$$

$$\Rightarrow \tau_L = 3.29 \times 10^{-2} \text{ s}, \quad f_L = 4.83 \text{ Hz}$$

$$f_H = \frac{1}{2\pi\tau_H}, \quad \tau_H = (R_C \| R_L) C_L = (5.1 \| 500) \times 10^3 \times (10 \times 10^{-12})$$

$$\Rightarrow \tau_H = 5.05 \times 10^{-8} \text{ s}, \quad f_H = 3.15 \text{ MHz}$$

7.41

$$(a) \quad A_v = -g_m \left(R_D \| R_L \left\| \frac{1}{sC_L} \right. \right) = -g_m \left[\frac{(R_D \| R_L) \cdot \frac{1}{sC_L}}{(R_D \| R_L) + \frac{1}{sC_L}} \right]$$

$$A_v = -g_m (R_D \| R_L) \left[\frac{1}{1 + s(R_D \| R_L) C_L} \right]$$

$$(b) \quad \tau = (R_D \| R_L) C_L$$

$$(c) \quad 5 = I_D R_S + V_{SG} = K_p R_S (V_{SG} + V_{TP})^2 + V_{SG}$$

$$5 = (0.25)(3.2)(V_{SG} - 2)^2 + V_{SG}$$

We find $0.8V_{SG}^2 - 2.2V_{SG} - 1.8 = 0 \Rightarrow V_{SG} = 3.41 \text{ V}$

$$I_{DQ} = (0.25)(3.41 - 2)^2 = 0.497 \text{ mA}$$

$$\tau = (10\parallel 20) \times 10^3 \times 10 \times 10^{-12} = 6.67 \times 10^{-8} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau} = \frac{1}{2\pi(6.67 \times 10^{-8})} \Rightarrow f_H = 2.39 \text{ MHz}$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.25)(0.497)} = 0.705 \text{ mA/V}$$

$$A_v = -g_m (R_D \parallel R_L) = -(0.705)(10\parallel 20) = -4.7$$

7.42 Computer Analysis

7.43 Computer Analysis

7.44 Computer Analysis

7.45

$$g_m = \frac{0.25}{0.026} = 9.615 \text{ mA/V}$$

$$f_\beta = \frac{f_T}{\beta_o} = \frac{4 \times 10^9}{120} \Rightarrow f_\beta = 33.3 \text{ MHz}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} \Rightarrow C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{9.615 \times 10^{-3}}{2\pi(4 \times 10^9)}$$

$$\text{or } C_\pi + C_\mu = 0.3826 \text{ pF}$$

$$\text{Then } C_\pi = 0.3826 - 0.08 = 0.303 \text{ pF}$$

7.46

$$(a) \quad f_\beta = \frac{f_T}{\beta_o} = \frac{2 \times 10^9}{120} \Rightarrow f_\beta = 16.67 \text{ MHz}$$

$$g_m = \frac{0.4}{0.026} = 15.38 \text{ mA/V}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{15.38 \times 10^{-3}}{2\pi(2 \times 10^9)}$$

$$C_\pi + C_\mu = 1.224 \text{ pF}, \quad C_\pi = 1.224 - 0.075 = 1.15 \text{ pF}$$

$$(b) \quad |h_{fe}| = \frac{\beta_o}{\sqrt{1 + \left(\frac{f}{f_\beta}\right)^2}}$$

$$(i) \text{At } f = 10 \text{ MHz, } |h_{fe}| = \frac{120}{\sqrt{1 + \left(\frac{10}{16.67}\right)^2}} = 103$$

$$(ii) \text{At } f = 20 \text{ MHz, } |h_{fe}| = \frac{120}{\sqrt{1 + \left(\frac{20}{16.67}\right)^2}} = 76.8$$

$$(iii) \text{At } f = 50 \text{ MHz, } |h_{fe}| = \frac{120}{\sqrt{1 + \left(\frac{50}{16.67}\right)^2}} = 38.0$$

7.47

$$(a) f_\beta = \frac{f_T}{\beta_o} = \frac{540 \times 10^6}{120} \Rightarrow f_\beta = 4.5 \text{ MHz}$$

$$g_m = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{7.692 \times 10^{-3}}{2\pi(540 \times 10^6)} \Rightarrow C_\pi + C_\mu = 2.267 \text{ pF}$$

$$C_\pi = 2.267 - 0.4 = 1.87 \text{ pF}$$

$$(b) g_m = \frac{0.8}{0.026} = 30.77 \text{ mA/V}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} = \frac{30.77 \times 10^{-3}}{2\pi(2.267 \times 10^{-12})} \Rightarrow f_T = 2.16 \text{ GHz}$$

$$f_\beta = \frac{2.16 \times 10^9}{120} \Rightarrow f_\beta = 18.0 \text{ MHz}$$

7.48

(a)

$$V_0 = -g_m V_\pi R_L \text{ where}$$

$$\begin{aligned} V_\pi &= \frac{\frac{r_\pi}{sC_1}}{\left\| \frac{1}{sC_1} + r_b \right\|} \cdot V_i = \frac{\frac{r_\pi}{1 + sr_\pi C_1}}{\frac{r_\pi}{1 + sr_\pi C_1} + r_b} \cdot V_i \\ &= \frac{r_\pi}{r_\pi + r_b + sr_b r_\pi C_1} \cdot V_i = \left(\frac{r_\pi}{r_\pi + r_b} \right) \left(\frac{1}{1 + s(r_b \| r_\pi) C_1} \right) \cdot V_i \end{aligned}$$

$$\text{So } A_v(s) = \frac{V_0(s)}{V_i(s)} = -g_m R_L \left(\frac{r_\pi}{r_\pi + r_b} \right) \left(\frac{1}{1 + s(r_b \| r_\pi) C_1} \right)$$

(b) Midband gain: $r_\pi = \frac{(100)(0.026)}{1} = 2.6 \text{ k}\Omega$, $g_m = \frac{1}{0.026} = 38.46 \text{ mA/V}$

For $r_b = 100 \Omega$

(i) $A_{v1} = -(38.46)(4)\left(\frac{2.6}{2.6+0.1}\right) \Rightarrow A_{v1} = -148.1$

For $r_b = 500 \Omega$

(ii) $A_{v2} = -(38.46)(4)\left(\frac{2.6}{2.6+0.5}\right) \Rightarrow A_{v2} = -129.0$

(c) $f_{3-dB} = \frac{1}{2\pi\tau}, \quad \tau = (r_b \| r_\pi) C_1$

For $r_b = 100 \Omega$

(i) $\tau_1 = (0.1 \| 2.6) \times 10^3 (2.2 \times 10^{-12}) = 2.12 \times 10^{-10} \text{ s} \Rightarrow f_{3-dB} = 751 \text{ MHz}$

For $r_b = 500 \Omega$

(ii) $\tau_2 = (0.5 \| 2.6) \times 10^3 (2.2 \times 10^{-12}) = 9.23 \times 10^{-10} \text{ s} \Rightarrow f_{3-dB} = 173 \text{ MHz}$

7.49

$f = 10 \text{ kHz} = 10^4$

$$Z_i = 200 + \frac{2500(1-j(10^4)(1.333 \times 10^{-6}))}{1+(10^4)^2 (1.333 \times 10^{-6})^2}$$

(b) $= 200 + 2500 - j33.3 = 2700 - j33.3$

$f = 100 \text{ kHz} = 10^5$

$$Z_i = 200 + \frac{2500(1-j(10^5)(1.333 \times 10^{-6}))}{1+(10^5)^2 (1.333 \times 10^{-6})^2}$$

(c) $Z_i = 200 + 2456 - j327 = 2656 - j327$

$f = 1 \text{ MHz} = 10^6$

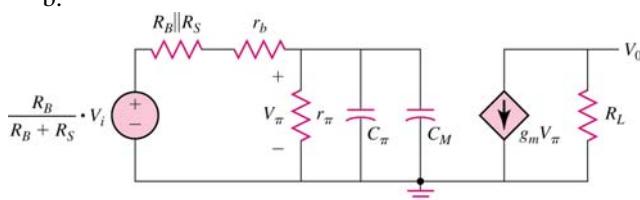
$$Z_i = 200 + \frac{2500(1-j(10^6)(1.333 \times 10^{-6}))}{1+(10^6)^2 (1.333 \times 10^{-6})^2}$$

(d) $Z_i = 200 + 900 - j1200 = 1100 - j1200$

7.50

a. $C_M = C_\mu (1 + g_m R_L)$

b.



$$V_0 = -g_m V_\pi R_L \quad \text{Let } C_\pi + C_M = C_i$$

$$V_\pi = \frac{r_\pi \left| \frac{1}{sC_i} \right|}{r_\pi \left| \frac{1}{sC_1} + R_B \| R_S + r_b \right|} \cdot \left(\frac{R_B}{R_B + R_S} \right) V_i$$

$$A_v(s) = \frac{V_o(s)}{V_i(s)}$$

$$\begin{aligned} &= -g_m R_L \left(\frac{R_B}{R_B + R_S} \right) \left[\frac{\frac{r_\pi \cdot \frac{1}{sC_i}}{r_\pi + \frac{1}{sC_i}}}{\frac{r_\pi \cdot \frac{1}{sC_i}}{r_\pi + \frac{1}{sC_i}} + R_B \| R_S + r_b} \right] \\ &= -g_m R_L \left(\frac{R_B}{R_B + R_S} \right) \cdot \left[\frac{r_\pi}{r_\pi + (1 + sr_\pi C_i)(R_B \| R_S + r_b)} \right] \end{aligned}$$

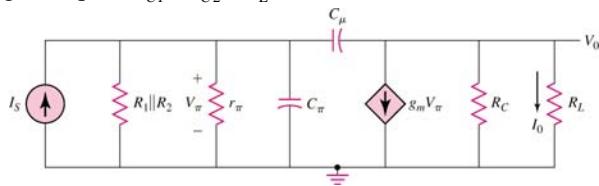
$$\text{Let } R_{eq} = (R_B \| R_S + r_b)$$

$$\begin{aligned} A_v(s) &= -\beta R_L \left(\frac{R_B}{R_B + R_S} \right) \times \left[\frac{1}{(r_\pi + R_{eq})(1 + s(r_\pi \| R_{eq})C_i)} \right] \\ A_v(s) &= \frac{-\beta R_L}{r_\pi + R_{eq}} \cdot \left(\frac{R_B}{R_B + R_S} \right) \cdot \frac{1}{1 + s(r_\pi \| R_{eq})C_i} \end{aligned}$$

$$\text{c. } f_H = \frac{1}{2\pi(r_\pi \| R_{eq})C_i}$$

7.51

High Freq. $\Rightarrow C_{C1}, C_{C2}, C_E \rightarrow$ short circuits



$$g_m = \frac{I_{CQ}}{V_T} = \frac{5}{0.026} = 192.3 \text{ mA/V}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} \Rightarrow 250 \times 10^6 = \frac{192 \times 10^{-3}}{2\pi(C_\pi + C_\mu)}$$

$$C_\pi + C_\mu = 122.4 \text{ pF} \Rightarrow C_\mu = 5 \text{ pF}, C_\pi = 117.4 \text{ pF}$$

$$\begin{aligned}
 C_M &= C_\mu \left(1 + g_m (R_C \| R_L) \right) \\
 &= 5 \left[1 + (192.3)(1\|1) \right] \Rightarrow C_M = 485.8 \text{ pF} \\
 C_i &= C_\pi + C_M = 117 + 485 = 603 \text{ pF} \\
 r_\pi &= \frac{(200)(0.026)}{5} = 1.04 \text{ k}\Omega \\
 R_{eq} &= R_1 \| R_2 \| r_\pi = 5 \| 1.04 = 0.861 \text{ k}\Omega \\
 \tau &= R_{eq} \cdot C_i = (0.861 \times 10^3) (603 \times 10^{-12}) \\
 &= 5.19 \times 10^{-7} \text{ s} \\
 f &= \frac{1}{2\pi\tau} = \frac{1}{2\pi(5.19 \times 10^{-7})} \Rightarrow f = 307 \text{ kHz}
 \end{aligned}$$

7.52

$$\begin{aligned}
 R_{TH} &= R_1 \| R_2 = 60 \| 5.5 = 5.04 \text{ k}\Omega \\
 V_{TH} &= \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{5.5}{5.5 + 60} \right) (15) = 1.26 \text{ V} \\
 I_{BQ} &= \frac{1.26 - 0.7}{5.04 + (101)(0.2)} = 0.0222 \text{ mA} \\
 I_{CQ} &= 2.22 \text{ mA} \\
 r_\pi &= \frac{(100)(0.026)}{2.22} = 1.17 \text{ k}\Omega \\
 g_m &= \frac{2.22}{0.026} = 85.4 \text{ mA/V}
 \end{aligned}$$

Lower 3 - dB frequency:

$$\begin{aligned}
 \tau_L &= R_{eq} \cdot C_{C1} \\
 R_{eq} &= R_S + R_1 \| R_2 \| r_\pi \\
 &= 2 + 60 \| 5.5 \| 1.17 = 2.95 \text{ k}\Omega \\
 \tau_L &= (2.95 \times 10^3) (0.1 \times 10^{-6}) = 2.95 \times 10^{-4} \text{ s} \\
 f_L &= \frac{1}{2\pi\tau_L} = \frac{1}{2\pi(2.95 \times 10^{-4})} \Rightarrow f_L = 540 \text{ Hz}
 \end{aligned}$$

Upper 3 - dB frequency:

$$\begin{aligned}
 f_T &= \frac{g_m}{2\pi(C_\pi + C_\mu)} \Rightarrow 400 \times 10^6 = \frac{85.4 \times 10^{-3}}{2\pi(C_\pi + C_\mu)} \\
 C_\pi + C_\mu &= 34 \text{ pF}; \quad C_\mu = 2 \text{ pF}; \quad C_\pi = 32 \text{ pF} \\
 C_M &= C_\mu (1 + g_m R_C) = 2 [1 + (85.4)(4)] \Rightarrow C_M = 685 \text{ pF} \\
 C_i &= C_\pi + C_M = 32 + 685 = 717 \text{ pF} \\
 R_{eq} &= R_S \| R_1 \| R_2 \| r_\pi = 2 \| 60 \| 5.5 \| 1.17 \Rightarrow R_{eq} = 0.644 \text{ k}\Omega \\
 \tau &= R_{eq} \cdot C_i = (0.644 \times 10^3) (717 \times 10^{-12}) \\
 &= 4.62 \times 10^{-7} \text{ s} \\
 f_H &= \frac{1}{2\pi\tau} \Rightarrow f_H = 344 \text{ kHz}
 \end{aligned}$$

7.53

$$R_{TH} = R_1 \parallel R_2 = 600 \parallel 55 = 50.38 \text{ K}$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) (15) = \left(\frac{55}{600 + 55} \right) (15) = 1.2595 \text{ V}$$

$$I_{BO} = \frac{1.26 - 0.7}{50.4 + (101)(2)} = 0.00222 \text{ mA}$$

$$I_{CQ} = 0.2217 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{0.222} = 11.73 \text{ K}$$

$$g_m = \frac{0.2217}{0.026} = 8.527 \text{ mA/V}$$

Lower – 3dB Freq

$$\begin{aligned} \tau_L &= R_{eq1} C_{cl}; R_{eq1} = R_S + R_{TH} \parallel r_\pi \\ &= 0.50 + 50.38 \parallel 11.73 = 10.0 \text{ K} \end{aligned}$$

$$\tau_L = (10 \times 10^3) (0.1 \times 10^{-6}) = 10^{-3} \text{ s}$$

$$f_L = \frac{1}{2\pi\tau_L} = \frac{1}{2\pi(10^{-3})} \Rightarrow f_L = 159 \text{ Hz}$$

Upper – 3dB Freq

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} = \frac{8.527 \times 10^{-3}}{2\pi(C_\pi + 2) \times 10^{-12}} = 400 \times 10^6$$

$$C_\pi + C_\mu = 3.393 \text{ pF} \Rightarrow C_\pi = 1.393 \text{ pF}$$

$$C_M = C_\mu (1 + g_m R_C) = 2 [1 + (8.527)(40)] = 684 \text{ pF}$$

$$C_T = C_\pi + C_M = 1.393 + 684 = 685.4 \text{ pF}$$

$$\begin{aligned} R_{eq2} &= R_S \parallel R_{TH} \parallel r_\pi = 0.5 \parallel 50.38 \parallel 11.73 \\ &= 50.38 \parallel 0.480 = 0.4750 \text{ K} \end{aligned}$$

$$\begin{aligned} \tau_H &= R_{eq2} \cdot C_T = (0.4750 \times 10^3) (685.4 \times 10^{-12}) \\ &= 3.256 \times 10^{-7} \text{ s} \end{aligned}$$

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi(3.256 \times 10^{-7})} \Rightarrow f_H = 489 \text{ KHz}$$

7.54

(a) $R_{TH} = R_1 \parallel R_2 = 33 \parallel 22 = 13.2 \text{ k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left(\frac{22}{22+33} \right)(5) = 2 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1+\beta)R_E} = \frac{2 - 0.7}{13.2 + (151)(4)} = 0.002106 \text{ mA}$$

$$I_{CQ} = 0.3159 \text{ mA}, \quad I_{EQ} = 0.3180 \text{ mA}$$

$$V_{CEQ} = 5 - (0.3159)(5) - (0.3180)(4) = 2.15 \text{ V}$$

(b) $f_\beta = \frac{f_T}{\beta_o} = \frac{800 \times 10^6}{150} \Rightarrow f_\beta = 5.33 \text{ MHz}$

$$g_m = \frac{0.3159}{0.026} = 12.15 \text{ mA/V}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{12.15 \times 10^{-3}}{2\pi(800 \times 10^6)} \Rightarrow C_\pi + C_\mu = 2.417 \text{ pF}$$

$$C_\pi = 2.417 - 0.45 = 1.97 \text{ pF}$$

$$C_M = C_\mu [1 + g_m R_C] = (0.45)[1 + (12.15)(5)] = 27.79 \text{ pF}$$

(c) $r_\pi = \frac{(150)(0.026)}{0.3159} = 12.35 \text{ k}\Omega, \quad R_{TH} \parallel r_\pi = 13.2 \parallel 12.35 = 6.38 \text{ k}\Omega$

$$f_{3-dB} = \frac{1}{2\pi(R_{TH} \parallel r_\pi)(C_\pi + C_M)} = \frac{1}{2\pi(6.38 \times 10^3)(1.97 + 27.79) \times 10^{-12}}$$

$$f_{3-dB} = 838 \text{ kHz}$$

7.55

$$g_{m1} = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)I_{DQ}} = 2\sqrt{\left(\frac{0.08}{2}\right)\left(\frac{4}{0.8}\right)(0.6)} = 0.6928 \text{ mA/V}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} = \frac{0.6928 \times 10^{-3}}{2\pi(50 + 10) \times 10^{-15}} \Rightarrow f_T = 1.84 \text{ GHz}$$

7.56

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2$$

$$0.12 = K_n(0.2)^2 \Rightarrow K_n = 3 \text{ mA/V}^2$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(3)(0.12)} = 1.2 \text{ mA/V}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} = \frac{1.2 \times 10^{-3}}{2\pi(40 + 10) \times 10^{-15}} \Rightarrow f_T = 3.82 \text{ GHz}$$

7.57

(a) $g_m = 2\sqrt{(1.5)(0.05)} = 0.5477 \text{ mA/V}$

$$f_T = \frac{0.5477 \times 10^{-3}}{2\pi(60 + 10) \times 10^{-15}} \Rightarrow f_T = 1.25 \text{ GHz}$$

$$(b) g_m = 2\sqrt{(1.5)(0.3)} = 1.342 \text{ mA/V}$$

$$f_T = \frac{1.342 \times 10^{-3}}{2\pi(60+10) \times 10^{-15}} \Rightarrow f_T = 3.05 \text{ GHz}$$

$$(c) 3 \times 10^9 = \frac{g_m}{2\pi(60+10) \times 10^{-15}} \Rightarrow g_m = 1.319 \text{ mA/V}$$

$$g_m = 2\sqrt{K_n I_{DQ}} \Rightarrow I_{DQ} = \frac{1}{K_n} \left(\frac{g_m}{2} \right)^2 = \frac{1}{1.5} \left(\frac{1.319}{2} \right)^2 = 0.29 \text{ mA}$$

$$(d) g_m = 2\sqrt{(1.5)(0.25)} = 1.225 \text{ mA/V}$$

$$2.5 \times 10^9 = \frac{1.225 \times 10^{-3}}{2\pi(C_{gs} + 8) \times 10^{-15}} \Rightarrow C_{gs} = 70 \text{ fF}$$

7.58

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

$$C_{gs} + C_{gd} = WLC_{ox}$$

$$g_m = 2K_n(V_{GS} - V_{TN}) = 2 \left(\frac{W}{L} \right) \left(\frac{\mu_n C_{ox}}{2} \right) (V_{GS} - V_{TN})$$

$$\text{Then } f_T = \frac{\left(\frac{W}{L} \right) (\mu_n C_{ox}) (V_{GS} - V_{TN})}{2\pi WLC_{ox}}$$

$$f_T = \frac{\mu_n (V_{GS} - V_{TN})}{2\pi L^2}$$

$$(a) f_T = \frac{450(0.5)}{2\pi(1.2 \times 10^{-4})^2} \Rightarrow f_T = 2.49 \text{ GHz}$$

$$(b) f_T = \frac{450(0.5)}{2\pi(0.18 \times 10^{-4})} \Rightarrow f_T = 111 \text{ GHz}$$

7.59

$$(a) C_M = C_{gd} [1 + g_m(r_o \| R_D)] = (12)[1 + (3)(120 \| 10)] = 344.3 \text{ fF}$$

$$(b) f_{3-dB} = \frac{1}{2\pi\tau}$$

$$\tau = r_i(C_{gs} + C_M) = (10^4)(80 + 344.3) \times 10^{-15} = 4.243 \times 10^{-9} \text{ s}$$

$$f_{3-dB} = \frac{1}{2\pi(4.243 \times 10^{-9})} \Rightarrow f_{3-dB} = 37.5 \text{ MHz}$$

7.60

$$f_T = \frac{g_m}{2\pi(C_{gst} + C_{gdT})} \quad (\text{Eq. (7.97)})$$

Let $C_{gdT} = 0$ and $C_{gst} = \left(\frac{2}{3}\right)(WLC_{ox})$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{\left(\frac{\mu_n C_{ox}}{2}\right) \left[\frac{W}{L}\right] I_D}$$

$$\text{So } f_T = \frac{2\sqrt{\left(\frac{1}{2}\mu_n C_{ox}\right) \left(\frac{W}{L}\right) I_D}}{2\pi \left(\frac{2}{3}\right) (WLC_{ox})}$$

$$= \frac{3}{2\pi L} \cdot \frac{\sqrt{\left(\frac{1}{2}\mu_n C_{ox}\right) \left(\frac{W}{L}\right) I_D}}{W C_{ox}}$$

$$\underline{f_T = \frac{3}{2\pi L} \cdot \sqrt{\frac{\mu_n I_D}{2W C_{ox} L}}}$$

7.61

$$(a) \quad K_n = \left(\frac{\mu_n C_{ox}}{2}\right) \left(\frac{W}{L}\right) = \frac{(400)(6.9 \times 10^{-9})}{2}(8) \Rightarrow K_n = 1.104 \text{ mA/V}^2$$

$$I_D = K_n (V_{GS} - V_{TN})^2 = (1.104)(3 - 0.4)^2 = 7.463 \text{ mA}$$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{(1.104)(7.463)} = 5.741 \text{ mA/V}$$

$$g'_m = \frac{g_m}{1 + g_m r_s}$$

$$(0.8)g_m = \frac{g_m}{1 + g_m r_s} \Rightarrow g_m r_s = \frac{1}{0.8} - 1 = 0.25$$

$$r_s = \frac{0.25}{5.741} \Rightarrow r_s = 43.5 \Omega$$

$$(b) \quad I_D = (1.104)(1 - 0.4)^2 = 0.3974 \text{ mA}$$

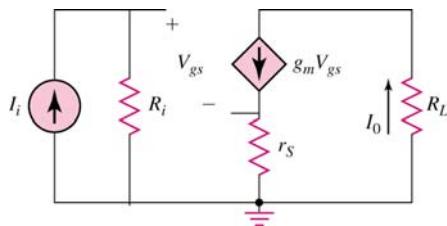
$$g_m = 2\sqrt{(1.104)(0.3974)} = 1.325 \text{ mA/V}$$

$$g'_m = \frac{1.325}{1 + (1.325)(0.04355)} = 1.253 \text{ mA/V}$$

$$\frac{g'_m}{g_m} \Rightarrow 94.5\%$$

7.62

a.

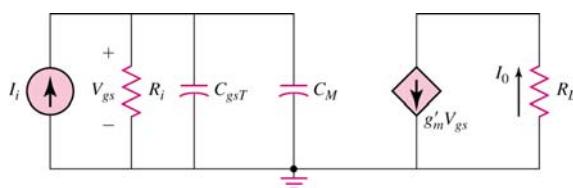


$$I_0 = g_m V_{gs} \text{ and } V_{gs} = I_i R_i - g_m V_{gs} r_s \text{ so } V_{gs} = \frac{I_i R_i}{1 + g_m r_s}$$

$$A_i = \frac{I_0}{I_i} = \frac{g_m R_i}{1 + g_m r_s}$$

Then

b. As an approximation, consider



In this case

$$A_i = \frac{I_0}{I_i} = g'_m R_i \cdot \frac{1}{1 + s R_i (C_{gsT} + C_M)} \text{ where } C_M = C_{gdT} (1 + g'_m R_L) \text{ and } g'_m = \frac{g_m}{1 + g_m r_s}$$

c. As r_s increases, C_M decreases, so the bandwidth increases, but the current gain magnitude decreases.

7.63

$$(b) V_{GS} = \left(\frac{225}{225 + 500} \right) (10) = 3.103 \text{ V}$$

$$I_{DQ} = (1)(3.103 - 2)^2 = 1.218 \text{ mA}$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(1)(1.218)} = 2.207 \text{ mA/V}$$

$$C_M = C_{gd} (1 + g_m R_D) = (8)[1 + (2.207)(5)] = 96.28 \text{ fF}$$

$$(c) f_{3-dB} = \frac{1}{2\pi\tau}, \quad \tau = (R_i \| R_1 \| R_2) C_{gs} + C_M$$

$$\text{Now } R_i \| R_1 \| R_2 = 1 \| 500 \| 225 = 0.9936 \text{ k}\Omega$$

$$\tau = (0.9936 \times 10^3) (50 + 96.28) \times 10^{-15} = 1.453 \times 10^{-10} \text{ s}$$

$$f_{3-dB} = \frac{1}{2\pi(1.453 \times 10^{-10})} \Rightarrow f_{3-dB} = 1.095 \text{ GHz}$$

$$A_v = -g_m R_D \left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_i} \right) = -(2.207)(5) \left(\frac{155.2}{155.2 + 1} \right) = -10.96$$

7.64

$$(a) C_M = C_{gd} (1 + |A_v|) = (0.04)(1+15) = 0.64 \text{ pF}$$

$$(b) f_H = \frac{1}{2\pi\tau}, \Rightarrow \tau = \frac{1}{2\pi f} = \frac{1}{2\pi(5 \times 10^6)} = 3.183 \times 10^{-8} \text{ s}$$

$$\tau = R_{eq} (C_{gs} + C_M)$$

$$\text{or } R_{eq} = \frac{\tau}{(C_{gs} + C_M)} = \frac{3.183 \times 10^{-8}}{(0.2 + 0.64) \times 10^{-12}} \Rightarrow R_{eq} = 37.9 \text{ k}\Omega$$

7.65

$$R_{TH} = R_1 \parallel R_2 = 33 \parallel 22 = 13.2 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right)(5) = \left(\frac{22}{22+33} \right)(5) = 2 \text{ V}$$

$$I_{BQ} = \frac{2 - 0.7}{13.2 + (121)(4)} = 0.00261 \text{ mA}$$

$$I_{CO} = 0.3138$$

$$r_\pi = \frac{(120)(0.026)}{0.3138} = 9.94 \text{ k}\Omega$$

$$g_m = \frac{0.3138}{0.026} = 12.07 \text{ mA/V}$$

$$r_0 = \frac{100}{0.3138} = 318 \text{ k}\Omega$$

a.

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{12.07 \times 10^{-3}}{2\pi(600 \times 10^6)}$$

$$C_\pi + C_\mu = 3.20 \text{ pF}; C_\mu = 1 \text{ pF} \Rightarrow C_\pi = 2.20 \text{ pF}$$

$$C_M = C_\mu \left[1 + g_m \left(r_o \parallel R_C \parallel R_L \right) \right]$$

$$= (1) \left[1 + (12.07)(318 \parallel 4 \parallel 5) \right]$$

$$C_M = 27.6 \text{ pF}$$

b.

$$\begin{aligned}
 \tau &= R_{eq} (C_\pi + C_M) \\
 R_{eq} &= R_1 \| R_2 \| R_S \| r_\pi = 33 \| 22 \| 2 \| r_\pi \\
 &= 1.74 \| 9.94 \Rightarrow R_{eq} = 1.48 \text{ k}\Omega \\
 \tau &= (1.48 \times 10^3) (2.20 + 27.6) \times 10^{-12} \\
 \tau &= 4.41 \times 10^{-8} \text{ s} \\
 f_H &= \frac{1}{2\pi\tau} = \frac{1}{2\pi(4.41 \times 10^{-8})} \Rightarrow f_H = 3.61 \text{ MHz} \\
 V_o &= -g_m V_\pi (r_o \| R_C \| R_L) \\
 V_\pi &= \frac{R_1 \| R_2 \| r_\pi}{R_1 \| R_2 \| r_\pi + R_S} \cdot V_i \\
 R_1 \| R_2 \| r_\pi &= 33 \| 22 \| 9.94 = 5.67 \text{ k}\Omega \\
 V_\pi &= \frac{5.67}{5.67 + 2} \cdot V_i = (0.739)V_i \\
 r_o \| R_C \| R_L &= 318 \| 4 \| 5 = 2.18 \text{ k}\Omega \\
 A_v &= -(12.07)(0.739)(2.18) \\
 A_v &= -19.7
 \end{aligned}$$

7.66

$$\begin{aligned}
 R_{TH} &= R_1 \| R_2 = 40 \| 5 = 4.44 \text{ k}\Omega \\
 V_{TH} &= \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{5}{5+40} \right) (10) = 1.111 \text{ V} \\
 I_{BQ} &= \frac{1.111 - 0.7}{4.44 + (121)(0.5)} = 0.00633 \text{ mA} \\
 I_{CQ} &= 0.760 \text{ mA} \\
 r_\pi &= \frac{(120)(0.026)}{0.760} = 4.11 \text{ k}\Omega \\
 g_m &= \frac{0.760}{0.026} = 29.23 \text{ mA/V} \\
 r_0 &= \infty \\
 f_T &= \frac{g_m}{2\pi(C_\pi + C_\mu)} \\
 C_\pi + C_\mu &= \frac{g_m}{2\pi f_T} = \frac{29.23 \times 10^{-3}}{2\pi(250 \times 10^6)} \\
 C_\pi + C_\mu &= 18.6 \text{ pF}; C_\mu = 3 \text{ pF} \Rightarrow C_\pi = 15.6 \text{ pF}
 \end{aligned}$$

a.

$$\begin{aligned}
 C_M &= C_\mu [1 + g_m (R_C \| R_L)] \\
 C_M &= 3 [1 + (29.2)(5 \| 2.5)] \Rightarrow C_M = 149 \text{ pF}
 \end{aligned}$$

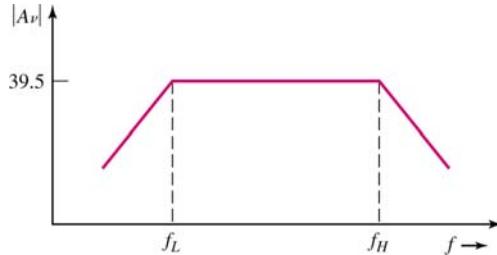
For upper frequency:

$$\begin{aligned}\tau_H &= R_{eq} (C_\pi + C_M) \\ R_{eq} &= r_\pi \| R_1 \| R_2 \| R_S = 4.11 \| 40 \| 5 \| 0.5 \\ R_{eq} &= 0.405 \text{ k}\Omega \\ \tau_H &= (0.405 \times 10^3) (15.6 + 149) \times 10^{-12} \\ &= 6.67 \times 10^{-8} \text{ s} \\ f_H &= \frac{1}{2\pi\tau_H} \Rightarrow f_H = 2.39 \text{ MHz}\end{aligned}$$

For lower frequency:

$$\begin{aligned}\tau_L &= R_{eq} C_{C1} \\ R_{eq} &= R_S + R_1 \| R_2 \| r_\pi = 0.5 + 40 \| 5 \| 4.11 \\ R_{eq} &= 2.64 \text{ k}\Omega \\ \tau_L &= (2.64 \times 10^3) (4.7 \times 10^{-6}) = 1.24 \times 10^{-2} \text{ s} \\ f_L &= \frac{1}{2\pi\tau_L} \Rightarrow f_L = 12.8 \text{ Hz}\end{aligned}$$

b.



$$\begin{aligned}V_0 &= -g_m V_\pi (R_C \| R_L) \\ V_\pi &= \left(\frac{R_1 \| R_2 \| r_\pi}{R_1 \| R_2 \| r_\pi + R_S} \right) V_i \\ V_\pi &= \left(\frac{2.135}{2.135 + 0.5} \right) V_i = 0.8102 V_i \\ |A_v| &= (29.23)(0.8102)(5 \| 2.5) \\ |A_v| &= 39.5\end{aligned}$$

7.67

$$I_D = K_P (V_{SG} + V_{TP})^2 = \frac{9 - V_{SG}}{R_s}$$

$$(2)(1.2)(V_{SG}^2 - 4V_{SG} + 4) = 9 - V_{SG}$$

$$2.4V_{SG}^2 - 8.6V_{SG} + 0.6 = 0$$

$$V_{SG} = \frac{8.6 \pm \sqrt{(8.6)^2 - 4(2.4)(0.6)}}{2(2.4)}$$

$$V_{SG} = 3.512 \text{ V}$$

$$g_m = 2K_P (V_{SG} + V_{TP}) = 2(2)(3.512 - 2)$$

$$g_m = 6.049 \text{ mA/V}$$

$$I_D = (2)(3.512 - 2)^2 = 4.572 \text{ mA}$$

$$r_o = \frac{1}{\lambda I_o} = \frac{1}{(0.01)(4.56)} \Rightarrow r_o = 21.9 \text{ k}\Omega$$

$$C_M = C_{gdT} (1 + g_m (r_o \| R_D))$$

$$\text{a. } C_M = (1)[1 + (6.04)(21.9 \| 1)] \Rightarrow C_M = 6.785 \text{ pF}$$

$$\text{b. } \tau_H = (R_i \| R_G) (C_{gsT} + C_M)$$

$$\tau_H = (2 \| 100) \times 10^3 \times (10 + 6.78) \times 10^{-12}$$

$$\tau_H = 3.29 \times 10^{-8} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau_H} \Rightarrow f_H = 4.84 \text{ MHz}$$

$$V_o = -g_m (r_o \| R_D) \cdot V_{gs}$$

$$V_{gs} = \left(\frac{R_G}{R_G + R_i} \right) \cdot V_i = \left(\frac{100}{102} \right) \cdot V_i$$

$$A_v = -(6.04) \left(\frac{100}{102} \right) (21.9 \| 1)$$

$$A_v = -5.67$$

7.68

$$\text{(a) } I_{DQ} = K_p (V_{SGQ} + V_{TP})^2$$

$$0.5 = 0.5(V_{SGQ} - 0.5)^2 \Rightarrow V_{SGQ} = 1.5 \text{ V}$$

$$R_s = \frac{3 - 1.5}{0.5} = 3 \text{ k}\Omega$$

$$V_D = 1.5 - 2 = -0.5 \text{ V}$$

$$R_D = \frac{-0.5 - (-3)}{0.5} = 5 \text{ k}\Omega$$

$$\text{(b) } g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.5)(0.5)} = 1 \text{ mA/V}$$

$$A_v = -g_m R_D \left(\frac{R_G}{R_G + R_i} \right) = -(1)(5) \left(\frac{200}{200 + 4} \right) = -4.90$$

$$(c) \quad C_M = C_{gd} (1 + g_m R_D) = (0.08)[1 + (1)(5)] = 0.48 \text{ pF}$$

$$(d) \quad f_{3-dB} = \frac{1}{2\pi\tau}$$

$$\text{where } \tau = R_{eq} \cdot C_{eq} = (R_i \| R_G) (C_{gs} + C_M) = (4 \| 200) \times 10^3 \times (0.8 + 0.48) \times 10^{-12}$$

$$\text{which yields } \tau = 5.02 \times 10^{-9} \text{ s}$$

$$\text{Then } f_{3-dB} = \frac{1}{2\pi(5.02 \times 10^{-9})} \Rightarrow f_{3-dB} = 31.7 \text{ MHz}$$

7.69

$$V_G = \left(\frac{R_2}{R_1 + R_2} \right) (20) - 10 = \left(\frac{22}{22+8} \right) (20) - 10$$

$$V_G = 4.67 \text{ V}$$

$$I_D = \frac{10 - V_{SG} - 4.67}{R_S} = K_p (V_{SG} + V_{TP})^2$$

$$5.33 - V_{SG} = (1)(0.5)(V_{SG}^2 - 4V_{SG} + 4)$$

$$0.5V_{SG}^2 - V_{SG} - 3.33 = 0$$

$$V_{SG} = \frac{1 \pm \sqrt{1 + 4(0.5)(3.33)}}{2(0.5)} \Rightarrow V_{SG} = 3.77 \text{ V}$$

$$g_m = 2K_p (V_{SG} + V_{TP}) = 2(1)(3.77 - 2)$$

$$g_m = 3.54 \text{ mA/V}$$

b.

$$C_M = C_{gdT} (1 + g_m (R_D \| R_L))$$

$$C_M = (3)[1 + (3.54)(2 \| 5)] \Rightarrow C_M = 18.2 \text{ pF}$$

a.

$$\tau = R_{eq} (C_{gsT} + C_M)$$

$$R_{eq} = R_i \| R_1 \| R_2 = 0.5 \| 8 \| 22 = 0.461 \text{ k}\Omega$$

$$\tau = (0.461 \times 10^3)(15 + 18.2) \times 10^{-12}$$

$$= 1.53 \times 10^{-8} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau} \Rightarrow f_H = 10.4 \text{ MHz}$$

c.

$$V_o = -g_m V_{gs} (R_D \| R_L)$$

$$V_{gs} = \left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_i} \right) \cdot V_i = \left(\frac{5.87}{5.87 + 0.5} \right) \cdot V_i = (0.9215) V_i$$

$$A_v = -(3.54)(0.9215)(2 \| 5) \Rightarrow A_v = -4.66$$

7.70

$$I_E = 0.5 \text{ mA} \Rightarrow I_{CQ} = \left(\frac{100}{101} \right) (0.5) = 0.495 \text{ mA}$$

$$g_m = \frac{0.495}{0.026} = 19.0 \text{ mA/V}$$

$$r_\pi = \frac{(100)(0.026)}{0.495} = 5.25 \text{ k}\Omega$$

a. Input: From Eq. (7.114(b))

$$\begin{aligned} \tau_{P\pi} &= \left[\frac{r_\pi}{1 + \beta} \| R_E \| R_S \right] C_\pi \\ &= \left[\frac{5.25}{101} \| 0.5 \| 0.05 \right] \times 10^3 \times (10 \times 10^{-12}) \\ &= 2.43 \times 10^{-10} \text{ s} \end{aligned}$$

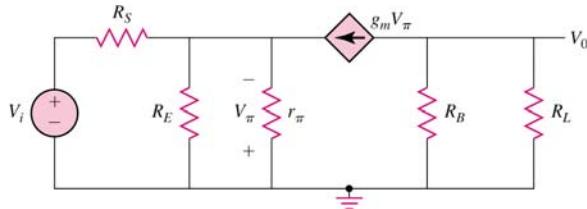
$$f_{H\pi} = \frac{1}{2\pi\tau_{P\pi}} \Rightarrow f_{H\pi} = 656 \text{ MHz}$$

Output: From Eq. (7.115(b))

$$\begin{aligned} \tau_{P\mu} &= (R_B \| R_L) C_\mu = (100 \| 1) \times 10^3 \times (10^{-12}) \\ &= 9.90 \times 10^{-10} \text{ s} \end{aligned}$$

$$f_{H\mu} = \frac{1}{2\pi\tau_{P\mu}} \Rightarrow f_{H\mu} = 161 \text{ MHz}$$

b.



$$V_o = -g_m V_\pi (R_B \| R_L)$$

$$g_m V_\pi + \frac{V_\pi}{r_\pi} + \frac{V_\pi}{R_E} + \frac{V_i - (-V_\pi)}{R_S} = 0$$

$$V_\pi \left[g_m + \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{R_S} \right] = -\frac{V_i}{R_S}$$

$$V_\pi \left[19 + \frac{1}{5.25} + \frac{1}{0.5} + \frac{1}{0.05} \right] = \frac{-V_i}{0.05}$$

$$V_\pi (41.19) = -V_i (20)$$

$$V_\pi = -(0.4856)V_i$$

$$\frac{V_o}{V_i} = -(19)(-0.4856)(100 \| 1)$$

$$A_v = 9.14$$

c.

$$\tau = (R_B \| R_L) C_L = (100 \| 1) \times 10^3 \times (15 \times 10^{-12})$$

$$\tau = 1.485 \times 10^{-8} \text{ s}$$

$$f = \frac{1}{2\pi\tau} \Rightarrow f = 10.7 \text{ MHz}$$

Since $f < f_{H_\mu} \Rightarrow$ 3-dB frequency dominated by C_L

7.71

$$I_{EQ} = \frac{20 - 0.7}{10} = 1.93 \text{ mA}$$

$$I_{CQ} = \left(\frac{100}{101} \right) (1.93) = 1.91 \text{ mA}$$

$$g_m = \frac{1.91}{0.026} = 73.5 \text{ mA/V}$$

$$r_\pi = \frac{(100)(0.026)}{1.91} = 1.36 \text{ k}\Omega$$

a. Input:

$$\begin{aligned} \tau_{P\pi} &= \left[\frac{r_\pi}{1 + \beta} \| R_E \| R_S \right] C_\pi \\ &= \left[\frac{1.36}{101} \| 10 \| 1 \right] \times 10^3 \times (10 \times 10^{-12}) \end{aligned}$$

$$\tau_{P\pi} = 1.327 \times 10^{-10} \text{ s}$$

$$f_{P\pi} = \frac{1}{2\pi\tau_{P\pi}} \Rightarrow f_{P\pi} = 1.20 \text{ GHz}$$

Output:

$$\tau_{P\mu} = (R_C \| R_L) C_\mu = (6.5 \| 5) \times 10^3 \times (10^{-12})$$

$$\tau_{P\mu} = 2.826 \times 10^{-9} \text{ s}$$

$$f_{P\mu} = \frac{1}{2\pi\tau_{P\mu}} \Rightarrow f_{P\mu} = 56.3 \text{ MHz}$$

b.

$$V_o = -g_m V_\pi (R_C \| R_L)$$

$$g_m V_\pi + \frac{V_\pi}{r_\pi} + \frac{V_\pi}{R_E} + \frac{V_i - (-V_\pi)}{R_S} = 0$$

$$V_\pi \left(g_m + \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{R_S} \right) = -\frac{V_i}{R_S}$$

$$V_\pi \left(73.5 + \frac{1}{1.36} + \frac{1}{10} + \frac{1}{1} \right) = -\frac{V_i}{1}$$

$$V_\pi (75.34) = -V_i \Rightarrow V_\pi = -(0.01327)V_i$$

$$V_o = -(73.5)(-0.01327)(6.5 \| 5)V_i$$

$$A_v = 2.76$$

c.

$$\tau = (R_C \| R_L) C_L = (6.5 \| 5) \times 10^3 \times (15 \times 10^{-12})$$

$$\tau = 4.24 \times 10^{-8} \text{ s}$$

$$f = \frac{1}{2\pi\tau} \Rightarrow f = 3.75 \text{ MHz}$$

Since $f < f_{P_\mu}$, 3-dB frequency is dominated by C_L

7.72

$$V_{GS} + I_D R_S = 5$$

$$I_D = \frac{5 - V_{GS}}{R_S} = K_n (V_{GS} - V_{TN})^2$$

$$5 - V_{GS} = (3)(10)(V_{GS}^2 - 2V_{GS} + 1)$$

$$30V_{GS}^2 - 59V_{GS} + 25 = 0$$

$$V_{GS} = \frac{59 \pm \sqrt{(59)^2 - 4(30)(25)}}{2(30)} \Rightarrow V_{GS} = 1.349 \text{ V}$$

$$g_m = 2K_n (V_{GS} - V_{TN}) = 2(3)(1.35 - 1)$$

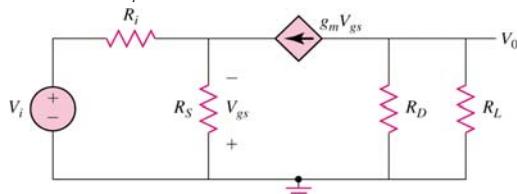
$$g_m = 2.093 \text{ mA/V}$$

On the output:

$$\tau_{P_\mu} = (R_D \| R_L) C_{gdT} = (5 \| 4) \times 10^3 \times (4 \times 10^{-12})$$

$$\tau_{P_\mu} = 8.89 \times 10^{-9} \text{ s}$$

$$f_{P_\mu} = \frac{1}{2\pi\tau_{P_\mu}} \Rightarrow f_{P_\mu} = 17.9 \text{ MHz}$$



$$V_0 = -g_m V_{gs} (R_D \| R_L)$$

$$g_m V_{gs} + \frac{V_{gs}}{R_S} + \frac{V_i - (-V_{gs})}{R_i} = 0$$

$$V_{gs} \left(g_m + \frac{1}{R_S} + \frac{1}{R_i} \right) = -\frac{V_i}{R_i}$$

$$V_{gs} \left(2.093 + \frac{1}{10} + \frac{1}{2} \right) = -\frac{V_i}{2}$$

$$V_{gs} = (0.1857)V_i$$

$$A_v = \frac{V_0}{V_i} = (2.093)(0.1857)(5 \| 4)$$

$$A_v = 0.864$$

7.73

dc analysis

$$I_D = \frac{V^+ - V_{SG}}{R_S} = K_P (V_{SG} + V_{TP})^2$$

$$5 - V_{SG} = (1)(4)(V_{SG} - 0.8)^2$$

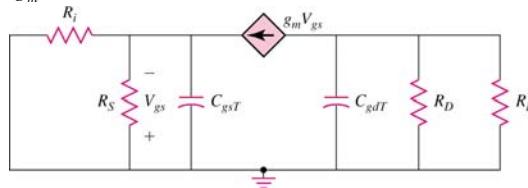
$$= 4(V_{SG}^2 - 1.6V_{SG} + 0.64)$$

$$4V_{SG}^2 - 5.4V_{SG} - 2.44 = 0$$

$$V_{SG} = \frac{5.4 \pm \sqrt{(5.4)^2 + 4(4)(2.44)}}{2(4)} = 1.707$$

$$g_m = 2K_P (V_{SG} + V_{TP}) = 2(1)(1.707 - 0.8)$$

$$g_m = 1.81 \text{ mA/V}$$



$$3 \cdot \text{dB frequency due to } C_{gsT} : R_{eq} = \frac{1}{g_m} \| R_s \| R_i$$

$$f_A = \frac{1}{2\pi R_{eq} \cdot C_{gsT}}$$

$$R_{eq} = \frac{1}{1.81} \| 4 \| 0.5 = 0.246 \text{ k}\Omega$$

$$f_A = \frac{1}{2\pi(246)(4 \times 10^{-12})} = 162 \text{ MHz}$$

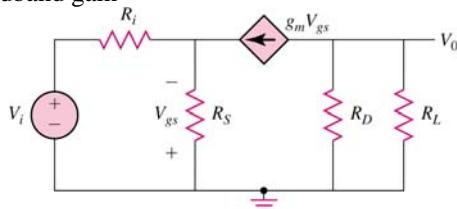
3-dB frequency due to C_{gdT}

$$f_B = \frac{1}{2\pi(R_D \| R_L) C_{gdT}}$$

$$= \frac{1}{2\pi(2 \| 4) \times 10^3 \times 10^{-12}}$$

$$\underline{f = 119 \text{ MHz}}$$

Midband gain



$$V_{gs} = \frac{-\frac{1}{g_m} \| R_s }{\frac{1}{g_m} \| R_s + R_i} \cdot V_i = \frac{-\frac{1}{1.81} \| 4 }{\frac{1}{1.81} \| 4 + 0.5} \cdot V_i$$

$$= -0.492 V_i$$

$$V_0 = -g_m V_{gs} (R_D \| R_L)$$

$$A_v = (0.492)(1.81)(4 \| 2) \Rightarrow \underline{A_v = 1.19}$$

7.74

$$r_\pi = \frac{(120)(0.026)}{1.02} = 3.059 \text{ k}\Omega$$

$$g_m = 39.23 \text{ mA/V}$$

a.

$$\text{Input: } f_{H\pi} = \frac{1}{2\pi\tau_\pi}$$

$$\tau_\pi = (R_S \| R_2 \| R_3 \| r_\pi) C_\pi + 2C_\mu$$

$$R_{eq} = 0.1 \| 20.5 \| 28.3 \| 3.06 = 0.096 \text{ k}\Omega$$

$$\tau_\pi = (96)[12 + 2(2)] \times 10^{-12} = 1.537 \times 10^{-9} \text{ s}$$

$$f_{H\pi} = \frac{1}{2\pi(1.537 \times 10^{-9})} \Rightarrow f_{H\pi} = 103.6 \text{ MHz}$$

$$\text{Output: } f_{H\mu} = \frac{1}{2\pi\tau_\mu}$$

$$\tau_\mu = (R_C \| R_L) C_\mu$$

$$= (15 \| 10) \times 10^3 \times (2 \times 10^{-12})$$

$$\tau_\mu = 6.67 \times 10^{-9} \text{ s}$$

$$f_{H\mu} = \frac{1}{2\pi(6.67 \times 10^{-9})} \Rightarrow f_{H\mu} = 23.9 \text{ MHz}$$

b.

$$A_v = g_m (R_C \| R_L) \left(\frac{R_2 \| R_3 \| r_\pi}{R_2 \| R_3 \| r_\pi + R_S} \right)$$

$$R_2 \| R_3 \| r_\pi = 20.5 \| 28.3 \| 3.059 = 2.433 \text{ k}\Omega$$

$$A_v = (39.23)(5 \| 10) \left(\frac{2.433}{2.433 + 0.1} \right) \Rightarrow A_v = 125.6$$

c. $C_L = 15 \text{ pF} > C_\mu \Rightarrow C_L$ dominates frequency response.

Chapter 8

8.1

- (b) (i) $R_D = \frac{24}{4} = 6\Omega$
 $P_{D,\max} = (12)(2) = 24\text{ W}$
- (ii) $P_{D,\max} = 30 = (20)I_{DQ} \Rightarrow I_{DQ} = 1.5\text{ A}$
 $I_{D,\max} = 2(1.5) = 3\text{ A}$
 $R_D = \frac{40}{3} = 13.3\Omega$
- (c) (i) $I_{D,\max} = 4\text{ A}$
(ii) $I_{D,\max} = 3\text{ A}$
-

8.2

- (a) $P_{Q,\max} = V_{CEQ} \cdot I_{CQ}$
 $25 = \left(\frac{24}{2}\right) \cdot I_{CQ} \Rightarrow I_{CQ} = 2.083\text{ A}$
 $R_L = \frac{24 - 12}{2.083} = 5.76\Omega$
 $I_{BQ} = \frac{2.083}{60} = 0.03472\text{ A}$
 $R_B = \frac{24 - 0.7}{0.03472} = 671\Omega$
- (b) $r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(60)(0.026)}{2.083} = 0.7489\Omega$
 $I_b = \frac{V_p}{r_\pi} = \frac{12\text{ mV}}{0.7489} = 16.02\text{ mA}$
 $I_c = \beta I_b = (60)(0.01602) = 0.9614\text{ A}$
 $\overline{P}_{avg} = \frac{1}{2} I_c^2 R_C = \frac{1}{2} (0.9614)^2 (5.76) = 2.66\text{ W}$

For the transistor,

$$P_Q = 25 - 2.66 = 22.34\text{ W}$$

8.3

- (a) $R_L = \frac{30}{1.2} = 25\Omega$
 $I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.6}{80} = 0.0075\text{ A}$
 $R_B = \frac{V_{CC} - V_{BE}(on)}{I_{BQ}} = \frac{30 - 0.7}{0.0075} \Rightarrow R_B = 3.91\text{k}\Omega$
 $P_{Q,\max} = I_{CQ} V_{CEQ} = (0.6)(15) = 9\text{ W}$

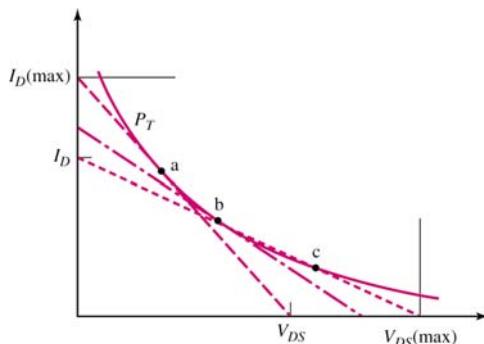
$$(b) \quad P_{Q,\max} = I_{CQ} V_{CEQ} = \left(\frac{V_{CC} - V_{CEQ}}{R_L} \right) \cdot V_{CEQ}$$

$$V_{CEQ} = \frac{1}{2} V_{CC}$$

$$5 = \frac{1}{25} \left(\frac{V_{CC}}{2} \right) \left(\frac{V_{CC}}{2} \right) \Rightarrow V_{CC} = 22.36 \text{ V}; \quad I_{C,\max} = \frac{22.36}{25} = 0.8944 \text{ A}$$

(c) For (a): $\Delta I_C = 0.6 \text{ A}; \quad \bar{P}_L = \frac{1}{2} (\Delta I_C)^2 R_L = \frac{1}{2} (0.6)^2 (25) = 4.5 \text{ W}$
 For (b): $\Delta I_C = I_{CQ} = \frac{0.8944}{2} = 0.4472 \text{ A}; \quad \bar{P}_L = \frac{1}{2} (0.4472)^2 (25) = 2.5 \text{ W}$

8.4



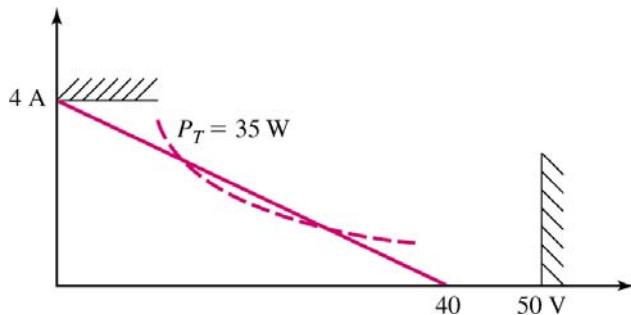
Point (b): Maximum power delivered to load.

Point (a): Will obtain maximum signal current output.

Point (c): Will obtain maximum signal voltage output.

8.5

a.



b.

$$V_{GG} = 5 \text{ V}, I_D = 0.25(5-4)^2 = 0.25 \text{ A}, V_{DS} = 37.5 \text{ V}, P = 9.375 \text{ W}$$

$$V_{GG} = 6 \text{ V}, I_D = 0.25(6-4)^2 = 1.0 \text{ A}, V_{DS} = 30 \text{ V}, P = 30 \text{ W}$$

$$V_{GG} = 7 \text{ V}, I_D = 0.25(7-4)^2 = 2.25 \text{ A}, V_{DS} = 17.5 \text{ V}, P = 39.375 \text{ W}$$

$$V_{GG} = 8 \text{ V}, I_D = 0.25[2(8-4)V_{DS} - V_{DS}^2]$$

$$= \frac{40 - V_{DS}}{10} \Rightarrow V_{DS} = 2.92$$

$$I_D = 3.71 \text{ A}, P = 10.8 \text{ W}$$

$$V_{GG} = 9 \text{ V}, I_D = 0.25[2(9-4)V_{DS} - V_{DS}^2]$$

$$= \frac{40 - V_{DS}}{10} \Rightarrow V_{DS} = 1.88 \text{ V}$$

$$I_D = 3.81 \text{ A}, P = 7.16 \text{ W}$$

c. Yes, at $V_{GG} = 7 \text{ V}, P = 39.375 \text{ W} > P_{D,\max} = 35 \text{ W}$

8.6

a.

$$\text{Set } V_{DSQ} = \frac{V_{DD}}{2} = 25 \text{ V}$$

$$I_{DQ} = \frac{50 - 25}{20} = 1.25 \text{ A}$$

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2$$

$$\sqrt{\frac{1.25}{0.2}} + 4 = V_{GS} = 6.5 \text{ V}$$

$$V_{GS} = \left(\frac{R_2}{R_1 + R_2} \right) V_{DD}$$

$$\text{Let } R_1 + R_2 = 100 \text{ k}\Omega$$

$$6.5 = \left(\frac{R_2}{100} \right)(50) \Rightarrow R_2 = 13 \text{ k}\Omega$$

$$\underline{R_1 = 87 \text{ k}\Omega}$$

$$\text{b. } P_D = I_{DQ}V_{DSQ} = (1.25)(25) \Rightarrow P_D = 31.25 \text{ W}$$

c.

$$I_{D,\max} = 2I_{DQ} \Rightarrow \underline{I_{D,\max} = 2.5 \text{ A}}$$

$$V_{DS,\max} = V_{DD} \Rightarrow \underline{V_{DS,\max} = 50 \text{ V}}$$

$$\underline{P_{D,\max} = 31.25 \text{ W}}$$

d.

$$\left| \frac{V_0}{V_i} \right| = g_m R_L$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.2)(1.25)} = 1 \text{ A/V}$$

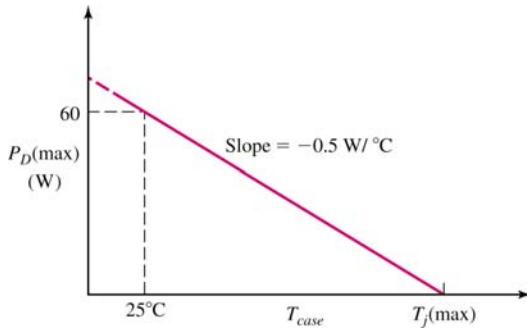
$$|V_0| = (1)(20)(0.5) = 10 \text{ V}$$

$$\overline{P_L} = \frac{1}{2} \cdot \frac{V_0^2}{R_L} = \frac{1}{2} \cdot \frac{(10)^2}{20} \Rightarrow \overline{P_L} = 2.5 \text{ W}$$

$$\overline{P_Q} = 31.25 - 2.5 \Rightarrow \overline{P_Q} = 28.75 \text{ W}$$

8.7

(a)



$$(b) P_D = P_{D,max} - (\text{Slope})(T_j - 25)$$

$$\text{At } P_D = 0, T_{j,max} = \frac{60}{0.5} + 25 \Rightarrow T_{j,max} = 145^\circ\text{C}$$

$$(c) \theta_{dev-amb} = \frac{T_{j,max} - T_{case}}{P_{D,max}} \quad \text{or} \quad \theta_{dev-amb} = \frac{145 - 25}{60} \Rightarrow \theta_{dev-amb} = 2^\circ\text{C/W}$$

8.8

$$P_{D,\text{rated}} = \frac{T_{j,max} - T_{amb}}{\theta_{dev-case}}$$

$$\text{or } \theta_{dev-case} = \frac{T_{j,max} - T_{amb}}{P_{D,\text{rated}}}$$

$$= \frac{150 - 25}{50} = 2.5^\circ\text{C/W}$$

$$\text{Then } T_{dev} - T_{amb} = P_D (\theta_{dev-case} + \theta_{case-amb})$$

$$150 - 25 = P_D (2.5 + \theta_{case-amb}) \Rightarrow 125 = P_D (2.5 + \theta_{case-amb})$$

8.9

$$(a) T_{j,max} - T_{amb} = P_T (\theta_{dev-case} + \theta_{snk-amb} + \theta_{case-snk})$$

$$120 - 25 = P_T (1.5 + 2.8 + 0.6) \Rightarrow P_T = 19.39 \text{ W}$$

(b) $T_{case} = 25 + (19.39)(0.6 + 2.8) = 90.9^\circ C$
(c) $T_{sink} = 25 + (19.39)(2.8) = 79.3^\circ C$

8.10

(a) $T_{j,max} - T_{amb} = P(\theta_{dev-case} + \theta_{case-amb})$
 $150 - 25 = 30(2.8 + \theta_{case-amb}) \Rightarrow \theta_{case-amb} = 1.37^\circ C / W$
(b) $T_{j,max} = 25 + 20(2.8 + 1.37) = 108^\circ C$

8.11

(a) $150 - 25 = P_T(3.8 + 1.5 + 4) \Rightarrow P_T = 13.4 \text{ W}$
(b) $P = I_{CQ}V_{CEQ}$
 $13.4 = (3) \cdot V_{CEQ} \Rightarrow V_{CEQ} = 4.48 \text{ V}$
 $V_{CC} = V_{CE,max} = 2V_{CEQ} = 8.96 \text{ V}$

8.12

$$\eta = \frac{\overline{P_L}}{\overline{P_S}}$$

$$\overline{P_S} = V_{CC} \cdot I_Q$$

$$\overline{P_L} = V_p \cdot I_p = \left(\frac{V_{CC}}{2} \right) (I_Q)$$

$$\eta = \frac{\frac{1}{2} \cdot V_{CC} \cdot I_Q}{V_{CC} \cdot I_Q} \Rightarrow \underline{\eta = 50\%}$$

8.13

(a) $A_v = \frac{(1+\beta)R_L}{r_\pi + (1+\beta)R_L} = \frac{(1+\beta)R_L}{\frac{\beta V_T}{I_C} + (1+\beta)R_L}$

We have $(1+\beta) \cong \beta$

$$A_v = \frac{I_C R_L}{I_C R_L + V_T} = \frac{R_L}{R_L + \frac{V_T}{I_C}} = \frac{R_L}{R_L + \frac{1}{g_m}}$$

(b) (i) $0.9 = \frac{8}{8 + \frac{1}{g_m}} \Rightarrow g_m = 1.125 \text{ A/V}, I_C = 29.25 \text{ mA}$

(ii) $0.95 = \frac{8}{8 + \frac{1}{g_m}} \Rightarrow g_m = 2.375 \text{ A/V}, I_C = 61.75 \text{ mA}$

(iii) $0.997 = \frac{8}{8 + \frac{1}{g_m}} \Rightarrow g_m = 41.54 \text{ A/V}, I_C = 108 \text{ mA}$

8.14

$$(a) \bar{P}_L = \frac{1}{2} \cdot \frac{V_p^2}{R_L} \Rightarrow 0.5 = \frac{1}{2} \cdot \frac{V_p^2}{8} \Rightarrow V_p = 2.828 \text{ V}$$

$$I_p = \frac{V_p}{R_L} = \frac{2.828}{8} = 0.3536 \text{ A}$$

(b) For $V_o = -V_p = -2.828 \text{ V}$

$$|I_L| = 0.3536 = (0.9)I_o \Rightarrow I_o = 0.393 \text{ A}$$

8.15

$$(a) V_o = 1.6 \text{ V}, \quad I_L = \frac{1.6}{8} = 0.20 \text{ A}$$

$$I_C \cong 0.2 + 0.25 = 0.45 \text{ A}$$

$$g_m = \frac{0.45}{0.026} = 17.31 \text{ A/V}$$

$$A_v = \frac{8}{8 + \frac{1}{17.31}} = 0.9928$$

(b) $V_o = 0, \quad I_L = 0, \quad \text{so } I_C = 0.25 \text{ A}$

$$g_m = \frac{0.25}{0.026} = 9.615 \text{ A/V}$$

$$A_v = \frac{8}{8 + \frac{1}{9.615}} = 0.9872$$

(c) $V_o = -1.6 \text{ V}, \quad I_L = -0.2 \text{ A}, \quad I_C \cong 0.25 - 0.2 = 0.05 \text{ A}$

$$g_m = \frac{0.05}{0.026} = 1.923 \text{ A/V}$$

$$A_v = \frac{8}{8 + \frac{1}{1.923}} = 0.939$$

8.16

$$v_o(\max) = 4.8 \text{ V}$$

$$i_{C3} = i_{C2} = \frac{-0.7 - (-5)}{1} = 4.3 \text{ mA}$$

$$v_I = v_o + 0.7 \quad i_L(\max) = -4.3 \text{ mA} = \frac{v_s(\min)}{1}$$

$$\text{so } -3.6 \leq v_I \leq 5.5 \text{ V} \quad \underline{v_o(\min) = -4.3 \text{ V}}$$

8.17

$$I_{D3} = K(V_{GS3} - V_{TN})^2 = \frac{0 - V_{GS3} - (-5)}{R}$$

$$12(V_{GS3} - 0.5)^2 = 5 - V_{GS3}$$

$$2V_{GS3}^2 - 11V_{GS3} - 2 = 0$$

$$V_{GS3} = \frac{11 \pm \sqrt{(11)^2 + 4(12)(2)}}{2(12)}$$

$$V_{GS3} = V_{GS2} = 1.072 \text{ V}$$

$$I_{D3} = I_{D2} = 12(1.072 - 0.5)^2 = 3.93 \text{ mA}$$

$$V_{DS2}(\text{sat}) = V_{GS2} - V_{TN} = 1.072 - 0.5 = 0.572 \text{ V}$$

$$v_o(\text{min}): i_2(\text{max}) = -3.93 = \frac{V_o(\text{min})}{1} \Rightarrow V_o(\text{min}) = -3.93 \text{ V}$$

$$v_I(\text{min}) = v_o(\text{min}) + V_{TN} = -3.93 + 0.5$$

$$\underline{v_I(\text{min}) = -3.43 \text{ V}}$$

$$v_o(\text{max}) = 5 - V_{DS}(\text{sat}) = 5 - 0.572$$

$$v_o(\text{max}) = 4.43 \text{ V}$$

$$I_{D1}(\text{max}) = 3.93 + \frac{4.43}{1} = 8.36 \text{ mA}$$

$$I_{D1} = 8.36 = 12(V_{GS1} - 0.5)^2 \Rightarrow V_{GS1} = 1.33 \text{ V}$$

$$v_I(\text{max}) = v_o + V_{GS1} = 4.43 + 1.33 \Rightarrow \underline{v_I(\text{max}) = 5.76 \text{ V}}$$

8.18

$$(a) \text{ For } v_o = -12 + 0.7 = -11.7 \text{ V}, \quad I_Q = \frac{11.3}{0.02} + 50 = 615 \text{ mA}$$

$$I_{REF} = \left(1 + \frac{2}{\beta}\right) \cdot I_Q = \left(1 + \frac{2}{40}\right)(615) = 645.75 \text{ mA}$$

$$R = \frac{0 - 0.7 - (-12)}{0.6475} \Rightarrow R = 17.5 \Omega$$

$$\text{For } v_o = 12 - 0.7 = +11.3 \text{ V}, \quad i_L = \frac{11.3}{0.02} = 565 \text{ mA}$$

$$i_{E1}(\text{max}) = I_Q + i_L = 615 + 565 \Rightarrow i_{E1}(\text{max}) = 1.18 \text{ A}$$

$$(b) \quad \bar{P}_L = \frac{1}{2} \cdot \frac{V_o^2}{R_L} = \frac{1}{2} \cdot \frac{(11.3)^2}{20} = 3.19 \text{ W}$$

$$P_S = I_Q(24) = (0.615)(24) = 14.76 \text{ W}$$

$$\text{Define } \eta = \frac{\bar{P}_L}{P_S} = \frac{3.19}{14.76} \times 100\% = 21.6\%$$

8.19

$$(a) V_o = -20 \text{ V}, i_L = \frac{-20}{200} = -0.10 \text{ A}$$

$$I_Q = 0.10 + i_{E1}(\text{min}) = 0.10 + 0.02 = 0.12 \text{ A}$$

$$I_{REF} = \left(1 + \frac{2}{\beta}\right) \cdot I_Q = \left(1 + \frac{2}{50}\right)(0.12) = 0.1248 \text{ A}$$

$$R = \frac{0 - 0.7 - (-24)}{0.1248} = 187 \Omega$$

$$(b) P_{Q1} = I_Q V_{CE1} = (0.12)(24) = 2.88 \text{ W}$$

$$P = I_Q V_{CE2} + I_{REF}(24) = (0.12)(24) + (0.1248)(24) = 5.88 \text{ W}$$

$$(c) \overline{P}_L = \frac{1}{2} \cdot \frac{V_o^2}{R_L} = \frac{1}{2} \cdot \frac{(20)^2}{200} = 1 \text{ W}$$

$$\eta = \frac{1}{2.88 + 5.88} \times 100\% = 11.4\%$$

8.20

$$I_{D1} = K_n (V_{GS} - V_{TN})^2 = 12(0 - (-1.8))^2$$

$$I_{D1} = 38.9 \text{ mA}$$

(a)

For $R_L = \infty$

$$v_o(\text{max}) = 4.8 \text{ V}$$

$$V_{DS}(\text{sat}) = V_{GS} - V_{TN} = 1.8 \text{ V}$$

$$v_o(\text{min}) = -5 + 1.8 = -3.2 \text{ V}$$

$$v_I = v_o + 0.7 \Rightarrow -2.5 \leq v_I \leq 5.5 \text{ V}$$

(b) For $R_L = 500 \Omega$ $v_o(\text{max}) = 4.8 \text{ V}$

$$\text{For } v_o < 0, v_o(\text{min}) = -3.2 \text{ V} \quad I'_2 = \frac{v_o}{R_L} = \frac{-3.2}{0.5} = -6.4 \text{ mA}$$

$$-2.5 \leq v_I \leq 5.5 \text{ V}$$

(c)

$$\text{For } v_o = -2V, I'_2(\text{max}) = -38.9 \text{ mA}$$

$$R_2(\text{min}) = \frac{-2}{-38.9} \Rightarrow R_L(\text{min}) = 51.4 \Omega$$

$$\overline{P}_L = \frac{1}{2} \cdot \frac{v_o^2}{R_L} = \frac{1}{2} \cdot \frac{(2)^2}{51.4} \Rightarrow \overline{P}_L = 38.9 \text{ mW}$$

$$P_L = 10(38.9) = 389 \text{ mW \%} = \frac{38.9}{389} = 10\%$$

8.21

$$\begin{aligned}\overline{P_L} &= \frac{V_p^2}{R_L} = \frac{(V^+)^2}{R_L} \\ \overline{P_S} &= \frac{1}{2} \cdot \frac{(V^+)^2}{R_L} + \frac{1}{2} \cdot \frac{(V^-)^2}{R_L}, \quad V^- = -V^+ \\ \text{So } \overline{P_S} &= \frac{(V^+)^2}{R_L} \\ \eta &= \frac{\overline{P_L}}{\overline{P_S}} \Rightarrow \eta = 100\%\end{aligned}$$

8.22

(a)

As maximum conversion efficiency

$$\eta = \frac{\pi}{4}, \quad \frac{V_p}{V_{CC}} = 0.785$$

$$\text{So } V_p(\max) = (0.785)(5)\left(\frac{4}{\pi}\right)$$

$$\underline{V_p(\max) = 5 \text{ V}}$$

$$V_p = \frac{2V_{CC}}{\pi} = \frac{2(5)}{\pi} = 3.183 \text{ V}$$

(b) Maximum power dissipation occurs when

$$\overline{P_\theta}(\max) = \frac{V_{CC}^2}{\pi^2 R_L}$$

$$2 = \frac{(5)^2}{\pi^2 R_L} \Rightarrow \underline{R_L = 1.27 \Omega}$$

(c)

8.23

$$\overline{P} = \frac{1}{2} \cdot \frac{V_p^2}{R_L}$$

$$(a) 50 = \frac{1}{2} \cdot \frac{V_p^2}{24} \Rightarrow V_p = 49 \text{ V} \Rightarrow \underline{V^+ = 52 \text{ V}, V^- = -52 \text{ V}}$$

$$(b) I_p = \frac{V_p}{R_L} = \frac{49}{24} = \underline{2.04 \text{ A}}$$

$$\eta = \frac{\pi}{4} \cdot \frac{V_p}{V_{CC}} = \frac{\pi}{4} \left(\frac{49}{52} \right)$$

$$(c) \underline{\eta = 74.0\%}$$

8.24

(a)

$$V_{DS} \geq V_{DS}(\text{sat}) = V_{GS} - V_{TN} = V_{GS}$$

$$V_{DS} = 10 - V_o(\text{max}) \text{ and } I_D = I_L = K_n(V_{GS})^2$$

$$\frac{V_o(\text{max})}{R_L} = K_n(V_{GS})^2$$

$$V_{GS} = \sqrt{\frac{V_o(\text{max})}{R_L \cdot K_n}}$$

$$\text{So } 10 - V_o(\text{max}) = \sqrt{\frac{V_o(\text{max})}{R_L \cdot K_n}} = \sqrt{\frac{V_o(\text{max})}{(5)(0.4)}}$$

$$[10 - V_o(\text{max})]^2 = \frac{V_o(\text{max})}{2}$$

$$100 - 20V_0(\text{max}) + V_0^2(\text{max}) = \frac{V_0(\text{max})}{2}$$

$$V_0^2(\text{max}) - 20.5V_0(\text{max}) + 100 = 0$$

$$V_0(\text{max}) = \frac{20.5 \pm \sqrt{(20.5)^2 - 4(100)}}{2} \Rightarrow V_0(\text{max}) = 8 \text{ V}$$

$$i_L = \frac{8}{5} \Rightarrow i_L = 1.6 \text{ mA}$$

$$V_{GS} = \sqrt{\frac{i_L}{K_n}} = \sqrt{\frac{1.6}{0.4}} = 2 \text{ V} \Rightarrow V_I = 10 \text{ V}$$

b.

$$\overline{P_L} = \frac{1}{2} \cdot \frac{(8)^2}{5} = 6.4 \text{ mW}$$

$$\overline{P_S} = \frac{20(1.6)}{\pi} = 10.2 \text{ mW}$$

$$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{6.4}{10.2} \Rightarrow \eta = 62.7\%$$

8.25

(b) $v_I = v_{GSn} + v_o$

Also $v_o = i_L R_L = i_{dn} R_L = KR_L(v_{GSn})^2$, but $v_{GSn} = v_I - v_o$

So $v_o = KR_L(v_I - v_o)^2$

$$\frac{dv_o}{dv_I} = 2KR_L(v_I - v_o) \left(1 - \frac{dv_o}{dv_I} \right)$$

$$\frac{dv_o}{dv_I} = \frac{2KR_L(v_I - v_o)}{1 + 2KR_L(v_I - v_o)}, \text{ also } v_I - v_o = \sqrt{\frac{v_o}{KR_L}}$$

$$\text{Then } \frac{dv_o}{dv_I} = A_v = \frac{2\sqrt{KR_L} \cdot \sqrt{v_o}}{1 + 2\sqrt{KR_L} \cdot \sqrt{v_o}}$$

We find $\sqrt{KR_L} = \sqrt{(4)(0.05)} = 0.4472$

(i) For $v_o = 0$, $A_v = 0$

$$(ii) \text{ For } v_o = 1 \text{ V}, \quad A_v = \frac{2(0.4472)(1)}{1+2(0.4472)(1)} = 0.472$$

$$(iii) \text{ For } v_o = 10 \text{ V}, \quad A_v = \frac{2(0.4472)\sqrt{10}}{1+2(0.4472)\sqrt{10}} = 0.739$$

8.26

$$(a) i_{Cn} = I_S \exp\left(\frac{V_{BE}}{V_T}\right) \Rightarrow V_{BE} = V_T \ln\left(\frac{i_{Cn}}{I_S}\right) = (0.026) \ln\left(\frac{10^{-3}}{2 \times 10^{-15}}\right) = 0.7004 \text{ V}$$

$$V_{BB} = 2V_{BE} = 1.40077 \text{ V}$$

$$P_Q = i_C \cdot v_{CE} = (1)(5) = 5 \text{ mW}$$

$$(b) \text{ For } v_o = -3.5 \text{ V}, \quad i_{Cp} \cong \frac{3.5}{1} = 3.5 \text{ mA} \cong i_L$$

$$v_{EB} = (0.026) \ln\left(\frac{3.5 \times 10^{-3}}{2 \times 10^{-15}}\right) = 0.732957 \text{ V}$$

$$v_{BE} = 1.40077 - 0.732957 = 0.66781 \text{ V}$$

$$i_{Cn} = (2 \times 10^{-15}) \exp\left(\frac{0.66781}{0.026}\right) \Rightarrow i_{Cn} = 0.2857 \text{ mA}$$

$$\text{Then } i_{Cp} \cong 0.2857 + 3.5 = 3.7857 \text{ mA}$$

$$v_{EB} = (0.026) \ln\left(\frac{3.7857 \times 10^{-3}}{2 \times 10^{-15}}\right) = 0.734997 \text{ V}$$

$$v_{BE} = 1.40077 - 0.734997 = 0.66577 \text{ V}$$

$$i_{Cn} = (2 \times 10^{-15}) \exp\left(\frac{0.66577}{0.026}\right) \Rightarrow i_{Cn} = 0.2642 \text{ mA}$$

$$i_{Cp} = 3.5 + 0.2642 = 3.764 \text{ mA}$$

$$v_I = v_o - v_{EB} + \frac{V_{BB}}{2} = -3.5 - 0.735 + 0.7004 = -3.535 \text{ V}$$

$$\text{For } R_L: \quad P_{RL} = i_L^2 R_L = (3.5)^2 (1) = 12.25 \text{ mW}$$

$$\text{For } Q_n: \quad P_{Qn} = i_{Cn} v_{CEn} = (0.2642)[5 - (-3.5)] = 2.25 \text{ mW}$$

$$\text{For } Q_p: \quad P_{Qp} = i_{Cp} v_{ECp} = (3.764)[-3.5 - (-5)] = 5.65 \text{ mW}$$

8.27

$$(a) (i) i_{Dn} = K_n (v_{GSn} - V_{TN})^2$$

$$1 = 4(v_{GSn} - 1.5)^2 \Rightarrow v_{GSn} = \frac{V_{BB}}{2} = 2 \text{ V}, \quad \Rightarrow V_{BB} = 4 \text{ V}$$

$$(ii) P = i_{Dn} \cdot v_{DSn} = (1)(12) = 12 \text{ mW}$$

(b) (i) $i_D = K[v_{DS}(\text{sat})]^2 = \frac{v_o}{R_L} = \frac{V_{DD} - v_{DS}(\text{sat})}{R_L}$

$$KR_L[v_{DS}(\text{sat})]^2 + v_{DS}(\text{sat}) - V_{DD} = 0$$

$$\text{Now } KR_L = 4 \quad (1) = 4$$

$$\text{We have } 4[v_{DS}(\text{sat})]^2 + v_{DS}(\text{sat}) - 12 = 0 \Rightarrow v_{DS}(\text{sat}) = 1.612 \text{ V}$$

$$v_o(\text{max}) = 12 - 1.612 = 10.39 \text{ V}$$

(ii) $i_{Dn} = i_L = 10.39 \text{ mA}$

$$i_{Dp} = 0$$

$$v_{GSn} = 3.112 \text{ V}$$

$$v_I = \frac{-V_{BB}}{2} + v_{GSn} + v_o = -2 + 3.112 + 10.39 = 11.5 \text{ V}$$

(iii) For $R_L : P_{RL} = i_L^2 R_L = (10.39)^2 (1) = 108 \text{ mW}$

$$\text{For } M_n : P_{Mn} = i_{Dn} v_{DSn} = (10.39)[12 - 10.39] = 16.7 \text{ mW}$$

$$\text{For } M_p : P_{Mp} = i_{Dp} v_{SDp} = 0$$

8.28

a.

$$v_0 = 24 \text{ V} \Rightarrow i_L = \frac{24}{8} \Rightarrow \underline{i_L \approx i_N = 3 \text{ A}}$$

$$i_{Bn} = \frac{3}{41} \Rightarrow i_{Bn} = 73.2 \text{ mA}$$

$$\text{For } i_D = 25 \text{ mA} \Rightarrow i_{R1} = 25 + 73.2 = 98.2 \text{ mA}$$

$$V_{BE} = V_T \ln\left(\frac{i_N}{I_s}\right) = (0.026) \ln\left(\frac{3}{6 \times 10^{-12}}\right) = 0.7004 \text{ V}$$

$$\text{Then } 98.2 = \frac{30 - (24 + 0.7)}{R_1} \Rightarrow R_1 = \frac{5.3}{98.2} \Rightarrow \underline{R_1 = 53.97 \Omega}$$

$$V_D = (0.026) \ln\left(\frac{25 \times 10^{-3}}{6 \times 10^{-12}}\right) = 0.5759 \text{ V}$$

$$V_{EB} = 2V_D - V_{BE} = 2(0.5759) - 0.7004 = 0.4514 \text{ V}$$

$$i_p = I_s \exp\left(\frac{V_{EB}}{V_T}\right) = (6 \times 10^{-12}) \exp\left(\frac{0.4514}{0.026}\right) \Rightarrow \underline{i_p = 0.208 \text{ mA}}$$

b. Neglecting base current

$$i_D \approx \frac{30 - 0.6}{R_1} = \frac{30 - 0.6}{53.97} \Rightarrow \underline{i_D \approx 545 \text{ mA}}$$

$$V_D = (0.026) \ln\left(\frac{0.545}{6 \times 10^{-12}}\right) = 0.656 \text{ V}$$

Approximation for i_D is okay.

$$\text{Diodes and transistors matched} \Rightarrow \underline{i_N = i_p = 545 \text{ mA}}$$

8.29

(a)

$$I_{D1} = K_1 (V_{GS1} - V_{TN})^2$$

$$V_{GS1} = \sqrt{\frac{5}{5}} + 1 = 2 \text{ V}$$

$$I_{D3} = K_3 (V_{GS3} - V_{TN})^2$$

$$200 = K_3 (2 - 1)^2 \Rightarrow K_{n3} = K_{p4} = 200 \text{ } \mu\text{A/V}^2$$

(b)

$$v_I + V_{SG4} + V_{GS3} - V_{GS1} = v_O$$

$$\text{For } v_o \text{ large, } i_L = i_l = K_{nl} (V_{GS1} - V_{TN})^2$$

$$V_{GS1} = \sqrt{\frac{i_L}{K_{nl}}} + V_{TN} = \sqrt{\frac{v_o}{R_L K_{nl}}} + V_{TN}$$

$$\text{So } v_I + 2 + 2 - \left(\sqrt{\frac{v_o}{(0.5)(5)}} + 1 \right) = v_0$$

$$v_I = v_0 + \sqrt{\frac{v_0}{2.5}} - 3$$

$$\frac{dv_L}{dv_I} = 1 = \frac{dv_0}{dv_I} + \frac{1}{2} \cdot \frac{1}{\sqrt{2.5v_0}} \cdot \frac{dv_0}{dv_I}$$

$$1 = \frac{dv_0}{dv_I} \left[1 + \frac{1}{2\sqrt{2.5v_0}} \right]$$

For $v_o = 5 \text{ V}$:

$$1 = \frac{dv_0}{dv_I} \left[1 + \frac{1}{2\sqrt{2.5(5)}} \right] = \frac{dv_0}{dv_I} (1.1414) \Rightarrow \underline{\underline{\frac{dv_0}{dv_I} = 0.876}}$$

8.30

$$v_O = v_I + \frac{V_{BB}}{2} - V_{GS} \text{ and } V_{GS} = \sqrt{\frac{I_{Dn}}{K_n}} + V_{TN}$$

$$v_O \approx 0, I_{Dn} = I_{DQ} + i_L = I_{DQ} + \frac{v_O}{R_L}$$

For

$$v_O = v_I + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ} + (v_O/R_L)}{K_n}} \quad \text{or} \quad v_O = v_I + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ}}{K_n}} \cdot \sqrt{1 + \frac{v_O}{I_{DQ} R_L}}$$

Then

$$v_O \cong v_I + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ}}{K_n}} \cdot \sqrt{1 + \frac{1}{2} \cdot \frac{v_O}{I_{DQ} R_L}}$$

For v_o small,

$$v_O \left[1 + \frac{1}{2} \cdot \sqrt{\frac{I_{DQ}}{K_n} \cdot \frac{1}{I_{DQ} R_L}} \right] = v_I + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ}}{K_n}}$$

Now

$$\frac{dv_o}{dv_I} = \frac{1}{1 + \frac{1}{2} \cdot \sqrt{\frac{I_{DQ}}{K_n}} \cdot \frac{1}{I_{DQ}R_L}} = 0.95$$

$$\text{So } \frac{1}{2} \cdot \sqrt{\frac{I_{DQ}}{K_n}} \cdot \frac{1}{I_{DQ}R_L} = \frac{1}{0.95} - 1 = 0.0526$$

$$R_L = 0.1 \text{ k}\Omega, \text{ then } \frac{1}{\sqrt{K_n I_{DQ}}} = 0.01052$$

For

$$\text{Or } \sqrt{K_n I_{DQ}} = 95.1$$

$$\text{We can write } g_m = 2\sqrt{K_n I_{DQ}} = 190 \text{ mA/V}$$

This is the required transconductance for the output transistor. This implies a very large transistor.

8.31

$$(a) R_{TH} = R_1 \| R_2 = 14 \| 10 = 5.833 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left(\frac{10}{10 + 14} \right) (12) = 5 \text{ V}$$

$$I_{BQ} = \frac{5 - 0.7}{5.833 + (91)(0.02)} = 0.5619 \text{ mA}, \quad I_{CQ} = 50.57 \text{ mA}$$

$$(b) R_L = \frac{V_{CC}}{I_{CQ}} = \frac{12}{50.57} \Rightarrow R_L = 237 \Omega$$

$$(c) \bar{P}_L(\max) = \frac{1}{2} \cdot \frac{(11)^2}{0.237} = 255 \text{ mW}$$

$$(d) \bar{P}_s = I_{CQ}V_{CC} = (50.57)(12) = 607 \text{ mW}$$

$$\eta = \frac{255}{607} \times 100\% = 42\%$$

8.32

$$I_{CQ} = \frac{V_{CC}}{R_L} = \frac{15}{1} = 15 \text{ mA}$$

$$I_{BQ} = \frac{15}{100} = 0.15 \text{ mA}$$

$$\overline{P}_L(\max) = \frac{1}{2} \cdot \frac{V_{CC}^2}{R_L} = \frac{(15)^2}{2(1)} \Rightarrow \overline{P}_L(\max) = 112.5 \text{ mW}$$

Let $R_{TH} = 10 \text{ k}\Omega$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE} + (1+\beta)I_{BQ}R_E \\ = (0.15)(10) + 0.7 + (101)(0.15)(0.1)$$

$$V_{TH} = 3.715 = \frac{1}{R_i} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_i} \cdot (10)(15)$$

$$\underline{R_i = 40.4 \text{ k}\Omega}$$

$$\underline{R_2 = 13.3 \text{ k}\Omega}$$

8.33

$$(a) R_{TH} = R_1 \| R_2 = 2.3 \| 1.75 = 0.9938 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left(\frac{1.75}{1.75 + 2.3} \right) (12) = 5.185 \text{ V}$$

$$I_{BQ} = \frac{5.185 - 0.7}{0.9938 + (41)(0.02)} = 2.473 \text{ mA}, \quad I_{CQ} = 98.91 \text{ mA}$$

$$(b) \text{ Want } R'_L = \frac{V_{CC}}{I_{CQ}} = \frac{12}{0.09891} = 121.3 \Omega = a^2 R_L = a^2 (8) \Rightarrow a = 3.89$$

$$(c) \quad \overline{P}_L = \frac{1}{2} \cdot \frac{(9)^2}{(0.1213)} = 333.9 \text{ mW}$$

$$\overline{P}_S = I_{CQ}V_{CC} = (98.91)(12) \Rightarrow 1.187 \text{ W}$$

$$(d) \quad \eta = \frac{0.3339}{1.187} \times 100\% = 28.1\%$$

8.34

- a. Assuming the maximum power is being delivered, then

$$V'_o(\text{peak}) = 36 \text{ V} \Rightarrow V_o = \frac{36}{4} = 9 \text{ V} \Rightarrow V_{rms} = \frac{9}{\sqrt{2}} \Rightarrow \underline{V_{rms} = 6.36 \text{ V}}$$

$$b. \quad V_o = \frac{36}{\sqrt{2}} \Rightarrow \underline{V_o = 25.5 \text{ V}}$$

$$c. \quad \text{Secondary} \quad I_{rms} = \frac{\overline{P}_L}{V_{rms}} = \frac{2}{6.36} \Rightarrow \underline{I_{rms} = 0.314 \text{ A}}$$

$$\text{Primary} \quad I_P = \frac{0.314}{4} \Rightarrow \underline{I_P = 78.6 \text{ mA}}$$

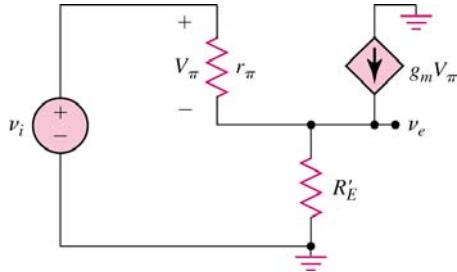
d.

$$\overline{P_s} = I_{CQ}V_{CC} = (0.15)(36) = 5.4 \text{ W}$$

$$\eta = \frac{2}{5.4} \Rightarrow \eta = 37\%$$

8.35

a.



$$v_e = \left(\frac{V_\pi}{r_\pi} + g_m V_\pi \right) R'_E = V_\pi \left(\frac{1}{r_\pi} + g_m \right) R'_E$$

$$= V_\pi \left(\frac{1+\beta}{r_\pi} \right) R'_E$$

$$v_i = V_\pi + v_e \Rightarrow V_\pi = v_i - v_e$$

$$v_e = (v_i - v_e) \left(\frac{1+\beta}{r_\pi} \right) R'_E$$

$$\frac{v_e}{v_i} = \frac{\frac{1+\beta}{r_\pi} \cdot R'_E}{1 + \frac{1+\beta}{r_\pi} \cdot R'_E} = \frac{(1+\beta)R'_E}{r_\pi + (1+\beta)R'_E} = \frac{v_e}{v_i} \text{ where } R'_E = \left(\frac{n_1}{n_2} \right)^2 R_L$$

$$v_0 = \frac{v_e}{\left(\frac{n_1}{n_2} \right)} \text{ so } v_e - v_0 \left(\frac{n_1}{n_2} \right)$$

$$\text{so } \frac{v_0}{v_i} = \frac{1}{\left(\frac{n_1}{n_2} \right)} \cdot \frac{(1+\beta)R'_E}{r_\pi + (1+\beta)R'_E}$$

b.

$$\overline{P_L} = \frac{1}{2} \cdot I_p^2 R_L, \quad a = \frac{n1}{n2}, \quad I_{CQ} = \frac{I_p}{a} \text{ so } \overline{P_L} = \frac{1}{2} \cdot a^2 I_{CQ}^2 R_L$$

$$\overline{P_s} = I_{CQ} V_{CC}$$

For $\eta = 50\%$:

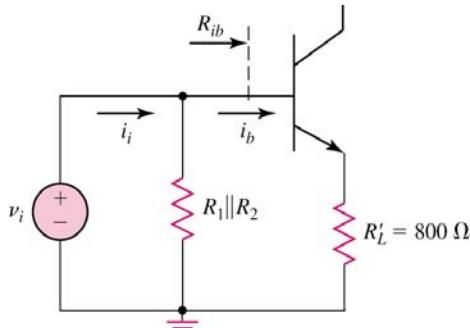
$$\frac{\overline{P_L}}{\overline{P_s}} = 0.5 = \frac{\frac{1}{2} \cdot a^2 I_{CQ}^2 R_L}{I_{CQ} \cdot V_{CC}} = \frac{a^2 I_{CQ} R_L}{2V_{CC}} \text{ so } a^2 = \frac{V_{CC}}{I_{CQ} \cdot R_L} = \frac{V_{CC}}{(0.1)(50)} \Rightarrow a^2 = \frac{V_{CC}}{5}$$

c.

$$R_0 = \frac{r_\pi}{1+\beta} = \frac{\beta V_T}{(1+\beta) I_{CQ}} = \frac{49(0.026)}{(50)(0.1)} \Rightarrow R_0 = 0.255 \Omega$$

8.36

- a. With a 10:1 transformer ratio, we need a current gain of 8 through the transistor.



$$i_e = (1 + \beta) i_b \text{ and } i_b = \left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_{ib}} \right) i_i \quad \frac{i_e}{i_i} = 8 = (1 + \beta) \left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_{ib}} \right) \text{ where}$$

$$R_{ib} = r_\pi + (1 + \beta) R'_L \approx (1 + \beta) R'_L = (101)(0.8) = 80.8$$

$$\text{Then } 8 = (101) \left(\frac{R_1 \| R_2}{R_1 \| R_2 + 80.8} \right)$$

$$\frac{R_1 \| R_2}{R_1 \| R_2 + 80.8} = 0.0792 \text{ or } R_1 \| R_2 = 6.95 \text{ k}\Omega$$

$$\text{Set } \frac{2V_{CC}}{2I_{CQ}} = R'_L \Rightarrow I_{CQ} = \frac{V_{CC}}{R'_L} = \frac{12}{0.8} = 15 \text{ mA}$$

$$I_{BQ} = \frac{15}{100} = 0.15 \text{ mA}$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE}$$

$$\frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = I_{BQ} R_{TH} + V_{BE}$$

$$\frac{1}{R_1} (6.95)(12) = (0.15)(6.95) + 0.7 \Rightarrow \underline{R_1 = 47.9 \text{ k}\Omega} \text{ then } \underline{R_2 = 8.13 \text{ k}\Omega}$$

b.

$$I_e = 0.9 I_{CQ} = 13.5 \text{ mA} = \frac{I_L}{a} \Rightarrow I_L = 135 \text{ mA}$$

$$\overline{P_L} = \frac{1}{2} (0.135)^2 (8) \Rightarrow \overline{P_L} = 72.9 \text{ mW}$$

$$\overline{P_S} = V_{CC} I_{CQ} = (12)(15) \Rightarrow \overline{P_S} = 180 \text{ mW}$$

$$\eta = \frac{\overline{P_L}}{\overline{P_S}} \Rightarrow \eta = 40.5\%$$

8.37

a.

$$V_P = \sqrt{2R_L P_L}$$

$$V_P = \sqrt{2(8)(2)} = 5.66 \text{ V} = \text{peak output voltage}$$

$$I_P = \frac{V_P}{R_L} = \frac{5.66}{8} = 0.708 \text{ A} = \text{peak output current}$$

Set $V_e = 0.9V_{CC} = aV_P$ to minimize distortion

$$\text{Then } a = \frac{(0.9)(18)}{5.66} \Rightarrow a = 2.86$$

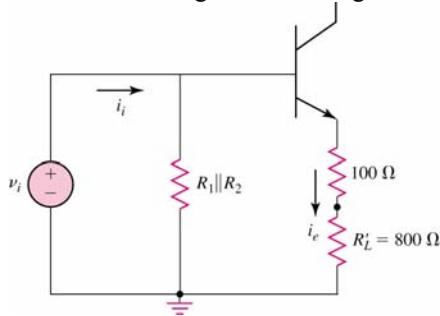
b.

$$\text{Now } I_{CQ} = \frac{1}{0.9} \left(\frac{I_P}{a} \right) = \frac{1}{0.9} \left(\frac{0.708}{2.86} \right) \Rightarrow I_{CQ} = 0.275 \text{ A}$$

$$\text{Then } P_Q = V_{CC} I_Q = (18)(0.275) \Rightarrow P_Q = 4.95 \text{ W Power rating of transistor}$$

8.38

a. Need a current gain of 8 through the transistor.



$$\frac{i_b}{i_i} = 8 = (1 + \beta) \left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_{ib}} \right) \text{ where } R_{ib} \approx (1 + \beta)(0.9) = 90.9 \text{ k}\Omega$$

$$\frac{8}{101} = \left(\frac{R_1 \| R_2}{R_1 \| R_2 + 90.9} \right) = 0.0792 \text{ or } R_1 \| R_2 = 7.82 \text{ k}\Omega$$

$$\text{Set } \frac{2V_{CC}}{2I_{CQ}} = 0.9 \text{ k}\Omega \Rightarrow I_{CQ} = \frac{12}{0.9} = 13.3 \text{ mA}$$

$$I_{BQ} = \frac{13.3}{100} = 0.133 \text{ mA}$$

$$\text{Then } \frac{1}{R_1} (7.82)(12) = (0.133)(7.82) + 0.7 \Rightarrow R_1 = 53.9 \text{ k}\Omega \text{ and } R_2 = 9.15 \text{ k}\Omega$$

b.

$$I_e = (0.9)I_{CQ} = 12 \text{ mA} = \frac{I_L}{a} \Rightarrow I_L = 120 \text{ mA}$$

$$\overline{P_L} = \frac{1}{2}(0.12)^2(8) \Rightarrow \overline{P_L} = 57.6 \text{ mW}$$

$$\overline{P_S} = V_{CC}I_{CQ} = (12)(13.3) \Rightarrow \overline{P_S} = 159.6 \text{ mW}$$

$$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{57.6}{159.6} \Rightarrow \eta = 36.1\%$$

8.39

$$(a) V_{BB} = 2V_T \ln\left(\frac{I_{Bias}}{I_{SD}}\right) = 2(0.026)\ln\left(\frac{10^{-3}}{5 \times 10^{-16}}\right) = 1.473 \text{ V}$$

$$(b) I_{CQ} = I_{SQ} \exp\left(\frac{1.473/2}{0.026}\right) \Rightarrow I_{CQ} = 14 \text{ mA}$$

8.40

$$(a) V_{BE} = V_T \ln\left(\frac{I_{CQ}}{I_{SQ}}\right) = (0.026)\ln\left(\frac{4 \times 10^{-3}}{2 \times 10^{-15}}\right) = 0.73643 = V_{BE_n} = V_{EB_p} = V_D$$

$$I_{Bias} = I_{SD} \exp\left(\frac{V_D}{V_T}\right) = (4 \times 10^{-16})\exp\left(\frac{0.73643}{0.026}\right) \Rightarrow I_{Bias} = 0.8 \text{ mA}$$

$$(b) V_{BB} = 2V_D = 1.473 \text{ V}$$

$$(c) v_I = -V_{EB_p} = -0.7364 \text{ V}$$

8.41

$$(a) V_{D1} = (0.026)\ln\left(\frac{0.5 \times 10^{-3}}{10^{-16}}\right) = 0.76025 \text{ V}$$

$$V_{D2} = (0.026)\ln\left(\frac{0.5 \times 10^{-3}}{4 \times 10^{-16}}\right) = 0.72421 \text{ V}$$

$$V_{BB} = V_{D1} + V_{D2} = 1.48446 \text{ V}$$

$$(b) i_{Cn} = i_{Cp} = I_{SQn} \exp\left(\frac{v_{BE_n}}{V_T}\right) = I_{SQp} \exp\left(\frac{v_{EB_p}}{V_T}\right)$$

$$\frac{I_{SQp}}{I_{SQn}} = \frac{\exp\left(\frac{v_{BE_n}}{V_T}\right)}{\exp\left(\frac{v_{EB_p}}{V_T}\right)} = \frac{\exp\left(\frac{v_{BE_n}}{V_T}\right)}{\exp\left(\frac{V_{BB} - v_{BE_n}}{V_T}\right)} = \exp\left(\frac{2v_{BE_n} - V_{BB}}{V_T}\right)$$

$$\frac{2v_{BE_n} - V_{BB}}{V_T} = \ln\left(\frac{I_{SQp}}{I_{SQn}}\right)$$

$$v_{BE_n} = \frac{1}{2} \left[V_{BB} + V_T \ln\left(\frac{I_{SQp}}{I_{SQn}}\right) \right] = \frac{1}{2} \left[1.48446 + (0.026)\ln\left(\frac{1.6 \times 10^{-15}}{8 \times 10^{-16}}\right) \right] = 0.75124 \text{ V}$$

$$\begin{aligned}v_{EBp} &= V_{BB} - v_{BEn} = 0.73322 \text{ V} \\(\text{c}) \quad I_{CQ} &= I_{SQn} \exp\left(\frac{v_{BEn}}{V_T}\right) = (8 \times 10^{-16}) \exp\left(\frac{0.75124}{0.026}\right) \\&\Rightarrow i_{Cn} = i_{Cp} = 2.828 \text{ mA} \\(\text{d}) \quad v_I &= -v_{EBp} = -0.73322 \text{ V}\end{aligned}$$

8.42

- a. All transistors are matched.

$$3 \text{ mA} = i_{E1} + i_{B3} = \left(\frac{1+\beta}{\beta}\right) i_C + \frac{i_C}{\beta}$$

$$3 = \left(\frac{61}{60} + \frac{1}{60}\right) i_C \Rightarrow i_C = 2.90 \text{ mA}$$

b.

For $v_o = 6 \text{ V}$, let $R_L = 200 \Omega$

$$i_o = \frac{6}{200} = 0.03 \text{ A} = 30 \text{ mA} \cong i_{E3}$$

$$i_{B3} = \frac{30}{61} = 0.492 \text{ mA}$$

$$i_{E1} = 3 - 0.492 = 2.508 \text{ mA}$$

$$i_{B1} = \frac{2.508}{61} \Rightarrow i_{B1} = 41.11 \mu\text{A}$$

$$i_{E2} \cong 3 \text{ mA} \Rightarrow i_{B2} = \frac{3}{61} \Rightarrow 49.18 \mu\text{A}$$

$$i_I = i_{B2} - i_{B1} = 49.18 - 41.11 \Rightarrow i_I = 8.07 \mu\text{A}$$

Current gain

$$A_i = \frac{30 \times 10^{-3}}{8.07 \times 10^{-6}} \Rightarrow A_i = 3.72 \times 10^3$$

$$V_{BE3} = V_T \ln\left(\frac{i_{E3}}{I_S}\right) = (0.026) \ln\left(\frac{30 \times 10^{-3}}{5 \times 10^{-13}}\right)$$

$$V_{BE3} = 0.6453 \text{ V}$$

$$V_{EB1} = V_T \ln\left(\frac{i_{E1}}{I_S}\right) = (0.026) \ln\left(\frac{2.508 \times 10^{-3}}{5 \times 10^{-13}}\right)$$

$$V_{EB1} = 0.5807 \text{ V}$$

$$v_I = v_0 + V_{BE3} - V_{EB1} = 6 + 0.6453 - 0.5807$$

$$v_I = 6.0646 \text{ V}$$

Voltage gain

$$A_v = \frac{v_0}{v_I} = \frac{6}{6.0646} \Rightarrow A_v = 0.989$$

8.43

a. For $i_0 = 1 \text{ A}$, $I_{B3} \cong \frac{1}{50} \Rightarrow 20 \text{ mA}$

$$\frac{10 - V_{EB1}}{R_l} = 2 \left[\frac{10 - (v_{o,\max} + V_{BE3})}{R_l} - 20 \right]$$

We can then write

$$\text{If, for simplicity, we assume } V_{EB1} = V_{BE3} = 0.7 \text{ V, then } \frac{10 - V_{BE}}{R_l} = \frac{2v_{o,\max}}{R_l} + 40$$

$$\text{If we assume } v_{o,\max} = 4 \text{ V, then } \frac{9.3}{R_l} = \frac{2(4)}{R_l} + 40 \quad \text{which yields } R_l = R_2 = 32.5 \Omega$$

b. For $v_I = 0$, $I_{E1} = \frac{9.3}{32.5} \Rightarrow I_{E1} = 0.286 \text{ A} = I_{E2}$

Since $I_{S3,4} = 10I_{S1,2}$, then $I_{E3} = I_{E4} = 2.86 \text{ A}$

c.

We can write

$$R_0 = \frac{1}{2} \left\{ \frac{r_{\pi 3} + R_l \left| \frac{r_{\pi 1}}{1 + \beta_1} \right|}{1 + \beta_3} \right\}$$

$$\text{Now } r_{\pi 3} = \frac{\beta_3 V_T}{I_{C3}} = \frac{(50)(0.026)}{2.86} = 0.4545 \Omega$$

$$r_{\pi 1} = \frac{\beta_1 V_T}{I_{C1}} = \frac{(120)(0.026)}{0.286} = 10.91 \Omega$$

So

$$R_0 = \frac{1}{2} \left\{ \frac{0.4545 + 32.5 \left| \frac{10.91}{121} \right|}{51} \right\}$$

$$32.5 \left| \frac{10.91}{121} \right| = 32.5 \parallel 0.0902 = 0.0900$$

$$\text{Then } R_0 = \frac{1}{2} \left\{ \frac{0.4545 + 0.0900}{51} \right\} \text{ or } R_0 = 0.00534 \Omega$$

8.44

$$R_i = \frac{1}{2} \left\{ r_{\pi 1} + (1 + \beta) \left[R_1 \left(r_{\pi 3} + (1 + \beta) 2R_L \right) \right] \right\}$$

$i_{C1} \approx 7.2 \text{ mA}$ and $i_{C3} \approx 7.2 \text{ mA}$

$$\text{Then } r_{\pi} = \frac{(60)(0.026)}{7.2} = 0.217 \text{ k}\Omega$$

$$\text{So } R_i = \frac{1}{2} \left\{ 0.217 + (61) \left[2 \left(0.217 + (61)(0.2) \right) \right] \right\}$$

$$= \frac{1}{2} \left\{ 0.217 + 61 [2 \parallel 12.4] \right\} \text{ or } R_i = 52.6 \text{ k}\Omega$$

8.45

(b) For M_{n3} ; $v_{GSn3} = \sqrt{\frac{I_{DQ}}{K_{n3}}} + V_{TN} = \sqrt{\frac{5}{5}} + 1 = 2 \text{ V}$

$$v_{SGp1} = v_{GSn3} = 2 \text{ V}$$

$$I_{DQ1} = 2(2 - 1)^2 = 2 \text{ mA}$$

$$R_1 = R_2 = \frac{10 - 2}{2} = 4 \text{ k}\Omega$$

(c) $I_{DQ1} = I_{DQ2} = 2 \text{ mA}$

(d) For $v_o = 3.5 \text{ V}$, Assume M_{p4} cutoff, so $I_{DQ4} = 0$

$$I_{Dn3} = i_o = \frac{v_o}{R_L} = \frac{3.5}{0.15} = 23.33 \text{ mA}$$

$$v_{GSn3} = \sqrt{\frac{23.33}{5}} + 1 = 3.160 \text{ V}$$

$$I_{Dp1} = I_{R1} = \frac{10 - (v_o + v_{GSn3})}{R_1} = \frac{10 - (3.16 + 3.5)}{4} = 0.835 \text{ mA}$$

$$v_{SGp1} = \sqrt{\frac{0.835}{2}} + 1 = 1.646 \text{ V}$$

$$v_I = v_o + v_{GSn3} - v_{SGp1} = 3.5 + 3.160 - 1.646 = 5.014 \text{ V}$$

$$v_I = v_{GSn2} + K_{n2} R_2 (v_{GSn2} - V_{TN})^2 + V^-$$

$$15.014 = v_{GSn2} + 8(v_{GSn2}^2 - 2v_{GSn2} + 1)$$

or $8v_{GSn2}^2 - 15v_{GSn2} - 7.014 = 0 \Rightarrow v_{GSn2} = 2.2625 \text{ V}$

$$I_{Dn2} = 2(2.2625 - 1)^2 = 3.188 \text{ mA}$$

$$v_{G4} = v_I - v_{GSn2} = 5.014 - 2.2625 = 2.75 \text{ V}$$

$$v_{SGp4} = v_o - v_{G4} = 3.5 - 2.75 = 0.75 \text{ V}, \Rightarrow M_{p4} \text{ cutoff}$$

$$\bar{P}_L = \frac{v_o^2}{R_L} = \frac{(3.5)^2}{0.15} = 81.7 \text{ mW}$$

8.46

For $v_I = -1.5 \text{ V}$ and $v_O = 0$, $\Rightarrow v_{SG2} = 1.5 \text{ V} = v_{GS1}$

$$i_{D1} = i_{D2} = 0.5 = \left(\frac{0.1}{2} \right) \left(\frac{W}{L} \right)_1 (1.5 - 0.8)^2 \Rightarrow \left(\frac{W}{L} \right)_1 = 20.4$$

$$0.5 = \left(\frac{0.04}{2} \right) \left(\frac{W}{L} \right)_2 (1.5 - 0.8)^2 \Rightarrow \left(\frac{W}{L} \right)_2 = 51.0$$

$$v_{SG4} = 1.5 \text{ V}, \quad 0.2 = \left(\frac{0.04}{2} \right) \left(\frac{W}{L} \right)_4 (1.5 - 0.8)^2 \Rightarrow \left(\frac{W}{L} \right)_4 = 20.4$$

$$0.2 = \left(\frac{0.1}{2} \right) \left(\frac{W}{L} \right)_3 (1.5 - 0.8)^2 \Rightarrow \left(\frac{W}{L} \right)_3 = 8.16$$

8.47

For $v_0 = 0$

$$I_Q = I_{C3} + I_{C2} + I_{E1}$$

$$I_{B3} = I_{E2} = \left(\frac{1 + \beta_n}{\beta_n} \right) I_{C2} = \frac{I_{C3}}{\beta_n}$$

$$I_{C3} = (1 + \beta_n) I_{C2}$$

$$I_{B2} = I_{C1} = \left(\frac{\beta_p}{1 + \beta_p} \right) I_{E1} = \frac{I_{C2}}{\beta_n}$$

$$I_{C2} = \beta_n \left(\frac{\beta_p}{1 + \beta_p} \right) I_{E1}$$

$$I_{C3} = (1 + \beta_n) \beta_n \left(\frac{\beta_p}{1 + \beta_p} \right) I_{E1}$$

$$I_Q = (1 + \beta_n) \beta_n \left(\frac{\beta_p}{1 + \beta_p} \right) I_{E1} + \beta_n \left(\frac{\beta_p}{1 + \beta_p} \right) I_{E1} + I_{E1}$$

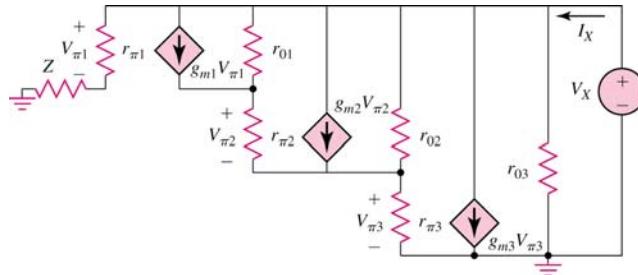
$$= (51)(50) \left(\frac{10}{11} \right) I_{E1} + (50) \left(\frac{10}{11} \right) I_{E1} + I_{E1}$$

$$I_Q = 2318.18 I_{E1} + 45.45 I_{E1} + I_{E1}$$

$$I_{E1} = 1.692 \mu\text{A} \Rightarrow \underline{I_{C1} = 1.534 \mu\text{A}}$$

$$I_{C2} = (50) \left(\frac{10}{11} \right) (1.692) \Rightarrow I_{C2} = 76.9 \mu\text{A}$$

$$I_{C3} = (51)(50) \left(\frac{10}{11} \right) (1.692) \Rightarrow \underline{I_{C3} = 3.92 \text{ mA}}$$



Because of $r_{\pi 1}$ and Z , neglect effect of r_0 . Then neglecting r_{01} , r_{02} and r_{03} , we find

$$I_x = g_{m3}V_{\pi 3} + g_{m2}V_{\pi 2} + g_{m1}V_{\pi 1} + \frac{V_x}{r_{\pi 1} + Z}$$

Now

$$V_{\pi 1} = \left(\frac{r_{\pi 1}}{r_{\pi 1} + Z} \right) V_x, \quad V_{\pi 2} \cong g_{m1}V_{\pi 1}r_{\pi 2}$$

and

$$\begin{aligned} V_{\pi 3} &= (g_{m1}V_{\pi 1} + g_{m2}V_{\pi 2})r_{\pi 3} \\ &= [g_{m1}V_{\pi 1} + g_{m2}(g_{m1}V_{\pi 1}r_{\pi 2})]r_{\pi 3} \\ V_{\pi 3} &= \left(\frac{r_{\pi 1}}{r_{\pi 1} + Z} \right) [g_{m1} + g_{m1}g_{m2}r_{\pi 2}]r_{\pi 3} \cdot V_x \\ V_{\pi 3} &= \frac{(\beta_1 + \beta_1\beta_2)r_{\pi 3}}{r_{\pi 1} + Z} \cdot V_x \end{aligned}$$

$$\text{and } V_{\pi 2} = g_{m1} \left(\frac{r_{\pi 1}}{r_{\pi 1} + Z} \right) r_{\pi 2} V_x = \left(\frac{\beta_1 r_{\pi 2}}{r_{\pi 1} + Z} \right) V_x$$

$$\text{Then } I_x = \frac{(\beta_1 + \beta_1\beta_2)\beta_3}{r_{\pi 1} + Z} \cdot V_x + \frac{\beta_1\beta_2}{r_{\pi 1} + Z} \cdot V_x + \frac{\beta_1}{r_{\pi 1} + Z} \cdot V_x + \frac{V_x}{r_{\pi 1} + Z}$$

Then

$$R_o = \frac{V_x}{I_x} = \frac{r_{\pi 1} + Z}{1 + \beta_1 + \beta_1\beta_2 + (\beta_1 + \beta_1\beta_2)\beta_3}$$

$$r_{\pi 1} = \frac{(10)(0.026)}{1.534} = 0.169 \text{ M}\Omega$$

$$Z = 25 \text{ k}\Omega$$

Then

$$R_o = \frac{169 + 25}{1 + (10) + (10)(50) + [10 + (10)(50)](50)}$$

$$R_o = \frac{194}{26,011} = 0.00746 \text{ k}\Omega \text{ or } R_o = 7.46 \Omega$$

8.48

- a Neglect base currents.

$$\begin{aligned} V_{BB} &= 2V_D = 2V_T \ln \left(\frac{I_{\text{Bias}}}{I_S} \right) \\ &= 2(0.026) \ln \left(\frac{5 \times 10^{-3}}{10^{-13}} \right) \Rightarrow V_{BB} = 1.281 \text{ V} \end{aligned}$$

$$V_{BE1} + V_{EB3} = V_{BB}$$

$$I_{E1} = I_{E3} + I_{C2}$$

$$I_{B2} = I_{C3} = \left(\frac{\beta_p}{1 + \beta_p} \right) I_{E3}$$

$$I_{C2} = \beta_n I_{B2} = \beta_n \left(\frac{\beta_p}{1 + \beta_p} \right) I_{E3}$$

$$I_{E1} = I_{E3} + \beta_n \left(\frac{\beta_p}{1 + \beta_p} \right) I_{E3}$$

$$I_{E1} = I_{E3} \left[1 + \beta_n \left(\frac{\beta_p}{1 + \beta_p} \right) \right]$$

$$\left(\frac{1 + \beta_n}{\beta_n} \right) I_{C1} = \left(\frac{1 + \beta_p}{\beta_p} \right) I_{C3} \left[1 + \beta_n \left(\frac{\beta_p}{1 + \beta_p} \right) \right]$$

$$V_{BE1} = V_T \ln \left[\frac{I_{C1}}{I_S} \right], \quad V_{EB3} = V_T \ln \left[\frac{I_{C3}}{I_S} \right]$$

$$(1.01) I_{C1} = \left(\frac{21}{20} \right) I_{C3} \left[1 + (100) \left(\frac{20}{21} \right) \right]$$

$$= I_{C3} \left[\frac{21}{20} + 100 \right] = 101.05 I_{C3}$$

$$I_{C1} = 100.05 I_{C3}$$

$$V_T \ln \left(\frac{100.05 I_{C3}}{I_S} \right) + V_T \ln \left(\frac{I_{C3}}{I_S} \right) = V_{BB}$$

$$V_T \ln \left(\frac{100.05 I_{C3}^2}{I_S^2} \right) = V_{BB}$$

$$\frac{100.05 I_{C3}^2}{I_S^2} = \exp \left(\frac{V_{BB}}{V_T} \right)$$

$$I_{C3} = \frac{I_S}{\sqrt{100.05}} \sqrt{\exp \left(\frac{V_{BB}}{V_T} \right)} = \frac{0.4995 \text{ mA}}{I_{C3}}$$

Then $I_{E3} = 0.5245 \text{ mA}$

Now $I_{C1} = 100.05 I_{C3} = \underline{49.97 \text{ mA}} = I_{C1}$

$$I_{C2} = (100) \left(\frac{20}{21} \right) (0.5245) = \underline{49.95 \text{ mA}} = I_{C2}$$

$$V_{BE1} = V_T \ln \left(\frac{I_{C1}}{I_S} \right) = 0.026 \ln \left(\frac{49.97 \times 10^{-3}}{10^{-13}} \right) \\ = 0.70037$$

$$V_{EB3} = V_T \ln \left(\frac{I_{C3}}{I_S} \right) = 0.026 \ln \left(\frac{0.4995 \times 10^{-3}}{10^{-13}} \right) \\ = 0.58062$$

Note: $V_{BE1} + V_{EB3} = 0.70037 + 0.58062 = 1.28099$

$$= V_{BB}$$

b.

$$v_0 = 10 \text{ V} \Rightarrow i_{E1} \approx \frac{10}{100} = 0.10 \text{ A} = i_{C1}$$

$$i_{B1} = \frac{100}{100} = 1 \text{ mA}$$

$$V_{BB} = 2(0.026) \ln\left(\frac{4 \times 10^{-3}}{10^{-13}}\right) = 1.2694 \text{ V}$$

$$V_{BE1} = (0.026) \ln\left(\frac{0.1}{10^{-13}}\right) = 0.7184$$

$$V_{EB3} = 1.2694 - 0.7184 = 0.55099 \text{ V}$$

$$I_{C3} = 10^{-13} \exp\left(\frac{0.55099}{0.026}\right) = 0.1598 \text{ mA}$$

$$\overline{P}_L = \frac{V_0^2}{R_L} = \frac{(10)^2}{100} \Rightarrow \overline{P}_L = 1 \text{ W}$$

$$P_{Q1} = i_{C1} \cdot v_{CE1} = (0.1)(12 - 10) \Rightarrow P_{Q1} = 0.2 \text{ W}$$

$$P_{Q3} = i_{C3} \cdot v_{EC3} = (0.1598)(10 - [0.7 - 12]) \Rightarrow P_{Q3} = 3.40 \text{ mW}$$

$$i_{C2} = (100)(i_{C3}) = (100)(0.1598) = 15.98 \text{ mA}$$

$$P_{Q2} = i_{C2} \cdot v_{CE2} = (15.98)(10 - [-12]) \Rightarrow P_{Q2} = 0.352 \text{ W}$$

8.49

a.

$$V_{BB} = 3(0.026) \ln\left(\frac{10 \times 10^{-3}}{2 \times 10^{-12}}\right) \Rightarrow V_{BB} = 1.74195 \text{ V}$$

$$V_{BE1} + V_{BE2} + V_{EB3} = V_{BB}$$

$$I_{C1} \approx \frac{I_{C2}}{\beta_n}, I_{C3} \approx \frac{I_{C2}}{\beta_n^2}$$

$$V_T \ln\left(\frac{I_{C1}}{I_S}\right) + V_T \ln\left(\frac{I_{C2}}{I_S}\right) + V_T \ln\left(\frac{I_{C3}}{I_S}\right) = V_{BB}$$

$$V_T \ln\left[\frac{I_{C2}^3}{\beta_n^3 I_S^3}\right] = V_{BB}$$

$$I_{C2} = \beta_n I_S \sqrt[3]{\exp\left(\frac{V_{BB}}{V_T}\right)}$$

$$= (20)(20 \times 10^{-12}) \sqrt[3]{\exp\left(\frac{1.74195}{0.026}\right)}$$

$$I_{C2} = 0.20 \text{ A}, I_{C1} \approx 10 \text{ mA}, I_{C3} \approx 0.5 \text{ mA}$$

$$V_{BE1} = (0.026) \ln\left(\frac{10 \times 10^{-3}}{2 \times 10^{-12}}\right) \Rightarrow V_{BE1} = 0.58065 \text{ V}$$

$$V_{BE2} = (0.026) \ln\left(\frac{0.2}{2 \times 10^{-12}}\right) \Rightarrow V_{BE2} = 0.6585 \text{ V}$$

$$V_{EB3} = (0.026) \ln\left(\frac{0.5 \times 10^{-3}}{2 \times 10^{-12}}\right) \Rightarrow V_{EB3} = 0.50276 \text{ V}$$

b.

$$\overline{P_L} = 10 \text{ W} = \frac{1}{2} \cdot \frac{V_0^2}{R_L} = \frac{1}{2} \cdot \frac{V_0^2}{20} \Rightarrow V_0 (\max) = 20 \text{ V}$$

For $v_0 (\max)$:

$$P_L = \frac{v_0^2}{R_L} = \frac{(20)^2}{20} \Rightarrow P_L = 20 \text{ W}$$

$$i_0 (\max) = -\frac{20}{20} = -1 \text{ A}$$

$$i_{C5} + i_{C4} + i_{E3} = -i_o (\max) = 1 \text{ A}$$

$$i_{C5} + \frac{i_{C5}}{\beta_n} \cdot \left(\frac{\beta_n}{1 + \beta_n} \right) + \frac{i_{C4}}{\beta_n} \left(\frac{1 + \beta_p}{\beta_p} \right) = 1$$

$$i_{C5} + \frac{i_{C5}}{\beta_n} \left(\frac{\beta_n}{1 + \beta_n} \right) + \frac{i_{C5}}{\beta_n} \left(\frac{\beta_n}{1 + \beta_n} \right) \left[\frac{1}{\beta_n} \left(\frac{1 + \beta_p}{\beta_p} \right) \right] = 1$$

$$i_{C5} \left[1 + \frac{1}{20} \left(\frac{20}{21} \right) \right] + \left(\frac{1}{21} \right) \left[\frac{1}{20} \left(\frac{6}{5} \right) \right] = 1$$

$$i_{C5} (1.05048) = 1 \quad \underline{i_{C5} = 0.952 \text{ A}}$$

$$\underline{i_{C4} = 0.0453 \text{ A}}$$

$$\underline{i_{E3} = 0.00272 \text{ A}}$$

$$i_{C3} = 0.00272 \left(\frac{5}{6} \right)$$

$$= \underline{0.002267 \text{ A}}$$

$$V_{EB3} = (0.026) \ln \left(\frac{2.267 \times 10^{-3}}{2 \times 10^{-12}} \right) = 0.54206 \text{ V}$$

$$V_{BE1} + V_{BE2} = 1.74195 - 0.54206 = 1.19989$$

$$V_T \ln \left(\frac{I_{C2}}{\beta_n I_S} \right) + V_T \ln \left(\frac{I_{C2}}{I_S} \right) = 1.19989$$

$$i_{C2} = \sqrt{\beta_n} \cdot I_S \sqrt{\exp \left(\frac{1.19989}{0.026} \right)}$$

$$= \sqrt{20} (18.83) \text{ mA}$$

$$\underline{i_{C2} = 93.9 \text{ mA}}$$

$$i_{C1} = \frac{i_{C2}}{\beta_n} \left(\frac{\beta_n}{1 + \beta_n} \right) = \frac{93.9}{21} = \underline{4.47 \text{ mA}}$$

$$P_{Q2} = I_{C2} (24 - (-20)) = (0.0939)(44) = \underline{4.13 \text{ W}}$$

$$P_{Q5} = (0.952)(-10 - (-24)) = \underline{13.3 \text{ W}}$$

Chapter 9

9.1

(a)

$$v_o = A_d(v_2 - v_1)$$

$$1 = A_d(10^{-3} - (-10^{-3})) \Rightarrow A_d = 500$$

(b)

$$1 = 500(v_2 - 10^{-3}) = 1 + 0.5 = 500v_2$$

$$\underline{v_2 = 3 \text{ mV}}$$

(c)

$$5 = 500(1 - v_1) \Rightarrow 500v_1 = 495$$

$$\underline{v_1 = 0.990 \text{ V}}$$

(d) $v_o = 0$

(e)

$$-3 = 500(v_2 - (-0.5))$$

$$-250 - 3 = 500v_2$$

$$\underline{v_2 = -0.506 \text{ V}}$$

9.2

(a) $v_2 = \frac{v_o}{A_{od}} = \frac{-2}{10^4} = -2 \times 10^{-4} \text{ V}$

$$v_2 = \left(\frac{1}{1+2000} \right) \cdot v_I$$

$$-2 \times 10^{-4} = \left(\frac{1}{2001} \right) \cdot v_I \Rightarrow v_I = -0.4002 \text{ V}$$

(b) $v_2 = \left(\frac{1}{1+2000} \right) \cdot v_I$

$$v_2 = \left(\frac{1}{2001} \right)(2) = 0.9995 \times 10^{-3} \text{ V}$$

$$v_o = 1 = A_{od}v_2 = A_{od}(0.9995 \times 10^{-3}) \Rightarrow A_{od} = 1000.5$$

9.3

(a) $v_o = A_{od}(v_2 - v_1) = (5 \times 10^3)(2.0000 - 2.0010) = -5 \text{ V}$

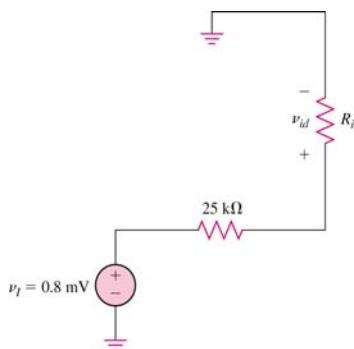
(b) $v_o = A_{od}(v_2 - v_1)$

$$-3.000 = (2 \times 10^4)(3.0025 - v_1) \Rightarrow v_1 = 3.00265 \text{ V}$$

(c) $v_o = A_{od}(v_2 - v_1)$

$$1.80 = A_{od}(0.01 - (-0.01)) \times 10^{-3} \Rightarrow A_{od} = 9 \times 10^4$$

9.4



$$v_{id} = \left(\frac{R_i}{R_i + 25} \right) v_I$$

$$0.790 = \left(\frac{R_i}{R_i + 25} \right) (0.80)$$

$$0.9875(R_i + 25) = R_i$$

$$24.6875 = 0.0125R_i$$

$$R_i = 1975 \text{ K}$$

9.5

$$(a) A_v = \frac{-R_2}{R_1} = \frac{-200}{20} = -10$$

$$(b) A_v = \frac{-120}{40} = -3$$

$$(c) A_v = \frac{-40}{40} = -1$$

9.6

$$\left. \begin{array}{l} A_v = -\frac{200}{20} = -10 \\ \text{and} \\ R_i = 20 \text{ k}\Omega \end{array} \right\} \text{for each case}$$

9.7

a.

$$A_v = -\frac{100}{10} = -10$$

$$R_i = R_l = 10 \text{ k}\Omega$$

b.

$$A_v = -\frac{100||100}{10} = -5$$

$$R_i = R_l = 10 \text{ k}\Omega$$

c.

$$A_v = -\frac{100}{10+10} = -5$$

$$R_i = 10+10 = 20 \text{ k}\Omega$$

9.8

$$(a) |i_2| = \left| \frac{0 - v_o}{R_2} \right| \Rightarrow R_2 = \frac{4}{20 \times 10^{-6}} \Rightarrow R_2 = 200 \text{ k}\Omega$$

$$A_v = \frac{-R_2}{R_1} \Rightarrow -12 = \frac{-200}{R_1} \Rightarrow R_1 = 16.67 \text{ k}\Omega$$

$$(b) i_2 = \frac{0 - v_o}{R_2} = \frac{0 - 1.5}{200 \times 10^3} \Rightarrow i_1 = i_2 = -7.5 \mu\text{A}$$

$$v_I = \frac{v_o}{A_v} = \frac{+1.5}{-12} \Rightarrow v_I = -0.125 \text{ V}$$

9.9

$$A_v = -\frac{R_2}{R_1}$$

$$(a) A_v = -10$$

$$(b) A_v = -1$$

$$(c) A_v = -0.20$$

$$(d) A_v = -10$$

$$(e) A_v = -2$$

$$(f) A_v = -1$$

9.10

$$(a) -3 = \frac{-R_2}{R_1} = \frac{-200}{R_1} \Rightarrow R_2 = 200 \text{ k}\Omega, R_1 = 66.67 \text{ k}\Omega$$

$$(b) -8 = \frac{-R_2}{R_1} = \frac{-200}{R_1} \Rightarrow R_2 = 200 \text{ k}\Omega, R_1 = 25 \text{ k}\Omega$$

$$(c) -20 = \frac{-R_2}{R_1} = \frac{-200}{R_1} \Rightarrow R_2 = 200 \text{ k}\Omega, R_1 = 10 \text{ k}\Omega$$

$$(d) -0.5 = \frac{-R_2}{R_1} = \frac{-R_2}{200} \Rightarrow R_2 = 100 \text{ k}\Omega, R_1 = 200 \text{ k}\Omega$$

9.11

$$(a) |i_1| = \frac{v_I}{R_1} \Rightarrow R_1 = \frac{0.25}{50 \times 10^{-6}} \Rightarrow R_1 = 5 \text{ k}\Omega$$

$$A_v = \frac{-R_2}{R_1} \Rightarrow -6.5 = \frac{-R_2}{5} \Rightarrow R_2 = 32.5 \text{ k}\Omega$$

$$(b) \quad v_I = \frac{v_o}{A_v} = \frac{-4}{-6.5} = 0.6154 \text{ V}$$

$$i_1 = i_2 = \frac{4}{32.5} = 0.123 \text{ mA}$$

9.12

$$(a) \quad A_v = \frac{-R_2}{R_1}$$

$$-20 = \frac{-R_2}{25} \Rightarrow R_1 = 25 \text{ k}\Omega, \quad R_2 = 500 \text{ k}\Omega$$

$$(b) \quad -20 = \frac{-1000}{R_1} \Rightarrow R_1 = 50 \text{ k}\Omega, \quad R_2 = 1 \text{ M}\Omega$$

$$(c) \quad \text{For (a), } i_1 = \frac{v_I}{R_1} = \frac{-0.2}{25} \Rightarrow i_1 = -8 \mu\text{A}$$

$$\text{For (b), } i_1 = \frac{v_I}{R_1} = \frac{-0.2}{50} \Rightarrow i_1 = -4 \mu\text{A}$$

9.13

a.

$$A_v = \frac{R_2}{R_1} \Rightarrow \frac{1.05R_2}{0.95R_1} = 1.105 \left(\frac{R_2}{R_1} \right)$$

$$\frac{0.95R_2}{1.05R_1} = 0.905 \left(\frac{R_2}{R_1} \right)$$

Deviation in gain is +10.5% and -9.5%

b.

$$A_v \Rightarrow \frac{1.01R_2}{0.99R_1} = 1.02 \left(\frac{R_2}{R_1} \right) \Rightarrow \frac{0.99R_2}{1.01R_1} = 0.98 \left(\frac{R_2}{R_1} \right)$$

Deviation in gain = ±2%

9.14

$$(a) \quad (i) \quad v_o = \frac{-R_2}{R_1} \cdot v_I = \frac{-15}{1} (-0.20) = 3 \text{ V}$$

$$(ii) \quad i_2 = \frac{0 - v_o}{R_2} = \frac{-3}{15} = -0.20 \text{ mA}$$

$$i_L = \frac{v_o}{R_L} = \frac{3}{4} = 0.75 \text{ mA}$$

$$i_o + i_2 = i_L \Rightarrow i_o = 0.75 - (-0.20) = 0.95 \text{ mA}$$

(b) (i) $v_o = \frac{-15}{1}(0.05) = -0.75 \text{ V}$

(ii) $i_2 = \frac{0 - (-0.75)}{15} = 0.05 \text{ mA}$

$$i_L = \frac{-0.75}{4} = -0.1875 \text{ mA}$$

$$i_o = i_L - i_2 = -0.1875 - 0.05 = -0.2375 \text{ mA}$$

(c) (i) $v_o = \frac{-15}{1}(8 \sin \omega t) \text{ (mV)} \Rightarrow v_o = -0.12 \sin \omega t \text{ (V)}$

(ii) $i_2 = \frac{0.12 \sin \omega t}{15} \Rightarrow i_2 = 8 \sin \omega t \text{ (\mu A)}$

$$i_L = \frac{-0.12 \sin \omega t}{4} \Rightarrow i_L = -30 \sin \omega t \text{ (\mu A)}$$

$$i_o = i_L - i_2 = -38 \sin \omega t \text{ (\mu A)}$$

9.15

$$A_v = -\frac{R_2}{R_1 + R_s}$$

$$A_v = -30 \pm 2.5\% \Rightarrow 29.25 \leq |A_v| \leq 30.75$$

$$\text{So } \frac{R_2}{R_1 + 2} = 29.25 \text{ and } \frac{R_2}{R_1 + 1} = 30.75$$

$$\text{We have } 29.25(R_1 + 2) = 30.75(R_1 + 1)$$

Which yields $R_1 = 18.5 \text{ k}\Omega$ and $R_2 = 599.6 \text{ k}\Omega$

For $v_I = 25 \text{ mV}$, then $0.731 \leq |v_o| \leq 0.769 \text{ V}$

9.16

$$v_{o1} = \frac{-R_2}{R_1} \cdot v_I = \frac{-80}{10}(-0.15) = 1.2 \text{ V}$$

$$v_o = \frac{-R_4}{R_3} \cdot v_{o1} = \frac{-100}{20}(1.2) = -6 \text{ V}$$

$$i_1 = i_2 = \frac{v_I}{R_1} = \frac{-(0.15)}{10} \Rightarrow i_1 = i_2 = -15 \text{ \mu A}$$

$$i_3 = i_4 = \frac{v_{o1}}{R_3} = \frac{1.2}{20} \Rightarrow i_3 = i_4 = 60 \text{ \mu A}$$

At v_{o1} : $i_2 + i_{o1} = i_3 \Rightarrow i_{o1} = 60 - (-15) = 75 \text{ \mu A}$; Out of Op-Amp

At v_o : $i_{o2} = i_4 = 60 \text{ \mu A}$; Into Op-Amp

9.17

$$\frac{v_o}{v_I} = \left(\frac{-R_2}{R_1} \right) \left(\frac{-R_4}{R_3} \right) = 100$$

For $v_I = 50 \text{ mV}$, $v_o = (100)(0.05) = 5 \text{ V}$

If $|i_4| = 50 \mu A$, $R_4 = \frac{5}{50 \times 10^{-6}} \Rightarrow R_4 = 100 k\Omega$

Set $R_1 = 10 k\Omega$

$$\text{Then } \frac{v_o}{v_i} = 100 = \left(\frac{R_2}{10} \right) \left(\frac{100}{R_3} \right) \Rightarrow \frac{R_2}{R_3} = 10$$

Set $R_2 = 100 k\Omega$, $R_3 = 10 k\Omega$

9.18

$$A_v = \left(\frac{-R_2}{R_1} \right) \left(\frac{-R_4}{R_3} \right) \left(\frac{-R_6}{R_5} \right) = -300$$

For $v_o = 6 V$, set $|i_6| = \frac{v_o}{R_6} \Rightarrow 60 \mu A = \frac{6}{R_6} \Rightarrow R_6 = 100 k\Omega$

Set $R_6 = 200 k\Omega$ so that $|i_6| = 30 \mu A$

Set $R_1 = 20 k\Omega$

$$\text{Now } 300 = \left(\frac{R_2}{20} \right) \left(\frac{R_4}{R_3} \right) \left(\frac{200}{R_5} \right)$$

For example, set $R_2 = 100 k\Omega$ and $R_5 = 20 k\Omega$

$$\text{Then } 300 = \left(\frac{100}{20} \right) \left(\frac{R_4}{R_3} \right) \left(\frac{200}{20} \right) = 50 \left(\frac{R_4}{R_3} \right)$$

Or $\frac{R_4}{R_3} = 6$, set $R_3 = 20 k\Omega$ and $R_4 = 120 k\Omega$

9.19

$$(a) v_o = \frac{-R_2}{R_1} \cdot v_i = -\left(\frac{22}{1} \right) (-0.40) = 8.8 V$$

$$(b) A_v = \frac{-R_2}{R_1} \cdot \frac{1}{1 + \frac{1}{A_{od}} \left(1 + \frac{R_2}{R_1} \right)} = -(22) \cdot \frac{1}{1 + \frac{1}{5 \times 10^3} (23)} = -21.8993$$

$$v_o = (-21.8993)(-0.40) = 8.7597 V$$

$$(c) A_v = -(0.998)(22) = -21.956$$

$$-21.956 = -(22) \cdot \frac{1}{1 + \frac{1}{A_{od}} (23)} \Rightarrow A_{od} = 1.1477 \times 10^4$$

9.20

(a)

$$A_v = -\frac{R_2}{R_1} \cdot \frac{1}{\left[1 + \frac{1}{A_{od}} \left(1 + \frac{R_2}{R_1}\right)\right]}$$

$$= -\frac{100}{25} \cdot \frac{1}{\left[1 + \frac{1}{5 \times 10^3} (5)\right]}$$

$$A_v = -3.9960$$

$$(b) \quad v_o = -3.9960(1.00) \Rightarrow v_o = \underline{-3.9960 \text{ V}}$$

$$(c) \quad \frac{4 - 3.9960}{4} \times 100\% = \underline{0.10\%}$$

(d)

$$v_o = A_{od} (v_2 - v_1) = -A_{od} v_1$$

$$v_1 = -\frac{v_o}{A_{od}} = \frac{-(-3.9960)}{5 \times 10^{+3}}$$

$$\underline{v_1 = 0.7992 \text{ mV}}$$

9.21

$$(a) \quad A_v = \frac{-100}{10} \cdot \frac{1}{\left[1 + \frac{1}{7 \times 10^3} \left(1 + \frac{100}{10}\right)\right]} = -9.98431$$

$$v_I = \frac{v_o}{A_v} = \frac{7}{-9.9843} = -0.7011 \text{ V}$$

$$v_1 = \frac{-v_o}{A_v} = \frac{-7}{7 \times 10^3} \Rightarrow v_1 = -1 \text{ mV}$$

$$(b) \quad A_{od} = \frac{-v_o}{v_1} = \frac{-(-5)}{0.2 \times 10^{-3}} = 2.5 \times 10^4$$

$$A_v = (-10) \cdot \frac{1}{\left[1 + \frac{1}{2.5 \times 10^4} (11)\right]} = -9.9956$$

$$v_I = \frac{v_o}{A_v} = \frac{-5}{-9.9956} = 0.50022 \text{ V}$$

9.22

$$(a) \quad A_v = \frac{-R_2}{R_1} \left(1 + \frac{R_3}{R_4} + \frac{R_3}{R_2}\right) = \frac{-50}{10} \left(1 + \frac{50}{5} + \frac{50}{50}\right) = -60$$

$$(b) \quad (i) \quad -100 = -5 \left(1 + \frac{50}{R_4} + \frac{50}{50}\right) \Rightarrow R_4 = 2.78 \text{ k}\Omega$$

$$(ii) \quad -150 = -5 \left(1 + \frac{50}{R_4} + \frac{50}{50}\right) \Rightarrow R_4 = 1.79 \text{ k}\Omega$$

9.23

a.

$$A_v = -\frac{R_2}{R_l} \left(1 + \frac{R_3}{R_4} + \frac{R_3}{R_2} \right)$$

$$R_l = 500 \text{ k}\Omega$$

$$80 = \frac{R_2}{500} \left(1 + \frac{R_3}{R_4} + \frac{R_3}{R_2} \right)$$

$$\text{Set } R_2 = R_3 = 500 \text{ k}\Omega$$

$$80 = 1 \left(1 + \frac{500}{R_4} + 1 \right) = 2 + \frac{500}{R_4} \Rightarrow R_4 = 6.41 \text{ k}\Omega$$

b.

$$\text{For } v_I = -0.05 \text{ V}$$

$$i_1 = i_2 = \frac{-0.05}{500 \text{ k}\Omega} \Rightarrow i_1 = i_2 = -0.1 \mu\text{A}$$

$$v_X = -i_2 R_2 = -(-0.1 \times 10^{-6})(500 \times 10^3) = 0.05$$

$$i_4 = -\frac{v_X}{R_4} = -\frac{0.05}{6.41} \Rightarrow i_4 = -7.80 \mu\text{A}$$

$$i_3 = i_2 + i_4 = -0.1 - 7.80 \Rightarrow i_3 = -7.90 \mu\text{A}$$

9.24

(a)

$$A_v = -1000 = \frac{-R_2}{R_l} = \frac{-500}{R_l}$$

$$\underline{R_l = 0.5 \text{ K}}$$

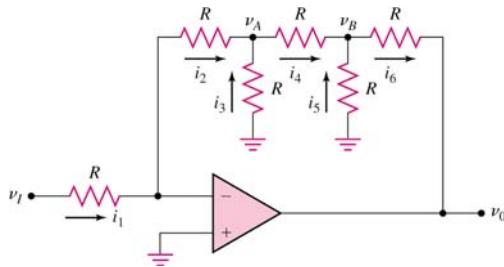
(b)

$$A_v = \frac{-R_2}{R_l} \left(1 + \frac{R_3}{R_4} + \frac{R_3}{R_2} \right)$$

$$-1000 = \frac{-250}{R_l} \left(1 + \frac{500}{250} + \frac{500}{250} \right) = \frac{-1250}{R_l}$$

$$\underline{R_l = 1.25 \text{ K}}$$

9.25



$$\begin{aligned}
 i_1 &= \frac{v_I}{R} = i_2 \\
 v_A &= -i_2 R = -\left(\frac{v_I}{R}\right) R = -v_I \\
 i_3 &= -\frac{v_A}{R} = \frac{v_I}{R} \\
 i_4 &= i_2 + i_3 = -\frac{v_A}{R} - \frac{v_A}{R} = -\frac{2v_A}{R} = \frac{2v_I}{R} \\
 v_B &= v_A - i_4 R = -v_I - \left(\frac{2v_I}{R}\right)(R) = -3v_I \\
 i_5 &= -\frac{v_B}{R} = -\frac{(-3v_I)}{R} = \frac{3v_I}{R} \\
 i_6 &= i_4 + i_5 = \frac{2v_I}{R} + \frac{3v_I}{R} = \frac{5v_I}{R} \\
 v_0 &= v_B - i_6 R = -3v_I - \left(\frac{5v_I}{R}\right) R \Rightarrow \underline{\underline{\frac{v_0}{v_I} = -8}}
 \end{aligned}$$

From Figure 9.12 $\Rightarrow \underline{\underline{A_v = -3}}$

9.26

$$\begin{aligned}
 \text{(a)} \quad A_v &= \frac{-R_2}{R_1} \cdot \frac{1}{1 + \frac{1}{A_{od}} \left(1 + \frac{R_2}{R_1} \right)} = \frac{-200}{20} \cdot \frac{1}{1 + \frac{1}{5 \times 10^4} \left(1 + \frac{200}{20} \right)} = -9.9978 \\
 \text{(b)} \quad v_I &= \frac{v_O}{A_v} = \frac{-4.80}{-9.9978} = 0.4801056 \text{ V} \\
 \text{(c)} \quad v_1 &= \frac{-v_O}{A_{od}} = \frac{-(-4.80)}{5 \times 10^4} \Rightarrow v_1 = 96 \mu \text{V} \\
 \text{(d)} \quad \frac{4.801056 - 4.8}{4.8} \times 100\% &= 0.022\%
 \end{aligned}$$

9.27

$$\begin{aligned}
 \text{(a)} \quad A_v &= \frac{-R_2}{R_1} \cdot \frac{1}{1 + \frac{1}{A_{od}} \left(1 + \frac{R_2}{R_1} \right)} = (-1) \cdot \frac{1}{1 + \frac{1}{2.5 \times 10^3} (2)} = -0.9992 \\
 \text{(i)} \quad v_O &= (-0.9992)(-0.8) = 0.7993605 \text{ V} \\
 \text{(ii)} \quad \frac{0.8 - 0.7993605}{0.8} \times 100\% &\cong 0.08\% \\
 \text{(b)} \quad A_v &= (-1) \cdot \frac{1}{1 + \frac{1}{2 \times 10^2} (2)} = -0.990099 \\
 \text{(i)} \quad v_O &= (-0.990099)(-0.8) = 0.79208 \text{ V} \\
 \text{(ii)} \quad \frac{0.8 - 0.79208}{0.9} \times 100\% &= 0.99\%
 \end{aligned}$$

9.28

$$(a) i_i = \frac{v_l}{R_1} = i_2 = -\frac{v_o}{R_2} \Rightarrow \frac{v_o}{v_l} = -\frac{R_2}{R_1}$$

(b)

$$i_2 = i_l = \frac{v_l}{R_1} = i_3 + \frac{v_o}{R_L} = i_3 + \frac{1}{R_L} \left(-\frac{R_2}{R_1} \cdot v_l \right)$$

$$\text{Then } i_3 = \frac{v_l}{R_1} \left(1 + \frac{R_2}{R_L} \right)$$

9.29

$$V_{X.\max} = \left(\frac{R_3 \| R_1}{R_3 \| R_1 + R_4} \right) \cdot V^+ = \left(\frac{0.1 \| 1}{0.1 \| 1 + 10} \right) (10) \Rightarrow V_{X.\max} = 0.09008 \text{ V}$$

$$|v_o| = \frac{R_2}{R_1} \cdot V_{X.\max}$$

$$10 = \frac{R_2}{R_1} (0.09008) \Rightarrow \frac{R_2}{R_1} = 111$$

$$\text{So } \underline{R_2 = 111 \text{ k}\Omega}$$

9.30

$$v_o = -\frac{R_F}{R_1} \cdot v_{I1} - \frac{R_F}{R_2} \cdot v_{I2} - \frac{R_F}{R_3} \cdot v_{I3} = -\frac{120}{40} \cdot v_{I1} - \frac{120}{20} \cdot v_{I2} - \frac{120}{60} \cdot v_{I3}$$

$$v_o = -3v_{I1} - 6v_{I2} - 2v_{I3}$$

$$(a) v_o = -3(-0.25) - 6(0.10) - 2(1.5) = -2.85 \text{ V}$$

$$(b) 0.5 = -3(v_{I1}) - 6(0.25) - 2(-1.2) \Rightarrow v_{I1} = 0.133 \text{ V}$$

9.31

$$(a) v_o = -2.5(1.2v_{I1} + 2.5v_{I2} + 0.25v_{I3}) = -3v_{I1} - 6.25v_{I2} - 0.625v_{I3}$$

$$\text{Then } \frac{R_F}{R_1} = 3, \quad \frac{R_F}{R_2} = 6.25, \quad \frac{R_F}{R_3} = 0.625$$

R_3 is the largest resistor, so set $R_3 = 400 \text{ k}\Omega$

Then $R_F = 250 \text{ k}\Omega$, $R_1 = 83.3 \text{ k}\Omega$, $R_2 = 40 \text{ k}\Omega$

$$(b) v_o = -3(-1) - 6.25(0.25) - 0.625(2) = 0.1875 \text{ V}$$

$$|i_F| = \frac{|v_o|}{R_F} = \frac{0.1875}{250} \Rightarrow |i_F| = 0.75 \mu \text{ A}$$

9.32

$$v_o = -2(v_{I1} + 3v_{I2}) = -2v_{I1} - 6v_{I2}$$

$$\text{Then } \frac{R_F}{R_1} = 2, \quad \frac{R_F}{R_2} = 6$$

For $v_{I1} = -1 \text{ V}$, $v_{I2} = -0.5 \text{ V}$

Then $v_o = -2(-1) - 6(-0.5) = 5 \text{ V}$

$$\text{Set } |i_F| = 80 \mu\text{A} = \frac{v_o}{R_F} = \frac{5}{R_F} \Rightarrow R_F = 62.5 \text{ k}\Omega$$

Then $R_1 = 31.25 \text{ k}\Omega$, $R_2 = 10.42 \text{ k}\Omega$

9.33

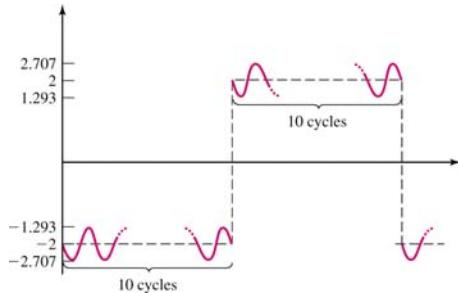
$$v_{I1} = (0.05)\sqrt{2} \sin(2\pi ft) = 0.0707 \sin(2\pi ft)$$

$$f = 1 \text{ kHz} \Rightarrow T = \frac{1}{10^3} \Rightarrow 1 \text{ ms} \quad v_{I2} \Rightarrow T_2 = \frac{1}{100} \Rightarrow 10 \text{ ms}$$

$$v_o = -\frac{R_F}{R_1} \cdot v_{I1} - \frac{R_F}{R_2} \cdot v_{I2} = -\frac{10}{1} \cdot v_{I1} - \frac{10}{5} \cdot v_{I2}$$

$$v_o = -(10)(0.0707 \sin(2\pi ft)) - (2)(\pm 1 \text{ V})$$

$$v_o = -0.707 \sin(2\pi ft) - (\pm 2 \text{ V})$$



9.34

$$v_o = -\frac{R_F}{R_1} \cdot v_{I1} - \frac{R_F}{R_2} \cdot v_{I2}$$

$$-0.5 \sin \omega t = -\frac{100}{R_1} (0.004 + 0.125 \sin \omega t) - \frac{100}{R_2} (-0.006)$$

$$\text{Set } -0.5 \sin \omega t = -\frac{100}{R_1} (0.125 \sin \omega t) \Rightarrow R_1 = 25 \text{ k}\Omega$$

$$\text{We have } 0 = -\frac{100}{R_1} (0.004) - \frac{100}{R_2} (-0.006)$$

$$0 = -\frac{0.4}{25} + \frac{0.6}{R_2} \Rightarrow R_2 = 37.5 \text{ k}\Omega$$

9.35

$$(a) \quad v_o = -2 \left[\frac{v_{I1}}{4} + 2v_{I2} + v_{I3} \right] = -\frac{1}{2}v_{I1} - 4v_{I2} - 2v_{I3}$$

$$\text{Then } \frac{R_F}{R_1} = \frac{1}{2}, \quad \frac{R_F}{R_2} = 4, \quad \frac{R_F}{R_3} = 2$$

Set $R_1 = 250 \text{ k}\Omega$, Then $R_F = 125 \text{ k}\Omega$, $R_2 = 31.25 \text{ k}\Omega$, $R_3 = 62.5 \text{ k}\Omega$

(b) For $v_{I1} = -2 \text{ V}$, $v_{I2} = 0$, $v_{I3} = -1 \text{ V}$

$$v_o = -\frac{1}{2}(-2) - 4(0) - 2(-1) = 3 \text{ V}$$

For $v_{I1} = 2 \text{ V}$, $v_{I2} = 0.5 \text{ V}$, $v_{I3} = 0$

$$v_o = -\frac{1}{2}(2) - 4(0.5) - 2(0) = -3 \text{ V}$$

Then $-3 \leq v_o \leq +3 \text{ V}$

$$|i_F|_{\max} = \frac{|v_o|_{\max}}{R_F} = \frac{3}{125} \Rightarrow |i_F|_{\max} = 24 \mu \text{A}$$

9.36

$$\begin{aligned} v_o &= -\frac{R_F}{R_1} \cdot v_{I1} - \frac{R_F}{R_2} \cdot v_{I2} - \frac{R_F}{R_3} \cdot v_{I3} \\ -6 \sin \omega t &= -\frac{R_F}{R_1}(2 + 2 \sin \omega t) - \frac{R_F}{R_2}(0.5 \sin \omega t) - \frac{R_F}{R_3}(-4) \end{aligned}$$

$$\text{We have } -\frac{R_F}{R_1}(2) + \frac{R_F}{R_3}(4) = 0 \Rightarrow R_3 = 2R_1$$

$$\text{Also } -6 = -\frac{R_F}{R_1}(2) - \frac{R_F}{R_2}(0.5)$$

$$\text{For } v_o = 6 \text{ V}, |i_F|_{\max} = 120 \mu \text{A} = \frac{6}{R_F} \Rightarrow R_F = 50 \text{ k}\Omega$$

$$\text{For } |v_{I1}|_{\max} = 4 \text{ V}, |i_1|_{\max} = 120 \mu \text{A} = \frac{4}{R_1} \Rightarrow R_1 = 33.33 \text{ k}\Omega \text{ and } R_3 = 2R_1 = 66.66 \text{ k}\Omega$$

$$\text{Now } 6 = \frac{(50)(2)}{R_1} + \frac{(50)(0.5)}{R_2} = \frac{100}{R_1} + \frac{25}{R_2} = \frac{100}{33.33} + \frac{25}{R_2} \Rightarrow R_2 = 8.33 \text{ k}\Omega$$

9.37

a.

$$v_0 = -\frac{R_F}{R_3} \cdot a_3(-5) - \frac{R_F}{R_2} \cdot a_2(-5) - \frac{R_F}{R_1} \cdot a_1(-5) - \frac{R_F}{R_0} \cdot a_0(-5)$$

$$\text{So } v_0 = \frac{R_F}{10} \left[\frac{a_3}{2} + \frac{a_2}{4} + \frac{a_1}{8} + \frac{a_0}{16} \right] (5)$$

$$\text{b. } v_0 = 2.5 = \frac{R_F}{10} \cdot \frac{1}{2} \cdot 5 \Rightarrow \underline{R_F = 10 \text{ k}\Omega}$$

c.

$$\text{i. } v_0 = \frac{10}{10} \cdot \frac{1}{16} \cdot 5 \Rightarrow \underline{v_0 = 0.3125 \text{ V}}$$

$$\text{ii. } v_0 = \frac{10}{10} \left[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \right] (5) \Rightarrow \underline{v_0 = 4.6875 \text{ V}}$$

9.38

(a) $v_o = \left(\frac{-10}{1} \right) \left(\frac{-20}{1} \right) \cdot v_{I1} - \left(\frac{20}{1} \right) \cdot v_{I2} = 200v_{I1} - 20v_{I2}$

(b) $v_o = (200)(5) - (20)(-25 - 50 \sin \omega t) = 1000 + 500 + 1000 \sin \omega t \text{ (mV)}$

$v_o = 1.5 + 1.0 \sin \omega t \text{ (V)}$

(c) For the 20 kΩ resistor:

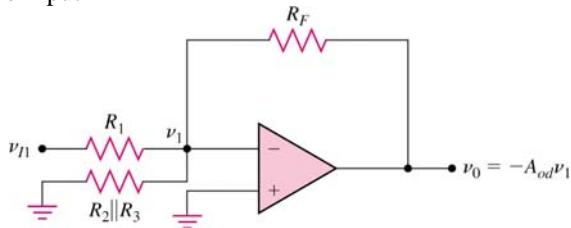
$$|i|_{\max} = \frac{2.5}{20} \Rightarrow |i|_{\max} = 0.125 \text{ mA}$$

For the 10 kΩ resistor:

$$|v_{o1}| = \left(\frac{10}{1} \right) (5) = 50 \text{ mV}, \quad |i|_{\max} = \frac{50 \text{ mV}}{10 \text{ k}\Omega} \Rightarrow |i|_{\max} = 5 \mu \text{A}$$

9.39

For one-input



$$v_1 = -\frac{v_o}{A_{od}}$$

$$\frac{v_{I1} - v_1}{R_1} = \frac{v_1}{R_2 \| R_3} + \frac{v_1 - v_o}{R_F}$$

$$\begin{aligned} \frac{v_{I1}}{R_1} &= v_1 \left[\frac{1}{R_1} + \frac{1}{R_2 \| R_3} + \frac{1}{R_F} \right] - \frac{v_o}{R_F} \\ &= -\frac{v_o}{A_{od}} \left[\frac{1}{R_1} + \frac{1}{R_2 \| R_3} + \frac{1}{R_F} \right] - \frac{v_o}{R_F} \\ &= -v_o \left[\frac{1}{A_{od} R_F} + \frac{1}{R_F} + \frac{1}{A_{od}} \left(\frac{1}{R_1} + \frac{1}{R_2 \| R_3} \right) \right] \\ &= -\frac{v_o}{R_F} \left[\frac{1}{A_{od}} + 1 + \frac{1}{A_{od}} \cdot \frac{R_F}{R_1 \| R_2 \| R_3} \right] \end{aligned}$$

$$v_o = -\frac{R_F}{R_1} \cdot v_{I1} \cdot \frac{1}{\left[1 + \frac{1}{A_{od}} \left(1 + \frac{R_F}{R_P} \right) \right]} \quad \text{where } R_P = R_1 \| R_2 \| R_3$$

Therefore, for three-inputs $v_o = \frac{-1}{1 + \frac{1}{A_{od}} \left(1 + \frac{R_F}{R_P} \right)} \cdot \left(\frac{R_F}{R_1} \cdot v_{I1} + \frac{R_F}{R_2} \cdot v_{I2} + \frac{R_F}{R_3} \cdot v_{I3} \right)$

9.40

$$(a) \quad A_v = \left(1 + \frac{R_2}{R_1}\right) = \left(1 + \frac{150}{15}\right) = 11$$

$$(b) \quad A_v = \left(1 + \frac{150}{50}\right) = 4$$

$$(c) \quad A_v = \left(1 + \frac{20}{50}\right) = 1.4$$

$$(d) \quad A_v = \left(1 + \frac{20}{20}\right) = 2$$

9.41

$$(a) \quad A_v = 15 = \left(1 + \frac{R_2}{R_1}\right) \Rightarrow \frac{R_2}{R_1} = 14$$

For $v_o = -7.5 \text{ V} \Rightarrow v_i = -0.5 \text{ V}$

$$|i| = 120 \mu \text{A} = \frac{7.5 - 0.5}{R_2} \Rightarrow R_2 = 58.33 \text{ k}\Omega, \quad R_1 = 4.17 \text{ k}\Omega$$

$$(b) \quad v_o = (15)(0.25) = 3.75 \text{ V}$$

$$i_1 = i_2 = \frac{0.25}{4.17} \Rightarrow i_1 = i_2 = 60 \mu \text{A}$$

9.42

$$(a) \quad A_v = \left(1 + \frac{R_2}{R_1}\right)$$

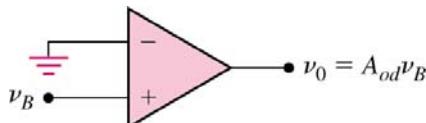
$$3 = \left(1 + \frac{R_2}{R_1}\right) \Rightarrow \frac{R_2}{R_1} = 2, \quad \text{Set } R_2 = 290 \text{ k}\Omega, \quad R_1 = 145 \text{ k}\Omega$$

$$(b) \quad 9 = \left(1 + \frac{R_2}{R_1}\right) \Rightarrow \frac{R_2}{R_1} = 8, \quad \text{Set } R_2 = 290 \text{ k}\Omega, \quad R_1 = 36.25 \text{ k}\Omega$$

$$(c) \quad 30 = \left(1 + \frac{R_2}{R_1}\right) \Rightarrow \frac{R_2}{R_1} = 29, \quad \text{Set } R_2 = 290 \text{ k}\Omega, \quad R_1 = 10 \text{ k}\Omega$$

$$(d) \quad 1 = \left(1 + \frac{R_2}{R_1}\right) \Rightarrow \frac{R_2}{R_1} = 0, \quad \text{Set } R_2 = 0, \quad R_1 = 290 \text{ k}\Omega$$

9.43



$$v_B = \left(\frac{1}{1+500}\right)v_I \quad v_0 = A_{od} \left(\frac{1}{501}\right)v_i$$

a. $2.5 = A_{od} \left(\frac{1}{501} \right) (5) \Rightarrow A_{od} = 250.5$

b. $v_0 = 5000 \left(\frac{1}{501} \right) (5) \Rightarrow v_0 = 49.9 \text{ V}$

9.44

$$v_0 = \left(1 + \frac{50}{50} \right) \left[\left(\frac{20}{20+40} \right) v_{I2} + \left(\frac{40}{20+40} \right) v_{II} \right]$$

$$\underline{v_0 = 1.33v_{II} + 0.667v_{I2}}$$

9.45

(a)

$$\frac{v_{II} - v_2}{20} + \frac{v_{I2} - v_2}{40} = \frac{v_2}{10}$$

$$v_o = \left(1 + \frac{100}{50} \right) v_2 = 3v_2$$

$$\text{Now } 2v_{II} - 2v_2 + v_{I2} - v_2 = 4v_2$$

$$2v_{II} + v_{I2} = 7v_2 = 7 \left(\frac{v_o}{3} \right)$$

$$\text{So } v_o = \frac{6}{7} \cdot v_{II} + \frac{3}{7} \cdot v_{I2}$$

(b) $v_o = \frac{6}{7} (0.2) + \left(\frac{3}{7} \right) (0.3) \Rightarrow \underline{v_o = 0.3 \text{ V}}$

(c) $v_o = \left(\frac{6}{7} \right) (0.25) + \left(\frac{3}{7} \right) (-0.4) \Rightarrow \underline{v_o = 42.86 \text{ mV}}$

9.46

(a) $v_2 = \left(\frac{R_4}{R_3 + R_4} \right) \cdot v_I$

$$v_o = \left(1 + \frac{R_2}{R_1} \right) \cdot v_2 = \left(1 + \frac{R_2}{R_1} \right) \cdot \frac{1}{\left(1 + \frac{R_3}{R_4} \right)} \cdot v_I$$

(b) $6 = \left(1 + \frac{R_2}{R_1} \right) \cdot \frac{1}{\left(1 + \frac{25}{50} \right)} \Rightarrow \left(1 + \frac{R_2}{R_1} \right) = 9 \Rightarrow \frac{R_2}{R_1} = 8$

Set $R_2 = 200 \text{ k}\Omega$, $R_1 = 25 \text{ k}\Omega$

9.47

(a)

$$\frac{v_o}{v_i} = \left(1 + \frac{50x}{(1-x)50} \right)$$

$$\frac{v_o}{v_i} = \left(1 + \frac{x}{1-x} \right) = \frac{1-x+x}{1-x}$$

$$A_v = \frac{v_o}{v_i} = \frac{1}{1-x}$$

(b) $1 \leq A_v \leq \infty$

(c) If $x = 1$, gain goes to infinity.

9.48

$$(a) v_x = \left(\frac{v_i}{R} \right) (2R) + v_i = 3v_i$$

$$\frac{v_x - v_i}{2R} + \frac{v_x}{R} + \frac{v_x - v_o}{2R} = 0$$

$$v_x \left(\frac{1}{2R} + \frac{1}{R} + \frac{1}{2R} \right) - \frac{v_i}{2R} = \frac{v_o}{2R}$$

$$3v_i \left(\frac{2}{R} \right) - \frac{v_i}{2R} = \frac{v_o}{2R}$$

$$\text{so } \frac{v_o}{v_i} = 11$$

(b) For $v_i = 0.25$ V, $\Rightarrow v_o = 2.75$ V

(c) $R = 30$ kΩ, $v_i = -0.15$ V

$$\text{For } R_1: |i| = \frac{0.15}{30} \Rightarrow 5 \mu\text{A}$$

$$\text{For } R_2: |i| = 5 \mu\text{A}$$

$$v_x = 3v_i = -0.45$$
 V

$$\text{For } R_4: |i| = \frac{0.45}{30} \Rightarrow 15 \mu\text{A}$$

$$v_o = (11)(-0.15) = -1.65$$
 V

$$\text{For } R_3: |i| = \frac{1.65 - 0.45}{60} \Rightarrow 20 \mu\text{A}$$

9.49

(a) $\frac{v_o}{v_i} = 1$

(b) From Exercise TYU9.7

$$\frac{v_o}{v_i} = \frac{\left(1 + \frac{R_2}{R_1}\right)}{\left[1 + \frac{1}{A_{od}}\left(1 + \frac{R_2}{R_1}\right)\right]}$$

But $R_2 = 0$, $R_1 = \infty$

$$\frac{v_o}{v_i} = \frac{1}{1 + \frac{1}{A_{od}}} = \frac{1}{1 + \frac{1}{1.5 \times 10^5}} \Rightarrow \frac{v_o}{v_i} = 0.999993$$

$$\text{Want } \frac{v_o}{v_i} = 0.990 = \frac{1}{1 + \frac{1}{A_{od}}} \Rightarrow A_{od} = 99$$

(b)

9.50

(a) $v_o = A_{od}(v_2 - v_1) = A_{od}(v_I - v_o)$
 $v_o(1 + A_{od}) = A_{od}v_I$

$$A_v = \frac{v_o}{v_I} = \frac{1}{1 + \frac{1}{A_{od}}} = \frac{1}{1 + \frac{1}{20}} = 0.9524$$

(b) $A_v = \frac{1}{1 + \frac{1}{200}} = 0.995$

(c) $A_v = \frac{1}{1 + \frac{1}{2000}} = 0.9995$

(d) $A_v = \frac{1}{1 + \frac{1}{20000}} = 0.99995$

9.51

(a) $A_{v1} = \frac{v_{o1}}{v_I} = \left(1 + \frac{R_2}{R_1}\right)$

$$A_{v2} = \frac{v_{o2}}{v_I} = -\left(1 + \frac{R_2}{R_1}\right)$$

(b) $v_{o1} = \left(1 + \frac{60}{20}\right)(-0.5) = -2 \text{ V}$

$$v_{o2} = -\left(1 + \frac{60}{20}\right)(-0.5) = 2 \text{ V}$$

$$(c) \quad v_{o1} - v_{o2} = 2 \left(1 + \frac{R_2}{R_1} \right) \cdot v_I = 2 \left(1 + \frac{60}{20} \right) (0.8) = 6.4 \text{ V}$$

9.52

$$(a) \quad i_L = \frac{v_I}{R_1}$$

(b)

$$v_{o1} = i_L R_L + v_I = i_L R_L + i_L R_1$$

$$v_{o1} (\text{max}) \cong 10 \text{ V} = i_L (1+9) = 10i_L$$

$$\text{So } i_L (\text{max}) \cong 1 \text{ mA}$$

$$\text{Then } v_I (\text{max}) \cong i_L R_1 = (1)(9) \Rightarrow v_I (\text{max}) \cong 9 \text{ V}$$

9.53

$$(a) \quad v_o = \left(\frac{20}{20+40} \right) \cdot v_I = (0.3333)v_I$$

$$(i) \quad v_o = 1 \text{ V}$$

$$(ii) \quad v_o = -1.67 \text{ V}$$

$$(b) \quad v_o = \left(\frac{20}{20+40} \right) \cdot v_I = (0.3333)v_I$$

$$(i) \quad v_o = 1 \text{ V}$$

$$(ii) \quad v_o = -1.67 \text{ V}$$

$$(c) \quad v_o = \left(1 + \frac{10}{10} \right) \left(\frac{6}{6+48} \right) \cdot v_I = (0.2222)v_I$$

$$(i) \quad v_o = 0.667 \text{ V}$$

$$(ii) \quad v_o = -1.111 \text{ V}$$

9.54

a.

$$R_{in} = \frac{v_1}{i_1} \text{ and } \frac{v_1 - v_0}{R_F} = i_1 \text{ and } v_0 = -A_{od}v_1$$

$$\text{So } i_1 = \frac{v_1 - (-A_{od}v_1)}{R_F} = \frac{v_1 (1 + A_{od})}{R_F}$$

$$\text{Then } R_{in} = \frac{v_1}{i_1} = \frac{R_F}{1 + A_{od}}$$

b.

$$i_1 = \left(\frac{R_s}{R_s + R_{in}} \right) i_s \text{ and } v_0 = -A_{od} \cdot \frac{R_F}{1 + A_{od}} \cdot i_1$$

$$\text{So } v_0 = -R_F \left(\frac{A_{od}}{1 + A_{od}} \right) \left(\frac{R_s}{R_s + R_{in}} \right) i_s$$

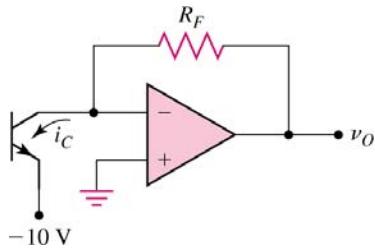
$$R_{in} = \frac{R_F}{1 + A_{od}} = \frac{10}{1001} = 0.009990$$

$$v_0 = -R_F \left(\frac{1000}{1001} \right) \left(\frac{R_s}{R_s + 0.009990} \right) i_s$$

$$\text{Want } \left(\frac{1000}{1001} \right) \left(\frac{R_s}{R_s + 0.009990} \right) \leq 0.990$$

which yields $R_s \geq 1.099 \text{ k}\Omega$

9.55



$$v_O = i_C R_F, \quad 0 \leq i_C \leq 8 \text{ mA}$$

For $v_O (\text{max}) = 8 \text{ V}$, Then $R_F = 1 \text{ k}\Omega$

9.56

$$i = \frac{v_I}{R} \text{ so } 1 = \frac{10}{R} \Rightarrow R = 10 \text{ k}\Omega$$

In the ideal op-amp, R_1 has no influence.

$$v_0 = \left(1 + \frac{R_2}{R} \right) v_I$$

Output voltage:

v_0 must remain within the bias voltages of the op-amp; the larger the R_2 , the smaller the range of input voltage v_I in which the output is valid.

9.57

$$(a) \quad i_L = \frac{-v_I}{R_2}; \quad R_2 = \frac{-(-5)}{5} = 1 \text{ k}\Omega$$

Set $\frac{R_F}{R_1 R_3} = \frac{1}{R_2}$; For example, set $R_1 = R_F = 10 \text{ k}\Omega$, $R_3 = 1 \text{ k}\Omega$

$$(b) \quad v_L = (5)(0.2) = 1 \text{ V} = v_1$$

$$i_1 = i_2 = \frac{v_I - v_1}{R_1} = \frac{-5 - 1}{10} = -0.6 \text{ mA}$$

$$v_o = v_1 - i_2 R_F = 1 - (-0.6)(10) = 7 \text{ V}$$

$$i_3 = \frac{v_o - v_L}{R_3} = \frac{7 - 1}{1} = 6 \text{ mA}$$

$$i_4 = \frac{v_L}{R_2} = \frac{1}{1} = 1 \text{ mA}$$

For the op-amp: $i_o + i_2 = i_3 \Rightarrow i_o = i_3 - i_2 = 6 - (-0.6) = 6.6 \text{ mA}$

9.58

(a)

$$i_1 = i_2 \text{ and } i_2 = \frac{v_x}{R_2} + i_D, \quad v_x = -i_2 R_F$$

$$\text{Then } i_1 = -i_1 \left(\frac{R_F}{R_2} \right) + i_D$$

$$\text{Or } i_D = i_1 \left(1 + \frac{R_F}{R_2} \right)$$

(b)

$$R_1 = \frac{v_I}{i_1} = \frac{5}{1} \Rightarrow R_1 = 5 \text{ k}\Omega$$

$$12 = (1) \left(1 + \frac{R_F}{R_2} \right) \Rightarrow \frac{R_F}{R_2} = 11$$

For example, $R_2 = 5 \text{ k}\Omega, R_F = 55 \text{ k}\Omega$

9.59

$$(1) \quad I_x = \frac{V_x}{R_2} + \frac{V_x - v_o}{R_3}$$

$$(2) \quad \frac{V_x}{R_1} + \frac{V_x - v_o}{R_F} = 0$$

$$\text{From (2)} \quad v_o = V_x \left(1 + \frac{R_F}{R_1} \right)$$

$$\text{Then (1)} \quad I_x = V_x \left(\frac{1}{R_2} + \frac{1}{R_3} \right) - \frac{1}{R_3} \cdot V_x \left(1 + \frac{R_F}{R_1} \right)$$

$$\frac{I_x}{V_x} = \frac{1}{R_0} = \frac{1}{R_2} + \frac{1}{R_3} - \frac{1}{R_3} - \frac{R_F}{R_1 R_3} = \frac{1}{R_2} - \frac{R_F}{R_1 R_3}$$

$$= \frac{R_1 R_3 - R_2 R_F}{R_1 R_2 R_3}$$

$$\text{or } R_o = \frac{R_1 R_2 R_3}{R_1 R_3 - R_2 R_F}$$

Note: If $\frac{R_F}{R_1 R_3} = \frac{1}{R_2} \Rightarrow R_2 R_F = R_1 R_3$ then $R_o = \infty$, which corresponds to an ideal current source.

9.60

(a) $R_{id} = R_1 + R_3 = 30 \text{ k}\Omega ; R_1 = R_3 = 15 \text{ k}\Omega$

$$\frac{R_2}{R_1} = \frac{R_4}{R_3} = 15 \Rightarrow R_2 = R_4 = 225 \text{ k}\Omega$$

(b) $v_o = i_L R_L = (0.25)(10) = 2.5 \text{ V}$

$$v_{I2} - v_{I1} = \frac{v_o}{A_d} = \frac{2.5}{15} = 0.1667 \text{ V}$$

(c) $v_o = A_d(v_{I2} - v_{I1}) = 15(1.2 - 1.5) = -4.5 \text{ V}$

$$i_L = \frac{v_o}{R_L} = \frac{-4.5}{10} = -0.45 \text{ mA}$$

(d) $v_o = (0.5)(10) = 5 \text{ V}$

$$v_{I2} - v_{I1} = \frac{v_o}{A_d} = \frac{5}{15} = 0.333 \text{ V}$$

$$v_{I1} = 2 - 0.333 = 1.667 \text{ V}$$

9.61

(a) $\frac{R_2}{R_1} = \frac{R_4}{R_3} = 40 ; \text{ Set } R_2 = R_4 = 250 \text{ k}\Omega , R_1 = R_3 = 6.25 \text{ k}\Omega$

(b) $\frac{R_2}{R_1} = \frac{R_4}{R_3} = 25 ; \text{ Set } R_2 = R_4 = 250 \text{ k}\Omega , R_1 = R_3 = 10 \text{ k}\Omega$

(c) $\frac{R_2}{R_1} = \frac{R_4}{R_3} = 5 ; \text{ Set } R_2 = R_4 = 250 \text{ k}\Omega , R_1 = R_3 = 50 \text{ k}\Omega$

(d) $\frac{R_2}{R_1} = \frac{R_4}{R_3} = 0.5 ; \text{ Set } R_2 = R_4 = 125 \text{ k}\Omega , R_1 = R_3 = 250 \text{ k}\Omega$

9.62

We have

$$v_o = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{R_4 / R_3}{1 + R_4 / R_3}\right) v_{I2} - \left(\frac{R_2}{R_1}\right) v_{I1} \text{ or } v_o = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{1}{1 + R_3 / R_4}\right) v_{I2} - \left(\frac{R_2}{R_1}\right) v_{I1}$$

Set $R_2 = 50(1+x)$, $R_1 = 50(1-x)$

$R_3 = 50(1-x)$, $R_4 = 50(1+x)$

$$v_o = \left[1 + \left(\frac{1+x}{1-x}\right)\right] \left[\frac{1}{1 + \left(\frac{1-x}{1+x}\right)}\right] v_{I2} - \left(\frac{1+x}{1-x}\right) v_{I1}$$

$$v_o = \left[\frac{1-x+(1+x)}{1-x}\right] \left[\frac{1+x}{1+x+(1-x)}\right] v_{I2} - \left(\frac{1+x}{1-x}\right) v_{I1}$$

$$= \left(\frac{1+x}{1-x}\right) v_{I2} - \left(\frac{1+x}{1-x}\right) v_{I1}$$

For $v_{I1} = v_{I2} \Rightarrow v_o = 0$

$$\text{Set } R_2 = 50(1+x) \quad R_1 = 50(1-x)$$

$$R_3 = 50(1+x) \quad R_4 = 50(1-x)$$

$$v_o = \left(1 + \frac{1+x}{1-x}\right) \left(\frac{1}{1 + \frac{1+x}{1-x}} \right) v_{I2} - \left(\frac{1+x}{1-x} \right) v_{I1}$$

$$= v_{I2} - \left(\frac{1+x}{1-x} \right) v_{I1}$$

$$v_{I1} = v_{I2} = v_{cm}$$

$$\frac{v_o}{v_{cm}} = 1 - \frac{1+x}{1-x} = \frac{1-x-(1+x)}{1-x} = \frac{-2x}{1-x}$$

$$\text{Set } R_2 = 50(1-x) \quad R_1 = 50(1+x)$$

$$R_3 = 50(1-x) \quad R_4 = 50(1+x)$$

$$v_o = \left(1 + \frac{1-x}{1+x}\right) \left(\frac{1}{1 + \frac{1-x}{1+x}} \right) v_{I2} - \left(\frac{1-x}{1+x} \right) v_{I1}$$

$$= \left(1 - \frac{1-x}{1+x}\right) v_{cm}$$

$$A_{cm} = \frac{1+x-(1-x)}{1+x} = \frac{2x}{1+x}$$

Worst common-mode gain

$$A_{cm} = \frac{-2x}{1-x}$$

(b)

$$\text{For } x = 0.01, \quad A_{cm} = \frac{-2x}{1-x} = \frac{-2(0.01)}{1-0.01} = -0.0202$$

$$\text{For } x = 0.02, \quad A_{cm} = \frac{-2(0.02)}{1-0.02} = -0.04082$$

$$\text{For } x = 0.05, \quad A_{cm} = \frac{-2(0.05)}{1-0.05} = -0.1053$$

$$\text{For this condition, set } v_{I2} = +\frac{1}{2}, \quad v_{I1} = -\frac{1}{2} \Rightarrow v_d = 1 \text{ V}$$

$$A_d = \frac{1}{2} \left[1 + \left(\frac{1+x}{1-x} \right) \right] = \frac{1}{2} \left[\frac{1-x+(1+x)}{1-x} \right] = \frac{1}{2} \cdot \frac{2}{1-x} = \frac{1}{1-x}$$

$$\text{For } x = 0.01 \quad A_d = 1.010 \quad CMRR_{dB} = 20 \log_{10} \left| \frac{1.010}{0.0202} \right| = 33.98 \text{ dB}$$

$$\text{For } x = 0.02, \quad A_d = \frac{1}{0.98} = 1.020 \quad CMRR_{dB} = 20 \log_{10} \left| \frac{1.020}{0.04082} \right| = 27.96 \text{ dB}$$

$$\text{For } x = 0.05, \quad A_d = \frac{1}{0.95} = 1.0526 \quad CMRR_{dB} = 20 \log_{10} \left| \frac{1.0526}{0.1053} \right| \approx 20 \text{ dB}$$

9.63

$$(a) v_o = 10(v_2 - v_1) = 10(1.4 - 1.8) = -4 \text{ V}$$

$$i_3 = i_4 = \frac{v_2}{R + 10R} = \frac{1.4}{(11)(10)} = 0.0127 \text{ mA}$$

$$v_x = v_y = \left(\frac{10}{11}\right) \cdot v_2 = \left(\frac{10}{11}\right)(1.4) = 1.273 \text{ V}$$

$$i_1 = i_2 = \frac{v_1 - v_x}{R} = \frac{1.8 - 1.273}{10} = 0.0527 \text{ mA}$$

$$(b) v_o = 10(v_2 - v_1) = 10(3.6 - 3.2) = 4 \text{ V}$$

$$v_x = v_y = \left(\frac{10}{11}\right)(3.6) = 3.273 \text{ V}$$

$$i_3 = i_4 = \frac{v_2}{(11)(10)} = \frac{3.6}{110} = 0.0327 \text{ mA}$$

$$i_1 = i_2 = \frac{3.2 - 3.273}{10} = -0.00727 \text{ mA}$$

$$(c) v_o = 10(-1.35 - (-1.20)) = -1.5 \text{ V}$$

$$i_3 = i_4 = \frac{-1.35}{(11)(10)} = -0.0123 \text{ mA}$$

$$v_x = v_y = \left(\frac{10}{11}\right)(-1.35) = -1.227 \text{ V}$$

$$i_1 = i_2 = \frac{-1.2 - (-1.227)}{10} = 0.00273 \text{ mA}$$

9.64

$$(a) I_E = (1 + \beta) \cdot I_B = (76)(1.2) = 91.2 \text{ mA}$$

$$R = \frac{10}{91.2} \Rightarrow R = 109.6 \Omega$$

$$(b) I_E = (101)(0.2) = 20.2 \text{ mA}$$

$$R = \frac{10}{20.2} = 0.495 \text{ k}\Omega$$

$$(c) I_E = \frac{6}{0.1096} = 54.74 \text{ mA}$$

$$I_o = \frac{54.74}{76} = 0.72 \text{ mA}$$

$$(d) I_E = \frac{4}{0.495} = 8.08 \text{ mA}$$

$$I_o = \frac{8.08}{101} = 0.080 \text{ mA}$$

9.65

$$(a) v_{o1} = \frac{-R_2}{R_1} \cdot v_{CM}$$

$$v_{o2} = \left(\frac{R_4}{R_3 + R_4} \right) \cdot v_{CM} \cdot \left(1 + \frac{R_2}{R_1} \right)$$

$$v_o = v_{o1} + v_{o2} = \left[\frac{\frac{R_4}{R_3} \cdot \left(1 + \frac{R_2}{R_1} \right) - \frac{R_2}{R_1}}{1 + \frac{R_4}{R_3}} \right] \cdot v_{CM}$$

$$A_{CM} = \frac{v_o}{v_{CM}} = \frac{\left(\frac{R_4}{R_3} \right) \left(1 + \frac{R_2}{R_1} \right) - \left(\frac{R_2}{R_1} \right) \left(1 + \frac{R_4}{R_3} \right)}{1 + \frac{R_4}{R_3}} = \frac{\frac{R_4}{R_3} - \frac{R_2}{R_1}}{1 + \frac{R_4}{R_3}}$$

$$(b) A_{CM} = \frac{\frac{86.4}{9.6} - \frac{62.4}{10.4}}{1 + \frac{86.4}{9.6}} = \frac{9 - 6}{10} = 0.3$$

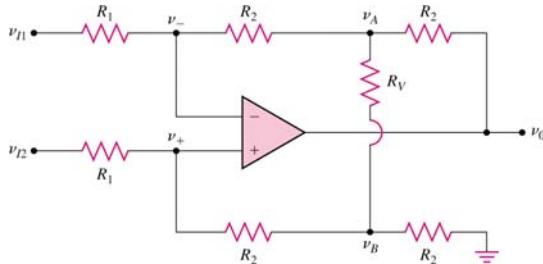
$$(c) A_{CM} = \frac{\frac{80.8}{19.8} - \frac{79.2}{20.2}}{1 + \frac{80.8}{19.8}} = 0.03149$$

or

$$A_{CM} = \frac{\frac{79.2}{20.2} - \frac{80.8}{19.8}}{1 + \frac{79.2}{20.2}} = -0.0325$$

$$\Rightarrow |A_{CM}|_{\max} = 0.0325$$

9.66



$$\frac{v_{I1} - v_A}{R_1 + R_2} = \frac{v_A - v_B}{R_V} + \frac{v_A - v_0}{R_2} \quad (1)$$

$$\frac{v_{I2} - v_B}{R_1 + R_2} = \frac{v_B - v_A}{R_V} + \frac{v_B}{R_2} \quad (2)$$

$$v_- = \left(\frac{R_1}{R_1 + R_2} \right) v_A + \left(\frac{R_2}{R_1 + R_2} \right) v_{I1} \quad (3)$$

$$v_+ = \left(\frac{R_1}{R_1 + R_2} \right) v_B + \left(\frac{R_2}{R_1 + R_2} \right) v_{I2} \quad (4)$$

Now $v_- = v_+ \Rightarrow R_1 v_A + R_2 v_{I1} = R_1 v_B + R_2 v_{I2}$

$$\text{So that } v_A = v_B + \frac{R_2}{R_1} (v_{I2} - v_{I1})$$

$$\frac{v_{I1}}{R_1 + R_2} = v_A \left(\frac{1}{R_1 + R_2} + \frac{1}{R_V} + \frac{1}{R_2} \right) - \frac{v_B}{R_V} - \frac{v_0}{R_2} \quad (1)$$

$$\frac{v_{I2}}{R_1 + R_2} = v_B \left(\frac{1}{R_1 + R_2} + \frac{1}{R_V} + \frac{1}{R_2} \right) - \frac{v_A}{R_V} \quad (2)$$

Then

$$\frac{v_{I1}}{R_1 + R_2} = v_B \left(\frac{1}{R_1 + R_2} + \frac{1}{R_V} + \frac{1}{R_2} \right) - \frac{v_B}{R_V} - \frac{v_0}{R_2} + \left(\frac{R_2}{R_1} \right) \left(\frac{1}{R_1 + R_2} + \frac{1}{R_V} + \frac{1}{R_2} \right) (v_{I2} - v_{I1}) \quad (1)$$

$$\frac{v_{I2}}{R_1 + R_2} = v_B \left(\frac{1}{R_1 + R_2} + \frac{1}{R_V} + \frac{1}{R_2} \right) - \frac{1}{R_V} \left[v_B + \frac{R_2}{R_1} (v_{I2} - v_{I1}) \right] \quad (2)$$

Subtract (2) from (1)

$$\frac{1}{R_1 + R_2} (v_{I1} - v_{I2}) = \left(\frac{R_2}{R_1} \right) \left(\frac{1}{R_1 + R_2} + \frac{1}{R_V} + \frac{1}{R_2} \right) (v_{I2} - v_{I1}) - \frac{v_0}{R_2} + \frac{1}{R_V} \cdot \frac{R_2}{R_1} (v_{I2} - v_{I1})$$

$$\frac{v_0}{R_2} = (v_{I2} - v_{I1}) \left\{ \left(\frac{R_2}{R_1} \right) \left(\frac{1}{R_1 + R_2} + \frac{1}{R_V} + \frac{1}{R_2} \right) + \frac{1}{R_1 + R_2} + \frac{1}{R_V} \cdot \frac{R_2}{R_1} \right\}$$

$$v_0 = (v_{I2} - v_{I1}) \left(\frac{R_2}{R_1} \right) \left\{ \frac{R_2}{R_1 + R_2} + \frac{R_2}{R_V} + 1 + \frac{R_1}{R_1 + R_2} + \frac{R_2}{R_V} \right\}$$

$$\underline{v_0 = \frac{2R_2}{R_1} \left(1 + \frac{R_2}{R_V} \right) (v_{I2} - v_{I1})}$$

9.67

$$(a) i_1 = \frac{v_{I1} - v_{I2}}{R_1} = \frac{(1.2 - 0.08 \sin \omega t) - (1.2 + 0.08 \sin \omega t)}{10} \Rightarrow i_1 = -16 \sin \omega t \text{ (\mu A)}$$

$$v_{O1} = (1.2 - 0.08 \sin \omega t) - (0.016 \sin \omega t)(40) = 1.2 - 0.72 \sin \omega t \text{ (V)}$$

$$v_{O2} = (1.2 + 0.08 \sin \omega t) - (-0.016 \sin \omega t)(40) = 1.2 + 0.72 \sin \omega t \text{ (V)}$$

$$v_O = \frac{R_4}{R_3} (v_{O2} - v_{O1}) = \left(\frac{120}{40} \right) (2)(0.72 \sin \omega t) = 4.32 \sin \omega t \text{ (V)}$$

(b) $i_i = \frac{(-0.65 + 0.05 \sin \omega t) - (-0.60 - 0.05 \sin \omega t)}{10}$

 $i_1 = -5 + 10 \sin \omega t \text{ } (\mu \text{A})$
 $v_{o1} = -0.65 + 0.05 \sin \omega t + (40)(-0.005 + 0.010 \sin \omega t) = -0.85 + 0.45 \sin \omega t \text{ (V)}$
 $v_{o2} = -0.60 - 0.05 \sin \omega t - (40)(-0.005 + 0.01 \sin \omega t) = -0.40 - 0.45 \sin \omega t \text{ (V)}$
 $v_o = \left(\frac{120}{40} \right) [(-0.40 - 0.45 \sin \omega t) - (-0.85 + 0.45 \sin \omega t)] = 1.35 - 2.7 \sin \omega t \text{ (V)}$

9.68

(a) $v_{OB} = \left(1 + \frac{40}{12} \right) v_I = 2.1667 \sin \omega t$

(b) $v_{OC} = -\frac{30}{12} v_I = -1.25 \sin \omega t$

(c) $v_o = v_{OB} - v_{OC} = 2.1667 \sin \omega t - (-1.25 \sin \omega t)$
 $v_o = 3.417 \sin \omega t$

(d) $\frac{v_o}{v_I} = \frac{3.417}{0.5} = 6.83$

9.69

(a) $i_o = \frac{v_{I1} - v_{I2}}{R}$

(b) $R = \frac{0.25 - (-0.25)}{5} \Rightarrow R = 100 \Omega$

(c) $v_{o1} = v_{I1} + i_o R_L = 0.25 + (5)(1) = 5.25 \text{ V}$
 $v_{o2} = v_{I2} = -0.25 \text{ V}$

(d) $i_o = \frac{v_{I1} - v_{I2}}{R} = \frac{1.25 - 1.75}{0.5} = -1 \text{ mA}$
 $v_{o1} = v_{I1} + i_o R_L = 1.25 - (1)(3) = -1.75 \text{ V}$
 $v_{o2} = v_{I2} = 1.75 \text{ V}$

9.70

$$A_d = \frac{v_o}{v_{I2} - v_{I1}} = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1} \right)$$
 $|v_o| = \frac{200}{50} \left(1 + \frac{2(115)}{R_1} \right) (0.06 \sin \omega t)$

For $|v_o| = 0.5 \quad \frac{230}{R_1} = 1.0833 \Rightarrow R_1 = 212.3 \text{ K}$

 $|v_o| = 8 \text{ V} \quad \frac{230}{R_1} = 32.33 \Rightarrow R_1 = 7.11 \text{ K} \Rightarrow R_{lf} = 7.11 \text{ K}, R_1(\text{potentiometer}) = 205.2 \text{ K}$

9.71

$$\text{For } v_o = 10 \text{ V}, \quad A_d = 200 \Rightarrow v_{I2} - v_{I1} = \frac{10}{200} = 0.05 \text{ V}$$

$$R_1(\text{fixed}) = \frac{0.05}{50 \times 10^{-6}} \Rightarrow R_1(\text{fixed}) = 1 \text{ k}\Omega$$

$$A_d = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1} \right)$$

$$200 = (2.5) \left(1 + \frac{2R_2}{1} \right) \Rightarrow R_2 = 39.5 \text{ k}\Omega$$

For $A_d = 5$

$$5 = (2.5) \left[1 + \frac{2(39.5)}{R_1} \right] \Rightarrow R_1 = 79 = R_1(\text{var}) + R_1(\text{fixed}) = R_1(\text{var}) + 1$$

$$R_1(\text{var}) = 78 \text{ k}\Omega$$

9.72

$$v_o(v_{o1}) = -\frac{R_4}{R'_3} \cdot v_{o1}$$

$$v_o(v_{o2}) = \left(\frac{R_4}{R_3 + R_4} \right) \left(1 + \frac{R_4}{R'_3} \right) \cdot v_{o2}$$

$$v_o = v_o(v_{o1}) + v_o(v_{o2}) \quad \text{and} \quad v_{o1} = v_{o2} \equiv v_{CM}$$

Then

$$A_{CM} = \frac{v_o}{v_{CM}} = \left(\frac{R_4}{R_3 + R_4} \right) \left(1 + \frac{R_4}{R'_3} \right) - \left(\frac{R_4}{R'_3} \right)$$

$$R_4 = 2R_3 = 60 \text{ k}\Omega, \quad R_3 = 30 \text{ k}\Omega, \quad R'_3 = 30 \text{ k}\Omega \pm 5\%$$

$$\text{For } R'_3 = 30 \text{ k}\Omega - 5\% = 28.5 \text{ k}\Omega$$

$$A_{CM} = \left(\frac{60}{60+30} \right) \left(1 + \frac{60}{28.5} \right) - \left(\frac{60}{28.5} \right) = -0.03509$$

$$\text{For } R'_3 = 30 \text{ k}\Omega + 5\% = 31.5 \text{ k}\Omega$$

$$A_{CM} = \left(\frac{60}{60+30} \right) \left(1 + \frac{60}{31.5} \right) - \left(\frac{60}{31.5} \right) = +0.03175$$

$$\text{Then } -0.03509 \leq A_{CM} \leq +0.03175$$

9.73

$$(a) \quad R_1 C_2 = (20 \times 10^3) (0.02 \times 10^{-6}) = 4 \times 10^{-4} \text{ s}$$

$$v_o = \frac{-1}{4 \times 10^{-4}} \int (0.25) \cos \omega t dt = \frac{-0.25}{(4 \times 10^{-4}) \omega} \cdot \sin \omega t$$

$$\text{For } |v_o| = 0.25 \Rightarrow (4 \times 10^{-4}) (2\pi f) = 1 \Rightarrow f = 398 \text{ Hz}$$

$$\text{Phase} = 90^\circ$$

$$(b) \quad (i) |v_o| = 1.5 = \frac{0.25}{2\pi f(4 \times 10^{-4})} \Rightarrow f = 66.3 \text{ Hz}$$

$$(ii) |v_o| = 0.15 = \frac{0.25}{2\pi f(4 \times 10^{-4})} \Rightarrow f = 663 \text{ Hz}$$

9.74

$$(a) \quad v_o = \frac{-1}{R_1 C_2} \int_0^t v_i(t') dt' = \frac{-1}{R_1 C_2} (0.25) t' \Big|_0^{1.2}$$

$$-5 = \frac{-(0.25)(1.2)}{R_1 C_2} \Rightarrow R_1 C_2 = 60 \text{ ms}$$

$$(b) \quad (i) \quad 0 = -5 + \frac{(0.10)}{0.06} \cdot t'' \Rightarrow t'' = 3 \text{ s}, \quad t = 4.2 \text{ s}$$

$$(ii) \quad 5 = -5 + \frac{(0.10)}{0.06} \cdot t'' \Rightarrow t'' = 6 \text{ s}, \quad t = 7.2 \text{ s}$$

9.75

$$(a) \quad A_v = \frac{-Z_2}{R_1}, \text{ where } Z_2 = R_2 \left| \frac{1}{j\omega C_2} \right. = \frac{R_2 \left(\frac{1}{j\omega C_2} \right)}{R_2 + \frac{1}{j\omega C_2}} = \frac{R_2}{1 + j\omega R_2 C_2}$$

$$A_v = \frac{-R_2}{R_1} \cdot \frac{1}{1 + j\omega R_2 C_2}$$

$$(b) \quad \text{At } \omega = 0, \quad A_v(0) = \frac{-R_2}{R_1}$$

$$(c) \quad |A_v| = \frac{R_2}{R_1} \cdot \frac{1}{\sqrt{1 + (\omega R_2 C_2)^2}}$$

$$\text{Set } 1 + (\omega R_2 C_2)^2 = 2 \Rightarrow \omega = \frac{1}{R_2 C_2} \Rightarrow f = \frac{1}{2\pi R_2 C_2}$$

9.76

$$(a) \quad R_1 = 20 \text{ k}\Omega$$

$$\frac{R_2}{R_1} = 15 \Rightarrow R_2 = 300 \text{ k}\Omega$$

$$\omega = \frac{1}{R_2 C_2} = 2\pi f$$

$$C_2 = \frac{1}{2\pi f R_2} = \frac{1}{2\pi (5 \times 10^3)(300 \times 10^3)} \Rightarrow C_2 = 106 \text{ pF}$$

(b) $R_1 = 15 \text{ k}\Omega$

$$\frac{R_2}{R_1} = 25 \Rightarrow R_2 = 375 \text{ k}\Omega$$

$$C_2 = \frac{1}{2\pi(15 \times 10^3)(375 \times 10^3)} \Rightarrow C_2 = 28.3 \text{ pF}$$

9.77

(a) $A_v = \frac{-R_2}{Z_1}$, where $Z_1 = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1}$

$$A_v = \frac{-j\omega R_2 C_1}{1 + j\omega R_1 C_1} = \frac{-R_2}{R_1} \cdot \frac{j\omega R_1 C_1}{1 + j\omega R_1 C_1}$$

(b) As $\omega \Rightarrow \infty$, $A_v = \frac{-R_2}{R_1}$

(c) $|A_v| = \frac{R_2}{R_1} \cdot \frac{\omega R_1 C_1}{\sqrt{1 + (\omega R_1 C_1)^2}}$

$$\text{Set } \frac{\omega R_1 C_1}{\sqrt{1 + (\omega R_1 C_1)^2}} = \frac{1}{\sqrt{2}} \Rightarrow \omega = \frac{1}{R_1 C_1} \Rightarrow f = \frac{1}{2\pi R_1 C_1}$$

9.78

(a) Set $R_2 = 350 \text{ k}\Omega$

$$\frac{R_2}{R_1} = 15 \Rightarrow R_1 = 23.33 \text{ k}\Omega$$

$$2\pi f = \frac{1}{R_1 C_1} \Rightarrow C_1 = \frac{1}{2\pi R_1 f} = \frac{1}{2\pi(23.33 \times 10^3)(20 \times 10^3)} \Rightarrow C_1 = 341 \text{ pF}$$

(b) Set $R_1 = 20 \text{ k}\Omega$

$$\frac{R_2}{R_1} = 25 \Rightarrow R_2 = 500 \text{ k}\Omega$$

$$C_1 = \frac{1}{2\pi(20 \times 10^3)(35 \times 10^3)} \Rightarrow C_1 = 227 \text{ pF}$$

9.79

Assuming the Zener diode is in breakdown,

$$v_o = -\frac{R_2}{R_1} \cdot V_z = -\frac{1}{1}(6.8) \Rightarrow v_o = -6.8 \text{ V}$$

$$i_2 = \frac{0 - v_o}{R_2} = \frac{0 - (-6.8)}{1} \Rightarrow i_2 = 6.8 \text{ mA}$$

$$i_z = \frac{10 - V_z}{R_s} - i_2 = \frac{10 - 6.8}{5.6} - 6.8 \Rightarrow i_z = -6.2 \text{ mA!!!}$$

Circuit is not in breakdown. Now

$$\frac{10 - 0}{R_s + R_1} = i_2 = \frac{10}{5.6 + 1} \Rightarrow i_2 = 1.52 \text{ mA}$$

$$v_o = -i_2 R_2 = -(1.52)(1) \Rightarrow v_o = -1.52 \text{ V}$$

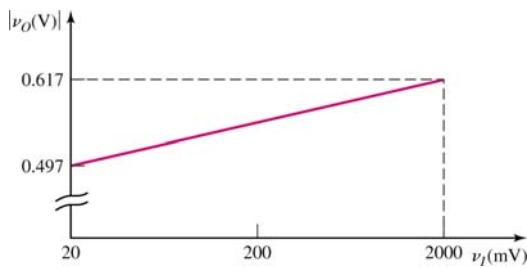
$$\underline{i_z = 0}$$

9.80

$$v_o = -V_T \ln\left(\frac{v_I}{I_s R_1}\right) = -\left(0.026\right) \ln\left[\frac{v_I}{\left(10^{-14}\right)\left(10^4\right)}\right] \Rightarrow v_o = -0.026 \ln\left(\frac{v_I}{10^{-10}}\right)$$

For $v_I = 20 \text{ mV}$, $|v_o| = 0.497 \text{ V}$

For $v_I = 2 \text{ V}$, $|v_o| = 0.617 \text{ V}$



9.81

$$v_0 = \left(\frac{333}{20} \right) (v_{01} - v_{02}) = 16.65 (v_{01} - v_{02})$$

$$v_{01} = -v_{BE1} = -V_T \ln \left(\frac{i_{C1}}{I_S} \right)$$

$$v_{02} = -v_{BE2} = -V_T \ln \left(\frac{i_{C2}}{I_S} \right)$$

$$v_{01} - v_{02} = -V_T \ln \left(\frac{i_{C1}}{i_{C2}} \right) = V_T \ln \left(\frac{i_{C2}}{i_{C1}} \right)$$

$$i_{C2} = \frac{v_2}{R_2}, \quad i_{C1} = \frac{v_1}{R_1}$$

$$\text{So } v_{01} - v_{02} = V_T \ln \left(\frac{v_2}{R_2} \cdot \frac{R_1}{v_1} \right)$$

Then

$$v_0 = (16.65)(0.026) \ln \left(\frac{v_2}{v_1} \cdot \frac{R_1}{R_2} \right)$$

$$v_0 = 0.4329 \ln \left(\frac{v_2}{v_1} \cdot \frac{R_1}{R_2} \right)$$

$$\ln(x) = \log_e(x) = [\log_{10}(x)] \cdot [\log_e(10)]$$

$$= 2.3026 \log_{10}(x)$$

$$\text{Then } v_0 \equiv (1.0) \log_{10} \left(\frac{v_2}{v_1} \cdot \frac{R_1}{R_2} \right)$$

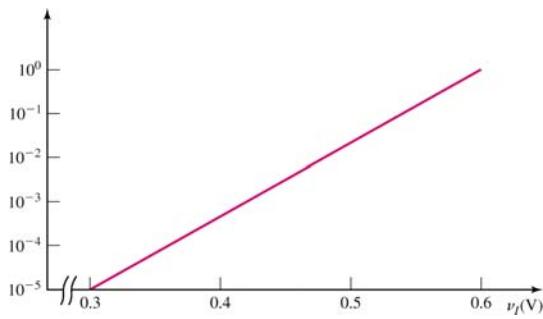
9.82

$$v_o = -I_s R \left(e^{v_I/V_T} \right) = -\left(10^{-14} \right) \left(10^4 \right) e^{v_I/V_T}$$

$$|v_o| = \left(10^{-10} \right) e^{v_I/0.026}$$

$$\text{For } v_I = 0.30 \text{ V, } |v_o| = 1.03 \times 10^{-5} \text{ V}$$

$$\text{For } v_I = 0.60 \text{ V, } |v_o| = 1.05 \text{ V}$$



9.83

From Figure 9.40

$$\begin{aligned} v_o &= -\frac{R_F}{R_1} \cdot v_{I2} - \frac{R_F}{R_2} \cdot v_{I4} + \left(1 + \frac{R_F}{R_N}\right) \left[\frac{R_P}{R_A} \cdot v_{I1} + \frac{R_P}{R_B} \cdot v_{I3} \right] \\ &= -10v_{I2} - v_{I4} + 2v_{I1} + 3v_{I3} \end{aligned}$$

$$\text{Then } \frac{R_F}{R_1} = 10, \quad \frac{R_F}{R_2} = 1, \quad \left(1 + \frac{R_F}{R_N}\right) \left(\frac{R_P}{R_A}\right) = 2, \quad \left(1 + \frac{R_F}{R_N}\right) \left(\frac{R_P}{R_B}\right) = 3$$

Set $R_F = 500 \text{ k}\Omega$, $R_1 = 50 \text{ k}\Omega$, $R_2 = 500 \text{ k}\Omega$

Now $R_N = R_1 \parallel R_2 = 50 \parallel 500 = 45.45 \text{ k}\Omega$

$$\text{Then } \left(1 + \frac{500}{45.45}\right) \left(\frac{R_P}{R_A}\right) = 12 \left(\frac{R_P}{R_A}\right) = 2, \quad \text{Also } \left(1 + \frac{500}{45.45}\right) \left(\frac{R_P}{R_B}\right) = 12 \left(\frac{R_P}{R_B}\right) = 3$$

$$\text{Let } R_A = 500 \text{ k}\Omega, \text{ then } R_B = \frac{2}{3} R_A = 333.3 \text{ k}\Omega$$

$$\text{Then } R_P = 83.33 \text{ k}\Omega = R_A \parallel R_B \parallel R_C$$

$$\text{We find } R_A \parallel R_B = 500 \parallel 333.3 = 200 \text{ k}\Omega$$

$$\text{So } 200 \parallel R_C = 83.33 \Rightarrow R_C = 142.8 \text{ k}\Omega$$

9.84

$$\begin{aligned} v_o &= \left(1 + \frac{R_F}{R_N}\right) \left[\frac{R_P}{R_A} \cdot v_{I1} + \frac{R_P}{R_B} \cdot v_{I2} + \frac{R_P}{R_C} \cdot v_{I3} \right] - \frac{R_R}{R_1} \cdot v_{I4} - \frac{R_F}{R_2} \cdot v_{I5} \\ &= 3v_{I1} + 1.5v_{I2} + 2v_{I3} - 4v_{I4} - 6v_{I5} \end{aligned}$$

$$\text{We have } \frac{R_F}{R_1} = 4, \quad \frac{R_F}{R_2} = 6; \quad \text{Set } R_F = 250 \text{ k}\Omega, \quad R_1 = 62.5 \text{ k}\Omega, \quad R_2 = 41.67 \text{ k}\Omega$$

$$\text{Now } R_N = R_1 \parallel R_2 = 62.5 \parallel 41.67 = 25 \text{ k}\Omega$$

$$\text{Also } \left(1 + \frac{R_F}{R_N}\right) = \left(1 + \frac{250}{25}\right) = 11$$

$$\text{Now } \frac{(11)R_P}{R_A} = 3, \quad \frac{(11)R_P}{R_B} = 1.5, \quad \frac{(11)R_P}{R_C} = 2 \Rightarrow \frac{R_A}{R_B} = \frac{1}{2}, \quad \frac{R_A}{R_C} = \frac{2}{3}$$

$$\text{Set } R_B = 250 \text{ k}\Omega, \quad R_A = 125 \text{ k}\Omega, \quad R_C = 187.5 \text{ k}\Omega$$

$$\text{This yields } R_P = 34.09 \text{ k}\Omega, \quad \text{We have } R_p = R_A \parallel R_B \parallel R_C \parallel R_D$$

$$\text{We find } R_A \parallel R_B \parallel R_C = 125 \parallel 250 \parallel 187.5 = 57.69 \text{ k}\Omega$$

$$\text{Then } 57.69 \parallel R_D = 34.09 \Rightarrow R_D = 83.3 \text{ k}\Omega$$

9.85

$$\frac{V_o}{V_z} = \left(1 + \frac{R_2}{R_1}\right) = \frac{12}{5.6} \Rightarrow \frac{R_2}{R_1} = 1.143$$

$$I_F = \frac{V_o - V_z}{R_F}; \quad \text{Set } I_F = I_Z(\text{min}) = 1.2 \text{ mA}$$

$$\text{Then } R_F = \frac{12 - 5.6}{1.2} = 5.33 \text{ k}\Omega$$

Set $I_{D1} = 0.15 \text{ mA}$

$$V'_z = V_z + V_\gamma = 5.6 + 0.7 = 6.3 \text{ V}$$

$$\text{Let } I_4 = 0.2 \text{ mA}, \Rightarrow R_4 = \frac{6.3}{0.2} = 31.5 \text{ k}\Omega$$

$$\text{Then } I_3 = 0.2 + 0.15 = 0.35 \text{ mA}$$

$$\text{So } R_3 = \frac{V_s - V'_z}{I_3} = \frac{10 - 6.3}{0.35} = 10.6 \text{ k}\Omega$$

9.86

$$R_1 = \frac{V_o - V_z}{I_z} = \frac{12 - 5.6}{2} = 3.2 \text{ k}\Omega$$

$$\frac{V_o}{V_z} = \left(1 + \frac{R_2}{R_3}\right) = \frac{12}{5.6} \Rightarrow \frac{R_2}{R_3} = 1.143$$

$$\text{Let } I_R = 2 \text{ mA}, \Rightarrow R_2 + R_3 = \frac{V_o}{I_R} = \frac{12}{2} = 6 \text{ k}\Omega$$

$$\text{Then } 1.143R_3 + R_3 = 6, \Rightarrow R_3 = 2.8 \text{ k}\Omega \text{ and } R_2 = 3.2 \text{ k}\Omega$$

$$\text{Let } I_{R4} = 4 \text{ mA}, R_4 = \frac{V_{IN} - V_o}{I_{R4}} = \frac{15 - 12}{4} = 0.75 \text{ k}\Omega$$

9.87

$$\text{Let } R_1 = R_2 = R_3 = 20 \text{ k}\Omega$$

$$\text{Let } R_T = 20(1+\delta) \text{ k}\Omega$$

$$\text{Now } v_{o1} = v_A = \left(\frac{R_3}{R_3 + R_1}\right) \cdot V^+ = \frac{1}{2}(10) = 5 \text{ V}$$

$$v_{o2} = v_B = \left(\frac{R_T}{R_T + R_2}\right) \cdot V^+ = \left[\frac{20(1+\delta)}{20(1+\delta) + 20}\right](10) = \frac{10(1+\delta)}{2+\delta}$$

$$\text{So } v_{OA} = v_A - v_B = 5 - \frac{10(1+\delta)}{2+\delta} = \frac{5(2+\delta) - 10(1+\delta)}{2+\delta} \cong -\frac{5\delta}{2} = -2.5\delta$$

$$\text{We have } \delta = y\left(\frac{T-300}{300}\right); \text{ At } T = 350, R_T = 21 \text{ k}\Omega, \Rightarrow 21 = 20(1+\delta) \Rightarrow \delta = 0.05$$

$$\text{Then } 0.05 = y\left(\frac{350-300}{300}\right) \Rightarrow y = 0.30$$

$$\text{For } \delta = 0.05, |v_{OA}| = (2.5)(0.05) = 0.125 \text{ V}$$

$$\text{For the instrumentation amplifier, } v_o = 5 = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1}\right)(0.125)$$

$$\text{For example, set } \frac{R_4}{R_3} = 4 \text{ and } \frac{R_2}{R_1} = 4.5$$

9.88

$$(a) \quad v_A = \left(\frac{R - \Delta R}{R - \Delta R + R + \Delta R} \right) \cdot V^+ = \left(\frac{R - \Delta R}{2R} \right) \cdot V^+$$

$$v_B = \left(\frac{R + \Delta R}{R + \Delta R + R - \Delta R} \right) \cdot V^+ = \left(\frac{R + \Delta R}{2R} \right) \cdot V^+$$

$$v_{o1} = v_A - v_B = \left[\frac{R - \Delta R}{2R} - \frac{R + \Delta R}{2R} \right] \cdot V^+ = -\frac{\Delta R}{R} \cdot V^+ = -\Delta R \left(\frac{9}{20 \times 10^3} \right)$$

or $v_{o1} = -(4.5 \times 10^{-4})(\Delta R)$

(b) For an instrumentation amplifier,

$$v_o = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1} \right) \cdot v_{o1}$$

For $\Delta R = 200 \Omega$, $v_o = -5 \text{ V}$

$$-5 = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1} \right) (-4.5 \times 10^{-4})(200)$$

$$\text{or } 55.55 = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1} \right)$$

For example, set $\frac{R_4}{R_3} = 6$ and $\frac{R_2}{R_1} = 4.13$

Chapter 10

10.1

$$I_1 = I_2 = \frac{0 - 2V_\gamma - V^-}{R_1 + R_2}$$

$$2V_\gamma + I_2 R_2 = V_{BE} + I_C R_3$$

$$2V_\gamma + \frac{R_2}{R_1 + R_2} (-2V_\gamma - V^-) = V_{BE} + I_C R_3$$

a. $I_C = \frac{1}{R_3} \left\{ 2V_\gamma - (2V_\gamma + V^-) \left(\frac{R_2}{R_1 + R_2} \right) - V_{BE} \right\}$

$V_\gamma = V_{BE}$ and $R_1 = R_2$

$$I_C = \frac{1}{R_3} \left\{ 2V_\gamma - \frac{1}{2} (2V_\gamma + V^-) - V_{BE} \right\}$$

b. or $I_C = \frac{-V^-}{2R_3}$

c. $I_C = 2 \text{ mA} = \frac{-(-10)}{2R_3} \Rightarrow R_3 = 2.5 \text{ k}\Omega$

$$I_1 = I_2 = 2 \text{ mA} = \frac{-2(0.7) - (-10)}{R_1 + R_2} \Rightarrow R_1 + R_2 = 4.3 \text{ k}\Omega \Rightarrow R_1 = R_2 = 2.15 \text{ k}\Omega$$

10.2

(a) (i) $I_O = 50 \mu \text{A}$, $V_{BE1} = (0.026) \ln \left(\frac{50 \times 10^{-6}}{10^{-16}} \right) = 0.7004 \text{ V}$

(ii) $I_O = 150 \mu \text{A}$, $V_{BE1} = (0.026) \ln \left(\frac{150 \times 10^{-6}}{10^{-16}} \right) = 0.7289 \text{ V}$

(iii) $I_O = 1.5 \text{ mA}$, $V_{BE1} = (0.026) \ln \left(\frac{1.5 \times 10^{-3}}{10^{-16}} \right) = 0.7888 \text{ V}$

(b) (i) $I_O = \frac{50}{1 + \frac{2}{50}} = 48.08 \mu \text{A}$, $V_{BE1} = (0.026) \ln \left(\frac{48.0769 \times 10^{-6}}{10^{-16}} \right) = 0.6994 \text{ V}$

(ii) $I_O = \frac{150}{1 + \frac{2}{50}} = 144.23 \mu \text{A}$, $V_{BE1} = (0.026) \ln \left(\frac{144.23 \times 10^{-6}}{10^{-16}} \right) = 0.7279 \text{ V}$

(iii) $I_O = \frac{1.5}{1 + \frac{2}{50}} = 1.4423 \text{ mA}$, $V_{BE1} = (0.026) \ln \left(\frac{1.4423 \times 10^{-3}}{10^{-16}} \right) = 0.7878 \text{ V}$

10.3

$$I_{C1} \cong \frac{I_{REF}}{1 + \frac{2}{\beta}} = \frac{200}{1 + \frac{2}{80}} = 195.12 \mu A$$

$$V_{BE1} = V_{BE2} = (0.026) \ln \left(\frac{195.12 \times 10^{-6}}{5 \times 10^{-15}} \right) = 0.6341 V$$

$$I_o = I_{S2} \exp \left(\frac{V_{BE2}}{V_T} \right) = (2 \times 10^{-15}) \exp \left(\frac{0.6341}{0.026} \right) \Rightarrow I_o = 78.05 \mu A$$

10.4

$$I_{C1} = \frac{I_{REF}}{1 + \frac{2}{\beta}} = \frac{150}{1 + \frac{2}{120}} = 147.54 \mu A$$

$$V_{BE1} = V_{BE2} = (0.026) \ln \left(\frac{147.54 \times 10^{-6}}{10^{-16}} \right) = 0.7285 V$$

$$I_o = I_{S2} \exp \left(\frac{V_{BE2}}{V_T} \right) = (3 \times 10^{-16}) \exp \left(\frac{0.7285}{0.026} \right) \Rightarrow I_o = 0.4426 mA$$

10.5

$$\text{Approximation: } I_{C1} \cong \frac{I_{REF}}{1 + \frac{2}{\beta}} = \frac{200}{1 + \frac{2}{40}} = 190.48 \mu A$$

$$V_{BE1} = (0.026) \ln \left(\frac{190.48 \times 10^{-6}}{5 \times 10^{-15}} \right) = 0.63345 V$$

$$I_{B2} \cong \frac{I_{C1}}{\beta} = \frac{190.48}{40} = 4.762 \mu A$$

$$V_{BE2} = V_{BE1} - I_{B2} R = 0.63345 - (0.004762)(2) = 0.62393 V$$

$$I_o = (5 \times 10^{-15}) \exp \left(\frac{0.62393}{0.026} \right) \Rightarrow I_o = 132.07 \mu A$$

10.6

$$I_{REF} = \frac{V^+ - V_{BE}(\text{on}) - V^-}{R_l} \Rightarrow 0.250 = \frac{3 - 0.7 - (-3)}{R_l}$$

$$\underline{R_l = 21.2 K}$$

$$I_{C1} = I_{C2} = \frac{I_{REF}}{1 + \frac{2}{\beta}} = \frac{0.250}{1 + \frac{2}{60}} \Rightarrow I_{C1} = I_{C2} = 0.2419 mA$$

$$I_{B1} = I_{B2} = 4.03 \mu A$$

10.7

$$I_{REF} = \frac{V^+ - V_{BE}(\text{on}) - V^-}{R_1} = \frac{5 - 0.7 - (-5)}{18.3}$$

$$I_{REF} = 0.5082 \text{ mA}$$

$$I_{C1} = I_{C2} = \frac{I_{REF}}{1 + \frac{2}{\beta}} = \frac{0.5082}{1 + \frac{2}{80}} \Rightarrow I_{C1} = I_{C2} = 0.4958 \text{ mA}$$

$$I_{B1} = I_{B2} = (6.198 \mu\text{A})$$

10.8

$$(a) P = (I_O + I_{REF})(V^+ - V^-)$$

$$1.8 = (0.25 + I_{REF})(5) \Rightarrow I_{REF} = 0.11 \text{ mA}$$

$$V_{BE1} = V_{BE2} = (0.026) \ln \left(\frac{0.25 \times 10^{-3}}{10^{-15}} \right) = 0.68236 \text{ V}$$

$$(b) 0.11 \times 10^{-3} = I_{S1} \exp \left(\frac{0.68236}{0.026} \right) \Rightarrow I_{S1} = 4.4 \times 10^{-16} \text{ A}$$

$$(c) R_1 = \frac{5 - 0.68236}{0.11} = 39.25 \text{ k}\Omega$$

10.9

$$(a) I_{REF} = \left(1 + \frac{2}{\beta} \right) \cdot I_O = \left(1 + \frac{2}{120} \right) (0.5) = 0.5083 \text{ mA}$$

$$R_1 = \frac{5 - 0.7 - (-5)}{0.5083} = 18.3 \text{ k}\Omega$$

$$(b) R_o = \frac{V_A}{I_O} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

$$\Delta I_O = \frac{\Delta V_{CE2}}{R_o} = \frac{7 - 0.7}{200} = 0.0315 \text{ mA}$$

$$\frac{\Delta I_O}{I_O} = \frac{0.0315}{0.5} \Rightarrow 6.3\%$$

10.10

$$I_0 = nI_{C1}$$

$$I_{REF} = I_{C1} + I_{B1} + I_{B2} = I_{C1} + \frac{I_{C1}}{\beta} + \frac{I_0}{\beta}$$

$$I_{REF} = I_{C1} \left(1 + \frac{1}{\beta} + \frac{n}{\beta} \right) = I_{C1} \left(1 + \frac{1+n}{\beta} \right)$$

$$= \frac{I_0}{n} \left(1 + \frac{1+n}{\beta} \right) \text{ or } I_0 = \frac{nI_{REF}}{\left(1 + \frac{1+n}{\beta} \right)}$$

10.11

$$I_o = \frac{I_{REF}}{1 + \frac{2}{\beta}} \Rightarrow I_{REF} = (0.20) \left(1 + \frac{2}{40} \right) = 0.210 \text{ mA}$$

$$R_l = \frac{5 - 0.7}{I_{REF}} = \frac{4.3}{0.21} \Rightarrow R_l = 20.5 \text{ k}\Omega$$

10.12

$$(a) \quad I_{REF} = \left(1 + \frac{2}{\beta} \right) \cdot I_o = \left(1 + \frac{2}{80} \right) (0.12) = 0.123 \text{ mA}$$

$$R_l = \frac{5 - 0.7}{0.123} = 34.96 \text{ k}\Omega$$

$$(b) \quad r_o = \frac{V_A}{I_o} = \frac{80}{0.12} = 666.7 \text{ k}\Omega$$

$$(i) \quad \Delta I_o = \frac{\Delta V_{EC}}{r_o} = \frac{2 - 0.7}{666.7} \Rightarrow \Delta I_o = 1.95 \mu\text{A}$$

$$(ii) \quad \Delta I_o = \frac{4 - 0.7}{666.7} \Rightarrow \Delta I_o = 4.95 \mu\text{A}$$

10.13

$$a. \quad I_{REF} = 1 = \frac{5 - 0.7 - (-5)}{R_l} \Rightarrow R_l = 9.3 \text{ k}\Omega$$

$$b. \quad I_0 = 2I_{REF} \Rightarrow I_0 = 2 \text{ mA}$$

$$c. \quad \text{For } V_{EC2} (\text{min}) = 0.7 \Rightarrow R_{C2} = \frac{5 - 0.7}{2} \Rightarrow R_{C2} = 2.15 \text{ k}\Omega$$

10.14

$$I_o = 0.50 \text{ mA} \Rightarrow I_{OA} = I_{OB} = 0.25 \text{ mA}$$

$$I_{REF} = I_{OA} \left(1 + \frac{3}{\beta} \right) = 0.25 \left(1 + \frac{3}{60} \right)$$

$$I_{REF} = 0.2625 \text{ mA}$$

$$R_l = \frac{2.5 - 0.7}{0.2625} \Rightarrow R_l = 6.86 \text{ k}\Omega$$

10.15

Similar to Figure P10.14 biased at V^+ and V^- .

$$R_l = \frac{2.5 - 0.7 - (-2.5)}{0.2} = 21.5 \text{ k}\Omega$$

10.16

$$I_2 = 2I_1 \text{ and } I_3 = 3I_1$$

$$(a) I_2 = 1.0 \text{ mA}, I_3 = 1.5 \text{ mA}$$

$$(b) I_1 = 0.25 \text{ mA}, I_3 = 0.75 \text{ mA}$$

$$(c) I_1 = 0.167 \text{ mA}, I_2 = 0.333 \text{ mA}$$

10.17

a.

$$I_0 = I_{C1} \text{ and } I_{REF} = I_{C1} + I_{B3} = I_{C1} + \frac{I_{E3}}{1+\beta}$$

$$I_{E3} = I_{B1} + I_{B2} + \frac{V_{BE}}{R_2} = \frac{2I_{C1}}{\beta} + \frac{V_{BE}}{R_2}$$

$$I_{REF} = I_{C1} + \frac{2I_{C1}}{\beta(1+\beta)} + \frac{V_{BE}}{(1+\beta)R_2}$$

$$I_{REF} - \frac{V_{BE}}{(1+\beta)R_2} = I_0 \left(1 + \frac{2}{\beta(1+\beta)} \right)$$

$$I_0 = \frac{I_{REF} - \frac{V_{BE}}{(1+\beta)R_2}}{\left(1 + \frac{2}{\beta(1+\beta)} \right)}$$

$$I_{REF} = (0.70) \left(1 + \frac{2}{(80)(81)} \right) + \frac{0.7}{(81)(10)}$$

$$I_{REF} = 0.700216 + 0.000864$$

$$I_{REF} = 0.7011 \text{ mA} = \frac{10 - 2(0.7)}{R_1} \Rightarrow R_1 = 12.27 \text{ k}\Omega$$

10.18

a.

$$I_{0i} = I_{CR} \text{ and } I_{REF} = I_{CR} + I_{BS} = I_{CR} + \frac{I_{ES}}{1+\beta}$$

$$\begin{aligned} I_{ES} &= I_{BR} + I_{B1} + I_{B2} + \dots + I_{BN} = (1+N)I_{BR} \\ &= \frac{(1+N)I_{CR}}{\beta} \end{aligned}$$

$$\text{Then } I_{REF} = I_{CR} + \frac{(1+N)I_{CR}}{\beta(1+\beta)}$$

$$\text{or } I_{0i} = \frac{I_{REF}}{\left(1 + \frac{(1+N)}{\beta(1+\beta)} \right)}$$

$$I_{REF} = (0.5) \left[1 + \frac{6}{(50)(51)} \right] = 0.5012 \text{ mA}$$

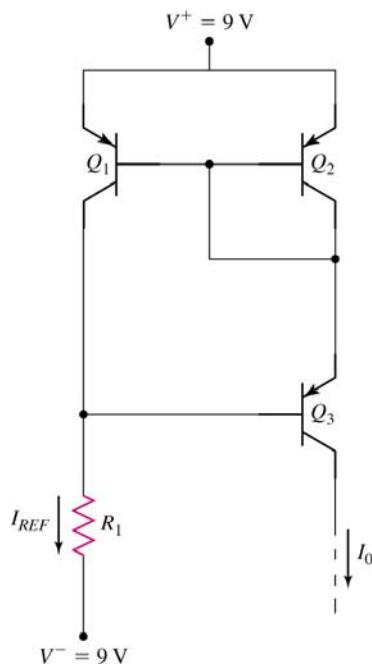
b. $R_l = \frac{5 - 2(0.7) - (-5)}{0.5012} \Rightarrow R_l = 17.16 \text{ k}\Omega$

10.19

$$I_{REF} = I_o \left[1 + \frac{2}{\beta(1+\beta)} \right] = (0.15) \left[1 + \frac{2}{(40)(41)} \right] = 0.15018 \text{ mA}$$

$$R_l = \frac{3 - 0.7 - 0.7 - (-3)}{0.15018} = 30.63 \text{ k}\Omega$$

10.20



$$I_0 = I_{REF} \cdot \frac{1}{\left(1 + \frac{2}{\beta(2+\beta)} \right)}$$

For $I_0 = 0.8 \text{ mA}$

$$I_{REF} = (0.8) \left(1 + \frac{2}{25(27)} \right) \Rightarrow I_{REF} = 0.8024 \text{ mA}$$

$$R_l = \frac{18 - 2(0.7)}{0.8024} \Rightarrow R_l = 20.69 \text{ k}\Omega$$

10.21

The analysis is exactly the same as in the text. We have

$$I_0 = I_{REF} \cdot \frac{1}{\left(1 + \frac{2}{\beta(2 + \beta)}\right)}$$

10.22

$$(a) \quad I_O = 0.5 \text{ mA}, \quad I_{B2} = \frac{0.5}{90} = 0.005556 \text{ mA}$$

$$I_{B1} = \frac{I_{B2}}{2} = 0.002778 \text{ mA}$$

$$I_{E3} = I_{B1} + I_{B2} = 0.008334 \text{ mA}$$

$$I_{B3} = \frac{0.008334}{61} = 0.0001366 \text{ mA}$$

$$I_{C1} = \frac{I_O}{2} = 0.25 \text{ mA}$$

$$I_{REF} = I_{C1} + I_{B3} = 0.2501366 \text{ mA}$$

$$R_1 = \frac{5 - 0.6 - 0.7 - (-5)}{0.2501366} = 34.78 \text{ k}\Omega$$

$$(b) \quad I_{B1} = 0.002778 \text{ mA}$$

$$I_{B2} = 0.005556 \text{ mA}$$

$$I_{E3} = 0.008334 \text{ mA}$$

$$I_{B3} = 0.0001366 \text{ mA}$$

10.23

(a)

$$\text{Assuming } R_o \approx \frac{\beta r_{o3}}{2}$$

$$r_{o3} = \frac{V_A}{I_O} = \frac{V_A}{I_{REF}} = \frac{100}{0.25} = 400 \text{ K}$$

$$R_o = \frac{(100)(400)}{2} \Rightarrow R_o = 20 \text{ M}\Omega$$

(b)

$$R_o = \frac{\Delta V}{\Delta I_o} \Rightarrow \Delta I_o = \frac{\Delta V}{20 \text{ M}\Omega} = \frac{5}{20 \text{ M}\Omega}$$

$$\underline{\Delta I_o = 0.25 \mu\text{A}}$$

10.24

$$I_{REF} = \frac{V^+ - V_{BE1} - V^-}{R_i} = \frac{5 - 0.7}{9.3}$$

$$I_{REF} = 0.4624 \text{ mA}$$

$$I_o = \frac{V_T}{R_E} \ln\left(\frac{I_{REF}}{I_o}\right) = \frac{0.026}{1.5} \ln\left(\frac{0.4624}{I_o}\right)$$

$$I_o = 0.01733 \ln\left(\frac{0.4624}{I_o}\right)$$

By trial and error

$$I_o = 41.7 \mu\text{A}$$

$$V_{BE2} = 0.7 - I_o R_E = 0.7 - (0.0417)(1.5)$$

$$V_{BE2} = 0.6375 \text{ V}$$

10.25

(a) $V_{BE1} = (0.026) \ln\left(\frac{200 \times 10^{-6}}{5 \times 10^{-15}}\right) = 0.6347 \text{ V}$

$$I_o R_E = V_T \ln\left(\frac{I_{REF}}{I_o}\right)$$

$$I_o(0.5) = (0.026) \ln\left(\frac{0.2}{I_o}\right)$$

$$I_o \approx 61.4 \mu\text{A}$$

$$V_{BE2} = (0.026) \ln\left(\frac{61.4 \times 10^{-6}}{5 \times 10^{-15}}\right) = 0.6040 \text{ V}$$

(b) $V_{BE1} = (0.026) \ln\left(\frac{200 \times 10^{-6}}{5 \times 10^{-15}}\right) = 0.6347 \text{ V}$

$$V_{BE1} - V_{BE2} = I_o R_E$$

$$V_T \left[\ln\left(\frac{I_{REF}}{I_{S1}}\right) - \ln\left(\frac{I_o}{I_{S2}}\right) \right] = I_o R_E$$

$$V_T \ln\left(\frac{I_{REF}}{I_o} \cdot \frac{I_{S2}}{I_{S1}}\right) = I_o R_E$$

$$(0.026) \ln\left[\left(\frac{0.2}{I_o}\right) \left(\frac{7}{5}\right)\right] = I_o(0.5)$$

$$I_o \approx 71.2 \mu\text{A}$$

$$V_{BE2} = (0.026) \ln\left(\frac{71.2 \times 10^{-6}}{7 \times 10^{-15}}\right) = 0.5991 \text{ V}$$

10.26

$$(a) V_{BE1} = (0.026) \ln\left(\frac{100 \times 10^{-6}}{5 \times 10^{-15}}\right) = 0.61669 \text{ V}$$

$$V_{BE2} = V_{BE1} + I_{REF} R_E = 0.61669 + (0.1)(0.7) = 0.68669 \text{ V}$$

$$I_o = (5 \times 10^{-15}) \exp\left(\frac{0.68669}{0.026}\right) \Rightarrow I_o = 1.477 \text{ mA}$$

$$(b) V_{BE1} = 0.61669 \text{ V}$$

$$V_{BE2} = 0.68669 \text{ V}$$

$$I_o = (2 \times 10^{-15}) \exp\left(\frac{0.68669}{0.026}\right) \Rightarrow I_o = 0.5906 \text{ mA}$$

10.27

$$(a) I_{REF} = \frac{5 - 0.7 - (-5)}{50} = 0.186 \text{ mA}$$

$$I_o R_E = V_T \ln\left(\frac{I_{REF}}{I_o}\right)$$

$$I_o (3) = (0.026) \ln\left(\frac{0.186}{I_o}\right)$$

$$I_o \cong 19.53 \mu \text{A}$$

$$V_{BE2} = V_{BE1} - I_o R_E = 0.7 - (0.01953)(3) = 0.6414 \text{ V}$$

$$(b) r_{o2} = \frac{V_A}{I_o} = \frac{80}{0.01953} \Rightarrow r_{o2} = 4.096 \text{ M}\Omega$$

$$g_{m2} = \frac{0.01953}{0.026} = 0.7512 \text{ mA/V}$$

$$r_{\pi2} = \frac{(120)(0.026)}{0.01953} = 159.8 \text{ k}\Omega$$

$$R_E \parallel r_{\pi2} = 3 \parallel 159.8 = 2.945 \text{ k}\Omega$$

$$R_o = (4.096)[1 + (0.7512)(2.945)] = 13.16 \text{ M}\Omega$$

10.28

$$R_o = r_{o2} [1 + g_{m2} (R_E \parallel r_{\pi2})]$$

From 10.27, $I_o = 19.53 \mu \text{A}$

$$g_{m2} = \frac{I_o}{V_T} = \frac{0.01953}{0.026} = 0.7512 \text{ mA/V}$$

$$r_{\pi2} = \frac{\beta V_T}{I_o} = \frac{(80)(0.026)}{0.01953} = 106.5 \text{ k}\Omega$$

$$r_{o2} = \frac{V_A}{I_o} = \frac{80}{0.01953} \Rightarrow 4.096 \text{ M}\Omega$$

$$\text{Then } R_o = (4.096)[1 + (0.7512)(3 \parallel 106.5)] = 13.07 \text{ M}\Omega$$

$$\Delta I_o = \frac{\Delta V_o}{R_o} = \frac{5}{13.07} = 0.382 \mu \text{A}$$

10.29

$$I_{REF} = \frac{5 - 0.7 - (-5)}{R_1} = 0.50$$

$$\underline{R_1 = 18.6 \text{ K}}$$

$$I_O R_E = V_T \ln\left(\frac{I_{REF}}{I_O}\right)$$

$$R_E = \frac{0.026}{0.050} \ln\left(\frac{0.50}{0.050}\right)$$

$$(a) \quad R_E = 1.20 \text{ K}$$

$$R_O = r_{c2} [1 + R'_E g_{m2}]$$

$$R'_E = R_E \| r_{\pi 2}$$

$$r_{\pi 2} = \frac{(75)(0.026)}{0.050} = 39 \text{ K} \quad g_{m2} = \frac{0.050}{0.026} = 1.923 \text{ mA/V}$$

$$r_{o2} = \frac{V_A}{I_O} = \frac{100}{0.05} \Rightarrow 2 \text{ M}\Omega \quad R'_E = 1.20 \| 39 = 1.164 \text{ K}$$

$$(b) \quad R_O = 2[1 + (1.164)(1.923)] \Rightarrow R_O = (6.477) \text{ M}\Omega$$

$$\Delta I_O = \frac{\Delta V}{R_O} = \frac{5}{6.477} = 0.772 \mu\text{A}$$

$$\frac{\Delta I_O}{I_O} \times 100\% = \frac{0.772}{50} \times 100 = 1.54\%$$

(c)

10.30

$$\text{Let } R_1 = 10 \text{ k}\Omega$$

$$\text{Then } I_{REF} = \frac{3 - 0.7 - (-3)}{10} = 0.53 \text{ mA}$$

$$R_E = \frac{V_T}{I_O} \ln\left(\frac{I_{REF}}{I_O}\right) = \frac{(0.026)}{(0.05)} \ln\left(\frac{0.53}{0.05}\right) \Rightarrow R_E = 1.228 \text{ k}\Omega$$

10.31

$$V_{BE} = V_T \ln\left(\frac{I_{REF}}{I_S}\right)$$

$$0.7 = (0.026) \ln\left(\frac{10^{-3}}{I_S}\right) \Rightarrow I_S = 2.03 \times 10^{-15} \text{ A}$$

$$\text{At } 2 \text{ mA, } V_{BE} = (0.026) \ln\left(\frac{2 \times 10^{-3}}{2.03 \times 10^{-15}}\right) \\ = 0.718 \text{ V}$$

$$R_l = \frac{15 - 0.718}{2} \Rightarrow \underline{R_l = 7.14 \text{ k}\Omega}$$

$$R_E = \frac{V_T}{I_0} \ln\left(\frac{I_{REF}}{I_0}\right) = \frac{0.026}{0.050} \cdot \ln\left(\frac{2}{0.050}\right) \Rightarrow \underline{R_E = 1.92 \text{ k}\Omega}$$

10.32

$$(a) I_{REF} = \frac{3 - 0.7 - (-3)}{20} = 0.265 \text{ mA}$$

$$(b) R_E = \frac{V_T}{I_O} \ln\left(\frac{I_{REF}}{I_O}\right) = \frac{(0.026)}{(0.1)} \ln\left(\frac{0.265}{0.1}\right) \Rightarrow R_E = 253 \Omega$$

10.33

$$I_{REF} \approx \frac{10 - 0.7 - (-10)}{40} = 0.4825 \text{ mA}$$

$$V_{BE} \cong V_T \ln\left(\frac{I_{REF}}{I_S}\right)$$

$$0.7 = (0.026) \ln\left(\frac{10^{-3}}{I_S}\right) \Rightarrow I_S = 2.03 \times 10^{-15} \text{ A}$$

Now

$$V_{BE} = (0.026) \ln\left(\frac{0.4825 \times 10^{-3}}{2.03 \times 10^{-15}}\right) = 0.681 \text{ V}$$

$$\underline{V_{BE1} = 0.681 \text{ V}}$$

So

$$I_{REF} \cong \frac{10 - 0.681 - (-10)}{40} \Rightarrow \underline{I_{REF} = 0.483 \text{ mA}}$$

$$I_0 R_E = V_T \ln\left(\frac{I_{REF}}{I_0}\right)$$

$$I_0 (12) = (0.026) \ln\left(\frac{0.483}{I_0}\right)$$

By trial and error.

$$\Rightarrow \underline{I_0 \cong 8.7 \mu\text{A}}$$

$$V_{BE2} = V_{BE1} - I_0 R_E = 0.681 - (0.0087)(12) \Rightarrow \underline{V_{BE2} = 0.5766 \text{ V}}$$

10.34

$$V_{BE1} + I_{REF} R_{E1} = V_{BE2} + I_0 R_{E2}$$

$$V_{BE1} - V_{BE2} = I_0 R_{E2} - I_{REF} R_{E1}$$

For matched transistors

$$V_{BE1} = V_T \ln\left(\frac{I_{REF}}{I_S}\right)$$

$$V_{BE2} = V_T \ln\left(\frac{I_0}{I_S}\right)$$

$$\text{Then } V_T \ln\left(\frac{I_{REF}}{I_0}\right) = I_0 R_{E2} - I_{REF} R_{E1}$$

Output resistance looking into the collector of Q_2 is increased.

10.35

$$(a) V_{BE1} = V_T \ln\left(\frac{I_{REF}}{I_S}\right) = (0.026) \ln\left(\frac{0.5 \times 10^{-3}}{10^{-15}}\right) = 0.70038 \text{ V}$$

$$V^+ = I_{REF} R_1 + V_{BE1} + I_{REF} R_{E1} + V^-$$

$$\text{Then } R_1 = \frac{3 - 0.70038 - (0.5)(0.5) - (-3)}{0.5} = 10.1 \text{ k}\Omega$$

$$V_{BE2} = (0.026) \ln\left(\frac{0.2 \times 10^{-3}}{10^{-15}}\right) = 0.67656 \text{ V}$$

$$V_{BE1} + I_{REF} R_{E1} = V_{BE2} + I_O R_{E2}$$

$$\text{Then } R_{E2} = \frac{0.70038 + (0.5)(0.5) - 0.67656}{0.2} = 1.37 \text{ k}\Omega$$

$$(b) R_1 = 10.1 \text{ k}\Omega$$

$$V_{BE2} = (0.026) \ln\left(\frac{0.2 \times 10^{-3}}{2 \times 10^{-15}}\right) = 0.65854 \text{ V}$$

$$R_{E2} = \frac{0.70038 + (0.5)(0.5) - 0.65854}{0.2} = 1.46 \text{ k}\Omega$$

10.36

Assume all transistors are matched.

a.

$$2V_{BE1} = V_{BE3} + I_0 R_E$$

$$V_{BE1} = V_T \ln\left(\frac{I_{REF}}{I_S}\right)$$

$$V_{BE3} = V_T \ln\left(\frac{I_0}{I_S}\right)$$

$$2V_T \ln\left(\frac{I_{REF}}{I_S}\right) - V_T \ln\left(\frac{I_0}{I_S}\right) = I_0 R_E$$

$$V_T \left[\ln\left(\frac{I_{REF}}{I_S}\right)^2 - \ln\left(\frac{I_0}{I_S}\right) \right] = I_0 R_E$$

$$\underline{V_T \ln\left(\frac{I_{REF}^2}{I_0 I_S}\right) = I_0 R_E}$$

b.

$$V_{BE} = 0.7 \text{ V at } 1 \text{ mA} \Rightarrow 10^{-3} = I_S \exp\left(\frac{0.7}{0.026}\right) \text{ or } I_S = 2.03 \times 10^{-15} \text{ A}$$

$$V_{BE} \text{ at } 0.1 \text{ mA} \Rightarrow V_{BE} = (0.026) \ln\left(\frac{0.1 \times 10^{-3}}{2.03 \times 10^{-15}}\right) = 0.640 \text{ V}$$

$$\text{Since } I_0 = I_{REF}, \text{ then } V_{BE} = I_0 R_E \Rightarrow R_E = \frac{0.640}{0.1} \text{ or } \underline{R_E = 6.4 \text{ k}\Omega}$$

10.37

$$(a) I_{REF} = \frac{5 - 0.7 - (-5)}{10} = 0.93 \text{ mA}$$

$$I_{O2} R_{E2} = V_T \ln\left(\frac{I_{REF}}{I_{O2}}\right)$$

$$I_{O2}(1) = (0.026) \ln\left(\frac{0.93}{I_{O2}}\right)$$

$$I_{O2} \cong 68 \mu \text{A}$$

$$I_{O3}(2) = (0.026) \ln\left(\frac{0.93}{I_{O3}}\right)$$

$$I_{O3} \cong 40.7 \mu \text{A}$$

$$(b) R_{E2} = \frac{V_T}{I_{O2}} \ln\left(\frac{I_{REF}}{I_{O2}}\right) = \frac{(0.026)}{(0.02)} \ln\left(\frac{0.93}{0.02}\right) \Rightarrow R_{E2} = 4.99 \text{ k}\Omega$$

$$R_{E3} = \frac{(0.026)}{(0.08)} \ln\left(\frac{0.93}{0.08}\right) \Rightarrow R_{E3} = 0.797 \text{ k}\Omega$$

10.38

(a)

$$V_{BE1} = V_{BE2}$$

$$I_{REF} = \frac{V^+ - 2V_{BE1} - V^-}{R_1 + R_2}$$

Now

$$2V_{BE1} + I_{REF} R_2 = V_{BE3} + I_O R_E$$

or

$$\underline{I_O R_E = 2V_{BE1} - V_{BE3} + I_{REF} R_2}$$

We have

$$V_{BE1} = V_T \ln\left(\frac{I_{REF}}{I_S}\right) \text{ and } V_{BE3} = V_T \ln\left(\frac{I_O}{I_S}\right)$$

(b)

$$\text{Let } R_1 = R_2 \text{ and } I_O = I_{REF} \Rightarrow V_{BE1} = V_{BE3} \equiv V_{BE}$$

Then

$$V_{BE} = I_O R_E - I_{REF} R_2 = I_O (R_E - R_2)$$

so

$$I_{REF} = I_O = \frac{V^+ - V^- - 2I_O (R_E - R_2)}{2R_2}$$

$$= \frac{V^+ - V^-}{2R_2} - I_O \left(\frac{R_E}{R_2} \right) + I_O$$

Then

$$I_O = \frac{V^+ - V^-}{2R_e}$$

(c)

Want $I_o = 0.5 \text{ mA}$

$$\text{So } R_E = \frac{5 - (-5)}{2(0.5)} \Rightarrow R_E = 10 \text{ k}\Omega$$

$$2R_2 = \frac{5 - 2(0.7) - (-5)}{0.5} = 17.2 \text{ k}\Omega$$

Then $R_1 = R_2 = 8.6 \text{ k}\Omega$

10.39

a.

$$I_{REF} = \frac{20 - 0.7 - 0.7}{12} = 1.55 \text{ mA}$$

$$I_{o1} = 2I_{REF} = 3.1 \text{ mA}$$

$$I_{o2} = I_{REF} = 1.55 \text{ mA}$$

$$I_{o3} = 3I_{REF} = 4.65 \text{ mA}$$

b.

$$V_{CE1} = -I_{o1}R_{C1} - (-10) = -(3.1)(2) + 10 \Rightarrow V_{CE1} = 3.8 \text{ V}$$

$$V_{EC2} = 10 - I_{o2}R_{C2} = 10 - (1.55)(3) \Rightarrow V_{EC2} = 5.35 \text{ V}$$

$$V_{EC3} = 10 - I_{o3}R_{C3} = 10 - (4.65)(1) \Rightarrow V_{EC3} = 5.35 \text{ V}$$

10.40

$$(a) I_{REF} = \frac{10 - 0.7 - 0.7 - (-10)}{24} = 0.775 \text{ mA}$$

$$I_{o1} = 2I_{REF} = 1.55 \text{ mA}$$

$$I_{o2} = I_{REF} = 0.775 \text{ mA}$$

$$I_{o3} = 3I_{REF} = 2.325 \text{ mA}$$

$$(b) R_{C1} = \frac{0 - 0.7 - (-10)}{1.55} = 6 \text{ k}\Omega$$

$$R_{C2} = \frac{10 - 0.7}{0.775} = 12 \text{ k}\Omega$$

$$R_{C3} = \frac{10 - 0.7}{2.325} = 4 \text{ k}\Omega$$

10.41

$$I_{C1} = I_{C2} = \frac{10 - 0.7 - 0.7 - (-10)}{10} = 1.86 \text{ mA}$$

$$I_{C3} = I_{C4} = 1.86 \text{ mA}$$

$$I_{C5}(0.5) = 0.026 \ln\left(\frac{1.86}{I_{C5}}\right)$$

By Trial and error.

$$\Rightarrow I_{C5} = 0.136 \text{ mA} = I_{C6} = I_{C7}$$

$$2I_{C3}(0.8) + V_{CE3} = 10 \Rightarrow V_{CE3} = 10 - 2(1.86)(0.8)$$

$$\underline{V_{CE3} = 7.02 \text{ V}}$$

$$5 = V_{EB6} + V_{CE5} + I_{C5}(0.5) - 10$$

$$V_{CE5} = 5 + 10 - 0.7 - (0.136)(0.5)$$

$$\underline{V_{CE5} = 14.2 \text{ V}}$$

$$5 = V_{EC7} + I_{C7}(0.8)$$

$$V_{EC7} = 5 - (0.136)(0.8)$$

$$\underline{V_{EC7} = 4.89 \text{ V}}$$

10.42

$$I_{C1} = I_{C2} = \frac{10 - 0.7 - 0.7 - (-10)}{10} \Rightarrow I_{C1} = I_{C2} = 1.86 \text{ mA}$$

$$I_{C4} = I_{C5} = 1.86 \text{ mA}$$

$$I_{C3}R_{E1} = V_T \ln\left(\frac{I_{C1}}{I_{C3}}\right) \Rightarrow I_{C3}(0.3) = 0.026 \ln\left(\frac{1.86}{I_{C3}}\right)$$

$$\text{By trial and error } I_{C3} = 0.195 \text{ mA}$$

$$I_{C6}R_{E2} = V_T \ln\left(\frac{I_{C5}}{I_{C6}}\right) \Rightarrow I_{C6}(0.5) = 0.026 \ln\left(\frac{1.86}{I_{C6}}\right)$$

$$\text{By trial and error } I_{C6} = 0.136 \text{ mA}$$

10.43

$$I_{REF} = \frac{10 - 0.7}{6.3 + 3} = 1 \text{ mA}$$

$$V_{BE}(Q_R) = 0.7 \text{ V as assumed}$$

$$V_{RER} = I_{REF} \cdot R_{ER} = (1)(3) = 3 \text{ V}$$

$$V_{RE1} = 3 \text{ V} \Rightarrow R_{E1} = \frac{V_{RE1}}{I_{01}} = \frac{3}{1} \Rightarrow \underline{R_{E1} = 3 \text{ k}\Omega}$$

$$V_{RE2} = 3 \text{ V} \Rightarrow R_{E2} = \frac{V_{RE2}}{I_{02}} = \frac{3}{2} \Rightarrow \underline{R_{E2} = 1.5 \text{ k}\Omega}$$

$$V_{RE3} = 3 \text{ V} \Rightarrow R_{E3} = \frac{V_{RE3}}{I_{03}} = \frac{3}{4} \Rightarrow \underline{R_{E3} = 0.75 \text{ k}\Omega}$$

$$I_{01} = 1 \text{ mA}$$

$$I_{02} = 2 \text{ mA}$$

$$I_{03} = 4 \text{ mA}$$

10.44

$$I_{REF} = \frac{2.5 - V_{GS}}{15} = \left(\frac{0.08}{2} \right) (6)(V_{GS} - 0.5)^2$$

$$2.5 - V_{GS} = 3.6(V_{GS}^2 - V_{GS} + 0.25)$$

$$3.6V_{GS}^2 - 2.6V_{GS} - 1.6 = 0$$

$$V_{GS} = \frac{2.6 \pm \sqrt{6.76 + 23.04}}{2(3.6)}$$

$$V_{GS} = 1.12 \text{ V}(1.1193)$$

$$I_{REF} = \frac{2.5 - 1.1193}{15} \Rightarrow I_{REF} = 92.0 \mu\text{A} (92.05)$$

$$I_o = 92.0 \mu\text{A}$$

$$V_{DS2}(\text{sat}) = V_{GS} - V_{TN} = 1.1193 - 0.5$$

$$V_{DS2}(\text{sat}) = 0.619 \text{ V}$$

10.45

(a)

$$I_{REF} = 50 = \left(\frac{80}{2} \right) \left(\frac{W}{L} \right)_1 (V_{GS} - 0.5)^2$$

$$I_{REF} = 0.050 = \frac{2.0 - V_{GS}}{R}$$

Design such that $V_{DS2}(\text{sat}) = 0.25 = V_{GS} - 0.5$

$$V_{GS} = 0.75 \text{ V}$$

$$\text{So } 0.050 = \frac{2 - 0.75}{R} \Rightarrow R = 25 \text{ K}$$

$$50 = \left(\frac{80}{2} \right) \left(\frac{W}{L} \right)_1 (0.75 - 0.5)^2 \Rightarrow \left(\frac{W}{L} \right)_1 = 20$$

$$\frac{\left(\frac{W}{L} \right)_1}{\left(\frac{W}{L} \right)_2} = \frac{I_{REF}}{I_o} \Rightarrow \frac{20}{\left(\frac{W}{L} \right)_2} = \frac{50}{100} \Rightarrow \left(\frac{W}{L} \right)_2 = 40$$

$$(b) R_o = \frac{1}{\lambda I_o} = \frac{1}{(0.015)(0.1)} \Rightarrow R_o = 667 \text{ K}$$

$$\Delta I_o = \frac{\Delta V}{R_o} = \frac{1}{666} \Rightarrow 1.5 \mu\text{A}$$

$$(c) \frac{\Delta I_o}{I_o} \times 100\% = \left(\frac{1.5}{100} \right) \times 100\% \Rightarrow \underline{1.5\%}$$

10.46

$$I_{REF} = 250 = \left(\frac{80}{2}\right)(3)(V_{GS} - 1)^2$$

$$V_{GS} = 2.44 \text{ V}$$

$$I_o = 250 \mu\text{A} \text{ at } V_{DS2} = V_{GS} = 2.44 \text{ V}$$

$$(a) R_o = \frac{1}{\lambda I_o} = \frac{1}{(0.02)(0.25)} = 200 \text{ K}$$

$$\Delta I_o = \frac{\Delta V}{R_o} = \frac{3 - 2.44}{200} \Rightarrow 2.8 \mu\text{A}$$

$$(i) \quad \underline{I_o = 252.8 \mu\text{A}}$$

$$\Delta I_o = \frac{\Delta V}{R_o} = \frac{4.5 - 2.44}{200} \Rightarrow 10.3 \mu\text{A}$$

$$(ii) \quad \underline{I_o = 260.3 \mu\text{A}}$$

$$\Delta I_o = \frac{\Delta V}{R_o} = \frac{6 - 2.44}{200} \Rightarrow 17.8 \mu\text{A}$$

$$(iii) \quad \underline{I_o = 267.8 \mu\text{A}}$$

$$I_o = \frac{4.5}{3}(250) = 375 \mu\text{A} \text{ at } V_{DS} = 2.44 \text{ V}$$

$$(b) \quad R_o = \frac{1}{\lambda I_o} = \frac{1}{(0.02)(0.375)} = 133.3 \text{ K}$$

$$\Delta I_o = \frac{\Delta V}{R_o} = \frac{3 - 2.44}{133.3} \Rightarrow 4.20 \mu\text{A}$$

$$(i) \quad \underline{I_o = 379.2 \mu\text{A}}$$

$$\Delta I_o = \frac{\Delta V}{R_o} = \frac{4.5 - 2.44}{133.3} \Rightarrow 15.5 \mu\text{A}$$

$$(ii) \quad \underline{I_o = 390.5 \mu\text{A}}$$

$$\Delta I_o = \frac{\Delta V}{R_o} = \frac{6 - 2.44}{133.3} \Rightarrow 26.7 \mu\text{A}$$

$$(iii) \quad \underline{I_o = 401.7 \mu\text{A}}$$

10.47

$$(a) \quad I_{REF} = K_{n1} (V_{GS1} - V_{TN1})^2$$

$$0.2 = 0.2(V_{GS1} - 0.4)^2 \Rightarrow V_{GS1} = V_{GS2} = 1.4 \text{ V}$$

$$I_o = K_{n2} (V_{GS2} - V_{TN2})^2$$

$$\text{Now } I_o = (0.2 - 0.01)(1.4 - 0.4)^2 = 0.19 \text{ mA}$$

$$I_o = (0.2 + 0.01)(1.4 - 0.4)^2 = 0.21 \text{ mA}$$

$$\text{So } 0.19 \leq I_o \leq 0.21 \text{ mA}$$

$$(b) I_o = (0.2)[1.4 - (0.4 - 0.02)]^2 = 0.2081 \text{ mA}$$

$$I_o = (0.2)[1.4 - (0.4 + 0.02)]^2 = 0.1921 \text{ mA}$$

So $0.1921 \leq I_o \leq 0.2081 \text{ mA}$

10.48

$$(a) I_{REF} = K_{n1}(V_{GS1} - V_{TN})^2$$

$$0.2 = 0.2(V_{GS1} - 0.5)^2 \Rightarrow V_{GS1} = 1.5 \text{ V}$$

$$V_{GS1} = V_{GS2} + I_o R_s = V_{GS2} + K_{n2} R_s (V_{GS2} - V_{TN})^2$$

$$1.5 = V_{GS2} + (0.2)(10)(V_{GS2}^2 - V_{GS2} + 0.25)$$

$$\text{or } 2V_{GS2}^2 - V_{GS2} - 1 = 0 \Rightarrow V_{GS2} = 1.0 \text{ V}$$

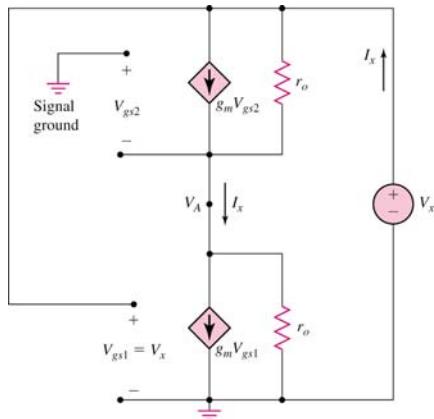
$$I_o = \frac{V_{GS1} - V_{GS2}}{R_s} = \frac{1.5 - 1.0}{10} \Rightarrow I_o = 50 \mu \text{A}$$

$$(b) I_o = 0.5 I_{REF} = 0.1 \text{ mA}$$

$$0.1 = 0.2(V_{GS2} - 0.5)^2 \Rightarrow V_{GS2} = 1.207 \text{ V}$$

$$R_s = \frac{V_{GS1} - V_{GS2}}{I_o} = \frac{1.5 - 1.207}{0.1} = 2.93 \text{ k}\Omega$$

10.49



$$(1) I_x = \frac{V_x - V_A}{r_o} + g_m V_{gs2}$$

$$I_x = \frac{V_A}{r_o} + g_m V_{gs1}$$

$$(2) V_{gs1} = V_x, V_{gs2} = -V_A$$

So

$$(1) I_x = \frac{V_x}{r_o} - V_A \left(\frac{1}{r_o} + g_m \right)$$

$$(2) I_x = \frac{V_A}{r_o} + g_m V_x \Rightarrow V_A = r_o [I_x - g_m V_x]$$

Then

$$I_x = \frac{V_x}{r_o} - r_o (I_x - g_m V_x) \left(\frac{1}{r_o} + g_m \right)$$

$$I_x = \frac{V_x}{r_o} - r_o \left[\frac{I_x}{r_o} + g_m I_x - \frac{g_m}{r_o} \cdot V_x - g_m^2 V_x \right]$$

$$I_x = \frac{V_x}{r_o} - I_x - g_m r_o I_x + g_m V_x + g_m^2 r_o V_x$$

$$I_x [2 + g_m r_o] = V_x \left[\frac{1}{r_o} + g_m + g_m^2 r_o \right]$$

$$\text{Since } g_m \gg \frac{1}{r_o}$$

$$I_x [2 + g_m r_o] \approx V_x (g_m) (1 + g_m r_o)$$

$$\text{Then } \frac{V_x}{I_x} = R_o = \frac{2 + g_m r_o}{g_m (1 + g_m r_o)}$$

$$\text{Usually, } g_m r_o \gg 2, \text{ so that } R_o \approx \frac{1}{g_m}$$

10.50

$$V_{GS2} = V_{DS2}(\text{sat}) + V_{TN} = 1 + 0.5 = 1.5 \text{ V}$$

$$I_o = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right)_2 (V_{GS2} - V_{TN})^2$$

$$0.15 = \left(\frac{0.08}{2} \right) \left(\frac{W}{L} \right)_2 (1.5 - 0.5)^2 \Rightarrow \left(\frac{W}{L} \right)_2 = 3.75$$

$$I_{REF} = 0.5 = \left(\frac{0.08}{2} \right) \left(\frac{W}{L} \right)_1 (1.5 - 0.5)^2 \Rightarrow \left(\frac{W}{L} \right)_1 = 12.5$$

$$V_{GS3} = (V^+ - V^-) - V_{GS1} = 1.8 - (-1.8) - 1.5 = 2.1 \text{ V}$$

$$I_{REF} = 0.5 = \left(\frac{0.08}{2} \right) \left(\frac{W}{L_3} \right) (2.1 - 0.5)^2 \Rightarrow \left(\frac{W}{L} \right)_3 = 4.88$$

10.51

(a)

$$I_{REF} = \left(\frac{60}{2}\right)(20)(V_{GS1} - 0.7)^2 = \left(\frac{60}{2}\right)(3)(V_{GS3} - 0.7)^2$$

$$V_{GS1} + V_{GS3} = 5$$

$$\sqrt{\frac{20}{3}}(V_{GS1} - 0.7) = 5 - V_{GS1} - 0.7$$

$$3.582V_{GS1} = 6.107 \Rightarrow V_{GS1} = V_{GS2} = 1.705 \text{ V}$$

$$I_o = \left(\frac{60}{2}\right)(12)(1.705 - 0.7)^2 = 363.6 \mu\text{A} \text{ at } V_{DS2} = 1.705 \text{ V}$$

$$I_{REF} = \left(\frac{60}{2}\right)(20)(1.705 - 0.7)^2 = 606 \mu\text{A}$$

$$R_o = \frac{1}{\lambda I_o} = \frac{1}{(0.015)(0.3636)} = 183.4 \text{ K}$$

$$\Delta I_o = \frac{\Delta V}{R_o} = \frac{1.5 - 1.705}{183.4} \Rightarrow -1.12 \mu\text{A}$$

(b) $I_o = 362.5 \mu\text{A}$

$$\Delta I_o = \frac{\Delta V}{R_o} = \frac{3 - 1.705}{183.4} \Rightarrow 7.06 \mu\text{A}$$

(c) $I_o = 370.7 \mu\text{A}$

10.52

$$I_{REF} = \left(\frac{50}{2}\right)(15)(V_{SG1} - 0.5)^2 = \left(\frac{50}{2}\right)(3)(V_{SG3} - 0.5)^2$$

$$V_{SG1} + V_{SG3} = 10 \Rightarrow V_{SG3} = 10 - V_{SG1}$$

$$\sqrt{\frac{15}{3}}(V_{SG1} - 0.5) = 10 - V_{SG1} - 0.5$$

$$3.236V_{SG1} = 10.618 \Rightarrow V_{SG1} = 3.28 \text{ V}$$

$$I_{REF} = \left(\frac{50}{2}\right)(15)(3.28 - 0.5)^2 \Rightarrow I_{REF} = 2.90 \text{ mA}$$

$$I_o = I_{REF} = 2.90 \text{ mA}$$

$$V_{SD2}(\text{sat}) = V_{SG2} + V_{TP} = 3.28 - 0.5 \Rightarrow V_{SD2}(\text{sat}) = 2.78 \text{ V}$$

10.53

$$V_{SD2}(\text{sat}) = 1.2 = V_{SG2} - 0.35 \Rightarrow V_{SG2} = 1.55 \text{ V}$$

$$I_o = 100 = \left(\frac{50}{2}\right) \left(\frac{W}{L}\right)_2 (1.55 - 0.35)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 2.78$$

$$\frac{I_{REF}}{I_o} = \frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2} \Rightarrow \frac{200}{100} = \frac{\left(\frac{W}{L}\right)_1}{2.78} \Rightarrow \left(\frac{W}{L}\right)_1 = 5.56$$

$$V_{SG1} + V_{SG3} = 4 \Rightarrow V_{SG3} = 2.45 \text{ V}$$

$$I_{REF} = 200 = \left(\frac{50}{2}\right) \left(\frac{W}{L}\right)_3 (2.45 - 0.35)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 1.81$$

10.54

$$I_{REF} = \left(\frac{80}{2}\right) (25) (V_{SG1} - 1.2)^2 = \left(\frac{80}{2}\right) (4) (V_{SG3} - 1.2)^2$$

$$V_{SG1} + 2V_{SG3} = 10 \Rightarrow V_{SG3} = \frac{10 - V_{SG1}}{2}$$

$$\text{Then } \sqrt{\frac{25}{4}} (V_{SG1} - 1.2) = \frac{10 - V_{SG1}}{2} - 1.2$$

$$3V_{SG1} = 6.8 \Rightarrow V_{SG1} = 2.27 \text{ V}$$

$$I_{REF} = \left(\frac{80}{2}\right) (25) (2.267 - 1.2)^2 \Rightarrow I_{REF} = I_o = 1.14 \text{ mA}$$

$$V_{SD2}(\text{sat}) = V_{SG2} + V_{TP} = 2.27 - 1.2 \Rightarrow V_{SD2}(\text{sat}) = 1.07 \text{ V}$$

10.55

$$V_{SG2} = V_{SD2}(\text{sat}) - V_{TP} = 1 + 0.5 = 1.5 \text{ V}$$

$$I_o = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_2 (V_{SG2} + V_{TP})^2$$

$$80 = \left(\frac{60}{2}\right) \left(\frac{W}{L}\right)_2 (1.5 - 0.5)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 2.67$$

$$I_{REF} = 250 = \left(\frac{60}{2}\right) \left(\frac{W}{L}\right)_1 (1.5 - 0.5)^2 \Rightarrow \left(\frac{W}{L}\right)_1 = 8.33$$

$$V_{SG3} = V_{SG4}$$

Then $2V_{SG3} = V^+ - V^- - V_{SG1} = 3 - (-3) - 1.5 = 4.5 \text{ V}$

So $V_{SG3} = 2.25 \text{ V}$

$$250 = \left(\frac{60}{2}\right) \left(\frac{W}{L}\right)_{3,4} (2.25 - 0.5)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 2.72$$

10.56

$$(a) I_{REF} = \left(\frac{k'_p}{2} \right) \left(\frac{W}{L} \right)_1 (V_{SG1} + V_{TP})^2 = \left(\frac{k'_p}{2} \right) \left(\frac{W}{L} \right)_3 (V_{SG3} + V_{TP})^2$$

$$V_{SG3} = 3 - V_{SG1}$$

$$\sqrt{25}(V_{SG1} - 0.4) = \sqrt{5}(3 - V_{SG1} - 0.4)$$

$$3.236V_{SG1} = 3.4944 \Rightarrow V_{SG1} = 1.08 \text{ V} \text{ and } V_{SG3} = 1.92 \text{ V}$$

$$I_{REF} = \left(\frac{60}{2} \right) (25)(1.08 - 0.4)^2 \Rightarrow I_{REF} = 0.347 \text{ mA}$$

$$I_o = \left(\frac{60}{2} \right) (15)(1.08 - 0.4)^2 \Rightarrow I_o = 0.208 \text{ mA}$$

$$(b) V_{SD2}(\text{sat}) = V_{SG2} + V_{TP} = 1.08 - 0.4 = 0.68 \text{ V}$$

$$R = \frac{3 - 0.68}{0.208} = 11.15 \text{ k}\Omega$$

10.57

$$V_{SD2}(\text{sat}) = 0.35 = V_{SG2} + V_{TP} = V_{SG2} - 0.4 \Rightarrow V_{SG2} = 0.75 \text{ V}$$

$$I_{REF} = 220 = \left(\frac{60}{2} \right) \left(\frac{W}{L} \right)_1 (0.75 - 0.4)^2 \Rightarrow \left(\frac{W}{L} \right)_1 = 59.9$$

$$I_o = 80 = \left(\frac{60}{2} \right) \left(\frac{W}{L} \right)_2 (0.75 - 0.4)^2 \Rightarrow \left(\frac{W}{L} \right)_2 = 21.8$$

$$V_{SG3} = 3 - 0.75 = 2.25 \text{ V}$$

$$220 = \left(\frac{60}{2} \right) \left(\frac{W}{L} \right)_3 (2.25 - 0.4)^2 \Rightarrow \left(\frac{W}{L} \right)_3 = 2.14$$

10.58

$$(a) I_{REF} = 100 = 100(V_{GS1} - 0.5)^2 \Rightarrow V_{GS1} = 1.5 \text{ V}$$

$$\text{For } V_{D4} = -2 \text{ V}, V_{DS4} + V_{DS2} = 3 \text{ V} = V_{DS3} + V_{DS1}$$

$$\text{Then } I_o = 100 \mu\text{A}$$

$$(b) R_o = r_{o4} + r_{o2}(1 + g_m r_{o4})$$

$$g_m = 2\sqrt{K_n I_o} = 2\sqrt{(0.1)(0.1)} = 0.2 \text{ mA/V}$$

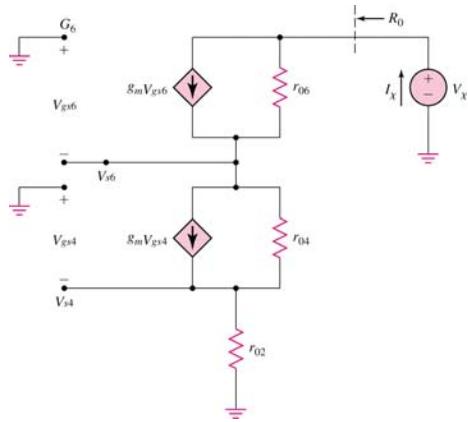
$$r_{o2} = r_{o4} = \frac{1}{\lambda I_o} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$R_o = 500 + 500[1 + (0.2)(500)] \Rightarrow R_o = 51 \text{ M}\Omega$$

$$\Delta I_o = \frac{\Delta V_{D4}}{R_o} = \frac{4}{51} = 0.07843 \mu\text{A}$$

$$\frac{\Delta I_o}{I_o} \times 100\% = \left(\frac{0.07843}{100} \right) \times 100\% = 0.07843\%$$

10.59



$$V_{gs4} = -I_x r_{02}$$

$$V_{S6} = (I_x - g_m V_{gs4}) r_{04} + I_x r_{02}$$

$$= (I_x + g_m I_x r_{02}) r_{04} + I_x r_{02}$$

$$V_{S6} = I_x [r_{02} + (1 + g_m r_{02}) r_{04}] = -V_{gs6}$$

$$I_x = g_m V_{gs6} + \frac{V_x - V_{S6}}{r_{06}} = \frac{V_x}{r_{06}} - V_{S6} \left(g_m + \frac{1}{r_{06}} \right)$$

$$I_x = \frac{V_x}{r_{06}} - I_x \left(g_m + \frac{1}{r_{06}} \right) [r_{02} + (1 + g_m r_{02}) r_{04}]$$

$$I_x \left\{ 1 + \left(g_m + \frac{1}{r_{06}} \right) [r_{02} + (1 + g_m r_{02}) r_{04}] \right\} = \frac{V_x}{r_{06}}$$

$$\frac{V_x}{I_x} = R_0 = r_{06} + (1 + g_m r_{06}) [r_{02} + (1 + g_m r_{02}) r_{04}]$$

$$I_0 \approx I_{REF} = 0.2 \text{ mA} = 0.2(V_{GS} - 1)^2$$

$$V_{GS} = 2 \text{ V}$$

$$g_m = 2K_n (V_{GS} - V_{TN}) = 2(0.2)(2-1) = 0.4 \text{ mA/V}$$

$$r_{02} = r_{04} = r_{06} = \frac{1}{\lambda I_0} = \frac{1}{(0.02)(0.2)} = 250 \text{ k}\Omega$$

$$R_0 = 250 + [1 + (0.4)(250)] \times \{250 + [1 + (0.4)(250)](250)\}$$

$$R_0 = 2575750 \text{ k}\Omega \Rightarrow \underline{R_0 = 2.58 \times 10^9 \Omega}$$

10.60

$$I_{REF} = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right)_3 (V_{GS3} - V_{TN})^2 = \left(\frac{k'_p}{2} \right) \left(\frac{W}{L} \right)_4 (V_{SG4} + V_{TP})^2$$

$$\left(\frac{100}{2} \right) (5)(V_{GS3} - 0.4)^2 = \left(\frac{60}{2} \right) (10)(V_{SG4} - 0.4)^2$$

We find $V_{SG4} = 0.91287 V_{GS3} + 0.03485$

$$I_{REF} = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right)_1 (V_{GS1} - V_{TN})^2 = \left(\frac{k'_p}{2} \right) \left(\frac{W}{L} \right)_3 (V_{GS3} - V_{TN})^2$$

$$\sqrt{20}(V_{GS1} - 0.4) = \sqrt{5}(V_{GS3} - 0.4)$$

$$\text{Now } V_{SG4} + V_{GS3} + V_{GS1} = 6$$

$$0.91287V_{GS3} + 0.03485 + V_{GS3} + V_{GS1} = 6$$

$$\text{Then } V_{GS3} = 3.1184 - 0.52277V_{GS1}$$

$$\text{And } 2(V_{GS1} - 0.4) = 3.1184 - 0.52277V_{GS1} - 0.4$$

$$\text{So } V_{GS1} = 1.395 \text{ V}$$

$$V_{GS3} = 2.389 \text{ V}$$

$$V_{SG4} = 2.216 \text{ V}$$

$$I_{REF} = \left(\frac{0.1}{2} \right) (20)(1.395 - 0.4)^2 = 0.99 \text{ mA}$$

$$I_o = \left(\frac{0.1}{2} \right) (20)(1.395 - 0.4)^2 = 0.99 \text{ mA}$$

$$V_{DS2}(\text{sat}) = 1.395 - 0.4 = 0.995 \text{ V}$$

10.61

$$V_{DS2}(\text{sat}) = 0.5 \text{ V} = V_{GS2} - V_{TN} = V_{GS2} - 0.4 \Rightarrow V_{GS2} = 0.9 \text{ V}$$

$$I_o = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right)_2 (V_{GS2} - V_{TN})^2$$

$$50 = \left(\frac{100}{2} \right) \left(\frac{W}{L} \right)_2 (0.9 - 0.4)^2 \Rightarrow \left(\frac{W}{L} \right)_2 = 4$$

$$I_{REF} = 500 = \left(\frac{100}{2} \right) \left(\frac{W}{L} \right)_1 (0.9 - 0.4)^2 \Rightarrow \left(\frac{W}{L} \right)_1 = 40$$

$$V_{GS3} = V_{SG4}$$

$$6 = V_{GS1} + V_{GS3} + V_{SG4} = 0.9 + 2V_{GS3} \Rightarrow V_{GS3} = V_{SG4} = 2.55 \text{ V}$$

$$500 = \left(\frac{100}{2} \right) \left(\frac{W}{L} \right)_3 (2.55 - 0.4)^2 \Rightarrow \left(\frac{W}{L} \right)_3 = 2.16$$

$$500 = \left(\frac{60}{2} \right) \left(\frac{W}{L} \right)_4 (2.55 - 0.4)^2 \Rightarrow \left(\frac{W}{L} \right)_4 = 3.61$$

10.62

a. As a first approximation

$$I_{REF} = 80 = 80(V_{GS1} - 1)^2 \Rightarrow V_{GS1} = 2 \text{ V}$$

$$\text{Then } V_{DS1} \approx 2(2) = 4 \text{ V}$$

The second approximation

$$80 = 80(V_{GS1} - 1)^2 [1 + (0.02)(4)]$$

$$\text{Or } \frac{80}{86.4} = (V_{GS1} - 1)^2 \Rightarrow V_{GS1} = 1.962$$

Then

$$I_o = K_n (V_{GS1} - V_{TN})^2 (1 + \lambda_n V_{GS1}) \\ = 80(1.962 - 1)^2 [1 + (0.02)(1.962)]$$

Or $I_o = 76.94 \mu\text{A}$

- b. From a PSpice analysis, $I_o = 77.09 \mu\text{A}$ for $V_{D3} = -1 \text{ V}$ and $I_o = 77.14 \mu\text{A}$ for $V_{D3} = 3 \text{ V}$.
The change is $\Delta I_o \approx 0.05 \mu\text{A}$ or 0.065%.

10.63

- a. For a first approximation,

$$I_{REF} = 80 = 80(V_{GS4} - 1)^2 \Rightarrow V_{GS4} = 2 \text{ V}$$

As a second approximation

$$I_{REF} = 80 = 80(V_{GS4} - 1)^2 [1 + (0.02)(2)]$$

Or $V_{GS4} = 1.98 \text{ V} = V_{GS1}$

$$I_o = K_n (V_{GS2} - V_{TN})^2 (1 + \lambda V_{GS2})$$

To a very good approximation $I_o = 80 \mu\text{A}$

- b. From a PSpice analysis, $I_o = 80.00 \mu\text{A}$ for $V_{D3} = -1 \text{ V}$ and the output resistance is $R_o = 76.9 \text{ M}\Omega$.

Then

For $V_D = +3 \text{ V}$

$$\Delta I_o = \frac{1}{R_o} \cdot V_{D3} = \frac{4}{76.9} = 0.052 \mu\text{A}$$

$I_o = 80.05 \mu\text{A}$

10.64

$$V_{DS3}(\text{sat}) = V_{GS3} - V_{TN} \text{ or } V_{GS3} = V_{DS3}(\text{sat}) + V_{TN} = 0.2 + 0.8 = 1.0$$

$$I_D = \frac{k'_n}{2} \left(\frac{W}{L} \right) (V_{GS3} - V_{TN})^2$$

$$(a) \quad 50 = 48 \left(\frac{W}{L} \right) (0.2)^2 \Rightarrow \underline{\left(\frac{W}{L} \right)_3} = 26$$

$$V_{GS5} - V_{TN} = 2(V_{GS3} - V_{TN})$$

$$(b) \quad V_{GS5} = 0.8 + 2(0.2) \Rightarrow \underline{V_{GS5} = 1.2 \text{ V}}$$

$$(c) \quad V_{D1}(\text{min}) = 2V_{DS}(\text{sat}) = 2(0.2) \Rightarrow \underline{V_{D1}(\text{min}) = 0.4 \text{ V}}$$

10.65

(a)

$$K_{n1} = \frac{k'_n}{2} \left(\frac{W}{L} \right)_1 = 50(5) = 250 \mu A/V^2$$

$$\begin{aligned} R &= \frac{1}{\sqrt{K_{n1} I_{D1}}} \left(1 - \sqrt{\frac{(W/L)_1}{(W/L)_2}} \right) \\ &= \frac{1}{\sqrt{(0.25)(0.05)}} \left(1 - \sqrt{\frac{5}{50}} \right) = (8.944)(0.6838) \end{aligned}$$

$$R = 6.12 k\Omega$$

(b)

$$V^+ - V^- = V_{SD3}(\text{sat}) + V_{GS1}$$

$$V_{SD3}(\text{sat}) = V_{SG3} + V_{TP}$$

$$I_{D1} = 50 = 20(5)(V_{SG3} - 0.5)^2 \Rightarrow V_{SG3} = 1.207 V$$

$$\text{Then } V_{SD3}(\text{sat}) = 1.21 - 0.5 = 0.707 V$$

$$\text{Also } I_{D1} = 50 = 50(5)(V_{GS1} - 0.5)^2 \Rightarrow V_{GS1} = 0.9472 V$$

$$\text{Then } (V^+ - V^-)_{\min} = 0.71 + 0.947 = 1.66 V$$

(c)

$$I_{O1} = 25 = 50 \left(\frac{W}{L} \right)_5 (0.947 - 0.5)^2 \Rightarrow \left(\frac{W}{L} \right)_5 = 2.5$$

$$I_{O2} = 75 = 20 \left(\frac{W}{L} \right)_6 (1.207 - 0.5)^2 \Rightarrow \left(\frac{W}{L} \right)_6 = 7.5$$

10.66

$$V_{GS3} = V_{GS4} = V_{GS5} \Rightarrow V_{GS3} = \frac{5}{3} V$$

$$I_{REF} = 0.1 = \left(\frac{0.08}{2} \right) \left(\frac{W}{L} \right)_3 \left(\frac{5}{3} - 0.7 \right)^2 \Rightarrow \left(\frac{W}{L} \right)_{3,4,5} = 2.68$$

$$I_{O1} = 0.2 = \left(\frac{0.08}{2} \right) \left(\frac{W}{L} \right)_1 \left(\frac{5}{3} - 0.7 \right)^2 \Rightarrow \left(\frac{W}{L} \right)_1 = 5.35$$

$$I_{O2} = 0.3 = \left(\frac{0.08}{2} \right) \left(\frac{W}{L} \right)_2 \left(\frac{5}{3} - 0.7 \right)^2 \Rightarrow \left(\frac{W}{L} \right)_2 = 8.03$$

10.67

$$P = (I_{REF} + I_{O1} + I_{O2} + I_{O3})(V^+ - V^-)$$

$$5 = (I_{REF} + 0.1 + 0.2 + 0.4)[1.8 - (-1.8)] \Rightarrow I_{REF} = 0.689 \text{ mA}$$

$$V_{DS2}(\text{sat}) = 0.4 = V_{GS2} - 0.4 \Rightarrow V_{GS2} = 0.8 V = V_{GS}$$

$$I_{REF} = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right)_1 (V_{GS1} - V_{TN})^2$$

$$0.689 = \left(\frac{0.1}{2} \right) \left(\frac{W}{L} \right)_1 (0.8 - 0.4)^2 \Rightarrow \left(\frac{W}{L} \right)_1 = 86.1$$

$$I_{O1} = 0.1 = \left(\frac{0.1}{2} \right) \left(\frac{W}{L} \right)_2 (0.8 - 0.4)^2 \Rightarrow \left(\frac{W}{L} \right)_2 = 12.5$$

$$I_{O2} = 0.2 = \left(\frac{0.1}{2} \right) \left(\frac{W}{L} \right)_3 (0.8 - 0.4)^2 \Rightarrow \left(\frac{W}{L} \right)_3 = 25$$

$$I_{O3} = 0.4 = \left(\frac{0.1}{2} \right) \left(\frac{W}{L} \right)_4 (0.8 - 0.4)^2 \Rightarrow \left(\frac{W}{L} \right)_4 = 50$$

10.68

$$24 = I_{REF} R + V_{SG} + V_{GS}$$

$$V_{SG} = \sqrt{\left(\frac{k'_p}{2} \right)(1)} I_{REF} - V_{TP} = 5.7735 \sqrt{I_{REF}} + 0.8$$

$$V_{GS} = \sqrt{\left(\frac{k'_n}{2} \right)(1)} I_{REF} + V_{TN} = 4.472 \sqrt{I_{REF}} + 0.8$$

$$\text{So } 24 = I_{REF} (100) + 5.7735 \sqrt{I_{REF}} + 0.8 + 4.472 \sqrt{I_{REF}} + 0.8$$

$$\text{Let } \sqrt{I_{REF}} = x$$

$$\text{Then } 100x^2 + 10.2456x - 22.4 = 0 \Rightarrow x = 0.4248 \Rightarrow x^2 = I_{REF} = 0.1805 \text{ mA}$$

$$I_1 = (0.2)I_{REF} = 0.0361 \text{ mA}$$

$$I_2 = (1.25)I_{REF} = 0.2256 \text{ mA}$$

$$I_3 = (0.8)I_{REF} = 0.1444 \text{ mA}$$

$$I_4 = (4)I_{REF} = 0.722 \text{ mA}$$

10.69

$$10 = I_{REF} R + V_{SG} + V_{GS}$$

$$V_{SG} = \sqrt{\left(\frac{0.06}{2} \right)(1)} I_{REF} + 0.8 = 5.7735 \sqrt{I_{REF}} + 0.8$$

$$V_{GS} = \sqrt{\left(\frac{0.1}{2} \right)(1)} I_{REF} + 0.8 = 4.472 \sqrt{I_{REF}} + 0.8$$

$$8.4 = 100I_{REF} + 10.2456 \sqrt{I_{REF}}$$

$$\text{Let } \sqrt{I_{REF}} = x$$

$$\text{Then } 100x^2 + 10.2456x - 8.4 = 0 \Rightarrow x = 0.2431 \Rightarrow x^2 = I_{REF} = 59.09 \mu \text{ A}$$

$$I_1 = (0.2)I_{REF} = 11.82 \mu \text{ A}, \quad I_2 = (1.25)I_{REF} = 73.87 \mu \text{ A}$$

$$I_3 = (0.8)I_{REF} = 47.27 \mu \text{ A}, \quad I_4 = (4)I_{REF} = 236.4 \mu \text{ A}$$

10.70

$$I_{D2} = \frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1} \cdot I_{REF} = \frac{9}{15}(200) \Rightarrow I_{D2} = 120 \mu A$$

$$I_o = \frac{\left(\frac{W}{L}\right)_4}{\left(\frac{W}{L}\right)_3} \cdot I_{D2} = \left(\frac{20}{9}\right)(120) \Rightarrow I_o = 267 \mu A$$

$$I_o = 266.7 = \left(\frac{40}{2}\right)(20)(V_{SG4} - 0.6)^2$$

$$V_{SG4} = 1.416 \text{ V}$$

$$V_{SD4}(\text{sat}) = 1.416 - 0.6 \Rightarrow V_{SD4}(\text{sat}) = 0.816 \text{ V}$$

10.71

$$\text{For } M_1: I_{REF} = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TN})^2$$

$$100 = \left(\frac{100}{2}\right)(4)(V_{GS1} - 0.4)^2 \Rightarrow V_{GS1} = V_{GS2} = 1.107 \text{ V}$$

$$I_{D2} = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TN})^2 = \left(\frac{100}{2}\right)(2.5)(1.107 - 0.4)^2 \Rightarrow I_{D2} = 62.5 \mu A$$

$$\text{Also } I_{D2} = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_3 (V_{SG3} + V_{TP})^2$$

$$62.5 = \left(\frac{60}{2}\right)(6)(V_{SG3} - 0.4)^2 \Rightarrow V_{SG3} = V_{SG4} = 0.9893 \text{ V}$$

$$I_o = \left(\frac{60}{2}\right)(4)(0.9893 - 0.4)^2 = 41.67 \mu A$$

10.72

$$I_{REF} = 50 = \left(\frac{40}{2}\right) \left(\frac{W}{L}\right)_1 (V_{SG1} - 0.6)^2$$

$$I_{REF} = \frac{1.75 - V_{SG1}}{R} = 50$$

$$V_{SD2}(\text{sat}) = 0.35 = V_{SG2} - 0.6 \Rightarrow V_{SG2} = 0.95 \text{ V}$$

$$R = \frac{1.75 - 0.95}{0.05} \Rightarrow R = 16 \text{ K}$$

$$50 = \left(\frac{40}{2}\right) \left(\frac{W}{L}\right)_1 (0.95 - 0.6)^2 \Rightarrow \left(\frac{W}{L}\right)_1 = 20.4$$

$$\frac{I_{O1}}{I_{REF}} = \frac{120}{50} = \frac{(W/L)_2}{(20.4)} \Rightarrow \underline{\left(\frac{W}{L}\right)_2 = 49}$$

$$\frac{I_{D3}}{I_{REF}} = \frac{25}{50} = \frac{(W/L)_3}{(20.4)} \Rightarrow \underline{\left(\frac{W}{L}\right)_3 = 10.2}$$

$$V_{DS5}(\text{sat}) = 0.35 = V_{GS5} - 0.4 \Rightarrow V_{GS5} = 0.75 \text{ V}$$

$$I_{O2} = \left(\frac{100}{2}\right) \left(\frac{W}{L}\right)_5 (0.75 - 0.4)^2 = 150 \Rightarrow \underline{\left(\frac{W}{L}\right)_5 = 24.5}$$

$$\frac{I_{D4}}{I_{O2}} = \frac{I_{D3}}{I_{O2}} = \frac{25}{150} = \frac{(W/L)_4}{24.5} \Rightarrow \underline{\left(\frac{W}{L}\right)_4 = 4.08}$$

10.73

For $v_{GS} = 0$, $i_D = I_{DSS}(1 + \lambda v_{DS})$

$$V_D = -5 \text{ V}, \quad v_{DS} = 5$$

a. $i_D = (2)[1 + (0.05)(5)] \Rightarrow \underline{i_D = 2.5 \text{ mA}}$

$$V_D = 0, \quad v_{DS} = 10$$

b. $i_D = (2)[1 + (0.05)(10)] \Rightarrow \underline{i_D = 3 \text{ mA}}$

$$V_D = 5 \text{ V}, \quad v_{DS} = 15 \text{ V}$$

c. $i_D = (2)[1 + (0.05)(15)] \Rightarrow \underline{i_D = 3.5 \text{ mA}}$

10.74

$$I_0 = I_{DSS} \left(1 - \frac{V_{GS}}{V_p}\right)^2$$

$$2 = 4 \left(1 - \frac{V_{GS}}{V_p}\right)^2$$

$$\frac{V_{GS}}{V_p} = 1 - \sqrt{\frac{2}{4}} = 0.293$$

$$\text{So } V_{GS} = (0.293)(-4) = -1.17 \text{ V}$$

$$\text{Then } I_0 = \frac{V_s}{R} \text{ and } V_s = -V_{GS}$$

$$R = \frac{-V_{GS}}{I_0} = -\frac{(-1.17)}{2} \Rightarrow \underline{R = 0.586 \text{ k}\Omega}$$

$$\text{Need } v_{DS} \geq v_{DS}(\text{sat}) = v_{GS} - V_p$$

$$= -1.17 - (-4)$$

$$v_{DS} \geq 2.83 \text{ V}$$

$$\text{So } V_D \geq v_{DS}(\text{sat}) + V_s = 2.83 + 1.17 \Rightarrow \underline{V_D \geq 4 \text{ V}}$$

10.75

$$(a) \quad A_v = \frac{-\frac{1}{V_T}}{\frac{1}{V_{AN}} + \frac{1}{V_{AP}}} = \frac{-\frac{1}{0.026}}{\frac{1}{120} + \frac{1}{80}} = \frac{-38.46}{0.00833 + 0.0125} \Rightarrow A_v = -1846$$

$$(b) \quad I_{CQ} = I_{S1} \left[\exp \left(\frac{V_I}{V_T} \right) \right] \left[1 + \frac{V_{CE1}}{V_{AN}} \right]$$

$$200 \times 10^{-6} = (5 \times 10^{-16}) \left[\exp \left(\frac{V_I}{V_T} \right) \right] \left[1 + \frac{1.25}{120} \right]$$

$$\text{or } V_I = (0.026) \ln \left[\frac{200 \times 10^{-6}}{(5 \times 10^{-16})(1 + 1.25/120)} \right] = 0.6943 \text{ V}$$

$$(c) \quad 200 \times 10^{-6} = (10^{-15}) \left[\exp \left(\frac{V_{EB}}{V_T} \right) \right] \left[1 + \frac{1.25}{80} \right]$$

$$V_{EB} = (0.026) \ln \left[\frac{200 \times 10^{-6}}{(10^{-15})(1 + 1.25/80)} \right] = 0.6762 \text{ V}$$

$$V_{EB} = 0.6762 = V^+ - V_B \Rightarrow V_B = 1.824 \text{ V}$$

10.76

$$(a) \quad A_v = -g_{m1}(r_{o1} \| r_{o2})$$

$$r_{o1} = \frac{1}{\lambda_1 I_{DQ}} = \frac{1}{(0.02)(0.2)} = 250 \text{ k}\Omega, \quad r_{o2} = \frac{1}{\lambda_2 I_{DQ}} = \frac{1}{(0.03)(0.2)} = 166.7 \text{ k}\Omega$$

$$-100 = -g_{m1}(250 \| 166.7) \Rightarrow g_{m1} = 1 \text{ mA/V}$$

$$g_{m1} = 1 = 2 \sqrt{\left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_1} I_{DQ} = 2 \sqrt{\left(\frac{0.1}{2}\right) \left(\frac{W}{L}\right)_1 (0.2)} \Rightarrow \left(\frac{W}{L}\right)_1 = 25$$

$$(b) \quad I_{DQ} = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_1 (V_I - V_{TN})^2 (1 + \lambda_1 V_{DS1})$$

$$0.2 = \left(\frac{0.1}{2}\right) (25) (V_I - 0.5)^2 [1 + (0.02)(1.25)] \Rightarrow V_I = 0.895 \text{ V}$$

$$(c) \quad K_{n1} = K_{p2} \Rightarrow \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_1 = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_2$$

$$\text{Then } \left(\frac{W}{L}\right)_2 = \left(\frac{100}{60}\right) (25) = 41.67$$

$$I_{DQ} = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_2 (V_{SG2} + V_{TP})^2 (1 + \lambda_2 V_{SD2})$$

$$0.2 = \left(\frac{0.06}{2}\right) (41.67) (V_{SG2} - 0.5)^2 [1 + (0.03)(1.25)] \Rightarrow V_{SG2} = 0.8927 \text{ V}$$

$$V_{SG2} = 0.8927 = V^+ - V_G \Rightarrow V_G = 1.607 \text{ V}$$

10.77

$$I_{REF} = I_{S1} \exp\left(\frac{V_{EB1}}{V_T}\right)$$

a. or $V_{EB1} = V_T \ln\left(\frac{I_{REF}}{I_{S1}}\right) = (0.026) \ln\left(\frac{1 \times 10^{-3}}{5 \times 10^{-13}}\right) \Rightarrow V_{EB1} = 0.5568$

b. $R_i = \frac{5 - 0.5568}{1} \Rightarrow R_i = 4.44 \text{ k}\Omega$

c. From Equations (10.79) and (10.80) and letting $V_{CE0} = V_{EC2} = 2.5 \text{ V}$

$$10^{-12} \exp\left(\frac{V_I}{V_T}\right) \left[1 + \frac{2.5}{120}\right] = 10^{-3} \left(\frac{1 + \frac{2.5}{80}}{1 + \frac{0.5568}{80}} \right)$$

$$1.0208333 \times 10^{-12} \exp\left(\frac{V_I}{V_T}\right) = (10^{-3}) \left(\frac{1.03125}{1.00696} \right)$$

Then $V_I = 0.026 \ln(1.003222 \times 10^9)$

So $V_I = 0.5389 \text{ V}$

$$A_v = \frac{-(1/V_T)}{(1/V_{AN}) + (1/V_{AP})}$$

$$A_v = \frac{-\frac{1}{0.026}}{\frac{1}{120} + \frac{1}{80}} = \frac{-38.46}{0.00833 + 0.0125}$$

d. $\underline{A_v = -1846}$

10.78

a. $V_{BE} = V_T \ln\left(\frac{I_{REF}}{I_{S1}}\right) = (0.026) \ln\left(\frac{0.5 \times 10^{-3}}{10^{-12}}\right) \Rightarrow V_{BE} = 0.5208$

b. $R_1 = \frac{5 - 0.5208}{0.5} \Rightarrow R_1 = 8.96 \text{ k}\Omega$

c. Modify Eqs. 10.79 and 10.80 to apply to pnp and npn, and set the two equation equal to each other.

$$I_{CO} = I_{SO} \exp\left(\frac{V_{EBO}}{V_T}\right) \left(1 + \frac{V_{ECO}}{V_{AP}}\right) = I_{C2} = I_{S2} \exp\left(\frac{V_{BE}}{V_T}\right) \left(1 + \frac{V_{CE2}}{V_{AN}}\right)$$

$$5 \times 10^{-13} \exp\left(\frac{V_{EBO}}{V_T}\right) \left(1 + \frac{2.5}{80}\right) = 10^{-12} \exp\left(\frac{V_{BE}}{V_T}\right) \left(1 + \frac{2.5}{120}\right)$$

$$5.15625 \times 10^{-13} \exp\left(\frac{V_{EBO}}{V_T}\right) = 1.020833 \times 10^{-12} \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$\frac{\exp\left(\frac{V_{EBO}}{V_T}\right)}{\exp\left(\frac{V_{BE}}{V_T}\right)} = 1.9798 = \exp\left(\frac{V_{EBO} - V_{BE}}{V_T}\right)$$

$$V_{EBO} = V_{BE} + V_T \ln(1.9798) = 0.5208 + (0.026) \ln(1.9798)$$

$$V_{EBO} = 0.5386 \Rightarrow V_I = 5 - 0.5386 \Rightarrow V_I = 4.461 \text{ V}$$

$$A_v = \frac{-(1/V_T)}{(1/V_{AN}) + (1/V_{AP})}$$

$$A_v = \frac{-\frac{1}{0.026}}{\frac{1}{120} + \frac{1}{80}} = \frac{-38.46}{0.00833 + 0.0125}$$

d. $A_v = -1846$

10.79

(a) For M_O : $I_{DQ} = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_o (V_I - V_{TN})^2 (1 + \lambda_n V_{DSO})$

$$100 = \left(\frac{100}{2}\right) \left(\frac{W}{L}\right)_o (1.2 - 0.5)^2 [1 + (0.02)(1.5)] \Rightarrow \left(\frac{W}{L}\right)_o = 3.96$$

For M_1, M_2 : For $I_{REF} = I_O \Rightarrow V_{SD2} = V_{SD1} = V_{SG} = 1.5 \text{ V}$

$$100 = \left(\frac{60}{2}\right) \left(\frac{W}{L}\right)_2 (1.5 - 0.5)^2 [1 + (0.02)(1.5)] \Rightarrow \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_1 = 3.24$$

For M_3 : $V_{SG3} = 3 - 1.5 = 1.5 \text{ V} = V_{SD3} \Rightarrow \left(\frac{W}{L}\right)_3 = 3.24$

(b) $A_v = -g_{mo}(r_{oo} \parallel r_{o2})$

$$g_{mo} = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_o} I_{DQ} = 2\sqrt{\left(\frac{0.1}{2}\right)(3.96)(0.1)} = 0.2814 \text{ mA/V}$$

$$r_{oo} = r_{o2} = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$A_v = -(0.2814)(500 \parallel 500) = -70.35$$

10.80

(a) $I_{REF} = K_p (V_{SG} + V_{TP})^2 (1 + \lambda_p V_{SD1})$
 $0.080 = 0.1(V_{SG} - 0.5)^2 [1 + (0.02)V_{SG}]$
 Approximation, for $\lambda = 0 \Rightarrow V_{SG} = 1.394 \text{ V}$

(b) $I_o = I_2$
 $K_n (V_I - V_{TN})^2 (1 + \lambda_n V_{DSO}) = K_p (V_{SG} + V_{TP})^2 (1 + \lambda_p V_{SD2})$
 $K_n = K_p, V_{DSO} = V_{SD2}, \lambda_n = \lambda_p$
 So $V_I = 1.394 \text{ V}$

(c) $I_o \cong I_{REF} = 80 \mu \text{A}$
 $A_v = -g_{mo}(r_{oo} \parallel r_{o2})$
 $g_{mo} = 2\sqrt{K_n I_o} = 2\sqrt{(0.1)(0.08)} = 0.1789 \text{ mA/V}$
 $r_{oo} = r_{o2} = \frac{1}{\lambda I_o} = \frac{1}{(0.02)(0.08)} = 625 \text{ k}\Omega$
 $A_v = -(0.1789)(625 \parallel 625) = -55.9$

10.81

(a) $I_{REF} = I_{CO} = \frac{3 - 0.6}{47} \Rightarrow I_{CO} = 51.06 \mu \text{A}$

 $r_{oo} = \frac{V_{AN}}{I_{CO}} = \frac{120}{0.05106} \Rightarrow r_{oo} = 2.35 \text{ M}\Omega$
 $r_{o2} = \frac{V_{AP}}{I_{CO}} = \frac{90}{0.05106} \Rightarrow r_{o2} = 1.763 \text{ M}\Omega$
 $g_m = \frac{I_{CO}}{V_T} = \frac{0.05106}{0.026} = 1.964 \text{ mA/V}$
 $A_v = -g_m(r_{oo} \parallel r_{o2}) = -(1.964)(2350 \parallel 1763) = -1978$

(b) $A_v = -g_m(r_{oo} \parallel r_{o2} \parallel R_L) = -(1.964)(2350 \parallel 1763 \parallel 300) = -454$

(c) $A_v = -(1.964)(2350 \parallel 1763 \parallel 150) = -256$

10.82

$$I_{REF} = \frac{5 - 0.6}{35} = 0.1257 \text{ mA}$$

Then

$$I_{CO} = 2I_{REF} = 0.2514 \text{ mA}$$

From Eq. 10.96

$$A_v = \frac{\frac{-0.2514}{0.026}}{\frac{0.2514}{120} + \frac{0.2514}{80} + \frac{1}{R_L}} = \frac{-9.6692}{0.002095 + 0.0031425 + \frac{1}{R_L}}$$

$$A_v = \frac{-9.6692}{0.0052375 + \frac{1}{R_L}}$$

(a) $R_L = \infty, A_v = -1846$

(b) $R_L = 250 \text{ K}, A_v = -1047$

10.83

(a) To a good approximation, output resistance is the same as the widlar current source.

$$R_0 = r_{02} [1 + g_{m2} (r_{\pi2} \parallel R_E)]$$

(b) $A_v = -g_{m0} (r_0 \parallel R_L \parallel R_0)$

10.84

Output resistance of Wilson source

$$R_0 \equiv \frac{\beta r_{03}}{2}$$

Then

$$A_v = -g_m (r_0 \parallel R_0)$$

$$r_{03} = \frac{V_{AP}}{I_{REF}} = \frac{80}{0.2} = 400 \text{ k}\Omega$$

$$r_0 = \frac{V_{AN}}{I_{REF}} = \frac{120}{0.2} = 600 \text{ k}\Omega$$

$$g_m = \frac{I_{REF}}{V_T} = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$$

$$A_v = -7.69 \left[600 \left| \frac{(80)(400)}{2} \right| \right] = -7.69 [600 \parallel 16,000] \Rightarrow A_v = -4448$$

10.85

(a) $I_{D2} = I_{D0} = I_{REF} = 200 \mu\text{A}$

$$\text{For } M_2; \quad r_{o2} = \frac{1}{\lambda_p I_{D2}} = \frac{1}{(0.02)(0.2)} = 250 \text{ K}$$

$$g_{m2} = 2\sqrt{K_p I_{D2}} = 2\sqrt{\left(\frac{0.04}{2}\right)(35)(0.2)}$$

$$g_{m2} = 0.748 \text{ mA/V}$$

$$\text{For } M_0; \quad r_\infty = \frac{1}{\lambda_n I_{D0}} = \frac{1}{(0.015)(0.2)} = 333 \text{ K}$$

$$g_{mo} = 2\sqrt{\left(\frac{0.08}{2}\right)(20)(0.2)} \Rightarrow g_{mo} = 0.80 \text{ mA/V}$$

$$A_v = -g_{mo} (r_{o2} \parallel r_{oo}) = -(0.80)(250 \parallel 333)$$

(b) $A_v = -114.3$

$$\text{Want } A_v = -57.15 = -0.80(142.8 \parallel R_L)$$

$$142.8 \parallel R_L = 71.375 = \frac{142.8 R_L}{142.8 + R_L} \Rightarrow R_L = 143 \text{ K}$$

(c)

10.86

Assume M_1, M_2 matched

$$I_{REF} = I_{D2} = I_{D0} = 200 \mu\text{A}$$

$$r_{o2} = \frac{1}{\lambda_p I_{D2}} = \frac{1}{(0.02)(0.2)} = 250 \text{ K}$$

$$r_{oo} = \frac{1}{\lambda_n I_{D0}} = \frac{1}{(0.015)(0.2)} = 333 \text{ K}$$

$$A_v = -g_{mo} (r_{o2} \parallel r_{oo})$$

$$-100 = -g_{mo} (250 \parallel 333) \Rightarrow g_{mo} = 0.70 \text{ mA/V}$$

$$g_{mo} = 2\sqrt{\left(\frac{0.08}{2}\right)\left(\frac{W}{L}\right)_0 (0.2)} = 0.70$$

$$\left(\frac{W}{L}\right)_0 = 15.3$$

$$\text{Now } \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_0 = \left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_2$$

$$\left(\frac{80}{2}\right)(15.3) = \left(\frac{40}{2}\right)\left(\frac{W}{L}\right)_2$$

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_1 = 30.6$$

10.87

$$g_{m1} = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_1 I_{D1}} = 2\sqrt{\left(\frac{0.1}{2}\right)(20)(0.1)} = 0.6325 \text{ mA/V}$$

$$r_{o1} = \frac{1}{\lambda_n I_{D1}} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$R_{o2} = r_{o2} + r_{o3}(1 + g_{m2}r_{o2})$$

$$g_{m2} = 2\sqrt{\left(\frac{0.06}{2}\right)(80)(0.1)} = 0.9798 \text{ mA/V}$$

$$r_{o2} = r_{o3} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$R_{o2} = 500 + 500[1 + (0.9798)(500)] = 245,949 \text{ k}\Omega$$

$$A_v = -g_{m1}(r_{o1} \| R_{o2}) = -(0.6325)(500 \| 245,949) = -316$$

10.88

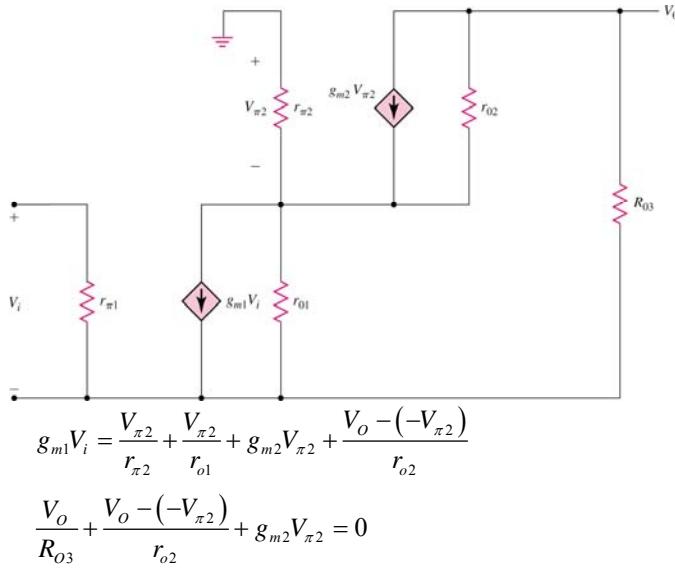
$$A_v = \frac{-g_m^2}{\frac{1}{r_{o3}r_{o4}} + \frac{1}{r_{o1}r_{o2}}}$$

$$g_m = 2\sqrt{\left(\frac{0.1}{2}\right)(25)(0.08)} = 0.6325 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.04)(0.08)} = 312.5 \text{ k}\Omega$$

$$A_v = \frac{-0.6325^2}{\frac{1}{(312.5)^2} + \frac{1}{(312.5)^2}} = \frac{-0.40}{2(0.00001024)} = -19,531$$

10.89



$$(1) \quad g_{m1}V_i = \frac{V_{\pi2}}{r_{\pi2}} + \frac{V_{\pi2}}{r_{o1}} + g_{m2}V_{\pi2} + \frac{V_o - (-V_{\pi2})}{r_{o2}}$$

$$(2) \quad \frac{V_o}{R_{o3}} + \frac{V_o - (-V_{\pi2})}{r_{o2}} + g_{m2}V_{\pi2} = 0$$

$$(1) \quad g_{m1}V_i = V_{\pi2} \left(\frac{1}{r_{\pi2}} + \frac{1}{r_{o1}} + g_{m2} + \frac{1}{r_{o2}} \right) + \frac{V_o}{r_{o2}}$$

$$(2) \quad V_o \left(\frac{1}{R_{o3}} + \frac{1}{r_{o2}} \right) + V_{\pi2} \left(\frac{1}{r_{o2}} + g_{m2} \right) = 0$$

$$g_m \gg \frac{1}{r_o}$$

$$(1) \quad g_{m1}V_i = V_{\pi2} \left(\frac{1+\beta}{r_{\pi2}} \right) + \frac{V_o}{r_{o2}}$$

$$(2) \quad V_o \left(\frac{1}{R_{o3}} + \frac{1}{r_{o2}} \right) + V_{\pi2} \cdot g_{m2} = 0$$

$$V_{\pi2} = -\frac{V_o}{g_{m2}} \left(\frac{1}{R_{o3}} + \frac{1}{r_{o2}} \right)$$

Then

$$(1) \quad g_{m1}V_i = -\frac{V_o}{g_{m2}} \left(\frac{1}{R_{o3}} + \frac{1}{r_{o2}} \right) \left(\frac{1+\beta}{r_{\pi2}} \right) + \frac{V_o}{r_{o2}}$$

$$= -V_o \left(\frac{1}{R_{o3}} + \frac{1}{r_{o2}} \right) \left(\frac{1+\beta}{\beta} \right) + \frac{V_o}{r_{o2}}$$

$$\approx -\frac{V_o}{R_{o3}} \left(\frac{1+\beta}{\beta} \right)$$

$$\frac{V_o}{V_i} = -g_{m1}R_{o3} \left(\frac{\beta}{1+\beta} \right)$$

From Equation (10.20) $R_{o3} \approx \beta r_{o3}$
So

$$A_v = \frac{V_o}{V_i} = \frac{-g_{m1}r_{o3}\beta^2}{1+\beta} \quad g_m = \frac{0.25}{0.026} = 9.615 \text{ mA/V}$$

$$r_{o3} = \frac{80}{0.25} = 320 \text{ K}$$

$$A_v = \frac{-(9.615)(320)(120)^2}{121} = -366,165$$

10.90 Design Problem

10.91

Let $(W/L)_{1,2} = 50$ and $(W/L) = 25$ for all other transistors

Let $I_{REF} = 80 \mu A$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.04)(0.08)} = 312.5 \text{ k}\Omega$$

$$g_m = 2\sqrt{\left(\frac{0.06}{2}\right)(50)(0.08)} = 0.6928 \text{ mA/V}$$

$$A_v = \frac{-g_m^2}{\frac{1}{r_{o3}r_{o4}} + \frac{1}{r_{o1}r_{o2}}} = \frac{- (0.6928)^2}{\frac{1}{(312.5)^2} + \frac{1}{(312.5)^2}} = \frac{-0.480}{2(0.00001024)}$$

$$A_v = -23,438$$

Chapter 11

11.1

- (a) $CMRR_{dB} = \infty \Rightarrow v_o = A_d v_d = (250)(1.5 \sin \omega t)$ (mV)
 $v_o = 0.375 \sin \omega t$ (V)
- (b) $CMRR_{dB} = 80 \text{ dB} \Rightarrow CMRR = 10^4 = \frac{250}{A_{cm}} \Rightarrow A_{cm} = 0.025$
 $v_o = (250)(0.0015 \sin \omega t) + (0.025)(3 \sin \omega t)$
 $v_o = 0.45 \sin \omega t$ (V)
- (c) $CMRR_{dB} = 50 \text{ dB} \Rightarrow CMRR = 316.2 = \frac{250}{A_{cm}} \Rightarrow A_{cm} = 0.791$
 $v_o = (250)(0.0015 \sin \omega t) + (0.791)(3 \sin \omega t)$
 $v_o = 2.75 \sin \omega t$ (V)
-

11.2

- (a) (i) $v_{o1} = -g_m R v_1 = -(1)(5)(0.7 + 0.1 \sin \omega t)$
 $v_{o1} = -3.5 - 0.5 \sin \omega t$ (V)
- (ii) $v_{o2} = -g_m R v_2 = -(1)(5)(0.7 - 0.1 \sin \omega t)$
 $v_{o2} = -3.5 + 0.5 \sin \omega t$ (V)
- (iii) $v_{o1} - v_{o2} = -1.0 \sin \omega t$ (V)
- (b) $\Delta(v_1 - v_2) = (0.7 + 0.1 \sin \omega t) - (0.7 - 0.1 \sin \omega t) = 0.2 \sin \omega t$
- (c) (i) $A_{d1} = \frac{-0.5}{0.2} = -2.5$
(ii) $A_{d2} = \frac{+0.5}{0.2} = +2.5$
(iii) $A_{d3} = \frac{-1}{0.2} = -5$
-

11.3

- (a) Neglect dc base currents
- $I_E = I_{C1} + I_{C2} = 0.2 \text{ mA} = \frac{-0.7 - (-3)}{R_E} \Rightarrow R_E = 11.5 \text{ k}\Omega$
- $v_{o1} = 1.2 \text{ V}, \Rightarrow R_C = \frac{3 - 1.2}{0.1} = 18 \text{ k}\Omega$
- (c) For $V_{CB} = 0 \Rightarrow V_{CE} = 0.7 \text{ V}$
 $3 = I_C(18) + 0.7 + 2I_C(11.5) - 3$
So $I_C = 0.1293 \text{ mA}$
 $v_{o1} = v_{cm+} = 3 - (0.1293)(18) = 0.673 \text{ V}$
 $v_{cm-} = -2.3 \text{ V}$
So $-2.3 \leq v_{cm} \leq 0.673 \text{ V}$
-

11.4

a.

$$I_1 = \frac{10 - 2(0.7)}{8.5} \Rightarrow I_1 = 1.01 \text{ mA}$$

$$I_{C2} = \frac{I_1}{1 + \frac{2}{\beta(1+\beta)}} = \frac{1.01}{1 + \frac{2}{(100)(101)}} \Rightarrow I_{C2} \cong 1.01 \text{ mA}$$

$$I_{C4} = \left(\frac{100}{101} \right) \left(\frac{1.01}{2} \right) \Rightarrow I_{C4} \cong 0.50 \text{ mA}$$

$$V_{CE2} = (0 - 0.7) - (-5) \Rightarrow V_{CE2} = 4.3 \text{ V}$$

$$V_{CE4} = [5 - (0.5)(2)] - (-0.7) \Rightarrow V_{CE4} = 4.7 \text{ V}$$

b.

$$\text{For } V_{CE4} = 2.5 \text{ V} \Rightarrow V_{C4} = -0.7 + 2.5 = 1.8 \text{ V}$$

$$I_{C4} = \frac{5 - 1.8}{2} \Rightarrow I_{C4} = 1.6 \text{ mA}$$

$$I_{C2} + \left(\frac{1 + \beta}{\beta} \right) (2I_{C4}) = \left(\frac{101}{100} \right) (2)(1.6) \Rightarrow I_{C2} = 3.23 \text{ mA}$$

$$I_1 \approx I_{C2} = 3.23 \text{ mA}$$

$$R_l = \frac{10 - 2(0.7)}{3.23} \Rightarrow R_l = 2.66 \text{ k}\Omega$$

11.5

a. Neglecting base currents

$$I_1 = I_3 = 400 \mu\text{A} \Rightarrow R_l = \frac{30 - 0.7}{0.4} \Rightarrow R_l = 73.25 \text{ k}\Omega$$

$$V_{CE1} = 10 \text{ V} \Rightarrow V_{C1} = 9.3 \text{ V}$$

$$R_C = \frac{15 - 9.3}{0.2} \Rightarrow R_C = 28.5 \text{ k}\Omega$$

b.

$$r_\pi = \frac{(100)(0.026)}{0.2} = 13 \text{ k}\Omega$$

$$r_0(Q_3) = \frac{50}{0.4} = 125 \text{ k}\Omega$$

We have

$$A_d = \frac{\beta R_C}{2(r_\pi + R_B)} = \frac{(100)(28.5)}{2(13 + 10)} \Rightarrow A_d = 62$$

$$A_{cm} = -\frac{\beta R_C}{r_\pi + R_B} \left\{ \frac{1}{1 + \frac{2r_0(1+\beta)}{r_\pi + R_B}} \right\}$$

$$= -\frac{(100)(28.5)}{13 + 10} \left\{ \frac{1}{1 + \frac{2(125)(101)}{13 + 10}} \right\} \Rightarrow A_{cm} = -0.113$$

$$CMRR_{dB} = 20 \log_{10} \left(\frac{62}{0.113} \right) \Rightarrow CMRR_{dB} = 54.8 \text{ dB}$$

c.

$$\begin{aligned} R_{id} &= 2(r_\pi + R_B) = 2(13 + 10) \Rightarrow \underline{R_{id} = 46 \text{ k}\Omega} \\ R_{icm} &= \frac{1}{2} [r_\pi + R_B + 2(1 + \beta)r_0] \\ &= \frac{1}{2} [13 + 10 + 2(101)(125)] \Rightarrow \underline{R_{icm} = 12.6 \text{ M}\Omega} \end{aligned}$$

11.6

(a) $v_{CM}(\max) \Rightarrow V_{CB} = 0$ so that $v_{CM}(\max) = 5 - \frac{I_O}{2}(R_C) = 5 - \frac{(0.5)}{2}(8)$

$$\underline{v_{CM}(\max) = 3 \text{ V}}$$

(b)

$$\Delta I = g_m \cdot \frac{V_d}{2} = \left(\frac{I_{CO}}{V_T} \right) \cdot \frac{V_d}{2} = \left(\frac{0.25}{0.026} \right) \left(\frac{0.018}{2} \right) = 0.08654 \text{ mA}$$

$$\Delta V_{C2} = \Delta I \cdot R_C = (0.08654)(8) = 0.692 \text{ V}$$

(c)

$$\Delta I = \left(\frac{0.25}{0.026} \right) \left(\frac{0.010}{2} \right) = 0.04808 \text{ mA}$$

$$\Delta V_{C2} = (0.04808)(8) = 0.385 \text{ V}$$

11.7

$$P = (I_1 + I_{C4})(V^+ - V^-)$$

$$I_1 \cong I_{C4} \text{ so } 1.2 = 2I_1 \Rightarrow \underline{I_1 = I_{C4} = 0.1 \text{ mA}}$$

$$R_i = \frac{3 - 0.7 - (-3)}{0.1} \Rightarrow \underline{R_i = 53 \text{ k}\Omega}$$

$$\text{For } v_{CM} = +1V \Rightarrow V_{C1} = V_{C2} = 1 \text{ V} \Rightarrow R_c = \frac{3 - 1}{0.05} \Rightarrow \underline{R_c = 40 \text{ k}\Omega}$$

One-sided output

$$A_d = \frac{1}{2} g_m R_c \text{ where } g_m = \frac{0.05}{0.026} = 1.923 \text{ mA/V}$$

Then

$$A_d = \frac{1}{2} (1.923)(40) \Rightarrow \underline{A_d = 38.5}$$

11.8

a.

$$0 = 0.7 + \frac{I_E}{2}(2) + I_E(85) - 5$$

$$I_E = \frac{5 - 0.7}{85 + 1} \Rightarrow \underline{I_E = 0.050 \text{ mA}}$$

$$I_{C1} = I_{C2} = \left(\frac{\beta}{1 + \beta} \right) \left(\frac{I_E}{2} \right) = \left(\frac{100}{101} \right) \left(\frac{0.050}{2} \right)$$

$$\text{Or } \underline{I_{C1} = I_{C2} = 0.0248 \text{ mA}}$$

$$V_{CE1} = V_{CE2} = [5 - I_{C1}(100)] - (-0.7)$$

$$\text{So } \underline{V_{CE1} = V_{CE2} = 3.22 \text{ V}}$$

b. $v_{cm}(\max)$ for $V_{CB} = 0$ and $V_C = 5 - I_{C1}(100) = 2.52$ V

So $\underline{v_{cm}(\max) = 2.52 \text{ V}}$

$v_{cm}(\min)$ for Q_1 and Q_2 at the edge of cutoff $\Rightarrow \underline{v_{cm}(\min) = -4.3 \text{ V}}$

(c) Differential-mode half circuits

$$\begin{aligned} -\frac{v_d}{2} &= V_\pi + \left(\frac{V_\pi}{r_\pi} + g_m V_\pi \right) R'_E \\ &= V_\pi \left[1 + \frac{(1+\beta)}{r_\pi} R'_E \right] \end{aligned}$$

Then

$$V_\pi = \frac{-(v_d/2)}{\left[1 + \frac{(1+\beta)}{r_\pi} R'_E \right]}$$

$$v_o = -g_m V_\pi R_C \Rightarrow A_d = \frac{1}{2} \cdot \frac{\beta R_C}{r_\pi + (1+\beta) R'_E}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.0248} = 105 \text{ k}\Omega \quad R'_E = 2 \text{ k}\Omega$$

Then

$$A_d = \frac{1}{2} \cdot \frac{(100)(100)}{105 + (101)(2)} \Rightarrow \underline{A_d = 16.3}$$

11.9

a. For $v_1 = v_2 = 0$ and neglecting base currents

$$R_E = \frac{-0.7 - (-10)}{0.15} \Rightarrow \underline{R_E = 62 \text{ k}\Omega}$$

b.

$$A_d = \frac{v_{o2}}{v_d} = \frac{\beta R_C}{2(r_\pi + R_B)}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.075} = 34.7 \text{ k}\Omega$$

$$A_d = \frac{(100)(50)}{2(34.7 + 0.5)} \Rightarrow \underline{A_d = 71.0}$$

$$A_{cm} = -\frac{\beta R_C}{r_\pi + R_B} \left[\frac{1}{1 + \frac{2R_E(1+\beta)}{r_\pi + R_B}} \right]$$

$$= -\frac{(100)(50)}{34.7 + 0.5} \left[\frac{1}{1 + \frac{2(62)(101)}{34.7 + 0.5}} \right] \Rightarrow \underline{A_{cm} = -0.398}$$

$$CMRR_{dB} = 20 \log_{10} \left| \frac{71.0}{0.398} \right| \Rightarrow \underline{CMRR_{dB} = 45.0 \text{ dB}}$$

c.

$$R_{id} = 2(r_\pi + R_B)$$

$$R_{id} = 2(34.7 + 0.5) \Rightarrow R_{id} = 70.4 \text{ k}\Omega$$

Common-mode input resistance

$$R_{icm} = \frac{1}{2} [r_\pi + R_B + 2(1 + \beta)R_E]$$

$$= \frac{1}{2} [34.7 + 0.5 + 2(101)(62)] \Rightarrow R_{icm} = 6.28 \text{ M}\Omega$$

11.10

(a) $I_{E1} = I_{E2} = \left(\frac{81}{80}\right)(50) = 50.625 \Rightarrow I_E = 101.25 \mu\text{A}$

$$R_E = \frac{3 - 0.6}{0.10125} = 23.7 \text{ k}\Omega$$

$$R_C = \frac{-1.5 - (-3)}{0.05} = 30 \text{ k}\Omega$$

(b) (i) $I_E = \frac{3 - (1 + 0.6)}{23.7} \Rightarrow I_E = 59.07 \mu\text{A}$

$$I_{C1} = I_{C2} = \frac{1}{2} \left(\frac{80}{81}\right)(59.07) = 29.17 \mu\text{A}$$

$$v_{C1} = v_{C2} = I_C R_C - 3 = (0.02917)(30) - 3 = -2.125 \text{ V}$$

(ii) $g_m = \frac{0.02917}{0.026} = 1.122 \text{ mA/V}$

$$v_d = 12 \text{ mV}, \quad \frac{v_d}{2} = 6 \text{ mV}$$

$$v_{C2} = -2.125 - (1.122)(30)(0.006) = -2.327 \text{ V}$$

$$v_{C1} = -2.125 + (1.122)(30)(0.006) = -1.923 \text{ V}$$

11.11

(a)

$$v_1 = v_2 = 0$$

$$I_{E1} = I_{E2} \cong 6 \mu\text{A}$$

$$\beta = 60$$

$$I_{C1} = I_{C2} = 5.90 \mu\text{A}$$

$$v_{C1} = v_{C2} = (5.90)(0.360) - 3$$

$$= -0.875 \text{ V}$$

$$V_{EC1} = V_{EC2} = +0.6 - (-0.875)$$

$$= 1.475 \text{ V}$$

(b)

(i)

$$g_m = \frac{5.90}{0.026} \Rightarrow 0.227 \text{ mA/V}$$

$$A_d = g_m R_C = (0.227)(360) = 81.7$$

$$A_{cm} = 0$$

(ii)

$$A_d = \frac{g_m R_C}{2} = 40.8 \quad r_\pi = \frac{(60)(0.026)}{0.0059} = 264 \text{ K}$$

$$A_{cm} = \frac{-0.227(360)}{1 + \frac{2(61)(4000)}{264}} = -0.0442$$

11.12

$$I_Q = 0.2 \text{ mA}, \Rightarrow I_{C1} = I_{C2} = \frac{1}{2} \left(\frac{80}{81} \right) (0.2) = 0.09877 \text{ mA}$$

(a) For $V_{CB} = 0$, $v_{C1} = v_{C2} = v_{CM} = -2.5 = I_C R_C - 5$

$$\text{So } R_C = \frac{5 - 2.5}{0.09877} = 25.3 \text{ k}\Omega$$

$$(b) A_d = g_m R_C, \quad g_m = \frac{0.09877}{0.026} = 3.799 \text{ mA/V}$$

$$A_d = (3.799)(25.3) = 96.1$$

$$(c) v_d = 14 \text{ mV}, \quad \frac{v_d}{2} = 7 \text{ mV}$$

$$v_{C1} = -2.5 - g_m R_C \cdot \frac{v_d}{2} = -2.5 - (3.799)(25.3)(0.007) = -3.173 \text{ V}$$

$$v_{C2} = -2.5 + (3.799)(25.3)(0.007) = -1.827 \text{ V}$$

(d) $CMRR_{dB} = 60 \text{ dB} \Rightarrow CMRR = 1000$

$$1000 = \frac{1}{2} \left[1 + \frac{(81)(0.2)R_o}{(0.026)(80)} \right] \Rightarrow R_o = 257 \text{ k}\Omega$$

11.13

(a) Neglect dc base currents

$$I_E = I_{C1} + I_{C2} = 240 \mu\text{A}$$

$$R_E = \frac{-0.7 - (-5)}{0.24} = 17.9 \text{ k}\Omega$$

$$R_C = \frac{5 - 3}{0.12} = 16.7 \text{ k}\Omega$$

$$(b) g_m = \frac{0.12}{0.026} = 4.615 \text{ mA/V}$$

$$A_d = g_m R_C = (4.615)(16.667) = 76.9$$

$$(c) A_{cm} \cong \frac{\Delta R_C}{R_E} = \frac{0.5}{17.9} = 0.0279$$

$$CMRR_{dB} = 20 \log_{10} \left(\frac{76.9}{0.0279} \right) = 68.8 \text{ dB}$$

11.14

$$v_1 = v_2 = 0$$

$$I_E = \frac{-0.7 - (-10)}{75}$$

$$= 0.124 \text{ mA}$$

$$I_{C1} = I_{C2} = 0.0615 \text{ mA}$$

$$g_m = \frac{0.0615}{0.026} = 2.365 \text{ mA/V}$$

$$\frac{\Delta g_m}{g_m} = 0.01$$

$$g_{m1} = 2.377 \text{ mA/V}$$

$$g_{m2} = 2.353 \text{ mA/V}$$

$$r_\pi = \frac{(120)(0.026)}{0.0615} = 50.7 \text{ K}$$

$$\Delta I = g_m \frac{V_d}{2}$$

$$\Delta v_{C1} = -g_{m1} \frac{V_d}{2} R_c$$

$$\Delta v_{C2} = +g_{m2} \frac{V_d}{2} R_c$$

$$v_o = \Delta v_{C1} - \Delta v_{C2} = -g_{m1} \frac{V_d}{2} R_c - g_{m2} \frac{V_d}{2} R_c$$

$$= -\frac{V_d}{2} R_c (g_{m1} + g_{m2})$$

$$A_d = -\frac{R_C}{2} (g_{m1} + g_{m2}) = \frac{-50}{2} (2.377 + 2.353) \Rightarrow A_d = -118.25$$

Common-Mode

$$\Delta v_{C1} = \frac{-g_{m1} R_C v_{cm}}{1 + \left(\frac{1 + \beta}{r_\pi} \right) (2R_E)} \quad \Delta v_{C2} = \frac{-g_{m2} R_C v_{cm}}{1 + \left(\frac{1 + \beta}{r_\pi} \right) (2R_E)}$$

$$\frac{v_o}{v_{cm}} = A_{cm} = \frac{-(g_{m1} - g_{m2}) R_C}{1 + \left(\frac{1 + \beta}{r_\pi} \right) (2R_E)} = \frac{-(2.377 - 2.353)(50)}{1 + \left(\frac{121}{50.7} \right)(2)(75)}$$

$$= \frac{-1.2}{358.99} \Rightarrow A_{cm} = -0.003343$$

$$CMRR_{dB} = 91 \text{ dB}$$

11.15

(a)

$$v_1 = v_2 = 0$$

$$v_E = +0.7 \text{ V}$$

$$I_E = \frac{5 - 0.7}{1} = 4.3 \text{ mA}$$

$$I_{C1} = I_{C2} = 2.132 \text{ mA}$$

$$v_{C1} = v_{C2} = (2.132)(1) - 5 \\ = -2.87 \text{ V}$$

$$v_1 = 0.5, \quad v_2 = 0 \quad Q_2 \text{ on}$$

$$Q_1 \text{ off}$$

(b)

$$I_{C1} = 0, \quad I_{C2} = 4.3 \left(\frac{120}{121} \right) \text{ mA} = 4.264 \text{ mA}$$

$$\underline{v_{C1} = -5 \text{ V}} \quad v_{C2} = (4.264)(1) - 5$$

$$\underline{\underline{v_{C2} = -0.736 \text{ V}}}$$

$$(c) \quad \underline{v_E \approx 0.7 \text{ V}} \quad g_m = \frac{2.132}{0.026} = 82.0 \text{ mA/V}$$

$$\Delta I = g_m \frac{v_d}{2} \quad \Delta v_C = \Delta I \cdot R_C = g_m \frac{V_d}{2} \cdot R_C = \frac{(82.0)}{2} \cdot V_d(1) = 41.0 V_d$$

$$V_d = 0.015 \Rightarrow \Delta v_c = 0.615 \text{ V}$$

$$v_{C2} \downarrow v_{C1} \uparrow$$

$$v_{C1} = -2.87 + 0.615 = -2.255 \text{ V}$$

$$v_{C2} = -2.87 - 0.615 = -3.485 \text{ V}$$

11.16

(a)

$$g_m = \frac{I_C}{V_T} = \frac{1}{0.026} = 38.46 \text{ mA/V}$$

$$A_d = \frac{v_o}{v_d} = \frac{1}{0.01} = 100$$

$$A_d = g_m R_C$$

$$100 = 38.46 R_C$$

$$\underline{\underline{R_c = 2.6 \text{ K}}}$$

(b)

With $v_1 = v_2 = 0$

$$v_{C1} = v_{C2} = 10 - (1)(2.6) = 7.4 \text{ V} \Rightarrow v_{cm}(\max) = 7.4 \text{ V}$$

11.17

(a) $I_O = I_{E1} + I_{E2} \Rightarrow I_{E1} = I_{E2} = 0.4 \text{ mA}$

(i) $v_{O1} - v_{O2} = [V^+ - I_{E1}R_{C1}] - [V^+ - I_{E2}R_{C2}] = I_{E2}R_{C2} - I_{E1}R_{C1}$
 $v_{O1} - v_{O2} = 0$

(ii) $v_{O1} - v_{O2} = (0.4)(7.6 - 7.4) = 0.08 \text{ V}$

(b) $V_{BE1} = V_{BE2}$

$$0.8 = (2.9 \times 10^{-15} + 3.1 \times 10^{-15}) \cdot \exp\left(\frac{V_{BE}}{V_T}\right) \Rightarrow \exp\left(\frac{V_{BE}}{V_T}\right) = 1.333 \times 10^{14}$$

Then $I_{E1} = (2.9 \times 10^{-15})(1.333 \times 10^{14}) = 0.3867 \text{ mA}$

$I_{E2} = (3.1 \times 10^{-15})(1.333 \times 10^{14}) = 0.4133 \text{ mA}$

(i) $v_{O1} - v_{O2} = (0.4133 - 0.3867)(7.5) = 0.1995 \text{ V}$

(ii) $v_{O1} - v_{O2} = (0.4133)(7.6) - (0.3867)(7.4) = 0.2795 \text{ V}$

11.18

(a) $\frac{i_{C1}}{I_Q} = 0.20 = \frac{1}{1 + \exp\left(\frac{-v_d}{V_T}\right)} \Rightarrow \exp\left(\frac{-v_d}{V_T}\right) = 4$

so that $-v_d = (0.026)\ln(4) \Rightarrow v_d = -36.0 \text{ mV}$

(b) $\frac{i_{C2}}{I_Q} = 0.90 = \frac{1}{1 + \exp\left(\frac{+v_d}{V_T}\right)} \Rightarrow \exp\left(\frac{+v_d}{V_T}\right) = 0.1111$

so that $v_d = (0.026)\ln(0.1111) = -57.1 \text{ mV}$

11.19

(a) $\frac{\left[0.5I_Q + \left(\frac{I_Q}{4V_T}\right) \cdot v_d(\max)\right] - \frac{I_Q}{1 + \exp(-v_d(\max)/V_T)}}{\left[0.5I_Q + \left(\frac{I_Q}{4V_T}\right) \cdot v_d(\max)\right]} = 0.005$

$$(0.995) \left[0.5 + \left(\frac{1}{4V_T} \right) \cdot v_d(\max) \right] = \frac{1}{1 + \exp(-v_d(\max)/V_T)}$$

By trial and error, $v_d(\max) \approx 14 \text{ mV}$

(b) $(0.985) \left[0.5 + \left(\frac{1}{4V_T} \right) \cdot v_d(\max) \right] = \frac{1}{1 + \exp(-v_d(\max)/V_T)}$

By trial and error, $v_d(\max) \approx 21.2 \text{ mV}$

11.20

For $V_{CM} = 3.5 \text{ V}$ and a maximum peak-to-peak swing in the output voltage of 2 V , we need the quiescent collector voltage to be

$$V_c = 3.5 + 1 = 4.5 \text{ V}$$

Assume the bias is $\pm 10 \text{ V}$, and $I_Q = 0.5 \text{ mA}$.

Then $I_C = 0.25 \text{ mA}$

$$\text{Now } R_C = \frac{10 - 4.5}{0.25} \Rightarrow R_C = 22 \text{ k}\Omega$$

$$r_\pi = \frac{(100)(0.026)}{0.25} = 10.4 \text{ k}\Omega$$

In this case,

Then

$$A_u = \frac{(100)(22)}{2(10.4 + 0.5)} = 101$$

So gain specification is met.

For $CMRR_{dB} = 80 \text{ dB} \Rightarrow$

$$CMRR = 10^4 = \frac{1}{2} \left[1 + \frac{(1 + \beta) I_Q R_o}{V_T \beta} \right] = \frac{1}{2} \left[1 + \frac{(101)(0.5) R_o}{(0.026)(100)} \right] \Rightarrow R_o = 1.03 \text{ M}\Omega$$

Need to use a Modified Widlar current source.

$$R_o = r_o [1 + g_m (R_{E1} \| r_\pi)]$$

$$\text{If } V_A = 100 \text{ V, then } r_o = \frac{100}{0.5} = 200 \text{ k}\Omega$$

$$r_\pi = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

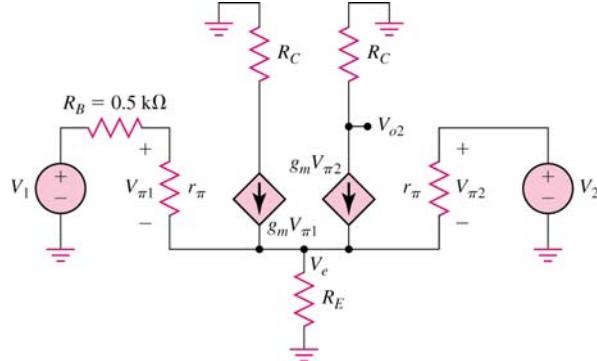
$$g_m = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$\text{Then } 1030 = 200 [1 + (19.23)(R_{E1} \| r_\pi)] \Rightarrow R_{E1} \| r_\pi = 0.216 \text{ k}\Omega \Rightarrow R_{E1} \| 5.2 = 0.216$$

So, $R_{E1} = 225 \text{ }\Omega$; also $I_{REF} \cong 0.5 \text{ mA}$

11.21

$$(a) \quad R_E = \frac{-0.7 - (-10)}{0.25} \Rightarrow R_E = 37.2 \text{ k}\Omega$$



(b)

$$\frac{V_{\pi 1}}{r_{\pi}} + g_m V_{\pi 1} + \frac{V_{\pi 2}}{r_{\pi}} + g_m V_{\pi 2} = \frac{V_e}{R_E} \text{ or } (1) \left(\frac{1+\beta}{r_{\pi}} \right) (V_{\pi 1} + V_{\pi 2}) = \frac{V_e}{R_E}$$

$$\frac{V_{\pi 1}}{r_{\pi}} = \frac{V_1 - V_e}{R_B + r_{\pi}} \Rightarrow V_{\pi 1} = \left(\frac{r_{\pi}}{r_{\pi} + R_B} \right) (V_1 - V_e)$$

$$V_{\pi 2} = V_2 - V_e$$

Then

$$(1) \left(\frac{1+\beta}{r_{\pi}} \right) \left[\frac{r_{\pi}}{r_{\pi} + R_B} (V_1 - V_e) + (V_2 - V_e) \right] = \frac{V_e}{R_E}$$

From this, we find

$$V_e = \frac{V_1 + \frac{r_{\pi} + R_B}{r_{\pi}} \cdot V_2}{\left[\frac{r_{\pi} + R_B}{R_E (1+\beta)} + 1 + \frac{r_{\pi} + R_B}{r_{\pi}} \right]}$$

Now

$$V_o = -g_m V_{\pi 2} R_C = -g_m R_C (V_2 - V_e)$$

We have

$$r_{\pi} = \frac{(120)(0.026)}{0.125} \approx 25 \text{ k}\Omega, \quad g_m = \frac{0.125}{0.026} = 4.81 \text{ mA/V}$$

(i)

$$\text{Set } V_1 = \frac{V_d}{2} \text{ and } V_2 = -\frac{V_d}{2}$$

Then

$$V_e = \frac{\frac{V_d}{2} \left(1 - \left(\frac{25+0.5}{25} \right) \right)}{\left[\frac{25+0.5}{(37.2)(121)} + 1 + \frac{25+0.5}{25} \right]} = \frac{\frac{V_d}{2} (-0.02)}{2.026}$$

So

$$V_e = -0.00494 V_d$$

Now

$$V_o = -(4.81)(50) \left(-\frac{V_d}{2} - (-0.00494) V_d \right) \Rightarrow A_d = \frac{V_o}{V_d} = 119$$

(ii)

$$\text{Set } V_1 = V_2 = V_{cm}$$

Then

$$V_e = \frac{V_{cm} \left(1 + \frac{25+0.5}{25} \right)}{\left[\frac{25+0.5}{(37.2)(121)} + 1 + \frac{25+0.5}{25} \right]} = \frac{V_{cm} (-2.02)}{2.02567}$$

$$V_e = V_{cm} (0.9972)$$

Then

$$V_o = -(4.81)(50) [V_{cm} - V_{cm} (0.9972)]$$

$$\text{or } A_{cm} = \frac{V_o}{V_{cm}} = -0.673$$

11.22

(a) $A_d = g_m R_C$

$$A_d = \frac{1.2}{0.016} = 75$$

$$g_m = \frac{0.125}{0.026} = 4.808 \text{ mA/V}$$

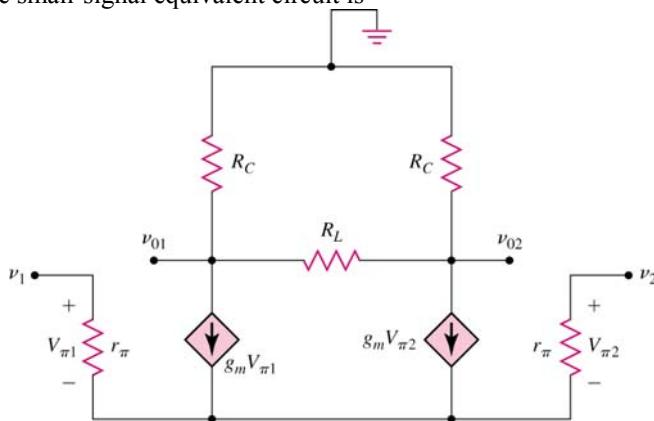
$$\text{Then } R_C = \frac{75}{4.808} = 15.6 \text{ k}\Omega$$

(b) For $V_{CB} = 0 \Rightarrow v_{C1} = v_{C2} = v_{CM} = 3 - (0.125)(15.6) = 1.05 \text{ V}$

(c) $CMRR = \frac{1}{2} \left[1 + \frac{(0.25)(4000)}{(0.026)} \right] = 19,231 \Rightarrow CMRR_{dB} = 85.7 \text{ dB}$

11.23

The small-signal equivalent circuit is



A KVL equation: $v_1 = V_{\pi 1} - V_{\pi 2} + v_2$
 $v_1 - v_2 = V_{\pi 1} - V_{\pi 2}$

A KCL equation

$$\frac{V_{\pi 1}}{r_\pi} + g_m V_{\pi 1} + \frac{V_{\pi 2}}{r_\pi} + g_m V_{\pi 2} = 0$$

$$(V_{\pi 1} + V_{\pi 2}) \left(\frac{1}{r_\pi} + g_m \right) = 0 \Rightarrow V_{\pi 1} = -V_{\pi 2}$$

$$\text{Then } v_1 - v_2 = 2V_{\pi 1} \Rightarrow V_{\pi 1} = \frac{1}{2}(v_1 - v_2) \text{ and } V_{\pi 2} = -\frac{1}{2}(v_1 - v_2)$$

At the v_{01} node:

$$\frac{v_{01}}{R_C} + \frac{v_{01} - v_{02}}{R_L} + g_m V_{\pi 1} = 0$$

$$v_{01} \left(\frac{1}{R_C} + \frac{1}{R_L} \right) - v_{02} \left(\frac{1}{R_L} \right) = \frac{1}{2} g_m (v_2 - v_1) \quad (1)$$

At the v_{02} node:

$$\frac{v_{02}}{R_C} + \frac{v_{02} - v_{01}}{R_L} + g_m V_{\pi 2} = 0$$

$$v_{02} \left(\frac{1}{R_C} + \frac{1}{R_L} \right) - v_{01} \left(\frac{1}{R_L} \right) = \frac{1}{2} g_m (v_1 - v_2) \quad (2)$$

From (1):

$$v_{02} = v_{01} \left(1 + \frac{R_L}{R_C} \right) - \frac{1}{2} g_m R_L (v_2 - v_1)$$

Substituting into (2)

$$v_{01} \left(1 + \frac{R_L}{R_C} \right) \left(\frac{1}{R_C} + \frac{1}{R_L} \right) - \frac{1}{2} g_m R_L (v_2 - v_1) \left(\frac{1}{R_C} + \frac{1}{R_L} \right) - v_{01} \left(\frac{1}{R_L} \right) = \frac{1}{2} g_m (v_1 - v_2)$$

$$v_{01} \left(\frac{1}{R_C} + \frac{R_L}{R_C^2} + \frac{1}{R_C} \right) = \frac{1}{2} g_m (v_1 - v_2) \left[1 - \left(\frac{R_L}{R_C} + 1 \right) \right]$$

$$\frac{v_{01}}{R_C} \left(2 + \frac{R_L}{R_C} \right) = -\frac{1}{2} g_m \left(\frac{R_L}{R_C} \right) (v_1 - v_2)$$

For $v_1 - v_2 = v_d$

$$A_{v1} = \frac{v_{01}}{v_d} = \frac{-\frac{1}{2} g_m R_L}{\left(2 + \frac{R_L}{R_C} \right)}$$

$$A_{v2} = \frac{v_{02}}{v_d} = \frac{\frac{1}{2} g_m R_L}{\left(2 + \frac{R_L}{R_C} \right)}$$

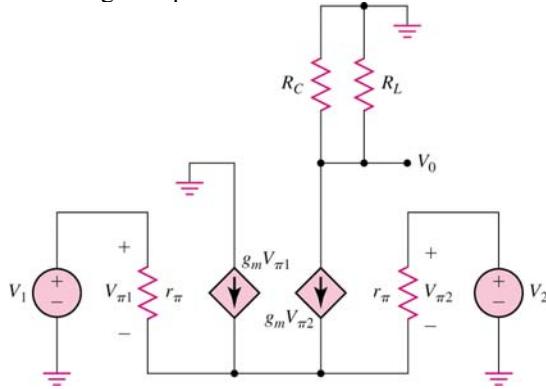
From symmetry:

$$A_v = \frac{v_{02} - v_{01}}{v_d} = \frac{g_m R_L}{\left(2 + \frac{R_L}{R_C} \right)}$$

Then

11.24

The small-signal equivalent circuit is



KVL equation: $v_1 = V_{\pi 1} - V_{\pi 2} + v_2$ or $v_1 - v_2 = V_{\pi 1} - V_{\pi 2}$

KCL equation:

$$\frac{V_{\pi 1}}{r_{\pi}} + g_m V_{\pi 1} + g_m V_{\pi 2} + \frac{V_{\pi 2}}{r_{\pi}} = 0$$

$$(V_{\pi 1} + V_{\pi 2}) \left(\frac{1}{r_{\pi}} + g_m \right) = 0 \Rightarrow V_{\pi 1} = -V_{\pi 2}$$

Then $v_1 - v_2 = -2V_{\pi 2}$, or $V_{\pi 2} = -\frac{1}{2}(v_1 - v_2)$

Now $v_o = -g_m V_{\pi 2} (R_C \| R_L)$

$$= \frac{1}{2} g_m (R_C \| R_L) (v_1 - v_2)$$

For $v_1 - v_2 \equiv v_d \Rightarrow A_d = \frac{v_o}{v_d} = \frac{1}{2} g_m (R_C \| R_L)$

11.25

We have $V_{C2} = -g_m V_{\pi 2} R_C = -g_m (V_{b2} - V_e) R_C$

and

$V_{C1} = -g_m V_{\pi 1} R_C = -g_m (V_{b1} - V_e) R_C$

Then

$$V_0 = V_{C2} - V_{C1}$$

$$= -g_m (V_{b2} - V_e) R_C - [-g_m (V_{b1} - V_e) R_C]$$

$$= g_m R_C (V_{b1} - V_{b2})$$

$$A_d = \frac{V_0}{V_{b1} - V_{b2}} = g_m R_C$$

Differential gain $\frac{A_d}{V_{b1} - V_{b2}}$

Common-mode gain $\frac{A_{cm}}{V_{b1} - V_{b2}} = 0$

11.26

(a)

$$v_{cm} = 3 \text{ V} \Rightarrow V_{C1} = V_{C2} = 3 \text{ V}$$

$$\text{Then } R_C = \frac{10 - 3}{0.1} \Rightarrow R_C = 70 \text{ k}\Omega$$

(b)

$$CMRR_{dB} = 75 \text{ dB} \Rightarrow CMRR = 5623$$

Now

$$CMRR = \frac{1}{2} \left[1 + \frac{(1+\beta) I_Q R_o}{\beta V_T} \right]$$

$$5623 = \frac{1}{2} \left[1 + \frac{(151)(0.2) R_o}{(150)(0.026)} \right] \Rightarrow R_o = 1.45 \text{ M}\Omega$$

Use a Widlar current source.

$$R_o = r_o [1 + g_m R'_E]$$

Let V_A of current source transistor be 100 V.

$$\text{Now } r_o = \frac{100}{0.2} = 500 \text{ k}\Omega, \quad g_m = \frac{0.2}{0.026} = 7.69 \text{ mA/V}$$

$$r_\pi = \frac{(150)(0.026)}{0.2} = 19.5 \text{ k}\Omega$$

$$\text{So } 1450 = 500[1 + (7.69)R'_E] \Rightarrow R'_E = 0.247 \text{ k}\Omega$$

$$\text{Now } R'_E = R_E \| r_\pi \Rightarrow 0.247 = R_E \| 19.5 \Rightarrow R_E = 250 \Omega$$

$$\text{We have } I_Q R_E = V_T \ln\left(\frac{I_{REF}}{I_Q}\right)$$

$$(0.2)(0.250) = (0.026) \ln\left[\frac{I_{REF}}{(0.2)}\right] \Rightarrow I_{REF} = 1.37 \text{ mA}$$

$$\text{Then } R_i = \frac{10 - 0.7 - (-10)}{1.37} \Rightarrow R_i = 14.1 \text{ k}\Omega$$

11.27

$$v_A = \left(\frac{R_A}{R_A + R} \right) \cdot V^+ = \left[\frac{R(1+\delta)}{R(1+\delta) + R} \right] \cdot V^+ = \left(\frac{1+\delta}{2+\delta} \right) \cdot V^+$$

$$v_B = \frac{1}{2} V^+ = 2.5 \text{ V}$$

$$\text{For } \delta = +0.01, \quad v_A = \left(\frac{1.01}{2.01} \right)(5) = 2.5124 \text{ V}$$

$$\text{For } \delta = -0.01, \quad v_A = \left(\frac{0.99}{1.99} \right)(5) = 2.4874 \text{ V}$$

$$v_d \cong 12.5 \text{ mV}$$

$$R_B \cong R \| R = 20 \text{ k}\Omega$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{0.1} = 31.2 \text{ k}\Omega$$

$$A_d = \frac{\beta R_C}{2(r_\pi + R_B)} = \frac{(120)(15)}{2(31.2 + 20)} = 17.58$$

$$v_o = A_d \cdot v_d = (17.58)(0.0125) = 0.22 \text{ V}$$

$$-0.22 \leq v_{o2} \leq +0.22 \text{ V}$$

11.28

$$(a) \quad R_{id} = 2r_\pi = \frac{2\beta V_T}{I_{CQ}} = \frac{2(120)(0.026)}{0.125} = 49.9 \text{ k}\Omega$$

$$(b) \quad R_{icm} = \frac{1}{2} [r_\pi + (1 + \beta)(2R_o)] = \frac{1}{2} \left[\frac{(120)(0.026)}{0.125} + (121)(2)(8000) \right]$$

$$R_{icm} = 968 \text{ M}\Omega$$

11.29

(a)

$$I_1 = \frac{10 - 0.7 - (-10)}{R_1} = 0.5 \Rightarrow R_1 = 38.6 \text{ K}$$

$$R_2 = \frac{0.026}{0.14} \ln\left(\frac{0.5}{0.14}\right) \Rightarrow R_2 = 236 \Omega$$

(b)

$$R_{icm} \approx (1 + \beta)R_o$$

$$R_o = r_{o4}(1 + g_{m4}R'_E) \quad g_{m4} = \frac{0.14}{0.026} = 5.385 \text{ mA/V}$$

$$r_{\pi 4} = \frac{(180)(0.026)}{0.14} = 33.4 \text{ K}$$

$$R'_E = 33.4 \parallel 0.236 = 0.234 \text{ K}$$

$$r_{o4} = \frac{100}{0.14} = 714 \text{ K}$$

$$R_o = 714[1 + (5.385)(0.234)]$$

$$= 1614 \text{ K}$$

$$R_{icm} = (181)(1614) \approx 292 \text{ M}\Omega$$

(c)

$$A_{cm} = \frac{-g_{m1}R_C}{1 + \frac{2(1 + \beta)R_o}{r_{\pi 1}}} \quad g_{m1} = \frac{0.07}{0.026} = 2.692 \text{ mA/V}$$

$$r_{\pi 1} = \frac{(180)(0.026)}{0.07} = 66.86 \text{ K}$$

$$A_{cm} = \frac{-(2.692)(40)}{1 + \frac{2(181)(1614)}{66.86}}$$

$$\underline{A_{cm} = -0.0123}$$

11.30

$$A_{d1} = g_{m1}(R_1 \parallel r_{\pi 3})$$

$$g_{m1} = \frac{I_{Q1}/2}{V_T} = 19.23I_{Q1}$$

$$r_{\pi 3} = \frac{\beta V_T}{I_{Q2}/2} = \frac{2(100)(0.026)}{I_{Q2}} = \frac{5.2}{I_{Q2}}$$

$$A_{d2} = \frac{g_{m3}R_2}{2}, \quad g_{m3} = \frac{I_{Q2}/2}{V_T} = 19.23I_{Q2}$$

$$\text{Then } 30 = \frac{(19.23)I_{Q2}}{2} \cdot R_2 \Rightarrow I_{Q2}R_2 = 3.12 \text{ V}$$

Maximum $v_{o2} - v_{o1} = \pm 18 \text{ mV}$ for linearity

$$v_{o3}(\max) = (\pm 18)(30) \text{ mV} \Rightarrow \pm 0.54 \text{ V}$$

so $I_{Q2}R_2 = 3.12 \text{ V}$ is OK.

From A_{d1} :

$$20 = 19.23I_{Q1}(R_1 \parallel r_{\pi 3}) = 19.23I_{Q1} \left[\frac{R_1 \left(\frac{5.2}{I_{Q2}} \right)}{R_1 + \left(\frac{5.2}{I_{Q2}} \right)} \right]$$

$$20 = \frac{19.23I_{Q1}R_1(5.2)}{I_{Q2}R_1 + 5.2}$$

$$\text{Let } \frac{I_{Q1}}{2} \cdot R_1 = 5 \text{ V, } \Rightarrow I_{Q1}R_1 = 10 \text{ V}$$

$$\text{Then } 20 = \frac{19.23(10)(5.2)}{I_{Q2}R_1 + 5.2} \Rightarrow I_{Q2}R_1 = 44.8 \text{ V}$$

$$\text{Now } I_{Q1}R_1 = 10 \Rightarrow R_1 = \frac{10}{I_{Q1}}$$

$$\text{So } I_{Q2} \left(\frac{10}{I_{Q1}} \right) = 44.8 \Rightarrow \frac{I_{Q2}}{I_{Q1}} = 4.48$$

$$\text{Let } I_{Q1} = 100 \mu \text{A, then } I_{Q2} = 448 \mu \text{A}$$

Then

$$I_{Q2}R_2 = 3.12 \Rightarrow R_2 = 6.96 \text{ k}\Omega$$

$$I_{Q1}R_1 = 10 \Rightarrow R_1 = 100 \text{ k}\Omega$$

11.31

a.

$$I_1 = \frac{20 - V_{GS3}}{50} = 0.25(V_{GS3} - 2)^2$$

$$20 - V_{GS3} = 12.5(V_{GS3}^2 - 4V_{GS3} + 4)$$

$$12.5V_{GS3}^2 - 49V_{GS3} + 30 = 0$$

$$V_{GS3} = \frac{49 \pm \sqrt{(49)^2 - 4(12.5)(30)}}{2(12.5)} \Rightarrow V_{GS3} = 3.16 \text{ V}$$

$$I_1 = \frac{20 - 3.16}{50} \Rightarrow I_1 = I_Q = 0.337 \text{ mA}$$

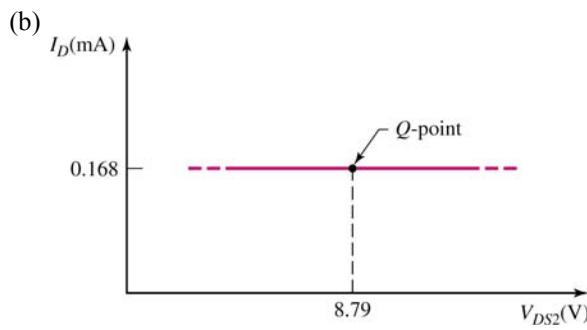
$$I_{D1} = \frac{I_Q}{2} \Rightarrow I_{D1} = 0.168 \text{ mA}$$

$$0.168 = 0.25(V_{GS1} - 2)^2 \Rightarrow V_{GS1} = 2.82 \text{ V}$$

$$V_{DS4} = -2.82 - (-10) \Rightarrow V_{DS4} = 7.18 \text{ V}$$

$$V_{D1} = 10 - (0.168)(24) = 5.97 \text{ V}$$

$$V_{DS1} = 5.97 - (-2.82) \Rightarrow V_{DS1} = 8.79 \text{ V}$$



(c)

$$\text{Max } v_{CM} \Rightarrow V_{DS1} = V_{DS2} = V_{DS}(\text{sat}) = V_{GS1} - V_{TN}$$

$$2.82 - 2 = 0.82 \text{ V}$$

$$\text{Now } V_{D1} = 10 - (0.168)(24) = 5.97 \text{ V}$$

$$V_s(\text{max}) = 5.97 - V_{DS1}(\text{sat}) = 5.97 - 0.82$$

$$V_s(\text{max}) = 5.15 \text{ V}$$

$$v_{CM}(\text{max}) = V_s(\text{max}) + V_{GS1} = 5.15 + 2.82$$

$$\underline{v_{CM}(\text{max}) = 7.97 \text{ V}}$$

$$v_{CM}(\text{min}) = V^- + V_{DS4}(\text{sat}) + V_{GS1}$$

$$V_{DS4}(\text{sat}) = V_{GS4} - V_{TN} = 3.16 - 2 = 1.16 \text{ V}$$

$$\text{Then } v_{CM}(\text{min}) = -10 + 1.16 + 2.82 \Rightarrow \underline{v_{CM}(\text{min}) = -6.02 \text{ V}}$$

11.32

(a) $I_{D1} = K_{n1}(V_{GS1} - V_{TN})^2$

$$60 = 100(V_{GS1} - 0.3)^2 \Rightarrow V_{GS1} = 1.075 \text{ V}$$

$$V_{D1} = V_{DS1} - V_{GS1} + V_1 = 4 - 1.075 - 1.15 = 1.775 \text{ V}$$

$$R_D = \frac{3 - 1.775}{0.06} = 20.4 \text{ k}\Omega$$

(i) $I_Q = I_1 = I_{D1} + I_{D2} = 120 \mu\text{A}$

$$I_1 = K_3(V_{GS3} - V_{TN})^2$$

$$120 = 200(V_{GS3} - 0.3)^2 \Rightarrow V_{GS3} = 1.075 \text{ V}$$

(ii) $R_1 = \frac{3 - 1.075 - (-3)}{0.12} = 41 \text{ k}\Omega$

(iii) $V_{GS1} = V_{GS4} = 1.075 \text{ V}$

(b) $r_o = \frac{1}{\lambda_4 I_Q} = \frac{1}{(0.01)(0.12)} = 833.3 \text{ k}\Omega$

$$\Delta I_Q = \frac{\Delta V_{DS4}}{r_o} = \frac{2.3}{833.3} \Rightarrow \Delta I_Q = 2.76 \mu\text{A}$$

11.33

(a)

$$I_Q = 160 \mu\text{A}$$

$$I_D = \frac{k'_n}{2} \left(\frac{W}{L} \right) (V_{GS} - V_{TN})^2$$

$$80 = \frac{80}{2} (4)(V_{os} - 0.5)^2$$

$$80 = 160(V_{os} - 0.5)^2$$

$$V_{GS} = \sqrt{\frac{80}{160}} + 0.5 = 1.207 \text{ V}$$

$$R_D = \frac{5-2}{0.08} = 37.5 \text{ k}\Omega \quad V_{DS} = 2 - (-1.207) = 3.21 \text{ V}$$

(c)

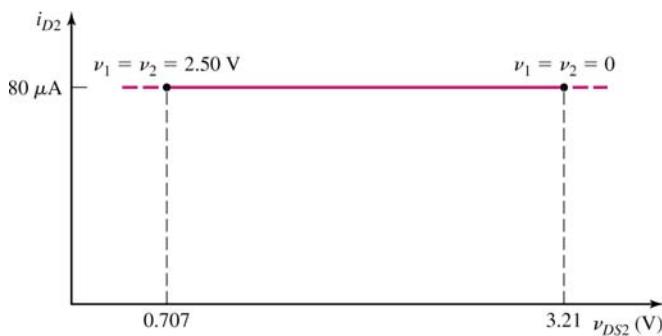
$$V_{DS}(\text{sat}) = V_{GS} - V_{TN} = 1.207 - 0.5 = 0.707 \text{ V}$$

$$\text{Then } V_S = V_{O2} - V_{DS}(\text{sat}) = 2 - 0.707 = +1.29 \text{ V}$$

$$\text{And } v_1 = v_2 = v_{cm} = V_{GS} + V_S = 1.207 + 1.29$$

$$v_{cm} = 2.50 \text{ V}$$

(b)



11.34

$$(a) v_{CM}(\text{max}) = v_{D2} - v_{DS2}(\text{sat}) + v_{GS2}$$

$$i_{D2} = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) (v_{GS2} - V_{TN})^2$$

$$0.09 = \left(\frac{0.1}{2} \right) (4)(v_{GS2} - 0.35)^2 \Rightarrow v_{GS2} = 1.021 \text{ V}$$

$$v_{DS2}(\text{sat}) = 1.0208 - 0.35 = 0.6708 \text{ V}$$

$$\text{Then } 2.25 = v_{D2} - 0.6708 + 1.021 \Rightarrow v_{D2} = 1.90 \text{ V}$$

$$R_D = \frac{3-1.90}{0.09} = 12.2 \text{ k}\Omega$$

$$(b) (i) v_{D2} = 1.90 \text{ V}$$

$$(ii) g_m = 2 \sqrt{\left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) I_{DQ}} = 2 \sqrt{\left(\frac{0.1}{2} \right) (4)(0.09)} = 0.2683 \text{ mA/V}$$

$$v_{D2} = 1.90 + g_m \cdot \frac{v_d}{2} \cdot R_D = 1.90 + (0.2683) \left(\frac{0.12}{2} \right) (12.2) = 2.096 \text{ V}$$

$$(iii) v_{D2} = 1.90 - g_m \cdot \frac{v_d}{2} \cdot R_D = 1.90 - (0.2683) \left(\frac{0.05}{2} \right) (12.2) = 1.818 \text{ V}$$

11.35

$$(a) I_Q = I_1 = K_{n3} (V_{GS3} - V_{TN})^2$$

$$0.25 = 0.20(V_{GS3} - 0.4)^2 \Rightarrow V_{GS3} = 1.518 \text{ V}$$

$$R_1 = \frac{5 - 1.518 - (-5)}{0.25} = 33.9 \text{ k}\Omega$$

$$(b) I_{D1} = I_{D2} = \frac{0.25}{2} = 0.125 \text{ mA}$$

$$v_{o1} - v_{o2} = (V^+ - I_{D1}R_{D1}) - (V^+ - I_{D2}R_{D2}) = I_{D2}R_{D2} - I_{D1}R_{D1}$$

$$(i) v_{o1} - v_{o2} = 0$$

$$(ii) v_{o1} - v_{o2} = (0.125)(15.5 - 14.5) = 0.125 \text{ V}$$

$$(c) I_Q = I_{D1} + I_{D2} = (K_{n1} + K_{n2})(V_{GS} - V_{TN})^2$$

$$0.25 = (0.125 + 0.115)(V_{GS} - V_{TN})^2 \Rightarrow (V_{GS} - V_{TN})^2 = 1.04166$$

$$I_{D1} = (0.125)(1.04166) = 0.1302 \text{ mA}$$

$$I_{D2} = (0.115)(1.04166) = 0.1198 \text{ mA}$$

$$(i) v_{o1} - v_{o2} = (15)(0.1198 - 0.1302) = -0.156 \text{ V}$$

$$(ii) v_{o1} - v_{o2} = (0.1198)(15.5) - (0.1302)(14.5) = -0.031 \text{ V}$$

11.36

$$(a) \frac{i_{D1}}{I_Q} = \frac{1}{2} + \sqrt{\frac{K_n}{2I_Q} \cdot v_d \sqrt{1 - \left(\frac{K_n}{2I_Q} \right) \cdot v_d^2}}$$

$$0.20 = 0.50 + \sqrt{\frac{0.20}{2(0.15)} \cdot v_d \sqrt{1 - \left(\frac{0.20}{2(0.15)} \right) \cdot v_d^2}}$$

$$-0.3674 = v_d \sqrt{1 - (0.6667)v_d^2}$$

$$0.135 = v_d^2 [1 - (0.6667)v_d^2]$$

$$0.6667v_d^4 - v_d^2 + 0.135 = 0 \Rightarrow v_d^2 = 0.15 \Rightarrow v_d = -0.3873 \text{ V}$$

$$(b) 0.80 = 0.50 - \sqrt{\frac{0.20}{2(0.15)} \cdot v_d \sqrt{1 - \left(\frac{0.20}{2(0.15)} \right) v_d^2}}$$

$$0.3674 = -v_d \sqrt{1 - (0.6667)v_d^2} \Rightarrow v_d = -0.3873 \text{ V}$$

$$(c) v_d = v_{d,\max} = \sqrt{\frac{I_Q}{K_n}} = \sqrt{\frac{0.15}{0.20}} = +0.866 \text{ V}$$

11.37

$$\frac{i_{D1}}{I_Q} \Big|_{Lin} = 0.5 + \frac{g_f v_d}{I_Q}, \text{ Now } g_f = \sqrt{\frac{K_n I_Q}{2}} = \sqrt{\frac{(0.2)(0.10)}{2}} = 0.10 \text{ mA/V}$$

$$= 0.5 + \frac{(0.1)}{(0.1)} \cdot v_d = 0.5 + v_d$$

$$\text{We have } \frac{K_n}{2I_Q} = \frac{(0.2)}{2(0.1)} = 1$$

$$(a) \frac{0.5 + v_d - [0.5 + v_d \sqrt{1 - v_d^2}]}{0.5 + v_d} = 0.005$$

$$0.0025 = v_d \left[0.995 - \sqrt{1 - v_d^2} \right], \Rightarrow v_d \approx 0.19 \text{ V}$$

$$(b) \frac{0.5 + v_d - [0.5 + v_d \sqrt{1 - v_d^2}]}{0.5 + v_d} = 0.015$$

$$0.0075 = v_d \left[0.985 - \sqrt{1 - v_d^2} \right], \Rightarrow v_d \approx 0.285 \text{ V}$$

11.38

(b)

$$g_m = 2\sqrt{K_p I_D} = 2\sqrt{(0.05)(0.008696)}$$

$$= 0.0417 \text{ mA/V}$$

$$\Delta I = g_m \frac{V_d}{2} = (0.0417)(0.05) = 0.002085 \text{ mA}$$

$$\Delta v_D = (0.002085)(510) = 1.063$$

$$v_{D2} \uparrow \Rightarrow v_{D2} = 1.063 - 4.565 = -3.502 \text{ V}$$

$$v_{D1} = -1.063 - 4.565 = -5.628 \text{ V}$$

$$9 = I_s R_s + V_{SG} + 1$$

$$I_s = 2I_D$$

$$8 = 2K_p R_s (V_{SG} + V_{TP})^2 + V_{SG}$$

$$8 = (2)(0.05)(390)(V_{SG} - 0.8)^2 + V_{SG}$$

$$8 = 39(V_{SG}^2 - 1.6V_{SG} + 0.64) + V_{SG}$$

$$39V_{SG}^2 - 61.4V_{SG} + 16.96 = 0$$

$$V_{SG} = \frac{61.4 \pm \sqrt{3769.96 - 4(39)(16.96)}}{2(39)}$$

$$= 1.217 \text{ V} \quad V_s = 2.217$$

$$I_s = 0.01739 \text{ mA} \quad I_{D1} = I_{D2} \Rightarrow 8.696 \mu\text{A}$$

$$v_{D1} = v_{D2} = (8.696)(0.510) - 9 = -4.565 \text{ V}$$

(b)

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.05)(0.008696)} = 0.0417 \text{ mA/V}$$

$$\Delta v_D = \Delta I_D \cdot R_D \quad \Delta I_D = g_m \cdot \frac{V_d}{2} = (0.0417)(0.05) = 0.002085 \text{ mA}$$

$$\Delta v_D = (0.002085)(510) = 1.063 \text{ V}$$

$$\begin{aligned}v_1 &\uparrow, I_{D1} \downarrow, v_{D1} \downarrow \\v_{D1} &= -4.565 - 1.063 = -5.628 \text{ V} \\v_{D2} &= -4.565 + 1.063 = -3.502 \text{ V}\end{aligned}$$

11.39

(a)

$$\begin{aligned}v_1 &= v_2 = 0 \\I_D &= K_n (V_{SG} + V_{TP})^2 \\I_D &= 6 \mu\text{A} \\\sqrt{\frac{6}{30}} + 0.4 &= V_{SG} \\V_{SG} &= 0.847 \text{ V} \\V_s &= +0.847 \text{ V} \\v_D &= I_D R_D - 3 \\&= (6)(0.36) - 3 = -0.84 \text{ V} \\V_{SD} &= V_s - v_D = 0.847 - (-0.84) \\v_{SD} &= 1.69 \text{ V}\end{aligned}$$

(b)

(i)

$$\begin{aligned}A_d &= g_m R_D \quad g_m = 2\sqrt{K_n I_D} \\&= 2\sqrt{(30)(6)} = 26.83 \mu\text{A/V} \\A_d &= (26.83)(0.36) \Rightarrow A_d = 9.66 \\A_{cm} &= 0\end{aligned}$$

(ii)

$$\begin{aligned}A_d &= \frac{g_m R_D}{2} = \frac{(26.83)(0.36)}{2} \Rightarrow A_d = 4.83 \\A_{cm} &= \frac{-g_m R_D}{1+2g_m R_o} = \frac{-(26.83)(0.36)}{1+2(26.83)(4)} = -0.0448\end{aligned}$$

11.40

(a) $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.12)(0.075)} = 0.1897 \text{ mA/V}$

(i) $v_{D1} - v_{D2} = g_m R_D v_d = (0.1897)(30)(0.1) = 0.5692 \text{ V}$

(ii) $v_{D1} - v_{D2} = g_m R_D v_d = (0.1897)(30)(0.2) = 1.138 \text{ V}$

(b) (i) $\Delta v_{D2} = g_m R_D \cdot \frac{v_d}{2} = (0.1897)(30)(0.1) = 0.5692 \text{ V}$

(ii) $A_{cm} = \frac{-g_m R_D}{1+2g_m R_o} = \frac{-(0.1897)(30)}{1+2(0.1897)(4000)} = -0.003748$

$$A_d = \frac{g_m R_D}{2} = \frac{(0.1897)(30)}{2} = 2.846$$

$\Delta v_{D2} = A_d v_d + A_{cm} v_{cm}$, let $v_{cm} = 1 \text{ V}$, $v_d = 0.2 \text{ V}$

Then $\Delta v_{D2} = (2.846)(0.2) - (0.003748)(1) = 0.5655 \text{ V}$

11.41

For $v_1 = v_2 = 0$

$$0 = V_{GS} + 2I_D R_S - 10$$

$$\begin{aligned} 10 &= V_{GS} + 2K_n R_S (V_{GS} - V_{TN})^2 \\ &= V_{GS} + 2(0.15)(75)(V_{GS} - 1)^2 \end{aligned}$$

$$22.5V_{GS}^2 - 44V_{GS} + 12.5 = 0$$

So $V_{GS} = 1.61$ V and $I_D = (0.15)(1.61 - 1)^2 = 55.9 \mu\text{A}$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{(0.15)(0.0559)}$$

$$g_m = 0.1831 \text{ mA/V}$$

Use Half-circuits – Differential gain

$$v_{D1} = -g_m \left(\frac{V_d}{2} \right) \left(R_D + \frac{\Delta R}{2} \right)$$

$$v_{D2} = g_m \left(\frac{V_d}{2} \right) \left(R_D - \frac{\Delta R}{2} \right)$$

$$v_o = v_{D1} - v_{D2} = -g_m V_d R_D$$

$$A_d = \frac{v_o}{V_d} = -g_m R_D$$

Now – Common-Mode Gain

$$V_i = V_{gs} + g_m V_{gs} (2R_S) = V_{cm}$$

$$V_{gs} = \frac{V_{cm}}{1 + g_m (2R_S)}$$

$$v_{D1} = \frac{-g_m \left(R_D + \frac{\Delta R_D}{2} \right) V_{cm}}{1 + g_m (2R_S)}$$

$$v_{D2} = \frac{-g_m \left(R_D - \frac{\Delta R_D}{2} \right) V_{cm}}{1 + g_m (2R_S)}$$

$$v_o = v_{D1} - v_{D2}$$

$$\text{So } v_o = \frac{-g_m (\Delta R_D) V_{cm}}{1 + g_m (2R_S)}$$

$$A_{cm} = \frac{v_o}{V_{cm}} = \frac{-g_m (\Delta R_D)}{1 + g_m (2R_S)}$$

Then

$$A_d = -(0.1831)(50) = -9.16$$

$$A_{cm} = \frac{-(0.1831)(0.5)}{1 + (0.1831)(2)(75)} = -0.003216$$

$$CMRR_{bb} = 69.1 \text{ dB}$$

11.42

From 11.41

$$I_{D1} = I_{D2} = 55.9 \mu\text{A}$$

$$g_m = 0.183 \text{ mA/V}$$

$$A_d : \Delta v_{D1} = -g_{m1} \frac{V_d}{2} \cdot R_D \quad \Delta v_{D2} = +g_{m2} \left(\frac{+V_d}{2} \right) R_D$$

$$v_o = \Delta v_{D1} - \Delta v_{D2} = -g_{m1} \frac{V_d}{2} R_D - g_{m2} \frac{V_d}{2} R_D$$

$$v_o = \frac{-V_d}{2} \cdot R_D (g_{m2} + g_{m1}) = \frac{-V_d}{2} \cdot R_D \left(g_m - \frac{\Delta g_m}{2} + \left(g_m - \frac{\Delta g_m}{2} \right) \right)$$

$$A_d = -g_m R_D = -(0.183)(50) = \underline{\underline{-9.15}}$$

$$A_{CM} : v_o = \Delta v_{D1} - \Delta v_{D2} = \frac{-\left(g_m + \frac{\Delta g_m}{2} \right) R_D v_{cm}}{1 + g_m (2R_s)} + \frac{\left(g_m - \frac{\Delta g_m}{2} \right) R_D v_{CM}}{1 + g_m (2R_s)}$$

$$A_{cm} = \frac{v_o}{v_{cm}} = \frac{-\Delta g_m R_D}{1 + g_m (2R_s)} \quad \Delta g_m = (0.01)(0.183) = 0.00183$$

$$A_{cm} = \frac{-(0.00183)(50)}{1 + (0.183)(2)(75)} = -0.003216$$

$$CMRR_{dB} = 69.1 \text{ dB}$$

11.43

$$(a) 5 = I_S R_S + V_{SG} = 2K_p (V_{SG} + V_{TP})^2 R_S + V_{SG}$$

$$5 = 2(1.2)(2)(V_{SG}^2 - 1.2V_{SG} + 0.36) + V_{SG}$$

$$4.8V_{SG}^2 - 4.76V_{SG} - 3.272 = 0 \Rightarrow V_{SG} = 1.459 \text{ V} = v_s$$

$$I_S = 2(1.2)(1.459 - 0.6)^2 = 1.77 \text{ mA}$$

$$I_{D1} = I_{D2} = 0.885 \text{ mA}$$

$$v_{D1} = v_{D2} = I_D R_D - 5 = (0.885)(1) - 5 = -4.115 \text{ V}$$

$$(b) 5 = 2(1.2)(2)(V_{SG}^2 - 1.2V_{SG} + 0.36) + V_{SG} + 1$$

$$4.8V_{SG}^2 - 4.76V_{SG} - 2.272 = 0 \Rightarrow V_{SG} = 1.344 \text{ V}$$

$$v_s = V_{SG} + 1 = 2.344 \text{ V}$$

$$I_{D1} = I_{D2} = (1.2)(1.344 - 0.6)^2 = 0.664 \text{ mA}$$

$$v_{D1} = v_{D2} = (0.664)(1) - 5 = -4.336 \text{ V}$$

$$(c) \text{ Using part (a), } g_m = 2\sqrt{(1.2)(0.885)} = 2.061 \text{ mA/V}$$

$$v_{D1} = -4.115 + g_m R_D \cdot \frac{v_d}{2} = -4.115 + (2.061)(1)(0.1) = -3.909 \text{ V}$$

$$v_{D2} = -4.115 - (2.061)(1)(0.1) = -4.321 \text{ V}$$

$$(d) \text{ Using part (b), } g_m = 2\sqrt{(1.2)(0.664)} = 1.785 \text{ mA/V}$$

$$v_{D1} = -4.336 + g_m R_D \cdot \frac{v_d}{2} = -4.336 + (1.785)(1)(0.1) = -4.158 \text{ V}$$

$$v_{D2} = -4.336 - (1.785)(1)(0.1) = -4.515 \text{ V}$$

11.44

(a)

$$g_m = 2\sqrt{K_n I_D} \\ = 2\sqrt{(0.4)(1)} \\ = 1.265 \text{ mA/V}$$

$$A_d = \frac{v_o}{v_d} = \frac{1}{0.1} = 10$$

$$A_d = g_m R_D \\ 10 = (1.265) R_D \\ R_D = 7.91 \text{ k}\Omega$$

(b)

$$\text{Quiescent } v_1 = v_2 = 0$$

$$v_{D1} = v_{D2} = 10 - (1)(7.91) = 2.09 \text{ V}$$

$$V_{GS} = \sqrt{\frac{I_D}{K_n}} + V_{TN} = \sqrt{\frac{1}{0.4}} + 0.8 = 2.38 \text{ V}$$

$$V_{DS}(\text{sat}) = 2.38 - 0.8 = 1.58$$

$$\text{So } v_{cm} = v_D - V_{DS}(\text{sat}) + V_{GS} \\ = 2.09 - 1.58 + 2.38$$

$$v_{cm} = 2.89 \text{ V}$$

11.45

$$A_d = \frac{g_m R_D}{2}$$

$$\text{For } v_{CM} = 2.5 \text{ V}$$

$$I_{D1} = I_{D2} = \frac{I_Q}{2} = 0.25 \text{ mA}$$

$$\text{Let } V_{D1} = V_{D2} = 3 \text{ V, then } R_D = \frac{10 - 3}{0.25} \Rightarrow R_D = 28 \text{ k}\Omega$$

$$\text{Then } 100 = \frac{g_m(28)}{2} \Rightarrow g_m = 7.14 \text{ mA/V}$$

$$\text{And } g_m = 2\sqrt{\frac{k'_n}{2} \left(\frac{W}{L} \right) I_D}$$

$$7.14 = 2\sqrt{\left(\frac{0.080}{2} \right) \left(\frac{W}{L} \right) (0.25)} \Rightarrow$$

$$\left(\frac{W}{L} \right)_1 = \left(\frac{W}{L} \right)_2 = 1274 \text{ (Extremely large transistors to meet the gain requirement.)}$$

$$\text{Need } |A_{CM}| = 0.10$$

From Eq. (11.82(b))

$$|A_{CM}| = \frac{g_m R_D}{1 + 2g_m R_o}$$

$$\text{So } 0.10 = \frac{(7.14)(28)}{1 + 2(7.14)R_o} \Rightarrow R_o = 140 \text{ k}\Omega$$

For the basic 2-transistor current source

$$R_o = r_o = \frac{1}{\lambda I_Q} = \frac{1}{(0.01)(0.5)} = 200 \text{ k}\Omega$$

This current source is adequate to meet common-mode gain requirement.

11.46

Not in detail, Approximation looks good.

a.

$$I_S = \frac{-V_{GS1} - (-5)}{R_s} \text{ and } I_S = 2I_D = 2K_n(V_{GS1} - V_{TN})^2$$

$$\frac{5 - V_{GS1}}{20} = 2(0.050)(V_{GS1} - 1)^2$$

$$5 - V_{GS1} = 2(V_{GS1}^2 - 2V_{GS1} + 1)$$

$$2V_{GS1}^2 - 3V_{GS1} - 3 = 0$$

$$V_{GS1} = \frac{3 \pm \sqrt{(3)^2 + 4(2)(3)}}{2(2)} \Rightarrow V_{GS1} = 2.186 \text{ V}$$

$$I_S = \frac{5 - 2.186}{20} \Rightarrow I_S = 0.141 \text{ mA}$$

$$I_{D1} = I_{D2} = \frac{I_S}{2} \Rightarrow I_{D1} = I_{D2} = 0.0704 \text{ mA}$$

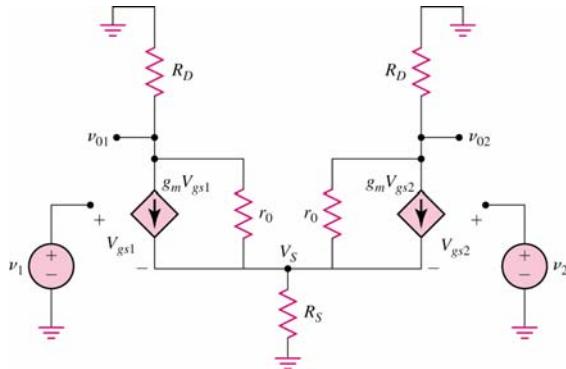
$$v_{02} = 5 - (0.0704)(25) \Rightarrow v_{02} = 3.24 \text{ V}$$

b.

$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.05)(2.186 - 1)$$

$$g_m = 0.119 \text{ mA/V}$$

$$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.0704)} = 710 \text{ k}\Omega$$



$$V_{gs1} = v_1 - V_s, \quad V_{gs2} = v_2 - V_s$$

$$\frac{v_{01}}{R_D} + g_m V_{gs1} + \frac{v_{01} - V_s}{r_0} = 0$$

$$v_{01} \left(\frac{1}{R_D} + \frac{1}{r_0} \right) + g_m (v_1 - V_s) - \frac{V_s}{r_0} = 0 \quad (1)$$

$$\frac{v_{02}}{R_D} + g_m V_{gs2} + \frac{v_{02} - V_s}{r_0} = 0$$

$$v_{02} \left(\frac{1}{R_D} + \frac{1}{r_0} \right) + g_m (v_2 - V_s) - \frac{V_s}{r_0} = 0 \quad (2)$$

$$\begin{aligned} g_m V_{gs1} + \frac{v_{01} - V_s}{r_0} + \frac{v_{02} - V_s}{r_0} + g_m V_{gs2} &= \frac{V_s}{R_s} \\ g_m(v_1 - V_s) + \frac{v_{01}}{r_0} + \frac{v_{02}}{r_0} - \frac{2V_s}{r_0} + g_m(v_2 - V_s) &= \frac{V_s}{R_s} \\ g_m(v_1 + v_2) + \frac{v_{01}}{r_0} + \frac{v_{02}}{r_0} &= V_s \left\{ 2g_m + \frac{2}{r_0} + \frac{1}{R_s} \right\} \end{aligned} \quad (3)$$

From (1)

$$v_{01} = \frac{V_s \left(g_m + \frac{1}{r_0} \right) - g_m v_1}{\left(\frac{1}{R_D} + \frac{1}{r_0} \right)}$$

Then

$$\begin{aligned} g_m(v_1 + v_2) + \frac{V_s \left(g_m + \frac{1}{r_0} \right) - g_m v_1}{r_0 \left(\frac{1}{R_D} + \frac{1}{r_0} \right)} + \frac{v_{02}}{r_0} &= V_s \left\{ 2g_m + \frac{2}{r_0} + \frac{1}{R_s} \right\} \\ g_m(v_1 + v_2) r_0 \left(\frac{1}{R_D} + \frac{1}{r_0} \right) + V_s \left(g_m + \frac{1}{r_0} \right) - g_m v_1 + v_{02} \left(\frac{1}{R_D} + \frac{1}{r_0} \right) &= V_s \left\{ 2g_m + \frac{2}{r_0} + \frac{1}{R_s} \right\} \cdot r_0 \left(\frac{1}{R_D} + \frac{1}{r_0} \right) \\ g_m(v_1 + v_2) \left(1 + \frac{r_0}{R_D} \right) - g_m v_1 + v_{02} \left(\frac{1}{R_D} + \frac{1}{r_0} \right) &= V_s \left\{ \left(2g_m + \frac{2}{r_0} + \frac{1}{R_s} \right) \left(1 + \frac{r_0}{R_D} \right) - \left(g_m + \frac{1}{r_0} \right) \right\} \\ g_m \left(v_1 \cdot \frac{r_0}{R_D} + v_2 \cdot \frac{r_0}{R_D} \right) + v_{02} \left(\frac{1}{R_D} + \frac{1}{r_0} \right) &= V_s \left\{ 2g_m + \frac{2}{r_0} + \frac{1}{R_s} + 2g_m \cdot \frac{r_0}{R_D} + \frac{2}{R_D} + \frac{r_0}{R_s R_D} - g_m - \frac{1}{r_0} \right\} \\ g_m \left(v_1 \cdot \frac{r_0}{R_D} + v_2 \cdot \frac{r_0}{R_D} \right) + v_{02} \left(\frac{1}{R_D} + \frac{1}{r_0} \right) &= V_s \left\{ 2g_m + \frac{1}{r_0} + \frac{1}{R_s} \left(1 + \frac{r_0}{R_D} \right) + \frac{2}{R_D} \left(1 + g_m r_0 \right) \right\} \end{aligned} \quad (4)$$

$$v_{02} \left(\frac{1}{R_D} + \frac{1}{r_0} \right) + g_m v_2 = V_s \left(g_m + \frac{1}{r_0} \right)$$

Then substituting into (2),

$$\begin{aligned} (0.119) \left[v_1 \frac{710}{25} + v_2 + v_2 \frac{710}{25} \right] + v_{02} \left[\frac{1}{25} + \frac{1}{710} \right] \\ = V_s \left\{ 0.119 + \frac{1}{710} + \frac{1}{20} \left(1 + \frac{710}{25} \right) + \frac{2}{25} [1 + (0.119)(710)] \right\} \end{aligned} \quad (4)$$

Substitute numbers:

$$\begin{aligned} (0.119)[28.4v_1 + 29.4v_2] + (0.0414)v_{02} &= V_s \{0.1204 + 1.470 + 6.8392\} \\ &= V_s (8.4296) \end{aligned}$$

$$\text{or } V_s = 0.4010v_1 + 0.4150v_2 + 0.00491v_{02}$$

$$\text{Then } v_{02} \left(\frac{1}{25} + \frac{1}{710} \right) + (0.119)v_2 = V_s \left(0.119 + \frac{1}{710} \right) \quad (2)$$

$$\begin{aligned} v_{02}(0.0414) + v_2(0.119) &= (0.1204)[0.401v_1 + 0.4150v_2 + 0.00491v_{02}] \\ v_{02}(0.0408) &= (0.04828)v_1 - (0.0690)v_2 \\ v_{02} &= (1.183)v_1 - (1.691)v_2 \end{aligned}$$

$$\text{Now } v_1 = v_{cm} + \frac{v_d}{2}$$

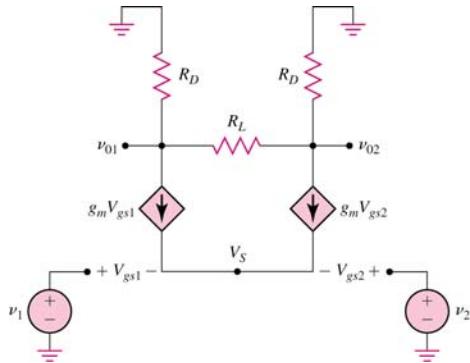
$$v_2 = v_{cm} - \frac{v_d}{2}$$

$$\text{So } v_{02} = (1.183)\left(v_{cm} + \frac{v_d}{2}\right) - (1.691)\left(v_{cm} - \frac{v_d}{2}\right)$$

$$\text{Or } v_{02} = 1.437v_d - 0.508v_{cm} \Rightarrow A_d = 1.437, \quad A_{cm} = -0.508$$

$$CMRR_{dB} = 20\log_{10}\left(\frac{1.437}{0.508}\right) \Rightarrow CMRR_{dB} = 9.03 \text{ dB}$$

11.47



KVL:

$$v_1 = V_{gs1} - V_{gs2} + v_2$$

$$\text{So } v_1 - v_2 = V_{gs1} - V_{gs2}$$

KCL:

$$g_m V_{gs1} + g_m V_{gs2} = 0 \Rightarrow V_{gs1} = -V_{gs2}$$

$$\text{So } V_{gs1} = \frac{1}{2}(v_1 - v_2), \quad V_{gs2} = -\frac{1}{2}(v_1 - v_2)$$

Now

$$\begin{aligned} \frac{v_{02}}{R_D} + \frac{v_{02} - v_{01}}{R_L} &= -g_m V_{gs2} \\ &= v_{02} \left(\frac{1}{R_D} + \frac{1}{R_L} \right) - \frac{v_{01}}{R_L} \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{v_{01}}{R_D} + \frac{v_{01} - v_{02}}{R_L} &= -g_m V_{gs1} \\ &= v_{01} \left(\frac{1}{R_D} + \frac{1}{R_L} \right) - \frac{v_{02}}{R_L} \end{aligned} \quad (2)$$

$$v_{01} = v_{02} \left(1 + \frac{R_L}{R_D} \right) + g_m R_L V_{gs2}$$

From (1):

Substitute into (2):

$$\begin{aligned} -g_m V_{gs1} &= v_{02} \left(1 + \frac{R_L}{R_D} \right) \left(\frac{1}{R_D} + \frac{1}{R_L} \right) + g_m R_L \left(\frac{1}{R_D} + \frac{1}{R_L} \right) V_{gs2} - \frac{v_{02}}{R_L} \\ -g_m \cdot (v_1 - v_2) + g_m \left(1 + \frac{R_L}{R_D} \right) \left(\frac{1}{2} \right) (v_1 - v_2) &= v_{02} \left(\frac{1}{R_D} + \frac{R_L}{R_D^2} + \frac{1}{R_D} \right) \\ \frac{1}{2} g_m \left(\frac{R_L}{R_D} \right) (v_1 - v_2) &= \frac{v_{02}}{R_D} \left(2 + \frac{R_L}{R_D} \right) \Rightarrow A_{d2} = \frac{v_{02}}{v_1 - v_2} = \frac{\frac{1}{2} \cdot g_m R_L}{\left(2 + \frac{R_L}{R_D} \right)} \\ A_{d1} &= \frac{v_{01}}{v_1 - v_2} = \frac{-\frac{1}{2} \cdot g_m R_L}{\left(2 + \frac{R_L}{R_D} \right)} \end{aligned}$$

From symmetry

$$A_v = \frac{v_{02} - v_{01}}{v_1 - v_2} = \frac{g_m R_L}{\left(2 + \frac{R_L}{R_D} \right)}$$

Then

11.48

$$(a) v_o = -g_m (R_D \| R_L) \left(\frac{-v_d}{2} \right)$$

$$A_d = \frac{1}{2} g_m (R_D \| R_L)$$

$$(b) v_o = -g_m (R_D \| R_L) v_{gs}$$

$$v_{cm} = v_{gs} + g_m v_{gs} (2R_o)$$

$$v_{gs} = \frac{v_{cm}}{1 + 2g_m R_o}$$

$$A_{cm} = \frac{-g_m (R_D \| R_L)}{1 + 2g_m R_o}$$

11.49

$$(a) A_d = \frac{\Delta v_o}{\Delta v_d} = \frac{0.5}{0.1} = 5$$

$$A_d = 5 = \frac{g_m R_D}{2}, \quad g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.15)(0.1)} = 0.2449 \text{ mA/V}$$

$$R_D = \frac{2(5)}{0.2449} = 40.8 \text{ k}\Omega$$

$$(b) v_{CM}(\max) = v_o - v_{DS(sat)} + v_{GS}$$

$$i_D = K_n (v_{GS} - V_{TN})^2$$

$$0.1 = 0.15(v_{GS} - 0.4)^2 \Rightarrow v_{GS} = 1.216 \text{ V}$$

$$v_{DS(sat)} = 1.216 - 0.4 = 0.8165 \text{ V}$$

$$v_o = 5 - (0.1)(40.8) = 0.92 \text{ V}$$

$$v_{CM}(\max) = 0.92 - 0.8165 + 1.216 = 1.32 \text{ V}$$

11.50

$$\begin{aligned} V_{d1} &= -g_m V_{gs1} R_D = -g_m R_D (V_1 - V_s) \\ V_{d2} &= -g_m V_{gs2} R_D = -g_m R_D (V_2 - V_s) \\ \text{Now } V_o &= V_{d2} - V_{d1} = -g_m R_D (V_2 - V_s) - (-g_m R_D (V_1 - V_s)) \\ V_o &= g_m R_D (V_1 - V_2) \\ \text{Define } V_1 - V_2 &\equiv V_d \\ \text{Then } A_d &= \frac{V_o}{V_d} = g_m R_D \text{ and } A_{cm} = 0 \end{aligned}$$

11.51

$$\begin{aligned} \text{(a)} \quad v_{CM} &= v_o - v_{DS}(\text{sat}) + v_{GS} \\ i_D &= K_n (v_{GS} - V_{TN})^2 \\ 0.1 &= \left(\frac{0.1}{2}\right)(10)(v_{GS} - 0.4)^2 \Rightarrow v_{GS} = 0.8472 \text{ V} \\ v_{DS}(\text{sat}) &= 0.8472 - 0.4 = 0.4472 \text{ V} \end{aligned}$$

$$\begin{aligned} \text{Then } v_o &= 1.5 + 0.4472 - 0.8472 = 1.1 \text{ V} \\ v_o &= 3 - (0.1)R_D = 1.1 \Rightarrow R_D = 19 \text{ k}\Omega \end{aligned}$$

$$\text{Now } A_d = \frac{g_m R_D}{2}, \text{ where } g_m = 2\sqrt{\left(\frac{0.1}{2}\right)(10)(0.1)} = 0.4472 \text{ mA/V}$$

$$A_d = \frac{(0.4472)(19)}{2} = 4.248$$

$$CMRR_{dB} = 50 \text{ dB} \Rightarrow CMRR = 316.2$$

$$CMRR = \frac{1}{2} \left[1 + 2\sqrt{K_n I_Q} \cdot R_o \right] \Rightarrow 316.2 = \frac{1}{2} \left[1 + 2\sqrt{\left(\frac{0.1}{2}\right)(10)(0.2)} \cdot R_o \right] \Rightarrow R_o = 998 \text{ k}\Omega$$

(b) Use cascode current source similar to Figure 10.18 with $v_{DS2}(\text{sat}) = 0.3 \text{ V}$.

11.52

(a) From Problem 11.27, $v_d \cong 12.5 \text{ mV}$.

$$A_d = \frac{g_m R_D}{2}, \text{ where } g_m = 2\sqrt{(1)(0.1)} = 0.6325 \text{ mA/V}$$

$$A_d = \frac{(0.6325)(20)}{2} = 6.325$$

$$v_o = (6.325)(0.0125) = 0.0791 \text{ V}$$

$$\text{So } -0.0791 \leq v_o \leq 0.0791 \text{ V}$$

11.53

From previous results

$$A_{d1} = \frac{v_{o2} - v_{o1}}{v_1 - v_2} = g_{m1}R_1 = \sqrt{2K_{n1}I_{Q1}} \cdot R_1 = 20$$

$$\text{and } A_{d2} = \frac{v_{o3}}{v_{o2} - v_{o1}} = \frac{1}{2}g_{m3}R_2 = \frac{1}{2}\sqrt{2K_{n3}I_{Q2}} \cdot R_2 = 30$$

$$\text{Set } \frac{I_{Q1}R_1}{2} = 5 \text{ V and } \frac{I_{Q2}R_2}{2} = 2.5 \text{ V}$$

$$\text{Let } I_{Q1} = I_{Q2} = 0.1 \text{ mA}$$

$$\text{Then } R_1 = 100 \text{ k}\Omega, \quad R_2 = 50 \text{ k}\Omega$$

$$\text{Then } 2\left(\frac{0.06}{2}\right)\left(\frac{W}{L}\right)_1(0.1) = \left(\frac{20}{100}\right)^2 \Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 6.67$$

$$\text{and } 2\left(\frac{0.060}{2}\right)\left(\frac{W}{L}\right)_3(0.1) = \left(\frac{2(30)}{50}\right)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 240$$

11.54

$$\text{a. } i_{D1} = I_{DSS} \left(1 - \frac{v_{GS1}}{V_p}\right)^2$$

$$i_{D2} = I_{DSS} \left(1 - \frac{v_{GS2}}{V_p}\right)^2$$

$$\sqrt{i_{D1}} - \sqrt{i_{D2}} = \sqrt{I_{DSS}} \left(1 - \frac{v_{GS1}}{V_p}\right) - \sqrt{I_{DSS}} \left(1 - \frac{v_{GS2}}{V_p}\right)$$

$$= \frac{\sqrt{I_{DSS}}}{V_p} (v_{GS2} - v_{GS1})$$

$$= -\frac{\sqrt{I_{DSS}}}{V_p} \cdot v_d = \frac{\sqrt{I_{DSS}}}{(-V_p)} \cdot v_d$$

$$i_{D1} + i_{D2} = I_Q \Rightarrow i_{D2} = I_Q - i_{D1}$$

$$\left(\sqrt{i_{D1}} - \sqrt{I_Q - i_{D1}}\right)^2 = \frac{I_{DSS}}{(-V_p)^2} \cdot v_d^2$$

$$i_{D1} - 2\sqrt{i_{D1}(I_Q - i_{D1})} + (I_Q - i_{D1}) = \frac{I_{DSS}}{(-V_p)^2} \cdot v_d^2$$

$$\text{Then } \sqrt{i_{D1}(I_Q - i_{D1})} = \frac{1}{2} \left[I_Q - \frac{I_{DSS}}{(-V_p)^2} \cdot v_d^2 \right]$$

Square both sides

$$i_{D1}^2 - i_{D1}I_Q + \frac{1}{4} \left[I_Q - \frac{I_{DSS}}{(-V_p)^2} \cdot v_d^2 \right]^2 = 0$$

$$i_{D1} = \frac{I_Q \pm \sqrt{I_Q^2 - 4\left(\frac{1}{4}\right)\left[I_Q - \frac{I_{DSS}}{(-V_p)^2} \cdot v_d^2 \right]^2}}{2}$$

$$i_{D1} = \frac{I_Q}{2} \pm \frac{1}{2} \sqrt{I_Q^2 - \left[I_Q^2 - \frac{2I_Q I_{DSS} v_d^2}{(-V_p)^2} + \left(\frac{I_{DSS} v_d^2}{(-V_p)^2} \right)^2 \right]}$$

Use + sign

$$i_{D1} = \frac{I_Q}{2} + \frac{1}{2} \sqrt{\frac{2I_Q I_{DSS}}{(-V_p)^2} \cdot v_d^2 - \left(\frac{I_{DSS}}{(-V_p)^2} \cdot v_d^2 \right)^2}$$

$$i_{D1} = \frac{I_Q}{2} + \frac{1}{2} \frac{I_Q}{(-V_p)} v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q} \right)^2 \left(\frac{v_d}{V_p} \right)^2}$$

Or

$$\frac{i_{D1}}{I_Q} = \frac{1}{2} + \left(\frac{1}{-2V_p} \right) \cdot v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q} \right)^2 \left(\frac{v_d}{V_p} \right)^2}$$

We had

$$i_{D2} = I_Q - i_{D1}$$

Then

$$\frac{i_{D2}}{I_Q} = \frac{1}{2} - \left(\frac{1}{-2V_p} \right) \cdot v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q} \right)^2 \left(\frac{v_d}{V_p} \right)^2}$$

b.

If $i_{D1} = I_Q$, then

$$1 = \frac{1}{2} + \left(\frac{1}{-2V_p} \right) \cdot v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q} \right)^2 \left(\frac{v_d}{V_p} \right)^2}$$

$$|V_p| = v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q} \right)^2 \left(\frac{v_d}{V_p} \right)^2}$$

Square both sides

$$|V_p|^2 = v_d^2 \left[\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q} \right)^2 \left(\frac{v_d}{V_p} \right)^2 \right]$$

$$\left(\frac{I_{DSS}}{I_Q} \right)^2 \left(\frac{1}{V_p} \right)^2 (v_d^2)^2 - \frac{2I_{DSS}}{I_Q} \cdot v_d^2 + |V_p|^2 = 0$$

$$v_d^2 = \frac{\frac{2I_{DSS}}{I_Q} \pm \sqrt{\left(\frac{2I_{DSS}}{I_Q} \right)^2 - 4 \left(\frac{I_{DSS}}{I_Q} \right)^2 \left(\frac{1}{V_p} \right)^2 (V_p)^2}}{2 \left(\frac{2I_{DSS}}{I_Q} \right)^2 \left(\frac{1}{V_p} \right)^2}$$

$$v_d^2 = (V_p)^2 \left(\frac{I_Q}{I_{DSS}} \right)$$

$$\text{Or } |v_d| = |V_p| \left(\frac{I_Q}{I_{DSS}} \right)^{1/2}$$

c. For v_d small,

$$i_{D1} \approx \frac{I_Q}{2} + \frac{1}{2} \cdot \frac{I_Q}{(-V_p)} \cdot v_d \sqrt{\frac{2I_{DSS}}{I_Q}}$$

$$g_f = \frac{di_{D1}}{d_{vd}} \Big|_{v_d \rightarrow 0} = \frac{1}{2} \cdot \frac{I_Q}{(-V_p)} \cdot \sqrt{\frac{2I_{DSS}}{I_Q}}$$

$$\text{Or } \Rightarrow g_f (\max) = \left(\frac{1}{-V_p} \right) \sqrt{\frac{I_Q I_{DSS}}{2}}$$

11.55

a.

$$I_Q = I_{D1} + I_{D2} \Rightarrow I_Q = 1 \text{ mA}$$

$$v_0 = 7 = 10 - (0.5)R_D \Rightarrow R_D = 6 \text{ k}\Omega$$

b.

$$g_f(\max) = \left(\frac{1}{-V_p} \right) \sqrt{\frac{I_Q \cdot I_{DSS}}{2}}$$

$$g_f(\max) = \left(\frac{1}{4} \right) \sqrt{\frac{(1)(2)}{2}} \Rightarrow g_f(\max) = 0.25 \text{ mA/V}$$

c.

$$A_d = \frac{g_m R_D}{2} = g_f(\max) \cdot R_D$$

$$A_d = (0.25)(6) \Rightarrow A_d = 1.5$$

11.56

a.

$$I_s = \frac{-V_{GS} - (-5)}{R_s} = (2)I_{DSS} \left(1 - \frac{V_{GS}}{V_p} \right)^2$$

$$5 - V_{GS} = (2)(0.8)(20) \left(1 - \frac{V_{GS}}{(-2)} \right)^2$$

$$5 - V_{GS} = (2)16 \left(1 + V_{GS} + \frac{1}{4}V_{GS}^2 \right)$$

$$8V_{GS}^2 + 33V_{GS} + 27 = 0$$

$$V_{GS} = \frac{-33 \pm \sqrt{1089 - 4(8)(27)}}{2(8)}$$

$$= -1.125 \text{ V}$$

$$I_s = \frac{5 - (-1.125)}{20}$$

$$= 0.306 \text{ mA}$$

$$I_{D1} = I_{D2} = 0.153 \text{ mA}$$

$$v_{o2} = 1.17 \text{ V}$$

(b)

11.57

Equivalent circuit and analysis is identical to that in problem 11.47.

$$A_{d2} = \frac{\frac{1}{2} \cdot g_m R_L}{\left(2 + \frac{R_L}{R_D}\right)}$$

$$A_{d1} = \frac{-\frac{1}{2} \cdot g_m R_L}{\left(2 + \frac{R_L}{R_D}\right)}$$

$$A_v = \frac{v_{02} - v_{01}}{v_d} = \frac{g_m R_L}{\left(2 + \frac{R_L}{R_D}\right)}$$

11.58

$$(a) A_d = g_m (r_{o2} \| r_{o4})$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$$

$$r_{o2} = \frac{V_{A2}}{I_{CQ}} = \frac{120}{0.2} = 600 \text{ k}\Omega$$

$$r_{o4} = \frac{V_{A4}}{I_{CQ}} = \frac{80}{0.2} = 400 \text{ k}\Omega$$

$$A_d = (7.692)(600 \| 400) = 1846$$

$$(b) R_o = r_{o2} \| r_{o4} = 600 \| 400 = 240 \text{ k}\Omega$$

$$(c) A_d = g_m (r_{o2} \| r_{o4} \| R_L)$$

$$A_d = (0.75)(1846) = 1384.5 = (7.692)(240 \| R_L) \Rightarrow R_L = 720 \text{ k}\Omega$$

11.59

(a)

$$\begin{aligned} I_Q &= 250 \mu\text{A} & I_{REF} &= I_Q \left(1 + \frac{2}{\beta}\right) \\ &&&= 250 \left(1 + \frac{2}{180}\right) = 252.8 \mu\text{A} \end{aligned}$$

$$R_l = \frac{5 - (0.7) - (-5)}{0.2528} \Rightarrow R_l = 36.8 \text{ K}$$

(b)

$$A_d = g_m (r_{o2} \| r_{o4}) \quad g_m = \frac{0.125}{0.026} = 4.808 \text{ mA/V}$$

$$r_{o2} = \frac{150}{0.125} = 1200 \text{ K}$$

$$A_d = (4.808)(1200 \| 800) \quad r_{o4} = \frac{100}{0.125} = 800 \text{ K}$$

$$\underline{A_d = 2308}$$

(c)

$$R_{id} = 2r_\pi = \frac{2(180)(0.026)}{0.125} \Rightarrow R_{id} = 74.9 \text{ K}$$

$$R_o = r_{o2} \parallel r_{o4} = 1200 \parallel 800 = 480 \text{ K} = R_o$$

(d)

$$v_{cm}(\max) = 5 - 0.7 = 4.3 \text{ V}$$

$$v_{cm}(\min) = 0.7 + 0.7 - 5 = -3.6 \text{ V}$$

11.60

a.

$$I_0 = I_{B3} + I_{B4} \approx 2 \left(\frac{I_Q}{2} \right) \left(\frac{1}{\beta} \right)$$

$$I_0 = \frac{I_Q}{\beta} = \frac{0.2}{100} \Rightarrow I_0 = 2 \mu\text{A}$$

b.

$$r_{02} = r_{04} = \frac{V_A}{I_{CQ}} = \frac{100}{0.1} = 1000 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$A_d = g_m(r_{02} \parallel r_{04}) = (3.846)(1000 \parallel 1000) \Rightarrow A_d = 1923$$

c.

$$A_d = g_m(r_{02} \parallel r_{04} \parallel R_L)$$

$$A_d = (3.846)(1000 \parallel 1000 \parallel 250) \Rightarrow A_d = 641$$

11.61

$$(a) A_d = \frac{\frac{I_Q}{2V_T}}{\frac{I_Q}{2V_{A2}} + \frac{I_Q}{2V_{A4}} + \frac{1}{R_L}}$$

$$1000 = \frac{\frac{I_Q}{2(0.026)}}{\frac{I_Q}{2(90)} + \frac{I_Q}{2(60)} + \frac{1}{250}}$$

$$1000(I_Q)(0.005556 + 0.008333) + 4 = I_Q(19.23) \Rightarrow I_Q = 0.749 \text{ mA}$$

$$(b) v_{CM}(\max) = V^+ - 2V_{EB}(on) = 5 - 2(0.6) = 3.8 \text{ V}$$

11.62

$$(b) v_{o1} = -g_{m1}(r_{o1} \parallel R_{o3}) \left(\frac{v_d}{2} \right)$$

$$A_{d1} = \frac{v_{o1}}{v_d} = -\frac{1}{2} g_{m1} (r_{o1} \parallel R_{o3})$$

$$g_{m1} = \frac{I_{CQ}}{V_T} = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$$

$$r_{o1} = \frac{V_{A1}}{I_{CQ}} = \frac{120}{0.2} = 600 \text{ k}\Omega$$

$$R_{o3} = r_{o3} \parallel r_{\pi3} \left\| \frac{1}{g_{m3}} \right., \text{ where } g_{m3} = 7.692 \text{ mA/V},$$

$$r_{\pi3} = \frac{(120)(0.026)}{0.2} = 15.6 \text{ k}\Omega, \quad r_{o3} = 400 \text{ k}\Omega$$

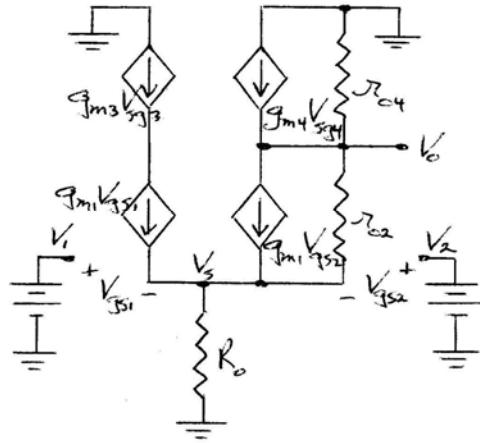
$$\text{Then } R_{o3} = 400 \left\| 15.6 \right\| \frac{1}{7.692} = 15.014 \parallel 0.130 = 0.1289 \text{ k}\Omega$$

$$A_{d1} = -\frac{1}{2} (7.692) (600 \parallel 0.1289) = -0.4956$$

$$(c) A_{d2} = +\frac{1}{2} g_{m2} (r_{o2} \parallel R_{o4}) = +0.4956$$

$$(d) A_{d3} = +g_{m1} (r_{o1} \parallel R_{o3}) = 2(0.4956) = 0.9912$$

11.63



$$g_{m4} V_{sg4} = g_{m3} V_{sg3} = g_{m1} V_{gs1}$$

$$V_{gs1} = V_1 - V_s, \text{ and } V_{gs2} = V_2 - V_s$$

$$g_{m1} V_{gs1} + g_{m1} V_{gs2} + \frac{V_o - V_s}{r_{o2}} = \frac{V_s}{R_o}$$

$$g_{m1} (V_1 + V_2 - 2V_s) + \frac{V_o}{r_{o2}} = V_s \left(\frac{1}{R_o} + \frac{1}{r_{o2}} \right)$$

$$g_{m1}(V_1 + V_2) + \frac{V_o}{r_{o2}} = V_s \left(2g_{m1} + \frac{1}{R_o} + \frac{1}{r_{o2}} \right)$$

$$\text{Now } g_{m1} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.1)} = 0.4472 \text{ mA/V}$$

$$r_{o2} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$\text{Then } (0.4472)(V_1 + V_2) + \frac{V_o}{500} = V_s \left[2(0.4472) + \frac{1}{2000} + \frac{1}{500} \right] = V_s (0.8969)$$

$$\text{We have (1) } V_s = (0.4986)(V_1 + V_2) + V_o (0.0022299)$$

$$\text{Also (2) } \frac{V_o - V_s}{r_{o2}} + \frac{V_o}{r_{o4}} + g_{m1}V_{gs2} - g_{m1}V_{gs1} = 0$$

$$V_o \left(\frac{1}{r_{o2}} + \frac{1}{r_{o4}} \right) - \frac{V_s}{r_{o2}} + g_{m1} (V_2 - V_s - (V_1 - V_s)) = 0$$

$$V_o \left(\frac{1}{r_{o2}} + \frac{1}{r_{o4}} \right) - \frac{V_s}{r_{o2}} + g_{m1} (V_2 - V_1) = 0$$

$$\text{We find } r_{o4} = \frac{1}{(0.03)(0.1)} = 333.3 \text{ k}\Omega$$

$$\text{Then } V_o \left(\frac{1}{500} + \frac{1}{333.3} \right) - \frac{1}{500} [(0.4986)(V_1 + V_2) + V_o (0.0022299)] + (0.4472)(V_2 - V_1) = 0$$

$$V_o [(0.005) - (0.00000446)] - (0.0009972)(V_1 + V_2) + (0.4472)(V_2 - V_1) = 0$$

(a) Let $V_1 = V_d$ and $V_2 = 0$

$$V_o [0.00499554] - V_d (0.0009972) - V_d (0.4472) = 0$$

$$A_d = \frac{V_o}{V_d} = 89.72$$

(b) Let $V_1 = 0$ and $V_2 = -V_d$

$$V_o [0.00499554] + V_d (0.0009972) - V_d (0.4472) = 0$$

$$A_d = \frac{V_o}{V_d} = 89.32$$

(c) Let $V_1 = \frac{V_d}{2}$ and $V_2 = -\frac{V_d}{2}$

$$V_o [0.00499554] = V_d (0.4472)$$

$$A = \frac{V_o}{V_d} = 89.52$$

11.64

a. From symmetry.

$$V_{GS3} = V_{GS4} = V_{DS3} = V_{DS4} = \sqrt{\frac{0.1}{0.1}} + 1$$

$$\text{Or } V_{DS3} = V_{DS4} = 2 \text{ V}$$

$$V_{SG1} = V_{SG2} = \sqrt{\frac{0.1}{0.1}} + 1 = 2 \text{ V}$$

$$\begin{aligned} V_{SD1} &= V_{SD2} = V_{SG1} - (V_{DS3} - 10) \\ &= 2 - (2 - 10) \end{aligned}$$

$$\text{Or } V_{SD1} = V_{SD2} = 10 \text{ V}$$

b.

$$r_{on} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.01)(0.1)} \Rightarrow 1 \text{ M}\Omega$$

$$r_{op} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.015)(0.1)} \Rightarrow 0.667 \text{ M}\Omega$$

$$\begin{aligned} g_m &= 2K_p (V_{SG} + V_{TP}) \\ &= 2(0.1)(2 - 1) = 0.2 \text{ mA/V} \end{aligned}$$

$$A_d = g_m (r_{on} \| r_{op}) = (0.2)(1000 \| 667) \Rightarrow A_d = 80$$

(c)

$$I_{D2} = I_{D1} = \frac{I_Q}{2} = 0.1 \text{ mA}$$

$$r_{o4} = \frac{1}{\lambda_n I_{D4}} = \frac{1}{(0.01)(0.1)} = 1000 \text{ k}\Omega$$

$$r_{o2} = \frac{1}{\lambda_p I_{D2}} = \frac{1}{(0.015)(0.1)} = 667 \text{ k}\Omega$$

$$R_o = r_{o2} \| r_{o4} = 667 \| 1000 = 400 \text{ k}\Omega$$

11.65

$$(a) A_d = g_m (r_{o2} \| r_{o4})$$

$$g_m = 2 \sqrt{\left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_n} I_{DQ} = 2 \sqrt{\left(\frac{0.1}{2}\right) (8)(0.06)} = 0.3098 \text{ mA/V}$$

$$r_{o2} = \frac{1}{(0.025)(0.06)} = 666.7 \text{ k}\Omega$$

$$r_{o4} = \frac{1}{(0.04)(0.06)} = 416.7 \text{ k}\Omega$$

$$A_d = (0.3098)(666.7 \| 416.7) = 79.44$$

$$(b) i_{D3} = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_p (V_{SG3} + V_{TP})^2$$

$$0.06 = \left(\frac{0.04}{2}\right) (10) (V_{SG3} - 0.3)^2 \Rightarrow V_{SG3} = 0.8477 \text{ V}$$

$$V_O = 1.8 - 0.8477 = 0.9523$$

$$V_{CM}(\max) = V_O - V_{DS}(sat) + V_{GS}$$

$$0.06 = \left(\frac{0.1}{2}\right) (8) (V_{GS} - 0.3)^2 \Rightarrow V_{GS} = 0.6873 \text{ V}, \quad V_{DS}(sat) = 0.3873 \text{ V}$$

$$v_{CM}(\max) = 0.9523 - 0.3873 + 0.6873 = 1.25 \text{ V}$$

11.66

$$R_{04} = r_{04} [1 + g_{m4}(R \parallel r_{\pi 4})]$$

$$r_{04} = \frac{80}{0.1} = 800 \text{ K}$$

$$g_{m4} = \frac{0.1}{0.026} = 3.846$$

$$r_{\pi 4} = \frac{(100)(0.026)}{0.1} \\ = 26 \text{ K}$$

$$R \parallel r_{\pi 4} = 1 \parallel 26 = 0.963 \text{ K}$$

Assume $\beta = 100$

$$r_{\pi 3} = \frac{(100)(0.026)}{0.1} = 26 \text{ k}\Omega$$

$$g_{m3} = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$R_{04} = 800 [1 + (3.846)(0.963)] \Rightarrow 3.763 \text{ M}\Omega$$

$$\Rightarrow R_0 = 3.763 \text{ M}\Omega$$

Then

$$A_v = -g_m (r_{02} \parallel R_0)$$

$$r_{02} = \frac{120}{0.1} = 1200 \text{ k}\Omega$$

$$g_m = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$A_v = -(3.846) [1200 \parallel 3763] \Rightarrow A_v = -3499$$

b.

For

$$R = 0, \quad r_{04} = \frac{80}{0.1} = 800 \text{ k}\Omega$$

$$A_v = -g_m (r_{02} \parallel r_{04}) \\ = -(3.846) [1200 \parallel 800] \Rightarrow A_v = -1846$$

(c) For part (a), $R_o = (3.763 \parallel 1.2) = 0.910 M\Omega$

$$\text{For part (b), } R_o = (1.2 \parallel 0.8) = 0.48 M\Omega$$

11.67

$$I_{B5} = \frac{I_{E5}}{1+\beta} = \frac{I_{B3} + I_{B4}}{1+\beta} = \frac{I_{C3} + I_{C4}}{\beta(1+\beta)}$$

Now $I_{C3} + I_{C4} \approx I_Q$

$$\text{So } I_{B5} \approx \frac{I_Q}{\beta(1+\beta)}$$

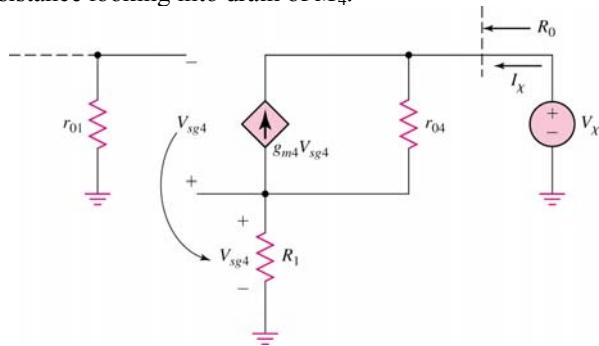
$$I_{B^6} = \frac{I_{E6}}{1+\beta} = \frac{I_{Q1}}{\beta(1+\beta)}$$

For balance, we want $I_{B6} = I_{B5}$

So that $I_{Q^1} = I_Q$

11.68

Resistance looking into drain of M_4 .



$$V_{sg\,4} \cong I_X R_1$$

$$I_X + g_{m4} V_{sg4} = \frac{V_X - V_{sg4}}{r_{o4}}$$

$$I_X \left[1 + g_{m4} R_1 + \frac{R_1}{r_{o4}} \right] = \frac{V_X}{r_{o4}}$$

$$\text{Or } R_o = r_{o4} \left[1 + g_{m4} R_1 + \frac{R_1}{r_{o4}} \right]$$

a.

$$A_d = g_{m2} (r_{o2} \| R_o)$$

$$g_{m2} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.080)(0.1)} = 0.179 \text{ mA/V}$$

$$r_{o2} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.015)(0.1)} = 667 \text{ k}\Omega$$

$$g_{m4} = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.080)(0.1)} = 0.179 \text{ mA/V}$$

$$r_{o4} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$R_0 = 500 \left[1 + (0.179)(1) + \frac{1}{500} \right] = 590.5 \text{ k}\Omega$$

$$A_d = (0.179) [667 \| 590.5] \Rightarrow \underline{A_d = 56.06}$$

b.

When $R_l = 0$, $R_0 = r_{o4} = 500 \text{ k}\Omega$

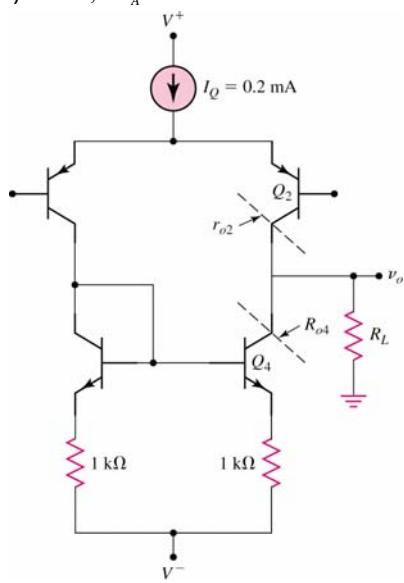
$$A_d = (0.179) [667 \| 500] \Rightarrow \underline{A_d = 51.15}$$

(c) For part (a), $R_o = r_{o2} \| R_o = 667 \| 590.5 \Rightarrow \underline{R_o = 313 \text{ k}\Omega}$

For part (b), $R_o = r_{o2} \| r_{o4} = 667 \| 500 \Rightarrow \underline{R_o = 286 \text{ k}\Omega}$

11.69

Let $\beta = 100$, $V_A = 100 \text{ V}$



$$r_{o2} = \frac{V_A}{I_{CQ}} = \frac{100}{0.1} = 1000 \text{ k}\Omega$$

$$R_{o4} = r_{o4} [1 + g_m R'_E] \text{ where } R'_E = r_\pi \| R_E$$

Now

$$r_\pi = \frac{(100)(0.026)}{0.1} = 26 \text{ k}\Omega$$

$$g_m = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

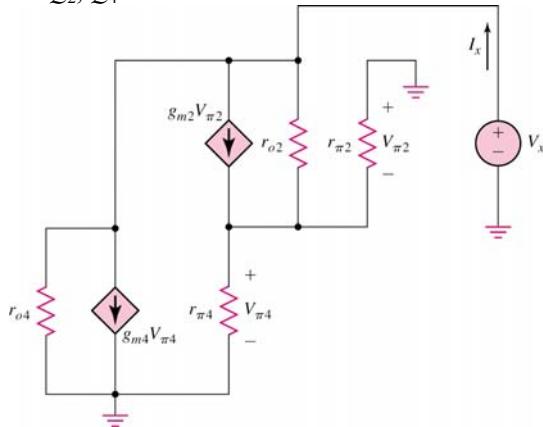
$$R'_E = 26 \| 1 = 0.963 \text{ k}\Omega$$

$$\text{Then } R_{o4} = 1000 [1 + (3.846)(0.963)] = 4704 \text{ k}\Omega$$

$$A_d = g_m (r_{o2} \| R_{o4}) = 3.846 (1000 \| 4704) \Rightarrow A_d = 3172$$

11.70

(a) For Q_2, Q_4



$$(1) \quad I_x = \frac{V_x - V_{\pi 4}}{r_{o2}} + g_{m2} V_{\pi 2} + g_{m4} V_{\pi 4} + \frac{V_x}{r_{o4}}$$

$$(2) \quad g_{m2} V_{\pi 2} + \frac{V_x - V_{\pi 4}}{r_{o2}} = \frac{V_{\pi 4}}{r_{\pi 4} \| r_{\pi 2}}$$

$$(3) \quad V_{\pi 4} = -V_{\pi 2}$$

$$\text{From (2)} \quad \frac{V_x}{r_{o2}} = V_{\pi 4} \left[\frac{1}{r_{\pi 4} \| r_{\pi 2}} + \frac{1}{r_{o2}} + g_{m2} \right]$$

Now

$$I_{C4} = \left(\frac{\beta}{1+\beta} \right) \left(\frac{I_Q}{2} \right) = \left(\frac{120}{121} \right) (0.5) = 0.496 \text{ mA}$$

$$I_{C2} = \left(\frac{I_Q}{2} \right) \left(\frac{1}{1+\beta} \right) \left(\frac{\beta}{1+\beta} \right) = (0.5) \left(\frac{120}{(121)^2} \right) \Rightarrow I_{C2} = 0.0041 \text{ mA}$$

So

$$r_{\pi 2} = \frac{(120)(0.026)}{0.0041} = 761 \text{ k}\Omega$$

$$g_{m2} = \frac{0.0041}{0.026} = 0.158 \text{ mA/V}$$

$$r_{o2} = \frac{100}{0.0041} \Rightarrow 24.4 \text{ M}\Omega$$

$$r_{\pi 4} = \frac{(120)(0.026)}{0.496} = 6.29 \text{ k}\Omega$$

$$g_{m4} = \frac{0.496}{0.026} = 19.08 \text{ mA/V}$$

$$r_{o4} = \frac{100}{0.496} = 202 \text{ k}\Omega$$

Now

$$\frac{V_x}{r_{o2}} = V_{\pi 4} \left[\frac{1}{6.29 \parallel 761} + \frac{1}{24400} + 0.158 \right] \Rightarrow \text{which yields } V_{\pi 4} = \frac{V_x}{(0.318)r_{o2}}$$

From (1),

$$I_x = \frac{V_x}{r_{o2}} + \frac{V_x}{r_{o4}} + V_{\pi 4} \left(g_{m4} - g_{m2} - \frac{1}{r_{o2}} \right)$$

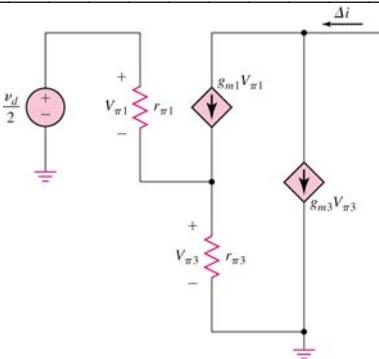
$$\frac{I_x}{V_x} = \left[\frac{1}{24400} + \frac{1}{202} + \frac{\left(19.08 - 0.158 - \frac{1}{24400} \right)}{(0.318)(24400)} \right] \text{ which yields } R_{o2} = \frac{V_x}{I_x} = 135 \text{ k}\Omega$$

$$\text{Now } r_{o6} = \frac{80}{0.5} = 160 \text{ k}\Omega$$

$$\text{Then } R_o = R_{o2} \parallel r_{o6} = 135 \parallel 160 \Rightarrow R_o = 73.2 \text{ k}\Omega$$

(b)

$$A_d = g_m^c R_o \text{ where } g_m^c = \frac{\Delta i}{v_d / 2}$$



$$\Delta i = g_{m1}V_{\pi1} + g_{m3}V_{\pi3} \text{ and } V_{\pi1} + V_{\pi3} = \frac{v_d}{2}$$

$$\text{Also } \left(\frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} \right) r_{\pi3} = V_{\pi3}$$

$$\text{So } V_{\pi1} \left(\frac{1+\beta}{r_{\pi1}} \right) r_{\pi3} = V_{\pi3}$$

$$\text{Or } V_{\pi1} \left(\frac{121}{761} \right) (6.29) = V_{\pi3} \equiv V_{\pi1}$$

$$\text{Then } 2V_{\pi1} = \frac{v_d}{2} \Rightarrow V_{\pi1} = \frac{v_d}{4}$$

$$\text{So } \Delta i = (g_{m1} + g_{m3})V_{\pi1} = (0.158 + 19.08) \left(\frac{v_d}{4} \right) = 9.62 \left(\frac{v_d}{2} \right)$$

$$\text{So } g_m^c = \frac{\Delta i}{v_d / 2} = 9.62 \Rightarrow A_d = (9.62)(73.2) \Rightarrow \underline{A_d = 704}$$

Now $R_{id} = 2R_i$ where $R_i = r_{\pi1} + (1+\beta)r_{\pi3}$

$$R_i = 761 + (121)(6.29) = 1522 \text{ k}\Omega$$

$$\text{Then } \underline{R_{id} = 3.044 \text{ M}\Omega}$$

11.71 - Design Problem

11.72

Input: $-8 \leq V_d \leq 8 \text{ mV}$

Output: $-0.8 \leq V_o \leq 0.8 \text{ V}$

$$A_d = \frac{V_o}{V_d} = \frac{0.8}{0.008} = 100$$

$$A_d = g_m (r_{o2} \| r_{o4})$$

Let $I_Q = 0.5 \text{ mA}$, $I_{DQ} = 0.25 \text{ mA}$

$$r_{o2} = \frac{1}{(0.025)(0.25)} = 160 \text{ k}\Omega$$

$$r_{o4} = \frac{1}{(0.04)(0.25)} = 100 \text{ k}\Omega$$

$$100 = g_m (160 \| 100) \Rightarrow g_m = 1.625 \text{ mA/V}$$

$$g_m = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_n} I_{DQ}$$

$$1.625 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_n (0.25)} \Rightarrow \left(\frac{W}{L}\right)_n = 52.8$$

11.73

For current source, $I_{REF} = I_Q$

$$P = (2I_Q)(V^+ - V^-)$$

$$0.8 = (2I_Q)(6) \Rightarrow I_Q \approx 66 \mu A$$

$$A_d = g_m(r_{o2} \| r_{o4})$$

$$r_{o2} = \frac{1}{(0.02)\left(\frac{0.066}{2}\right)} = 1515 \text{ k}\Omega$$

$$r_{o4} = \frac{1}{(0.03)\left(\frac{0.066}{2}\right)} = 1010 \text{ k}\Omega$$

$$240 = g_m(1515 \| 1010) \Rightarrow g_m = 0.396 \text{ mA/V}$$

$$g_m = 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_n} I_{DQ}$$

$$0.396 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_n (0.033)} \Rightarrow \left(\frac{W}{L}\right)_n = 23.76$$

11.74

$$A_d = g_m(r_{o2} \| R_o)$$

$$\approx g_m r_{o2}$$

$$r_{o2} = \frac{1}{\lambda_n I_D}$$

$$= \frac{1}{(0.015)(0.1)} = 666.7 \text{ K}$$

$$A_d = 400 = g_m(666.7)$$

$$g_m = 0.60 \text{ mA/V}$$

$$= 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_n} I_D$$

$$0.60 = 2\sqrt{\left(\frac{0.08}{2}\right)\left(\frac{W}{L}\right)(0.1)}$$

$$0.090 = 0.004\left(\frac{W}{L}\right)$$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 22.5$$

11.75

$$A_d = g_m (R_{o4} \| R_{o6})$$

where

$$R_{o4} = r_{o4} + r_{o2} [1 + g_{m4} r_{o4}]$$

$$R_{o6} = r_{o6} + r_{o8} [1 + g_{m6} r_{o6}]$$

We have

$$r_{o2} = r_{o4} = \frac{1}{(0.015)(0.040)} = 1667 \text{ k}\Omega$$

$$r_{o6} = r_{o8} = \frac{1}{(0.02)(0.040)} = 1250 \text{ k}\Omega$$

$$g_{m4} = 2\sqrt{\left(\frac{0.060}{2}\right)(15)(0.040)} = 0.268 \text{ mA/V}$$

$$g_{m6} = 2\sqrt{\left(\frac{0.025}{2}\right)(10)(0.040)} = 0.141 \text{ mA/V}$$

Then

$$R_{o4} = 1667 + 1667 [1 + (0.268)(1667)] \Rightarrow 748 \text{ M}\Omega$$

$$R_{o6} = 1250 + 1250 [1 + (0.141)(1250)] \Rightarrow 222.8 \text{ M}\Omega$$

(a)

$$R_o = R_{o4} \| R_{o6} = 748 \| 222.8 \Rightarrow \underline{R_o = 172 \text{ M}\Omega}$$

(b)

$$A_d = g_{m4} (R_{o4} \| R_{o6}) = (0.268)(172000) \Rightarrow \underline{A_d = 46096}$$

11.76

(a) $A_d = g_m (r_{o2} \| r_{o4})$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.2)(0.06)} = 0.2191 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{1}{(0.025)(0.06)} = 666.7 \text{ k}\Omega$$

$$A_d = (0.2191)(666.7 \| 666.7) = 73.0$$

(b) $R_o = r_{o2} \| r_{o4} = 333.3 \text{ k}\Omega$

(c) $i_{D3} = K_p (v_{SG3} + V_{TP})^2$

$$0.06 = 0.2(v_{SG3} - 0.3)^2 \Rightarrow v_{SG3} = 0.8477 \text{ V}$$

$$v_o = V^+ - v_{SG3} = 2.8 - 0.8477 = 1.9523 \text{ V}$$

$$i_{D1} = K_n (v_{GS1} - V_{TN})^2$$

$$0.06 = 0.2(v_{GS1} - 0.3)^3 \Rightarrow v_{GS1} = 0.8477 \text{ V}, \Rightarrow v_{DS1}(sat) = 0.5477 \text{ V}$$

$$v_{CM}(\max) = v_o - v_{DS1}(sat) + v_{GS1} = 1.9523 - 0.5477 + 0.8477 = 2.25 \text{ V}$$

11.77

(a) $i_{D3} = K_p (v_{SG3} + V_{TP})^2$
 $0.25 = 0.25(v_{SG3} - 0.4)^2 \Rightarrow v_{SG3} = 1.4 \text{ V}$

Then $v_{GS1} = 1.4 \text{ V}$, $v_{DS1}(\text{sat}) = 1.0 \text{ V}$

$$v_{CM} = v_O - v_{DS1}(\text{sat}) + v_{GS1}$$

$$3 = (V^+ - 1.4) - 1.0 + 1.4 \Rightarrow V^+ = 4 \text{ V} = -V^-$$

(b) $A_d = g_m (r_{o2} \| r_{o4})$
 $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.25)(0.25)} = 0.50 \text{ mA/V}$
 $r_{o2} = r_{o4} = \frac{1}{(0.02)(0.25)} = 200 \text{ k}\Omega$
 $A_d = (0.50)(200 \| 200) = 50$

11.78

(a) For $v_{cm} = +2 \text{ V} \Rightarrow V^+ = 2.7 \text{ V}$
If I_Q is a 2-transistor current source,

$$V^- = v_{cm} - 0.7 - 0.7$$

$$V^- = -3.4 \text{ V} \Rightarrow V^+ = -V^- = 3.4 \text{ V}$$

(b)

$$A_d = g_m (r_{o2} \| r_{o4}) \quad r_{o2} = \frac{100}{0.1} = 1000 \text{ K}$$

$$r_{o4} = \frac{60}{0.1} = 600 \text{ K}$$

$$g_m = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$A_d = (3.846)(1000 \| 600) \Rightarrow A_d = 1442$$

11.79

(a) $V^+ = -V^- = 3.4 \text{ V}$

(b)

$$r_{o2} = \frac{75}{0.06} = 1250 \text{ K}$$

$$r_{o4} = \frac{40}{0.06} = 666.7 \text{ K}$$

$$g_m = \frac{0.06}{0.026} = 2.308 \text{ mA/V}$$

$$A_d = (2.308)(1250 \| 666.7)$$

$$\underline{A_d = 1004}$$

11.80

(a) $I_{E2} = 0.25 \text{ mA}$, $I_{B2} = 0.001656 \text{ mA}$, $I_{C2} = 0.2483 \text{ mA}$

$$I_{D1} = 0.25 + 0.001656 = 0.25166 \text{ mA}$$

$$g_{m1} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.2)(0.25166)} = 0.4487 \text{ mA/V}$$

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{0.2483}{0.026} = 9.55 \text{ mA/V}$$

$$r_{\pi_2} = \frac{(150)(0.026)}{0.2483} = 15.71 \text{ k}\Omega$$

$$g_m^C = \frac{(0.4487)[1 + (9.55)(15.71)]}{1 + (0.4487)(15.71)} = 8.42 \text{ mA/V}$$

(b) $I_{E2} = 0.45 \text{ mA}$, $I_{B2} = 0.002980 \text{ mA}$, $I_{C2} = 0.4470 \text{ mA}$

$$I_{D1} = 0.05 + 0.00298 = 0.05298 \text{ mA}$$

$$g_{m1} = 2\sqrt{(0.2)(0.05298)} = 0.2059 \text{ mA/V}$$

$$g_{m2} = \frac{0.4470}{0.026} = 17.19 \text{ mA/V}$$

$$r_{\pi_2} = 8.725 \text{ k}\Omega$$

$$g_m^C = \frac{(0.2059)[1 + (17.19)(8.725)]}{1 + (0.2059)(8.725)} = 11.12 \text{ mA/V}$$

11.81

$$r_0(M_2) = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.01)(0.2)} = 500 \text{ k}\Omega$$

$$r_0(Q_2) = \frac{V_A}{I_{CQ}} = \frac{80}{0.2} = 400 \text{ k}\Omega$$

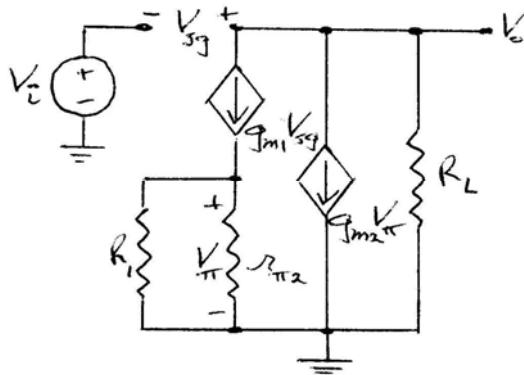
$$g_m(M_2) = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.2)(0.2)} \\ = 0.4 \text{ mA/V}$$

$$A_d = g_m(M_2) [r_0(M_2) \| r_0(Q_2)] \\ = 0.4 [500 \| 400] \Rightarrow A_d = 88.9$$

If the I_Q current source is ideal, $\underline{A_{cm} = 0}$ and $\underline{CMRR_{dB} = \infty}$

11.82

(a)



$$(b) I_{R1} = \frac{0.7}{8} = 0.0875 \text{ mA}$$

$$I_Q = 0.5 = I_{R1} + I_{E2} = 0.0875 + I_{E2} \Rightarrow I_{E2} = 0.4125 \text{ mA}$$

$$I_{B2} = 0.002279 \text{ mA}, \quad I_{C2} = 0.41022 \text{ mA}$$

$$I_{D1} = I_{R1} + I_{B2} = 0.0875 + 0.002279 = 0.08978 \text{ mA}$$

$$g_{m1} = 2\sqrt{(0.7)(0.08978)} = 0.5014 \text{ mA/V}$$

$$g_{m2} = \frac{0.41022}{0.026} = 15.78 \text{ mA/V}$$

$$r_{\pi_2} = 11.41 \text{ k}\Omega$$

$$(c) V_o = -(g_{m1}V_{sg} + g_{m2}V_\pi)R_L$$

$$V_\pi = (g_{m1}V_{sg})(R_1 \| r_{\pi 2})$$

$$V_o = -[g_{m1}V_{sg} + g_{m2}g_{m1}V_{sg}(R_1 \| r_{\pi 2})]R_L$$

$$-V_o = V_{sg}[g_{m1} + g_{m2}g_{m1}(R_1 \| r_{\pi 2})]R_L$$

$$\text{Now } V_{sg} = V_o - V_i$$

$$\text{So } V_o[1 + g_{m1}(1 + g_{m2}(R_1 \| r_{\pi 2}))R_L] = V_i g_{m1}[1 + g_{m2}(R_1 \| r_{\pi 2})]R_L$$

$$A_v = \frac{V_o}{V_i} = \frac{g_{m1}[1 + g_{m2}(R_1 \| r_{\pi 2})]R_L}{1 + g_{m1}[1 + g_{m2}(R_1 \| r_{\pi 2})]R_L}$$

$$A_v = \frac{(0.5014)[1 + (15.78)(8 \| 11.41)]R_L}{1 + (0.5014)[1 + (15.78)(8 \| 11.41)]R_L} = \frac{37.71R_L}{1 + 37.71R_L}$$

(i) For $R_L = 10 \text{ k}\Omega$,

$$A_v = \frac{(37.71)(10)}{1 + (37.71)(10)} = 0.99736$$

(ii) For $R_L = 100 \text{ k}\Omega$,

$$A_v = \frac{(37.71)(100)}{1 + (37.71)(100)} = 0.99973$$

11.83

$$(a) I_{R1} = \frac{0.7}{8} = 0.0875 \text{ mA}$$

$$I_Q = 0.8 = I_{R1} + I_{E2} = 0.0875 + I_{E2} \Rightarrow I_{E2} = 0.7125 \text{ mA}$$

$$I_{B2} = 0.003936 \text{ mA}, \quad I_{C2} = 0.7086 \text{ mA}$$

$$I_{D1} = I_{R1} + I_{B2} = 0.0875 + 0.003936 = 0.091436 \text{ mA}$$

$$g_{m1} = 2\sqrt{(0.7)(0.091436)} = 0.506 \text{ mA/V}$$

$$g_{m2} = \frac{0.7086}{0.026} = 27.25 \text{ mA/V}$$

$$r_{\pi 2} = 6.605 \text{ k}\Omega$$

$$(b) I_x = g_{m2}V_\pi + g_{m1}V_{sg}$$

$$V_\pi = (g_{m1}V_{sg})(R_1 \| r_{\pi 2})$$

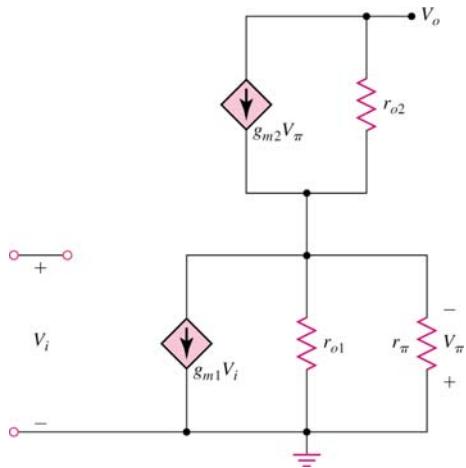
$$\text{Now } V_{sg} = V_x$$

$$\frac{I_x}{V_x} = \frac{1}{R_o} = g_{m1} + g_{m2}g_{m1}(R_1 \| r_{\pi 2}) = 0.506 + (0.506)(27.25)(8 \| 6.605) = 50.39 \text{ mA/V}$$

$$\text{Or } R_o = 19.8 \Omega$$

11.84

(a)



$$(1) \quad g_{m2}V_\pi + \frac{V_o - (-V_\pi)}{r_{o2}} = 0$$

$$(2) \quad g_{m2}V_\pi + \frac{V_o - (-V_\pi)}{r_{o2}} = g_{m1}V_i + \frac{-V_\pi}{r_{o1}} + \frac{-V_\pi}{r_\pi} \text{ or } 0 = g_{m1}V_i - V_\pi \left(\frac{1}{r_{o1}} + \frac{1}{r_\pi} \right)$$

$$V_\pi = \frac{g_{m1}V_i}{\left(\frac{1}{r_{o1}} + \frac{1}{r_\pi} \right)}$$

Then

From (1)

$$\begin{aligned} \left(g_{m2} + \frac{1}{r_{o2}} \right) V_\pi + \frac{V_o}{r_{o2}} &= 0 \\ V_o = -r_{o2} \left(g_{m2} + \frac{1}{r_{o2}} \right) V_\pi &= -r_{o2} g_{m1} V_i \frac{\left(g_{m2} + \frac{1}{r_{o2}} \right)}{\left(\frac{1}{r_{o1}} + \frac{1}{r_\pi} \right)} \\ A_v = \frac{V_o}{V_i} &= \frac{-g_{m1} r_{o2} \left(g_{m2} + \frac{1}{r_{o2}} \right)}{\left(\frac{1}{r_{o1}} + \frac{1}{r_\pi} \right)} \end{aligned}$$

Now

$$g_{m1} = 2\sqrt{K_n I_Q} = 2\sqrt{(0.25)(0.025)} = 0.158 \text{ mA/V}$$

$$g_{m2} = \frac{I_Q}{V_T} = \frac{0.025}{0.026} = 0.9615 \text{ mA/V}$$

$$r_{o1} = \frac{1}{\lambda I_Q} = \frac{1}{(0.02)(0.025)} = 2000 \text{ k}\Omega$$

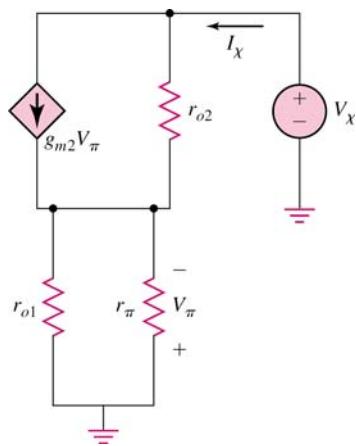
$$r_{o2} = \frac{V_A}{I_Q} = \frac{50}{0.025} = 2000 \text{ k}\Omega$$

$$r_\pi = \frac{\beta V_T}{I_Q} = \frac{(100)(0.026)}{0.025} = 104 \text{ k}\Omega$$

Then

$$A_v = \frac{-(0.158)(2000)\left(0.9615 + \frac{1}{2000}\right)}{\left(\frac{1}{2000} + \frac{1}{104}\right)} \Rightarrow A_v = -30039$$

To find R_o ; set $V_i = 0 \Rightarrow g_{m1} V_i = 0$



$$I_x = g_{m2}V_\pi + \frac{V_x - (-V_\pi)}{r_{o2}}$$

$$V_\pi = -I_x(r_{o1} \| r_\pi)$$

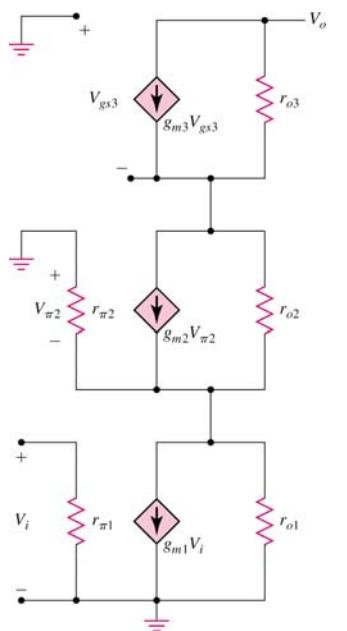
Then

$$I_x = \left(g_{m2} + \frac{1}{r_{o2}} \right) (-I_x)(r_{o1} \| r_\pi) + \frac{V_x}{r_{o2}}$$

Combining terms,

$$\begin{aligned} R_o &= \frac{V_x}{I_x} = r_{o2} \left[1 + (r_{o1} \| r_\pi) \left(g_{m2} + \frac{1}{r_{o2}} \right) \right] \\ &= 2000 \left[1 + (2000 \| 104) \left(0.9615 + \frac{1}{2000} \right) \right] \Rightarrow R_o = 192.2 M\Omega \end{aligned}$$

(b)



$$(1) \quad g_{m3}V_{gs3} + \frac{V_o - (-V_{gs3})}{r_{o3}} = 0$$

$$(2) \quad g_{m3}V_{gs3} + \frac{V_o - (-V_{gs3})}{r_{o3}} = g_{m2}V_{\pi2} + \frac{-V_{gs3} - (-V_{\pi2})}{r_{o2}} \text{ or } 0 = V_{\pi2} \left(g_{m2} + \frac{1}{r_{o2}} \right) - \frac{V_{gs3}}{r_{o2}}$$

$$(3) \quad \frac{V_{\pi2}}{r_{\pi2}} + g_{m2}V_{\pi2} + \frac{-V_{gs3} - (-V_{\pi2})}{r_{o2}} = g_{m1}V_i + \frac{(-V_{\pi2})}{r_{o1}}$$

$$V_{\pi2} = \frac{V_{gs3}}{r_{o2} \left(g_{m2} + \frac{1}{r_{o2}} \right)}$$

From (2),

Then

$$(3) \quad V_{\pi2} \left(\frac{1}{r_{\pi2}} + g_{m2} + \frac{1}{r_{o2}} + \frac{1}{r_{o1}} \right) = g_{m1}V_i + \frac{V_{gs3}}{r_{o2}}$$

or

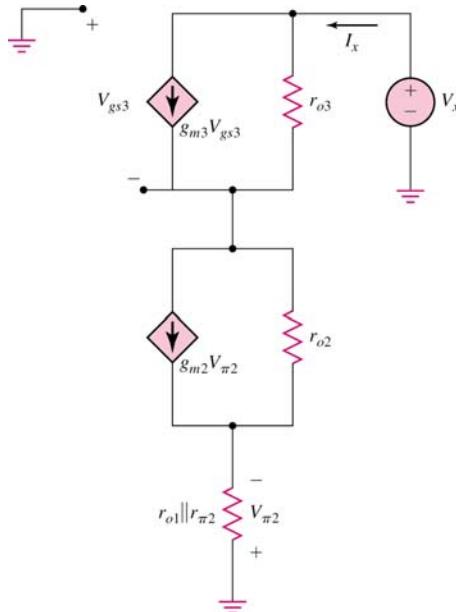
$$\frac{V_{gs3}}{r_o2 \left(g_{m2} + \frac{1}{r_{o1}} \right)} \left[\frac{1}{r_{\pi2}} + g_{m2} + \frac{1}{r_{o2}} + \frac{1}{r_{o1}} \right] = g_{m1} V_i + \frac{V_{gs3}}{r_{o2}}$$

$$\frac{V_{gs3}}{2000 \left(0.9615 + \frac{1}{2000} \right)} \left[\frac{1}{104} + 0.9615 + \frac{1}{2000} + \frac{1}{2000} \right] = 0.9615 V_i + \frac{V_{gs3}}{2000}$$

Then $V_{gs3} = 1.83 \times 10^5 V_i$

From (1), $\left(g_{m3} + \frac{1}{r_{o3}} \right) V_{gs3} = -\frac{V_o}{r_{o3}}$ or $V_o = -2000 \left(0.158 + \frac{1}{2000} \right) (1.83 \times 10^5) V_i$

$$A_v = \frac{V_o}{V_i} = -5.80 \times 10^7$$



To find R_o

$$(1) \quad I_x = g_{m3} V_{gs3} + \frac{V_x - (-V_{gs3})}{r_{o3}}$$

$$(2) \quad g_{m3} V_{gs3} + \frac{V_x - (-V_{gs3})}{r_{o3}} = g_{m2} V_{\pi2} + \frac{-V_{gs3} - (-V_{\pi2})}{r_{o2}}$$

$$(3) \quad V_{\pi2} = -I_x (r_{o1} \| r_{\pi2})$$

From (1) $I_x = V_{gs3} \left(g_{m3} + \frac{1}{r_{o3}} \right) + \frac{V_x}{r_{o3}}$

$$I_x = V_{gs3} \left(0.158 + \frac{1}{2000} \right) + \frac{V_x}{2000}$$

$$\text{So } V_{gs3} = \frac{I_x - \frac{V_x}{2000}}{0.1585}$$

From (2),

$$V_{gs3} \left[g_{m3} + \frac{1}{r_{o3}} + \frac{1}{r_{o2}} \right] + \frac{V_x}{r_{o3}} = V_{\pi 2} \left(g_{m2} + \frac{1}{r_{o2}} \right)$$

$$V_{gs3} \left[0.158 + \frac{1}{2000} + \frac{1}{2000} \right] + \frac{V_x}{2000} = V_{\pi 2} \left(0.9615 + \frac{1}{2000} \right)$$

$$\text{Then } \left[\frac{I_x - V_x / 2000}{0.1585} \right] (0.159) + \frac{V_x}{2000} = -I_x (2000 \| 104) (0.962)$$

$$\text{We find } R_o = \frac{V_x}{I_x} = 6.09 \times 10^{10} \Omega$$

11.85

Assume emitter of Q_1 is capacitively coupled to signal ground.

$$I_{CQ} = 0.2 \left(\frac{80}{81} \right) = 0.1975 \text{ mA}$$

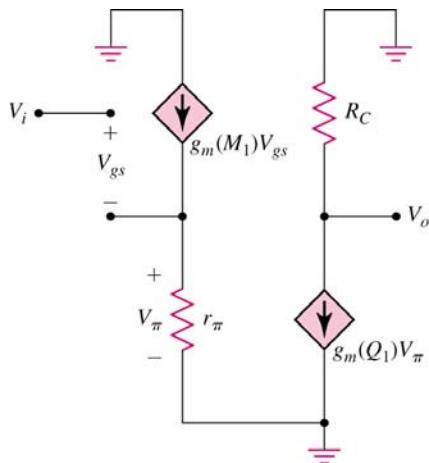
$$I_{DQ} = \frac{0.2}{81} = 0.00247 \text{ mA}$$

$$r_{\pi} = \frac{(80)(0.026)}{0.1975} = 10.5 \text{ k}\Omega$$

$$g_m(Q_1) = \frac{0.1975}{0.026} = 7.60 \text{ mA/V}$$

$$g_m(M_1) = 2\sqrt{K_n I_D} = 2\sqrt{(0.2)(0.00247)}$$

$$g_m(M_1) = 0.0445 \text{ mA/V}$$



$$V_i = V_{gs} + V_\pi \text{ and } V_\pi = g_m(M_1)V_{gs}r_\pi \text{ or } V_{gs} = \frac{V_\pi}{g_m(M_1)r_\pi}$$

$$\text{Then } V_i = V_\pi \left(1 + \frac{1}{g_m(M_1)r_\pi} \right) \text{ or } V_\pi = \frac{V_i}{\left(1 + \frac{1}{g_m(M_1)r_\pi} \right)}$$

$$V_o = -g_m(Q_1)V_\pi R_C \Rightarrow A_v = \frac{V_o}{V_i} = \frac{-g_m(Q_1)R_C}{\left(1 + \frac{1}{g_m(M_1)r_\pi} \right)}$$

$$\text{Then } A_v = \frac{-7.60(20)}{\left(1 + \frac{1}{(0.0445)(10.5)} \right)} \Rightarrow A_v = -48.4$$

11.86

$$r_{\pi 11} = \frac{(120)(0.026)}{0.2} = 15.6 \text{ k}\Omega$$

$$R'_E = R_3 \| r_{\pi 11} = 0.2 \| 15.6 = 0.1975 \text{ k}\Omega$$

$$g_{m11} = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$$

$$r_{o11} = \frac{V_{A11}}{I_{C11}} = \frac{120}{0.2} = 600 \text{ k}\Omega$$

$$R_{C11} = r_{o11}(1 + g_{m11}R'_E) = 600[1 + (7.692)(0.1975)] = 1512 \text{ k}\Omega$$

$$R_{C7} = r_{o7} = \frac{V_{A7}}{I_{C7}} = \frac{60}{0.2} = 300 \text{ k}\Omega$$

$$Z = R_{C7} \| R_{C11} = 300 \| 1512 = 250 \text{ k}\Omega$$

$$r_{\pi 8} = \frac{(120)(0.026)}{1} = 3.12 \text{ k}\Omega$$

$$I_{C9} = \frac{1}{120} \left(\frac{120}{121} \right) = 0.008264 \text{ mA}$$

$$r_{\pi 9} = \frac{(120)(0.026)}{0.008264} = 377.5 \text{ k}\Omega$$

$$\text{Now } R_o = R_4 \left| \frac{r_{\pi 8} + \left(\frac{r_{\pi 9} + Z}{121} \right)}{121} \right| = 5 \left| \frac{3.12 + \left(\frac{377.5 + 250}{121} \right)}{121} \right| = 5 \| 0.06864$$

$$\text{Or } R_o = 67.7 \text{ }\Omega$$

11.87

$$R_i = r_{\pi 1} + (1 + \beta) r_{\pi 2}$$

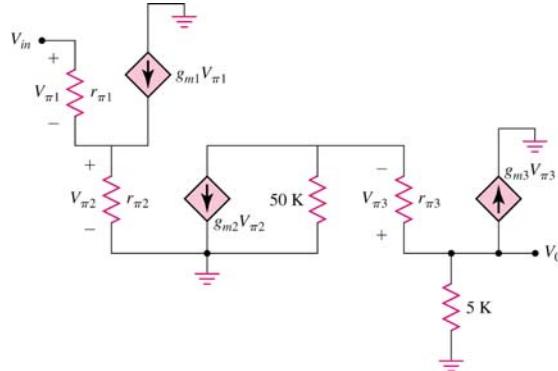
$$r_{\pi 2} = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$r_{\pi 1} = \frac{(100)(0.026)}{(0.5/100)} = \frac{(100)^2 (0.026)}{0.5} = 520 \text{ k}\Omega$$

$$R_i = 520 + (101)(5.2) \Rightarrow R_i \approx 1.05 \text{ M}\Omega$$

$$R_0 = 5 \left| \frac{r_{\pi 3} + 50}{101} \right|, \quad r_{\pi 3} = \frac{(100)(0.026)}{1} = 2.6 \text{ k}\Omega$$

$$R_0 = 5 \left| \frac{2.6 + 50}{101} \right| = 5 \parallel 0.521 \Rightarrow R_0 = 0.472 \text{ k}\Omega$$



$$V_0 = - \left(\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3} V_{\pi 3} \right) (5)$$

$$V_0 = -V_{\pi 3} \left(\frac{1 + \beta}{r_{\pi 3}} \right) (5) \quad (1)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} = g_{m2} V_{\pi 2} + \frac{(V_0 - V_{\pi 3})}{50}$$

$$g_{m2} V_{\pi 2} = V_{\pi 3} \left(\frac{1}{r_{\pi 3}} + \frac{1}{50} \right) - \frac{V_0}{50} \quad (2)$$

$$V_{\pi 2} = \left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} \right) r_{\pi 2}$$

$$= V_{\pi 1} \left(\frac{1 + \beta}{r_{\pi 1}} \right) r_{\pi 2} \quad (3)$$

and

$$V_{in} = V_{\pi 1} + V_{\pi 2} \quad (4)$$

$$g_{m2} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

Then

$$V_0 = -V_{\pi 3} \left(\frac{101}{2.6} \right) (5) \Rightarrow V_{\pi 3} = -V_0 (0.005149) \quad (1)$$

And

$$19.23V_{\pi 2} = -V_0(0.005149) \left(\frac{1}{2.6} + \frac{1}{50} \right) - \frac{V_0}{50}$$

$$= -V_0(0.02208)$$

Or $V_{\pi 2} = -V_0(0.001148)$ (2)

And

$$V_{\pi 1} = V_{in} - V_{\pi 2} = V_{in} + V_0(0.001148) \quad (4)$$

So

$$-V_0(0.001148) = [V_{in} + V_0(0.001148)] \left(\frac{101}{520} \right)(5.2) \quad (3)$$

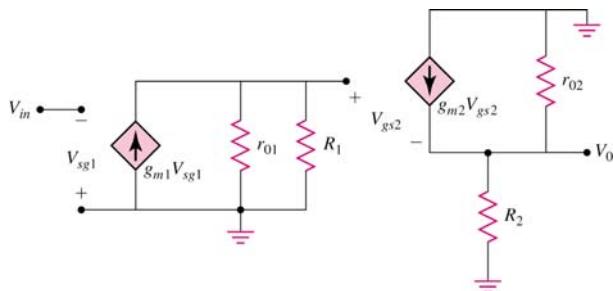
$$-V_0(0.001148) - V_0(0.001159) = V_{in}(1.01) \Rightarrow A_v = \frac{V_0}{V_{in}} = -438$$

11.88

$$I_2 = \frac{5}{5} = 1 \text{ mA}$$

$$V_{GS2} = \sqrt{\frac{1}{0.5}} + 0.8 = 2.21 \text{ V}$$

$$I_1 = \frac{2.21 - (-5)}{35} = 0.206 \text{ mA}$$



$$V_0 = (g_{m2} V_{gs2}) (R_2 \| r_{o2})$$

$$V_{gs2} = (g_{m1} V_{sg1}) (r_{o1} \| R_l) - V_0 \text{ and } V_{sg1} = -V_{in}$$

$$\text{So } V_{gs2} = -(g_{m1} V_{in}) (r_{o1} \| R_l) - V_0$$

Then

$$V_0 = g_{m2} (R_2 \| r_{o2}) [- (g_{m1} V_{in}) (r_{o1} \| R_l) - V_0]$$

$$A_v = \frac{V_0}{V_{in}} = \frac{-g_{m2} (R_2 \| r_{o2}) g_{m1} (r_{o1} \| R_l)}{1 + g_{m2} (R_2 \| r_{o2})}$$

$$g_{m2} = 2\sqrt{K_{n2} I_{D2}} = 2\sqrt{(0.5)(1)} = 1.414 \text{ mA/V}$$

$$g_{m1} = 2\sqrt{K_{p1} I_{D1}} = 2\sqrt{(0.2)(0.206)} = 0.406 \text{ mA/V}$$

$$r_{o1} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.01)(0.206)} = 485 \text{ k}\Omega$$

$$r_{o2} = \frac{1}{\lambda_2 I_{D2}} = \frac{1}{(0.01)(1)} = 100 \text{ k}\Omega$$

$$R_2 \| r_{o2} = 5 \| 100 = 4.76 \text{ k}\Omega$$

$$R_1 \| r_{o1} = 35 \| 485 = 32.6 \text{ k}\Omega$$

$$\text{Then } A_v = \frac{-(1.414)(4.76)(0.406)(32.6)}{1 + (1.414)(4.76)}$$

$$\text{So } \underline{A_v = -11.5}$$

Output Resistance—From the results for a source follower in Chapter 4.

$$R_0 = \frac{1}{g_{m2}} \| R_2 \| r_{o2} = \frac{1}{1.414} \| 5 \| 100 = 0.707 \| 4.76$$

$$\text{So } \underline{R_0 = 0.616 \text{ k}\Omega}$$

11.89

$$(a) R_2 = \frac{5-0}{1} = 5 \text{ k}\Omega$$

$$I_2 = K_p (V_{SG} + V_{TP})^2 \Rightarrow 1 = 1(V_{SG} - 0.8)^2 \Rightarrow V_{SG} = 1.8 \text{ V}$$

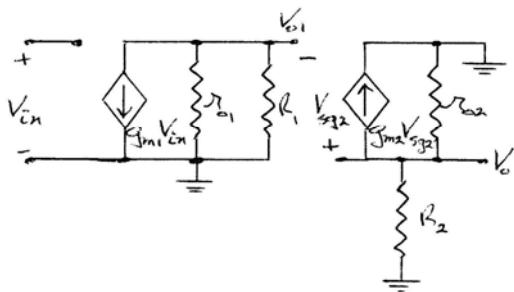
$$R_1 = \frac{5 - (-1.8)}{0.25} = 27.2 \text{ k}\Omega$$

$$(b) g_{m1} = 2\sqrt{(0.5)(0.25)} = 0.7071 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{(1)(1)} = 2 \text{ mA/V}$$

$$r_{o1} = \frac{1}{(0.02)(0.25)} = 200 \text{ k}\Omega, \quad r_{o2} = \frac{1}{(0.02)(1)} = 50 \text{ k}\Omega$$

(c)



$$V_{o1} = -g_{m1}V_{in}(r_{o1}\parallel R_1)$$

$$V_{sg2} = V_o - V_{o1}$$

$$\frac{V_o}{R_2} + \frac{V_o}{r_{o2}} + g_{m2}V_{sg2} = 0$$

$$V_o\left(\frac{1}{R_2} + \frac{1}{r_{o2}}\right) + g_{m2}[V_o + g_{m1}V_{in}(r_{o1}\parallel R_1)] = 0$$

$$V_o\left(\frac{1}{R_2} + \frac{1}{r_{o2}} + g_{m2}\right) = -g_{m1}g_{m2}(r_{o1}\parallel R_1)V_{in}$$

$$A_v = \frac{V_o}{V_{in}} = \frac{-g_{m1}g_{m2}(r_{o1}\parallel R_1)}{\left(\frac{1}{R_2} + \frac{1}{r_{o2}} + g_{m2}\right)} = \frac{-(0.7071)(2)(200\parallel 27.2)}{\left(\frac{1}{5} + \frac{1}{50} + 2\right)} = -15.25$$

$$(d) \quad I_x = \frac{V_x}{r_{o2}} + \frac{V_x}{R_2} + g_{m2}V_x$$

$$R_o = r_{o2}\parallel R_2 \left\| \frac{1}{g_{m2}} \right\| = 50\left\| 5 \right\| \left\| \frac{1}{2} \right\| = 4.545\left\| 0.5 \right\|$$

$$R_o = 0.450 \text{ k}\Omega$$

11.90

$$(a) \quad R = \frac{5-0}{0.25} = 20 \text{ k}\Omega$$

$$R_{E1} = \frac{-0.7 - (-5)}{0.25} = 17.2 \text{ k}\Omega$$

$$R_C = \frac{5-0.7}{0.25} = 17.2 \text{ k}\Omega$$

$$R_{E2} = \frac{0 - (-5)}{2} = 2.5 \text{ k}\Omega$$

$$(b) \quad A_{d1} = \frac{V_{o2}}{V_d} = \frac{g_{m1}}{2} (R\parallel r_{\pi3})$$

$$g_{m1} = \frac{0.25}{0.026} = 9.615 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(120)(0.026)}{0.25} = 12.48 \text{ k}\Omega$$

$$A_{d1} = \frac{(9.615)}{2} (20 \| 12.48) = 36.94$$

$$A_3 = -g_{m3} (R_C \| R_{i4})$$

$$R_{i4} = r_{\pi 4} + (1 + \beta)R_{E2}$$

$$g_{m3} = \frac{0.25}{0.026} = 9.615 \text{ mA/V}$$

$$r_{\pi 4} = \frac{(120)(0.026)}{2} = 1.56 \text{ k}\Omega$$

$$R_{i4} = 1.56 + (121)(2.5) = 304 \text{ k}\Omega$$

$$A_3 = -(9.615)(17.2 \| 304) = -156.5$$

$$A_4 = \frac{(1 + \beta)R_{E2}}{r_{\pi 4} + (1 + \beta)R_{E2}} = \frac{(121)(2.5)}{1.56 + (121)(2.5)} = 0.995$$

Now $A_d = \frac{V_o}{V_d} = A_{d1} \cdot A_3 \cdot A_4 = (36.94)(-156.5)(0.995) = -5752$

(c) $A_{cm1} = \frac{-g_{m1}(R \| r_{\pi 3})}{1 + \frac{2(1 + \beta)R_o}{r_{\pi 1}}}$

$$r_{\pi 1} = \frac{(120)(0.026)}{0.25} = 12.48 \text{ k}\Omega$$

$$A_{cm1} = \frac{-(9.615)(20 \| 12.48)}{1 + \frac{2(121)(200)}{12.48}} = -0.01905$$

$$A_{cm} = A_{cm1} \cdot A_3 \cdot A_4 = (-0.01905)(-156.5)(0.995) = 2.966$$

$$CMRR_{dB} = 20 \log_{10} \left(\frac{5752}{2.966} \right) = 65.8 \text{ dB}$$

11.91

a.

$$R_{C1} = \frac{10 - v_{01}}{I_{C1}} = \frac{10 - 2}{0.1} \Rightarrow R_{C1} = 80 \text{ k}\Omega$$

$$R_{C2} = \frac{10 - v_{04}}{I_{C4}} = \frac{10 - 6}{0.2} \Rightarrow R_{C2} = 20 \text{ k}\Omega$$

b.

$$A_{d1} = \frac{v_{01} - v_{02}}{v_d} = -g_{m1} (R_{C1} \| r_{\pi3})$$

$$g_{m1} = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$r_{\pi3} = \frac{(180)(0.026)}{0.2} = 23.4 \text{ k}\Omega$$

$$A_{d1} = -(3.846)(80 \| 23.4) \Rightarrow A_{d1} = -69.6$$

$$A_{d2} = \frac{v_{04}}{v_{01} - v_{02}} = \frac{1}{2} g_{m4} R_{C2}$$

$$g_{m4} = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$$

$$A_{d2} = \frac{1}{2}(7.692)(20) = 76.9$$

$$\text{Then } A_d = (76.9)(-69.6) \Rightarrow A_d = -5352$$

11.92

- a. Neglect the effect of r_0 in determining the differential-mode gain.

$$A_{d1} = \frac{v_{02}}{v_d} = \frac{1}{2} g_{m2} (R_C \| R_{i3}) \text{ where } R_{i3} = r_{\pi3} + (1 + \beta) R_E$$

$$A_2 = \frac{-\beta R_{C2}}{r_{\pi3} + (1 + \beta) R_E}$$

$$I_1 = \frac{12 - 0.7 - (-12)}{R_1} = \frac{23.3}{12} = 1.94 \text{ mA} \approx I_{C5}$$

$$g_{m2} = \frac{\frac{1}{2} \cdot (1.94)}{0.026} = 37.3 \text{ mA/V}$$

$$r_{\pi3} = \frac{(200)(0.026)}{I_{C3}}$$

$$v_{02} = 12 - \frac{1}{2}(1.94)(8) = 4.24 \text{ V}$$

$$I_{C3} = \frac{4.24 - 0.7}{3.3} = 1.07 \text{ mA}$$

$$r_{\pi3} = \frac{(200)(0.026)}{1.07} = 4.86 \text{ k}\Omega$$

$$R_{i3} = 4.86 + (201)(3.3) = 668 \text{ k}\Omega$$

$$A_{d1} = \frac{1}{2}(37.3)[8 \| 668] = 147.4$$

Then

$$A_d = A_{d1} \cdot A_2 = (147.4)(-1.197) \Rightarrow \underline{A_d = -176}$$

$$R_0 = r_{05} = \frac{V_A}{I_{C5}} = \frac{80}{1.94} = 41.2 \text{ k}\Omega$$

$$A_{cm1} = \frac{-g_{m2}(R_C \| R_{i3})}{1 + \frac{2(1+\beta)R_0}{r_{\pi2}}}$$

$$r_{\pi2} = \frac{(200)(0.026)}{\frac{1}{2} \cdot (1.94)} = 5.36 \text{ k}\Omega$$

$$A_{cm1} = \frac{-(37.3)(8 \| 668)}{1 + \frac{2(201)(41.2)}{5.36}} = -0.09539$$

$$A_2 = -1.197$$

$$A_{cm} = (-0.09539)(-1.197) \Rightarrow \underline{A_{cm} = 0.114}$$

b.

$$v_d = v_1 - v_2 = 2.015 \sin \omega t - 1.985 \sin \omega t$$

$$v_d = 0.03 \sin \omega t (\text{V})$$

$$v_{cm} = \frac{v_1 + v_2}{2} = 2.0 \sin \omega t$$

$$\begin{aligned} v_{03} &= A_d v_d + A_{cm} v_{cm} \\ &= (-176)(0.03) + (0.114)(2) \end{aligned}$$

Or $\underline{v_{03} = -5.052 \sin \omega t}$

Ideal, $A_{cm} = 0$

So

$$v_{03} = A_d v_d = (-176)(0.03)$$

$$\underline{v_{03} = -5.28 \sin \omega t}$$

c.

$$R_{id} = 2r_{\pi2} = 2(5.36) \Rightarrow \underline{R_{id} = 10.72 \text{ k}\Omega}$$

$$2R_{icm} \cong 2(1+\beta)R_0 \| (1+\beta)r_0$$

$$r_0 = \frac{V_A}{I_{C2}} = \frac{80}{\frac{1}{2} \cdot (1.94)} = 82.5 \text{ k}\Omega$$

$$\begin{aligned} 2R_{icm} &= [2(201)(41.2)] \| [(201)(82.5)] \\ &= 16.6 \text{ M}\Omega \| 16.6 \text{ M}\Omega \end{aligned}$$

$$\text{So} \Rightarrow \underline{R_{icm} = 4.15 \text{ M}\Omega}$$

11.93

a.

$$I_1 = \frac{24 - V_{GS4}}{R_1} = k_n (V_{GS4} - V_{Th})^2$$

$$24 - V_{GS4} = (55)(0.2)(V_{GS4} - 2)^2$$

$$24 - V_{GS4} = 11(V_{GS4}^2 - 4V_{GS4} + 4)$$

$$11V_{GS4}^2 - 43V_{GS4} + 20 = 0$$

$$V_{GS4} = \frac{43 \pm \sqrt{(43)^2 - 4(11)(20)}}{2(11)} = 3.37 \text{ V}$$

$$I_1 = \frac{24 - 3.37}{55} = 0.375 \text{ mA} = I_Q$$

$$v_{02} = 12 - \left(\frac{0.375}{2} \right)(40) = 4.5 \text{ V}$$

$$\frac{v_{02} - V_{GS3}}{R_5} = I_{D3} = k_n (V_{GS3} - V_{Th})^2$$

$$4.5 - V_{GS3} = (0.2)(6)(V_{GS3}^2 - 4V_{GS3} + 4)$$

$$1.2V_{GS3}^2 - 3.8V_{GS3} + 0.3 = 0$$

$$V_{GS3} = \frac{3.8 \pm \sqrt{(3.8)^2 - 4(1.2)(0.3)}}{2(1.2)} = 3.09 \text{ V}$$

$$I_{D3} = \frac{4.5 - 3.09}{6} = 0.235 \text{ mA}$$

$$g_{m2} = 2\sqrt{K_n I_{D2}} = 2\sqrt{(0.2)\left(\frac{0.375}{2}\right)} \\ = 0.387 \text{ mA/V}$$

$$A_{d1} = \frac{1}{2} g_{m2} R_D = \frac{1}{2}(0.387)(40) \Rightarrow A_{d1} = 7.74$$

$$A_2 = \frac{-g_{m3} R_{D2}}{1 + g_{m3} R_5}$$

$$g_{m3} = 2\sqrt{K_n I_{D3}} = 2\sqrt{(0.2)(0.235)} \\ = 0.434 \text{ mA/V}$$

$$A_2 = \frac{-(0.434)(4)}{1 + (0.434)(6)} = -0.482$$

$$\text{So } A_d = A_{d1} \cdot A_2 = (7.74)(-0.482) \Rightarrow A_d = -3.73$$

$$R_0 = r_{05} = \frac{1}{\lambda I_Q} = \frac{1}{(0.02)(0.375)} = 133 \text{ k}\Omega$$

$$A_{cm1} = \frac{-g_{m2} R_D}{1 + 2g_{m2} R_0} = \frac{-(0.387)(40)}{1 + 2(0.387)(133)} \\ = -0.149$$

$$A_{cm} = (-0.149)(-0.482) \Rightarrow A_{cm} = 0.0718$$

b.

$$v_d = v_1 - v_2 = 0.3 \sin \omega t$$

$$v_{cm} = \frac{v_1 + v_2}{2} = 2 \sin \omega t$$

$$v_{03} = A_d v_d + A_{cm} v_{cm} \\ = (-3.73)(0.3) + (0.0718)(2) \Rightarrow v_{03} = -0.975 \sin \omega t (\text{V})$$

Ideal, $A_{cm} = 0$

$$v_{03} = A_d v_d = (-3.73)(0.3)$$

Or

$$\Rightarrow v_{03} = -1.12 \sin \omega t (\text{V})$$

11.94

$$(a) A_d = \frac{\beta R_C}{r_\pi + R_B}$$

$$\text{Assuming } I_{CQ} \cong I_{EQ}, \quad r_\pi = \frac{(150)(0.026)}{0.4} = 9.75 \text{ k}\Omega$$

$$A_d = \frac{(150)(10)}{9.75 + 0.5} = 146$$

(b) $C_M \equiv C_\mu (1 + g_m R_C)$

$$g_m = \frac{0.4}{0.026} = 15.38 \text{ mA/V}$$

$$C_M = (0.2)[1 + (15.38)(10)] = 30.96 \text{ pF}$$

(c) $f_H = \frac{1}{2\pi(r_\pi \| R_B)(C_\pi + C_M)} = \frac{1}{2\pi(9.75 \| 0.5) \times 10^3 (1.2 + 30.96) \times 10^{-12}}$

$$f_H = 10.4 \text{ MHz}$$

11.95

(a) $f_Z = \frac{1}{2\pi R_o C_o} = \frac{1}{2\pi(10 \times 10^6)(0.4 \times 10^{-12})} \Rightarrow f_Z = 39.8 \text{ kHz}$

(b) $r_\pi = \frac{(150)(0.026)}{0.4} = 9.75 \text{ k}\Omega$

$$R_{eq} = \frac{R_o \left(1 + \frac{R_B}{r_\pi} \right)}{1 + \frac{R_B}{r_\pi} + \frac{2(1 + \beta)R_o}{r_\pi}} = \frac{\left(10 \right) \left(1 + \frac{0.5}{9.75} \right)}{1 + \frac{0.5}{9.75} + \frac{2(151)(10,000)}{9.75}}$$

Or $R_{eq} = 33.94 \Omega$

$$f_P = \frac{1}{2\pi R_{eq} C_o} = \frac{1}{2\pi(33.94)(0.4 \times 10^{-12})}$$

$$f_P = 11.7 \text{ GHz}$$

11.96

a. From Equation (7.73), $f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$

$$g_m = \frac{1}{0.026} = 38.46 \text{ mA/V}$$

$$\text{Then } 800 \times 10^6 = \frac{38.46 \times 10^{-3}}{2\pi(C_\pi + C_\mu)}$$

$$\text{Or } C_\pi + C_\mu = 7.65 \times 10^{-12} \text{ F} = 7.65 \text{ pF}$$

$$\text{And } C_\pi = 6.65 \text{ pF}$$

$$C_M = C_\mu (1 + g_m R_C) = 1 [1 + (38.46)(10)] = 386 \text{ pF}$$

$$f_H = \frac{1}{2\pi[r_\pi \| R_B](C_\pi + C_M)}$$

$$r_\pi = \frac{(120)(0.026)}{1} = 3.12 \text{ k}\Omega$$

$$f_H = \frac{1}{2\pi[3.12 \| 1] \times 10^3 \times (6.65 + 386) \times 10^{-12}}$$

$$\text{Or } f_H = 535 \text{ kHz}$$

$$f_z = \frac{1}{2\pi R_0 C_0} = \frac{1}{2\pi(10 \times 10^6)(10^{-12})}$$

b. From Equation (11.140),

$$\text{Or } f_z = 15.9 \text{ kHz}$$

11.97

$$(a) A_v = \frac{\frac{1}{2}\beta R_C}{r_\pi + (1 + \beta)R_E}$$

$$r_\pi = \frac{(120)(0.026)}{0.25} = 12.48 \text{ k}\Omega$$

$$A_v = \frac{\frac{1}{2}(120)(8)}{12.48 + (121)(0.1)} = 19.5$$

$$(b) A_v = \frac{\frac{1}{2}(120)(8)}{12.48 + (121)(0.25)} = 11.2$$

Chapter 12

12.1

$$(a) \quad A_f = \frac{A}{1+A\beta} \Rightarrow 100 = \frac{5 \times 10^4}{1 + (5 \times 10^4)\beta} \Rightarrow \beta = 9.98 \times 10^{-3}$$

$$(b) \quad 80 = \frac{A}{1+A(0.012)} \Rightarrow A = 2000$$

12.2

$$(a) \quad A_f = \frac{A}{1+A\beta} \Rightarrow -80 = \frac{-10^5}{1 + (-10^5)\beta} \Rightarrow \beta = -0.01249$$

$$(b) \quad A_f = \frac{-5 \times 10^4}{1 + (-5 \times 10^4)(-0.015)} = -66.58$$

12.3

$$(a) \quad A_f = \frac{A}{1+A\beta} \quad \beta = 0.15$$

$$T = A\beta$$

$$(i) \quad T = \infty$$

$$(ii) \quad A = 80 \text{ dB} \Rightarrow A = 10^4 \Rightarrow T = 1.5 \times 10^3$$

$$(iii) \quad T = 15$$

$$(i) \quad A_f = \frac{1}{\beta} = 6.667$$

$$(ii) \quad A_f = 6.662$$

$$(iii) \quad A_f = 6.25$$

(b)

$$(i) \quad T = \infty$$

$$(ii) \quad T = 2.5 \times 10^3$$

$$(iii) \quad T = 25$$

$$(i) \quad A_f = \frac{1}{\beta} = 4.00$$

$$(ii) \quad A_f = 3.9984$$

$$(iii) \quad A_f = 3.846$$

12.4

(a)

$$A_f = \frac{A}{1+A\beta} \approx \frac{1}{\beta} = 125$$

$$\beta = 0.0080$$

(b)

$$A_f = (125)(0.9975) = 124.6875$$

$$124.6875 = \frac{A}{1 + (0.008)A}$$

$$124.6875[1 + (0.008)A] = A$$

$$124.6875 = A[1 - 0.9975]$$

$$A = 49,875$$

12.5

$$(a) A_f = \frac{A}{1 + A\beta} \Rightarrow -80 = \frac{-2 \times 10^4}{1 + (-2 \times 10^4)\beta} \Rightarrow \beta = -0.01245$$

$$(b) \frac{dA_f}{A_f} = \frac{1}{(1 + A\beta)} \cdot \frac{dA}{A}$$

$$0.01 = \frac{1}{[1 + (-2 \times 10^4)(-0.01245)]} \cdot \frac{dA}{A} \Rightarrow \frac{dA}{A} = 2.5\%$$

12.6

$$\frac{dA_f}{A_f} = \frac{1}{1 + \beta A} \cdot \frac{dA}{A}$$

$$0.001 = \frac{1}{1 + \beta A} \cdot (0.10) \Rightarrow 1 + \beta A = 100$$

$$\beta = \frac{100 - 1}{5 \times 10^4} \Rightarrow \beta = 1.98 \times 10^{-3}$$

$$\text{Now } A_f = \frac{A}{1 + \beta A} = \frac{5 \times 10^4}{1 + (1.98 \times 10^{-3})(5 \times 10^4)} \Rightarrow A_f = 500$$

12.7

$$(a) A_{vf} = \left(\frac{A_1}{1 + A_1 \beta_1} \right) \left(\frac{A_2}{1 + A_2 \beta_1} \right)$$

$$50 = \left(\frac{200}{1 + 200 \beta_1} \right) \left(\frac{10}{1 + 10 \beta_1} \right)$$

$$(1 + 200 \beta_1)(1 + 10 \beta_1) = \frac{(200)(10)}{50} = 40$$

$$\text{Then } 2000 \beta_1^2 + 210 \beta_1 - 39 = 0 \Rightarrow \beta_1 = 0.096685$$

$$A_{vf} = \frac{A_1 A_2}{1 + A_1 A_2 \beta_2}$$

$$50 = \frac{2000}{1 + 2000 \beta_2} \Rightarrow \beta_2 = 0.0195$$

(b) From circuit (a):

$$A_{vf} = \left(\frac{200}{1 + (200)(0.096685)} \right) \left(\frac{9}{1 + (9)(0.096685)} \right) = 47.33 \Rightarrow (-5.43\%)$$

For circuit (b):

$$A_{vf} = \frac{1800}{1 + (1800)(0.0195)} = 49.86 \Rightarrow (-0.28\%)$$

12.8

$$V_o = (-10)(-15)(-20)V_e = -3000V_e$$

$$(a) V_e = \beta V_o + V_s$$

$$\text{So } V_o = -3000(\beta V_o + V_s)$$

We find

$$A_{vf} = \frac{V_o}{V_s} = \frac{-3000}{1 + 3000\beta}$$

$$A_{vf} = -120 = \frac{-3000}{1 + 3000\beta} \Rightarrow \beta = 0.008$$

For

$$(b) \text{ Now } V_o = (-9)(-13.5)(-18)V_e = -2187V_e$$

Then

$$A_{vf} = \frac{-2187}{1 + 2187\beta} = \frac{-2187}{1 + 2187(0.008)} = -118.24$$

$$\% \text{ change} = \frac{120 - 118.24}{120} \times 100 \Rightarrow \underline{1.47\% \text{ change}}$$

12.9

$$(a) A_v f_H = A_{vf} f_C$$

$$(5 \times 10^4)(10) = (25)f_C \Rightarrow f_C = 20 \text{ kHz}$$

$$(b) A_{vf} = \frac{A_v f_H}{f_C} = \frac{(10^5)(8)}{20 \times 10^3} = 40$$

12.10

$$(a) f_C = \frac{(5 \times 10^4)(10)}{50} \Rightarrow f_C = 10 \text{ kHz}$$

$$(b) f_C = \frac{(10^4)(10)}{50} \Rightarrow f_C = 2 \text{ kHz}$$

12.11

$$(a) (i) A_v = \frac{A_{vf} f_C}{f_H} = \frac{(75)(35 \times 10^3)}{5} = 5.25 \times 10^5$$

$$(ii) 75 = \frac{5.25 \times 10^5}{1 + (5.25 \times 10^5)\beta} \Rightarrow \beta = 0.01333$$

$$(b) \quad A = (5.25 \times 10^5)(0.9) = 4.725 \times 10^5$$

$$A_{vf} = \frac{4.725 \times 10^5}{1 + (4.725 \times 10^5)(0.01333)} = 74.99$$

$$f_C = \frac{A_v f_H}{A_{vf}} = \frac{(4.725 \times 10^5)(5)}{74.99} \Rightarrow f_C = 31.5 \text{ kHz}$$

12.12

$$A_f = \frac{A_0}{1 + A_0 \beta}$$

Low freq.

$$100 = \frac{5000}{1 + (5000)\beta} \Rightarrow \beta = 0.0098$$

Freq. response

$$\begin{aligned} A_f &= \frac{A}{1 + A\beta} = \frac{\frac{5000}{(1 + j\frac{f}{f_1})(1 + j\frac{f}{f_2})}}{1 + \frac{(5000)(0.0098)}{(1 + j\frac{f}{f_1})(1 + j\frac{f}{f_2})}} \\ &= \frac{5000}{(1 + j\frac{f}{f_1})(1 + j\frac{f}{f_2}) + 49} \\ &= \frac{5000}{1 + j\frac{f}{f_1} + j\frac{f}{f_2} + \left(\frac{jf}{f_1}\right)\left(\frac{jf}{f_2}\right) + 49} \\ &= \frac{5000}{50 + j\frac{f}{f_1} + j\frac{f}{f_2} + \left(\frac{jf}{f_1}\right)\left(\frac{jf}{f_2}\right)} \end{aligned}$$

Also

$$A_f = \frac{A_{f0}}{(1 + j\frac{f}{f_A})(1 + j\frac{f}{f_B})} = \frac{100}{1 + j\frac{f}{f_A} + j\frac{f}{f_B} + \left(j\frac{f}{f_A}\right)\left(j\frac{f}{f_B}\right)}$$

So

$$\frac{100}{1 + j\frac{f}{f_A} + j\frac{f}{f_B} + \left(j\frac{f}{f_A}\right)\left(j\frac{f}{f_B}\right)} = \frac{100}{1 + j\frac{f}{50f_1} + j\frac{f}{50f_2} + \frac{1}{50}\left(\frac{jf}{f_1}\right)\left(\frac{jf}{f_2}\right)}$$

Then

$$\frac{1}{f_A} + \frac{1}{f_B} = \frac{1}{50f_1} + \frac{1}{50f_2}$$

$$\text{and } \frac{1}{f_A f_B} = \frac{1}{50f_1 f_2}$$

$$f_1 = 10 \text{ and } f_2 = 2000$$

$$\frac{1}{f_A} + \frac{1}{f_B} = \frac{1}{50(10)} + \frac{1}{50(2000)} = 0.002 + 0.000010 = 0.002010$$

and

$$\frac{1}{f_A f_B} = \frac{1}{(50)(10)(2000)} \Rightarrow \frac{1}{f_A} = \frac{f_B}{10^6}$$

$$\frac{f_B}{10^6} + \frac{1}{f_B} = 0.002010$$

Then

$$10^{-6} f_B^2 + 1 = 2.01 + 10^{-3} f_B$$

$$10^{-6} f_B^2 - 2.01 \times 10^{-3} f_B + 1 = 0$$

$$f_B = \frac{2.01 \times 10^{-3} \pm \sqrt{4.0401 \times 10^{-6} - 4(10^{-6})(1)}}{2(10^{-6})}$$

$$f_B = \frac{2.01 \times 10^{-3} \pm 2.0025 \times 10^{-4}}{2(10^{-6})}$$

+ sign $f_B = 1.105 \times 10^3 \text{ Hz}$
+ sign $f_A = 9.05 \times 10^2 \text{ Hz}$

12.13

(a) Fig. P12.7(a)

$$A_f = \left[\frac{\left(\frac{200}{1+j\frac{f}{f_1}} \right)}{1+\left(\frac{200}{1+j\frac{f}{f_1}} \right)(0.1126)} \right] \left[\frac{10}{1+(10)(0.1126)} \right]$$

$$= \left[\frac{200}{\left(1+j\frac{f}{f_1} \right) + 22.52} \right] (4.704)$$

$$= \frac{940.73}{23.52 + j\frac{f}{f_1}} = \frac{940.73}{23.52} \cdot \frac{1}{1+j\frac{f}{(23.52)f_1}}$$

$$= \frac{40}{1+\frac{jf}{(23.52)f_1}} \quad f_{-3dB} = (23.52)(100) \Rightarrow 2.352 \text{ kHz}$$

Fig P12.7(b)

$$A_f = \frac{\frac{(200)(10)}{1+j\frac{f}{f_1}}}{1+\frac{(0.0245)(200)(10)}{1+j\frac{f}{f_1}}} = \frac{2000}{1+j\frac{f}{f_1} + 49}$$

$$= \frac{2000}{50} \cdot \frac{1}{1+j\frac{f}{(50)f_1}} \quad f_{-3dB} = (50)(100) \Rightarrow 5 \text{ KHz}$$

(b) Overall feedback \Rightarrow wider bandwidth.

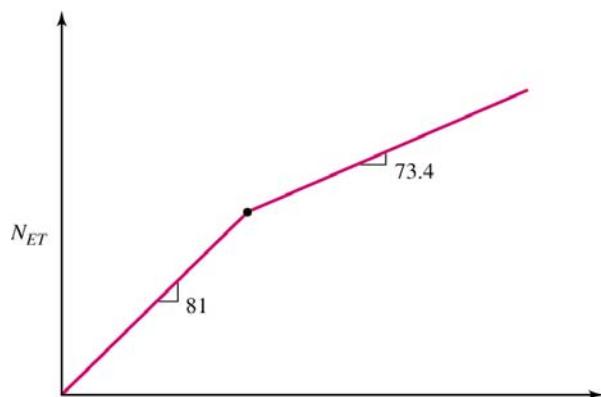
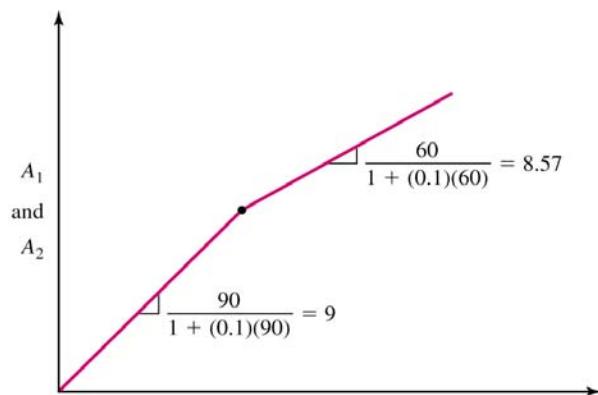
12.14

$$v_0 = A_1 A_2 v_i + A_1 v_n$$

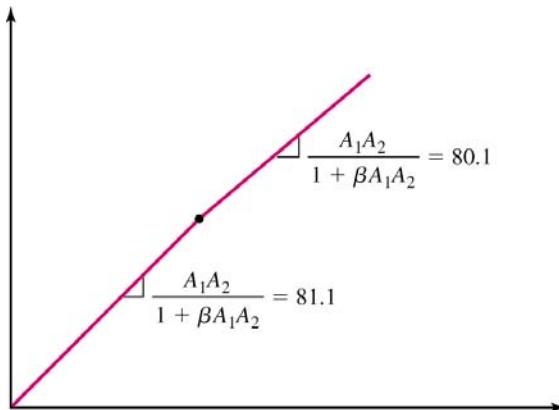
$$v_0 = (100)v_i + (1)v_n = (100)(10) + (1)(1) \Rightarrow \underline{\underline{\frac{S_0}{N_0} = \frac{1000}{1} = 1000}}$$

12.15

(a)



(b)



Circuit (b) – less distortion

12.16

$$A_{vf} = \frac{A_v}{1 + \beta_v A_v} = \frac{5 \times 10^3}{1 + (0.0080)(5 \times 10^3)} \Rightarrow A_{vf} = 121.95$$

$$R_{if} = R_i (1 + \beta_v A_v) = (10) [1 + (0.0080)(5 \times 10^3)] \Rightarrow R_{if} = 410 \text{ k}\Omega$$

$$R_{of} = \frac{R_o}{1 + \beta_v A_v} = \frac{1 \times 10^3}{1 + (0.0080)(5 \times 10^3)} \Rightarrow R_{of} = 24.4 \Omega$$

12.17

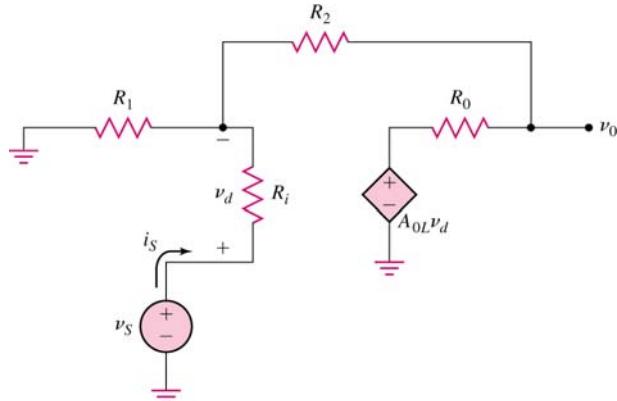
$$V_{fb} = \beta_v V_o = (0.0096)(2.5) \Rightarrow V_{fb} = 24 \text{ mV}$$

$$V_e = V_i - V_{fb} = 25 - 24 = 1 \text{ mV}$$

$$V_o = A_v V_e \Rightarrow A_v = \frac{V_o}{V_e} = \frac{2.5}{0.001} = 2.5 \times 10^3 \text{ V/V}$$

$$A_{vf} = \frac{V_o}{V_i} = \frac{2.5}{0.025} = 100 \text{ V/V}$$

12.18



$$A_{vf} \approx \left(1 + \frac{R_2}{R_1}\right) = 20 \Rightarrow \frac{R_2}{R_1} = 19$$

$$v_d = i_s R_i$$

$$i_s = \frac{v_s - v_d}{R_1} + \frac{(v_s - v_d) - v_0}{R_2} \quad (1)$$

$$\frac{v_0 - A_{0L} v_d}{R_0} + \frac{v_0 - (v_s - v_d)}{R_2} = 0 \quad (2)$$

$$v_0 \left(\frac{1}{R_0} + \frac{1}{R_2} \right) = \frac{A_{0L} v_d}{R_0} + \frac{(v_s - v_d)}{R_2}$$

$$v_0 = \frac{\frac{A_{0L} v_d}{R_0} + \frac{(v_s - v_d)}{R_2}}{\left(\frac{1}{R_0} + \frac{1}{R_2} \right)}$$

From (1):

$$\begin{aligned}
 i_s &= \frac{v_s - v_d}{R_1} + \frac{v_s - v_d}{R_2} - \frac{1}{R_2} \cdot \frac{\left[\frac{A_{0L}v_d}{R_0} + \frac{(v_s - v_d)}{R_2} \right]}{\left(\frac{1}{R_0} + \frac{1}{R_2} \right)} \\
 i_s &= v_s \left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{\frac{1}{R_2}}{1 + \frac{R_2}{R_0}} \right) - v_d \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{\frac{A_{0L}}{R_0} - \frac{1}{R_2}}{1 + \frac{R_2}{R_0}} \right) \\
 v_d &= i_s R_i \\
 i_s &= v_s \left\{ 1 + \frac{R_i \left[\left(\frac{1}{R_1} + \frac{1}{R_2} \right) \left(1 + \frac{R_2}{R_0} \right) + \frac{A_{0L}}{R_0} - \frac{1}{R_2} \right]}{1 + \frac{R_2}{R_0}} \right\} = v_s \left[\frac{\left(\frac{1}{R_1} + \frac{1}{R_2} \right) \left(1 + \frac{R_2}{R_0} \right) - \frac{1}{R_2}}{1 + \frac{R_2}{R_0}} \right] \\
 i_s &= v_s \left\{ 1 + \frac{R_2}{R_0} + R_i \left[\frac{1}{R_1} + \frac{R_2}{R_1} \cdot \frac{1}{R_0} + \frac{1}{R_0} + \frac{A_{0L}}{R_0} \right] \right\} = v_s \left[\frac{1}{R_1} + \frac{R_2}{R_1} \cdot \frac{1}{R_0} + \frac{1}{R_0} \right] \\
 i_s &= R_0 + R_2 + R_i \left[\frac{R_0}{R_1} + \left(1 + \frac{R_2}{R_1} \right) + A_{0L} \right] = v_s \left[\frac{R_0}{R_1} + \left(1 + \frac{R_2}{R_1} \right) \right] \quad (1)
 \end{aligned}$$

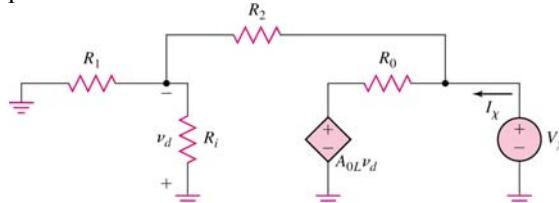
Let $R_2 = 190 \text{ k}\Omega$, $R_i = 10 \text{ k}\Omega$

$$i_s \left\{ 0.1 + 190 + 100 \cdot \left[\frac{0.1}{10} + 20 + 10^5 \right] \right\} = v_s \left[\frac{0.1}{10} + 20 \right]$$

$$i_s (1.000219 \times 10^7) = v_s (20.01)$$

$$R_{if} = \frac{v_s}{i_s} \cong 5 \times 10^5 \text{ k}\Omega \Rightarrow R_{if} \cong 500 \text{ M}\Omega$$

Output Resistance



$$I_x = \frac{V_x - A_{0L}v_d}{R_0} + \frac{V_x}{R_2 + R_1 \parallel R_i}$$

$$v_d = \frac{-R_1 \parallel R_i}{R_1 \parallel R_i + R_2} \cdot V_x$$

$$\frac{I_x}{V_x} = \frac{1}{R_{0f}} = \frac{1}{R_0} + \frac{A_{0L} \cdot R_1 \parallel R_i}{R_0(R_1 \parallel R_i + R_2)} + \frac{1}{R_2 + R_1 \parallel R_i}$$

$$R_1 \parallel R_i = 10 \parallel 100 = 9.09$$

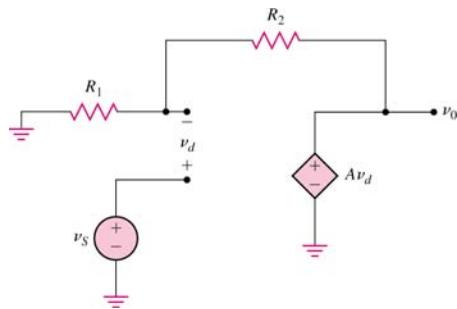
$$\frac{1}{R_{0f}} = \frac{1}{0.1} + \frac{10^5}{0.1} \cdot \left(\frac{9.09}{9.09 + 190} \right) + \frac{1}{190 + 9.09}$$

$$= 10 + 4.566 \times 10^4 + 0.00502$$

$$R_{0f} = 2.19 \times 10^{-5} \text{ k}\Omega \Rightarrow R_{0f} = 0.0219 \Omega$$

12.19

a.



$$\frac{v_s - v_d}{R_1} = \frac{v_0 - (v_s - v_d)}{R_2} \quad \text{and} \quad v_d = \frac{v_0}{A}$$

$$\frac{v_s}{R_1} + \frac{v_s}{R_2} = \frac{v_0}{R_2} + v_d \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$= \frac{v_0}{R_2} + \frac{v_0}{A} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$v_s \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{v_0}{R_2} \left[1 + \frac{1}{A} \left(1 + \frac{R_2}{R_1} \right) \right]$$

$$\frac{v_0}{v_s} = \frac{\left(1 + \frac{R_2}{R_1} \right)}{1 + \frac{1}{A} \left(1 + \frac{R_2}{R_1} \right)}$$

which can be written as

$$A_{vf} = \frac{v_0}{v_s} = \frac{A}{1 + \left[A / \left(1 + \frac{R_2}{R_1} \right) \right]}$$

$$\beta = \frac{1}{1 + \frac{R_2}{R_1}}$$

b.

$$20 = \frac{10^5}{1 + (10^5)\beta}$$

$$\text{So } \beta = \frac{\frac{10^5}{20} - 1}{10^5} \Rightarrow \underline{\beta = 0.04999}$$

$$\frac{R_2}{R_1} = \frac{1}{\beta} - 1 = \frac{1}{0.04999} - 1 \Rightarrow \underline{\frac{R_2}{R_1} = 19.004}$$

Then $\underline{\underline{A_f = 9 \times 10^4}}$

d. $A \rightarrow 9 \times 10^4$

$$A_f = \frac{9 \times 10^4}{1 + (9 \times 10^4)(0.04999)} = 19.99956$$

$$\frac{\Delta A_f}{A_f} = \frac{-4.444 \times 10^{-4}}{20} = -2.222 \times 10^{-3}\% \Rightarrow \underline{\underline{\frac{\Delta A_f}{A_f} = -0.005\%}}$$

12.20

$$I_e = I_i - I_{fb} = 20 - 19 = 1 \mu A$$

$$I_{fb} = \beta_i I_o \Rightarrow I_o = \frac{19}{0.0095} \Rightarrow I_o = 2 \text{ mA}$$

$$A_i = \frac{I_o}{I_e} = \frac{2 \times 10^{-3}}{1 \times 10^{-6}} = 2 \times 10^3 \text{ A/A}$$

$$A_{if} = \frac{I_o}{I_i} = \frac{2}{0.020} = 100 \text{ A/A}$$

$$R_{if} = \frac{R_i}{1 + \beta_i A_i} = \frac{500}{1 + (0.0095)(2 \times 10^3)} = 25 \Omega$$

$$R_{of} = (1 + \beta_i A_i) R_o = [1 + (0.0095)(2 \times 10^3)][20 \times 10^3] \Rightarrow R_{of} = 400 \text{ k}\Omega$$

12.21

$$I_{fb} = I_i - I_e = 25 - 0.8 = 24.2 \mu A$$

$$I_o = A_{if} I_i = (125)(25) \Rightarrow I_o = 3.125 \text{ mA}$$

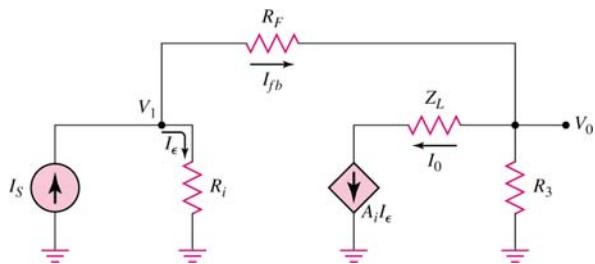
$$\beta_i = \frac{I_{fb}}{I_o} = \frac{24.2}{3125} = 0.007744 \text{ A/A}$$

$$A_{if} = \frac{A_i}{1 + \beta_i A_i}$$

$$125 = \frac{A_i}{1 + (0.007744)A_i} \Rightarrow A_i = 3906 \text{ A/A}$$

12.22

a.



Assume that V_1 is at virtual ground.

$$V_0 = -I_{fb} R_F$$

Now

$$I_{fb} = I_0 + \frac{V_0}{R_3} = I_0 - \frac{I_{fb} R_F}{R_3}$$

$$I_{fb} = I_s - I_e$$

and

$$I_0 = A_i I_e = \frac{I_0}{A_i}$$

so

$$I_{fb} = I_s - \frac{I_0}{A_i}$$

From above

$$\begin{aligned} I_{fb} \left(1 + \frac{R_F}{R_3} \right) &= I_0 \\ \left(I_s - \frac{I_0}{A_i} \right) \left(1 + \frac{R_F}{R_3} \right) &= I_0 \\ I_s \left(1 + \frac{R_F}{R_3} \right) &= I_0 \left[1 + \frac{1}{A_i} \left(1 + \frac{R_F}{R_3} \right) \right] \end{aligned}$$

or

$$\begin{aligned} A_{if} = \frac{I_0}{I_s} &= \frac{\left(1 + \frac{R_F}{R_3} \right)}{\left[1 + \frac{1}{A_i} \left(1 + \frac{R_F}{R_3} \right) \right]} \\ &= \frac{A_i}{1 + \frac{A_i}{\left(1 + \frac{R_F}{R_3} \right)}} = A_{if} \end{aligned}$$

b. $\beta_i = \frac{1}{\left(1 + \frac{R_F}{R_3} \right)}$

c. $25 = \frac{10^5}{1 + (10^5)\beta_i}$

so $\beta_i = \frac{10^5 - 1}{25 \cdot 10^5} \Rightarrow \beta_i = 0.03999$

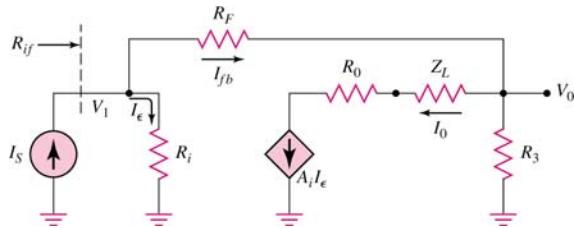
so $\frac{R_F}{R_3} = \frac{1}{\beta_i} - 1 = \frac{1}{0.03999} - 1 \Rightarrow \frac{R_F}{R_3} = 24.0$

d. $A_i = 10^5 - (0.15)(10^5) = 8.5 \times 10^4$

so $A_{if} = \frac{8.5 \times 10^4}{1 + (8.5 \times 10^4)(0.03999)} = 24.9989$

so $\frac{\Delta A_{if}}{A_{if}} = -\frac{1.10 \times 10^{-3}}{25} = -4.41 \times 10^{-5} \Rightarrow -4.41 \times 10^{-3}\%$

12.23



$$I_S = I_e + I_{fb}, V_1 = I_e R_i$$

$$I_{fb} = I_0 + \frac{V_0}{R_3} \quad \text{and} \quad V_0 = V_1 - I_{fb} R_F$$

$$I_0 = A_i I_e \Rightarrow I_e = \frac{I_0}{A_i}$$

Now

$$I_{fb} = A_i I_e + \frac{1}{R_3} (V_1 - I_{fb} R_F)$$

$$I_{fb} \left[1 + \frac{R_F}{R_3} \right] = A_i I_e + \frac{V_1}{R_3}$$

$$I_{fb} = I_S - I_e$$

$$(I_S - I_e) \left[1 + \frac{R_F}{R_3} \right] = A_i I_e + \frac{V_1}{R_3}$$

$$I_S \left[1 + \frac{R_F}{R_3} \right] = I_e \left[\left(1 + \frac{R_F}{R_3} \right) + A_i \right] + \frac{V_1}{R_3}$$

$$I_e = \frac{V_1}{R_i}$$

$$I_S \left[1 + \frac{R_F}{R_3} \right] = V_1 \left\{ \frac{1}{R_i} \cdot \left[\left(1 + \frac{R_F}{R_3} \right) + A_i \right] + \frac{1}{R_3} \right\}$$

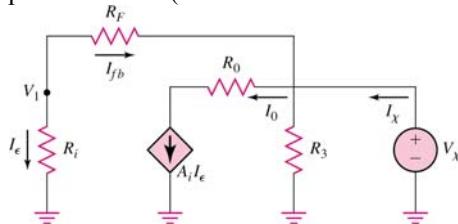
$$R_{if} = \frac{V_1}{I_S} = \frac{\left(1 + \frac{R_F}{R_3} \right)}{\left\{ \frac{1}{R_i} \cdot \left[\left(1 + \frac{R_F}{R_3} \right) + A_i \right] + \frac{1}{R_3} \right\}}$$

The $1/R_3$ term in the denominator will be negligible. Then

$$R_{if} = \frac{25}{\left\{ \frac{1}{2} [(25) + 10^5] \right\}}$$

$$R_{if} \cong 5 \times 10^{-4} \text{ k}\Omega \Rightarrow R_{if} = 0.5 \Omega$$

Output Resistance (Let $Z_L = 0$)



$$I_x = \frac{V_x}{R_3} + A_i I_e + \frac{V_x}{R_F + R_i}$$

$$I_e = \frac{V_x}{R_F + R_i}$$

so

$$\frac{I_x}{V_x} = \frac{1}{R_{0f}} = \frac{1}{R_3} + \frac{A_i + 1}{R_F + R_i}, \frac{R_F}{R_3} = 24$$

Let $R_F = 240 \text{ k}\Omega$, $R_3 = 10 \text{ k}\Omega$

$$\frac{1}{R_{of}} = \frac{1}{10} + \frac{10^5 + 1}{240 + 2}$$

$$R_{of} \approx \frac{R_F + R_i}{A_i + 1} = \frac{240 + 2}{10^5 + 1} \Rightarrow R_{of} \approx 2.42 \times 10^{-3} \text{ k}\Omega \text{ or } \underline{R_{of} \approx 2.42 \Omega}$$

so

12.24

$$V_\epsilon = V_i - V_{fb} = 0.2 - 0.195 \Rightarrow V_\epsilon = 5 \mu\text{V}$$

$$\beta_z = \frac{V_{fb}}{I_o} = \frac{0.195 \times 10^{-3}}{5 \times 10^{-6}} = 0.039 \text{ V/A}$$

$$A_g = \frac{I_o}{V_\epsilon} = \frac{5 \times 10^{-3}}{5 \times 10^{-6}} = 1000 \text{ A/V}$$

$$A_{gf} = \frac{A_g}{1 + A_g \beta_z} = \frac{1000}{1 + (1000)(0.039)} = 25 \text{ A/V}$$

$$R_{if} = R_i (1 + \beta_z A_g) = (20 \times 10^3) [1 + (0.039)(1000)] \Rightarrow R_{if} = 800 \text{ k}\Omega$$

$$R_{of} = R_o (1 + \beta_z A_g) = (10 \times 10^3) [1 + (0.039)(1000)] \Rightarrow R_{of} = 400 \text{ k}\Omega$$

12.25

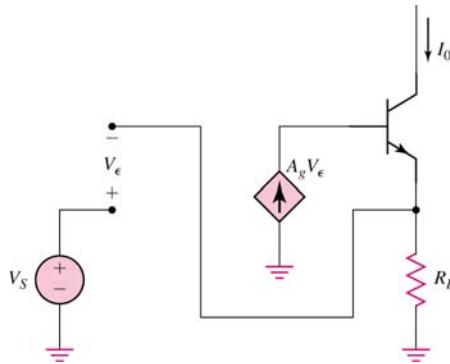
$$A_{gf} = \frac{A_g}{1 + \beta_z A_g} = \frac{2000}{1 + (2000)(0.0245)} = 40 \text{ A/V}$$

$$I_o = A_{gf} V_i = (40)(150 \times 10^{-6}) \Rightarrow I_o = 6 \text{ mA}$$

$$V_{fb} = \beta_z I_o = (0.0245)(6 \times 10^{-3}) \Rightarrow V_{fb} = 147 \mu\text{V}$$

$$V_\epsilon = V_i - V_{fb} = 150 - 147 \Rightarrow V_\epsilon = 3 \mu\text{V}$$

12.26



$$I_E = \frac{(1 + h_{FE})}{h_{FE}} \cdot I_0 = \frac{V_S - V_\epsilon}{R_E}$$

$$\text{Also } I_0 = h_{FE}(A_g V_\epsilon) \text{ so } V_\epsilon = \frac{I_0}{h_{FE} A_g}$$

Then

$$\frac{1+h_{FE}}{h_{FE}} \cdot I_0 = \frac{V_S}{R_E} - \frac{I_0}{h_{FE}A_g R_E}$$

$$\left[\frac{1+h_{FE}}{h_{FE}} + \frac{1}{h_{FE}A_g R_E} \right] I_0 = \frac{V_S}{R_E}$$

$$\left[\frac{A_g(1+h_{FE})R_E + 1}{h_{FE}A_g R_E} \right] I_0 = \frac{V_S}{R_E}$$

$$\frac{I_0}{V_S} = \frac{1}{R_E} \cdot \left[\frac{h_{FE}A_g R_E}{1+A_g(1+h_{FE})R_E} \right] \Rightarrow \frac{I_0}{V_S} \approx \frac{h_{FE}A_g}{1+(h_{FE}A_g)R_E}$$

b. $\beta_z = R_E$

c. $10 = \frac{5 \times 10^5}{1 + (5 \times 10^5)\beta_z}$

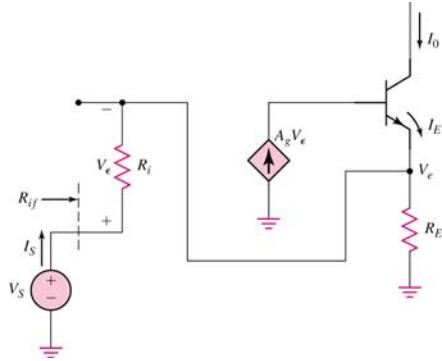
$$\beta_z = \frac{\frac{5 \times 10^5}{10} - 1}{5 \times 10^5} \Rightarrow \beta_z = R_E = 0.099998 \text{ k}\Omega$$

d. If $A_g \rightarrow 5.5 \times 10^5$ then

$$A_{gf} = \frac{5.5 \times 10^5}{1 + (5.5 \times 10^5)(0.099998)} = 10.0000182$$

$$\frac{\Delta A_{gf}}{A_{gf}} = \frac{1.82 \times 10^{-5}}{10} \Rightarrow 1.82 \times 10^{-4}\%$$

12.27



$$I_E = (1+h_{FE})A_g V_e, \quad I_E = \frac{V_e}{R_E} - I_S \quad \text{and} \quad V_e = I_S R_i, \quad V_e = V_S - V_\epsilon = V_S - I_S R_i$$

$$(1+h_{FE})A_g I_S R_i = \frac{1}{R_E} \cdot (V_S - I_S R_i) - I_S$$

Now

$$\left[(1+h_{FE})A_g R_i + \frac{R_i}{R_E} + 1 \right] I_S = \frac{V_S}{R_E}$$

$$R_{if} = \frac{V_S}{I_S} = R_E \left[(1+h_{FE})A_g R_i + \frac{R_i}{R_E} + 1 \right]$$

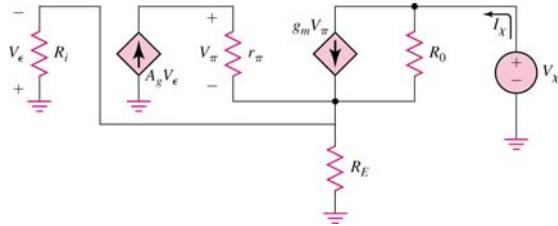
We have

$$(1+h_{FE})A_g \approx h_{FE}A_g = 5 \times 10^5 \text{ mS}$$

$$R_E \approx 0.1 \text{ k}\Omega$$

$$\text{so } R_{if} = (0.1) \left[(5 \times 10^5)(20) + \frac{20}{0.1} + 1 \right]$$

or $R_{if} = 10^6 \text{ k}\Omega$



$$\frac{V_\pi}{r_\pi} = A_g V_\epsilon$$

$$I_x = g_m V_\pi + \frac{V_x - (-V_\epsilon)}{R_0} \quad (1)$$

$$V_\epsilon = -(I_x + A_g V_\epsilon)(R_E \parallel R_i) \quad (2)$$

or $V_\epsilon [1 + A_g (R_E \parallel R_i)] = -I_x (R_E \parallel R_i)$

Now:

$$I_x = g_m A_g r_\pi V_\epsilon + \frac{V_x}{R_0} + \frac{V_\epsilon}{R_0} \quad (1)$$

$$I_x = \left(g_m A_g r_\pi + \frac{1}{R_0} \right) \left[\frac{-I_x (R_E \parallel R_i)}{1 + A_g (R_E \parallel R_i)} \right] + \frac{V_x}{R_0}$$

$$R_{of} = \frac{V_x}{I_x}$$

$$= R_0 \left\{ 1 + \left(g_m A_g r_\pi + \frac{1}{R_0} \right) \left[\frac{(R_E \parallel R_i)}{1 + A_g (R_E \parallel R_i)} \right] \right\}$$

$$g_m r_\pi A_g = h_{FE} A_g = 5 \times 10^5 \text{ mS}$$

$$\text{Let } h_{FE} = 100 \text{ so } A_g = 5 \times 10^3 \text{ mS}$$

$$R_E \parallel R_i = 0.1 \parallel 20 \approx 0.1 \text{ k}\Omega$$

Then

$$R_{of} = 50 \left\{ 1 + \left(5 \times 10^5 + \frac{1}{50} \right) \left[\frac{0.1}{1 + (5 \times 10^3)(0.1)} \right] \right\}$$

or $R_{of} = 5.04 \text{ M}\Omega$

12.28

$$A_{zf} = \frac{A_z}{1 + \beta_g A_z}$$

$$0.20 \times 10^6 = \frac{A_z}{1 + (4.25 \times 10^{-6}) A_z} \Rightarrow A_z = 1.333 \text{ V/}\mu\text{A}$$

$$R_{if} = \frac{R_i}{1 + \beta_g A_z} = \frac{500}{1 + (4.25 \times 10^{-6})(1.333 \times 10^6)} \Rightarrow R_{if} = 75 \Omega$$

$$R_{of} = \frac{R_o}{1 + \beta_g A_z} = 75 \Omega$$

12.29

$$I_e = I_i - I_{fb} = 40 - 38 = 2 \mu\text{A}$$

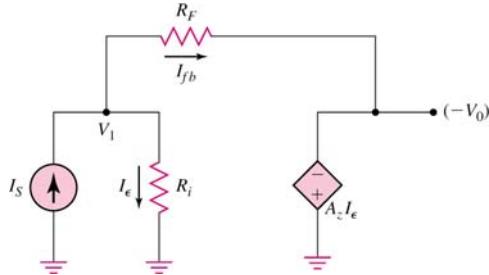
$$A_z = \frac{V_o}{I_e} = \frac{8}{2} = 4 \frac{\text{V}}{\mu\text{A}}$$

$$\beta_g = \frac{I_{fb}}{V_o} = \frac{38}{8} = 4.75 \frac{\mu\text{A}}{\text{V}}$$

$$A_{af} = \frac{V_o}{I_i} = \frac{8}{40} = 0.2 \frac{\text{V}}{\mu\text{A}}$$

12.30

a.



Assuming V_1 is at virtual ground

$$(-V_0) = -I_{fb}R_F \quad \text{and} \quad (-V_0) = -A_z I_e \Rightarrow I_e = \frac{V_0}{A_z}$$

$$I_{fb} = I_s - I_e$$

$$\text{So } V_0 = (I_s - I_e)R_F = I_sR_F - \left(\frac{V_0}{A_z}\right)R_F$$

$$V_0 \left[1 + \frac{R_F}{A_z}\right] = I_sR_F$$

$$A_{af} = \frac{V_0}{I_s} = \frac{R_F}{\left[1 + \frac{R_F}{A_z}\right]} = \frac{A_z R_F}{A_z + R_F}$$

so

$$A_{af} = \frac{A_z}{1 + A_z \left(\frac{1}{R_F}\right)} = \frac{A_z}{1 + A_z \beta_g}$$

or

$$\beta_g = \frac{1}{R_F}$$

$$\text{c. } 5 \times 10^4 = \frac{5 \times 10^6}{1 + (5 \times 10^6)\beta_g}$$

$$\beta_g = \frac{\frac{5 \times 10^6}{5 \times 10^4} - 1}{5 \times 10^6} \Rightarrow \underline{\beta_g = 1.98 \times 10^{-5}}$$

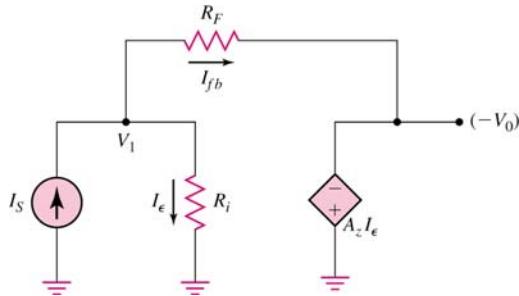
$$R_F = \frac{1}{\beta_g} \Rightarrow \underline{R_F = 50.5 \text{ k}\Omega}$$

d. $A_z = (0.9)(5 \times 10^6) = 4.5 \times 10^6$

$$A_{zf} = \frac{4.5 \times 10^6}{1 + (4.5 \times 10^6)(1.98 \times 10^{-5})} = 4.994 \times 10^4$$

$$\frac{\Delta A_{zf}}{A_{zf}} = -\frac{55.4939}{5 \times 10^4} = -1.11 \times 10^{-3} \Rightarrow -0.111\%$$

12.31



$$V_1 = I_\epsilon R_i, -V_0 = -A_z I_\epsilon \Rightarrow V_0 = A_z I_\epsilon$$

$$I_{fb} = I_s - I_\epsilon \text{ and } -V_0 = V_1 - I_{fb} R_F$$

$$-A_z I_\epsilon = V_1 - (I_s - I_\epsilon) R_F$$

$$-A_z \left(\frac{V_1}{R_i} \right) = V_1 - I_s R_F + \left(\frac{V_1}{R_i} \right) R_F$$

$$I_s R_F = V_1 \left[1 + \frac{A_z}{R_i} + \frac{R_F}{R_i} \right]$$

$$R_{if} = \frac{V_1}{I_s} = \frac{R_F}{\left[1 + \frac{A_z}{R_i} + \frac{R_F}{R_i} \right]}$$

We have

$$R_{if} = \frac{50.5 \times 10^3}{\left[1 + \frac{5 \times 10^6}{10 \times 10^3} + \frac{50.5 \times 10^3}{10 \times 10^3} \right]} \\ = \frac{50.5 \times 10^3}{[1 + 500 + 5.05]} \Rightarrow R_{if} = 99.79 \Omega$$

12.32

- (a) Low input $R \Rightarrow$ Shunt input
Low output $R \Rightarrow$ Shunt output
Or a Shunt-Shunt circuit
- (b) High input $R \Rightarrow$ Series input
High output $R \Rightarrow$ Series output
Or a series-Series circuit
- (c) Shunt-Series circuit
- (d) Series-Shunt circuit

12.33

(a) $R_i(\max) = R_i(1+T) = 10(1+10^4) \Rightarrow R_i(\max) \approx 10^5 \text{ k}\Omega$

$$R_i(\min) = \frac{R_i}{1+T} = \frac{10}{1+10^4} \approx 10^{-3} \text{ k}\Omega$$

Or $R_i(\min) = 1\Omega$

(b) $R_o(\max) = R_o(1+T) = 1(1+10^4) \Rightarrow R_o(\max) \approx 10^4 \text{ k}\Omega$

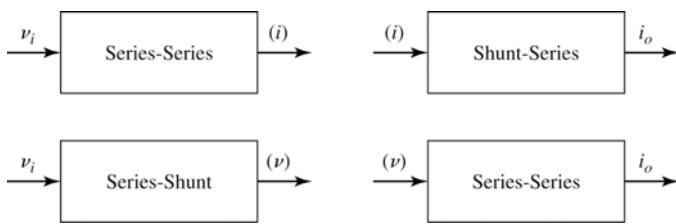
$$R_o(\min) = \frac{R_o}{1+T} = \frac{1}{1+10^4} \approx 10^{-4} \text{ k}\Omega$$

Or $R_o(\min) = 0.1\Omega$

12.34

$$A_g = \frac{i_o}{v_i}$$

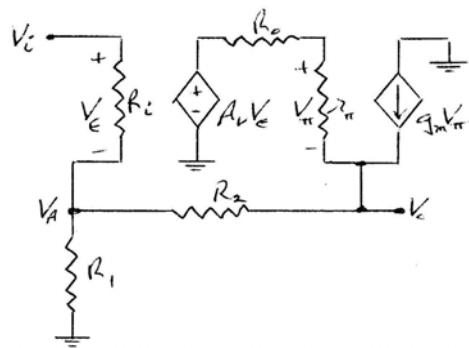
Overall Transconductance Amplifier, Series output = current signal and Shunt input = current signal. Also, Shunt output = voltage signal and Series input = voltage signal. Two possible solutions are shown.



12.35

$$I_{CQ} = 0.2 \text{ mA}, \quad r_\pi = \frac{(140)(0.026)}{0.2} = 18.2 \text{ k}\Omega, \quad g_m = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$$

(a)



$$(1) \quad \frac{V_\pi}{r_\pi} + g_m V_\pi = \frac{V_o - V_A}{R_2} = V_\pi \left(\frac{1 + h_{FE}}{r_\pi} \right)$$

$$(2) \quad \frac{V_\pi}{r_\pi} = \frac{A_v V_\epsilon - V_o}{R_o + r_\pi}$$

$$(3) \quad \frac{V_A - V_o}{R_2} + \frac{V_A}{R_1} + \frac{V_A - V_i}{R_i} = 0$$

$$(4) \quad V_\epsilon = V_i - V_A$$

From (1), $\frac{V_o - V_A}{R_2} = \left(\frac{1 + h_{FE}}{r_\pi} \right) \left[r_\pi \left(\frac{A_v V_e - V_o}{R_o + r_\pi} \right) \right] = (1 + h_{FE}) \left(\frac{A_v V_e - V_o}{R_o + r_\pi} \right)$

Using (4), $\frac{V_o - V_i + V_e}{R_2} = (1 + h_{FE}) \left(\frac{A_v V_e - V_o}{R_o + r_\pi} \right)$ (Eq. 5)

From (3), $V_A \left(\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_i} \right) = \frac{V_o}{R_2} + \frac{V_i}{R_i}$

$$(V_i - V_e) \left(\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_i} \right) = \frac{V_o}{R_2} + \frac{V_i}{R_i}$$

$$(V_i - V_e) \left(\frac{1}{10} + \frac{1}{1} + \frac{1}{30} \right) = \frac{V_o}{10} + \frac{V_i}{30}$$

$$(V_i - V_e)(1.1333) = V_o(0.10) + V_i(0.0333)$$

We find, $V_e = V_i(0.9706) - V_o(0.08824)$

From Eq. (5) above,

$$\frac{V_o - V_i}{R_2} + \frac{(1 + h_{FE})V_o}{R_o + r_\pi} = \frac{(1 + h_{FE})}{R_o + r_\pi} \cdot A_v V_e - \frac{V_e}{R_2}$$

$$\frac{V_o - V_i}{10} + \frac{141}{0.5 + 18.2} \cdot V_o = \frac{141}{0.5 + 18.2} (10^5) V_e - \frac{V_e}{10}$$

$$7.640 V_o - V_i(0.1) = 7.540 \times 10^5 V_e = 7.540 \times 10^5 [V_i(0.9706) - V_o(0.08824)]$$

$$6.653705 \times 10^4 V_o = V_i(7.318235 \times 10^5)$$

Then $\frac{V_o}{V_i} = 11.0$

(b) From (2), $V_\pi = r_\pi \left(\frac{A_v V_e - V_o}{R_o + r_\pi} \right)$

From (1), $V_o = V_A + R_2 V_\pi \left(\frac{1 + h_{FE}}{r_\pi} \right)$

$$V_o = (V_i - V_e) + R_2 (1 + h_{FE}) \left(\frac{A_v V_e - V_o}{R_o + r_\pi} \right)$$

$$V_o \left[1 + \frac{R_2 (1 + h_{FE})}{R_o + r_\pi} \right] = V_i + V_e \left[\frac{R_2 (1 + h_{FE}) A_v}{R_o + r_\pi} - 1 \right]$$
 (Eq. 6)

From (3),

$$V_A \left(\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_i} \right) = \frac{V_o}{R_2} + \frac{V_i}{R_i}$$

$$(V_i - V_e) \left(\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_i} \right) = \frac{V_o}{R_2} + \frac{V_i}{R_i}$$

Then, using Eq. (6),

$$V_i \left(\frac{1}{R_2} + \frac{1}{R_1} \right) - V_e \left(\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_i} \right) = \frac{1}{R_2} \left\{ \frac{V_i + V_e \left[\frac{R_2 (1 + h_{FE}) A_v}{R_o + r_\pi} - 1 \right]}{1 + \frac{R_2 (1 + h_{FE})}{R_o + r_\pi}} \right\}$$

$$V_i(1.10) - V_e(1.1333) = \frac{1}{10} \left\{ \frac{V_i + V_e (7.540106 \times 10^6)}{75.401} \right\}$$

$$V_i(1.098674) = V_e(1.1333 + 1.0 \times 10^4)$$

So $\frac{V_i}{V_e} = 9.103 \times 10^3$

Then $R_{if} = R_i \left(\frac{V_i}{V_e} \right) = (30 \times 10^3)(9.103 \times 10^3) \Rightarrow R_{if} = 273 \text{ M}\Omega$

(c) $I_x + g_m V_\pi + \frac{V_x}{r_\pi} = \frac{V_x - V_A}{R_2}$

From (2), $V_\pi = r_\pi \left(\frac{A_v V_e - V_x}{R_o + r_\pi} \right)$

Then, $I_x + \frac{(1+h_{FE})}{R_o + r_\pi} (A_v V_e - V_x) = \frac{V_x - V_A}{R_2}$

Now, $V_A = -V_e$ and $V_e = -V_x (0.08824)$

So, $I_x + \frac{141}{0.5+18.2} [10^5 (-0.08824 V_x) - V_x] = \frac{V_x}{10} - \frac{V_x (0.08824)}{10}$

$$I_x = V_x (0.091176 + 6.654 \times 10^4)$$

$$R_{of} = \frac{V_x}{I_x} = 15 \mu \Omega$$

12.36

a. Neglecting base currents

$$I_{C2} = 0.5 \text{ mA}, V_{C2} = 12 - (0.5)(22.6) = 0.7 \text{ V}$$

$$I_{C1} = 0.5 \text{ mA} \Rightarrow v_0 = 0$$

Then $I_{C3} = 2 \text{ mA}$

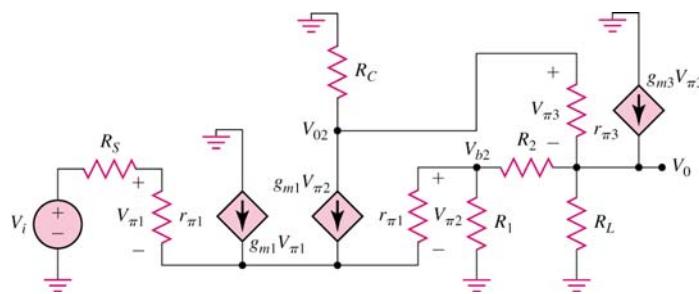
b.

$$r_{\pi 1} = r_{\pi 2} = \frac{h_{FE} \cdot V_T}{I_{C1}} = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$g_{m1} = g_{m2} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(100)(0.026)}{2} = 1.3 \text{ k}\Omega$$

$$g_{m3} = \frac{2}{0.026} = 76.92 \text{ mA/V}$$



$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} + g_{m1}V_{\pi 2} + \frac{V_{\pi 2}}{r_{\pi 1}} = 0$$

$$(V_{\pi 1} + V_{\pi 2})\left(\frac{1}{r_{\pi 1}} + g_{m1}\right) = 0 \Rightarrow V_{\pi 1} = -V_{\pi 2} \quad (1)$$

$$V_i = \frac{V_{\pi 1}}{r_{\pi 1}}(R_s + r_{\pi 1}) - V_{\pi 2} + V_{b2}$$

$$V_i = V_{\pi 1}\left(1 + \frac{R_s}{r_{\pi 1}}\right) - V_{\pi 2} + V_{b2}$$

or

$$\text{But } V_{\pi 2} = -V_{\pi 1}$$

so

$$V_i = V_{\pi 1}\left(2 + \frac{R_s}{r_{\pi 1}}\right) + V_{b2} \quad (2)$$

$$\frac{V_{02}}{R_C} + g_{m1}V_{\pi 2} + \frac{V_{02} - V_0}{r_{\pi 3}} = 0 \quad (3)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3}V_{\pi 3} = \frac{V_0}{R_L} + \frac{V_0 - V_{b2}}{R_2}$$

$$V_{\pi 3} = V_{02} - V_0$$

so

$$(V_{02} - V_0)\left(\frac{1 + h_{FE}}{r_{\pi 3}}\right) = V_0\left(\frac{1}{R_L} + \frac{1}{R_2}\right) - \frac{V_{b2}}{R_2} \quad (4)$$

$$\frac{V_{b2} - V_0}{R_2} + \frac{V_{b2}}{R_1} + \frac{V_{\pi 2}}{r_{\pi 1}} = 0 \quad (5)$$

Substitute numbers into (2), (3), (4) and (5):

$$V_i = -V_{\pi 2}\left(2 + \frac{1}{5.2}\right) + V_{b2}$$

$$V_i = -V_{\pi 2}(2.192) + V_{b2} \quad (2)$$

$$V_{02}\left(\frac{1}{22.6} + \frac{1}{1.3}\right) + (19.23)V_{\pi 2} - V_0\left(\frac{1}{1.3}\right) = 0$$

$$V_{02}(0.8135) + (19.23)V_{\pi 2} - (0.7692)V_0 = 0 \quad (3)$$

$$V_{02}\left(\frac{101}{1.3}\right) = V_0\left(\frac{101}{1.3} + \frac{1}{4} + \frac{1}{50}\right) - V_{b2}\left(\frac{1}{50}\right)$$

$$V_{02}(77.69) = V_0(77.96) - V_{b2}(0.02) \quad (4)$$

$$V_{b2}\left(\frac{1}{50} + \frac{1}{10}\right) - V_0\left(\frac{1}{50}\right) + V_{\pi 2}\left(\frac{1}{5.2}\right) = 0$$

$$V_{b2}(0.120) - V_0(0.020) + V_{\pi 2}(0.1923) = 0 \quad (5)$$

From (2): $V_{b2} = V_i + V_{\pi 2}(2.192)$. Substitute in (4) and (5) to obtain:

$$V_{02}(77.69) = V_0(77.96) - [V_i + V_{\pi 2}(2.192)](0.02) \quad (4')$$

$$[V_i + V_{\pi 2}(2.192)](0.120) - V_0(0.020) + V_{\pi 2}(0.1923) = 0 \quad (5')$$

So we now have the following three equations:

$$V_{02}(0.8135) + (19.23)V_{\pi 2} - (0.7692)V_0 = 0 \quad (3)$$

$$V_{02}(77.69)$$

$$= V_0(77.96) - V_i(0.02) - V_{\pi 2}(0.04384) \quad (4')$$

$$(0.120)V_i + V_{\pi 2}(0.4553) - V_0(0.020) = 0 \quad (5')$$

From (3): $V_{02} = V_0(0.9455) - V_{\pi 2}(23.64)$. Substitute for V_{02} in (4') to obtain:

$$(77.69)[V_o(0.9455) - V_{\pi 2}(23.64)] = V_0(77.96) - V_i(0.02) - V_{\pi 2}(0.04384)$$

or

$$0 = V_0(4.504) - V_i(0.02) + V_{\pi 2}(1836.5)$$

Next, solve (5') for $V_{\pi 2}$:

$$(0.120)V_i + V_{\pi 2}(0.4553) - V_0(0.020) = 0$$

$$V_{\pi 2} = V_0(0.04393) - V_i(0.2636)$$

Finally,

$$0 = V_0(4.504) - V_i(0.02) + (1836.5)[V_0(0.04393) - V_i(0.2636)]$$

$$0 = V_0(85.18) - V_i(484.12)$$

So

$$A_{vf} = \frac{V_0}{V_i} = \frac{484.12}{85.18} \Rightarrow A_{vf} = 5.68$$

12.37

a. $R_{TH} = R_1 \parallel R_2 = 400 \parallel 75 = 63.2 \text{ k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_{CC} = \left(\frac{75}{75 + 400} \right)(10) = 1.579 \text{ V}$$

$$I_{BQ1} = \frac{1.579 - 0.7}{63.2 + (121)(0.5)} = 0.007106 \text{ mA}$$

$$I_{CQ1} = 0.853 \text{ mA}$$

$$V_{C1} = 10 - (0.853)(8.8) = 2.49 \text{ V}$$

$$I_{C2} \approx \frac{2.49 - 0.7}{3.6} = 0.497 \text{ mA}$$

$$V_{C2} = 10 - (0.497)(13) = 3.54 \text{ V}$$

$$I_{C3} \approx \frac{3.54 - 0.7}{1.4} = 2.03 \text{ mA}$$

Then

$$r_{\pi 1} = \frac{(120)(0.026)}{0.853} = 3.66 \text{ k}\Omega$$

$$g_{m1} = \frac{0.853}{0.026} = 32.81 \text{ mA/V}$$

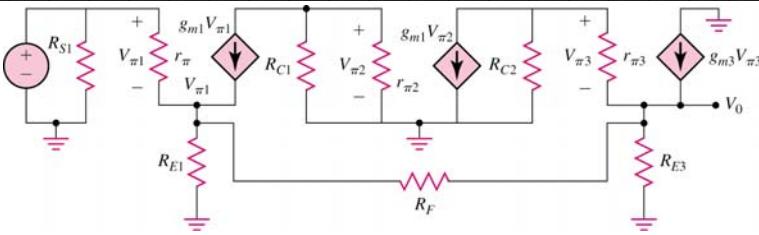
$$r_{\pi 2} = \frac{(120)(0.026)}{0.497} = 6.28 \text{ k}\Omega$$

$$g_{m2} = \frac{0.497}{0.026} = 19.12 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(120)(0.026)}{2.03} = 1.54 \text{ k}\Omega$$

$$g_{m3} = \frac{2.03}{0.026} = 78.08 \text{ mA/V}$$

b.



$$V_i = V_{\pi1} + V_{e1} \Rightarrow V_{e1} = V_i - V_{\pi1} \quad (1)$$

$$\frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} = \frac{V_{e1}}{R_{E1}} + \frac{V_{e1} - V_0}{R_F} \quad (2)$$

$$V_{\pi2} = -(g_{m1}V_{\pi1})(R_{C1} \parallel r_{\pi2}) \quad (3)$$

$$g_{m2}V_{\pi2} + \frac{V_{\pi3} + V_0}{R_{C2}} + \frac{V_{\pi3}}{r_{\pi3}} = 0 \quad (4)$$

$$\frac{V_{\pi3}}{r_{\pi3}} + g_{m3}V_{\pi3} = \frac{V_0}{R_{E3}} + \frac{V_0 - V_{e1}}{R_F} \quad (5)$$

Substitute numbers in (2), (3), (4) and (5):

$$V_{\pi1} \left(\frac{1}{3.66} + 32.81 \right) = (V_i - V_{\pi1}) \left(\frac{1}{0.5} + \frac{1}{10} \right) - \frac{V_0}{10}$$

$$\text{or } V_{\pi1}(35.18) = V_i(2.10) - V_0(0.10) \quad (2)$$

$$V_{\pi2} = -(32.81)V_{\pi1}(88 \parallel 6.28)$$

$$\text{or } V_{\pi2} = -V_{\pi1}(120.2) \quad (3)$$

$$(19.12)V_{\pi2} + \frac{V_{\pi3}}{13} + \frac{V_0}{13} + \frac{V_{\pi3}}{1.54} = 0$$

or

$$V_{\pi2}(19.12) + V_{\pi3}(0.7263) + V_0(0.07692) = 0 \quad (4)$$

$$V_{\pi3} \left(\frac{1}{1.54} + 78.08 \right) = V_0 \left(\frac{1}{1.4} + \frac{1}{10} \right) - \frac{V_i - V_{\pi1}}{10}$$

or

$$V_{\pi3}(78.73) = V_0(0.8143) - V_i(0.10) + V_{\pi1}(0.10) \quad (5)$$

Now substituting $V_{\pi2} = -V_{\pi1}(120.2)$ in (4):

$$(19.12)[-V_{\pi1}(120.2)] + V_{\pi3}(0.7263) + V_0(0.07692) = 0$$

or

$$-V_{\pi1}(2298.2) + V_{\pi3}(0.7263) + V_0(0.07692) = 0$$

Then

$$V_{\pi3} = V_{\pi1}(3164.3) - V_0(0.1059)$$

Substituting $V_{\pi3} = V_{\pi1}(3164.3) - V_0(0.1059)$ in (5):

$$(78.73)[V_{\pi1}(3164.3) - V_0(0.1059)] = V_0(0.8143) - V_i(0.10) + V_{\pi1}(0.10)$$

$$\text{or } V_{\pi1}(2.49 \times 10^5) - V_0(9.152) = -V_i(0.10)$$

Then

$$V_{\pi1} = V_0(3.674 \times 10^{-5}) - V_i(4.014 \times 10^{-7})$$

Now substituting $V_{\pi1} = V_0(3.674 \times 10^{-5})$

$-V_i(4.014 \times 10^{-7})$ in (2):

$$(35.18)[V_0(3.674) \times 10^{-5} - V_i(4.014 \times 10^{-7})]$$

$$= V_i(2.10) - V_0(0.10)$$

$$\text{or } V_0(0.1013) = V_i(2.10)$$

$$\text{So } \frac{V_0}{V_i} = 20.7$$

$$\text{c. } R_{if} = \frac{V_i}{I_i} \text{ and } I_i = I_{RB1} + I_{b1}$$

$$I_{RB1} = \frac{V_i}{R_{B1}}$$

$$I_{b1} = \frac{V_{\pi 1}}{r_{\pi 1}}$$

Now

$$V_{\pi 1} = (20.7V_i)(3.674 \times 10^{-5}) - V_i(4.014 \times 10^{-7})$$

$$V_{\pi 1} = V_i(7.60 \times 10^{-4})$$

Then

$$R_{if} = \frac{V_i}{\frac{V_i}{63.2} + \frac{V_i(7.60 \times 10^{-4})}{3.66}}$$

$$= \frac{1}{0.01582 + 2.077 \times 10^{-4}}$$

$$\text{or } R_{if} = 62.4 \text{ k}\Omega$$

d. To determine R_{0f} :

Equation (1) is modified to $V_{\pi 1} + V_{e1} = 0$ ($V_i = 0$) Equation (5) is modified to:

$$V_{\pi 3}(78.73) + I_x = V_0(0.8143) + V_{\pi 1}(0.10) \quad (5)$$

Now

$$V_{\pi 1}(35.18) = -V_0(0.10) \quad (2)$$

$$V_{\pi 2} = -V_{\pi 1}(120.2) \quad (3)$$

$$V_{\pi 2}(19.12) + V_{\pi 3}(0.7263) + V_0(0.07692) = 0 \quad (4)$$

Now

$$V_{\pi 1} = -V_0(0.002843)$$

so

$$V_{\pi 2} = -(-V_0)(0.002843)(120.2)$$

$$V_{\pi 2} = V_0(0.3417)$$

Then

$$V_0(0.3417)(19.12) + V_{\pi 3}(0.7263) + V_0(0.07692) = 0$$

$$\text{or } V_{\pi 3} = -V_0(9.101) \quad (4)$$

So then

$$\begin{aligned} -V_0(9.101)(78.73) + I_x \\ = V_0(0.8143) + (0.10)(-V_0)(0.002843) \end{aligned}$$

or

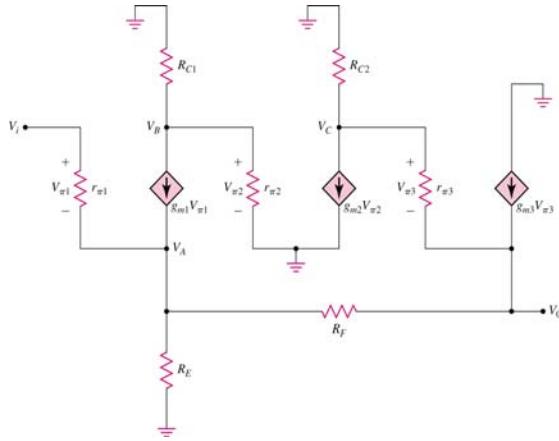
$$I_x = V_0(717.3) \quad (5)$$

or

$$R_{0f} = \frac{V_0}{I_x} = 0.00139 \text{ k}\Omega \Rightarrow R_{0f} = 1.39 \Omega$$

12.38

(a)



$$(1) \frac{V_i - V_A}{r_{\pi 1}} + g_{m1}V_{\pi 1} = \frac{V_A}{R_E} + \frac{V_A - V_O}{R_F}$$

$$(2) \frac{V_B}{R_{C1}} + g_{m1}V_{\pi 1} + \frac{V_B}{r_{\pi 2}} = 0$$

$$(3) \frac{V_C}{R_{C2}} + g_{m2}V_{\pi 2} + \frac{V_C - V_O}{r_{\pi 3}} = 0$$

$$(4) g_{m3}V_{\pi 3} + \frac{V_C - V_O}{r_{\pi 3}} = \frac{V_O - V_A}{R_F}$$

$$V_{\pi 1} = V_i - V_A \quad V_{\pi 2} = V_B \quad V_{\pi 3} = V_C - V_O$$

$$(1) (V_i - V_A) \left(\frac{1}{r_{\pi 1}} + g_{m1} \right) = \frac{V_A}{R_E} + \frac{V_A - V_O}{R_F}$$

$$(2) V_B \left(\frac{1}{R_{C1}} + \frac{1}{r_{\pi 2}} \right) + g_{m1}(V_i - V_A) = 0$$

$$(3) V_C \left(\frac{1}{R_{C2}} + \frac{1}{r_{\pi 3}} \right) + g_{m2}V_B - \frac{V_O}{r_{\pi 3}} = 0$$

$$(4) (V_C - V_O) \left(g_{m3} + \frac{1}{r_{\pi 3}} \right) = \frac{V_O - V_A}{R_F}$$

$$r_{\pi 1} = \frac{(100)(0.026)}{14.3} = 0.182 \text{ K} \quad g_{m1} = \frac{14.3}{0.026} = 550 \text{ mA/V}$$

$$r_{\pi 2} = \frac{(100)(0.026)}{4.62} = 0.563 \text{ K} \quad g_{m2} = \frac{4.62}{0.026} = 178 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(100)(0.026)}{4.47} = 0.582 \text{ K} \quad g_{m3} = \frac{4.47}{0.026} = 172 \text{ mA/V}$$

$$(1) (V_i - V_A) \left(\frac{1}{0.182} + 550 \right) = \frac{V_A}{0.05} + \frac{V_A - V_O}{1.2}$$

$$(2) V_B \left(\frac{1}{0.3} + \frac{1}{0.563} \right) + (550)(V_i - V_A) = 0$$

$$(3) \quad V_c \left(\frac{1}{0.65} + \frac{1}{0.582} \right) + 178 V_B - \frac{V_o}{0.582} = 0$$

$$(4) \quad (V_c - V_o) \left(172 + \frac{1}{0.582} \right) = \frac{V_o - V_A}{1.2}$$

$$(1) \quad (V_i - V_A)(555.5) = V_A(20) + (V_A - V_o)(0.8333)$$

$$(2) \quad V_B(5.109) + 550(V_i - V_A) = 0$$

$$(3) \quad V_c(3.257) + 178V_B - V_o(1.718) = 0$$

$$(4) \quad (V_c - V_o)(173.7) = (V_o - V_A)(0.8333)$$

$$(1) \quad V_i(555.5) + V_o(0.8333) = V_A(576.3)$$

$$(2) \quad V_B(5.109) + 550V_i - V_A(550) = 0$$

$$(3) \quad V_c(3.257) + 178V_B - V_o(1.718) = 0$$

$$(4) \quad V_c(173.7) + V_A(0.8333) = V_o(174.5)$$

$$\text{From (2)} \quad V_B = V_A(107.7) - V_i(107.7)$$

$$\text{From (4)} \quad V_c = V_o(1.0046) - V_A(0.004797)$$

Substitute into (3)

$$(3.257)[V_o(1.0046) - V_A(0.004797)] + (178)[V_A(107.7) - V_i(107.7)] - V_o(1.718) = 0$$

$$V_o(3.272) - V_A(0.01562) + V_A(19170.6) - V_i(19170.6) - V_o(1.718) = 0$$

$$V_A(19170.6) = V_i(19170.6) - V_o(1.554)$$

$$V_A = V_i(1.00) - V_o(0.00008106)$$

Substitute into (1)

$$V_i(555.5) + V_o(0.8333) = (576.3)[V_i(1.00) - V_o(0.00008106)] \\ = V_i(576.3) - V_o(0.0467)$$

$$V_o(0.880) = V_i(20.8)$$

$$\frac{V_o}{V_i} = A_{vf} = 23.6$$

Ideal

$$A_{vf} = \frac{R_F + R_E}{R_E} = \frac{1.2 + 0.05}{0.05} = 25.0$$

$$(b) \quad R_{if} = \frac{V_i}{I_i} \quad \text{and} \quad I_i = \frac{V_{\pi 1}}{r_{\pi 1}} = \frac{V_i - V_A}{r_{\pi 1}}$$

We have

$$V_A = V_i(1.00) - V_o(0.00008106) \\ = V_i(1.00) - (23.6)V_i(0.00008106)$$

$$V_A = V_i(0.99809)$$

$$I_i = \frac{V_i(1 - 0.99809)}{0.182} = V_i(0.01051)$$

Then

$$R_{if} = \frac{V_i}{V_i(0.01051)} \Rightarrow \underline{\underline{R_{if} = 95.1 \text{ K}}}$$

To find R_{of} , set $V_i = 0$

$$I_x + g_{m3}V_{\pi3} + \frac{V_{\pi3}}{r_{\pi3}} = \frac{V_x - V_A}{R_F}$$

$$V_{\pi3} = V_C - V_x$$

$$I_x + (V_C - V_x)(g_{m3} + \frac{1}{r_{\pi3}}) = \frac{V_x - V_A}{R_F}$$

For $V_i = 0$, we have

$$V_C = V_x(1.0046) - V_A(0.004797)$$

$$V_A(576.3) = V_x(0.8333)$$

$$V_A = V_x(0.001446)$$

$$V_C = V_x(1.0046) - V_x(0.001446)(0.004797)$$

$$V_C = V_x(1.0046)$$

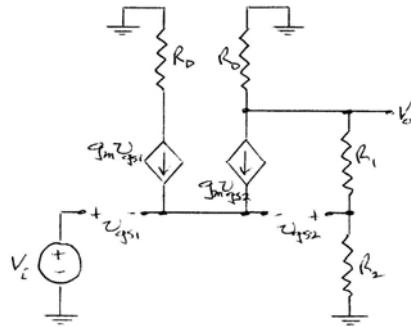
$$I_x + V_x(1.0046 - 1.0)\left(172 + \frac{1}{0.582}\right) = \frac{V_x(1 - 0.004797)}{1.2}$$

$$I_x + V_x(0.7991) = V_x(0.8293)$$

$$I_x = V_x(0.03024)$$

$$R_{of} = \frac{V_x}{I_x} = 33.1 \text{ K}$$

12.39



$$g_m(V_{gs1} + V_{gs2}) = 0 \Rightarrow V_{gs2} = -V_{gs1}$$

$$V_i = V_{gs1} - V_{gs2} + \left(\frac{R_2}{R_1 + R_2}\right) \cdot V_o$$

$$V_i = -2V_{gs2} + \frac{1}{2}V_o$$

$$\text{Also } \frac{V_o}{R_1 + R_2} + g_m V_{gs2} + \frac{V_o}{R_D} = 0$$

$$V_{gs2} = -\frac{V_o}{g_m} \left(\frac{1}{R_D} + \frac{1}{R_1 + R_2} \right)$$

$$\text{Then } V_i = \frac{2V_o}{g_m} \left(\frac{1}{R_D} + \frac{1}{R_1 + R_2} \right) + \frac{1}{2}V_o$$

We have $R_1 + R_2 \gg R_D$

$$V_i \equiv V_o \left(\frac{2}{g_m R_D} + \frac{1}{2} \right)$$

$$A_{vf} = \frac{V_o}{V_i} = \frac{1}{\left(\frac{2}{g_m R_D} + \frac{1}{2} \right)} = \frac{g_m R_D}{\left(2 + \frac{1}{2} g_m R_D \right)}$$

$$\text{Now } g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.5)} = 1.0 \text{ mA/V}$$

$$A_{vf} = \frac{(1)(7)}{\left[2 + \frac{1}{2}(1)(7) \right]} = 1.273$$

12.40

- (a) Neglect base currents

$$\frac{V_{GG} - V_{GS}}{R_L} = I_{D1} + I_{C2}$$

$$I_{D1} = \frac{5 - V_D}{R_{D1}} = \frac{5 - V_D}{5}$$

$$I_{C2} = \frac{5 - (V_D + 0.7)}{R_{E2}} = \frac{4.3 - V_D}{1.6}$$

$$V_D = 5 - 5I_{D1} \Rightarrow I_{C2} = \frac{4.3 - (5 - 5I_{D1})}{1.6} = -0.4375 + 3.125I_{D1}$$

$$\text{Then } \frac{2.5 - V_{GS}}{1.2} = I_{D1} + (-0.4375 + 3.125I_{D1})$$

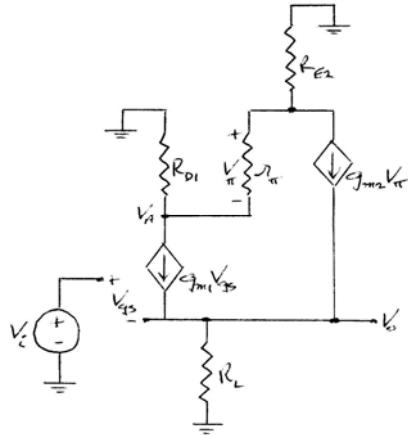
$$3.025 = V_{GS} + 4.95I_{D1} = V_{GS} + 4.95(K_n)(V_{GS} - 0.5)^2$$

$$\text{We find } 7.425V_{GS}^2 - 6.425V_{GS} - 1.16875 = 0 \Rightarrow V_{GS} = 1.0197 \text{ V}$$

$$\text{Then } I_{D1} = (1.5)(1.0197 - 0.5)^2 = 0.405 \text{ mA}$$

$$I_{C2} = -0.4375 + 3.125(0.405) = 0.828 \text{ mA}$$

(b)



$$g_{m1} = 2\sqrt{K_n I_{D1}} = 2\sqrt{(1.5)(0.405)} = 1.559 \text{ mA/V}$$

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{0.828}{0.026} = 31.85 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta V_T}{I_{C2}} = \frac{(120)(0.026)}{0.828} = 3.768 \text{ k}\Omega$$

$$(1) \quad V_o = (g_{m1}V_{gs} + g_{m2}V_\pi)R_L$$

$$(2) \quad V_i = V_{gs} + V_o \Rightarrow V_{gs} = V_i - V_o$$

$$(3) \quad \frac{V_A}{R_{D1}} + g_{m1}V_{gs} = \frac{V_\pi}{r_\pi}$$

$$(4) \quad \frac{V_A + V_\pi}{R_{E2}} + \frac{V_\pi}{r_\pi} + g_{m2}V_\pi = 0$$

$$\text{So } \frac{-V_A}{1.6} = V_\pi \left(\frac{1}{1.6} + \frac{1}{3.768} + 31.85 \right) = V_\pi (32.74)$$

$$V_A = -52.385V_\pi$$

$$(3) \quad \frac{-52.385V_\pi}{5} + 1.559V_{gs} = \frac{V_\pi}{3.768}$$

$$\text{or } 1.559V_{gs} = V_\pi (0.2654 + 10.477)$$

$$\text{so } V_\pi = 0.1451V_{gs} = 0.1451(V_i - V_o)$$

$$(1) \quad V_o = [1.559(V_i - V_o) + 31.85(0.1451)(V_i - V_o)]/(1.2)$$

$$\text{Then } A_v = \frac{V_o}{V_i} = 0.8812$$

(c) Set $V_i = 0$

$$I_x + g_{m2}V_\pi + g_{m1}V_{gs} = \frac{V_x}{R_L}$$

$$V_{gs} = -V_x$$

$$V_\pi = 0.1451V_{gs} = -0.1451V_x$$

$$I_x = V_x \left[\frac{1}{1.2} + (31.85)(0.1451) + (1.559) \right] = V_x (7.014)$$

$$R_o = \frac{V_x}{I_x} = 143 \Omega$$

12.41

$$(a) \quad (i) \quad A_{vf} = \frac{g_m R_s}{1 + g_m R_s}$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(1.5)(1.2)} = 2.683 \text{ mA/V}$$

$$A_{vf} = \frac{(2.683)(1.5)}{1 + (2.683)(1.5)} = 0.801$$

$$(ii) \quad R_{of} = \frac{1}{g_m} \| R_s = \frac{1}{2.683} \| 1.5 = 0.3727 \| 1.5$$

$$R_{of} = 299 \Omega$$

$$(b) \quad (i) \quad g_m = 2\sqrt{(2.25)(1.2)} = 3.286 \text{ mA/V}$$

$$A_{vf} = \frac{(3.286)(1.5)}{1 + (3.286)(1.5)} = 0.8313 \text{ mA/V}$$

$$\frac{\Delta A_{vf}}{A_{vf}} \times 100\% = +3.78\%$$

$$(ii) \quad R_{of} = \frac{1}{3.286} \| 1.5 = 0.3043 \| 1.5$$

$$R_{of} = 253 \Omega$$

$$\frac{\Delta R_{of}}{R_{of}} \times 100\% = -15.4\%$$

12.42

dc analysis:

$$R_{TH1} = 150 \| 47 = 35.8 \text{ k}\Omega,$$

$$V_{TH1} = \left(\frac{47}{47+150} \right) (25) = 5.96 \text{ V}$$

$$R_{TH2} = 33 \| 47 = 19.4 \text{ k}\Omega,$$

$$V_{TH2} = \left(\frac{33}{33+47} \right) (25) = 10.3 \text{ V}$$

$$I_{B1} = \frac{5.96 - 0.7}{35.8 + (51)(4.8)} = 0.0187 \text{ mA}$$

$$I_{C1} = (50)(0.0187) = 0.935 \text{ mA}$$

$$I_{B2} = \frac{10.3 - 0.7}{19.4 + (51)(4.7)} = 0.03705 \text{ mA}$$

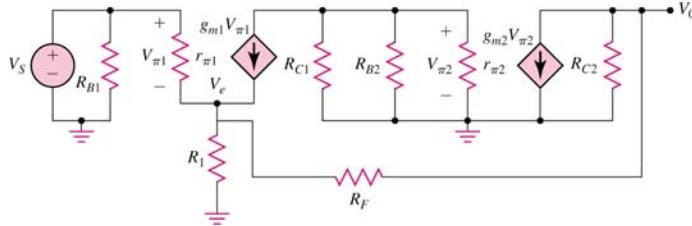
$$I_{C2} = (50)(0.03705) = 1.85 \text{ mA}$$

$$r_{\pi 1} = \frac{(50)(0.026)}{0.935} = 1.39 \text{ k}\Omega;$$

$$r_{\pi 2} = \frac{(50)(0.026)}{1.85} = 0.703 \text{ k}\Omega$$

$$g_{m1} = \frac{0.935}{0.026} = 35.96 \text{ mA/V}$$

$$g_{m2} = \frac{1.85}{0.026} = 71.15 \text{ mA/V}$$



$$V_s = V_{\pi 1} + V_e \quad (1)$$

$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} = \frac{V_e}{R_l} + \frac{V_e - V_0}{R_F} \quad (2)$$

$$g_{m1}V_{\pi1} + \frac{V_{\pi2}}{R_{C1}} + \frac{V_{\pi2}}{R_{B2}} + \frac{V_{\pi2}}{r_{\pi2}} = 0 \quad (3)$$

$$g_{m2}V_{\pi2} + \frac{V_0}{R_{C2}} + \frac{V_0 - V_e}{R_f} = 0 \quad (4)$$

Substitute numerical values in (2), (3) and (4):

$$V_e = V_s - V_{\pi1} \quad (1)$$

$$\frac{V_{\pi1}}{1.39} + (35.96)V_{\pi1} = (V_s - V_{\pi1})\left(\frac{1}{0.1} + \frac{1}{4.7}\right) - V_0\left(\frac{1}{4.7}\right)$$

or

$$V_{\pi1}(46.89) = V_s(10.213) - V_0(0.2128) \quad (2)$$

$$(35.96)V_{\pi1} + V_{\pi2}\left(\frac{1}{10} + \frac{1}{19.4} + \frac{1}{0.703}\right) = 0$$

or

$$(35.96)V_{\pi1} + V_{\pi2}(1.574) = 0 \quad (3)$$

$$(71.15)V_{\pi2} + V_0\left(\frac{1}{4.7} + \frac{1}{4.7}\right) - (V_s - V_{\pi1})\left(\frac{1}{4.7}\right) = 0$$

or

$$(71.15)V_{\pi2} + V_0(0.4255) - V_s(0.2128) + V_{\pi1}(0.2128) = 0 \quad (4)$$

From (3): $V_{\pi2} = -V_{\pi1}(22.85)$

Then substitute in (4):

$$-(71.15)V_{\pi1}(22.85) + V_0(0.4255) - V_s(0.2128) + V_{\pi1}(0.2128) = 0$$

or

$$-V_{\pi1}(1625.6) + V_0(0.4255) - V_s(0.2128) = 0$$

From (2): $V_{\pi1} = V_s(0.2178) - V_0(0.004538)$

Then

$$-(1625.6)[V_s(0.2178) - V_0(0.004538)] + V_0(0.4255) - V_s(0.2128) = 0$$

or $-V_s(354.3) + V_0(7.802) = 0$

Finally

$$\Rightarrow \frac{V_0}{V_s} = 45.4$$

12.43

For example, use a 2-stage amplifier. Each stage is shown in Fig. 12.29.

12.44

$$I_{DQ1} = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right) (V_{GS} - V_{TN})^2$$

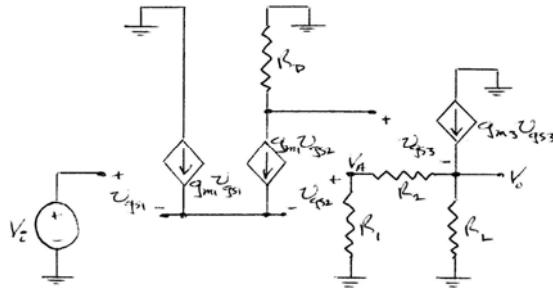
$$\text{Let all } \left(\frac{W}{L}\right) = 20$$

$$0.5 = \left(\frac{0.1}{2}\right)(20)(V_{GS} - 1.5)^2 \Rightarrow V_{GS1,2} = 2.207 \text{ V}$$

$$I_{DQ3} = 2 = \left(\frac{0.1}{2}\right)(20)(V_{GS3} - 1.5)^2 \Rightarrow V_{GS3} = 2.914 \text{ V}$$

Want $V_{G3} = 2.914 \text{ V}$, then

$$R_D = \frac{12 - 2.914}{0.5} = 18.2 \text{ k}\Omega$$



$$g_{m1}v_{gs1} + g_{m1}v_{gs2} = 0 \Rightarrow v_{gs2} = -v_{gs1}$$

$$V_i = v_{gs1} - v_{gs2} + V_A = -2v_{gs2} + V_A$$

$$V_A = \left(\frac{R_1}{R_1 + R_2} \right) \cdot V_o$$

$$\frac{V_o}{R_L} + \frac{V_o}{R_1 + R_2} = g_{m3}v_{gs3}$$

$$v_{gs3} = -(g_{m1}v_{gs2}R_D) - V_o$$

$$V_o \left(\frac{1}{R_L} + \frac{1}{R_1 + R_2} \right) = g_{m3}(-g_{m1}v_{gs2}R_D - V_o)$$

$$v_{gs2} = \frac{1}{2}(V_A - V_i) = \frac{1}{2} \left(\frac{R_1}{R_1 + R_2} \right) \cdot V_o - \frac{1}{2} \cdot V_i$$

$$V_o \left(\frac{1}{R_L} + \frac{1}{R_1 + R_2} \right) = -g_{m1}g_{m3}R_D \left[\frac{1}{2} \left(\frac{R_1}{R_1 + R_2} \right) \cdot V_o - \frac{1}{2} \cdot V_i \right] - g_{m3}V_o$$

$$V_o \left[\frac{1}{R_L} + \frac{1}{R_1 + R_2} + \frac{1}{2} g_{m1}g_{m3}R_D \left(\frac{R_1}{R_1 + R_2} \right) + g_{m3} \right] = \frac{1}{2} g_{m1}g_{m3}R_D V_i$$

$$A_{vf} = \frac{V_o}{V_i} = \frac{\frac{1}{2} g_{m1}g_{m3}R_D}{\left[\frac{1}{R_L} + \frac{1}{R_1 + R_2} + \frac{1}{2} g_{m1}g_{m3}R_D \left(\frac{R_1}{R_1 + R_2} \right) + g_{m3} \right]}$$

$$g_{m1} = 2\sqrt{\left(\frac{0.1}{2}\right)(20)(0.5)} = 1.414 \text{ mA/V}$$

$$g_{m3} = 2\sqrt{\left(\frac{0.1}{2}\right)(20)(2)} = 2.828 \text{ mA/V}$$

$$\text{Then } 8 = \frac{\frac{1}{2}(1.414)(2.828)(18.2)}{\left[\frac{1}{10} + \frac{1}{R_1 + R_2} + \frac{1}{2}(1.414)(2.828)(18.2)\left(\frac{15}{R_1 + R_2} \right) + 2.828 \right]}$$

$$8 = \frac{36.389}{\left[2.928 + \frac{546.8}{R_1 + R_2} \right]}$$

$$\Rightarrow R_1 + R_2 = 337 \text{ k}\Omega \quad \Rightarrow R_2 = 322 \text{ k}\Omega$$

12.45

$$\frac{I_o}{I_s} = \frac{5}{0.06} = 83.33 = 1 + \frac{R_1}{R_2} \Rightarrow \frac{R_1}{R_2} = 82.33$$

For example, let $R_2 = 3 \text{ k}\Omega$, $R_1 = 247 \text{ k}\Omega$

12.46

(a) (1) $V_A = (I_{D1} + I_{D2})R_{D2}$

(2) $I_{D1}R_{D1} = V_{SG2}$

(3) $\frac{(V_G - V_{GS1}) - V_A}{R_F} = I_{D1}$

Now $V_G - V_{GS1} - (I_{D1} + I_{D2})R_{D2} = I_{D1}R_F$

And $V_{GS2} = \sqrt{\frac{I_{D2}}{K_p}} - V_{TP} = I_{D1}R_{D1}$

$$\sqrt{I_{D2}}(0.3162) + 1 = I_{D1}(0.525)$$

$$\sqrt{I_{D2}} = I_{D1}(1.660) - 3.162$$

$$I_{D2} = [I_{D1}(1.660) - 3.162]^2$$

Then $V_G - V_{GS1} - [I_{D1}(1.660) - 3.162]^2 R_{D2} = I_{D1}(R_{D2} + R_F)$

$$I_{D1} = K_n(V_{GS1} - V_{TN})^2$$

$$7.6 - V_{GS1} - [I_{D1}(1.660) - 3.162]^2 (0.25) = I_{D1}(0.75)$$

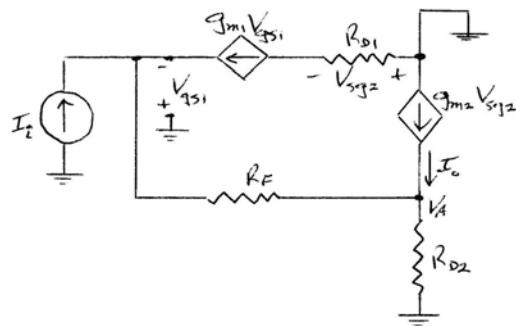
$$I_{D1} = 10(V_{GS1} - 1)^2$$

By trial and error, $I_{D1} \approx 3.98 \text{ mA}$

Then $V_{SG2} = I_{D1}R_{D1} = (3.98)(0.525) = 2.0895 \text{ V}$

$$I_{D1} = 10(2.0895 - 1)^2 = 11.87 \text{ mA}$$

(b)



$$I_o = g_{m2} V_{sg2}$$

$$V_{sg2} = g_{m1} V_{gs1} R_{D1}$$

$$I_o = \frac{V_A}{R_{D2}} + \frac{V_A - (-V_{gs1})}{R_F}$$

$$I_i + g_{m1} V_{gs1} + \frac{V_A + V_{gs1}}{R_F} = 0$$

$$\text{or } V_A = -R_F \left[I_i + g_{m1} V_{gs1} + \frac{V_{gs1}}{R_F} \right]$$

$$I_o = V_A \left(\frac{1}{R_{D2}} + \frac{1}{R_F} \right) + \frac{V_{gs1}}{R_F}$$

$$I_o = -R_F \left(I_i + g_{m1} V_{gs1} + \frac{V_{gs1}}{R_F} \right) \left(\frac{1}{R_{D2}} + \frac{1}{R_F} \right) + \frac{V_{gs1}}{R_F}$$

$$\text{Now } V_{gs1} = \frac{V_{sg2}}{g_{m1} R_{D1}} = \frac{I_o}{g_{m2}} \cdot \frac{1}{g_{m1} R_{D1}}$$

$$I_o = -R_F I_i \left(\frac{1}{R_{D2}} + \frac{1}{R_F} \right) - R_F V_{gs1} \left[\left(g_{m1} + \frac{1}{R_F} \right) \left(\frac{1}{R_{D2}} + \frac{1}{R_F} \right) - \frac{1}{R_F^2} \right]$$

$$\text{Then } I_o \left[1 + \frac{R_F}{g_{m1} g_{m2} R_{D1}} \left(\frac{g_{m1}}{R_{D2}} + \frac{g_{m1}}{R_F} + \frac{1}{R_F R_{D2}} \right) \right] = -I_i \left(1 + \frac{R_F}{R_{D2}} \right)$$

$$\text{So } A_i = \frac{I_o}{I_i} = \frac{-g_{m2} R_{D1}}{1 + \frac{1}{g_{m1} (R_F + R_{D2})} + \frac{g_{m2} R_{D1} R_{D2}}{R_F + R_{D2}}}$$

$$(c) g_{m1} = 2\sqrt{K_n I_{D1}} = 2\sqrt{(10)(3.98)} = 12.62 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{K_p I_{D2}} = 2\sqrt{(10)(11.87)} = 21.79 \text{ mA/V}$$

$$A_i = \frac{-(21.79)(0.525)}{1 + \frac{1}{(12.62)(0.75)} + \frac{(21.79)(0.525)(0.25)}{0.75}}$$

$$A_i = -2.33$$

12.47

$$(a) I_Q = I_{D1} + I_{D2}$$

$$V_{SG2} = I_{D1} R_D$$

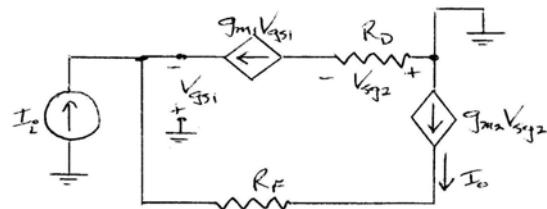
$$I_{D2} = K_p (V_{SG2} + V_{TP})^2 = K_p (I_{D1} R_D + V_{TP})^2$$

$$I_Q = I_{D1} + K_p [I_{D1}^2 R_D^2 - 2I_{D1} R_D + 1]$$

$$\text{We find } 2.756 I_{D1}^2 - 9.5 I_{D1} - 6 = 0$$

$$\Rightarrow I_{D1} = 3.99 \text{ mA and } I_{D2} = 12.01 \text{ mA}$$

(b)



$$I_i + g_{m1}V_{gs1} + I_o = 0$$

$$V_{sg2} = g_{m1}V_{gs1}R_D \Rightarrow V_{gs1} = \frac{V_{sg2}}{g_{m1}R_D}$$

$$I_o = g_{m2}V_{sg2}$$

$$\text{Then } V_{gs1} = \frac{I_o}{g_{m1}g_{m2}R_D}$$

$$I_i + g_{m1}\left(\frac{I_o}{g_{m1}g_{m2}R_D}\right) + I_o = 0$$

$$\text{So that } A_i = \frac{I_o}{I_i} = \frac{-1}{\left(1 + \frac{1}{g_{m2}R_D}\right)} = \frac{-g_{m2}R_D}{1 + g_{m2}R_D}$$

$$\text{We find } g_{m2} = 2\sqrt{K_p I_{D2}} = 2\sqrt{(10)(12.01)} = 21.92 \text{ mA/V}$$

$$A_i = \frac{-21.92(0.525)}{1 + 21.92(0.525)} = -0.920$$

12.48

- (a) Neglect base currents

$$I_Q = I_{C1} + I_{C2}$$

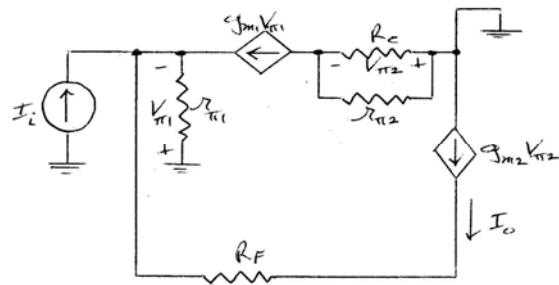
$$V_{EB2} = I_{C1}R_C$$

$$I_Q = I_{C1} + I_{S2} \exp\left(\frac{I_{C1}R_C}{V_T}\right)$$

$$16 \times 10^{-3} = I_{C1} + (10^{-15}) \exp\left[\frac{I_{C1}(200)}{0.026}\right]$$

By trial and error, $I_{C1} = 3.92 \text{ mA}$ and $I_{C2} = 12.08 \text{ mA}$

(b)



$$I_o = g_{m2}V_{\pi2}$$

$$V_{\pi2} = g_{m1}V_{\pi1}(r_{\pi2}\parallel R_C)$$

$$\text{Then } I_o = g_{m1}g_{m2}V_{\pi1}(r_{\pi2}\parallel R_C)$$

$$I_i + I_o + g_{m1}V_{\pi1} + \frac{V_{\pi1}}{r_{\pi1}} = 0$$

$$I_i + I_o + V_{\pi1}\left(\frac{1+\beta_1}{r_{\pi1}}\right) = 0$$

$$\text{Now } V_{\pi1} = \frac{I_o}{g_{m1}g_{m2}(r_{\pi2}\parallel R_C)}$$

$$\text{Then } I_i + I_o \left[1 + \frac{1}{g_{m1}g_{m2}(r_{\pi2}\parallel R_C)} \left(\frac{1+\beta_1}{r_{\pi1}} \right) \right] = 0$$

$$\text{So } A_i = \frac{I_o}{I_i} = \frac{-\left(\frac{\beta_1}{1+\beta_1}\right)(g_{m2})(r_{\pi2}\parallel R_C)}{1+\left(\frac{\beta_1}{1+\beta_1}\right)(g_{m2})(r_{\pi2}\parallel R_C)}$$

$$(c) \quad g_{m2} = \frac{I_{C2}}{V_T} = \frac{12.08}{0.026} = 464.6 \text{ mA/V}$$

$$r_{\pi2} = \frac{\beta_2 V_T}{I_{C2}} = \frac{(180)(0.026)}{12.08} = 0.3874 \text{ k}\Omega$$

$$r_{\pi2}\parallel R_C = 387.4\parallel 200 = 131.9 \Omega$$

$$A_i = \frac{-\left(\frac{180}{181}\right)(464.6)(0.1319)}{1+\left(\frac{180}{181}\right)(464.6)(0.1319)} = -0.984$$

12.49

$$I_{C1} = \left(\frac{h_{FE}}{1+h_{FE}} \right) I_{E1} = \left(\frac{100}{101} \right) (0.2) = 0.198 \text{ mA}$$

$$V_{C1} = 10 - (0.198)(40) = 2.08 \text{ V}$$

$$I_{E2} = \frac{2.08 - 0.7}{1} = 1.38 \text{ mA}$$

$$(a) \quad I_{C2} = \left(\frac{100}{101} \right) (1.38) = 1.37 \text{ mA}$$

For Q_1 :

$$r_{\pi 1} = \frac{(100)(0.026)}{0.198} = 13.1 \text{ k}\Omega$$

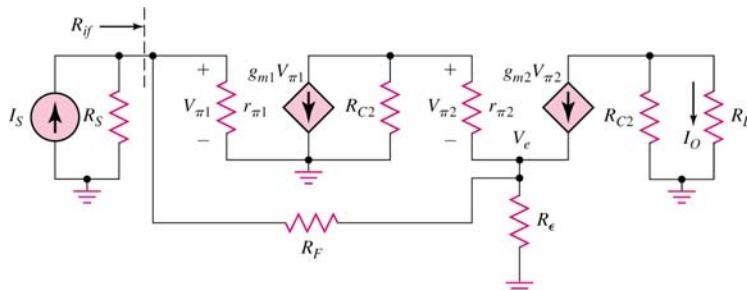
$$g_{m1} = \frac{0.198}{0.026} = 7.62 \text{ mA/V}$$

For Q_2 :

$$r_{\pi 2} = \frac{(100)(0.026)}{1.37} = 1.90 \text{ k}\Omega$$

$$g_{m2} = \frac{1.37}{0.026} = 52.7 \text{ mA/V}$$

(b)



$$I_S = \frac{V_{\pi 1}}{R_S} + \frac{V_{\pi 1}}{r_{\pi 1}} + \frac{V_{\pi 1} - V_e}{R_F} \quad (1)$$

$$g_{m1}V_{\pi 1} + \frac{V_{\pi 2} + V_e}{R_{C1}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \quad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2}V_{\pi 2} = \frac{V_e}{R_E} + \frac{V_e - V_{\pi 1}}{R_F} \quad (3)$$

Substitute numerical values in (1), (2), and (3):

$$I_S = V_{\pi 1} \left(\frac{1}{10} + \frac{1}{13.1} + \frac{1}{10} \right) - V_e \left(\frac{1}{10} \right)$$

$$I_S = V_{\pi 1}(0.2763) - V_e(0.10) \quad (1)$$

$$(7.62)V_{\pi 1} + V_{\pi 2} \left(\frac{1}{40} + \frac{1}{1.90} \right) + V_e \left(\frac{1}{40} \right) = 0$$

$$(7.62)V_{\pi 1} + V_{\pi 2}(0.5513) + V_e(0.025) = 0 \quad (2)$$

$$V_{\pi 2} \left(\frac{1}{1.90} + 52.7 \right) = V_e \left(\frac{1}{1} + \frac{1}{10} \right) - V_{\pi 1} \left(\frac{1}{10} \right)$$

$$V_{\pi 2}(53.23) = V_e(1.10) - V_{\pi 1}(0.10) \quad (3)$$

From (3),

$$V_e = V_{\pi 2}(48.39) + V_{\pi 1}(0.0909)$$

Substituting into (1),

$$I_s = V_{\pi 1}(0.2763) - (0.10)[V_{\pi 2}(48.39) + V_{\pi 1}(0.0909)]$$

or

$$I_s = V_{\pi 1}(0.2672) - V_{\pi 2}(4.839) \quad (1')$$

and substituting into (2),

$$(7.62)V_{\pi 1} + V_{\pi 2}(0.5513) + (0.025)[V_{\pi 2}(48.39) + V_{\pi 1}(0.0909)] = 0$$

or

$$(7.622)V_{\pi 1} + V_{\pi 2}(1.761) = 0 \Rightarrow V_{\pi 1} = -V_{\pi 2}(0.2310) \quad (2')$$

Then substituting (2') into (1'), we obtain

$$I_s = (0.2672)(-V_{\pi 2})(0.2310) - V_{\pi 2}(4.839)$$

or

$$I_s = -V_{\pi 2}(4.901)$$

Now

$$\begin{aligned} I_o &= -g_{m2}V_{\pi 2}\left(\frac{R_{C2}}{R_{C2} + R_L}\right) \\ &= -(52.7)\left(\frac{2}{2+0.5}\right)V_{\pi 2} = -(42.16)V_{\pi 2} \end{aligned}$$

Then

$$I_o = -(42.16)\left(\frac{-I_s}{4.901}\right)$$

or

$$A_{if} = \frac{I_o}{I_s} = 8.60$$

$$(c) \quad R_i = \frac{V_{\pi 1}}{I_s} \quad \text{and} \quad R_i = R_S \parallel R_{if}$$

We had

$$V_{\pi 1} = -V_{\pi 2}(0.2310) \quad \text{and} \quad I_s = -V_{\pi 2}(4.901)$$

so

$$I_s = -\left(\frac{-V_{\pi 1}}{0.2310}\right)(4.901) = V_{\pi 1}(21.22)$$

Then

$$R_i = \frac{V_{\pi 1}}{I_s} = \frac{1}{21.22} = 0.04713$$

Finally

$$0.04713 = \frac{10R_{if}}{10 + R_{if}} \Rightarrow R_{if} = 47.4 \Omega$$

12.50

(a)

$$(1) \quad I_i = \frac{V_{\pi 1}}{R_S \parallel R_{B1} \parallel r_{\pi 1}} + \frac{V_{\pi 1} - V_{e2}}{R_F}$$

$$(2) \quad g_{m1}V_{\pi 1} + \frac{V_{C1}}{R_{C1} \parallel R_{B2}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0$$

$$(3) \quad \frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2}V_{\pi 2} = \frac{V_{e2}}{R_{E2}} + \frac{V_{e2} - V_{\pi 1}}{R_F}$$

$$(4) \quad I_o = -(g_{m2}V_{\pi2}) \left(\frac{R_{C2}}{R_{C2} + R_L} \right)$$

Now

$$(1)' \quad I_i = \frac{V_{\pi1}}{R_S \| R_{B1} \| r_{\pi1} \| R_F} - \frac{V_{e2}}{R_F}$$

$$\text{So } V_{e2} = \left(\frac{R_F}{R_S \| R_{B1} \| r_{\pi1} \| R_F} \right) V_{\pi1} - I_i R_F$$

Now, from (2)

$$(2)' \quad g_{m1}V_{\pi1} + \frac{V_{\pi2} + V_{e2}}{R_{c1} \| R_{B2}} + \frac{V_{\pi2}}{r_{\pi2}} = 0$$

$$\left(g_{m1} + \frac{1}{r_{\pi2}} \right) V_{\pi1} + \frac{V_{\pi2}}{R_{c1} \| R_{B2} \| r_{\pi2}} + \frac{V_{e2}}{R_{c1} \| R_{B2}} = 0$$

Also

$$(3)' \quad \left(g_{m2} + \frac{1}{r_{\pi2}} \right) V_{\pi2} + \frac{V_{\pi1}}{R_F} = V_{e2} \left(\frac{1}{R_{E2}} + \frac{1}{R_F} \right)$$

And

$$(4)' \quad V_{\pi2} = - \left(\frac{I_o}{g_{m2}} \right) \left(\frac{R_{C2} + R_L}{R_{C2}} \right)$$

Substitute (1)' into (2)' and (3)'

$$(2)'' \quad \left(g_{m1} + \frac{1}{r_{\pi2}} \right) V_{\pi1} + \frac{V_{\pi2}}{R_{c1} \| R_{B2} \| r_{\pi2}} + \frac{1}{R_{c1} \| R_{B2}} \left[\frac{R_F}{R_S \| R_{B1} \| r_{\pi1} \| R_F} - V_{\pi1} - I_i R_F \right] = 0$$

$$(3)'' \quad \left(g_{m2} + \frac{1}{r_{\pi2}} \right) V_{\pi2} + \frac{V_{\pi1}}{R_F} = \left(\frac{1}{R_{E2}} + \frac{1}{R_F} \right) \left[\frac{R_F}{R_S \| R_{B1} \| r_{\pi1} \| R_F} - V_{\pi1} - I_i R_F \right]$$

Solve for $V_{\pi1}$ from (2)'' and substitute into (3)''. Also use Equation (4)'.

$$(b) \quad R_{B1} = R_1 \| R_2 = 20 \| 80 = 16 \text{ K}$$

$$V_{TH1} = \left(\frac{20}{100} \right) (10) = 2 \text{ V}$$

$$I_{BQ1} = \frac{2 - 0.7}{16 + (10)(1)} = 0.0111 \text{ mA}$$

$$I_{CQ1} = 1.11 \text{ mA}$$

$$R_{TH2} = 15 \| 85 = 12.75 \text{ K}$$

$$V_{TH2} = \left(\frac{15}{100} \right) (10) = 1.5 \text{ V}$$

$$I_{BQ2} = \frac{1.5 - 0.7}{12.75 + (10)(0.5)} = 0.01265 \text{ mA}$$

$$I_{CQ2} = 1.265 \text{ mA}$$

$$g_{m1} = \frac{1.11}{0.026} = 42.69 \text{ mA/V}$$

$$g_{m2} = \frac{1.265}{0.026} = 48.65 \text{ mA/V}$$

$$r_{\pi 1} = \frac{(100)(0.026)}{1.11} = 2.34 \text{ K}$$

$$r_{\pi 2} = \frac{(100)(0.026)}{1.265} = 2.06 \text{ K}$$

Now $R_{C1} \| R_{B2} = 2 \| 12.75 = 1.729 \text{ K}$

$$R_s \| R_{B1} \| r_{\pi 1} \| R_F \equiv R_{B1} \| r_{\pi 1} \| R_F = 16 \| 2.34 \| 10 = 1.695 \text{ K}$$

$$R_{C1} \| R_{B2} \| r_{\pi 2} = 1.729 \| 2.06 = 0.940 \text{ K}$$

Now

$$\left(42.69 + \frac{1}{2.06} \right) V_{\pi 1} + \frac{V_{\pi 2}}{0.940} + \frac{1}{1.729} \left[\frac{10}{1.695} \cdot V_{\pi 1} - I_i(10) \right]$$

$$(2)' 46.587 V_{\pi 1} + 1.064 V_{\pi 2} - 5.784 I_i = 0$$

$$\left(\frac{101}{2.06} \right) V_{\pi 2} + \frac{V_{\pi 1}}{10} = \left(\frac{1}{0.5} + \frac{1}{10} \right) \left[\frac{10}{1.695} \cdot V_{\pi 1} - I_i(10) \right]$$

$$(3)' 49.03 V_{\pi 2} = 12.29 V_{\pi 1} - 21 I_i$$

$$\text{From (2)'' } V_{\pi 1} = (0.1242) I_i - (0.02284) V_{\pi 2}$$

Then

$$(3)'' 49.03 V_{\pi 2} = 12.29 [(0.1242) I_i - (0.02284) V_{\pi 2}] - 21 I_i$$

$$49.31 V_{\pi 2} = -19.47 I_i$$

$$V_{\pi 2} = - \left(\frac{I_o}{48.65} \right) \left(\frac{4+4}{4} \right) = -(0.0411) I_o$$

From (4)'

Then

$$(49.31) [-(0.0411) I_o] = -19.47 I_i$$

$$\frac{I_o}{I_i} = A_i = 9.61$$

12.51

$$R_{TH} = 13.5 \| 38.3 = 9.98 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{13.5}{13.5 + 38.3} \right) (10) = 2.606 \text{ V}$$

$$\text{a. } I_{C1} = \frac{(120)(2.606 - 0.7)}{9.98 + (121)(1)} = 1.75 \text{ mA}$$

$$V_{C1} = 10 - (1.75)(3) = 4.75 \text{ V}$$

$$I_{C2} \approx \frac{4.75 - 0.7}{8.1} = 0.50 \text{ mA}$$

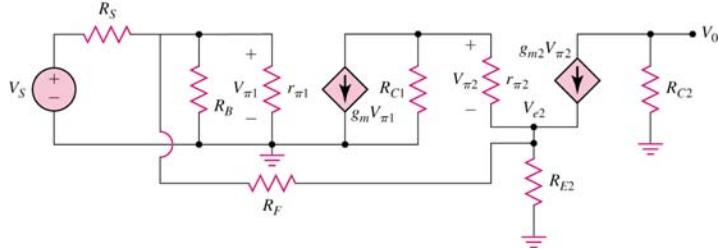
$$r_{\pi 1} = \frac{(120)(0.026)}{1.75} = 1.78 \text{ k}\Omega$$

$$g_{m1} = \frac{1.75}{0.026} = 67.31 \text{ mA/V}$$

$$r_{\pi 2} = \frac{(120)(0.026)}{0.50} = 6.24 \text{ k}\Omega$$

$$g_{m2} = \frac{0.50}{0.026} = 19.23 \text{ mA/V}$$

b.



$$\frac{V_s - V_{\pi 1}}{R_s} = \frac{V_{\pi 1}}{R_B \parallel r_{\pi 1}} + \frac{V_{\pi 1} - V_{e2}}{R_F} \quad (1)$$

$$g_{m1} V_{\pi 1} + \frac{V_{\pi 2} + V_{e2}}{R_{C1}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \quad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2} V_{\pi 2} = \frac{V_{e2}}{R_{E2}} + \frac{V_{e2} - V_{\pi 1}}{R_F} \quad (3)$$

and

$$V_0 = -(g_{m2} V_{\pi 2}) R_{C2} \quad (4)$$

Substitute numerical values in (1), (2), and (3)

$$\frac{V_s}{0.6} = V_{\pi 1} \left[\frac{1}{0.6} + \frac{V_{\pi 1}}{9.98 \parallel 1.78} + \frac{1}{1.2} \right] - \frac{V_{e2}}{1.2}$$

$$V_s(1.67) = V_{\pi 1}(4.011) - V_{e2}(0.8333) \quad (1)$$

$$(67.31)V_{\pi 1} + V_{\pi 2} \left(\frac{1}{3} + \frac{1}{6.24} \right) + \frac{V_{e2}}{3} = 0$$

or

$$V_{\pi 1}(67.31) + V_{\pi 2}(0.4936) + V_{e2}(0.3333) = 0 \quad (2)$$

$$V_{\pi 1} \left(\frac{1}{6.24} + 19.23 \right) = \frac{V_{e2}}{8.1} + \frac{V_{e2}}{1.2} - \frac{V_{\pi 2}}{1.2}$$

or

$$V_{\pi 2}(19.39) = V_{e2}(0.9568) - V_{\pi 1}(0.8333) \quad (3)$$

From (1)

$$V_{e2} = V_{\pi 1}(4.813) - V_s(2.00)$$

Then

$$V_{\pi 1}(67.31) + V_{\pi 2}(0.4936) + (0.3333)[V_{\pi 1}(4.813) - V_s(2.00)] = 0$$

or

$$V_{\pi 1}(68.91) + V_{\pi 2}(0.4936) - V_s(0.6666) = 0 \quad (2')$$

and

$$V_{\pi 2}(19.39) = (0.9568)[V_{\pi 1}(4.813) - V_s(2.00)] - V_{\pi 1}(0.8333)$$

or

$$V_{\pi 2}(19.39) = V_{\pi 1}(3.772) - V_s(1.914) \quad (3')$$

We find

$$V_{\pi 1} = V_s(0.009673) - V_{\pi 2}(0.007163)$$

Then

$$V_{\pi 2}(19.39) = (3.772)[V_s(0.009673) - V_{\pi 2}(0.007163)] - V_s(1.914)$$

$$V_{\pi 2}(19.42) = V_s(-1.878) \text{ or } V_{\pi 2} = -V_s(0.09670)$$

so that

$$V_0 = -(19.23)(4)(-V_s)(0.09670)$$

Then

$$\frac{V_0}{V_s} = 7.44$$

12.52

$$R_{if} = \frac{V_{\pi 1}}{I_s}$$

Using the circuit from Problem 12.51, we have

$$I_s = \frac{V_s - V_{\pi 1}}{R_s}$$

Where

From Problem 12.51

$$\begin{aligned} V_{\pi 1} &= V_s(0.009673) - V_{\pi 2}(0.007163) \\ &= V_s(0.009673) - (0.007163)(-V_s)(0.09670) \\ &= V_s(0.01037) \end{aligned}$$

So

$$R_{if} = \frac{V_s(0.01037) \cdot (0.6)}{V_s - V_s(0.01037)} = 0.00629 \text{ k}\Omega$$

or

$$\underline{R_{if} = 6.29 \Omega}$$

12.53

$$R_{TH} = 1.4 \parallel 17.9 = 1.298 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{1.4}{1.4 + 17.9} \right) (10) = 0.7254 \text{ V}$$

$$I_{B1} = \frac{0.7254 - 0.7}{1.298} = 0.0196 \text{ mA}$$

$$I_{C1} = (50)(0.0196) = 0.98 \text{ mA}$$

Neglecting dc base currents,

$$V_{B2} = 10 - (0.98)(7) = 3.14 \text{ V}$$

$$I_{E2} = \frac{3.14 - 0.7}{0.25 + 0.5} = 3.25 \text{ mA}$$

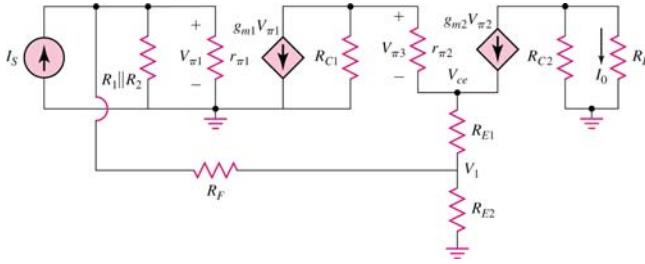
$$I_{C2} = \left(\frac{50}{51} \right) (3.25) = 3.19 \text{ mA}$$

$$r_{\pi 1} = \frac{(50)(0.026)}{0.98} = 1.33 \text{ k}\Omega$$

$$g_{m1} = \frac{0.98}{0.026} = 37.7 \text{ mA/V}$$

$$r_{\pi 2} = \frac{(50)(0.026)}{3.19} = 0.408 \text{ k}\Omega$$

$$g_{m2} = \frac{3.19}{0.026} = 123 \text{ mA/V}$$



$$I_s = \frac{V_{\pi 1}}{R_1 \parallel R_2 \parallel r_{\pi 1}} + \frac{V_{\pi 1} - V_1}{R_F} \quad (1)$$

$$g_{m1}V_{\pi 1} + \frac{V_{\pi 2}}{r_{\pi 2}} + \frac{V_{\pi 2} + V_{e2}}{R_{C1}} = 0 \quad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2}V_{\pi 2} = \frac{V_{e2} - V_1}{R_{E1}} \quad (3)$$

$$\frac{V_{e2} - V_{\pi 1}}{R_{E2}} = \frac{V_1}{R_{E2}} + \frac{V_1 - V_{\pi 1}}{R_F} \quad (4)$$

Enter numerical values in (1), (2), (3) and (4):

$$I_s = \frac{V_{\pi 1}}{17.9 \parallel 1.4 \parallel 1.33} + \frac{V_{\pi 1} - V_1}{5}$$

or

$$I_s = V_{\pi 1}(1.722) - V_1(0.20) \quad (1)$$

$$(37.7)V_{\pi 1} + \frac{V_{\pi 2}}{0.408} + \frac{V_{\pi 2} + V_{e2}}{7} = 0$$

or

$$V_{\pi 1}(37.7) + V_{\pi 2}(2.594) + V_{e2}(0.1429) = 0 \quad (2)$$

$$\frac{V_{\pi 2}}{0.408} + (123)V_{\pi 2} = \frac{V_{e2} - V_1}{0.25}$$

or

$$V_{\pi 2}(125.5) = V_{e2}(4) - V_1(4) \quad (3)$$

$$\frac{V_{e2} - V_1}{0.25} = \frac{V_1}{0.50} + \frac{V_1 - V_{\pi 1}}{5}$$

or

$$V_{e2}(4) = V_1(6.20) - V_{\pi 1}(0.20) \quad (4)$$

From (4):

$$V_{e2} = V_1(1.55) - V_{\pi 1}(0.05)$$

Then substituting in (3):

$$V_{\pi 2}(125.5) = (4)[V_1(1.55) - V_{\pi 1}(0.05)] - V_1(4)$$

or

$$V_{\pi 2}(125.5) = V_1(2.20) - V_{\pi 1}(0.20) \quad (3')$$

and substituting in (2):

$$V_{\pi 1}(37.7) + V_{\pi 2}(2.594) + (0.1429)[V_1(1.55) - V_{\pi 1}(0.05)] = 0$$

or

$$V_{\pi 1}(37.69) + V_{\pi 2}(2.594) + V_1(0.2215) = 0$$

Now

$$V_1 = -V_{\pi 1}(170.16) - V_{\pi 2}(11.71)$$

Then substituting in (1):

$$I_s = V_{\pi 1}(1.722) - (0.20)[-V_{\pi 1}(170.16) - V_{\pi 2}(11.71)]$$

or

$$I_s = V_{\pi 1}(35.75) + V_{\pi 2}(2.342)$$

and substituting in (3'):

$$V_{\pi 2}(125.5) = (2.20)[-V_{\pi 1}(170.16) - V_{\pi 2}(11.71)] - V_{\pi 1}(0.20)$$

$$V_{\pi 2}(151.3) = -V_{\pi 1}(374.55)$$

or $V_{\pi 1} = -V_{\pi 2}(0.4040)$ so that

Then

$$I_s = (35.75)[-V_{\pi 2}(0.4040)] + V_{\pi 2}(2.342)$$

$$I_s = -V_{\pi 2}(12.10)$$

$$I_0 = -(g_m V_{\pi 2}) \left(\frac{R_{C2}}{R_{C2} + R_L} \right)$$

$$= -(123) \left(\frac{2.2}{2.2 + 2} \right) V_{\pi 2} = -(64.43) V_{\pi 2}$$

or $V_{\pi 2} = -(0.01552) I_0$

Then

$$\frac{I_0}{I_s} = \frac{1}{(0.01552)(12.10)} \Rightarrow \underline{\frac{I_0}{I_s} = 5.33}$$

12.54

For example, use the circuit shown in Figure P12.49

12.55

$$(a) \frac{V_i - V_L}{R_1} = \frac{V_L - V_O}{R_F}$$

$$\frac{V_O}{R_F} = V_L \left(\frac{1}{R_1} + \frac{1}{R_F} \right) - \frac{V_i}{R_1}$$

$$\text{So (1)} \quad V_O = V_L \left(1 + \frac{R_F}{R_1} \right) - \left(\frac{R_F}{R_1} \right) V_i$$

$$\frac{V_O - V_L}{R_3} = \frac{V_L}{R_L} + \frac{V_L}{R_2}$$

$$\text{So (2)} \quad V_O = V_L \left(1 + \frac{R_3}{R_L} + \frac{R_3}{R_2} \right)$$

Then, from (1) = (2)

$$V_L \left(1 + \frac{R_F}{R_1} \right) - \left(\frac{R_F}{R_1} \right) V_i = V_L \left(1 + \frac{R_3}{R_L} + \frac{R_3}{R_2} \right)$$

$$V_L \left(1 + \frac{R_F}{R_1} - 1 - \frac{R_3}{R_L} - \frac{R_3}{R_2} \right) = \left(\frac{R_F}{R_1} \right) V_i$$

$$\text{Now } V_L = I_O R_L$$

$$I_O R_L \left(\frac{R_F}{R_1} - \frac{R_3}{R_L} - \frac{R_3}{R_2} \right) = \left(\frac{R_F}{R_1} \right) V_i$$

$$A_{gf} = \frac{I_o}{V_i} = \frac{\frac{R_F}{R_1}}{\left(\frac{R_L R_F}{R_1} - R_3 - \frac{R_L R_3}{R_2} \right)}$$

(b) Set $\frac{R_F}{R_1} = \frac{R_3}{R_2}$

$$\text{Then } A_{gf} = \frac{\cancel{R_F}/\cancel{R_1}}{-R_3} = \frac{-R_F}{R_1 R_3}$$

$$\text{Or } A_{gf} = \frac{I_o}{V_i} = \frac{-1}{R_2}$$

(c) For $\frac{I_o}{V_i} = -0.5 \text{ mA/V} = \frac{-1}{R_2} \Rightarrow R_2 = 2 \text{ k}\Omega$

$$\text{For } \frac{R_F}{R_1} = \frac{R_3}{R_2}, \text{ set } R_3 = 2 \text{ k}\Omega \text{ and } R_1 = R_F = 10 \text{ k}\Omega$$

12.56

(a) $V_{D1} = 3 - (1)(1.6) = 1.40 \text{ V}, V_{SG3} = (1)(1.6) = 1.6 \text{ V}$

$$I_{DQ3} = K_p (V_{SG3} + V_{TP})^2 = (10)(1.6 - 0.5)^2 = 12.1 \text{ mA}$$

$$V_G = (12.1)(0.248) - 3 \approx 0$$

(b) $g_{m1}V_{gs1} + g_{m1}V_{gs2} = 0 \Rightarrow V_{gs2} = -V_{gs1}$

$$V_{sg3} = g_{m1}V_{gs1}R_D$$

$$I_o = g_{m3}V_{sg3} = g_{m1}g_{m3}R_DV_{gs1}$$

$$V_i = V_{gs1} - V_{gs2} + I_oR_L = 2V_{gs1} + I_oR_L$$

$$V_{gs1} = \frac{V_i - I_oR_L}{2}$$

$$I_o = g_{m1}g_{m3}R_D \left(\frac{V_i - I_oR_L}{2} \right)$$

$$I_o \left(1 + \frac{g_{m1}g_{m3}R_D R_L}{2} \right) = \frac{g_{m1}g_{m3}R_D}{2} \cdot V_i$$

$$A_{gf} = \frac{I_o}{V_i} = \frac{g_{m1}g_{m3}R_D}{2 + g_{m1}g_{m3}R_D R_L}$$

(c) $g_{m1} = 2\sqrt{K_n I_{DQ1}} = 2\sqrt{(2)(1)} = 2.828 \text{ mA/V}$

$$g_{m3} = 2\sqrt{K_p I_{DQ3}} = 2\sqrt{(10)(12.1)} = 22 \text{ mA/V}$$

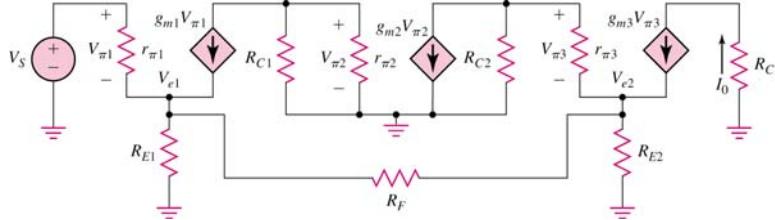
$$A_{gf} = \frac{(2.828)(22)(1.6)}{2 + (2.828)(22)(1.6)(0.248)} = 3.73 \text{ mA/V}$$

12.57

$$r_{\pi 1} = 6.24 \text{ k}\Omega, r_{\pi 2} = 3.12 \text{ k}\Omega, r_{\pi 3} = 1.56 \text{ k}\Omega$$

$$g_{m1} = 19.23 \text{ mA/V}, g_{m2} = 38.46 \text{ mA/V},$$

$$g_{m3} = 76.92 \text{ mA/V}$$



$$V_s = V_{\pi 1} + V_{e1} \quad (1)$$

$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} = \frac{V_{e1}}{R_{E1}} + \frac{V_{e1} - V_{e3}}{R_F} \quad (2)$$

$$V_{\pi 2} = -g_{m1}V_{\pi 1}(R_{C1} \parallel r_{\pi 2}) \quad (3)$$

$$g_{m2}V_{\pi 2} + \frac{V_{\pi 3} + V_{e3}}{R_{C2}} + \frac{V_{\pi 3}}{r_{\pi 3}} = 0 \quad (4)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3}V_{\pi 3} = \frac{V_{e3}}{R_{E2}} + \frac{V_{e3} - V_{e1}}{R_F} \quad (5)$$

Enter numerical values in (2)-(5):

$$\frac{V_{\pi 1}}{6.24} + (19.23)V_{\pi 1} = V_{e1} \left(\frac{1}{0.1} + \frac{1}{0.8} \right) - V_{e3} \left(\frac{1}{0.8} \right)$$

or

$$V_{\pi 1}(19.39) = V_{e1}(11.25) - V_{e3}(1.25) \quad (2)$$

$$V_{\pi 2} = -(19.23)V_{\pi 1}(5 \parallel 3.12) = -(36.94)V_{\pi 1} \quad (3)$$

$$(38.46)V_{\pi 2} + V_{\pi 3} \left(\frac{1}{2} + \frac{1}{1.56} \right) + V_{e3} \left(\frac{1}{2} \right) = 0 \quad (4)$$

or

$$V_{\pi 2}(38.46) + V_{\pi 3}(1.141) + V_{e3}(0.5) = 0 \quad (4)$$

$$V_{\pi 3} \left(\frac{1}{1.56} + 76.92 \right) = V_{e3} \left(\frac{1}{0.1} + \frac{1}{0.8} \right) - V_{e3} \left(\frac{1}{0.8} \right)$$

or

$$V_{\pi 3}(77.56) = V_{e3}(11.25) - V_{e1}(1.25) \quad (5)$$

From (1) $V_{\pi 1} = V_s - V_{e1}$

Then

$$(V_s - V_{e1})(19.39) = V_{e1}(11.25) - V_{e3}(1.25)$$

$$\text{or } V_s(19.39) = V_{e1}(30.64) - V_{e3}(1.25) \quad (2')$$

$$V_{\pi 2} = -V_s(36.94) + V_{e1}(36.94) \quad (3')$$

$$(38.46)[-V_s(36.94) + V_{e1}(36.94)] + V_{\pi 3}(1.141) + V_{e3}(0.5) = 0 \quad (4')$$

From (5): $V_{e3} = V_{\pi 3}(6.894) + V_{e1}(0.1111)$

Then

$$V_s(19.39) = V_{e1}(30.64) - (1.25)[V_{\pi 3}(6.894) + V_{e1}(0.1111)]$$

or

$$V_s(19.39) = V_{e1}(30.50) - V_{\pi 3}(8.6175) \quad (2'')$$

and

$$-V_s(1420.7) + V_{el}(1420.7) + V_{\pi_3}(1.141) + (0.5)[V_{\pi_3}(6.894) + V_{el}(0.1111)] = 0$$

or

$$-V_s(1420.7) + V_{el}(1420.76) + V_{\pi_3}(4.588) = 0 \quad (4'')$$

From (2''):

$$V_{el} = V_s(0.6357) + V_{\pi_3}(0.2825)$$

Then substituting in (4''):

$$-V_s(1420.7) + (1420.76)[V_s(0.6357) + V_{\pi_3}(0.2825)] + V_{\pi_3}(4.588) = 0$$

$$-V_s(517.5) + V_{\pi_3}(405.95) = 0$$

Now

$$I_0 = g_{m3}V_{\pi_3} = 76.92V_{\pi_3} \text{ or } V_{\pi_3} = I_0(0.0130)$$

$$\text{Then } -V_s(517.5) + I_0(0.0130)(405.95) = 0$$

or

$$\frac{I_0}{V_s} = 98.06 \text{ mA/V}$$

12.58

Computer Analysis

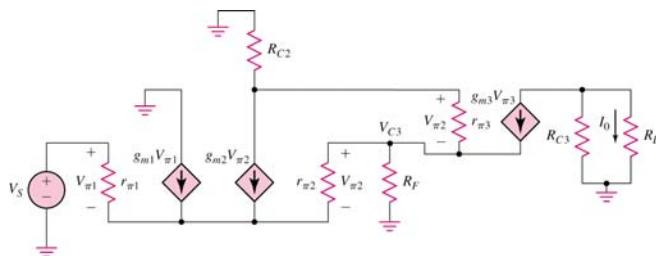
12.59

$$r_{\pi 1} = r_{\pi 2} = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$g_{m1} = g_{m2} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(100)(0.026)}{2} = 1.3 \text{ k}\Omega$$

$$g_{m3} = \frac{2}{0.026} = 76.92 \text{ mA/V}$$



$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} + g_{m2}V_{\pi 2} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \quad (1)$$

Since $r_{\pi 1} = r_{\pi 2}$ and $g_{m1} = g_{m2}$, then $V_{\pi 1} = -V_{\pi 2}$

$$V_s = V_{\pi 1} - V_{\pi 2} + V_{e3} = -2V_{\pi 2} + V_{e3} \quad (2)$$

$$g_{m2}V_{\pi 2} + \frac{V_{\pi 3}}{r_{\pi 3}} + \frac{V_{\pi 3} + V_{e3}}{R_{C2}} = 0 \quad (3)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3}V_{\pi 3} = \frac{V_{e3}}{R_F} + \frac{V_{\pi 2}}{r_{\pi 2}} \quad (4)$$

$$I_0 = -\left(\frac{R_{C3}}{R_{C3} + R_L}\right)(g_{m3}V_{\pi3}) \quad (5)$$

From (2): $V_{e3} = V_s + 2V_{\pi2}$

$$(19.23) \quad V_{\pi2} + \frac{V_{\pi3}}{1.3} + \frac{V_{\pi3}}{18.6} + \frac{1}{18.6}(V_s + 2V_{\pi2}) = 0$$

or

$$(19.23)V_{\pi2} + (0.8230)V_{\pi3} + (0.05376)V_s = 0 \quad (3')$$

$$V_{\pi3} \left(\frac{1}{1.3} + 76.92 \right) = \left(\frac{1}{10} \right) (V_s + 2V_{\pi2}) + \frac{V_{\pi2}}{5.2}$$

or

$$(77.69)V_{\pi3} = (0.3923)V_{\pi2} + (0.1)V_s \quad (4')$$

$$I_0 = -\left(\frac{2}{2+1}\right)(76.92)V_{\pi3} = -(51.28)V_{\pi3} \quad (5')$$

From (3'):

$$V_{\pi2} = -(0.04255)V_{\pi3} - (0.002780)V_s$$

Then

$$(77.69)V_{\pi3} = (0.3923)[-(0.04255)V_{\pi3} - (0.002780)V_s] + (0.1)V_s$$

$$(77.71)V_{\pi3} = (0.0989)V_s$$

or

$$V_{\pi3} = (0.001273)V_s$$

so that

$$I_0 = -(51.28)(0.001273)V_s$$

or

$$\frac{I_0}{V_s} = -(0.0653) \text{ mA/V}$$

12.60

$$(a) \quad A_{gf} = \frac{I_o}{V_i} = \frac{1}{R_E} = 3 \text{ mA/V} \Rightarrow R_E = 333.3 \Omega$$

$$(b) \quad A_{gf} = \frac{I_o}{V_i} = \frac{h_{FE}A_g}{1 + (h_{FE})A_g R_E} = \frac{(80)(1000)}{1 + (80)(1000)(0.3333)} = 2.9998875 \text{ mA/V}$$

$$I_o = A_{gf}(5) = 14.9994377$$

$$\text{Error} = \frac{14.9994377 - 15.0}{15.0} \times 100\% = -0.00375\%$$

12.61

$$(a) \quad 5 = (1 + h_{FE})I_{BQ}R_E + V_{EB}(\text{on}) + I_{BQ}R_B$$

$$I_{BQ} = \frac{5 - 0.7}{100 + (81)(0.5)} = 0.030605 \text{ mA}$$

$$I_{CQ} = 2.448 \text{ mA, and } I_{EQ} = 2.479 \text{ mA}$$

$$V_{ECQ} = 5 - (2.479)(0.5) - (2.448)(1) = 1.31 \text{ V}$$

$$(b) \frac{V_o}{R_C} + \frac{V_o}{r_o} + h_{FE} I_b + \frac{V_o - V_\pi}{R_F} = 0$$

$$V_o \left(\frac{1}{R_C} + \frac{1}{r_o} + \frac{1}{R_F} \right) + h_{FE} I_b - \frac{V_\pi}{R_F} = 0$$

$$V_\pi = I_b r_\pi$$

$$I_s = \frac{V_\pi}{R_B} + \frac{V_\pi}{r_\pi} + \frac{V_\pi - V_o}{R_F}$$

$$I_s = I_b \left(1 + \frac{r_\pi}{R_B} + \frac{r_\pi}{R_F} \right) - \frac{V_o}{R_F}$$

$$I_b = \frac{I_s + \frac{V_o}{R_F}}{\left(1 + \frac{r_\pi}{R_B} + \frac{r_\pi}{R_F} \right)}$$

$$V_o \left(\frac{1}{R_C} + \frac{1}{r_o} + \frac{1}{R_F} \right) + \left(h_{FE} - \frac{r_\pi}{R_F} \right) \left[\frac{I_s + \frac{V_o}{R_F}}{1 + \frac{r_\pi}{R_B} + \frac{r_\pi}{R_F}} \right] = 0$$

$$V_o \left(\frac{1}{R_C} + \frac{1}{r_o} + \frac{1}{R_F} \right) \left(1 + \frac{r_\pi}{R_B} + \frac{r_\pi}{R_F} \right) + \left(h_{FE} - \frac{r_\pi}{R_F} \right) \cdot I_s + \left(h_{FE} - \frac{r_\pi}{R_F} \right) \cdot \frac{V_o}{R_F} = 0$$

$$\text{Now } r_o = \frac{V_A}{I_{CQ}} = \frac{100}{2.448} = 40.85 \text{ k}\Omega$$

$$r_\pi = \frac{h_{FE} V_T}{I_{CQ}} = \frac{(80)(0.026)}{2.448} = 0.8497 \text{ k}\Omega$$

$$\text{Then } V_o \left(\frac{1}{1 + \frac{1}{40.85} + \frac{1}{10}} \right) \left(1 + \frac{0.8497}{100} + \frac{0.8497}{10} \right) + \left(80 - \frac{0.8497}{10} \right) \cdot I_s + \left(80 - \frac{0.8497}{10} \right) \cdot \frac{V_o}{10} = 0$$

$$V_o (9.22114) + I_s (79.915) = 0$$

$$A_{zf} = \frac{V_o}{I_s} = -8.666 \text{ V/mA}$$

$$(c) R_{if} = \frac{V_\pi}{I_s} = \frac{I_b r_\pi}{I_s} = r_\pi \left(\frac{1 + \frac{V_o}{I_s} \cdot \frac{1}{R_F}}{1 + \frac{r_\pi}{R_B} + \frac{r_\pi}{R_F}} \right) = (0.8497) \left(\frac{1 - \frac{8.666}{10}}{1 + \frac{0.8497}{100} + \frac{0.8497}{10}} \right)$$

$$R_{if} = 0.1037 \text{ k}\Omega$$

$$(d) I_x = \frac{V_x}{R_C} + \frac{V_x}{r_o} + h_{FE} I_b + \frac{V_x}{R_F + R_B \| r_\pi }$$

$$I_b = \frac{V_\pi}{r_\pi}, \text{ and } V_\pi = \left(\frac{R_B \| r_\pi}{R_B \| r_\pi + R_F} \right) \cdot V_x$$

$$\text{Then } \frac{I_x}{V_x} = \frac{1}{R_{of}} = \frac{1}{R_C} + \frac{1}{r_o} + \left(\frac{h_{FE}}{r_\pi} \right) \left(\frac{R_B \| r_\pi}{R_B \| r_\pi + R_F} \right) + \frac{1}{R_B \| r_\pi + R_F}$$

$$R_B \| r_\pi = 100 \| 0.8497 = 0.8425 \text{ k}\Omega$$

$$\frac{1}{R_{of}} = \frac{1}{1} + \frac{1}{40.85} + \left(\frac{80}{0.8497} \right) \left(\frac{0.8425}{0.8425+10} \right) + \frac{1}{0.8425+10}$$

Then $R_{of} = 0.1186 \text{ k}\Omega$

12.62

(a) (i) $V^+ = I_{DQ}R_D + V_{GS} = K_n R_D (V_{GS} - V_{TN})^2 + V_{GS}$

$$3 = (0.5)(5)(V_{GS}^2 - 0.8V_{GS} + 0.16) + V_{GS}$$

$$\text{We find } 2.5V_{GS}^2 - V_{GS} - 2.6 = 0 \Rightarrow V_{GS} = 1.239 \text{ V}$$

$$I_{DQ} = (0.5)(1.239 - 0.4)^2 = 0.3522 \text{ mA}$$

$$(ii) g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.3522)} = 0.8392 \text{ mA/V}$$

(b) (1) $\frac{V_o}{R_D} + g_m V_{gs} + \frac{V_o - V_{gs}}{R_F} = 0$

(2) $I_s = \frac{V_{gs} - V_o}{R_F} \Rightarrow V_{gs} = I_s R_F + V_o$

Then $V_o \left(\frac{1}{R_D} + \frac{1}{R_F} \right) + \left(g_m - \frac{1}{R_F} \right) (I_s R_F + V_o) = 0$

$$V_o \left(\frac{1}{R_D} + g_m \right) + I_s (g_m R_F - 1) = 0$$

$$A_{zf} = \frac{V_o}{I_s} = \frac{1 - g_m R_F}{\frac{1}{R_D} + g_m} = \frac{1 - (0.8392)(25)}{\frac{1}{5} + 0.8392}$$

$$A_{zf} = -19.23 \text{ V/mA}$$

(c) $I_x = \frac{V_x}{R_D} + g_m V_{gs} \quad \text{and} \quad V_{gs} = V_x$

$$\frac{I_x}{V_x} = \frac{1}{R_{of}} = \frac{1}{R_D} + g_m = \frac{1}{5} + 0.8392$$

$$R_{of} = 0.962 \text{ k}\Omega$$

12.63

(a) $A_{zf} = \frac{1 - g_m R_F}{\frac{1}{R_D} + g_m} = \frac{\frac{1}{g_m} - R_F}{\frac{1}{g_m R_D} + 1}$

As $g_m \rightarrow \infty$, $A_{zf} = -R_F$

$$(b) -0.95R_F = \frac{1-g_m R_F}{\frac{1}{R_D} + g_m}$$

$$-23.75 = \frac{1-g_m(25)}{0.2+g_m}$$

So that $g_m = 4.6 \text{ mA/V}$

12.64

dc analysis

$$R_{TH} = 24 \parallel 150 = 20.7 \text{ k}\Omega$$

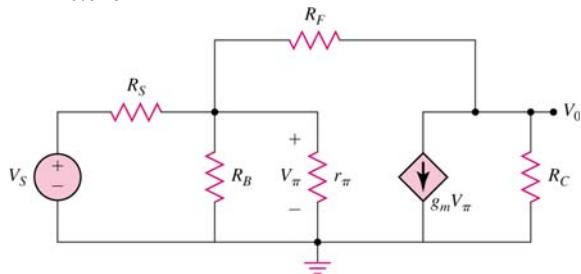
$$V_{TH} = \left(\frac{24}{24+150} \right)(12) = 1.655 \text{ V}$$

$$I_{BO} = \frac{1.655 - 0.7}{20.7 + (151)(1)} = 0.00556 \text{ mA}$$

$$\text{so } I_{CO} = 0.834 \text{ mA}$$

$$r_\pi = \frac{(150)(0.026)}{0.834} = 4.68 \text{ k}\Omega$$

$$g_m = \frac{0.834}{0.026} = 32.08 \text{ mA/V}$$



$$\frac{V_S - V_\pi}{R_S} = \frac{V_\pi}{R_B \parallel r_\pi} + \frac{V_\pi - V_0}{R_F} \quad (1)$$

$$g_m V_\pi + \frac{V_0}{R_C} + \frac{V_0 - V_\pi}{R_F} = 0 \quad (2)$$

From (1):

$$\frac{V_S}{5} = V_\pi \left[\frac{1}{5} + \frac{1}{20.7 \parallel 4.68} + \frac{1}{R_F} \right] - \frac{V_0}{R_F}$$

or

$$V_S (0.20) = V_\pi \left(0.4620 + \frac{1}{R_F} \right) - \frac{V_0}{R_F}$$

From (2):

$$\left(32.08 - \frac{1}{R_F} \right) V_\pi + V_0 \left(\frac{1}{6} + \frac{1}{R_F} \right) = 0$$

so

$$V_\pi = \frac{-V_0 \left(0.1667 + \frac{1}{R_F} \right)}{\left(32.08 - \frac{1}{R_F} \right)} \quad (2)$$

Then

$$V_s(0.20) = \left(0.4620 + \frac{1}{R_F} \right) \left[\frac{-V_0 \left(0.1667 + \frac{1}{R_F} \right)}{\left(32.08 - \frac{1}{R_F} \right)} \right] - \frac{V_0}{R_F}$$

Neglect the R_F in the denominator term. Now

$$\begin{aligned} \frac{V_0}{V_s} &= -5 \Rightarrow V_s = -\frac{V_0}{5} = -V_0(0.20) \\ -V_0(0.20)(0.20)R_F &= (0.4620R_F + 1) \left[\frac{-V_0(0.1667R_F + 1)}{32.08R_F} \right] - V_0 \\ -1.283R_F^2 &= -(0.4620R_F + 1)(0.1667R_F + 1) - 32.08R_F \\ 1.206R_F^2 - 32.71R_F - 1 &= 0 \\ R_F &= \frac{32.71 \pm \sqrt{(32.71)^2 + 4(1.206)(1)}}{2(1.206)} \end{aligned}$$

so that

$$\underline{R_F = 27.2 \text{ k}\Omega}$$

12.65

dc analysis

$$R_{TH} = 4 \parallel 15 = 3.16 \text{ k}\Omega = R_B$$

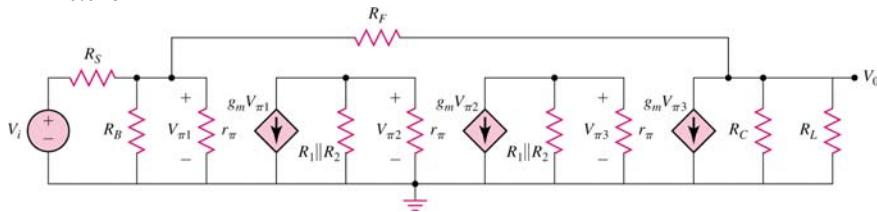
$$V_{TH} = \left(\frac{4}{4+15} \right) 12 = 2.526 \text{ V}$$

$$I_{BQ} = \frac{2.526 - 0.7}{3.16 + (181)(4)} = 0.00251$$

$$I_{CQ} = 0.452 \text{ mA}$$

$$r_\pi = \frac{(180)(0.026)}{0.452} = 10.4 \text{ k}\Omega$$

$$g_m = \frac{0.452}{0.026} = 17.4 \text{ mA/V}$$



$$\frac{V_i - V_{\pi 1}}{R_S} = \frac{V_{\pi 1}}{R_B \parallel r_\pi} + \frac{V_{\pi 1} - V_0}{R_F} \quad (1)$$

$$g_m V_{\pi 1} + \frac{V_{\pi 2}}{R_C \parallel R_B \parallel r_\pi} = 0 \quad (2)$$

$$g_m V_{\pi 2} + \frac{V_{\pi 3}}{R_C \parallel R_B \parallel r_\pi} = 0 \quad (3)$$

$$g_m V_{\pi 3} + \frac{V_0}{R_C} + \frac{V_0 - V_{\pi 1}}{R_L} = 0 \quad (4)$$

Now

$$R_C \parallel R_B \parallel r_\pi = 8 \parallel 3.16 \parallel 10.4 = 1.86 \text{ k}\Omega$$

$$R_B \parallel r_\pi = 3.16 \parallel 10.4 = 2.42 \text{ k}\Omega$$

Now substituting in (2):

$$(17.4)V_{\pi 1} + \frac{V_{\pi 2}}{1.86} = 0 \quad \text{or} \quad V_{\pi 2} = -(32.36)V_{\pi 1}$$

and substituting in (3):

$$(17.4)V_{\pi 2} + \frac{V_{\pi 3}}{1.86} = 0$$

$$(17.4)[-(32.36)V_{\pi 1}] + \frac{V_{\pi 3}}{1.86} = 0$$

$$\text{or } V_{\pi 3} = (1047.3)V_{\pi 1}$$

Substitute numerical values in (1):

$$\frac{V_i}{10} = V_{\pi 1} \left(\frac{1}{10} + \frac{1}{2.42} + \frac{1}{R_F} \right) - \frac{V_0}{R_F}$$

or

$$V_i(0.10) = V_{\pi 1} \left(0.513 + \frac{1}{R_F} \right) - \frac{V_0}{R_F}$$

Substitute numerical values in (4):

$$(17.4)(1047.3)V_{\pi 1} + V_0 \left(\frac{1}{8} + \frac{1}{4} + \frac{1}{R_F} \right) - \frac{V_{\pi 1}}{R_F} = 0$$

$$V_{\pi 1} \left(1.822 \times 10^4 - \frac{1}{R_F} \right) + V_0 \left(0.375 + \frac{1}{R_F} \right) = 0$$

$$V_{\pi 1} = \frac{-V_0 \left(0.375 + \frac{1}{R_F} \right)}{1.822 \times 10^4 - \frac{1}{R_F}}$$

so that

$$V_i(0.10) = \left(0.513 + \frac{1}{R_F} \right) \left[\frac{-V_0 \left(0.375 + \frac{1}{R_F} \right)}{1.822 \times 10^4 - \frac{1}{R_F}} \right] - \frac{V_0}{R_F}$$

$$\frac{V_0}{V_i} = -80 \quad \text{or} \quad V_i = -\frac{V_0}{80}$$

$$-\frac{(0.10)}{80} = \left(0.513 + \frac{1}{R_F} \right) \left[\frac{-\left(0.375 + \frac{1}{R_F} \right)}{1.822 \times 10^4 - \frac{1}{R_F}} \right] - \frac{1}{R_F}$$

Neglect that $1/R_F$ term in the denominator.

$$-(0.00125R_F) = -\frac{(0.513R_F + 1)(0.375R_F + 1)}{1.822 \times 10^4 R_F} - 1$$

$$22.775R_F^2 = (0.513R_F + 1)(0.375R_F + 1) + 1.822 \times 10^4 R_F$$

We find

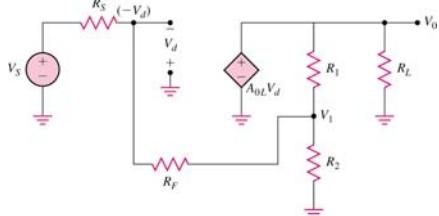
$$22.58R_F^2 - 1.822 \times 10^4 R_F - 1 = 0$$

$$R_F = \frac{1.822 \times 10^4 \pm \sqrt{(1.822 \times 10^4)^2 + 4(22.58)(1)}}{2(22.58)}$$

or

$$\underline{R_F = 0.807 \text{ M}\Omega}$$

12.66



a. $\frac{V_s - (-V_d)}{R_s} = \frac{-V_d - V_1}{R_F}$

or

$$V_d \left(\frac{1}{R_s} + \frac{1}{R_F} \right) + \frac{V_s}{R_s} + \frac{V_1}{R_F} = 0 \quad (1)$$

$$\frac{V_0 - V_1}{R_1} = \frac{V_1}{R_2} + \frac{V_1 - (-V_d)}{R_F}$$

or

$$\frac{V_0}{R_1} = V_1 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_F} \right) + \frac{V_d}{R_F} \quad (2)$$

$$\text{and } V_0 = A_{OL} V_d \text{ or } V_d = \frac{V_0}{A_{OL}}$$

Substitute numerical values in (1) and (2):

$$\frac{V_0}{10^4} \cdot \left(\frac{1}{5} + \frac{1}{10} \right) + \frac{V_s}{5} + \frac{V_1}{10} = 0$$

or

$$V_0(0.3 \times 10^{-4}) + V_s(0.20) + V_1(0.10) = 0 \quad (1)$$

$$\frac{V_0}{50} = V_1 \left(\frac{1}{50} + \frac{1}{10} + \frac{1}{10} \right) + \frac{V_0}{10^4} \cdot \left(\frac{1}{10} \right)$$

or

$$V_0(0.02 - 10^{-5}) = V_1(0.22) \quad (2)$$

$$V_1 = V_0 \left(\frac{0.02 - 10^{-5}}{0.22} \right)$$

Then

and

$$V_0(0.3 \times 10^{-4}) + V_s(0.20) + (0.10) \left[V_0 \left(\frac{0.02 - 10^{-5}}{0.22} \right) \right] = 0$$

$$V_0 \left[0.3 \times 10^{-4} - 0.4545 \times 10^{-5} + 0.00909 \right] + V_s(0.20) = 0$$

$$\frac{V_0}{V_s} = \frac{-0.20}{9.115 \times 10^{-3}} \Rightarrow \underline{\underline{\frac{V_0}{V_s} = -21.94}}$$

Then

$$R_{if} = \frac{-V_d}{V_s - (-V_d)} = \frac{-V_d \cdot R_s}{V_s + V_d}$$

b.

$$V_d = \frac{V_0}{A_{0L}} = -\frac{21.94V_s}{10^4}$$

Now

$$R_{if} = \frac{(21.94 \times 10^{-4})(5)}{1 - 21.94 \times 10^{-4}}$$

Then

$$\text{or } R_{if} = 1.099 \times 10^{-2} \text{ k}\Omega \Rightarrow R_{if} = 10.99 \Omega$$

c. Because of the $A_{0L}V_d$ source,

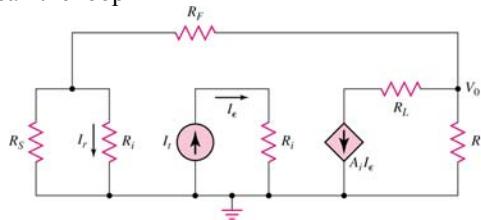
$$\underline{R_{0f} = 0}$$

12.67

For example, use the circuit shown in Figure 12.41

12.68

Break the loop



$$I_t = I_e$$

$$A_t I_t + \frac{V_0}{R_i} + \frac{V_0}{R_F + R_S \| R_i} = 0$$

Now

$$I_r = \left(\frac{R_S}{R_S + R_i} \right) \cdot \frac{V_0}{R_F + R_S \| R_i}$$

$$V_0 = I_r \left(\frac{R_S + R_i}{R_S} \right) \cdot (R_F + R_S \| R_i)$$

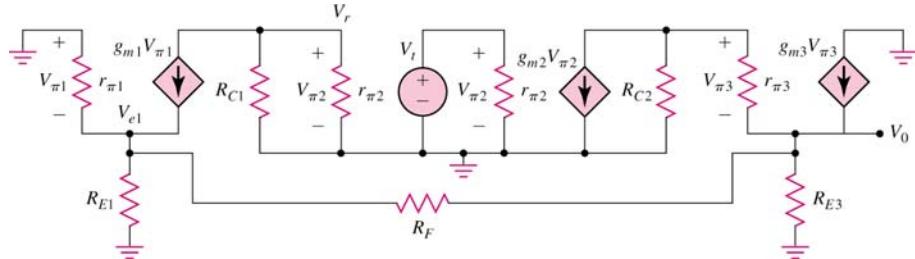
or

Then

$$A_t I_t + \left(\frac{1}{R_i} + \frac{1}{R_F + R_S \| R_i} \right) \times \left[I_r \left(\frac{R_S + R_i}{R_S} \right) (R_F + R_S \| R_i) \right] = 0$$

$$T = -\frac{I_r}{I_t} \Rightarrow T = \frac{A_t}{\left[\frac{1}{R_i} + \frac{1}{R_F + R_S \| R_i} \right] \left(\frac{R_S + R_i}{R_S} \right) (R_F + R_S \| R_i)}$$

12.69



$$\frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} = \frac{V_{e1}}{R_{E1}} + \frac{V_{e1} - V_0}{R_F} \quad (1)$$

$$g_{m1}V_{\pi1} + \frac{V_r}{R_{C1}\parallel r_{\pi2}} = 0 \Rightarrow V_r = -(g_{m1}V_{\pi1})(R_{C1}\parallel r_{\pi2}) \quad (2)$$

$V_{\pi2} = V_t$ so that

$$g_{m2}V_t + \frac{V_{\pi3} + V_0}{R_{C2}} + \frac{V_{\pi3}}{r_{\pi3}} = 0 \quad (3)$$

$$\frac{V_{\pi3}}{r_{\pi3}} + g_{m3}V_{\pi3} = \frac{V_0}{R_{E3}} + \frac{V_0 - V_{e1}}{R_F} \quad (4)$$

From (4):

$$V_0 \left(\frac{1}{R_{E3}} + \frac{1}{R_F} \right) = V_{\pi3} \left(\frac{1}{r_{\pi3}} + g_{m3} \right) + \frac{V_{e1}}{R_F}$$

But $V_{e1} = -V_{\pi1}$

$$V_0 = \frac{V_{\pi3} \left(\frac{1}{r_{\pi3}} + g_{m3} \right) - \frac{V_{\pi1}}{R_F}}{\left(\frac{1}{R_{E3}} + \frac{1}{R_F} \right)}$$

so

Then

$$V_{\pi1} \left[\left(\frac{1}{r_{\pi1}} + g_{m1} \right) - \left(\frac{1}{R_{E1}} + \frac{1}{R_F} \right) \right] = \frac{-V_{\pi3} \left(\frac{1}{r_{\pi3}} + g_{m3} \right) + \frac{V_{\pi1}}{R_F}}{R_F \cdot \left(\frac{1}{R_{E3}} + \frac{1}{R_F} \right)} \quad (1')$$

and

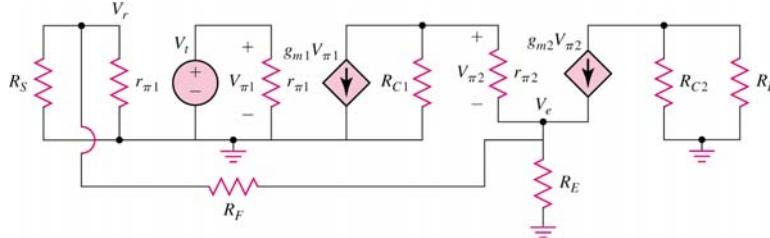
$$g_{m2}V_t + V_{\pi3} \left(\frac{1}{R_{C2}} + \frac{1}{r_{\pi3}} \right) + \frac{V_{\pi3} \left(\frac{1}{r_{\pi3}} + g_{m3} \right) - \frac{V_{\pi1}}{R_F}}{R_{C2} \cdot \left(\frac{1}{R_{E3}} + \frac{1}{R_F} \right)} = 0 \quad (3')$$

From (3'), solve for $V_{\pi3}$ and substitute into (1'). Then from (1'), solve for $V_{\pi1}$ and substitute into (2).

$$T = -\frac{V_r}{V_t}$$

Then

12.70



$$\frac{V_r}{R_S} + \frac{V_r}{r_{\pi 1}} + \frac{V_r - V_e}{R_F} = 0 \quad (1)$$

$$g_{m1} V_t + \frac{V_{\pi 2} + V_e}{R_{C1}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \quad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2} V_{\pi 2} = \frac{V_e}{R_E} + \frac{V_e - V_r}{R_F} \quad (3)$$

Assume $r_{\pi 1} = 15.8 \text{ k}\Omega$, $g_{m1} = 7.62 \text{ mA/V}$, $r_{\pi 2} = 2.28 \text{ k}\Omega$, $g_{m2} = 52.7 \text{ mA/V}$

$$V_r \left(\frac{1}{10} + \frac{1}{15.8} + \frac{1}{10} \right) - \frac{V_e}{10} = 0$$

or

$$V_r(0.2633) = V_e(0.10) \quad (1)$$

$$(7.62)V_t + V_{\pi 2} \left(\frac{1}{40} + \frac{1}{2.28} \right) + \frac{V_e}{40} = 0$$

or

$$V_t(7.62) + V_{\pi 2}(0.4636) + V_e(0.025) = 0 \quad (2)$$

$$V_{\pi 2} \left(\frac{1}{2.28} + 52.7 \right) = V_e \left(\frac{1}{1} + \frac{1}{10} \right) - \frac{V_r}{10}$$

or

$$V_{\pi 2}(53.14) = V_e(1.10) - V_r(0.10)$$

Then

$$V_{\pi 2} = V_e(0.0207) - V_r(0.001882) \quad (3)$$

Substituting in (2):

$$V_t(7.62) + (0.4636)[V_e(0.0207) - V_r(0.001882)] + V_e(0.025) = 0$$

or

$$V_t(7.62) + V_e(0.03460) - V_r(0.0008725) = 0$$

From (1) $V_e = V_r(2.633)$

Then

$$V_t(7.62) + V_r(2.633)(0.03460) - V_r(0.0008725) = 0$$

$$V_t(7.62) + V_r(0.09023) = 0$$

$$\frac{V_r}{V_t} = -84.45$$

or $\frac{V_t}{V_r}$

Now

$$T = -\frac{V_r}{V_t} \Rightarrow T = \underline{84.45}$$

12.71

$$R_{TH} = R_1 \parallel R_2 = 24 \parallel 150 = 20.69 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left(\frac{24}{24+150} \right) (12) = 1.655 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1+h_{FE})R_E} = \frac{1.655 - 0.7}{20.69 + (51)(1)} = 0.0133 \text{ mA}$$

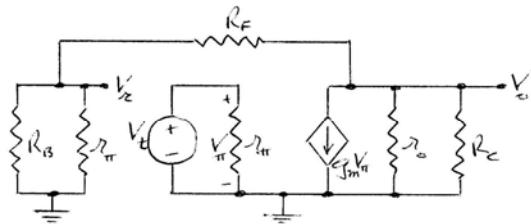
$$I_{CQ} = 0.666 \text{ mA}$$

$$g_m = \frac{0.666}{0.026} = 25.62 \text{ mA/V}$$

$$r_\pi = \frac{(50)(0.026)}{0.666} = 1.951 \text{ k}\Omega$$

$$r_o = \frac{100}{0.666} = 150 \text{ k}\Omega$$

From Problem 12.64, let $R_F = 27.2 \text{ k}\Omega$



We see that $V_\pi = V_t$. Let $R_B = R_1 \parallel R_2 \parallel R_S$

$$(1) \quad g_m V_t + \frac{V_o}{r_o \parallel R_C} + \frac{V_o}{R_F + r_\pi \parallel R_B} = 0$$

$$(2) \quad V_r = \left(\frac{R_B \parallel r_\pi}{R_B \parallel r_\pi + R_F} \right) \cdot V_o$$

$$\text{Now } R_B = 24 \parallel 150 \parallel 5 = 20.69 \parallel 5 = 4.027 \text{ k}\Omega; \quad r_\pi \parallel R_B = 1.951 \parallel 4.027 = 1.314 \text{ k}\Omega$$

$$r_o \parallel R_C = 150 \parallel 6 = 5.769 \text{ k}\Omega$$

$$(1) \quad 25.62 V_t + V_o \left[\frac{1}{5.769} + \frac{1}{27.2 + 1.314} \right] = 0 \Rightarrow V_o = -122.93 V_t$$

$$(2) \quad V_r = \left(\frac{1.314}{1.314 + 27.2} \right) \cdot V_o = (0.04608)(-122.93) V_t$$

$$T = -\frac{V_r}{V_t} = 5.66$$

12.72

$$(a) \quad \phi = -\tan^{-1} \left(\frac{f}{10^3} \right) - 2 \tan^{-1} \left(\frac{f}{5 \times 10^4} \right)$$

$$\text{At } f_{180} = 5.1 \times 10^4 \text{ Hz}, \quad \phi \cong -180^\circ$$

$$(b) |T| = 0.25 = \frac{\beta(5 \times 10^4)}{\sqrt{1 + \left(\frac{5.1 \times 10^4}{10^3}\right)^2} \left[1 + \left(\frac{5.1 \times 10^4}{5 \times 10^4}\right)^2\right]}$$

$$0.25 = \frac{\beta(5 \times 10^4)}{(51.0098)(2.0404)}$$

$$\beta = 5.2 \times 10^{-4}$$

12.73

$$(a) \phi = -2 \tan^{-1} \left(\frac{f}{5 \times 10^3} \right) - \tan^{-1} \left(\frac{f}{5 \times 10^5} \right)$$

At $f_{180} = 7.1 \times 10^4$ Hz, $\phi \approx -180^\circ$

$$(b) |T| = 1 = \frac{\beta(10^4)}{\sqrt{1 + \left(\frac{7.1 \times 10^4}{5 \times 10^3}\right)^2} \sqrt{1 + \left(\frac{7.1 \times 10^4}{5 \times 10^5}\right)^2}}$$

$$1 = \frac{\beta(10^4)}{(202.64)(1.01)} \Rightarrow \beta = 0.0205$$

$$(c) A_{vf}(0) = \frac{10^4}{1 + (0.0205)(10^4)} = 48.54$$

(d) Smaller

12.74

$$\phi = -\tan^{-1} \left(\frac{f}{10^4} \right) - \tan^{-1} \left(\frac{f}{5 \times 10^4} \right) - \tan^{-1} \left(\frac{f}{10^5} \right)$$

At $f = 8.1 \times 10^4$ Hz, $\phi = -180.28^\circ$

Determine $|T(f)|$ at this frequency.

$$|T| = \beta(10^3) \times \frac{1}{\sqrt{1 + \left(\frac{8.1 \times 10^4}{10^4}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{8.1 \times 10^4}{5 \times 10^4}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{8.1 \times 10^4}{10^5}\right)^2}}$$

$$= \frac{\beta(10^3)}{(8.161)(1.904)(1.287)}$$

a. For $\beta = 0.005$

$$|T(f)| = 0.250 < 1 \Rightarrow \text{Stable}$$

b. For $\beta = 0.05$

$$|T(f)| = 2.50 > 1 \Rightarrow \text{Unstable}$$

12.75

(b) Phase margin = $80^\circ \Rightarrow \phi = -100^\circ$

$$\phi = -100 = -2 \tan^{-1} \left(\frac{f}{10^3} \right) - \tan^{-1} \left(\frac{f}{5 \times 10^4} \right)$$

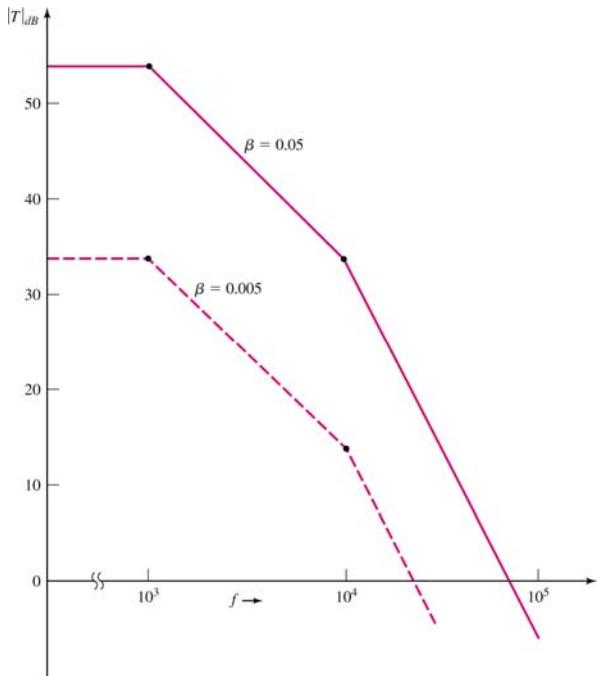
By trial and error, $f = 1.16 \times 10^3$ Hz

Then

$$|T| = 1 = \frac{\beta(5 \times 10^3)}{\left(\sqrt{1 + \left(\frac{1.16 \times 10^3}{10^3} \right)^2} \right)^2 \cdot \sqrt{1 + \left(\frac{1.16 \times 10^3}{5 \times 10^4} \right)^2}}$$

$$= \frac{\beta(5 \times 10^3)}{(2.35)(1.00)} \Rightarrow \underline{\beta = 4.7 \times 10^{-4}}$$

12.76



c. For $\beta = 0.005$,

$|T(f)| = 1$ (0 dB) at $f \approx 2.10 \times 10^4$ Hz

Then

$$\phi = -\tan^{-1}\left(\frac{2.10 \times 10^4}{10^3}\right) - \tan^{-1}\left(\frac{2.10 \times 10^4}{10^4}\right) - \tan^{-1}\left(\frac{2.10 \times 10^4}{10^5}\right)$$

$$= -87.27 - 64.54 - 11.86$$

or

$$\phi = -163.7$$

System is stable.

Phase margin = 16.3°

For $\beta = 0.05$,

$|T(f)| = 1$ (0 dB) at $f \approx 6.44 \times 10^4$ Hz

Then

$$\phi = -\tan^{-1}\left(\frac{6.44 \times 10^4}{10^3}\right) - \tan^{-1}\left(\frac{6.44 \times 10^4}{10^4}\right) - \tan^{-1}\left(\frac{6.44 \times 10^4}{10^5}\right)$$

$$= -89.11 - 81.17 - 32.78$$

or

$\phi = -203.1^\circ \Rightarrow$ System is unstable.

12.77

$$(a) \phi = -\tan^{-1}\left(\frac{f}{10^3}\right) - \tan^{-1}\left(\frac{f}{10^5}\right) - \tan^{-1}\left(\frac{f}{10^7}\right)$$

At $f_{180} = 10^6 \text{ Hz}$, $\phi \approx -180^\circ$

$$(b) \phi = -135^\circ = -\tan\left(\frac{f_{135}}{10^3}\right) - \tan\left(\frac{f_{135}}{10^5}\right) - \tan\left(\frac{f_{135}}{10^7}\right)$$

$$f_{135} \approx 10^5 \text{ Hz}$$

$$|T(f_{135})| = 1 = \frac{\beta(5 \times 10^4)}{\sqrt{1 + \left(\frac{10^5}{10^3}\right)^2} \sqrt{1 + \left(\frac{10^5}{10^5}\right)^2} \sqrt{1 + \left(\frac{10^5}{10^7}\right)^2}}$$

$$1 = \frac{\beta(5 \times 10^4)}{(100)(1.414)(1)} \Rightarrow \beta = 2.83 \times 10^{-3}$$

$$(c) A_{vf}(0) = \frac{5 \times 10^4}{1 + (2.83 \times 10^{-3})(5 \times 10^4)} = 351$$

12.78

$$(a) 100 = \frac{10^5}{1 + (10^5)\beta} \Rightarrow \beta = 9.99 \pm 10^{-3}$$

$$|T| = 1 = \beta A_v = \frac{(9.99 \times 10^{-3})(10^5)}{\sqrt{1 + \left(\frac{f}{10^3}\right)^2} \sqrt{1 + \left(\frac{f}{10^5}\right)^2}}$$

$$1 = \frac{999}{\sqrt{1 + \left(\frac{f}{10^3}\right)^2} \sqrt{1 + \left(\frac{f}{10^5}\right)^2}}$$

$$f = 3.08 \times 10^5 \text{ Hz}$$

Phase

$$\begin{aligned} \phi &= -\tan^{-1}\frac{f}{10^3} - \tan^{-1}\frac{f}{10^5} \\ &= -\tan^{-1}\frac{3.08 \times 10^5}{10^3} - \tan^{-1}\frac{3.08 \times 10^5}{10^5} \\ &= -89.81 - 72.01 \end{aligned}$$

$$\phi = -161.8$$

Stable

$$(b) \text{ Phase Margin} = 180 - 161.8 = 18.2^\circ$$

12.79

$$(a) \phi = -\tan^{-1}\left(\frac{f}{10^4}\right) - \tan^{-1}\left(\frac{f}{10^5}\right) - \tan^{-1}\left(\frac{f}{10^6}\right)$$

At $f_{180} = 3.33 \times 10^5$ Hz, $\phi \equiv -180^\circ$

$$(b) (i) |T(f_{180})| = \frac{(0.019)(10^3)}{\sqrt{1+(33.3)^2} \sqrt{1+(3.33)^2} \sqrt{1+(0.333)^2}}$$

$$|T(f_{180})| = \frac{(0.019)(10^3)}{(33.315)(3.477)(1.054)} = 0.156$$

$$(ii) |T| = 1 = \frac{(0.019)(10^3)}{\sqrt{1+\left(\frac{f}{10^4}\right)^2} \sqrt{1+\left(\frac{f}{10^5}\right)^2} \sqrt{1+\left(\frac{f}{10^6}\right)^2}} \Rightarrow f = 1.2 \times 10^5$$

$$\phi = -\tan^{-1}\left(\frac{1.2 \times 10^5}{10^4}\right) - \tan^{-1}\left(\frac{1.2 \times 10^5}{10^5}\right) - \tan^{-1}\left(\frac{1.2 \times 10^5}{10^6}\right)$$

$$= -85.236^\circ - 50.194^\circ - 6.843^\circ$$

$$\phi = -142.3^\circ$$

$$(c) A_f(0) = \frac{10^3}{1+(0.019)(10^3)} = 50$$

12.80

$$\phi = -180 = -\tan^{-1} \frac{f_{180}}{5 \times 10^3} - 2 \tan^{-1} \frac{f_{180}}{10^5}$$

$$(a) f_{180} = 1.05 \times 10^5$$

$$|T| = \frac{(0.0045)(2 \times 10^3)}{\sqrt{1+\left(\frac{1.05 \times 10^5}{5 \times 10^3}\right)^2} \left[1+\left(\frac{1.05 \times 10^5}{10^5}\right)^2\right]}$$

$$= \frac{9}{(21.02)(2.1025)}$$

$$(b) |T| = f_{180}^0 = 0.204$$

System is stable

$$|T| = 1 = \frac{9}{\sqrt{1+\left(\frac{f}{5 \times 10^3}\right)^2} \left[1+\left(\frac{f}{10^5}\right)^2\right]}$$

$$f = 3.88 \times 10^4$$

$$\phi = -\tan^{-1} \frac{3.88 \times 10^4}{5 \times 10^3} - 2 \tan^{-1} \frac{3.88 \times 10^4}{10^5}$$

$$= -82.66 - 42.41$$

$$\phi = -125.1^\circ$$

$$|T| = \frac{(0.15)(2 \times 10^3)}{\sqrt{1 + \left(\frac{1.05 \times 10^5}{5 \times 10^3}\right)^2} \left[1 + \left(\frac{1.05 \times 10^5}{10^5}\right)^2\right]} \\ = \frac{300}{(21.02)(2.1025)}$$

(c) $|T| = 6.79$

System is unstable

$$|T| = 1 = \frac{300}{\sqrt{1 + \left(\frac{f}{5 \times 10^3}\right)^2} \left[1 + \left(\frac{f}{10^5}\right)^2\right]} \\ f = 2.33 \times 10^5 \text{ Hz}$$

$$\phi = -\tan^{-1} \frac{2.33 \times 10^5}{5 \times 10^3} - 2 \tan^{-1} \frac{2.33 \times 10^5}{10^5} \\ = -88.77 - 133.54 \\ \phi = -222.3^\circ$$

12.81

Phase Margin $= 45^\circ \Rightarrow \phi = -135^\circ$

$$\phi = -135^\circ \\ = -\tan^{-1} \left(\frac{f}{10^3}\right) - \tan^{-1} \left(\frac{f}{10^4}\right) - \tan^{-1} \left(\frac{f}{10^5}\right) - \tan^{-1} \left(\frac{f}{10^6}\right)$$

At $f = 10^4$ Hz, $\phi = -135.6^\circ$

$$|T| = 1 \\ = \beta(10^3) \times \frac{1}{\sqrt{1 + \left(\frac{10^4}{10^3}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{10^4}{10^4}\right)^2}} \times \\ \times \frac{1}{\sqrt{1 + \left(\frac{10^4}{10^5}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{10^4}{10^6}\right)^2}} \\ 1 = \frac{\beta(10^3)}{(10.05)(1.414)(1.005)(1.00)}$$

or

$$\underline{\beta = 0.01428}$$

12.82

$$\phi = -120^\circ = -\tan^{-1} \left(\frac{f_{120}}{f_{PD}}\right) - \tan^{-1} \left(\frac{f_{120}}{4 \times 10^5}\right) - \tan^{-1} \left(\frac{f_{120}}{4 \times 10^6}\right) - \tan^{-1} \left(\frac{f_{120}}{4 \times 10^7}\right)$$

$$f_{120} \cong 2.31 \times 10^5 \text{ Hz}$$

$$|T(f_{120})| = 1 = \frac{4000}{\sqrt{1 + \left(\frac{2.31 \times 10^5}{f_{PD}}\right)^2} \sqrt{1 + \left(\frac{2.31 \times 10^5}{4 \times 10^5}\right)^2} \sqrt{1 + \left(\frac{2.31 \times 10^5}{4 \times 10^6}\right)^2} \sqrt{1 + \left(\frac{2.31 \times 10^5}{4 \times 10^7}\right)^2}}$$

$$1 = \frac{4000}{\sqrt{1 + \left(\frac{2.31 \times 10^5}{f_{PD}}\right)^2} (1.1548)(1.00167)(1.0)}$$

$$\frac{2.31 \times 10^5}{f_{PD}} = \frac{4000}{(1.1548)(1.00167)(1.09)}$$

$$f_{PD} = 66.8 \text{ Hz}$$

12.83

(a) $f_{180} \Rightarrow -180^\circ = -2 \tan^{-1}\left(\frac{f_{180}}{10^4}\right) - \tan^{-1}\left(\frac{f_{180}}{10^6}\right)$

$$f_{180} = 1.42 \times 10^5 \text{ Hz}$$

$$|T(f_{180})| = \frac{10^3}{\sqrt{1 + (14.2)^2} \sqrt{1 + (0.142)^2}} = \frac{10^3}{(202.64)(1.01)} = 4.89$$

$|T(f_{180})| > 1 \Rightarrow \text{Unstable}$

(b) $\phi = -135^\circ = -\tan^{-1}\left(\frac{f_{135}}{f_{PD}}\right) - 2 \tan^{-1}\left(\frac{f_{135}}{10^4}\right) - \tan^{-1}\left(\frac{f_{135}}{10^6}\right)$

$$f_{135} \cong 0.414 \times 10^4 \text{ Hz}$$

$$|T(f_{135})| = 1 = \frac{10^3}{\sqrt{1 + \left(\frac{0.414 \times 10^4}{f_{PD}}\right)^2} \left[1 + \left(\frac{0.414 \times 10^4}{10^4}\right)^2 \right] \sqrt{1 + \left(\frac{0.414 \times 10^4}{10^6}\right)^2}}$$

$$\frac{0.414 \times 10^4}{f_{PD}} = \frac{10^3}{(1.171)(1)}$$

$$f_{PD} = 4.85 \text{ Hz}$$

12.84

(a) $\phi = -180 = -\tan^{-1}\left(\frac{f_{180}}{10^4}\right) - \tan^{-1}\left(\frac{f_{180}}{5 \times 10^4}\right) - \tan^{-1}\left(\frac{f_{180}}{10^5}\right)$

$$f_{180} \cong 8.06 \times 10^4 \text{ Hz}$$

$$|T| = \frac{500}{\sqrt{1+\left(\frac{8.06 \times 10^4}{10^4}\right)^2} \sqrt{1+\left(\frac{8.06 \times 10^4}{5 \times 10^4}\right)^2} \sqrt{1+\left(\frac{8.06 \times 10^4}{10^5}\right)^2}}$$

$$= \frac{500}{(8.122)(1.897)(1.284)}$$

(b) $|T| = 25.3$

$$(c) T = \frac{500}{\left(1+j\frac{f}{f_{PD}}\right)\left(1+j\frac{f}{10^4}\right)\left(1+j\frac{f}{5 \times 10^4}\right)\left(1+j\frac{f}{10^5}\right)}$$

Phase Margin = $60^\circ \Rightarrow \phi = -120^\circ$

$$-120 = -\tan^{-1} \frac{f}{f_{PD}} - \tan^{-1} \frac{f}{10^4} - \tan^{-1} \frac{f}{5 \times 10^4} - \tan^{-1} \frac{f}{10^5}$$

$$\tan^{-1} \frac{f}{f_{PD}} \approx 90^\circ$$

Assume

Then $f \approx 4.2 \times 10^3$ Hz

$$|T| = 1 = \frac{500}{\sqrt{1+\left(\frac{4.2 \times 10^3}{f_{PD}}\right)^2} \sqrt{1+\left(\frac{4.2 \times 10^3}{10^4}\right)^2} \sqrt{1+\left(\frac{4.2 \times 10^3}{5 \times 10^4}\right)^2} \sqrt{1+\left(\frac{4.2 \times 10^3}{10^5}\right)^2}}$$

$$1 = \frac{500}{\sqrt{1+\left(\frac{4.2 \times 10^3}{f_{PD}}\right)^2} (1.085)(1.004)(1.0)}$$

$$\frac{4.2 \times 10^3}{f_{PD}} \approx \frac{500}{(1.0846)(1.0035)(1.0)}$$

$$f_{PD} = 9.14 \text{ Hz}$$

12.85

$$(a) 50 = \frac{10^4}{1+(10^4)\beta} \Rightarrow \beta = 0.0199$$

$$T = \frac{(0.0199)(10^4)}{\left(1+j\frac{f}{f_{PD}}\right)\left(1+j\frac{f}{10^5}\right)}$$

Phase margin = $45^\circ \Rightarrow \phi = -135^\circ$

$$-135 = -\tan^{-1} \frac{f}{f_{PD}} - \tan^{-1} \frac{f}{10^5}$$

$$f = 10^5 \text{ Hz}$$

$$|T| = 1 = \frac{(0.0199)(10^4)}{\sqrt{1 + \left(\frac{10^5}{f_{PD}}\right)^2} \sqrt{1 + \left(\frac{10^5}{10^5}\right)^2}}$$

$$\frac{10^5}{f_{PD}} = \frac{(0.0199)(10^4)}{1.414}$$

$$f_{PD} = 711 \text{ Hz}$$

$$20 = \frac{10^4}{1 + (10^4)\beta} \Rightarrow \beta = 0.0499$$

(b)

$$T = \frac{(0.0499)(10^4)}{\left(1 + j \frac{f}{711}\right) \left(1 + j \frac{f}{10^5}\right)}$$

$$|T| = 1 = \frac{(0.0499)(10^4)}{\sqrt{1 + \left(\frac{f}{711}\right)^2} \sqrt{1 + \left(\frac{f}{10^5}\right)^2}}$$

$$f = 1.76 \times 10^5 \text{ Hz}$$

$$\phi = -\tan^{-1}\left(\frac{1.76 \times 10^5}{711}\right) - \tan^{-1}\left(\frac{1.76 \times 10^5}{10^5}\right)$$

$$= -89.77 - 60.40$$

$$\phi = -150.2$$

Phase Margin = $180 - 150.2 = 29.8^\circ$

12.86

(a) $A_o = 100 \text{ dB} \Rightarrow A_o = 10^5$

$$20 = \frac{10^5}{1 + (10^5)\beta} \Rightarrow \beta = 0.04999$$

$$T = \frac{(0.04999)(10^5)}{\left(1 + j \frac{f}{f_{PD}}\right) \left(1 + j \frac{f}{10^6}\right) \left(1 + j \frac{f}{10^7}\right)}$$

Phase Margin = $45^\circ \Rightarrow \phi = -135^\circ$

$$-135 = -\tan^{-1}\frac{f}{f_{PD}} - \tan^{-1}\frac{f}{10^6} - \tan^{-1}\frac{f}{10^7}$$

$$f \approx 10^6 \text{ Hz}$$

$$|T| = 1 = \frac{(0.04999)(10^5)}{\sqrt{1 + \left(\frac{10^6}{f_{PD}}\right)^2} \sqrt{1 + \left(\frac{10^6}{10^6}\right)^2} \sqrt{1 + \left(\frac{10^6}{10^7}\right)^2}}$$

$$1 = \frac{(0.04999)(10^5)}{\sqrt{1 + \left(\frac{10^6}{f_{PD}}\right)^2} (1.414)(1.005)}$$

$$\frac{10^6}{f_{PD}} = \frac{(0.04999)(10^5)}{(1.414)(1.005)}$$

$$f_{PD} = 2.84 \text{ Hz}$$

(b) $5 = \frac{10^5}{1 + (10^5)\beta} \Rightarrow \beta = 0.19999$

$$|T| = 1 = \frac{(0.19999)(10^5)}{\sqrt{1 + \left(\frac{f}{284}\right)^2} \sqrt{1 + \left(\frac{f}{10^6}\right)^2} \sqrt{1 + \left(\frac{f}{10^7}\right)^2}}$$

$$f = 2.25 \times 10^6 \text{ Hz}$$

$$\begin{aligned} \phi &= -\tan^{-1}\left(\frac{2.25 \times 10^6}{284}\right) - \tan^{-1}\left(\frac{2.25 \times 10^6}{10^6}\right) - \tan^{-1}\left(\frac{2.25 \times 10^6}{10^7}\right) \\ &= -89.99 - 66.04 - 12.68 \end{aligned}$$

$$\phi = -168.7$$

Phase Margin = $180 - 168.7 = 11.3^\circ$

12.87

a. $T(0) = 100 \text{ dB} \Rightarrow T(0) = 10^5$

$$T(f) = \frac{10^5}{\left(1 + j\frac{f}{10}\right)\left(1 + j\frac{f}{5 \times 10^6}\right)\left(1 + j\frac{f}{10 \times 10^6}\right)}$$

$$\begin{aligned} |T| &= 1 = \\ &= 10^5 \times \frac{1}{\sqrt{1 + \left(\frac{f}{10}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{f}{5 \times 10^6}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{f}{10 \times 10^6}\right)^2}} \end{aligned}$$

By trial and error

$$\underline{f = 0.976 \text{ MHz}}$$

$$\phi = -\tan^{-1}\left(\frac{0.976 \times 10^6}{10}\right) - \tan^{-1}\left(\frac{0.976}{5}\right) - \tan^{-1}\left(\frac{0.976}{10}\right)$$

$$= -90^\circ - 11.05^\circ - 5.574^\circ = -106.6^\circ$$

Phase Margin = $180^\circ - 106.6^\circ = \underline{73.4^\circ}$

b. $f'_{p1} \propto \frac{1}{C_F} \quad \text{so} \quad \frac{10}{f'_{p1}} = \frac{75}{20}$

or

$$\underline{f'_{p1} = 2.67 \text{ Hz}}$$

Now

$$|T| = 1 =$$

$$\begin{aligned} &= 10^5 \times \frac{1}{\sqrt{1 + \left(\frac{f}{2.67}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{f}{5 \times 10^6}\right)^2}} \\ &\quad \times \frac{1}{\sqrt{1 + \left(\frac{f}{10 \times 10^6}\right)^2}} \end{aligned}$$

By trial and error

$$f \approx 2.66 \times 10^5 \text{ Hz}$$

then

$$\begin{aligned}\phi &= -\tan^{-1}\left(\frac{2.66 \times 10^5}{2.67}\right) - \tan^{-1}\left(\frac{0.266}{5}\right) - \tan^{-1}\left(\frac{0.266}{10}\right) \\ &= -90^\circ - 3.045^\circ - 1.524^\circ = -94.57^\circ \\ \text{Phase Margin} &= 180^\circ - 94.57^\circ = \underline{85.4^\circ}\end{aligned}$$

12.88

$$(a) f_P = \frac{1}{2\pi(R_{o1}\|R_{i2})C_i} = \frac{1}{2\pi(2\|0.75)\times 10^6 \times (1.2 \times 10^{-12})}$$

$$f_P = 243 \text{ kHz}$$

$$(b) f_{PD} = 6 = \frac{1}{2\pi(R_{o1}\|R_{i2})(C_i + C_M)} = \frac{1}{2\pi(2\|0.75)\times 10^6 \times (C_i + C_M)}$$

$$C_i + C_M = 4.863 \times 10^{-8} = 1.2 \times 10^{-12} + C_M$$

$$C_M = 0.0486 \mu\text{F}$$

$$(c) C_M = C_F(1+|A|)$$

$$0.0486 \times 10^{-6} = C_F(1001)$$

$$C_F = 48.6 \text{ pF}$$

12.89

$$\phi = -120^\circ = -\tan^{-1}\left(\frac{f_{120}}{f_{PD}}\right) - \tan^{-1}\left(\frac{f_{120}}{4 \times 10^6}\right) - \tan^{-1}\left(\frac{f_{120}}{4 \times 10^7}\right)$$

$$f_{120} \cong 2.05 \times 10^6 \text{ Hz}$$

$$|T(f_{120})| = 1 = \frac{4000}{\sqrt{1 + \left(\frac{2.05 \times 10^6}{f_{PD}}\right)^2} \sqrt{1 + \left(\frac{2.05 \times 10^6}{4 \times 10^6}\right)^2} \sqrt{1 + \left(\frac{2.05 \times 10^6}{4 \times 10^7}\right)^2}}$$

$$\frac{2.05 \times 10^6}{f_{PD}} = \frac{4000}{(1.1237)(1.0013)}$$

$$f_{PD} = 577 \text{ Hz}$$

12.90

$$(a) 40 = \frac{5 \times 10^5}{1 + \beta(5 \times 10^5)} \Rightarrow \beta = 0.024998$$

$$(b) \phi = -120^\circ = -\tan^{-1}\left(\frac{f_{120}}{f_{PD}}\right) - \tan^{-1}\left(\frac{f_{120}}{5 \times 10^5}\right) - \tan^{-1}\left(\frac{f_{120}}{10^7}\right)$$

$$f_{120} \cong 2.71 \times 10^5 \text{ Hz}$$
$$|T(f_{120})| = 1 = \frac{(0.024998)(5 \times 10^5)}{\sqrt{1 + \left(\frac{2.71 \times 10^5}{f_{PD}}\right)^2} \sqrt{1 + \left(\frac{2.71 \times 10^5}{5 \times 10^5}\right)^2} \sqrt{1 + \left(\frac{2.71 \times 10^5}{10^7}\right)^2}}$$
$$\frac{2.71 \times 10^5}{f_{PD}} = \frac{(0.024998)(5 \times 10^5)}{(1.1374)(1)}$$
$$f_{PD} = 24.66 \text{ Hz}$$

Chapter 13

13.1

$$(a) R_{D2} = \frac{0 - (-3)}{0.2} = 15 \text{ k}\Omega$$

$$I_{D3} = \left(\frac{k'_p}{2} \right) \left(\frac{W}{L} \right)_3 (V_{SG3} + V_{TP})^2$$

$$0.2 = \left(\frac{0.04}{2} \right) (40)(V_{SG3} - 0.4)^2 \Rightarrow V_{SG3} = 0.9 \text{ V}$$

$$R_{D1} = \frac{0.9}{0.1} = 9 \text{ k}\Omega$$

$$(b) (i) A_d = \frac{v_o}{v_d} = \frac{g_{m1}}{2} \cdot R_{D1}$$

$$g_{m1} = 2 \sqrt{\left(\frac{0.1}{2} \right) (20)(0.1)} = 0.6325 \text{ mA/V}$$

$$A_d = \left(\frac{0.6325}{2} \right) (9) = 2.846$$

$$(ii) A_2 = \frac{v_o}{v_{o1}} = -g_{m3} R_{D2}$$

$$g_{m3} = 2 \sqrt{\left(\frac{0.04}{2} \right) (40)(0.2)} = 0.8 \text{ mA/V}$$

$$A_2 = -(0.8)(15) = -12$$

$$(c) A = A_d \cdot A_2 = (2.846)(-12) = -34.15$$

13.2

$$(a) R_{C2} = \frac{3 - 0}{0.4} = 7.5 \text{ k}\Omega$$

$$V_{B3} = 0.7 + (0.4)(0.5) - 3 = -2.1 \text{ V}$$

$$R_{C1} = \frac{-2.1 - (-3)}{0.25} = 3.6 \text{ k}\Omega$$

$$(b) (i) A_d = \frac{v_o}{v_d} = \frac{g_{m1}}{2} (R_{C1} \| R_{i3})$$

$$g_{m1} = \frac{0.25}{0.026} = 9.615 \text{ mA/V}, r_{\pi3} = \frac{(180)(0.026)}{0.4} = 11.7 \text{ k}\Omega$$

$$R_{i3} = r_{\pi3} + (1 + \beta_n)R_E = 11.7 + (181)(0.5) = 102.2 \text{ k}\Omega$$

$$A_d = \left(\frac{9.615}{2} \right) (3.6 \| 102.2) = 16.72$$

$$(ii) A_2 = \frac{v_o}{v_{o1}} = \frac{-\beta_n R_{C2}}{r_{\pi3} + (1 + \beta_n)R_E} = \frac{-(180)(7.5)}{11.7 + (181)(0.5)} = -13.21$$

$$(c) A = A_d \cdot A_2 = (16.72)(-13.21) = -220.9$$

13.3 Computer Analysis

13.4 Computer Analysis

13.5

$$(a) \quad A_d = g_{m1} (r_{o2} \| r_{o4} \| R_{i6})$$

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{20}{0.026} \Rightarrow 0.769 \text{ mA/V}$$

$$r_{o2} = \frac{V_{A2}}{I_{C2}} = \frac{80}{20} = 4 \text{ M}\Omega$$

$$r_{o4} = \frac{V_{A4}}{I_{C2}} = \frac{80}{20} = 4 \text{ M}\Omega$$

$$R_{i6} = r_{\pi6} + (1 + \beta_n) [R_i \| r_{\pi7}]$$

$$r_{\pi7} = \frac{(120)(0.026)}{0.2} = 15.6 \text{ k}\Omega$$

$$I_{C6} \equiv \frac{V_{BE}(\text{on})}{R_i} = \frac{0.6}{20} = 0.030 \text{ mA}$$

$$r_{\pi6} = \frac{(120)(0.026)}{0.030} = 104 \text{ k}\Omega$$

Then

$$R_{i6} = 104 + (121)[20 \| 15.6] \Rightarrow 1.16 \text{ M}\Omega$$

Then

$$A_d = 769(4 \| 4 \| 1.16) \Rightarrow \underline{A_d = 565}$$

Now

$$V_o = -I_{c7} r_{o7} = -(\beta_n I_{b7}) r_{o7} = -\beta_n r_{o7} \left(\frac{R_i}{R_i + r_{\pi7}} \right) I_{c6}$$

$$= -\beta_n (1 + \beta_n) r_{o7} \left(\frac{R_i}{R_i + r_{\pi7}} \right) I_{b6} \text{ and } I_{b6} = \frac{V_{o1}}{R_{i6}}$$

Then

$$A_{v2} = \frac{V_o}{V_{o1}} = \frac{-\beta_n (1 + \beta_n) r_{o7}}{R_{i6}} \left(\frac{R_i}{R_i + r_{\pi7}} \right)$$

$$r_{o7} = \frac{V_A}{I_{C7}} = \frac{80}{0.2} = 400 \text{ k}\Omega$$

So

$$A_{v2} = \frac{-(120)(121)(400)}{1160} \left(\frac{20}{20 + 15.6} \right) \Rightarrow \underline{A_{v2} = -2813}$$

Overall gain = $A_d \cdot A_{v2} = (565)(-2813) \Rightarrow \underline{A = -1.59 \times 10^6}$

$$(b) \quad R_{id} = 2r_{\pi1} \text{ and } r_{\pi1} = \frac{(80)(0.026)}{0.020} = 104 \text{ k}\Omega$$

$$\underline{R_{id} = 208 \text{ k}\Omega}$$

(c) $f_{PD} = \frac{1}{2\pi R_{eq} C_M}$ and $C_M = (10)(1 + 2813) = 28,140 \text{ pF}$

$$R_{eq} = r_{o2} \| r_{o4} \| R_{i6} = 4 \| 4 \| 1.16 = 0.734 \text{ M}\Omega$$

$$f_{PD} = \frac{1}{2\pi(0.734 \times 10^6)(28,140 \times 10^{-12})} = 7.71 \text{ Hz}$$

Gain-Bandwidth Product = $(7.71)(1.59 \times 10^6) \Rightarrow 12.3 \text{ MHz}$

13.6

- a. Q_3 acts as the protection device.
- b. Same as part (a).

13.7

If we assume $V_{BE}(\text{on}) = 0.7 \text{ V}$, then $V_{in} = 0.7 + 0.7 + 50 + 5$

So breakdown voltage $\approx 56.4 \text{ V}$.

13.8

(a) $V_{EB12} = V_{BE11} = (0.026) \ln\left(\frac{0.5 \times 10^{-3}}{5 \times 10^{-16}}\right) = 0.7184 \text{ V}$

$$R_5 = \frac{15 - 0.7184 - 0.7184 - (-15)}{0.5} = 57.1 \text{ k}\Omega$$

$$R_4 = \frac{(0.026)}{(0.03)} \ln\left(\frac{0.5}{0.03}\right) = 2.438 \text{ k}\Omega$$

$$V_{BE10} = V_{BE11} - I_{C10} R_4 = 0.7184 - (0.03)(2.438) = 0.6453 \text{ V}$$

(b) $I_{REF} = \frac{15 - 0.6 - 0.6 - (-15)}{57.1} = 0.5044 \text{ mA}$

$$I_{C10}(2.438) = (0.026) \ln\left(\frac{0.5044}{I_{C10}}\right) \Rightarrow I_{C10} \approx 30.1 \mu\text{A}$$

(c) $I_{REF} : \left(\frac{0.5044 - 0.5}{0.5}\right) \times 100\% = 0.88\%$

$$I_{C10} : \left(\frac{30.1 - 30}{30}\right) \times 100\% = 0.33\%$$

13.9

(a) $R_5 = \frac{5 - 0.7184 - 0.7184 - (-5)}{0.5} = 17.13 \text{ k}\Omega$

$$R_4 = \frac{(0.026)}{(0.03)} \ln\left(\frac{0.5}{0.03}\right) = 2.438 \text{ k}\Omega$$

$$V_{EB12} = V_{BE11} = 0.7184 \text{ V}$$

$$V_{BE10} = 0.6453 \text{ V}$$

$$(b) \quad I_{REF} = \frac{5 - 0.6 - 0.6 - (-5)}{17.13} = 0.5137 \text{ mA}$$

$$I_{C10}(2.438) = (0.026) \ln\left(\frac{0.5137}{I_{C10}}\right) \Rightarrow I_{C10} \cong 30.22 \mu \text{A}$$

$$(c) \quad I_{REF} : \left(\frac{0.5137 - 0.5}{0.5}\right) \times 100\% = 2.74\%$$

$$I_{C10} : \left(\frac{30.22 - 30}{30}\right) \times 100\% = 0.733\%$$

13.10

$$(a) \quad I_{REF} = \frac{3 - V_{EB2} - V_{BE1} - (-3)}{80}$$

$$V_{EB2} = V_{BE1} = (0.026) \ln\left(\frac{I_{REF}}{3 \times 10^{-15}}\right)$$

By trial and error, $I_{REF} \cong 59.6 \mu \text{A}$

$$I_3(3.5) = (0.026) \ln\left(\frac{0.0596}{I_3}\right) \Rightarrow I_3 \cong 11.9 \mu \text{A}$$

$$I_4 = \frac{I_{S4}}{I_{S2}} \cdot I_{REF} = \frac{(3 \times 10^{-15})}{(5 \times 10^{-15})} \cdot (59.6) = 35.76 \mu \text{A}$$

$$I_5 = \frac{I_{S5}}{I_{S2}} \cdot I_{REF} = \frac{(1 \times 10^{-15})}{(5 \times 10^{-15})} \cdot (59.6) = 11.92 \mu \text{A}$$

$$(b) \quad I_{REF} = 59.6 \mu \text{A}, \quad I_3 = 11.9 \mu \text{A}$$

$$I_4 = \frac{I_{S4}}{I_{S2}} \cdot I_{REF} = \frac{(8 \times 10^{-15})}{(5 \times 10^{-15})} \cdot (59.6) = 95.36 \mu \text{A}$$

$$I_5 = \frac{I_{S5}}{I_{S2}} \cdot I_{REF} = \frac{(2 \times 10^{-15})}{(5 \times 10^{-15})} \cdot (59.6) = 23.84 \mu \text{A}$$

13.11

$$I_{REF} = \frac{5 - 0.6 - 0.6 - (-5)}{40} \Rightarrow I_{REF} = 0.22 \text{ mA}$$

$$I_{C10}R_4 = V_T \ln\left(\frac{I_{REF}}{I_{C10}}\right)$$

$$I_{C10}(5) = (0.026) \ln\left(\frac{0.22}{I_{C10}}\right)$$

By trial and error;

$$\underline{I_{C10} \cong 14.2 \mu \text{A}}$$

$$\underline{I_{C6} \cong \frac{I_{C10}}{2} \Rightarrow I_{C6} = 7.10 \mu \text{A}}$$

$$I_{C17} = 0.75I_{REF} \Rightarrow \underline{I_{C17} = 0.165 \text{ mA}}$$

$$I_{C13A} = 0.25I_{REF} \Rightarrow \underline{I_{C13A} = 0.055 \text{ mA}}$$

13.12

$$I_{C8} = I_{C9}$$

$$2I = I_{C8} + \frac{2I_{C9}}{\beta_p} = I_{C9} \left(1 + \frac{2}{\beta_p} \right)$$

$$I_{E1} = I_{E2} = I \left(\frac{1 + \beta_n}{\beta_n} \right)$$

$$I_{B3} = I_{B4} = I \left(\frac{1 + \beta_n}{\beta_n} \right) \left(\frac{1}{1 + \beta_p} \right)$$

$$I_{C10} = I_{C9} + 2I \left(\frac{1 + \beta_n}{\beta_n} \right) \left(\frac{1}{1 + \beta_p} \right)$$

$$I_{C10} = I_{C9} + I_{C9} \left(1 + \frac{2}{\beta_p} \right) \left(\frac{1 + \beta_n}{\beta_n} \right) \left(\frac{1}{1 + \beta_p} \right)$$

$$50 = I_{C9} \left[1 + \left(1 + \frac{2}{40} \right) \left(\frac{91}{90} \right) \left(\frac{1}{41} \right) \right] = I_{C9} (1.0259) \Rightarrow I_{C9} = 48.738 \mu A$$

$$I_{C2} = I = \frac{I_{C9}}{2} \left(1 + \frac{2}{\beta_p} \right) = \frac{48.738}{2} \left(1 + \frac{2}{40} \right) = 25.587 \mu A$$

$$I_{C4} = I_{C2} \left(\frac{\beta_p}{1 + \beta_p} \right) \left(\frac{1 + \beta_n}{\beta_n} \right) = (25.587) \left(\frac{40}{41} \right) \left(\frac{91}{90} \right) = 25.240 \mu A$$

$$I_{B9} = \frac{I_{C9}}{\beta_p} = \frac{48.738}{40} = 1.218 \mu A$$

$$I_{B4} = \frac{I_{C4}}{\beta_p} = \frac{25.240}{40} = 0.631 \mu A$$

13.13

$$\begin{aligned} V_{B5} - V^- &= V_{BE}(\text{on}) + I_{C5}(1) \\ &= 0.6 + (0.0095)(1) = 0.6095 \end{aligned}$$

$$I_{C7} = \frac{0.6095}{50} \Rightarrow I_{C7} = 12.2 \mu A$$

$$I_{C8} = I_{C9} = 19 \mu A$$

$$I_{REF} = 0.72 \text{ mA}$$

$$I_{E13} = I_{REF} = 0.72 \text{ mA}$$

$$I_{C14} = 138 \mu A$$

$$\text{Power} = (V^+ - V^-) [I_{C7} + I_{C8} + I_{C9} + I_{REF} + I_{E13} + I_{C14}]$$

$$= 30 [0.0122 + 0.019 + 0.019 + 0.72 + 0.72 + 0.138]$$

$$\Rightarrow \text{Power} = 48.8 \text{ mW}$$

$$\begin{aligned} \text{Current supplied by } V^+ \text{ and } V^- &= I_{C7} + I_{C8} + I_{C9} + I_{REF} + I_{E13} + I_{C14} \\ &= \underline{1.63 \text{ mA}} \end{aligned}$$

13.14

$$(a) v_{cm}(\text{min}) = -15 + 0.6 + 0.6 + 0.6 + 0.6 = -12.6 \text{ V}$$

$$v_{cm}(\text{max}) = +15 - .6 = 14.4 \text{ V}$$

So $-12.6 \leq v_{cm} \leq 14.4 \text{ V}$

$$(b) v_{cm}(\text{min}) = -5 + 4(0.6) = -2.6 \text{ V}$$

$$v_{cm}(\text{max}) = 5 - 0.6 = 4.4 \text{ V}$$

So $-2.6 \leq v_{cm} \leq 4.4 \text{ V}$

13.15

$$I_{C13A} = (0.25)I_{REF} = (0.25)(0.5) = 0.125 \text{ mA}$$

$$I_{R10} \cong \frac{0.6}{50} = 0.012 \text{ mA}$$

$$I_{C19} \cong I_{C13A} - I_{R10} = 0.125 - 0.012 = 0.113 \text{ mA}$$

$$I_{B19} = \frac{I_{C19}}{\beta_n} = \frac{0.113}{200} \Rightarrow I_{B19} = 0.565 \mu\text{A}$$

$$I_{C18} = I_{R10} + I_{B19} = 12 + 0.565 = 12.565 \mu\text{A}$$

$$V_{BE18} = (0.026) \ln \left(\frac{12.565 \times 10^{-6}}{10^{-14}} \right) = 0.54474 \text{ V}$$

$$V_{BE19} = (0.026) \ln \left(\frac{0.113 \times 10^{-3}}{10^{-14}} \right) = 0.60185 \text{ V}$$

$$\frac{V_{BB}}{2} = \frac{V_{BE18} + V_{BE19}}{2} = 0.57329 \text{ V}$$

$$I_{C14} = (3 \times 10^{-14}) \exp \left(\frac{0.57329}{0.026} \right) \Rightarrow I_{C14} = 113 \mu\text{A}$$

13.16

$$(a) V_{BB} = 2V_T \ln \left(\frac{I_{Bias}}{I_{SD}} \right) = 2(0.026) \ln \left(\frac{80 \times 10^{-6}}{5 \times 10^{-15}} \right) = 1.22178 \text{ V}$$

$$I_{CN} = I_{CP} = I_{SQ} \exp \left[\frac{1.22178/2}{0.026} \right] \Rightarrow I_{CN} = I_{CP} = 128 \mu\text{A}$$

$$(b) \text{ For } v_L = 3 \text{ V}, \quad v_o \cong 3 \text{ V}, \quad i_L \cong \frac{3}{10} = 0.3 \text{ mA}$$

First approximation:

$$I_{BN} \cong \frac{0.3}{120} \Rightarrow I_{BN} = 2.5 \mu\text{A}$$

$$I_D = 80 - 2.5 = 77.5 \mu\text{A}$$

$$V_{BB} = 2(0.026) \ln \left(\frac{77.5 \times 10^{-6}}{5 \times 10^{-15}} \right) = 1.22013 \text{ V}$$

$$V_{BEN} = (0.026) \ln \left(\frac{0.3 \times 10^{-3}}{8 \times 10^{-15}} \right) = 0.63304 \text{ V}$$

$$V_{EBP} = V_{BB} - V_{BEN} = 0.58709 \text{ V}$$

$$I_{CP} = \left(8 \times 10^{-15}\right) \exp\left(\frac{0.58709}{0.026}\right) \Rightarrow I_{CP} = 51.24 \mu A$$

Second approximation:

$$I_{CN} = \left(\frac{120}{121}\right)(0.3 + 0.05124) = 0.34834 \text{ mA}, \quad I_{BN} = 2.903 \mu A$$

$$I_D = 80 - 2.903 = 77.1 \mu A$$

$$V_{BB} = 2(0.026) \ln\left(\frac{77.1 \times 10^{-6}}{5 \times 10^{-15}}\right) = 1.219864 \text{ V}$$

$$V_{BEN} = (0.026) \ln\left(\frac{0.34834 \times 10^{-3}}{8 \times 10^{-15}}\right) = 0.636922 \text{ V}$$

$$V_{EBP} = 1.219864 - 0.636922 = 0.582942 \text{ V}$$

$$I_{CP} = \left(8 \times 10^{-15}\right) \exp\left(\frac{0.582942}{0.026}\right) \Rightarrow I_{CP} = 43.7 \mu A$$

13.17

$$R_1 + R_2 = \frac{V_{BB}}{(0.1)I_{Bias}} = \frac{1.160}{(0.1)(0.12)} = 96.67 \text{ k}\Omega$$

$$V_{BE} = V_T \ln\left(\frac{I_{CQ}}{I_S}\right) = (0.026) \ln\left(\frac{(0.9)(120 \times 10^{-6})}{5 \times 10^{-15}}\right) = 0.6187 \text{ V}$$

$$V_{BE} = \left(\frac{R_2}{R_1 + R_2}\right)(V_{BB}) \Rightarrow 0.6187 = \left(\frac{R_2}{96.67}\right)(1.16)$$

$$\text{So } R_2 = 51.56 \text{ k}\Omega, \quad R_1 = 45.11 \text{ k}\Omega$$

13.18

$$(a) \quad A_d = -g_m (r_{o4} \| r_{o6} \| R_{i2})$$

From example 13.4

$$g_m = \frac{9.5}{0.026} = 365 \mu A/V, \quad r_{o4} = 5.26 M\Omega$$

Now

$$r_{o6} = r_{o4} = 5.26 M\Omega$$

Assuming $R_s = 0$, we find

$$\begin{aligned} R_{i2} &= r_{\pi 16} + (1 + \beta_n) R'_E \\ &= 329 + (201)(50 \| 9.63) \Rightarrow 1.95 M\Omega \end{aligned}$$

Then

$$A_d = -(365)(5.26 \| 5.26 \| 1.95) \Rightarrow A_d = -409$$

(b) From Equation (13.20),

$$A_{v2} = \frac{-\beta_n(1 + \beta_n)R_g(R_{act2} \| R_{i3} \| R_{017})}{R_{i2} \{ R_g + [r_{\pi 17} + (1 + \beta_n)R_g] \}}$$

For $R_g = 0, R_{i2} = 1.95 M\Omega$

Using the results of Example 13.5

$$A_{v2} = \frac{-200(201)(50)(92.6 \| 4050 \| 92.6)}{(1950)\{50 + 9.63\}} \Rightarrow \underline{A_{v2} = -792}$$

13.19

Let $I_{C10} = 40 \mu\text{A}$, then $I_{C1} = I_{C2} = 20 \mu\text{A}$. Use the procedure in Example 13.4:

$$R_{i2} = 4.07 \text{ M}\Omega$$

$$r_{\pi 6} = \frac{(200)(0.026)}{0.020} = 260 \text{ k}\Omega$$

$$g_{m6} = \frac{0.020}{0.026} = 0.769 \text{ mA/V}$$

$$r_{06} = \frac{50}{0.02} \Rightarrow 2.5 \text{ M}\Omega$$

Then

$$R_{act1} = 2.5[1 + (0.769)(1 \| 260)] = 4.42 \text{ M}\Omega$$

$$r_{06} = \frac{50}{0.02} \Rightarrow 2.5 \text{ M}\Omega$$

Then

$$\begin{aligned} A_d &= -\left(\frac{I_{CQ}}{V_r}\right)(r_{04} \| R_{act1} \| R_{i2}) \\ &= -\left(\frac{20}{0.026}\right)(2.5 \| 4.42 \| 4.07) \end{aligned}$$

So

$$\underline{A_d = -882}$$

13.20

From Problem 13.11

$$I_1 = I_2 = 7.10 \mu\text{A}, I_{C17} = 0.165 \text{ mA}, I_{C13A} = 0.055 \text{ mA}$$

$$\begin{aligned} I_{C16} &\approx I_{B17} + \frac{I_{E17}R_8 + V_{BE17}}{R_9} = \frac{0.165}{200} + \frac{(0.165)(0.1) + 0.6}{50} \\ &= 0.000825 + 0.01233 \end{aligned}$$

$$I_{C16} = 0.0132 \text{ mA}$$

$$r_{\pi 17} = \frac{(200)(0.026)}{0.165} = 31.5 \text{ K}$$

$$R_E^1 = R_9 \| [r_{\pi 17} + (1 + \beta)R_8] = 50 \| [31.5 + (201)(0.1)]$$

$$= 50 \| 51.6 = 25.4 \text{ K}$$

$$r_{\pi 16} = \frac{(200)(0.026)}{0.0132} = 394 \text{ K}$$

Then

$$R_{i2} = r_{\pi 16} + (1 + \beta)R_E^1 = 394 + (201)(25.4) \Rightarrow 5.50 \text{ M}\Omega$$

Now

$$r_{\pi 6} = \frac{(200)(0.026)}{0.0071} = 732 \text{ K}$$

$$g_{m6} = \frac{0.0071}{0.026} = 0.273 \text{ mA/V}$$

$$r_{o6} = \frac{50}{0.0071} \Rightarrow 7.04 \text{ M}\Omega$$

$$R_{act1} = r_{o6}[1 + g_{m6}(R \| r_{\pi 6})]$$

$$= 7.04[1 + (0.273)(1 \| 732)] = 8.96 \text{ M}\Omega$$

$$r_{o4} = \frac{50}{0.0071} \Rightarrow 7.04 \text{ M}\Omega$$

Then

$$A_d = -g_{m1}(r_{o4} \| R_{act1} \| R_{i2})$$

$$= -\left(\frac{7.10}{0.026}\right)(7.04 \| 8.96 \| 5.5)$$

$$\underline{A_d = -627}$$

$$R_{act2} = \frac{50}{0.165} \Rightarrow 303 \text{ K} \quad R_{o17} = \frac{50}{0.165} = 303 \text{ K}$$

Now From Eq. (13.20), assuming $R_{i3} \rightarrow \infty$

$$A_{v2} \cong -\frac{\beta(1+\beta)R_9(R_{act2} \| R_{o17})}{R_{i2}\{R_9 + [r_{\pi 17} + (1+\beta)R_8]\}}$$

$$= \frac{-(200)(201)(50)(303 \| 303)}{(5500)[50 + 31.5 + (201)(0.1)]} = \frac{-3.045 \times 10^8}{5.588 \times 10^5}$$

$$A_{v2} = -545$$

Overall gain $A_v = (-627)(-545) = 341,715$

13.21

Using results from Problem 13.20

$$R_{i2} = 5.50 \text{ M}\Omega, R_{act1} \left(\frac{100}{0.0071} \right) [1 + (0.273)(1 \| 732)] \Rightarrow 17.93 \text{ M}\Omega$$

$$r_{o4} = \frac{100}{0.0071} \Rightarrow 14.08 \text{ M}\Omega$$

$$A_d = -\left(\frac{7.10}{0.026}\right)(14.08 \| 17.93 \| 5.50)$$

$$A_d = -885$$

Now

$$R_{act2} = \frac{100}{0.165} = 606 \text{ K} \quad R_{o17} = \frac{100}{0.165} = 606 \text{ K}$$

$$A_{v2} = \frac{-(200)(201)(50)(606 \| 606)}{(5500)[50 + 31.5 + (201)(0.1)]} = \frac{-6.09 \times 10^8}{5.588 \times 10^5}$$

$$A_{v2} = -1090$$

Overall gain

$$A_v = (-885)(-1090) = 964,650$$

13.22

(a) $I_{C18} + I_{C19} = I_{Bias}$
 $(0.25)I_{C19} + I_{C19} = 0.18 \text{ mA}, \Rightarrow I_{C19} = 0.144 \text{ mA}$

$$I_{C18} = 0.036 \text{ mA}$$

$$V_{BE19} = (0.026) \ln \left(\frac{0.144 \times 10^{-3}}{10^{-14}} \right) = 0.60815 \text{ V}$$

$$R_{10} = \frac{0.60815}{0.036} = 16.9 \text{ k}\Omega$$

(b) $V_{BE19} = 0.60815 \text{ V}$

$$V_{BE18} = (0.026) \ln \left(\frac{0.036 \times 10^{-3}}{10^{-14}} \right) = 0.5721 \text{ V}$$

13.23

(b) $I_{C19} = 0.168 \text{ mA}, g_{m19} = \frac{0.168}{0.026} = 6.462 \text{ mA/V},$

$$r_{\pi19} = \frac{(200)(0.026)}{0.168} = 30.95 \text{ k}\Omega, r_{o19} = \frac{50}{0.168} = 298 \text{ k}\Omega$$

$$I_{C18} = 12.84 \mu\text{A}, g_{m18} = \frac{0.01284}{0.026} = 0.4938 \text{ mA/V}$$

$$r_{\pi18} = \frac{(200)(0.026)}{0.01284} = 405 \text{ k}\Omega, r_{o18} = \frac{50}{0.01284} = 3894 \text{ k}\Omega$$

$$V_x = V_{\pi18} + V_{\pi19}$$

$$(1) \quad I_x = \frac{V_{\pi18}}{r_{\pi18}} + g_{m18}V_{\pi18} + \frac{V_{\pi18}}{r_{o18}} + g_{m19}V_{\pi19} + \frac{V_x}{r_{o19}}$$

$$(2) \quad \frac{V_{\pi18}}{r_{\pi18}} + g_{m18}V_{\pi18} + \frac{V_{\pi18}}{r_{o18}} = \frac{V_{\pi19}}{R_{10}} + \frac{V_{\pi19}}{r_{\pi19}}$$

$$\text{Then } V_{\pi18} \left(\frac{1}{405} + 0.4938 + \frac{1}{3894} \right) = V_{\pi19} \left(\frac{1}{50} + \frac{1}{30.95} \right)$$

$$V_{\pi18}(0.49653) = V_{\pi19}(0.05231) \Rightarrow V_{\pi18} = V_{\pi19}(0.10535)$$

$$\text{Then (1) } I_x = V_{\pi18}(0.49653) + (6.462)V_{\pi19} + \frac{V_x}{298}$$

$$I_x = V_{\pi19}(0.10535)(0.49653) + (6.462)V_{\pi19} + (0.003356)V_x$$

$$I_x = V_{\pi19}(6.5143) + V_x(0.003356)$$

$$\text{Now } V_x = V_{\pi18} + V_{\pi19} = V_{\pi19}(0.10535) + V_{\pi19} = (1.10535)V_{\pi19}$$

$$\text{Or } V_{\pi19} = (0.90469)V_x$$

$$\text{Then } I_x = (0.90469)V_x(6.5143) + V_x(0.003356)$$

$$\text{So } \frac{I_x}{V_x} = \frac{1}{R_{eq}} = 5.8968 \Rightarrow R_{eq} = 170 \Omega$$

13.24

Now

$$R_{e14} = \frac{r_{\pi14} + R_{01}}{1 + \beta_p} \quad \text{and} \quad R_0 = R_6 + R_{e14}$$

Assume series resistance of Q_{18} and Q_{19} is small. Then

$$R_{01} = r_{013A} \| R_{e22}$$

$$R_{e22} = \frac{r_{\pi22} + R_{017} \| r_{013B}}{1 + \beta_p}$$

where

$$\text{and } R_{017} = r_{017} [1 + g_{m17}(R_8 \| r_{\pi17})]$$

Using results from Example 13.6,

$$r_{\pi17} = 9.63 \text{ k}\Omega \quad r_{\pi22} = 7.22 \text{ k}\Omega$$

$$g_{m17} = 20.8 \text{ mA/V} \quad r_{017} = 92.6 \text{ k}\Omega$$

Then

$$R_{017} = 92.6 [1 + (20.8)(0.1 \| 9.63)] = 283 \text{ k}\Omega$$

$$r_{013B} = \frac{50}{0.54} = 92.6 \text{ k}\Omega$$

Then

$$R_{e22} = \frac{7.22 + 283 \| 92.6}{51} = 1.51 \text{ k}\Omega$$

$$R_{01} = r_{013A} \| R_{e22} = 278 \| 1.51 = 1.50 \text{ k}\Omega$$

$$r_{\pi14} = \frac{(50)(0.026)}{2} = 0.65 \text{ k}\Omega$$

Then

$$R_{e14} = \frac{0.65 + 1.50}{51} = 0.0422 \text{ k}\Omega$$

or

$$R_{e14} = 42.2 \text{ }\Omega$$

Then

$$R_0 = 42.2 + 27 \Rightarrow \underline{R_0 = 69.2 \text{ }\Omega}$$

13.25

$$R_{id} = 2 \left[r_{\pi1} + (1 + \beta_n) \left(\frac{r_{\pi3}}{1 + \beta_p} \right) \right]$$

$$\beta_n = 200, \quad \beta_p = 10$$

(a)

$$I_{C1} = 9.5 \mu\text{A}$$

$$r_{\pi1} = \frac{(200)(0.026)}{0.0095} = 547 \text{ K}$$

$$r_{\pi3} = \frac{(10)(0.026)}{0.0095} = 27.4 \text{ K}$$

Then

$$R_{id} = 2 \left[547 + \frac{(201)(27.4)}{11} \right]$$

$$\underline{R_{id} \Rightarrow 2.095 \text{ M}\Omega}$$

(b)

$$\begin{aligned}I_{C1} &= 7.10 \mu\text{A} \\r_{\pi 1} &= \frac{(200)(0.026)}{0.0071} = 732 \text{ K} \\r_{\pi 3} &= \frac{(10)(0.026)}{0.0071} = 36.6 \text{ K} \\R_{id} &= 2 \left[732 + \frac{(201)(36.6)}{11} \right] \\R_{id} &\Rightarrow 2.80 \text{ M}\Omega\end{aligned}$$

13.26

We can write

$$\begin{aligned}A(f) &= \frac{A_0}{\left(1 + j \frac{f}{f_{PD}}\right) \left(1 + j \frac{f}{f_1}\right)} \\&= \frac{181,260}{\left(1 + j \frac{f}{10.7}\right) \left(1 + j \frac{f}{f_1}\right)}\end{aligned}$$

Phase:

$$\phi = -\tan^{-1}\left(\frac{f}{10.7}\right) - \tan^{-1}\left(\frac{f}{f_1}\right)$$

For a Phase margin = 70°, $\phi = -110^\circ$

So

$$-110^\circ = -\tan^{-1}\left(\frac{f}{10.7}\right) - \tan^{-1}\left(\frac{f}{f_1}\right)$$

Assuming $f \gg 10.7 \text{ Hz}$, we have

$$\tan^{-1}\left(\frac{f}{f_1}\right) = 20^\circ \Rightarrow \frac{f}{f_1} = 0.364$$

At this frequency, $|A(f)| = 1$, so

$$\begin{aligned}1 &= \frac{181,260}{\sqrt{1 + \left(\frac{f}{10.7}\right)^2} \cdot \sqrt{1 + (0.364)^2}} \\&= \frac{170,327}{\sqrt{1 + \left(\frac{f}{10.7}\right)^2}}\end{aligned}$$

$$\text{or } \frac{f}{10.7} = 170,327 \Rightarrow f = 1.82 \text{ MHz}$$

Then, second pole at

$$f_1 = \frac{f}{0.364} \Rightarrow f_1 = 5 \text{ MHz}$$

13.27

$$(a) 10 \times 10^6 = f_{PD} (10^6) \Rightarrow f_{PD} = 10 \text{ Hz}$$

$$(b) f_{PD} = \frac{1}{2\pi R_{eq} C_M} \Rightarrow C_M = \frac{1}{2\pi (1.2 \times 10^6) (10)} = 1.326 \times 10^{-8} \text{ F}$$

$$C_M = C_F (1 + |A|) \Rightarrow C_F = \frac{1.326 \times 10^{-8}}{1001} \Rightarrow C_F = 13.25 \text{ pF}$$

13.28

$$\phi = -110 = -\tan^{-1}\left(\frac{f}{f_{PD}}\right) - 2\tan^{-1}\left(\frac{f}{f_{2,3}}\right) = -90 - 2\tan^{-1}\left(\frac{f}{f_{2,3}}\right)$$

$$\text{So } \left(\frac{f}{f_{2,3}}\right) = 0.1763$$

$$|A| = 1 = \frac{200,000}{\sqrt{1 + \left(\frac{f}{f_{PD}}\right)^2} \left[1 + \left(\frac{f}{f_{2,3}}\right)^2\right]} \cong \frac{200,000}{\left(\frac{f}{10}\right) [1 + (0.1763)^2]} \Rightarrow f = 1.9397 \text{ MHz}$$

$$\text{Then } f_{2,3} = \frac{1.9397 \times 10^6}{0.1763} \Rightarrow f_{2,3} = 11.0 \text{ MHz}$$

13.29

$$(a) I_{D3} = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_3 (V_{SG3} + V_{TP})^2$$

$$150 = \left(\frac{40}{2}\right) (50) (V_{SG3} - 0.4)^2 \Rightarrow V_{SG3} = 0.7873 \text{ V}$$

$$R_{D1} = \frac{0.7873}{0.1} = 7.87 \text{ k}\Omega$$

$$I_{D4} = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_4 (V_{GS4} - V_{TN})^2$$

$$200 = \left(\frac{100}{2}\right) (40) (V_{GS4} - 0.4)^2 \Rightarrow V_{GS4} = 0.7162 \text{ V}$$

$$R_{D2} = \frac{0.7162 - (-3)}{0.15} = 24.8 \text{ k}\Omega$$

$$R_S = \frac{0 - (-3)}{0.15} = 15 \text{ k}\Omega$$

$$(b) (i) A_{d1} = \left(\frac{g_{m1}}{2}\right) \cdot R_{D1}, \quad g_{m1} = 2\sqrt{\left(\frac{0.1}{2}\right)(20)(0.1)} = 0.6325 \text{ mA/V}$$

$$A_{d1} = \left(\frac{0.6325}{2}\right)(7.87) = 2.49$$

$$(ii) A_2 = -g_{m3} R_{D2}, \quad g_{m3} = 2\sqrt{\left(\frac{0.04}{2}\right)(50)(0.15)} = 0.7746 \text{ mA/V}$$

$$A_2 = -(0.7746)(24.8) = -19.21$$

$$(iii) A_3 = \frac{g_{m4}R_S}{1+g_{m4}R_S}, \quad g_{m4} = 2\sqrt{\left(\frac{0.1}{2}\right)(40)(0.2)} = 1.265 \text{ mA/V}$$

$$A_3 = \frac{(1.265)(15)}{1+(1.265)(15)} = 0.950$$

$$(c) A = A_{d1} \cdot A_2 \cdot A_3 = (2.49)(-19.21)(0.95) = -45.4$$

13.30

$$(a) I_{D3} = 100 = \left(\frac{100}{2}\right)(25)(V_{GS3} - 0.4)^2 \Rightarrow V_{GS3} = 0.6828 \text{ V}$$

$$R_{D1} = \frac{0.6828}{0.05} = 13.66 \text{ k}\Omega$$

$$I_{D4} = 200 = \left(\frac{40}{2}\right)(100)(V_{SG4} - 0.4)^2 \Rightarrow V_{SG4} = 0.7162 \text{ V}$$

$$R_{D3} = \frac{3 - (-0.7162)}{0.1} = 37.16 \text{ k}\Omega$$

$$R_S = \frac{3 - 0}{0.2} = 15 \text{ k}\Omega$$

$$(b) (i) A_d = \left(\frac{g_{m1}}{2}\right) \cdot R_{D1}, \quad g_{m1} = 2\sqrt{\left(\frac{0.04}{2}\right)(80)(0.05)} = 0.5657 \text{ mA/V}$$

$$A_d = \left(\frac{0.5657}{2}\right)(13.66) = 3.864$$

$$(ii) A_2 = -g_{m3}R_{D3}, \quad g_{m3} = 2\sqrt{\left(\frac{0.1}{2}\right)(25)(0.1)} = 0.7071 \text{ mA/V}$$

$$A_2 = -(0.7071)(37.16) = -26.28$$

$$(iii) A_3 = \frac{g_{m4}R_S}{1+g_{m4}R_S}, \quad g_{m4} = 2\sqrt{\left(\frac{0.04}{2}\right)(100)(0.2)} = 1.265 \text{ mA/V}$$

$$A_3 = \frac{(1.265)(15)}{1+(1.265)(15)} = 0.950$$

$$(c) A = A_d \cdot A_2 \cdot A_3 = (3.864)(-26.28)(0.95) = -96.5$$

13.31

a. Original g_{m1} and g_{m2}

$$K_{p1} = K_{p2} = \left(\frac{W}{L}\right)\left(\frac{\mu_p C_{ox}}{2}\right) = (12.5)(10) \\ = 125 \text{ }\mu\text{A/V}^2$$

So

$$g_{m1} = g_{m2} = 2\sqrt{K_{p1}\left(\frac{I_\phi}{2}\right)} = 2\sqrt{(0.125)(10)} \\ = 0.09975 \text{ mA/V}$$

If $\left(\frac{W}{L}\right)$ is increased to 50, then

$$K_{p1} = K_{p2} = (50)(10) = 500 \mu A/V^2$$

So

$$g_{m1} = g_{m2} = 2\sqrt{(0.5)(0.0199)} = 0.1995 \text{ mA/V}$$

b. Gain of first stage

$$A_d = g_{m1}(r_{o2} \| r_{o4}) = (0.1995)(5025 \| 5025)$$

or

$$\underline{A_d = 501}$$

Voltage gain of second stage remains the same, or

$$A_{v2} = 251$$

Then $A_v = A_d \cdot A_{v2} = (501)(251)$

or

$$\underline{A_d = 125,751}$$

13.32

$$v_{CM}(\max) = V^+ - v_{SD6}(\min) - v_{SG1} \\ v_{SD6}(\min) = v_{SD}(\text{sat}) + 0.2 = 0.9022 - 0.5 + 0.2 = 0.6022 \text{ V}$$

$$i_{D1} = K_{p1}(v_{SG1} + V_{TP})^2$$

$$20.2 = 250(v_{SG1} - 0.5)^2 \Rightarrow v_{SG1} = 0.7843 \text{ V}$$

$$v_{CM}(\max) = 5 - 0.6022 - 0.7843 = 3.61 \text{ V}$$

$$v_{CM}(\min) = V^- + v_{GS3} + v_{SD1}(\min) - v_{SG1}$$

$$i_{D3} = K_n(v_{GS3} - V_{TN})^2, \quad K_n = \left(\frac{0.1}{2}\right)(6.25) = 0.3125 \text{ mA/V}^2$$

$$20.2 = 312.5(v_{GS3} - 0.5)^2 \Rightarrow v_{GS3} = 0.7542 \text{ V}$$

$$v_{SD1}(\min) = 0.7843 - 0.5 + 0.2 = 0.4843 \text{ V}$$

$$v_{CM}(\min) = -5 + 0.7542 + 0.4843 - 0.7843 = -4.55 \text{ V}$$

So $-4.55 \leq v_{CM} \leq 3.61 \text{ V}$

13.33

$$(a) \quad K_{p5} = \left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_5 = \left(\frac{0.04}{2}\right)(50) = 1.0 \text{ mA/V}^2$$

$$K_{p5}(V_{SG5} + V_{TP})^2 = \frac{V^+ - V_{SG5} - V^-}{R_{set}}$$

$$(1)(50)(V_{SG5}^2 - 1.4V_{SG5} + 0.49) = 10 - V_{SG5}$$

$$50V_{SG5}^2 - 69V_{SG5} + 14.5 = 0 \Rightarrow V_{SG5} = 1.121 \text{ V}$$

$$I_{set} = I_Q = \frac{10 - 1.121}{50} = 0.1776 \text{ mA} = I_{D7}$$

$$(b) \quad A_d = g_{m1}(r_{o2} \| r_{o4}), \quad g_{m1} = 2\sqrt{\left(\frac{0.04}{2}\right)(50)\left(\frac{0.1776}{2}\right)} = 0.5960 \text{ mA/V}$$

$$r_{o2} = \frac{1}{(0.04)\left(\frac{0.1776}{2}\right)} = 281.5 \text{ k}\Omega, \quad r_{o4} = \frac{1}{(0.02)\left(\frac{0.1776}{2}\right)} = 563.1 \text{ k}\Omega$$

$$A_d = (0.5960)(281.5\|563.1) = 111.9$$

$$A_2 = -g_{m7}(r_{o7}\|r_{o8}), \quad g_{m7} = 2\sqrt{\left(\frac{0.1}{2}\right)(30)(0.1776)} = 1.032 \text{ mA/V}$$

$$r_{o7} = \frac{1}{(0.02)(0.1776)} = 281.5 \text{ k}\Omega, \quad r_{o8} = \frac{1}{(0.04)(0.1776)} = 140.8 \text{ k}\Omega$$

$$A_2 = -(1.032)(281.5\|140.8) = -96.86$$

$$A = A_d \cdot A_2 = (111.9)(-96.86) = -10,839$$

13.34

$$A_d = g_{m1}(r_{o2}\|r_{o4}), \quad g_{m1} = 2\sqrt{\left(\frac{0.04}{2}\right)(50)(0.1)} = 0.6325 \text{ mA/V}$$

$$r_{o2} = \frac{1}{(0.025)(0.1)} = 400 \text{ k}\Omega, \quad r_{o4} = \frac{1}{(0.015)(0.1)} = 666.7 \text{ k}\Omega$$

$$A_d = (0.6325)(400\|666.7) = 158.1$$

$$A_2 = -g_{m7}(r_{o7}\|r_{o8}), \quad g_{m7} = 2\sqrt{\left(\frac{0.1}{2}\right)(30)(0.2)} = 1.095 \text{ mA/V}$$

$$r_{o7} = \frac{1}{(0.015)(0.2)} = 333.3 \text{ k}\Omega, \quad r_{o8} = \frac{1}{(0.025)(0.2)} = 200 \text{ k}\Omega$$

$$A_2 = -(1.095)(333.3\|200) = -136.9$$

$$A = A_d \cdot A_2 = (158.1)(-136.9) = -21,644$$

13.35

$$f_{PD} = \frac{1}{2\pi R_{eq} C_i}$$

where $R_{eq} = r_{o2}\|r_{o4}$ and $C_i = C_1(1 + |A_{v2}|)$

We can find that

$$A_{v2} = 251 \text{ and } r_{o4} = r_{o2} = 5.025 \text{ M}\Omega$$

Now

$$R_{eq} = 5.025\|5.025 = 2.51 \text{ M}\Omega$$

and

$$C_i = 12(1 + 251) = 3024 \text{ pF}$$

So

$$f_{PD} = \frac{1}{2\pi(2.51 \times 10^6)(3024 \times 10^{-12})}$$

or

$$\underline{f_{PD} = 21.0 \text{ Hz}}$$

13.36

From Problem 13.33,

$$|A_2| = 96.86, \quad r_{o2} = 281.5 \text{ k}\Omega, \quad r_{o4} = 563.1 \text{ k}\Omega$$

$$f_{PD} = \frac{1}{2\pi(r_{o2} \| r_{o4})C_M}$$

$$\text{Then } C_M = \frac{1}{2\pi(8)(281.5 \| 563.1) \times 10^3} = 1.06 \times 10^{-7} \text{ F}$$

$$C_M = C_1(1 + |A_2|) \Rightarrow 1.06 \times 10^{-7} = C_1(97.86)$$

$$\text{Or } C_1 = 1.08 \times 10^{-9} \text{ F}$$

13.37

$$R_0 = r_{07} \| r_{08}$$

We can find that

$$r_{07} = r_{08} = 2.52 \text{ M}\Omega$$

Then

$$R_0 = 2.52 \| 2.52$$

or

$$\underline{R_0 = 1.26 \text{ M}\Omega}$$

13.38

$$(a) \quad V_{GS2} = \frac{6}{3} = 2 \text{ V}$$

$$I_D = 0.5 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(2 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_{2-5} = 3.91$$

$$(b) \quad A_v = \frac{g_{m1}(r_{o1} \| r_{o2})}{1 + g_{m1}(r_{o1} \| r_{o2})}, \quad r_{o1} = r_{o2} = \frac{1}{(0.025)(0.5)} = 80 \text{ k}\Omega, \quad r_{o1} \| r_{o2} = 40 \text{ k}\Omega$$

$$0.98 = \frac{g_{m1}(40)}{1 + g_{m1}(40)} \Rightarrow g_{m1} = 1.225 \text{ mA/V}$$

$$g_{m1} = 1.225 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_1(0.5)} \Rightarrow \left(\frac{W}{L}\right)_1 = 15$$

$$(c) \quad R_o = \frac{1}{g_{m1}} \| r_{o1} \| r_{o2} = \frac{1}{1.225} \| 40 = 0.8163 \| 40$$

$$R_o = 800 \text{ }\Omega$$

13.39

$$I_{Q2} = \left(\frac{80}{2}\right)(20)[1.1737 - 0.7]^2$$

$$(a) \quad I_{Q2} = 180 \mu\text{A}$$

$$I_{D6} = \left(\frac{80}{2}\right)(25)(V_{GS6} - 0.7)^2 = 25 \Rightarrow V_{GS6} = 0.8581 \text{ V}$$

$$(b) \quad I_{D7} = \left(\frac{40}{2}\right)(50)(V_{SG7} - 0.7)^2 = 25 \Rightarrow V_{SG7} = 0.8581 \text{ V}$$

Set

$$V_{SG8P} = V_{GS8N} = 0.8581 \text{ V}$$

$$180 = \left(\frac{40}{2}\right)\left(\frac{W}{L}\right)_{8P} (0.8581 - 0.7)^2 \Rightarrow \left(\frac{W}{L}\right)_{8P} = 360$$

$$180 = \left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_{8N} (0.8581 - 0.7)^2 \Rightarrow \left(\frac{W}{L}\right)_{8N} = 180$$

13.40

$$I_{REF} = I_{Q1} = 0.150 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_{11} (V_{GS11} - 0.5)^2 = \left(\frac{0.1}{2}\right)(20)(V_{GS11} - 0.5)^2 \Rightarrow V_{GS11} = 0.8873 \text{ V}$$

$$V_{GS,REF} = 5 - 0.8873 = 4.1127 \text{ V}$$

For three NMOS transistors in series:

$$V_{GS} = \frac{4.1127}{3} = 1.3709 \text{ V}$$

$$I_{REF} = 0.15 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(1.3709 - 0.5)^2 \Rightarrow \left(\frac{W}{L}\right) = 3.96$$

13.41

(a)

$$I_{Q2} = 250 \mu\text{A} = \left(\frac{80}{2}\right)(5)(V_{GS8} - 0.7)^2$$

$$\Rightarrow V_{GS8} = 1.818 \text{ V}$$

$$\Rightarrow V_{GS6} = V_{SG7} = \frac{1.818}{2} = 0.909 \text{ V}$$

$$I_{D6} = I_{D7} = \left(\frac{80}{2}\right)(25)(0.909 - 0.7)^2 = 43.7 \mu\text{A}$$

(b)

$$g_{m1} = 2\sqrt{\left(\frac{80}{2}\right)(15)\left(\frac{250}{2}\right)} \Rightarrow 0.5477 \text{ mA/V}$$

$$r_{o2} = \frac{1}{(0.01)(0.125)} = 800 \text{ K}$$

$$r_{o4} = \frac{1}{(0.015)(0.125)} = 533.3 \text{ K}$$

$$A_{d1} = g_{m1}(r_{o2} \| r_{o4}) = (0.5477)(800 \| 533.3)$$

$$A_{d1} = 175$$

Second stage:

$$A_2 = -g_{m5}(r_{o5} \| r_{o9})$$

$$g_{m5} = 2\sqrt{\left(\frac{40}{2}\right)(80)(250)} \Rightarrow 1.265 \text{ mA/V}$$

$$r_{o5} = \frac{1}{(0.015)(0.25)} = 266.7 \text{ K}$$

$$r_{o9} = \frac{1}{(0.01)(0.25)} = 400 \text{ K}$$

$$A_2 = -(1.265)(266.7 \| 400)$$

$$A_2 = -202$$

Assume the gain of the output stage ≈ 1 , then

$$A_v = A_{d1} \cdot A_2 = (175)(-202)$$

$$A_v = -35,350$$

13.42

$$(a) \quad A_d = g_{m1}(R_{o6} \| R_{o8})$$

$$g_{m1} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.025)} \Rightarrow 224 \mu\text{A/V}$$

$$g_{m1} = g_{m8}$$

$$g_{m6} = 2\sqrt{(0.5)(0.025)} \Rightarrow 224 \mu\text{A/V}$$

$$r_{o1} = r_{o6} = r_{o8} = r_{o10} = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.015)(25)} = 2.67 M\Omega$$

$$r_{o4} = \frac{1}{\lambda I_{D4}} = \frac{1}{(0.015)(50)} \Rightarrow 1.33 M\Omega$$

Now

$$R_{o8} = g_{m8}(r_{o8} r_{o10}) = (224)(2.67)(2.67) = 1597 M\Omega$$

$$R_{o6} = g_{m6}(r_{o6})(r_{o4} \| r_{o1}) = (224)(2.67)(2.67 \| 1.33) \Rightarrow R_{o6} = 531 M\Omega$$

Then

$$A_d = (224)(531 \| 1597) \Rightarrow A_d = 89,264$$

$$(b) \quad R_o = R_{o6} \| R_{o8} = 531 \| 1597 \Rightarrow R_o = 398 M\Omega$$

$$(c) \quad f_{PD} = \frac{1}{2\pi R_o C_L} = \frac{1}{2\pi(398 \times 10^6)(5 \times 10^{-12})} \Rightarrow f_{PD} = 80 \text{ Hz}$$

$$GBW = (89,264)(80) \Rightarrow GBW = 7.14 \text{ MHz}$$

13.43

(a)

$$r_{o1} = r_{o8} = r_{o10} = \frac{1}{\lambda_p I_D} = \frac{1}{(0.02)(25)} = 2 M\Omega$$

$$r_{o6} = \frac{1}{\lambda_n I_D} = \frac{1}{(0.015)(25)} = 2.67 M\Omega$$

$$r_{o4} = \frac{1}{\lambda_n I_{D4}} = \frac{1}{(0.015)(50)} = 1.33 M\Omega$$

$$g_{m1} = 2\sqrt{\left(\frac{35}{2}\right)\left(\frac{W}{L}\right)_1}(25) = 41.8\sqrt{\left(\frac{W}{L}\right)_1} = g_{m8}$$

$$g_{m6} = 2\sqrt{\left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_6}(25) = 63.2\sqrt{\left(\frac{W}{L}\right)_6}$$

$$R_o = R_{o6} \| R_{o8} = [g_{m6}(r_{o6})(r_{o4} \| r_{o1})] \| [g_{m8}(r_{o8}r_{o10})]$$

$$X_1 = \sqrt{\left(\frac{W}{L}\right)_1} \quad \text{and} \quad X_6 = \sqrt{\left(\frac{W}{L}\right)_6}$$

Define

Then

$$R_o = [63.2X_6(2.67)(1.33 \| 2)] \| [41.8X_1(2)(2)]$$

$$= 134.8X_6 \| 167.2X_1 = \frac{22,539X_1X_6}{134.8X_6 + 167.2X_1}$$

$$A_d = g_{m1}R_o = (41.8X_1)\left(\frac{22,539X_1X_6}{134.8X_6 + 167.2X_1}\right)$$

$$= 10,000$$

$$X_6 = \sqrt{\left(\frac{W}{L}\right)_6} = \sqrt{\frac{1}{2.2}\left(\frac{W}{L}\right)_1} = 0.674X_1$$

Now

We then find

$$\underline{X_1^2 = \left(\frac{W}{L}\right)_1 = 4.06 = \left(\frac{W}{L}\right)_p}$$

and

$$\underline{\left(\frac{W}{L}\right)_n = 1.85}$$

13.44

Let $V^+ = 5V$, $V^- = -5V$

$$P = I_T(10) = 3 \Rightarrow I_T = 0.3 \text{ mA} \Rightarrow I_{REF} = 0.1 \text{ mA} = 100 \mu\text{A}$$

$$r_{o1} = r_{o8} = r_{o10} = \frac{1}{(0.02)(50)} = 1 M\Omega$$

$$r_{o6} = \frac{1}{(0.015)(50)} = 1.33 M\Omega$$

$$r_{o4} = \frac{1}{(0.015)(100)} = 0.667 M\Omega$$

$$g_{m1} = 2\sqrt{\left(\frac{35}{2}\right)\left(\frac{W}{L}\right)_1}(50) = 59.2X_1 = g_{m8}$$

$$X_1 = \sqrt{\left(\frac{W}{L}\right)_1}$$

where

Assume all width-to-length ratios are the same.

$$g_{m6} = 2\sqrt{\left(\frac{80}{2}\right)\left(\frac{W}{L}\right)(50)} = 89.4X_1$$

Now

$$\begin{aligned} R_o &= R_{o6} \parallel R_{o8} = \left[g_{m6}(r_{o6})(r_{o4} \parallel r_{o1}) \right] \parallel \left[g_{m8}(r_{o8}r_{o10}) \right] \\ &= \left[89.4X_1(1.33)(0.667 \parallel 1) \right] \parallel \left[59.2X_1(1)(1) \right] \\ &= [47.6X_1] \parallel [59.2X_1] = \frac{(47.6X_1)(59.2X_1)}{47.6X_1 + 59.2X_1} \end{aligned}$$

$$\text{So } R_o = 26.4X_1$$

Now

$$A_d = g_{m1}R_o = (59.2X_1)(26.4X_1) = 25,000$$

$$X_1^2 = \frac{W}{L} = 16$$

So that $\frac{W}{L} = 16$ for all transistors

13.45

$$(a) A_d = Bg_{m1}(r_{o6} \parallel r_{o8})$$

$$g_{m1} = 2\sqrt{\left(\frac{0.1}{2}\right)(20)(0.06)} = 0.4899 \text{ mA/V}$$

$$r_{o6} = \frac{1}{(0.04)(0.06)(3)} = 138.9 \text{ k}\Omega$$

$$r_{o8} = \frac{1}{(0.02)(0.06)(3)} = 277.8 \text{ k}\Omega$$

$$A_d = (3)(0.4899)(138.9 \parallel 277.8) = 136.1$$

$$(b) R_o = r_{o6} \parallel r_{o8} = 138.9 \parallel 277.8 = 92.6 \text{ k}\Omega$$

$$(c) f_{PD} = \frac{1}{2\pi R_o C} = \frac{1}{2\pi(92.6 \times 10^3)(5 \times 10^{-12})} \Rightarrow f_{PD} = 343.7 \text{ kHz}$$

$$GBW = (136.1)(343.7 \times 10^3) \Rightarrow GBW = 46.8 \text{ MHz}$$

13.46

$$r_{o6} = \frac{1}{(0.02)(2.5)(40)} = 0.5 M\Omega$$

$$r_{o8} = \frac{1}{(0.015)(2.5)(40)} = 0.667 M\Omega$$

$$A_d = Bg_{m1}(r_{o6} \parallel r_{o8})$$

$$400 = (2.5)g_{m1}(0.5 \parallel 0.667) \Rightarrow g_{m1} = 560 \mu A/V$$

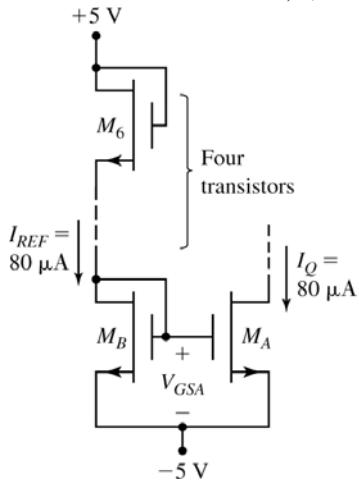
$$(a) g_{m1} = 560 = 2\sqrt{\left(\frac{80}{2}\right)\left(\frac{W}{L}\right)(40)} \Rightarrow \left(\frac{W}{L}\right) = 49$$

Assume all (W/L) ratios are the same except for

$$M_5 \text{ and } M_6 \cdot \left(\frac{W}{L} \right)_5 = \left(\frac{W}{L} \right)_6 = 122.5$$

(b) Assume the bias voltages are

$$V^+ = 5V, V^- = -5V.$$



$$\text{Assume } \left(\frac{W}{L} \right)_A = \left(\frac{W}{L} \right)_B = 49$$

$$I_Q = \left(\frac{80}{2} \right) (49)(V_{GSA} - 0.5)^2 = 80 \Rightarrow V_{GSA} = 0.702 \text{ V}$$

Then

$$I_{REF} = 80 = \left(\frac{80}{2} \right) \left(\frac{W}{L} \right)_C (V_{GSC} - 0.5)^2$$

For four transistors

$$V_{GSC} = \frac{10 - 0.702}{4} = 2.325 \text{ V}$$

$$80 = \left(\frac{80}{2} \right) \left(\frac{W}{L} \right)_C (2.325 - 0.5)^2 \Rightarrow \left(\frac{W}{L} \right)_C = 0.60$$

$$(c) f_{3-dB} = \frac{1}{2\pi R_o C} \quad R_o = 0.5 \| 0.667 = 0.286 \text{ M}\Omega$$

$$f_{3-dB} = \frac{1}{2\pi(286 \times 10^3)(3 \times 10^{-12})} = 185 \text{ kHz}$$

$$GBW = (400)(185 \times 10^3) \Rightarrow 74 \text{ MHz}$$

13.47

(a) From previous results, we can write

$$R_{o10} = g_{m10}(r_{o10} r_{o6})$$

$$R_{o12} = g_{m12}(r_{o12} r_{o8})$$

$$A_d = Bg_{m1}(R_{o10} \| R_{o12})$$

Now

$$r_{o10} = r_{o6} = \frac{1}{\lambda_p B(I_o/2)} = \frac{1}{(0.02)(2.5)(40)} = 0.5 \text{ M}\Omega$$

$$r_{o12} = r_{o8} = \frac{1}{\lambda_n B(I_Q/2)} = \frac{1}{(0.015)(2.5)(40)} = 0.667 \text{ M}\Omega$$

Assume all transistors have the same width-to-length ratios except for M_5 and M_6 .

$$\text{Let } \left(\frac{W}{L}\right) = X^2$$

Then

$$\begin{aligned} g_{m10} &= 2\sqrt{\left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_{10}(I_{DQ10})} = 2\sqrt{\left(\frac{35}{2}\right)X^2(2.5)(40)} \\ &= 83.67X \\ g_{m12} &= 2\sqrt{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_{12}(I_{DQ12})} = 2\sqrt{\left(\frac{80}{2}\right)X^2(2.5)(40)} \\ &= 126.5X \\ g_{m1} &= 2\sqrt{\left(\frac{80}{2}\right)X^2(40)} = 80X \end{aligned}$$

Then

$$R_{o10} = (83.67X)(0.5)(0.5) = 20.9X \text{ M}\Omega$$

$$R_{o12} = (126.5X)(0.667)(0.667) = 56.3X \text{ M}\Omega$$

We want

$$\begin{aligned} 20,000 &= (2.5)(80X)[20.9X \parallel 56.3X] \\ &= 200X \left[\frac{(20.9X)(56.3X)}{20.9X + 56.3X} \right] = 3048X^2 \end{aligned}$$

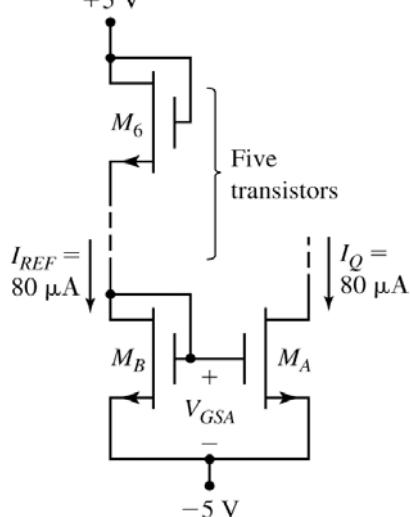
Then

$$X^2 = 6.56 = \left(\frac{W}{L}\right)$$

Then

$$\left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_5 = (2.5)(6.56) = 16.4$$

(b) Assume bias voltages are $V^+ = 5V$, $V^- = -5V$



$$\text{Assume } \left(\frac{W}{L}\right)_A = \left(\frac{W}{L}\right)_B = 6.56$$

$$I_Q = 80 = \left(\frac{80}{2}\right)(6.56)(V_{GSA} - 0.5)^2 \Rightarrow V_{GSA} = 1.052 \text{ V}$$

Need 5 transistors in series

$$V_{GSC} = \frac{10 - 1.052}{5} = 1.79 \text{ V}$$

Then

$$I_{REF} = 80 = \left(\frac{80}{2} \right) \left(\frac{W}{L} \right)_c (1.79 - 0.5)^2 \Rightarrow \left(\frac{W}{L} \right)_c = 1.20$$

$$(c) f_{3-dB} = \frac{1}{2\pi R_o C} \quad \text{where } R_o = R_{o10} \parallel R_{o12}$$

Now

$$R_{o10} = 20.9\sqrt{6.56} = 53.5 \text{ M}\Omega$$

$$R_{o12} = 56.3\sqrt{6.56} = 144 \text{ M}\Omega$$

Then

$$R_o = 53.5 \parallel 144 = 39 \text{ M}\Omega$$

$$f_{3-dB} = \frac{1}{2\pi(39 \times 10^6)(3 \times 10^{-12})} = 1.36 \text{ kHz}$$

$$GBW = (20,000)(1.36 \times 10^3) \Rightarrow GBW = 27.2 \text{ MHz}$$

13.48

$$(a) \Delta V_{O1} = 0.7 + (0.3)(0.4) = 0.82 \text{ V}$$

$$R_D = \frac{0.82}{0.15} = 5.47 \text{ k}\Omega$$

$$R_C = \frac{3 - 0}{0.3} = 10 \text{ k}\Omega$$

$$(b) A_d = \frac{g_m 1}{2} (R_D \parallel R_{iC}), \quad g_m 1 = 2 \sqrt{\left(\frac{0.04}{2} \right) (50)(0.15)} = 0.7746 \text{ mA/V}$$

$$R_{iC} = r_\pi + (1 + \beta)R_E, \quad r_\pi = \frac{(120)(0.026)}{0.3} = 10.4 \text{ k}\Omega$$

$$R_{iC} = 10.4 + (121)(0.4) = 58.8 \text{ k}\Omega$$

$$A_d = \left(\frac{0.7746}{2} \right) (5.47 \parallel 58.8) = 1.938$$

$$(c) A_2 = \frac{-\beta R_C}{r_\pi + (1 + \beta)R_E} = \frac{-(120)(10)}{10.4 + (121)(0.4)} = -20.41$$

$$(d) A_v = A_d \cdot A_2 = (1.938)(-20.41) = -39.6$$

13.49

$$(a) R_S = \frac{0 - (-3)}{0.3} = 10 \text{ k}\Omega$$

$$I_{D1} = K_n (V_{GS1} - V_{TN})^2$$

$$0.3 = 3(V_{GS1} - 0.4)^2 \Rightarrow V_{GS1} = 0.7162 \text{ V}$$

$$R_{C2} = \frac{0.7162 - (-3)}{0.3} = 12.39 \text{ k}\Omega$$

$$\Delta V_{O1} = (0.3)(0.5) + 0.7 = 0.85 \text{ V}$$

$$R_{C1} = \frac{0.85}{0.25} = 3.4 \text{ k}\Omega$$

$$(b) \quad g_{m1} = \frac{0.25}{0.026} = 9.615 \text{ mA/V}, \quad r_{\pi 1} = \frac{(120)(0.026)}{0.25} = 12.48 \text{ k}\Omega$$

$$g_{m3} = \frac{0.3}{0.026} = 11.54 \text{ mA/V}, \quad r_{\pi 3} = \frac{(120)(0.026)}{0.3} = 10.4 \text{ k}\Omega$$

$$A_d = \frac{g_{m1}}{2} (R_{C1} \| R_{i3})$$

$$R_{i3} = r_{\pi 3} + (1 + \beta)R_E = 10.4 + (121)(0.5) = 70.9 \text{ k}\Omega$$

$$A_d = \frac{9.615}{2} (3.4 \| 70.9) = 15.60$$

$$(c) \quad A_2 = \frac{-\beta R_{C2}}{r_{\pi 3} + (1 + \beta)R_E} = \frac{-(120)(12.39)}{10.4 + (121)(0.5)} = -20.97$$

$$(d) \quad A_3 = \frac{g_{mD} R_S}{1 + g_{mD} R_S}, \quad g_{mD} = 2\sqrt{(3)(0.3)} = 1.897 \text{ mA/V}$$

$$A_3 = \frac{(1.897)(10)}{1 + (1.897)(10)} = 0.950$$

$$(e) \quad A_v = A_d \cdot A_2 \cdot A_3 = (15.6)(-20.97)(0.95) = -310.8$$

13.50

(a) For PMOS:

$$g_{m1} = g_{m2} = 2\sqrt{\left(\frac{0.04}{2}\right)(40)(0.125)} = 0.6325 \text{ mA/V}$$

$$r_{oP} = \frac{1}{\lambda I_{D1}} = \frac{1}{(0.035)(0.125)} = 228.6 \text{ k}\Omega$$

For BJT:

$$r_{o2} = \frac{V_A}{I_C} = \frac{150}{0.125} = 1200 \text{ k}\Omega$$

$$(b) \quad A_d = g_{m2} (r_{oP} \| r_{o2}) = (0.6325)(228.6 \| 1200) = 121.5$$

13.51

(a) For NMOS:

$$g_{mN} = 2\sqrt{\left(\frac{0.1}{2}\right)(40)(0.125)} = 1.0 \text{ mA/V}$$

$$r_{oN} = \frac{1}{(0.02)(0.125)} = 400 \text{ k}\Omega$$

For BJT:

$$r_{o2} = \frac{100}{0.125} = 800 \text{ k}\Omega$$

$$(b) \quad A_d = g_{mN} (r_{oN} \| r_{o2}) = (1.0)(400 \| 800) = 266.7$$

13.52

$$I_{REF} = 200 \mu A \quad K_n = K_p = 0.5 mA/V^2 \\ \lambda_n = \lambda_p = 0.015 V^{-1}$$

$$(a) \quad A_d = g_{m1}(R_{o6} \| R_{o8})$$

where

$$R_{o8} = g_{m8}(r_{o8} r_{o10}) \\ R_{o6} = g_{m6}(r_{o6})(r_{o4} \| r_{o1})$$

Now

$$g_{m8} = 2\sqrt{K_p I_{D8}} = 2\sqrt{(0.5)(0.1)} = 0.447 mA/V$$

$$r_{o8} = \frac{1}{\lambda_p I_{D8}} = \frac{1}{(0.015)(0.1)} = 667 k\Omega$$

$$r_{o10} = \frac{1}{\lambda_p I_{D8}} = 667 k\Omega$$

$$g_{m6} = \frac{I_{C6}}{V_T} = \frac{0.1}{0.026} = 3.846 mA/V$$

$$r_{o6} = \frac{V_A}{I_{C6}} = \frac{80}{0.1} = 800 k\Omega$$

$$r_{o4} = \frac{1}{\lambda_n I_{D4}} = \frac{1}{(0.015)(0.2)} = 333 k\Omega$$

$$r_{o1} = \frac{1}{\lambda_p I_{D1}} = \frac{1}{(0.015)(0.1)} = 667 k\Omega$$

$$g_{m1} = 2\sqrt{K_p I_{D1}} = 2\sqrt{(0.5)(0.1)} = 0.447 mA/V$$

So

$$R_{o8} = (0.447)(667)(667) \Rightarrow 198.9 M\Omega$$

$$R_{o6} = (3.846)(800)(333 \| 667) \Rightarrow 683.4 M\Omega$$

Then

$$A_d = 447(198.9 \| 683.4) \Rightarrow \underline{A_d = 68,865}$$

13.53

Assume biased at $V^+ = 10V$, $V^- = -10V$.

$$P = 3I_{REF}(20) = 10 \Rightarrow I_{REF} = 167 \mu A$$

$$A_d = g_{m1}(R_{o6} \| R_{o8}) = 25,000$$

$$k'_n = 80 \mu A/V^2, k'_p = 35 \mu A/V^2$$

$$\lambda_n = 0.015 V^{-1}, \lambda_p = 0.02 V^{-1}$$

$$\text{Assume } \left(\frac{W}{L}\right)_p = 2.2 \left(\frac{W}{L}\right)_n$$

$$R_{o8} = g_{m8}(r_{o8} r_{o10})$$

$$R_{o6} = g_{m6}(r_{o6})(r_{o4} \| r_{o1})$$

$$r_{o8} = \frac{1}{\lambda_p I_{D8}} = \frac{1}{(0.02)(83.3)} = 0.60 M\Omega$$

$$r_{o10} = \frac{1}{\lambda_p I_{D8}} = 0.60 M\Omega$$

$$g_{m8} = 2\sqrt{\left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_8 I_{D8}} = 2\sqrt{\left(\frac{35}{2}\right)(2.2)X^2(83.3)}$$

$$= 113.3X$$

where $X^2 = \left(\frac{W}{L}\right)_n$

$$r_{o6} = \frac{V_A}{I_{C6}} = \frac{80}{83.3} = 0.960 M\Omega$$

$$r_{o4} = \frac{1}{\lambda_n I_{D4}} = \frac{1}{(0.015)(167)} = 0.40 M\Omega$$

$$r_{o1} = \frac{1}{\lambda_p I_{D1}} = \frac{1}{(0.02)(83.3)} = 0.60 M\Omega$$

$$g_{m6} = \frac{I_{C6}}{V_T} = \frac{83.3}{0.026} = 3204 \mu A/V$$

$$g_{m1} = 2\sqrt{\left(\frac{k'_p}{2}\right)\left(\frac{W}{L}\right)_1} I_{D1} = 2\sqrt{\left(\frac{35}{2}\right)(2.2)X^2(83.3)}$$

$$= 113.3X$$

Now

$$R_{o6} = (3204)(0.960)[0.40\|0.60] = 738 M\Omega$$

$$R_{o8} = (113.3X)(0.60)(0.60) = 40.8X M\Omega$$

Then

$$A_a = 25,000 = (113.3X)[738\|40.8X]$$

$$= (113.3X)\left[\frac{30,110X}{738+40.8X}\right]$$

which yields $X = 2.48$

or

$$X^2 = 6.16 = \left(\frac{W}{L}\right)_n$$

and

$$\left(\frac{W}{L}\right)_p + (2.2)(6.16) = 12.3$$

13.54

For v_{cm} (max), assume $V_{CB}(Q_5) = 0$. Then

$$V_S = 15 - 0.6 - 0.6 = 13.8 V$$

$$I_{D9} = I_{D10} = \frac{0.236}{2} = 0.118 \text{ mA}$$

Using parameters given in Example 13.12

$$V_{SG} = \sqrt{\frac{I_{D9}}{K_p}} - V_{TP} = \sqrt{\frac{0.118}{0.20}} + 1.4 = 2.17 V$$

Then

$$v_{cm}(\text{max}) = 13.8 - 2.17 \Rightarrow v_{cm}(\text{max}) = 11.6 V$$

For

$$v_{cm}(\text{min}), \text{ assume}$$

$$V_{SD}(M_9) = V_{SD}(\text{sat}) = V_{SG} + V_{TP} = 2.17 - 1.4 = 0.77 V$$

Now

$$V_{D10} = I_{D10}(0.5) + 0.6 + I_{D10}(0.5) - 15$$

$$= 0.118 + 0.6 - 15 \Rightarrow V_{D10} = -14.28 V$$

Then

$$\begin{aligned} v_{cm}(\text{min}) &= -14.28 + V_{SD}(\text{sat}) - V_{SG} \\ &= -14.28 + 0.77 - 2.17 = -15.68 \text{ V} \end{aligned}$$

Then, common-mode voltage range

$$\underline{-15.68 \leq v_{cm} \leq 11.6}$$

Or, assuming the input is limited to ± 15 V, then

$$\underline{-15 \leq v_{cm} \leq 11.6 \text{ V}}$$

13.55

$$I_1 = I_2$$

$$K_p(V_{SG} + V_{TP})^2 = \frac{V_{SG} - V_{BE7}}{R_1}$$

$$I_2 = 0.15 = \frac{V_{SG} - 0.6}{8} \Rightarrow V_{SG} = 1.8 \text{ V}$$

$$I_1 = 0.15 = K_p(1.8 - 1)^2 \Rightarrow K_p = 0.234 \text{ mA/V}^2$$

13.56

$$(a) \quad K_p(V_{SG} + V_{TP})^2 = \frac{V_{SG} - V_{BE7}}{R_1}$$

$$0.15(V_{SG} - 1.2)^2 = \frac{V_{SG} - 0.6}{8}$$

$$\text{We find } 1.2V_{SG}^2 - 3.88V_{SG} + 2.328 = 0, \Rightarrow V_{SG} = 2.437 \text{ V}$$

$$I_1 = I_2 = \frac{2.437 - 0.6}{8} = 0.2296 \text{ mA}$$

$$(b) \quad V_{C7} = V^+ - 0.6 - 0.6$$

$$V_{C6} = V^- + 2.437$$

$$\text{Set } V_{C6} = V_{C7}$$

$$\text{Then } V_S - 1.2 = -V_S + 2.437 \Rightarrow V_S = 1.82 \text{ V}$$

13.57

$$I_{C5} = I_{C4} = 300 \mu\text{A}$$

We have

$$R_{i2} = r_{\pi13} = \frac{\beta_n V_T}{I_{C13}} = \frac{(200)(0.026)}{0.3} = 17.3 \text{ k}\Omega$$

$$A_d = \sqrt{2K_n I_{Q5}} \cdot (R_{i2}) = \sqrt{2(0.6)(0.3)} \cdot (17.3)$$

or

$$\underline{A_d = 10.38}$$

Now

$$g_{m13} = \frac{I_{C13}}{V_T} = \frac{0.3}{0.026} = 11.5 \text{ mA/V}$$

$$r_{013} = \frac{V_A}{I_{C13}} = \frac{50}{0.3} = 167 \text{ k}\Omega$$

Then

$$|A_{v2}| = g_{m13} \cdot r_{013} = (11.5)(167)$$

or

$$\underline{|A_{v2}| = 1917}$$

Overall gain:

$$\underline{|A_v| = (10.38)(1917) = 19,895}$$

13.58

Assuming the resistances looking into Q_4 and into the output stage are very large, we have

$$|A_{v2}| = \frac{\beta R_{013}}{r_{\pi13} + (1+\beta)R_{E13}}$$

where $R_{013} = r_{013} [1 + g_{m13}(R_{E13} \| r_{\pi13})]$

$$I_{C13} = 300 \mu\text{A}, r_{013} = \frac{50}{0.3} = 167 \text{ k}\Omega$$

$$g_{m13} = \frac{0.3}{0.026} = 11.5 \text{ mA/V}$$

$$r_{\pi13} = \frac{(200)(0.026)}{0.3} = 17.3 \text{ k}\Omega$$

So

$$R_{013} = (167)[1 + (11.5)(1 \| 17.3)] \Rightarrow 1.98 \text{ M}\Omega$$

Then

$$|A_{v2}| = \frac{(200)(1.98)}{17.3 + (201)(1)} = 1814$$

Now

$$C_i = C_1(1 + |A_{v2}|) = 12[1 + 1814]$$

$$\Rightarrow C_i = 21,780 \text{ pF}$$

$$f_{PD} = \frac{1}{2\pi R_{eq} C_i}$$

$$R_{eq} = R_{i2} \| r_{012} \| r_{010}$$

Neglecting R_3 ,

$$r_{010} = \frac{1}{\lambda I_{D10}} = \frac{1}{(0.02)(0.15)} = 333 \text{ k}\Omega$$

Neglecting R_s ,

$$r_{012} = \frac{50}{0.15} = 333 \text{ k}\Omega$$

$$R_{i2} = r_{\pi13} + (1+\beta)R_{E13} = 17.3 + (201)(1) = 218 \text{ k}\Omega$$

Then

$$f_{PD} = \frac{1}{2\pi [218 \| 333 \| 333] \times 10^3 \times (21,780) \times 10^{-12}}$$

or

$$\underline{f_{PD} = 77.4 \text{ Hz}}$$

Unity-Gain Bandwidth

Gain of first stage:

$$\begin{aligned} A_d &= \sqrt{2K_n I_{Qs}} \cdot (R_{i2} \| r_{o12} \| r_{o10}) \\ &= \sqrt{2(0.6)(0.3)} \cdot (218 \| 333 \| 333) \\ &= (0.6)(218 \| 333 \| 333) \end{aligned}$$

or $A_d = 56.6$

Overall gain:

$$A_v = (56.6)(1814) = 102,672$$

Then unity-gain bandwidth = $(77.4)(102,672)$
 $\Rightarrow \underline{7.95 \text{ MHz}}$

13.59

Since $V_{GS} = 0$ in J_6 , $I_{REF} = I_{DSS}$
 $\Rightarrow \underline{I_{DSS} = 0.8 \text{ mA}}$

13.60

a. $R_{i2} = r_{\pi 5} + (1 + \beta)[r_{\pi 6} + (1 + \beta)R_E]$

$$r_{\pi 6} = \frac{(100)(0.026)}{0.2} = 13 \text{ k}\Omega$$

$$I_{C5} \cong \frac{I_{C6}}{\beta} = \frac{200 \mu\text{A}}{100} = 2 \mu\text{A}$$

So

$$r_{\pi 5} = \frac{(100)(0.026)}{0.002} = 1300 \text{ k}\Omega$$

Then

$$R_{i2} = 1300 + (101)[13 + (101)(0.3)]$$

or

$$R_{i2} = 5.67 \text{ M}\Omega$$

b. $A_v = g_{m2}(r_{02} \| r_{04} \| R_{i2})$

$$\begin{aligned} g_{m2} &= \frac{2}{V_p} \cdot \sqrt{I_D \cdot I_{DSS}} = \frac{2}{3} \cdot \sqrt{(0.1)(0.2)} \\ &= 0.0943 \text{ mA/V} \end{aligned}$$

$$r_{02} = \frac{1}{\lambda I_D} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$r_{04} = \frac{V_A}{I_{C4}} = \frac{5.0}{0.1} = 500 \text{ k}\Omega$$

Then

$$A_v = (0.0943)[500 \| 500 \| 5670]$$

or

$$\underline{A_v = 22.6}$$

13.61

a. Need $V_{SD}(Q_E) \geq V_{SD}(\text{sat}) = V_P$ For minimum bias ± 3 V

Set $\underline{V_p = 3 \text{ V}}$ and $\underline{V_{ZK} = 3 \text{ V}}$

$$I_{REF2} = \frac{V_{ZK} - V_{D1}}{R_3}$$

$$\text{so that } R_3 = \frac{3 - 0.6}{0.1} \Rightarrow R_3 = 24 \text{ k}\Omega$$

Set bias in $Q_E = I_{REF2} + I_{Z2} = 0.1 + 0.1 = 0.2 \text{ mA}$

Therefore,

$$\underline{I_{DSS} = 0.2 \text{ mA}}$$

b. Neglecting base currents

$$I_{01} = I_{REF1} = 0.5 \text{ mA} = \frac{12 - 0.6}{R_4}$$

so that

$$\underline{R_4 = 22.8 \text{ k}\Omega}$$

13.62

a. We have

$$g_{m2} = \frac{2}{|V_p|} \cdot \sqrt{I_D \cdot I_{DSS}} = \frac{2}{4} \cdot \sqrt{(0.5)(1)} = 0.354 \text{ mA/V}$$

$$r_{02} = \frac{1}{\lambda I_D} = \frac{1}{(0.02)(0.5)} = 100 \text{ k}\Omega$$

$$r_{04} = \frac{V_A}{I_D} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

$$g_{m4} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$r_{\pi 4} = \frac{(200)(0.026)}{0.5} = 10.4 \text{ k}\Omega$$

So

$$\begin{aligned} R_{04} &= r_{04} [1 + g_{m4} (r_{\pi 4} \| R_2)] \\ &= 200 [1 + (19.23)(10.4 \| 0.5)] \\ &= 2035 \text{ k}\Omega \end{aligned}$$

$$|A_d| = g_{m2} (r_{02} \| R_{04} \| R_L)$$

$$R_L \rightarrow \infty$$

$$\text{For } |A_d| = 0.354 (100 \| 2035) = 33.7$$

With these parameter values, gain can never reach 500.

b. Similarly for this part, gain can never reach 700.

Chapter 14

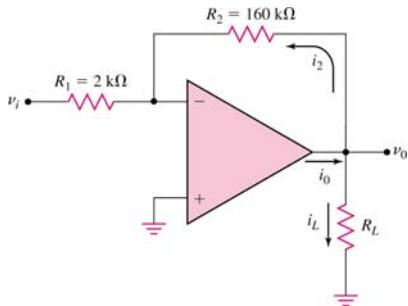
14.1

$$A_d = \frac{v_o}{v_i} = -80$$

$$v_o(\max) = 4.5 \Rightarrow v_i(\max) = 56.25 \text{ mV}$$

$$\text{So } v_i(\max)_{rms} = \frac{56.25}{\sqrt{2}} = \underline{\underline{39.77 \text{ mV}}}$$

14.2



(a)

$$i_2 = \frac{4.5}{160} = 0.028125 \text{ mA}$$

$$i_L = \frac{4.5}{1} = 4.5 \text{ mA}$$

Output Circuit = 4.528 mA

$$v_i = -\frac{v_o}{A} = \frac{-4.5}{80} \Rightarrow v_i = -0.05625 \text{ V}$$

(b)

$$i_o \approx 15 \text{ mA} = \frac{v_o}{R_L} = \frac{4.5}{R_L}$$

$$\Rightarrow \underline{R_L(\min) = 300 \Omega}$$

14.3

$$(1) \quad v_o = 2 \text{ V}$$

$$(2) \quad v_2 = 12.5 \text{ mV}$$

$$(3) \quad A_{OL} = 2 \times 10^4$$

$$(4) \quad v_1 = 8 \mu\text{V}$$

$$(5) \quad A_{OL} = 1000$$

14.4

$$(a) A_{CL}(\infty) = -\frac{R_2}{R_1} = -\frac{120}{5.6} = -21.42857$$

$$A_{CL} = \frac{-21.42857}{1 + \frac{22.42857}{10^5}} = -21.42376$$

$$\frac{-21.42376 - (-21.42857)}{-21.42857} \times 100\% = -0.0224\%$$

$$(b) A_{CL}(\infty) = -\frac{R_2}{R_1} = -\frac{120}{8.2} = -14.634146$$

$$A_{CL} = \frac{-14.634146}{1 + \frac{15.634146}{10^5}} = -14.63186$$

$$\frac{-14.63186 - (-14.634146)}{-14.634146} \times 100\% = -0.0156\%$$

14.5

$$(a) (i) A_{CL} = \frac{1 + \frac{47}{6.8}}{1 + \frac{\left(1 + \frac{47}{6.8}\right)}{2 \times 10^4}} = \frac{7.91176}{1 + \frac{7.91176}{2 \times 10^4}} = 7.90863$$

$$(ii) \frac{7.90863 - 7.91176}{7.91176} \times 100\% = -0.03956\%$$

$$(b) (i) A_{CL} = \frac{7.91176}{1 + \frac{7.91176}{10^3}} = 7.84966$$

$$(ii) \frac{7.84966 - 7.91176}{7.91176} \times 100\% = -0.785\%$$

14.6

$$(a) -15.0 = \frac{-R_2/R_1}{1 + \frac{\left(1 + R_2/R_1\right)}{2 \times 10^3}} = \frac{-R_2/R_1}{1.0005 + 5 \times 10^{-4} \left(R_2/R_1\right)} \Rightarrow \frac{R_2}{R_1} = 15.12091$$

$$(b) A_{CL} = \frac{-15.12091}{1 + \frac{15.12091}{5 \times 10^4}} = -15.1160$$

14.7

$$(1 - 0.0001)(90) = \frac{90}{1 + \frac{90}{A_{OL}}} \Rightarrow A_{OL} = 8.9991 \times 10^5$$

14.8

$$A_{CL} = (1 - 0.0002)(1) = \frac{1}{1 + \frac{1}{A_{OL}}} \Rightarrow A_{OL} = 4999$$

14.9

$$(a) |A| = + \frac{R_2}{R_1} = \frac{210(1 \pm 0.001)}{21(1 \pm 0.001)}$$

$$|A|_{\max} = \frac{210.21}{20.979} = 10.02$$

$$|A|_{\min} = \frac{209.79}{21.021} = 9.98$$

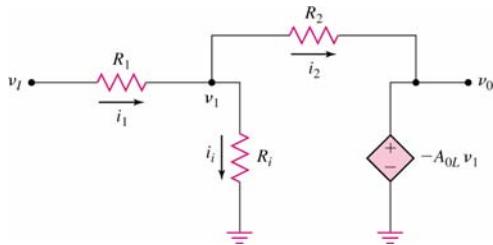
So $9.98 \leq |A| \leq 10.02$

$$(b) |A|_{\max} = \frac{10.02}{1 + \frac{11.02}{10^4}} = 10.009$$

$$|A|_{\min} = \frac{9.98}{1 + \frac{10.98}{10^4}} = 9.969$$

So $9.969 \leq |A| \leq 10.009$

14.10



$$\frac{v_I - v_1}{R_1} = \frac{v_1 - v_0}{R_2} + \frac{v_1}{R_i} \text{ and } v_0 = -A_{OL} v_1$$

$$\text{so that } v_1 = -\frac{v_0}{A_{OL}}$$

$$\frac{v_I}{R_1} + \frac{v_0}{R_2} = v_1 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_i} \right)$$

So

$$\frac{v_I}{R_1} = -v_0 \left[\frac{1}{R_2} + \frac{1}{A_{OL}} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_i} \right) \right]$$

Then

$$\frac{v_0}{v_I} = \frac{-(1/R_1)}{\left[\frac{1}{R_2} + \frac{1}{A_{OL}} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_i} \right) \right]} = A_{CL}$$

From Equation (14.20) for $R_L = \infty$ and $R_0 = 0$

$$\frac{1}{R_{if}} = \frac{1}{R_i} + \frac{1}{R_2} \cdot \frac{(1 + A_{0L})}{1}$$

a. For $R_i = 1 \text{ k}\Omega$

$$A_{CL} = \frac{-(1/20)}{\left[\frac{1}{100} + \frac{1}{10^3} \left(\frac{1}{20} + \frac{1}{100} + \frac{1}{1} \right) \right]}$$

$$= \frac{-0.05}{[0.01 + 1.06 \times 10^{-3}]}$$

or

$$\Rightarrow A_{CL} = -4.52$$

$$\frac{1}{R_{if}} = \frac{1}{1} + \frac{1+10^3}{100} \Rightarrow R_{if} = 90.8 \Omega$$

b. For $R_i = 10 \text{ k}\Omega$

$$A_{CL} = \frac{-(1/20)}{\left[\frac{1}{100} + \frac{1}{10^3} \left(\frac{1}{20} + \frac{1}{100} + \frac{1}{10} \right) \right]}$$

$$= \frac{-0.05}{[0.01 + 1.6 \times 10^{-4}]}$$

or

$$\Rightarrow A_{CL} = -4.92$$

$$\frac{1}{R_{if}} = \frac{1}{10} + \frac{1+10^3}{100} \Rightarrow R_{if} = 98.9 \Omega$$

c. For $R_i = 100 \text{ k}\Omega$

$$A_{CL} = \frac{-(1/20)}{\left[\frac{1}{100} + \frac{1}{10^3} \left(\frac{1}{20} + \frac{1}{100} + \frac{1}{100} \right) \right]}$$

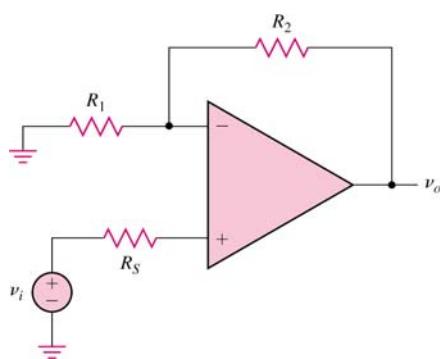
$$= \frac{-0.05}{[0.01 + 7 \times 10^{-5}]}$$

or

$$\Rightarrow A_{CL} = -4.965$$

$$\frac{1}{R_{if}} = \frac{1}{100} + \frac{1+10^3}{100} \Rightarrow R_{if} = 99.8 \Omega$$

14.11



$$A_{CL} = \frac{v_o}{v_i} = \frac{\left(1 + \frac{R_2}{R_1}\right)}{\left[1 + \frac{1}{A_{OL}}\left(1 + \frac{R_2}{R_1}\right)\right]}$$

For the ideal:

$$\left(1 + \frac{R_2}{R_1}\right) = \frac{0.10}{0.002} = 50$$

$$v_o(\text{actual}) = (0.10)(1 - 0.001) = 0.0999$$

So

$$\frac{0.0999}{0.002} = \frac{50}{1 + \frac{1}{A_{OL}}(50)} = 49.95$$

which yields

$$\underline{A_{OL} = 1000}$$

14.12

From Equation (14.18)

$$A_{vf1} = \frac{v_{o1}}{v_i} = \frac{-\left(\frac{A_{OL}}{R_o} - \frac{1}{R_2}\right)}{\left(\frac{1}{R_L} + \frac{1}{R_o} + \frac{1}{R_2}\right)}$$

Or

$$v_{o1} = \frac{-\left(\frac{5 \times 10^3}{1} - \frac{1}{100}\right)}{\left(\frac{1}{10} + \frac{1}{1} + \frac{1}{100}\right)} \cdot v_i = \frac{-(4.99999 \times 10^3)}{1.11} \cdot v_i$$

$$v_{o1} = -4.504495 \times 10^3 \cdot v_i$$

Now

$$\frac{i_1}{v_i} = \frac{v_i - v_1}{R_1 v_1} \equiv K$$

Then

$$v_i - v_1 = KR_1 v_1$$

which yields

$$v_1 = \frac{v_i}{KR_1 + 1}$$

Now, from Equation (14.20)

$$K = \frac{1}{10} + \frac{1}{100} \left[\frac{1 + 5 \times 10^3 + \frac{1}{10}}{1 + \frac{1}{10} + \frac{1}{100}} \right]$$

$$= (0.1) + (0.01) \left[\frac{5.0011 \times 10^3}{1.11} \right] = 45.15495$$

Then

$$v_1 = \frac{v_i}{(45.15495)(10) + 1} = \frac{v_i}{452.5495}$$

We find

$$v_{o1} = -4.504495 \times 10^3 \left[\frac{v_i}{452.5495} \right]$$

Or

$$A_{vf1} = \frac{v_{o1}}{v_i} = -9.9536$$

For the second stage, $R_L = \infty$

$$v_{o2} = \frac{-\left(\frac{5 \times 10^3}{1} - \frac{1}{100}\right)}{\left(\frac{1}{1} + \frac{1}{100}\right)} \cdot v'_1 = -4.950485 \times 10^3 \cdot v'_1$$

$$K \equiv \frac{1}{10} + \frac{1}{100} \left[\frac{1+5 \times 10^3}{1+\frac{1}{100}} \right] = 49.61485$$

$$v'_1 = \frac{v_{o1}}{KR_1 + 1} = \frac{v_{o1}}{(49.61485)(10) + 1} = \frac{v_{o1}}{497.1485}$$

Then

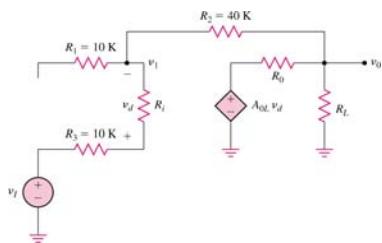
$$\frac{v_{o2}}{v_{o1}} = \frac{-4.950485 \times 10^3}{497.1485} = -9.95776$$

So

$$A_{vf} = \frac{v_{o2}}{v_i} = (-9.9536)(-9.95776) \Rightarrow A_{vf} = 99.12$$

14.13

a.



$$\frac{v_1 - v_L}{R_3 + R_i} + \frac{v_1}{R_1} + \frac{v_1 - v_0}{R_2} = 0 \quad (1)$$

$$v_1 \left[\frac{1}{R_3 + R_i} + \frac{1}{R_1} + \frac{1}{R_2} \right] = \frac{v_0}{R_2} + \frac{v_L}{R_3 + R_i}$$

$$\frac{v_0}{R_L} + \frac{v_0 - A_{0L}v_d}{R_0} + \frac{v_0 - v_1}{R_2} = 0 \quad (2)$$

or

$$v_0 \left[\frac{1}{R_L} + \frac{1}{R_0} + \frac{1}{R_2} \right] = \frac{v_1}{R_2} + \frac{A_{0L}v_d}{R_0}$$

$$v_d = \left(\frac{v_L - v_1}{R_3 + R_i} \right) \cdot R_i \quad (3)$$

So substituting numbers:

$$v_i \left[\frac{1}{10+20} + \frac{1}{10} + \frac{1}{40} \right] = \frac{v_0}{40} + \frac{v_I}{10+20} \quad (1)$$

or

$$\begin{aligned} v_i [0.15833] &= v_0 [0.025] + v_I [0.03333] \\ v_0 \left[\frac{1}{1} + \frac{1}{0.5} + \frac{1}{40} \right] &= \frac{v_1}{40} + \frac{(10^4)v_d}{0.5} \end{aligned} \quad (2)$$

or

$$\begin{aligned} v_0 [3.025] &= v_i [0.025] + (2 \times 10^4) v_d \\ v_d &= \left(\frac{v_I - v_i}{10+20} \right) \cdot 20 = 0.6667(v_I - v_i) \end{aligned} \quad (3)$$

So

$$v_0 [3.025] = v_i [0.025] + (2 \times 10^4)(0.6667)(v_I - v_i) \quad (2)$$

or

$$v_0 [3.025] = 1.333 \times 10^4 v_I - 1.333 \times 10^4 v_i$$

From (1):

$$v_i = v_0 (0.1579) + v_I (0.2105)$$

Then

$$\begin{aligned} v_0 [3.025] &= 1.333 \times 10^4 v_I - 1.333 \times 10^4 [v_0 (0.1579) + v_I (0.2105)] \\ v_0 [2.1078 \times 10^3] &= v_I [1.0524 \times 10^4] \end{aligned}$$

or

$$A_{cl} = \frac{v_0}{v_I} = 4.993$$

To find R_{if} : Use Equation (14.27)

$$\begin{aligned} i_I \left(1 + \frac{0.5}{1} + \frac{0.5}{40} \right) \\ = v_i \left\{ \left(\frac{1}{10} + \frac{1}{40} \right) \left(1 + \frac{0.5}{1} + \frac{0.5}{40} \right) - \frac{0.5}{(40)^2} \right\} - \frac{(10^3)v_d}{40} \\ i_I (1.5125) = v_i \{ (0.125)(1.5125) - 0.0003125 \} - 25v_d \end{aligned}$$

or

$$i_I (1.5125) = v_i \{ 0.18875 \} - 25v_d$$

Now

$$v_d = i_I R_i = i_I (20) \text{ and } v_i = v_I - i_I (20)$$

So

$$i_I (1.5125) = [v_I - i_I (20)] \cdot [0.18875] - 25i_I (20)$$

$$i_I [505.3] = v_I (0.18875)$$

or

$$\frac{v_I}{i_I} = 2677 \text{ k}\Omega$$

$$\text{Now } R_{if} = 10 + 2677 \Rightarrow R_{if} = 2.687 \text{ M}\Omega$$

To determine R_{of} : Using Equation (14.36)

$$\frac{1}{R'_{of}} = \frac{1}{R_0} \cdot \left[\frac{A_{0L}}{1 + \frac{R_2}{R_i \| R_i}} \right] = \frac{1}{0.5} \cdot \left[\frac{10^3}{1 + \frac{40}{10 \| 20}} \right]$$

or $R'_{0f} = 3.5 \Omega$

Then $R_{0f} = 1 \text{ k}\Omega \parallel 3.5 \Omega$

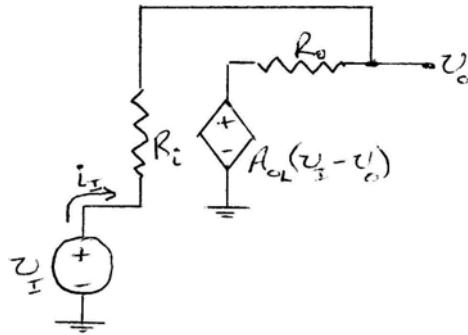
$$\Rightarrow R_{0f} = 3.49 \Omega$$

b. Using Equation (14.16)

$$\frac{dA_{CL}}{A_{CL}} = (-10) \left(\frac{5}{10^3} \right) \Rightarrow \underline{\underline{\frac{dA_{CL}}{A_{CL}} = -(0.05)\%}}$$

14.14

(a)



$$(b) (i) \frac{v_i - v_o}{R_i} = \frac{v_o - A_{OL}(v_i - v_o)}{R_o}$$

$$\frac{v_i}{R_i} + \frac{A_{OL}v_i}{R_o} = v_o \left(\frac{1}{R_i} + \frac{1}{R_o} + \frac{A_{OL}}{R_o} \right)$$

$$v_i \left(\frac{1}{10} + \frac{5 \times 10^3}{1} \right) = v_o \left(\frac{1}{10} + \frac{1}{1} + \frac{5 \times 10^3}{1} \right)$$

$$v_i (5.0001 \times 10^3) = v_o (5.0011 \times 10^3)$$

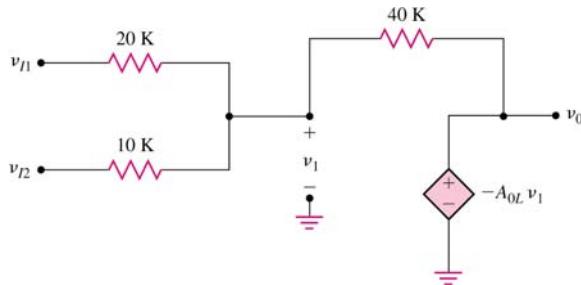
$$\frac{v_o}{v_i} = 0.9998$$

$$(ii) I_x = \frac{V_x - A_{OL}(-V_x)}{R_o} + \frac{V_x}{R_i}$$

$$\frac{I_x}{V_x} = \frac{1}{R_{of}} = \frac{1 + A_{OL}}{R_o} + \frac{1}{R_i} = \frac{1 + 5 \times 10^3}{1} + \frac{1}{10}$$

$$R_{of} \approx 0.2 \Omega$$

14.15



$$\frac{v_{I1} - v_1}{20} + \frac{v_{I2} - v_1}{10} = \frac{v_1 - v_0}{40}$$

$$\frac{v_{I1}}{20} + \frac{v_{I2}}{10} + \frac{v_0}{40} = v_1 \left[\frac{1}{20} + \frac{1}{10} + \frac{1}{40} \right]$$

$$v_1 = -\frac{v_0}{A_{0L}}$$

and $v_0 = -A_{0L} v_1$ so that

Then

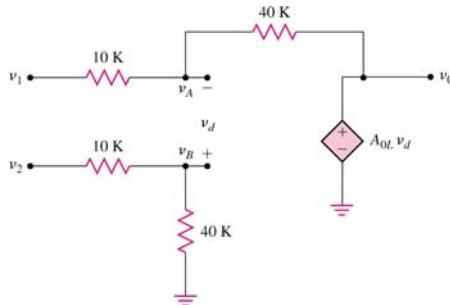
$$v_{I1}(0.05) + v_{I2}(0.10) = -v_0 \left\{ \frac{1}{40} + \frac{1}{2 \times 10^3} \cdot \left(\frac{7}{40} \right) \right\}$$

$$= -v_0 [2.50875 \times 10^{-2}]$$

$$\Rightarrow v_0 = -1.993v_{I1} - 3.986v_{I2}$$

$$\frac{\Delta v_0}{v_0} = \frac{2 - 1.993}{2} \Rightarrow \frac{\Delta v_0}{v_0} = 0.35\%$$

14.16



$$v_B = \left(\frac{40}{40+10} \right) v_2 = \left(\frac{4}{5} \right) v_2 = 0.8v_2 \quad (1)$$

$$\frac{v_1 - v_A}{10} = \frac{v_A - v_0}{40}$$

$$\frac{v_1}{10} + \frac{v_0}{40} = v_A \left(\frac{1}{10} + \frac{1}{40} \right)$$

$$v_1(0.1) + v_0(0.025) = v_A(0.125) \quad (2)$$

$$v_0 = A_{0L} v_d = A_{0L} (v_B - v_A) \quad (3)$$

or

$$\begin{aligned} v_0 &= A_{0L}[0.8v_2 - v_A] \\ \frac{v_0}{A_{0L}} - 0.8v_2 &= -v_A \\ \Rightarrow v_A &= 0.8v_2 - \frac{v_0}{A_{0L}} \end{aligned}$$

Then

$$\begin{aligned} v_1(0.1) + v_0(0.025) &= (0.125) \left[0.8v_2 - \frac{v_0}{A_{0L}} \right] \\ v_1(0.1) - v_2(0.1) &= -v_0 \left[0.025 + \frac{0.125}{10^3} \right] \\ &= -v_0 [2.5125 \times 10^{-2}] \\ \Rightarrow A_d &= \frac{v_0}{v_2 - v_1} = 3.9801 \\ \Rightarrow \frac{\Delta A_d}{A_d} &= \frac{0.0199}{4} \Rightarrow 0.4975\% \end{aligned}$$

14.17

a. Considering the second op-amp and Equation (14.20), we have

$$\frac{1}{R_{if2}} = \frac{1}{10} + \frac{1}{0.1} \cdot \left[\frac{1+100}{1 + \frac{1}{0.1}} \right] = 0.10 + \frac{101}{(0.1)(11)}$$

So $R_{if2} = 0.0109 \text{ k}\Omega$

The effective load on the first op-amp is then

$$R_{L1} = 0.1 + R_{if2} = 0.1109 \text{ k}\Omega$$

Again using Equation (14.20), we have

$$\frac{1}{R_f} = \frac{1}{10} + \frac{1}{1} \cdot \frac{1+100 + \frac{1}{0.1109}}{1 + \frac{1}{0.1109} + \frac{1}{1}} = 0.10 + \frac{110.017}{11.017}$$

so that

$$\underline{R_{if} = 99.1 \Omega}$$

b. To determine R_{0f} :

For the first op-amp, we can write, using Equation (14.36)

$$\frac{1}{R_{0f1}} = \frac{1}{R_0} \cdot \left[\frac{A_{0L}}{1 + \frac{R_2}{R_i \parallel R_i}} \right] = \frac{1}{1} \cdot \left[\frac{100}{1 + \frac{40}{1 \parallel 10}} \right]$$

which yields $\underline{R_{0f1} = 0.021 \text{ k}\Omega}$

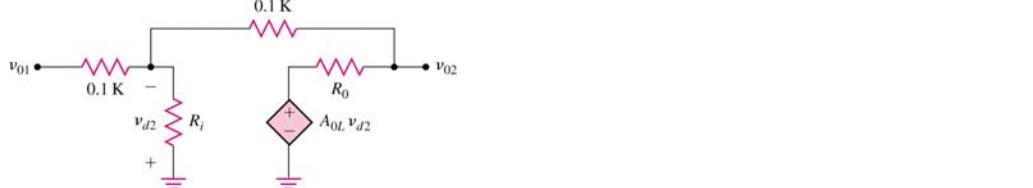
For the second op-amp, then

$$\frac{1}{R_{0f}} = \frac{1}{R_0} \cdot \left[\frac{\frac{A_{0L}}{R_2}}{1 + \frac{(R_i + R_{0f1}) \| R_i}{(R_i + R_{0f1}) \| R_i}} \right]$$

$$= \frac{1}{1} \cdot \left[\frac{\frac{100}{0.10}}{1 + \frac{(0.121) \| 10}{(0.121) \| 10}} \right]$$

$$\text{or } \frac{R_{0f} = 18.4 \Omega}{}$$

c. To find the gain, consider the second op-amp.



$$\frac{v_{01} - (-v_{d2})}{0.1} + \frac{v_{d2}}{R_i} = \frac{-v_{d2} - v_{02}}{0.1} \quad (1)$$

$$\frac{v_{01}}{0.1} + v_{d2} \left(\frac{1}{0.1} + \frac{1}{10} + \frac{1}{0.1} \right) = -\frac{v_{02}}{0.1}$$

or

$$v_{01}(10) + v_{d2}(20.1) = -v_{02}(10)$$

$$\frac{v_{02} - A_{0L} v_{d2}}{R_0} + \frac{v_{02} - (-v_{d2})}{0.1} = 0 \quad (2)$$

$$\frac{v_{02}}{1} - v_{d2} \left(\frac{100}{1} - \frac{1}{0.1} \right) + \frac{v_{02}}{0.1} = 0$$

$$v_{02}(11) - v_{d2}(90) = 0$$

or

$$v_{d2} = v_{02}(0.1222)$$

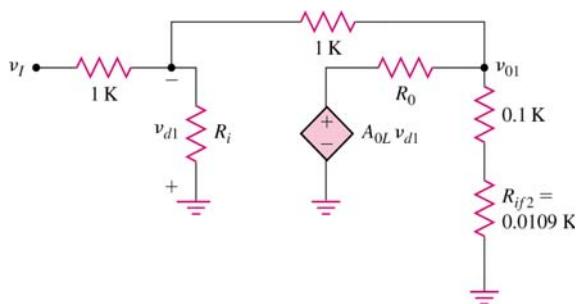
Then Equation (1) becomes

$$v_{01}(10) + v_{02}(0.1222)(20.1) = -v_{02}(10)$$

or

$$v_{01} = -v_{02}(1.246)$$

Now consider the first op-amp.



$$\frac{v_I - (-v_{d1})}{1} + \frac{v_{d1}}{R_i} = \frac{-v_{d1} - v_{01}}{1} \quad (1)$$

$$v_I(1) + v_{d1}\left(\frac{1}{1} + \frac{1}{10} + \frac{1}{1}\right) = -v_{01}(1)$$

or

$$v_I(1) + v_{d1}(2.1) = -v_{01}(1)$$

$$\frac{v_{01}}{0.1109} + \frac{v_{01} - A_{0L}v_{d1}}{R_0} + \frac{v_{01} - (-v_{d1})}{1} = 0 \quad (2)$$

$$v_{01}\left(\frac{1}{0.1109} + \frac{1}{1} + \frac{1}{1}\right) - v_{d1}\left(\frac{100}{1} - \frac{1}{1}\right) = 0$$

$$v_{01}(11.017) - v_{d1}(99) = 0$$

or

$$v_{d1} = v_{01}(0.1113)$$

Then Equation (1) becomes

$$v_I(1) + v_{01}(0.1113)(2.1) = -v_{01}$$

$$\text{or } v_I = -v_{01}(1.234)$$

$$\text{We had } v_{01} = -v_{02}(1.246)$$

$$\text{So } v_I = v_{02}(1.246)(1.234)$$

$$\text{or } \frac{v_{02}}{v_I} = 0.650$$

$$\text{d. Ideal } \frac{v_I}{v_I}$$

So ratio of actual to ideal = 0.650.

14.18

(a) For the op-amp. $A_{0L} \cdot f_{3dB} = 10^6$

$$f_{3dB} = \frac{10^6}{2 \times 10^4} = 50 \text{ Hz}$$

For the closed-loop amplifier.

$$f_{3dB} = \frac{10^6}{25} = 40 \text{ kHz}$$

(b) Open-loop amplifier.

$$A = \frac{2 \times 10^4}{1 + j \frac{f}{f_{3dB}}} \Rightarrow |A| = \sqrt{\frac{2 \times 10^4}{1 + \left(\frac{f}{f_{3dB}}\right)^2}}$$

$$f = 0.25 f_{3dB} \Rightarrow |A| = \frac{2 \times 10^4}{\sqrt{1 + (0.25)^2}} = 1.94 \times 10^4$$

$$f = 5 f_{3-dB} \Rightarrow |A| = \frac{2 \times 10^4}{\sqrt{1 + (5)^2}} = 3.92 \times 10^3$$

Closed-loop amplifier

$$f = 0.25 f_{3dB} \Rightarrow |A| = \frac{25}{\sqrt{1 + (0.25)^2}} = 24.25$$

$$f = 5 f_{3-dB} \Rightarrow |A| = \frac{25}{\sqrt{1 + (5)^2}} = 4.90$$

14.19

$$A_o = 100 \text{ dB}, \Rightarrow A_o = 10^5$$

$$A = 38 \text{ dB}, A = 79.43$$

$$\text{Then } 79.43 = \frac{10^5}{\sqrt{1 + \left(\frac{10^4}{f_{PD}}\right)^2}}$$

$$\frac{10^4}{f_{PD}} \approx \frac{10^5}{79.43} \Rightarrow f_{PD} = 7.94 \text{ Hz}$$

$$GBW = (10^5)(7.94) = 7.94 \times 10^5 \text{ Hz}$$

14.20

$$(a) A_{CLO} = \left(1 + \frac{R_2}{R_1}\right) = \left(1 + \frac{150}{15}\right) = 11$$

$$f_T = 1.2 \times 10^6 = (11)f_{3-dB} \Rightarrow f_{3-dB} = 109 \text{ kHz}$$

$$(b) A_{CLO} = \left[1 + \frac{(150)(1 \pm 0.05)}{(15)(1 \pm 0.05)}\right]$$

$$A_{CLO} (\max) = 1 + \frac{157.5}{14.25} = 12.05$$

$$A_{CLO} (\min) = 1 + \frac{142.5}{15.75} = 10.05$$

$$\text{Then } 10.05 \leq A_{CLO} \leq 12.05$$

$$f_T = 1.2 \times 10^6 = (12.05)f_{3-dB} \Rightarrow f_{3-dB} = 99.6 \text{ kHz}$$

$$f_T = 1.2 \times 10^6 = (10.05)f_{3-dB} \Rightarrow f_{3-dB} = 119.4 \text{ kHz}$$

$$\text{Then } 99.6 \leq f_{3-dB} \leq 119.4 \text{ kHz}$$

14.21

The open loop gain can be written as

$$A_{0L}(f) = \frac{A_0}{\left(1 + j \cdot \frac{f}{f_{PD}}\right)\left(1 + j \cdot \frac{f}{5 \times 10^6}\right)}$$

$$\text{where } A_0 = 2 \times 10^5.$$

The closed-loop response is

$$A_{CL} = \frac{A_{0L}}{1 + \beta A_{0L}}$$

At low frequency,

$$100 = \frac{2 \times 10^5}{1 + \beta(2 \times 10^5)}$$

$$\text{So that } \beta = 9.995 \times 10^{-3}.$$

Assuming the second pole is the same for both the open-loop and closed-loop, then

$$\phi = -\tan^{-1}\left(\frac{f}{f_{PD}}\right) - \tan^{-1}\left(\frac{f}{5 \times 10^6}\right)$$

For a phase margin of 80° , $\phi = -100^\circ$.

So

$$-100 = -90 - \tan^{-1}\left(\frac{f}{5 \times 10^6}\right)$$

or

$$f = 8.816 \times 10^5 \text{ Hz}$$

Then

$$\begin{aligned} |A_{0L}| &= 1 \\ &= \frac{2 \times 10^5}{\sqrt{1 + \left(\frac{8.816 \times 10^5}{f_{PD}}\right)^2} \sqrt{1 + \left(\frac{8.816 \times 10^5}{5 \times 10^6}\right)^2}} \end{aligned}$$

or

$$\frac{8.816 \times 10^5}{f_{PD}} \approx 1.9696 \times 10^5$$

or

$$f_{PD} = 4.48 \text{ Hz}$$

14.22

(a) 1st stage

$$(10)f_{3-dB} = 1 \text{ MHz} \Rightarrow f_{3-dB} = 100 \text{ kHz}$$

2nd stage

$$(50)f_{3-dB} = 1 \text{ MHz} \Rightarrow f_{3-dB} = 20 \text{ kHz}$$

Bandwidth of overall system $\approx 20 \text{ kHz}$

(b) If each stage has the same gain, so

$$K^2 = 500 \Rightarrow K = 22.36$$

Then bandwidth of each stage

$$(22.36)f_{3-dB} = 1 \text{ MHz} \Rightarrow f_{3-dB} = 44.7 \text{ kHz}$$

14.23

$$(a) A_{CLO} = \frac{-R_2/R_1}{1 + \frac{(1 + R_2/R_1)}{A_o}} = \frac{-10.0}{1 + \frac{11}{5 \times 10^4}} = -9.9978$$

$$f_T = 1.5 \times 10^6 = (9.9978)f_{3-dB} \Rightarrow f_{3-dB} = 150.033 \text{ kHz}$$

$$(b) A_{CLO} = (-9.9978)^3 = -999.34$$

$$\text{At } f_{3-dB}; \Rightarrow |A_{CL}| = \frac{999.34}{\sqrt{2}} = 706.64$$

Then $706.64 = \frac{999.34}{\left[\sqrt{1 + \left(\frac{f_{3-dB}}{150.033 \times 10^3} \right)^2} \right]^3}$

$$\left[1 + \left(\frac{f_{3-dB}}{150.033 \times 10^3} \right)^2 \right]^3 = \left(\frac{999.34}{706.64} \right)^2 \Rightarrow f_{3-dB} = 76.49 \text{ kHz}$$

14.24

$$(5 \times 10^4) f_{PD} = 10^6 \Rightarrow f_{PD} = 20 \text{ Hz}$$

$$(25) f_{3-dB} 10^6 \Rightarrow f_{3-dB} = 40 \text{ kHz}$$

$$A_v = \frac{A_{vo}}{1 + j \frac{f}{f_{3-dB}}} \Rightarrow |A_v| = \frac{25}{\sqrt{1 + \left(\frac{f}{40 \times 10^3} \right)^2}}$$

At $f = 0.5 f_{3-dB} = 20 \text{ kHz}$

$$|A_v| = \frac{25}{\sqrt{1 + (0.5)^2}} = 22.36$$

At $f = 2 f_{3-dB} = 80 \text{ kHz}$

$$|A_v| = \frac{25}{\sqrt{1 + (2)^2}} = 11.18$$

14.25

$$(20 \times 10^3) \cdot |A_{vf}|_{MAX} = 10^6 \Rightarrow |A_{vf}|_{MAX} = 50$$

14.26

$$(a) f_{max} = \frac{SR}{2\pi V_{PO}} = \frac{5 \times 10^6}{2\pi(5)} \Rightarrow f_{max} = 159 \text{ kHz}$$

$$(b) f_{max} = \frac{5 \times 10^6}{2\pi(1.5)} \Rightarrow f_{max} = 530.5 \text{ kHz}$$

$$(c) f_{max} = \frac{5 \times 10^6}{2\pi(0.4)} \Rightarrow f_{max} = 1.99 \text{ MHz}$$

14.27

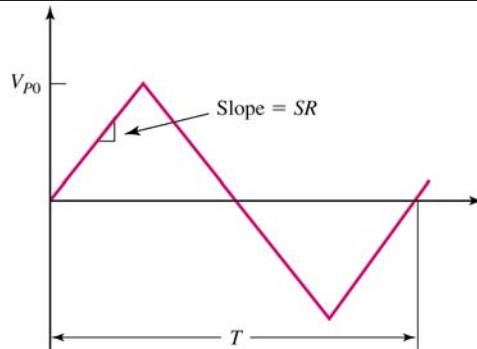
a. Using Equation (14.55),

$$V_{P0} = \frac{8 \times 10^6}{2\pi(250 \times 10^3)}$$

or

$$\underline{V_{P0} = 5.09 \text{ V}}$$

b.



$$\text{Period } T = \frac{1}{f} = \frac{1}{250 \times 10^3} = 4 \times 10^{-6} \text{ s}$$

One-fourth period = 1 μ s

$$\text{Slope} = \frac{V_{PO}}{1 \mu\text{s}} = SR = 8 \text{ V}/\mu\text{s}$$

$$\Rightarrow \underline{V_{PO} = 8 \text{ V}}$$

14.28

$$f_{\max} = \frac{SR}{2\pi V_{PO}}$$

$$SR = 2\pi(10)(12 \times 10^3) = 7.54 \times 10^5 \text{ V/s}$$

$$\text{Or } SR = 0.754 \text{ V}/\mu\text{s}$$

14.29

$$(a) f_{\max} = 20 \times 10^3 = \frac{0.63 \times 10^6}{2\pi V_{PO}} \Rightarrow V_{PO} = 5.0 \text{ V}$$

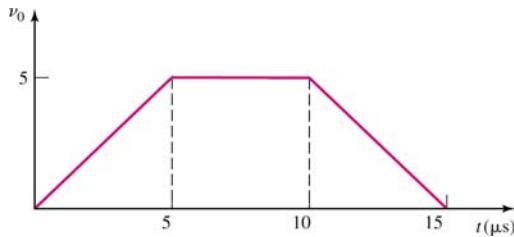
$$(b) V_{PO} = \frac{3 \times 10^6}{2\pi(20 \times 10^3)} = 23.87 \text{ V}$$

14.30

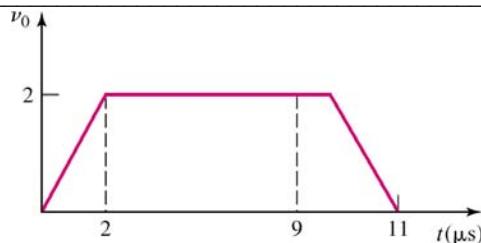
For input (a), maximum output is 5 V.

$$SR = 1 \text{ V}/\mu\text{s}$$

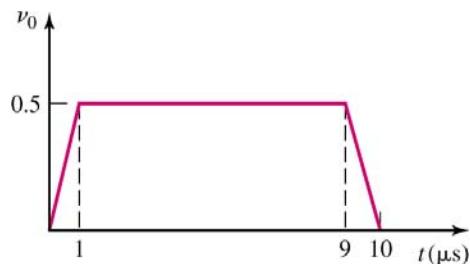
so



For input (b), maximum output is 2 V.

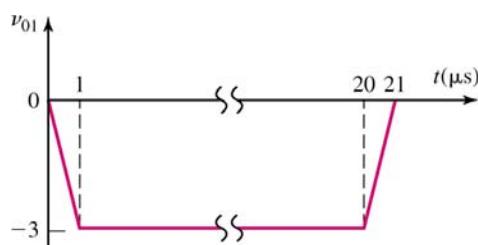


For input (c), maximum output is 0.5 V so the output is

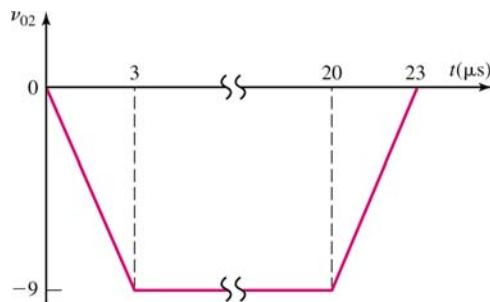


14.31

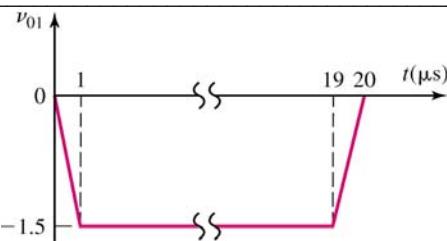
For input (a), $\max|v_{01}| = 3$ V.



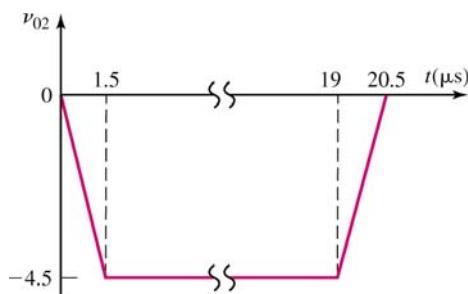
Then $|v_{02}|_{\max} = 3(3) = 9$ V



For input (b), $\max|v_{01}| = 1.5$ V.



Then $|v_{02}|_{\max} = 3(1.5) = 4.5 \text{ V}$



14.32

$$I_1 = I_{S1} \exp\left(\frac{V_{BE1}}{V_T}\right), \quad I_2 = I_{S2} \exp\left(\frac{V_{BE2}}{V_T}\right)$$

Want $I_1 = I_2$, so

$$\begin{aligned} \frac{I_1}{I_2} &= 1 = \frac{5 \times 10^{-14} (1+x) \exp\left(\frac{V_{BE1}}{V_T}\right)}{5 \times 10^{-14} (1-x) \exp\left(\frac{V_{BE2}}{V_T}\right)} \\ &= \frac{(1+x)}{(1-x)} \exp\left(\frac{V_{BE1} - V_{BE2}}{V_T}\right) \end{aligned}$$

Or

$$\begin{aligned} \frac{1+x}{1-x} &= \exp\left(\frac{V_{BE2} - V_{BE1}}{V_T}\right) = \exp\left(\frac{V_{os}}{V_T}\right) \\ &= \exp\left(\frac{0.0025}{0.026}\right) = 1.10 \end{aligned}$$

Now

$$\begin{aligned} 1+x &= (1-x)(1.10) \Rightarrow \\ x &= 0.0476 \Rightarrow 4.76\% \end{aligned}$$

14.33

- (a) Balanced circuit, $I_{S4} = 5 \times 10^{-15} \text{ A}$
- (b) From Eq. (14.62), $v_{CE1} = 5 \text{ V}$, $v_{CE2} = 5.6 - 1.2 = 4.4 \text{ V}$

$$\frac{1+\frac{5}{120}}{1+\frac{0.6}{80}} = \frac{I_{S3}}{I_{S4}} \cdot \frac{\left(1+\frac{4.4}{120}\right)}{\left(1+\frac{1.2}{80}\right)}$$

$$\frac{1.041667}{1.0075} = \frac{I_{S3}}{I_{S4}} \cdot \frac{(1.036667)}{(1.015)}$$

$$\frac{I_{S3}}{I_{S4}} = 1.0123 \Rightarrow I_{S4} = 4.939 \times 10^{-15} \text{ A}$$

(c) $v_{CE1} = 5 \text{ V}$, $v_{CE2} = 5.6 - 2.5 = 3.1 \text{ V}$

$$\frac{1+\frac{5}{120}}{1+\frac{0.6}{80}} = \frac{I_{S3}}{I_{S4}} \cdot \frac{\left(1+\frac{3.1}{120}\right)}{\left(1+\frac{2.5}{80}\right)}$$

$$\frac{1.041667}{1.0075} = \frac{I_{S3}}{I_{S4}} \cdot \frac{(1.025833)}{(1.03125)}$$

$$\frac{I_{S3}}{I_{S4}} = 1.03937 \Rightarrow I_{S4} = 4.811 \times 10^{-15} \text{ A}$$

14.34

$$K_n = 150 \mu\text{A/V}^2$$

$$\Delta K_n = 150(1+x) - 150(1-x) = 300x \mu\text{A/V}^2$$

$$|V_{OS}| = \frac{1}{2} \sqrt{\frac{I_Q}{2K_n}} \left(\frac{\Delta K_n}{K_n} \right)$$

$$15 \times 10^{-3} = \frac{1}{2} \sqrt{\frac{200}{2(150)}} \left(\frac{300x}{150} \right) = 0.8165x \Rightarrow x = 0.01837$$

14.35

$$(a) v_o = -30(10 \pm 2) \times 10^{-3} = (-300 \pm 60) \times 10^{-3} \text{ V}$$

$$\text{So } -0.360 \leq v_o \leq -0.240 \text{ V}$$

$$(b) v_o = -30(100 \pm 2) \times 10^{-3} = -3 \pm 0.06 \text{ V}$$

$$\text{So } -3.06 \leq v_o \leq -2.94 \text{ V}$$

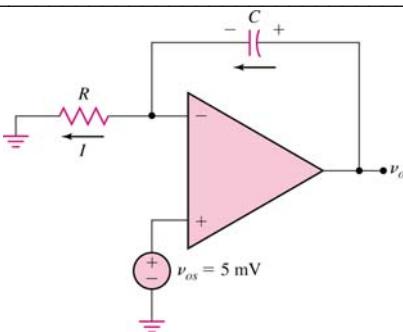
14.36

$$v_o = -30(25 \sin \omega t \pm 2) \text{ mV}$$

$$v_o = -0.75 \sin \omega t \pm 0.06 \text{ V}$$

$$\text{So } (-0.75 \sin \omega t - 0.06) \leq v_o \leq (-0.75 \sin \omega t + 0.06) \text{ V}$$

14.37



$$I = \frac{0.5 \times 10^{-3}}{10^4} = 5 \times 10^{-8} \text{ A}$$

Also

$$I = C \frac{dV_o}{dt} \Rightarrow V_o = \frac{1}{C} \int_0^t Idt = \frac{I}{C} \cdot t$$

Then

$$5 = \frac{5 \times 10^{-8}}{10 \times 10^{-6}} t \Rightarrow t = 10^3 \text{ s}$$

14.38

$$(a) \quad v_{o1} = \left(1 + \frac{100}{10}\right)(\pm 3) \text{ mV}, \quad -33 \leq v_{o1} \leq 33 \text{ mV}$$

$$v_{o2} = -\left(\frac{50}{10}\right)(\pm 33 \pm 3) \text{ mV}, \quad -180 \leq v_{o2} \leq 180 \text{ mV}$$

$$(b) \quad v_{o1} = (11)(10 \pm 3) \text{ mV}, \quad 77 \leq v_{o1} \leq 143 \text{ mV}$$

$$v_{o2} = -5(143 + 3) = -730 \text{ mV}$$

$$v_{o2} = -5(77 - 3) = -370 \text{ mV}$$

$$\text{So } -0.73 \leq v_{o2} \leq -0.37 \text{ V}$$

$$(c) \quad v_{o1} = (11)(100 \pm 3) \text{ mV}$$

$$1.067 \leq v_{o1} \leq 1.133 \text{ V}$$

$$v_{o2} = -5(1.133 + 0.003) = -5.68 \text{ V}$$

$$v_{o2} = -5(1.067 - 0.003) = -5.32 \text{ V}$$

$$\text{So } -5.68 \leq v_{o2} \leq -5.32 \text{ V}$$

14.39

v_o due to v_I

$$v_o = (0.5) \left(1 + \frac{1}{1.1}\right) = 0.9545 \text{ V}$$

Wiper arm at $V^+ = 10 \text{ V}$, (using superposition)

$$v_1 = \left(\frac{R_1 \parallel R_s}{R_1 \parallel R_s + R_4} \right) (10) = \left(\frac{0.0909}{0.0909 + 10} \right) (10) = 0.090$$

Then $v_{01} = -\left(\begin{matrix} 1 \\ 1 \end{matrix} \right) (0.090) = -0.090$

Wiper arm in center, $v_1 = 0$ and $v_{02} = 0$

Wiper arm at $V^- = -10$ V, $v_1 = -0.090$

So

$$v_{03} = 0.090$$

Finally, total output v_0 : (from superposition)

Wiper arm at V^+ ,

$$\underline{v_0 = 0.8645 \text{ V}}$$

Wiper arm in center,

$$\underline{v_0 = 0.9545 \text{ V}}$$

Wiper arm at V^- ,

$$\underline{v_0 = 1.0445 \text{ V}}$$

14.40

a. $R'_1 = R'_2 = 0.5 \parallel 25 = 0.490 \text{ k}\Omega$

or

$$\underline{R'_1 = R'_2 = 490 \Omega}$$

b. From Equation (14.75),

$$(0.026) \ln \left(\frac{125 \times 10^{-6}}{2 \times 10^{-14}} \right) + (0.125)R'_1 \\ = (0.026) \ln \left(\frac{125 \times 10^{-6}}{2.2 \times 10^{-14}} \right) + (0.125)R'_2$$

$$0.586452 + (0.125)R'_1 = 0.583974 + (0.125)R'_2$$

$$0.002478 = (0.125)(R'_2 - R'_1)$$

So $R'_2 - R'_1 = 0.0198 \text{ k}\Omega \Rightarrow 19.8 \Omega$

Then

$$\frac{R_2(1-x)R_x}{R_2 + (1-x)R_x} - \frac{R_1 \times R_x}{R_1 + xR_x} = 0.0198 \\ \frac{(0.5)(1-x)(50)}{(0.5) + (1-x)(50)} - \frac{(0.5)(50)x}{(0.5) + x(50)} = 0.0198 \\ \frac{25(1-x)}{50.5 - 50x} - \frac{25x}{0.5 + 50x} = 0.0198 \\ \frac{(0.5 + 50x)(25 - 25x) - (25x)(50.5 - 50x)}{(50.5 - 50x)(0.5 + 50x)} = 0.0198$$

$$25\{0.5 - 0.5x + 50x - 50x^2 - 50.5x + 50x^2\} = 0.0198\{25.25 + 2525x - 25x - 2500x^2\}$$

$$25\{0.5 - x\} = 0.0198\{25.25 + 2500x - 2500x^2\}$$

$$0.5 - x = 0.019998 + 1.98x - 1.98x^2$$

$$1.98x^2 - 2.98x + 0.48 = 0$$

$$x = \frac{2.98 \pm \sqrt{(2.98)^2 - 4(1.98)(0.48)}}{2(1.98)}$$

So

$$\underline{x = 0.183}$$

and

$$\underline{1-x = 0.817}$$

14.41

$$R'_1 = R_1 \parallel 15 = 0.5 \parallel 15 = 0.4839 \text{ k}\Omega$$

$$R'_2 = R_2 \parallel 35 = 0.5 \parallel 35 = 0.4930 \text{ k}\Omega$$

From Equation (14.75),

$$(0.026) \ln \left(\frac{i_{C1}}{I_{S3}} \right) + i_{C1} R'_1 = (0.026) \ln \left(\frac{i_{C2}}{I_{S4}} \right) + i_{C2} R'_2$$

$$(0.026) \ln \left(\frac{i_{C1}}{i_{C2}} \right) = i_{C2} R'_2 - i_{C1} R'_1$$

$$(0.026) \ln \left(\frac{i_{C1}}{i_{C2}} \right) = i_{C2} R'_2 \left[1 - \frac{i_{C1}}{i_{C2}} \cdot \frac{R'_1}{R'_2} \right]$$

$$(0.026) \ln \left(\frac{i_{C1}}{i_{C2}} \right) = i_{C2} (0.4930) \left[1 - (0.9815) \left(\frac{i_{C1}}{i_{C2}} \right) \right]$$

By trial and error:

$$\underline{i_{C1} = 252 \mu\text{A}} \quad \text{and} \quad \underline{i_{C2} = 248 \mu\text{A}}$$

or

$$\frac{i_{C1}}{i_{C2}} = 1.0155$$

14.42

$$(a) \quad v_O \Big|_{(1 \mu\text{A})} = (10^{-6})(200 \times 10^3) = 0.2 \text{ V}$$

Insert resistor R_3

$$v_O \Big|_{(2 \mu\text{A})} = -0.2 = -(2 \times 10^{-6}) R_3 \left(1 + \frac{200}{20} \right) \Rightarrow R_3 = 9.09 \text{ k}\Omega$$

$$(b) \quad v_O \Big|_{(0.8 \mu\text{A})} = (0.8 \times 10^{-6})(200 \times 10^3) = 0.16 \text{ V}$$

$$v_O \Big|_{(0.5 \mu\text{A})} = -0.16 = -(0.5 \times 10^{-6}) R_3 \left(1 + \frac{200}{20} \right) \Rightarrow R_3 = 29.09 \text{ k}\Omega$$

14.43

(a) $v_o = -I_B R_2 = -(2 \times 10^{-6})(150 \times 10^3) = -0.3 \text{ V}$

(b) $v_o = -\frac{150}{15}(0.02) - 0.3 = -0.5 \text{ V}$

(c) $v_o = -\frac{150}{15}(-0.02) - 0.3 = -0.1 \text{ V}$

(d) $v_o = -\frac{150}{15}(0.1) - 0.3 = -1.3 \text{ V}$

14.44

(a) $v_o = (0.6 \times 10^{-6})(250 \times 10^3) = 0.15 \text{ V}$

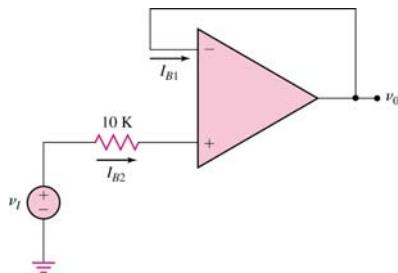
(b) $v_o = (41)(0.008) + 0.15 = 0.478 \text{ V}$

(c) $v_o = (41)(-0.0035) + 0.15 = 0.0065 \text{ V}$

(d) $v_o = (41)(0.005 \sin \omega t) + 0.15 = 0.205 \sin \omega t + 0.15 \text{ (V)}$

14.45

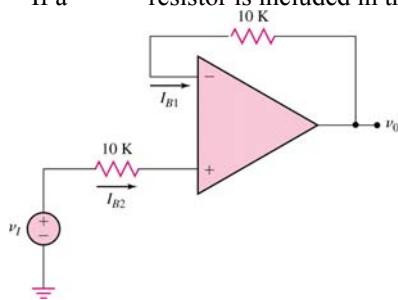
a.



For $I_{B2} = 1 \mu\text{A}$, then $v_o = -(10^{-6})(10^4)$

or $v_o = -0.010 \text{ V}$

b. If a 10 kΩ resistor is included in the feedback loop



Now $v_o = -I_{B2}(10) + I_{B1}(10) = 0$

Circuit is compensated if $I_{B1} = I_{B2}$.

14.46

From Equation (14.83), we have

$$v_0 = R_2 I_{0s}$$

where $R_2 = 40 \text{ k}\Omega$ and $I_{0s} = 3 \mu\text{A}$.

Then

$$v_0 = (40 \times 10^3)(3 \times 10^{-6})$$

or

$$\underline{v_0 = 0.12 \text{ V}}$$

14.47

- a. Assume all bias currents are in the same direction and into each op-amp.

$$v_{01} = I_{B1} (100 \text{ k}\Omega) - (10^{-6})(10^5) \Rightarrow \underline{v_{01} = 0.1 \text{ V}}$$

Then

$$\begin{aligned} v_{02} &= v_{01}(-5) + I_{B1}(50 \text{ k}\Omega) \\ &= (0.1)(-5) + (10^{-6})(5 \times 10^4) \\ &= -0.5 + 0.05 \end{aligned}$$

or

$$\underline{v_{02} = -0.45 \text{ V}}$$

- b. Connect $R_3 = 10 \parallel 100 = 9.09 \text{ k}\Omega$ resistor to noninverting terminal of first op-amp, and $R_3 = 10 \parallel 50 = 8.33 \text{ k}\Omega$ resistor to noninverting terminal of second op-amp.

14.48

- a. For a constant current through a capacitor.

$$v_0 = \frac{1}{C} \int_0^t I dt$$

$$\text{or } v_0 = \frac{0.1 \times 10^{-6}}{10^{-6}} \cdot t \Rightarrow \underline{v_0 = (0.1)t}$$

$$\text{b. At } t = 10 \text{ s, } \underline{v_0 = 1 \text{ V}}$$

c. Then

$$v_0 = \frac{100 \times 10^{-12}}{10^{-6}} \cdot t \Rightarrow v_0 = (10^{-4})t$$

$$\text{At } t = 10 \text{ s, } \underline{v_0 = 1 \text{ mV}}$$

14.49

$$(a) v_{o1} = (3 \times 10^{-6})(50 \times 10^3) = 0.15 \text{ V}$$

$$v_{o2} = 0.15 \text{ V}$$

$$v_{o3} = -\frac{20}{20}(0.15) + (3 \times 10^{-6})(20 \times 10^3) = -0.09 \text{ V}$$

$$(b) R_A = 10 \parallel 50 = 8.33 \text{ k}\Omega$$

$$R_B = 20 \parallel 20 = 10 \text{ k}\Omega$$

$$(c) v_{o1} = \pm(50 \times 10^3)(0.3 \times 10^{-6}) = \pm 0.015 \text{ V}$$

$$v_{o2} = \pm 0.015 \text{ V}$$

$$v_{o3} = \pm(20 \times 10^3)(0.3 \times 10^{-6}) \pm 0.015 = \pm 0.021 \text{ V}$$

14.50

a. Using Equation (14.79),

Circuit (a),

$$v_0 = (0.8 \times 10^{-6})(50 \times 10^3) - (0.8 \times 10^{-6})(25 \times 10^3) \left(1 + \frac{50}{50}\right)$$

or

$$\underline{v_0 = 0}$$

Circuit (b),

$$\begin{aligned} v_0 &= (0.8 \times 10^{-6})(50 \times 10^3) - (0.8 \times 10^{-6})(10^3) \left(1 + \frac{50}{50}\right) \\ &= 4 \times 10^{-2} - 1.6 \end{aligned}$$

or

$$\underline{v_0 = -1.56 \text{ V}}$$

b. Assume $I_{B1} = 0.7 \mu\text{A}$ and $I_{B2} = 0.9 \mu\text{A}$, then using Equation (14.79):

Circuit (a),

$$\begin{aligned} v_0 &= (0.7 \times 10^{-6})(50 \times 10^3) - (0.9 \times 10^{-6})(25 \times 10^3) \left(1 + \frac{50}{50}\right) \\ &= 0.035 - 0.045 \end{aligned}$$

or $\underline{v_0 = -0.010 \text{ V}}$

Circuit (b),

$$\begin{aligned} v_0 &= (0.7 \times 10^{-6})(50 \times 10^3) - (0.9 \times 10^{-6})(10^6) \left(1 + \frac{50}{50}\right) \\ &= 0.035 - 1.8 \end{aligned}$$

or $\underline{v_0 = -1.765 \text{ V}}$

14.51

(a) For V_{OS} : $v_o = \left(1 + \frac{100}{10}\right)(\pm 3) = \pm 33 \text{ mV}$

For I_B : $v_o(\text{max}) = (0.43 \times 10^{-6})(100 \times 10^3) = 0.043 \text{ V}$

$v_o(\text{max}) = (0.37 \times 10^{-6})(100 \times 10^3) = 0.037 \text{ V}$

So $4 \leq v_o \leq 76 \text{ mV}$

(b) For V_{OS} : $v_o = \pm 33 \text{ mV}$

For I_{OS} : $v_o = \pm (0.06 \times 10^{-6})(100 \times 10^3) = \pm 0.006 \text{ V}$

So $-39 \leq v_o \leq 39 \text{ mV}$

(c) $v_o = \left(1 + \frac{100}{10}\right)(0.2) \pm 0.039$

So $2.161 \leq v_o \leq 2.239 \text{ V}$

14.52

a. $\left(\frac{R_i}{R_i + R_2} \right) (15) = 0.010 \text{ V}$

$$\frac{15}{15 + R_2} = 0.0006667$$

$$15(1 - 0.0006667) = 0.0006667 R_2$$

Then

$$R_2 = 22.48 \text{ M}\Omega$$

b. $R_i = R_i \parallel R_F = 15 \parallel 10 \Rightarrow R_i = 6 \text{ k}\Omega$

14.53

- a. Assume the offset voltage polarities are such as to produce the worst case values, but the bias currents are in the same direction.

Use superposition:

Offset voltages

$$|v_{01}| = \left(1 + \frac{100}{10}\right)(10) = 110 \text{ mV} = |v_{01}|$$

$$|v_{02}| = (5)(110) + \left(1 + \frac{50}{10}\right)(10)$$

$$\Rightarrow |v_{02}| = 610 \text{ mV}$$

Bias Currents:

$$v_{01} = I_B(100 \text{ k}\Omega) = (2 \times 10^{-6})(100 \times 10^3) = 0.2 \text{ V}$$

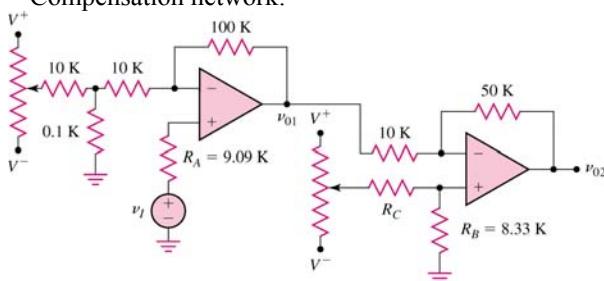
Then

$$v_{02} = (-5)(0.2) + (2 \times 10^{-6})(50 \times 10^3) = -0.9 \text{ V}$$

Worst case: v_{01} is positive and v_{02} is negative, then

$$v_{01} = 0.31 \text{ V} \text{ and } v_{02} = -1.51 \text{ V}$$

- b. Compensation network:



If we want

$$\left(\frac{R_B}{R_B + R_C} \right) V^+ = 20 \text{ mV} \text{ and } V^+ = 10 \text{ V}$$

$$\left(\frac{8.33}{8.33 + R_C} \right) (10) = 0.020$$

or $R_C \approx 4.15 \text{ M}\Omega$

14.54

(a) Offset voltage:

$$v_{o1} = \left(1 + \frac{50}{10}\right)(\pm 2) = \pm 12 \text{ mV}$$

$$v_{o2} = \pm 12 \pm 2 = \pm 14 \text{ mV}$$

$$v_{o3} = \left(-\frac{20}{20}\right)(\pm 12) + (2)(\pm 2) = \pm 16 \text{ mV}$$

Bias current:

$$v_{o1} = (0.21 \times 10^{-6})(50 \times 10^3) = 0.0105 \text{ V}$$

$$\text{or } v_{o1} = (0.19 \times 10^{-6})(50 \times 10^3) = 0.0095 \text{ V}$$

$$v_{o2} = v_{o1}$$

$$v_{o3} = (-1)(v_{o1}) + (0.21 \times 10^{-6})(20 \times 10^3) = -v_{o1} + 0.0042$$

$$\text{or } v_{o3} = -v_{o1} + (0.19 \times 10^{-6})(20 \times 10^3) = -v_{o1} + 0.0038$$

By superposition

$$-2.5 \leq v_{o1} \leq 22.5 \text{ mV}$$

$$-4.5 \leq v_{o2} \leq 24.5 \text{ mV}$$

$$-22.3 \leq v_{o3} \leq 10.7 \text{ mV}$$

(b) Bias currents:

$$v_{o1} = \pm I_{os}(50 \times 10^3) = \pm (0.02 \times 10^{-6})(50 \times 10^3) \Rightarrow v_{o1} = \pm 1 \text{ mV}$$

$$v_{o3} = \pm I_{os}(20 \times 10^3) = \pm (0.02 \times 10^{-6})(20 \times 10^3) \Rightarrow v_{o3} = \pm 0.4 \text{ mV}$$

$$\text{By superposition: } v_{o3} = \pm v_{o1} \pm 2(2) \pm 0.4$$

$$-13 \leq v_{o1} \leq 13 \text{ mV}$$

$$-15 \leq v_{o2} \leq 15 \text{ mV}$$

$$-17.4 \leq v_{o3} \leq 17.4 \text{ mV}$$

14.55

For circuit (a), effect of bias current:

$$v_0 = (50 \times 10^3)(100 \times 10^{-9}) \Rightarrow 5 \text{ mV}$$

Effect of offset voltage

$$v_0 = (2)\left(1 + \frac{50}{50}\right) = 4 \text{ mV}$$

So net output voltage is $\underline{v_0 = 9 \text{ mV}}$

For circuit (b), effect of bias current:

Let $I_{B2} = 550 \text{ nA}$, $I_{B1} = 450 \text{ nA}$, then from Equation (14.79),

$$\begin{aligned} v_0 &= (450 \times 10^{-9})(50 \times 10^3) - (550 \times 10^{-9})(10^6)\left(1 + \frac{50}{50}\right) \\ &= 2.25 \times 10^{-2} - 1.1 \end{aligned}$$

or

$$\underline{v_0 = -1.0775 \text{ V}}$$

If the offset voltage is negative, then

$$v_0 = (-2)(2) = -4 \text{ mV}$$

So the net output voltage is

$$\underline{v_0 = -1.0815 \text{ V}}$$

14.56

a. At $T = 25^\circ\text{C}$, $V_{0S} = 2 \text{ mV}$ so the output voltage for each circuit is

$$\underline{v_0 = 4 \text{ mV}}$$

b. For $T = 50^\circ\text{C}$, the offset voltage for is

$$V_{0S} = 2 \text{ mV} + (0.0067)(25) = 2.1675 \text{ mV}$$

so the output voltage for each circuit is

$$\underline{v_0 = 4.335 \text{ mV}}$$

14.57

a. At $T = 25^\circ\text{C}$, $V_{0S} = 1 \text{ mV}$, then

$$v_{01} = (1)\left(1 + \frac{50}{10}\right) \Rightarrow \underline{v_{01} = 6 \text{ mV}}$$

and

$$\begin{aligned} v_{02} &= v_{01}\left(1 + \frac{60}{20}\right) + (1)\left(1 + \frac{60}{20}\right) \\ &= 6(4) + (1)(4) \Rightarrow \underline{v_{02} = 28 \text{ mV}} \end{aligned}$$

b. At $T = 50^\circ\text{C}$, $V_{0S} = 1 + (0.0033)(25) = 1.0825 \text{ mV}$, then

$$v_{01} = (1.0825)(6) \Rightarrow \underline{v_{01} = 6.495 \text{ mV}}$$

and

$$v_{02} = (6.495)(4) + (1.0825)(4)$$

or

$$\underline{v_{02} = 30.31 \text{ mV}}$$

14.58

$$25^\circ\text{C}; I_B = 500 \text{ nA}, I_{0S} = 200 \text{ nA}$$

$$50^\circ\text{C}, I_B = 500 \text{ nA} + (8 \text{ nA}/^\circ\text{C})(25^\circ\text{C}) = 700 \text{ nA}$$

$$I_{0S} = 200 \text{ nA} + (2 \text{ nA}/^\circ\text{C})(25^\circ\text{C}) = 250 \text{ nA}$$

a. Circuit (a): For I_B , bias current cancellation, $v_0 = 0$

Circuit (b): For I_B , Equation (14.79),

$$\begin{aligned} v_0 &= (500 \times 10^{-9})(50 \times 10^3) - (500 \times 10^{-9})(10^6)\left(1 + \frac{50}{50}\right) \\ &= 0.025 - 1.00 \Rightarrow \underline{v_0 = -0.975 \text{ V}} \end{aligned}$$

b. Due to offset bias currents.

Circuit (a):

$$v_0 = (200 \times 10^{-9})(50 \times 10^3) \Rightarrow \underline{v_0 = 0.010 \text{ V}}$$

Circuit (b):

$$\text{Let } I_{B2} = 600 \text{ nA}$$

$$I_{B1} = 400 \text{ nA}$$

Then

$$\begin{aligned} v_0 &= (400 \times 10^{-9})(50 \times 10^3) - (600 \times 10^{-9})(10^6)\left(1 + \frac{50}{50}\right) \\ &= 0.020 - 1.20 \Rightarrow \underline{v_0 = -1.18 \text{ V}} \end{aligned}$$

c. Circuit (a): Due to I_B , $v = 0$

Circuit (b): Due to I_B ,

$$v_0 = (700 \times 10^{-9})(50 \times 10^3) - (700 \times 10^{-9})(10^6) \left(1 + \frac{50}{50}\right)$$

$$= 0.035 - 1.40 \Rightarrow v_0 = -1.365 \text{ V}$$

Circuit (a): Due to I_{0S} ,

$$v_0 = (250 \times 10^{-9})(50 \times 10^3) \Rightarrow v_0 = 0.0125 \text{ V}$$

Circuit (b): Due to I_{0S} ,

Let $I_{B2} = 825 \text{ nA}$

$$I_{B1} = 575 \text{ nA}$$

Then

$$v_0 = (575 \times 10^{-9})(50 \times 10^3) - (825 \times 10^{-9})(10^6) \left(1 + \frac{50}{50}\right)$$

$$= 0.02875 - 1.65 \Rightarrow v_0 = -1.62 \text{ V}$$

14.59

$$25^\circ\text{C}; I_B = 2 \mu\text{A}, \quad I_{0S} = 0.2 \mu\text{A}$$

$$50^\circ\text{C}, I_B = 2 \mu\text{A} + (0.020 \mu\text{A}/^\circ\text{C})(25^\circ\text{C}) = 2.5 \mu\text{A}$$

$$I_{0S} = 0.2 \mu\text{A} + (0.005 \mu\text{A}/^\circ\text{C})(25^\circ\text{C}) = 0.325 \mu\text{A}$$

a. Due to I_B : (Assume bias currents into op-amp).

$$v_{01} = I_B(50 \text{ k}\Omega) = (2 \times 10^{-6})(50 \times 10^3)$$

$$\Rightarrow v_{01} = 0.10 \text{ V}$$

$$v_{02} = v_{01} \left(1 + \frac{60}{20}\right) + I_B(60 \text{ k}\Omega) - I_B(50 \text{ k}\Omega) \left(1 + \frac{60}{20}\right)$$

$$= (0.1)(4) + (2 \times 10^{-6})(60 \times 10^3) - (2 \times 10^{-6})(60 \times 10^3)(4)$$

or $v_{02} = 0.12 \text{ V}$

b. Due to I_{0S} :

1st op-amp. Let $I_{B1} = 2.1 \mu\text{A}$

2nd op-amp. Let $I_{B1} = 2.1 \mu\text{A}$

$$I_{B2} = 1.9 \mu\text{A}$$

$$v_{01} = I_{B1}(50 \text{ k}\Omega) = (2.1 \times 10^{-6})(50 \times 10^3)$$

$$\Rightarrow v_{01} = 0.105 \text{ V}$$

$$v_{02} = v_{01} \left(1 + \frac{60}{20}\right) + I_{B1}(60 \text{ k}\Omega) - I_{B2}(50 \text{ k}\Omega) \left(1 + \frac{60}{20}\right)$$

$$= (0.105)(4) + (2.1 \times 10^{-6})(60 \times 10^3) - (1.9 \times 10^{-6})(50 \times 10^3)(4)$$

or

$$v_{02} = 0.166 \text{ V}$$

c. Due to I_B :

$$v_{01} = (2.5 \times 10^{-6})(50 \times 10^3) \Rightarrow v_{01} = 0.125 \text{ V}$$

$$v_{01} = v_{02} \left(1 + \frac{60}{20}\right) + I_B(60 \text{ k}\Omega) - I_B(50 \text{ k}\Omega) \left(1 + \frac{60}{20}\right)$$

$$= (0.125)(4) + (2.5 \times 10^{-6})(60 \times 10^3) - (2.5 \times 10^{-6})(50 \times 10^3)(4)$$

or $v_{02} = 0.15 \text{ V}$

Due to I_{0S} :

Let $I_{B1} = 2.625 \mu\text{A}$

$$I_{B2} = 2.3375 \mu\text{A}$$

$$v_{01} = I_{B1}(50 \text{ k}\Omega) = (2.6625 \times 10^{-6})(50 \times 10^3)$$

$$\Rightarrow v_{01} = 1.133 \text{ V}$$

$$v_{02} = v_{01} \left(1 + \frac{60}{20}\right) + I_{B1}(60 \text{ k}\Omega) - I_{B2}(50 \text{ k}\Omega) \left(1 + \frac{60}{20}\right)$$

$$= (0.133)(4) + (2.6625 \times 10^{-6})(60 \times 10^3) - (2.3375 \times 10^{-6})(50 \times 10^3)(4)$$

or

$$\underline{v_{02} = 0.224 \text{ V}}$$

14.60

$$(a) A_d = \frac{50}{10} = 5.0$$

For common-mode, $v_{I1} = v_{I2}$

From Chapter 9,

$$A_{cm} = \frac{\left(1 + \frac{R_2}{R_1}\right)}{\left(1 + \frac{R_3}{R_4}\right)} - \frac{R_2}{R_1}$$

$$\text{If } R_2 = 50(1.015) = 50.75, R_1 = 10(1 - 0.015) = 9.85$$

$$R_3 = 10(1 - 0.015) = 9.85, R_4 = 50(1.015) = 50.75$$

$$\text{Then } A_{cm} = \frac{1 + \frac{50.75}{9.85}}{1 + \frac{9.85}{50.75}} - \frac{50.75}{9.85} = \frac{6.15228}{1.19409} - 5.15228 = 5.046 \times 10^{-6}$$

$$\text{If } R_3 = 10(1.015) = 10.15, R_4 = 50(1 - 0.015) = 49.25$$

$$\text{Then } A_{cm} = \frac{1 + \frac{50.75}{9.85}}{1 + \frac{10.15}{49.25}} - \frac{50.75}{9.85} = \frac{6.15228}{1.20609} - 5.15228 = -0.051268$$

$$\text{If } R_2 = 49.25, R_1 = 10.15$$

$$\text{Then } A_{cm} = \frac{1 + \frac{49.25}{10.15}}{1 + \frac{9.85}{50.75}} - \frac{49.25}{10.15} = \frac{5.85222}{1.19409} - 4.85222 = +0.04877$$

$$\text{Now } CMRR_{dB} (\text{min}) = 20 \log_{10} \left(\frac{5}{0.051268} \right) = 39.8 \text{ dB}$$

$$(b) R_2 = 50(1.03) = 51.5, R_1 = 10(0.97) = 9.70$$

$$R_4 = 50(0.97) = 48.5, R_3 = 10(1.03) = 10.3$$

$$A_{cm} = \frac{1 + \frac{51.5}{9.70}}{1 + \frac{10.3}{48.5}} - \frac{51.5}{9.70} = \frac{6.30928}{1.21237} - 5.30928 = -0.10519$$

$$CMRR_{dB} = 20 \log_{10} \left(\frac{5}{0.10519} \right) = 33.5 \text{ dB}$$

14.61

(a) $CMRR_{dB} = 50 \text{ dB} \Rightarrow CMRR = 316.2$

$$|A_{cm}| = \frac{5}{316.2} = 0.01581$$

From Problem 14.60,

$$A_{cm} = -0.01581 = \frac{1 + \frac{50(1+x)}{10(1-x)}}{1 + \frac{10(1+x)}{50(1-x)}} - \frac{50(1+x)}{10(1-x)}$$

x is small, so that $\frac{1}{1-x} \approx 1+x$

$$\text{Then } -0.01581 = \frac{1 + 5(1+x)(1+x)}{1 + 0.2(1+x)(1+x)} - 5(1+x)(1+x)$$

Neglect x^2 ,

$$-0.01581 \approx \frac{1 + 5(1+2x)}{1 + 0.2(1+2x)} - 5(1+2x)$$

$$(-0.01581)[1 + 0.2(1+2x)] = 1 + 5(1+2x) - 5(1+2x)[1 + 0.2(1+2x)]$$

$$\text{We find } (1+2x)^2 - 0.003162(1+2x) - 1.01581 = 0 \Rightarrow (1+2x) = 1.009456$$

$$\text{Then } x = 0.004728 \Rightarrow x = 0.4728\%$$

(b) $CMRR_{dB} = 75 \text{ dB, } \Rightarrow CMRR = 5623.4$

$$A_{cm} = \frac{5}{5623.4} = 0.00088914$$

$$\text{Then } (-0.00088914)[1 + 0.2(1+2x)] = 1 - (1+2x)^2$$

$$(1+2x)^2 - 0.000177828(1+2x) - 1.00088914 = 0 \Rightarrow (1+2x) = 1.0005334 \Rightarrow x = 0.0267\%$$

Chapter 15

15.1

- (a) Noninverting amplifier

$$8 = 1 + \frac{R_2}{R_1} \Rightarrow \frac{R_2}{R_1} = 7 \Rightarrow R_2 = 210 \text{ k}\Omega, R_1 = 30 \text{ k}\Omega$$

At noninverting terminal

$$RC = \frac{1}{2\pi f} = \frac{1}{2\pi(30 \times 10^3)} = 5.305 \times 10^{-6}$$

Let input $C = 0.001 \mu\text{F}$, then $R = 5.305 \text{ k}\Omega$

- (b) Set $R_1 = 15 \text{ k}\Omega, R_2 = 300 \text{ k}\Omega$ for inverting amplifier

$$R_1 C = \frac{1}{2\pi f} = \frac{1}{2\pi(20 \times 10^3)} = 7.958 \times 10^{-6}$$

Put C in series with R_1

$$C = \frac{7.958 \times 10^{-6}}{15 \times 10^3} \Rightarrow C = 530.5 \text{ pF}$$

15.2

$$(a) |T| = \frac{1}{\sqrt{1+(1.5)^4}} = 0.4061 \Rightarrow -7.83 \text{ dB}$$

$$(b) |T| = \frac{1}{\sqrt{1+(1.5)^6}} = 0.2841 \Rightarrow -10.93 \text{ dB}$$

$$(c) |T| = \frac{1}{\sqrt{1+(1.5)^8}} = 0.1938 \Rightarrow -14.25 \text{ dB}$$

$$(d) |T| = \frac{1}{\sqrt{1+(1.5)^{10}}} = 0.1306 \Rightarrow -17.68 \text{ dB}$$

15.3

$$|T| = -6 \text{ dB}, \Rightarrow |T| = 0.50$$

$$0.50 = \frac{1}{\sqrt{1+\left(\frac{1}{0.9}\right)^{2N}}} = \frac{1}{\sqrt{1+(1.111)^{2N}}}$$

$$\text{or } (1.111)^{2N} = \left(\frac{1}{0.5}\right)^2 - 1 = 3$$

For N=6, $(1.111)^{(2)(6)} = 3.54$, \Rightarrow 6-pole filter

15.4

- (a) From Figure 15.8(a)

$$RC = \frac{1}{2\pi f_{3-dB}} = \frac{1}{2\pi(25 \times 10^3)} = 6.366 \times 10^{-6}$$

Let $C = 0.001 \mu F$, then $R = 6.366 k\Omega$

$$\text{And } R_3 = (0.707)R = 4.50 k\Omega$$

$$R_4 = (1.414)R = 9.0 k\Omega$$

$$(b) \quad (i) |T| = \frac{1}{\sqrt{1 + \left(\frac{25}{22}\right)^4}} = 0.6123 \Rightarrow -4.26 \text{ dB}$$

$$(ii) |T| = \frac{1}{\sqrt{1 + \left(\frac{25}{25}\right)^4}} = 0.707 \Rightarrow -3 \text{ dB}$$

$$(iii) |T| = \frac{1}{\sqrt{1 + \left(\frac{25}{28}\right)^4}} = 0.7819 \Rightarrow -2.14 \text{ dB}$$

15.5

$$(a) \quad RC = \frac{1}{2\pi f_{3-dB}} = \frac{1}{2\pi(20 \times 10^3)} = 7.958 \times 10^{-6}$$

Let $R = 20 k\Omega$, then $C = 397.9 \text{ pF}$

$$\text{And } C_1 = (3.546)C = 1411 \text{ pF}$$

$$C_2 = (1.392)C = 553.9 \text{ pF}$$

$$C_3 = (0.2024)C = 80.5 \text{ pF}$$

$$(b) \quad (i) |T| = \frac{1}{\sqrt{1 + \left(\frac{10}{20}\right)^6}} = 0.9923 \Rightarrow -0.0673 \text{ dB}$$

$$(ii) |T| = \frac{1}{\sqrt{1 + \left(\frac{15}{20}\right)^6}} = 0.9214 \Rightarrow -0.711 \text{ dB}$$

$$(iii) |T| = \frac{1}{\sqrt{1 + \left(\frac{20}{20}\right)^6}} = 0.7071 \Rightarrow -3.0 \text{ dB}$$

$$(iv) |T| = \frac{1}{\sqrt{1 + \left(\frac{25}{20}\right)^6}} = 0.4557 \Rightarrow -6.83 \text{ dB}$$

$$(v) |T| = \frac{1}{\sqrt{1 + \left(\frac{30}{20}\right)^6}} = 0.2841 \Rightarrow -10.9 \text{ dB}$$

15.6

From Equation (15.7).

$$T(s) = \frac{Y_1 Y_2}{Y_1 Y_2 + Y_4 (Y_1 + Y_2 + Y_3)}$$

For a high-pass filter, let $Y_1 = Y_2 = sC$,

$$Y_3 = \frac{1}{R_3}, \quad \text{and} \quad Y_4 = \frac{1}{R_4}$$

Then

$$\begin{aligned} T(s) &= \frac{s^2 C^2}{s^2 C^2 + \frac{1}{R_4} \left(sC + sC + \frac{1}{R_3} \right)} \\ &= \frac{1}{1 + \frac{1}{s R_4 C} \left(2 + \frac{1}{s R_3 C} \right)} \end{aligned}$$

Define $\tau_3 = R_3 C$ and $\tau_4 = R_4 C$

$$T(s) = \frac{1}{1 + \frac{1}{s \tau_4} \left(2 + \frac{1}{s \tau_3} \right)}$$

Set $s = j\omega$

$$\begin{aligned} T(j\omega) &= \frac{1}{1 + \frac{1}{j\omega \tau_4} \left(2 + \frac{1}{j\omega \tau_3} \right)} \\ &= \frac{1}{1 - \frac{j}{\omega \tau_4} \left(2 - \frac{j}{\omega \tau_3} \right)} \\ &= \frac{1}{\left(1 - \frac{1}{\omega^2 \tau_3 \tau_4} \right) - \frac{2j}{\omega \tau_4}} \\ |T(j\omega)| &= \left| \left(1 - \frac{1}{\omega^2 \tau_3 \tau_4} \right)^2 + \frac{4}{\omega^2 \tau_4^2} \right|^{-1/2} \end{aligned}$$

For a maximally flat filter, we want

$$\left. \frac{d|T|}{d\omega} \right|_{\omega \rightarrow \infty} = 0$$

Taking the derivative, we find

$$\frac{d|T(j\omega)|}{d\omega} = -\frac{1}{2} \left\{ \left(1 - \frac{1}{\omega^2 \tau_3 \tau_4} \right)^2 + \frac{4}{\omega^2 \tau_4^2} \right\}^{-3/2} \times \left[2 \left(1 - \frac{1}{\omega^2 \tau_3 \tau_4} \right) \left(\frac{2}{\omega^3 \tau_3 \tau_4} \right) + \frac{4(-2)}{\omega^3 \tau_4^2} \right]$$

or

$$\begin{aligned}\frac{d|T(j\omega)|}{d\omega} \Big|_{\omega \rightarrow \infty} &= 0 \\ &= \left[\left(\frac{4}{\omega^3 \tau_3 \tau_4} \right) \left(1 - \frac{1}{\omega^2 \tau_3 \tau_4} \right) - \frac{8}{\omega^3 \tau_4^2} \right] \\ &= \frac{4}{\omega^3} \left[\frac{1}{\tau_3 \tau_4} \left(1 - \frac{1}{\omega^2 \tau_3 \tau_4} \right) - \frac{2}{\tau_4^2} \right]\end{aligned}$$

Then

$$\left[\frac{1}{\tau_3 \tau_4} \left(1 - \frac{1}{\omega^2 \tau_3 \tau_4} \right) - \frac{2}{\tau_4^2} \right]_{\omega \rightarrow \infty} = 0$$

$$\text{So that } \frac{1}{\tau_3} = \frac{2}{\tau_4} \Rightarrow 2\tau_3 = \tau_4$$

Then the transfer function can be written as:

$$\begin{aligned}|T(j\omega)| &= \left\{ \left[1 - \frac{1}{\omega^2 (2\tau_3^2)} \right]^2 + \frac{4}{\omega^2 (4\tau_3^2)} \right\}^{-1/2} \\ &= \left\{ 1 - \frac{1}{\omega^2 \tau_3^2} + \frac{1}{4(\omega^2 \tau_3^2)^2} + \frac{1}{\omega^2 \tau_3^2} \right\}^{-1/2} \\ &= \left\{ 1 + \frac{1}{4(\omega^2 \tau_3^2)^2} \right\}^{-1/2}\end{aligned}$$

$3-dB$ frequency

$$2\omega^2 \tau_3^2 = 1 \text{ or } \omega = \frac{1}{\sqrt{2}(\tau_3)} = \frac{1}{\sqrt{2}(R_3 C)}$$

Define

$$\omega = \frac{1}{RC}$$

So that

$$R_3 = \frac{R}{\sqrt{2}}$$

We had $2\tau_3 = \tau_4$ or $2(R_3 C) = R_4 C \Rightarrow R_4 = 2R_3$

$$\text{So that } R_4 = \sqrt{2} \cdot R$$

15.7

$$-14 \text{ dB} \Rightarrow |T| = 0.1995$$

$$0.1995 = \frac{1}{\sqrt{1 + (1.2)^{2N}}} \Rightarrow (1.2)^{2N} = \left(\frac{1}{0.1995} \right)^2 - 1 = 24.1$$

$N=9$, 9th order filter

15.8

$$-12 \text{ dB} \Rightarrow |T| = 0.2512$$

$$0.2512 = \frac{1}{\sqrt{1 + \left(\frac{4}{3}\right)^{2N}}} \Rightarrow (1.333)^{2N} = \left(\frac{1}{0.2512}\right)^2 - 1 = 14.85$$

N=5, 5th order filter

15.9

$$|T| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3dB}}\right)^{2N}}}$$

At $f = 12 \text{ kHz}$, $|T| = 0.9$

$$0.9 = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3dB}}\right)^{2N}}} = \frac{1}{\sqrt{1 + \left(\frac{12}{f_{3dB}}\right)^{2N}}}$$

$$\left(\frac{12}{f_{3dB}}\right)^{2N} = \frac{1}{(0.9)^2} - 1 = 0.2346$$

Also

$$0.01 = \frac{1}{\sqrt{1 + \left(\frac{14}{f_{3dB}}\right)^{2N}}}$$

$$\left(\frac{14}{f_{3dB}}\right)^{2N} = \frac{1}{(0.01)^2} - 1 = 9999$$

$$\left(\frac{14}{f_{3dB}}\right)^{2N} = \left(\frac{14}{12}\right)^{2N} = \frac{9999}{0.2346} = 4.262 \times 10^4$$

$$(1.16667)^{2N} = 4.262 \times 10^4$$

$$\underline{N = 35}$$

Then

$$0.9 = \frac{1}{\sqrt{1 + \left(\frac{12}{f_{3dB}}\right)^{2N}}}$$

$$\left(\frac{12}{f_{3dB}}\right)^{2N} = 0.2346$$

$$\left(\frac{12}{f_{3dB}}\right) = (0.2346)^{\left(\frac{1}{2N}\right)} = (0.2346)^{0.014286}$$

$$= 0.9795$$

So

$$\underline{f_{3dB} = 12.25 \text{ kHz}}$$

15.10

$$(a) |T| = \frac{1}{\sqrt{1 + \left(\frac{1}{0.8}\right)^{2N}}} = \frac{1}{\sqrt{1 + (1.25)^{2N}}}$$

$$\text{For } N=3, |T| = \frac{1}{\sqrt{1 + (1.25)^6}} = 0.4557 \Rightarrow -6.83 \text{ dB}$$

$$(b) \text{ For } N=5, |T| = \frac{1}{\sqrt{1 + (1.25)^{10}}} = 0.3114 \Rightarrow -10.1 \text{ dB}$$

$$(c) \text{ For } N=7, |T| = \frac{1}{\sqrt{1 + (1.25)^{14}}} = 0.2053 \Rightarrow -13.8 \text{ dB}$$

15.11

$$(a) |T| = \frac{1}{\sqrt{1 + (1.4)^{2N}}}$$

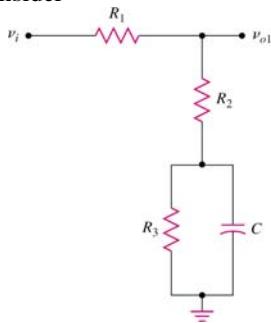
$$\text{For } N=3, |T| = \frac{1}{\sqrt{1 + (1.4)^6}} = 0.3424 \Rightarrow -9.31 \text{ dB}$$

$$(b) \text{ For } N=5, |T| = \frac{1}{\sqrt{1 + (1.4)^{10}}} = 0.1828 \Rightarrow -14.8 \text{ dB}$$

$$(c) \text{ For } N=7, |T| = \frac{1}{\sqrt{1 + (1.4)^{14}}} = 0.0944 \Rightarrow -20.5 \text{ dB}$$

15.12

Consider



$$\frac{v_o}{v_i} = \frac{R_2 + R_3}{R_1 + R_2 + R_3}$$

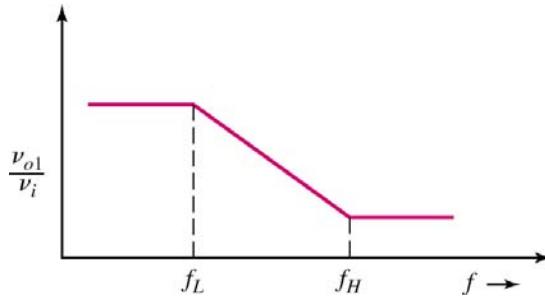
For low-frequency:

$$\frac{v_o}{v_i} = \frac{R_2}{R_1 + R_2}$$

For high-frequency:

So we need

$$\frac{R_2 + R_3}{R_1 + R_2 + R_3} = 25 \left(\frac{R_2}{R_1 + R_2} \right)$$

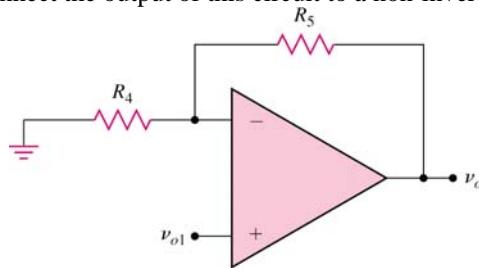


Let $R_1 + R_2 = 50 \text{ k}\Omega$ and $\frac{R_2}{R_1 + R_2} = 1.5 \text{ k}\Omega \Rightarrow R_1 = 48.5 \text{ k}\Omega$

Then

$$\frac{1.5 + R_3}{50 + R_3} = 25 \left(\frac{1.5}{50} \right) \Rightarrow R_3 = 144 \text{ k}\Omega$$

Connect the output of this circuit to a non-inverting op-amp circuit.



At low-frequency:

$$v_{o1} = \frac{R_2 + R_3}{R_1 + R_2 + R_3} \cdot v_i = \frac{1.5 + 144}{48.5 + 1.5 + 144} \cdot v_i = 0.75v_i$$

Need to have $v_o = 25$.

$$v_o = 25 = \left(1 + \frac{R_5}{R_4} \right) \cdot v_{o1} = \left(1 + \frac{R_5}{R_4} \right) (0.75)v_i \Rightarrow \frac{R_5}{R_4} = 32.3$$

To check at high-frequency.

$$v_{o1} = \frac{R_2}{R_1 + R_2} v_i = \frac{1.5}{1.5 + 48.5} v_i = 0.03v_i$$

$$v_o = (1 + 32.3)v_{o1} = (33.3)(0.03)v_i = (1.0)v_i$$

which meets the design specification

Consider the frequency response.

$$\frac{v_{o1}}{v_i} = \frac{R_2 + R_3}{R_1 + R_2 + R_3} \left| \frac{1}{sC} \right|$$

Now

$$R_3 \left| \frac{1}{sC} \right| = \frac{R_3}{1 + sR_3 C}$$

Then, we find

$$\frac{v_{o1}}{v_i} = \frac{R_3 + R_2(1+sR_3C)}{R_3 + (R_1 + R_2)(1+sR_3C)}$$

which can be rearranged as

$$\frac{v_{o1}}{v_i} = \frac{(R_2 + R_3)(1+s(R_2 \parallel R_3)C)}{(R_1 + R_2 + R_3)(1+s(R_3 \parallel (R_1 + R_2))C)}$$

So

$$f_L \cong \frac{1}{2\pi(R_2 \parallel R_3)C} = \frac{1}{2\pi(1.5 \parallel 144) \times 10^3 C} = \frac{1}{(9.33 \times 10^3)C}$$

$$\begin{aligned} f_H &\cong \frac{1}{2\pi(R_3 \parallel (R_1 + R_2))C} = \frac{1}{2\pi(144 \parallel 50) \times 10^3 C} \\ &= \frac{1}{(2.33 \times 10^5)C} \end{aligned}$$

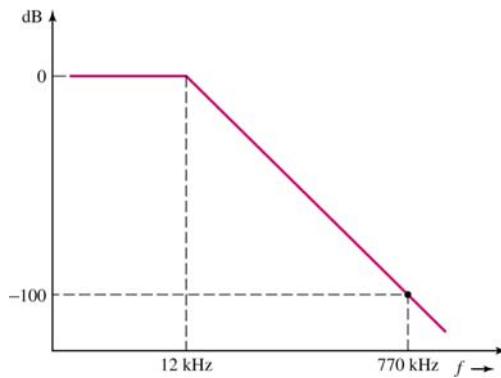
Set

$$25 \text{ kHz} = \frac{f_L + f_H}{2} = \frac{1}{2} \left[\frac{1}{(9.33 \times 10^3)C} + \frac{1}{(2.33 \times 10^5)C} \right]$$

Which yields

$$\underline{C = 2.23 \text{ nF}}$$

15.13



$$|A_v| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{-3-dB}}\right)^{2N}}} - 100 \text{ dB} \Rightarrow 10^{-5}$$

So

$$10^{-5} = \frac{1}{\sqrt{1 + \left(\frac{770}{12}\right)^{2N}}}$$

or

$$1 + (64.2)^{2N} = \left(\frac{1}{10^{-5}}\right)^2 = 10^{10}$$

or

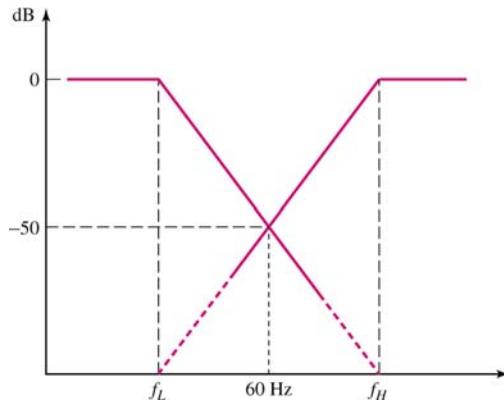
$$(64.2)^{2N} \cong 10^{10}$$

Now

<i>N</i>	<i>Left Side</i>
1	4.112×10^3
2	1.7×10^7
3	7×10^{10}

So, we need a 3rd order filter.

15.14



Low-pass: $-50 \text{ dB} \Rightarrow 3.16 \times 10^{-3}$

Then

$$3.16 \times 10^{-3} = \frac{1}{\sqrt{1 + \left(\frac{f}{f_L}\right)^4}} = \frac{1}{\sqrt{1 + \left(\frac{60}{f_L}\right)^4}}$$

We find $f_L = 3.37 \text{ Hz}$

High Pass:

$$3.16 \times 10^{-3} = \frac{1}{\sqrt{1 + \left(\frac{f_H}{f}\right)^4}} = \frac{1}{\sqrt{1 + \left(\frac{f_H}{60}\right)^4}}$$

We find $f_H = 1067 \text{ Hz}$

Bandwidth: $BW = f_H - f_L = 1067 - 3.37 \Rightarrow$

$$\underline{BW \cong 1064 \text{ Hz}}$$

15.15

a.

$$\frac{v_I}{R_4} = -\frac{v_{02}}{R_3} - \frac{v_0}{R_l} \left| \left(\frac{1}{sC} \right) \right| \quad (1)$$

$$\frac{v_0}{R_2} = -\frac{v_{01}}{\left(\frac{1}{sC} \right)} \quad (2)$$

$$\frac{v_{01}}{R_5} = -\frac{v_{02}}{R_5} \Rightarrow v_{01} = -v_{02} \quad (3)$$

Then

$$\frac{v_0}{R_2} = +\frac{v_{02}}{\left(\frac{1}{sC} \right)} \quad \text{or} \quad v_{02} = v_0 \left(\frac{1}{sR_2 C} \right) \quad (2)$$

And

$$\begin{aligned} \frac{v_I}{R_4} &= -\frac{v_0}{R_3} \cdot \left(\frac{1}{sR_2 C} \right) - \frac{v_0}{R_l} \left| \left(\frac{1}{sC} \right) \right| \\ &= -v_0 \left[\frac{1}{R_3(sR_2 C)} + \frac{1}{R_l \cdot (1/sC)} \right] \quad (1) \\ &= -v_0 \left[\frac{1}{R_3(sR_2 C)} + \frac{1+sR_l C}{R_l} \right] \\ &= -v_0 \left[\frac{R_l + (1+sR_l C)(sR_2 R_3 C)}{(sC)R_l R_2 R_3} \right] \end{aligned}$$

Then

$$\frac{v_0}{v_I} = -\frac{1}{R_4} \left[\frac{(sC)(R_l R_2 R_3)}{R_l + sR_2 R_3 C + s^2 R_l R_2 R_3 C^2} \right]$$

or

$$A_v(s) = \frac{v_0}{v_I} = \frac{-\frac{1}{R_4}}{\frac{1}{R_l} + sC + \frac{1}{sCR_2R_3}}$$

$$A_v(j\omega) = \frac{-\frac{1}{R_4}}{\frac{1}{R_l} + j\omega C + \frac{1}{j\omega CR_2R_3}}$$

b.

or

$$\begin{aligned} A_v(j\omega) &= \frac{-\frac{1}{R_4}}{\frac{1}{R_i} + j\left[\omega C - \frac{1}{\omega CR_2 R_3}\right]} \\ &= -\frac{R_i}{R_4} \cdot \frac{1}{\left\{1 + j\left[\omega R_i C - \frac{R_i}{\omega CR_2 R_3}\right]\right\}} \\ |A_v(j\omega)| &= \frac{R_i}{R_4} \cdot \frac{1}{\left\{1 + \left[\omega R_i C - \frac{R_i}{\omega CR_2 R_3}\right]^2\right\}^{-1/2}} \\ |A_v|_{\max} \text{ when } &\left[\omega R_i C - \frac{R_i}{\omega CR_2 R_3}\right] = 0 \end{aligned}$$

Then

$$|A_v|_{\max} = \frac{R_i}{R_4} = \frac{85}{3} \Rightarrow |A_v|_{\max} = 28.3$$

Now

$$\omega R_i C \left[1 - \frac{1}{\omega^2 C^2 R_2 R_3}\right] = 0 \quad \text{or} \quad \omega = \frac{1}{C \sqrt{R_2 R_3}}$$

Then

$$f = \frac{1}{2\pi C \sqrt{R_2 R_3}} = \frac{1}{2\pi(0.1 \times 10^{-6}) \sqrt{(300)^2}}$$

So

$$f = 5.305 \text{ kHz}$$

To find the two $3-dB$ frequencies,

$$\begin{aligned} \left[\omega R_i C - \frac{R_i}{\omega CR_2 R_3}\right] &= \pm 1 \\ \omega^2 R_i R_2 R_3 C^2 - R_i &= \pm \omega R_2 R_3 C \\ \omega^2 (85 \times 10^3) (300)^2 (0.1 \times 10^{-6})^2 - 85 \times 10^3 &= \pm \omega (300)^2 (0.1 \times 10^{-6}) \\ \omega^2 (7.65 \times 10^{-5}) - 85 \times 10^3 &= \pm \omega (9 \times 10^{-3}) \\ \omega^2 (7.65 \times 10^{-5}) \pm \omega (9 \times 10^{-3}) - 85 \times 10^3 &= 0 \\ \omega &= \frac{\pm (9 \times 10^{-3})}{2(7.65 \times 10^{-5})} \pm \frac{\sqrt{(9 \times 10^{-3})^2 + 4(7.65 \times 10^{-5})(85 \times 10^{-3})}}{2(7.65 \times 10^{-5})} \end{aligned}$$

We find $f = 5.315 \text{ kHz}$ and $f = 5.296 \text{ kHz}$

15.16

a.

$$\frac{v_I - v_A}{R} = \frac{v_A}{\left(\frac{1}{sC}\right)} \quad (1)$$

$$\frac{v_I - v_B}{R} = \frac{v_B - v_0}{R} \quad (2)$$

and $v_A = v_B$

So

$$\frac{v_I}{R} = v_A \left(\frac{1}{R} + sC \right) = v_A \left(\frac{1+sRC}{R} \right) \quad (1)$$

or

$$v_A = \frac{v_I}{1+sRC}$$

Then

$$v_I + v_0 = 2v_B = 2v_A = \frac{2v_I}{1+sRC} \quad (2)$$

$$v_0 = v_I \left[\frac{2}{1+sRC} - 1 \right] = v_I \left[\frac{1-sRC}{1+sRC} \right]$$

Now

$$\frac{v_0}{v_I} = A(j\omega) = \frac{1-j\omega RC}{1+j\omega RC}$$

$$|A| = \frac{\sqrt{1+\omega^2 R^2 C^2}}{\sqrt{1+\omega^2 R^2 C^2}} \Rightarrow |A| = 1$$

Phase:

$$\phi = -2 \tan^{-1}(\omega RC)$$

$$\text{b. } RC = (10^4)(15.9 \times 10^{-9}) = 1.59 \times 10^{-4}$$

f	ϕ
0	0
10^2	-11.4
5×10^3	-53.1
$1/2\pi RC = 10^3$ Hz	-90°
5×10^3	-157
10^4	-169

15.17

a.

$$\begin{aligned} \frac{V_i}{R_1} + \frac{V_i - V_0}{R_2 \parallel (1/sC)} &= 0 \\ \frac{V_i}{R_1} + \frac{V_i - V_0}{\left[\frac{R_2}{1+sR_2C} \right]} &= 0 \\ \frac{R_2}{R_1} \cdot \frac{1}{1+sR_2C} (V_i) + V_i &= V_0 \\ \frac{V_0}{V_i} = \frac{R_2 + R_1(1+sR_2C)}{R_1(1+sR_2C)} &= \frac{(R_2 + R_1)[1+s(R_1 \parallel R_2)C]}{R_1(1+sR_2C)} \\ \Rightarrow \frac{V_0}{V_i} &= \left(1 + \frac{R_2}{R_1} \right) \left[\frac{1+s(R_1 \parallel R_2)C}{(1+sR_2C)} \right] \\ \Rightarrow f_{3dB_1} &= \frac{1}{2\pi R_2 C} \\ \Rightarrow f_{3dB_2} &= \frac{1}{2\pi (R_1 \parallel R_2) C} \end{aligned}$$

b.

$$\begin{aligned} \frac{V_i}{R_1 \parallel (1/sC)} + \frac{V_i - V_0}{R_2} &= 0 \\ \frac{V_i}{\left(\frac{R_1}{1+sR_1C} \right)} + \frac{V_i}{R_2} &= \frac{V_0}{R_2} \\ V_i \left[\frac{R_2}{R_1} \cdot (1+sR_1C) + 1 \right] &= V_0 \\ \frac{V_i}{R_1} \cdot [R_2 + R_1 + sR_1R_2C] &= V_0 \\ \frac{V_0}{V_i} = \frac{R_2 + R_1}{R_1} \cdot [1+s(R_1 \parallel R_2)C] &\Rightarrow \frac{V_0}{V_i} = \left(1 + \frac{R_2}{R_1} \right) [1+s(R_1 \parallel R_2)C] \Rightarrow f_{3dB} = \frac{1}{2\pi (R_1 \parallel R_2) C} \end{aligned}$$

15.18

a.

$$\begin{aligned} \frac{V_i}{R_1 + (1/sC_1)} &= \frac{-V_0}{R_2 \parallel (1/sC_2)} \\ V_i \left(\frac{sC_1}{1+sR_1C_1} \right) &= -V_0 \left(\frac{1+sR_2C_2}{sC_2} \right) \\ \frac{V_0}{V_i} = \frac{-sR_2C_1}{(1+sR_1C_1)(1+sR_2C_2)} &= \frac{-sR_2C_1}{1+sR_1C_1 + sR_2C_2 + s^2R_1R_2C_1C_2} \\ \frac{V_0}{V_i} = -\frac{R_2}{R_1} \times \left[\frac{sC_1}{\frac{1}{R_1} + sC_1 \left(1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1} \right) + s^2R_2C_1C_2} \right] & \end{aligned}$$

or

$$T(s) = \frac{V_0}{V_i} = -\frac{R_2}{R_1} \cdot \left[\frac{1}{\frac{1}{sR_1C_1} + \left(1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1} \right) + sR_2C_2} \right]$$

b.

$$|T(j\omega)| = -\frac{R_2}{R_1} \times \frac{1}{\sqrt{\left(1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1} \right)^2 + \left(\omega R_2 C_2 - \frac{1}{\omega R_1 C_1} \right)^2}}$$

when $\left(\omega R_2 C_2 - \frac{1}{\omega R_1 C_1} \right) = 0$, we want

$$|T(j\omega)| = 50 = \frac{R_2}{R_1} \cdot \frac{1}{\left(1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1} \right)}$$

At the 3-dB frequencies, we want

$$\left(\omega R_2 C_2 - \frac{1}{\omega R_1 C_1} \right) = \pm \left(1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1} \right)$$

For $f = 5 \text{ kHz}$, use + sign and for $f = 200 \text{ Hz}$, use - sign.

$$\omega_1 = 2\pi(200) = 1257$$

$$\omega_2 = 2\pi(5 \times 10^3) = 3.142 \times 10^4$$

Define $\tau_2 = R_2 C_2$ and $\tau_1 = R_1 C_1$

Then

$$50 = \frac{R_2}{R_1} \cdot \frac{1}{1 + \frac{\tau_2}{\tau_1}} \quad (1)$$

$$\left(\omega_2 \tau_2 - \frac{1}{\omega_2 \tau_1} \right) = + \left(1 + \frac{\tau_2}{\tau_1} \right) \quad (2)$$

$$\left(\omega_1 \tau_2 - \frac{1}{\omega_1 \tau_1} \right) = - \left(1 + \frac{\tau_2}{\tau_1} \right) \quad (3)$$

From (2)

$$\frac{\omega_2^2 \tau_1 \tau_2 - 1}{\omega_2 \tau_1} = \frac{\tau_1 + \tau_2}{\tau_1}$$

or

$$\omega_2 \tau_1 \tau_2 - \frac{1}{\omega_2} = \tau_1 + \tau_2$$

$$\tau_1 (\omega_2 \tau_2 - 1) = \frac{1}{\omega_2} + \tau_2$$

So

$$\tau_1 = \frac{\frac{1}{\omega_2} + \tau_2}{\omega_2 \tau_2 - 1}$$

Substituting into (3), we find

$$\begin{aligned} \omega_1\tau_2 - \frac{1}{\omega_1 \cdot \frac{\left(\frac{1}{\omega_2} + \tau_2\right)}{\left(\frac{1}{\omega_2\tau_2} - 1\right)}} &= -\left[1 + \frac{\tau_2(\omega_2\tau_2 - 1)}{\frac{1}{\omega_2} + \tau_2} \right] \\ \omega_1\tau_2 \left[\frac{1}{\omega_2} + \tau_2 \right] - \frac{1}{\omega_1} (\omega_2\tau_2 - 1) &= -\left[\left(\frac{1}{\omega_2} + \tau_2 \right) + \tau_2(\omega_2\tau_2 - 1) \right] \\ \frac{\omega_1}{\omega_2} \cdot \tau_2 + \omega_1\tau_2^2 - \frac{\omega_2}{\omega_1} \cdot \tau_2 + \frac{1}{\omega_1} &= -\frac{1}{\omega_2} - \tau_2 - \omega_2\tau_2^2 + \tau_2 \\ (\omega_1 + \omega_2)\tau_2^2 + \left(\frac{\omega_1}{\omega_2} - \frac{\omega_2}{\omega_1} \right)\tau_2 + \left(\frac{1}{\omega_1} + \frac{1}{\omega_2} \right) &= 0 \\ (3.2677 \times 10^4)\tau_2^2 - 24.96\tau_2 + 8.273 \times 10^{-4} &= 0 \\ \tau_2 = \frac{24.96 \pm \sqrt{(24.96)^2 - 4(3.2677 \times 10^4)(8.273 \times 10^{-4})}}{2(3.2677 \times 10^4)} & \end{aligned}$$

Since ω_2 is large, τ_2 should be small so use minus sign:

$$\tau_2 = 3.47 \times 10^{-5} \text{ s}$$

Then

$$\tau_1 = \frac{3.18 \times 10^{-5} + 3.47 \times 10^{-5}}{9.09 \times 10^{-2}} \Rightarrow \tau_1 = 7.32 \times 10^{-4} \text{ s}$$

Now

$$50 = \frac{R_2}{R_1} \cdot \frac{1}{1 + \frac{3.47 \times 10^{-5}}{7.32 \times 10^{-4}}}$$

Then

$$\frac{R_2}{R_1} = 52.37 \quad \text{or} \quad R_2 = 524 \text{ k}\Omega$$

Also

$$\tau_1 = R_1 C_1 \text{ so that } C_1 = 0.0732 \mu\text{F}$$

$$\tau_2 = R_2 C_2 \text{ so that } C_2 = 66.3 \text{ pF}$$

15.19

Two noninverting amplifiers,

$$\left(1 + \frac{R_2}{R_1} \right)^2 = 30 \text{ dB} \Rightarrow 31.62$$

which gives $\frac{R_2}{R_1} = 4.62$, then $R_2 = 250 \text{ k}\Omega$, $R_1 = 54.1 \text{ k}\Omega$

For high-pass filter:

$$RC = \frac{1}{2\pi(20 \times 10^3)} = 7.958 \times 10^{-6}$$

Set $R = 250 \text{ k}\Omega$, then $C = 31.8 \text{ pF}$

For low-pass filter:

$$RC = \frac{1}{2\pi(150)} = 1.061 \times 10^{-3}$$

Set $R = 250 \text{ k}\Omega$, then $C = 0.00424 \mu\text{F}$

15.20

$$(a) R_{eq} = \frac{1}{(50 \times 10^3)(0.5 \times 10^{-12})} \Rightarrow R_{eq} = 40 \text{ M}\Omega$$

$$(b) R_{eq} = \frac{1}{(50 \times 10^3)(2 \times 10^{-12})} \Rightarrow R_{eq} = 10 \text{ M}\Omega$$

$$(c) R_{eq} = \frac{1}{(50 \times 10^3)(10 \times 10^{-12})} \Rightarrow R_{eq} = 2 \text{ M}\Omega$$

15.21

a. From Equation (15.28),

$$Q = \frac{V_1 - V_2}{R_{eq}} \cdot T_c$$

$$\text{and } f_c = 100 \text{ kHz} \text{ so that } T_c = \frac{1}{100 \times 10^3} \Rightarrow 10 \mu\text{s}$$

Now

$$R_{eq} = \frac{1}{f_c C} = \frac{1}{(100 \times 10^3)(10 \times 10^{-12})} \Rightarrow 1 \text{ M}\Omega$$

So

$$Q = \frac{(2-1)(10 \times 10^{-6})}{10^6} = 10 \times 10^{-12} \text{ C}$$

or

$$Q = 10 \text{ pC}$$

$$b. I_{eq} = \frac{Q}{T_c} = \frac{10 \times 10^{-12}}{10 \times 10^{-6}} \text{ or } I_{eq} = 1 \mu\text{A}$$

c. $Q = CV$ so find the time that V_o reaches 99% of its full value.

$$V_o = V_1(1 - e^{-t/\tau}) \text{ where } \tau = RC$$

Then $0.99 = 1 - e^{-t/\tau}$ or $e^{-t/\tau} = 0.01$

$$\text{or } t = \tau \ln(100)$$

$$\tau = RC = (10^3)(10 \times 10^{-12}) = 10^{-8} \text{ s}$$

Then

$$t = 4.61 \times 10^{-8} \text{ s}$$

15.22

$$\text{Low frequency gain} = -10 \Rightarrow \frac{C_1}{C_2} = 10$$

$$f_{3dB} = 10 \times 10^3 \text{ Hz} = \frac{f_c C_2}{2\pi C_f}$$

Set

$$f_C = 10 f_{3dB} = 100 \text{ kHz}$$

Then

$$\frac{C_2}{C_F} = \frac{2\pi(10 \times 10^3)}{100 \times 10^3} = 0.628$$

The largest capacitor is C_1 , so let

$$\underline{C_1 = 30 \text{ pF}}$$

Then

$$\underline{C_2 = 3 \text{ pF}}$$

and

$$\underline{C_F = 4.78 \text{ pF}}$$

15.23

- a. Time constant $\tau = R_{eq} C_F$ where

$$R_{eq} = \frac{1}{f_C C_1} = \frac{1}{(100 \times 10^3)(5 \times 10^{-12})} = 2 \times 10^6 \Omega$$

Then

$$\tau = (2 \times 10^6) (30 \times 10^{-12})$$

or

$$\tau = 60 \mu\text{s}$$

b. $v_o = -\frac{1}{\tau} \int v_I dt$

or

$$\Delta v_o = \frac{(1)(T_C)}{\tau} \text{ and } T_C = \frac{1}{f_C}$$

So

$$\Delta v_0 = \frac{1}{(60 \times 10^{-6})(100 \times 10^3)}$$

or

$$\underline{\Delta v_0 = 0.167 \text{ V}}$$

- c. Now $\Delta v_0 = 13 = N(0.167)$

or

$$\underline{N = 78 \text{ clock pulses}}$$

15.24

(a) $f_o = \frac{1}{2\pi\sqrt{3}RC} = \frac{1}{2\pi\sqrt{3}(20 \times 10^3)(0.001 \times 10^{-6})}$

$$f_o = 4.59 \text{ kHz}, \quad R_2 = 8R = 160 \text{ k}\Omega$$

(b) $C = \frac{1}{2\pi\sqrt{3}Rf_o} = \frac{1}{2\pi\sqrt{3}(20 \times 10^3)(25 \times 10^3)}$

$$C = 184 \text{ pF}, \quad R_2 = 160 \text{ k}\Omega$$

15.25

a.

$$v_1 = \frac{R}{R + (1/sC_V)} \cdot v_0 = \left(\frac{sRC_V}{1+sRC_V} \right) \cdot v_0$$

$$v_2 = \frac{R}{R + \frac{1}{sC}} \cdot v_1 = \left(\frac{sRC}{1+sRC} \right) \cdot v_1$$

$$v_3 = \frac{R}{R + \frac{1}{sC}} \cdot v_2 = \left(\frac{sRC}{1+sRC} \right) \cdot v_2$$

$$v_0 = -\frac{R_2}{R} \cdot v_3$$

Then

$$v_0 = -\frac{R_2}{R} \left(\frac{sRC}{1+sRC} \right)^2 \left(\frac{sRC_V}{1+sRC_V} \right) v_0$$

Set $s = j\omega$

$$1 = -\frac{R_2}{R} \left(\frac{-\omega^2 R^2 C^2}{1+2j\omega RC - \omega^2 R^2 C^2} \right) \left(\frac{j\omega RC_V}{1+j\omega RC_V} \right)$$

The real part of the denominator must be zero.

$$1 - \omega^2 R^2 C^2 - 2\omega^2 R^2 C C_V = 0$$

so

$$\omega_0 = \frac{1}{R\sqrt{C(C+2C_V)}}$$

b.

$$f_{0,\max} = \frac{1}{2\pi(10^4)\sqrt{(10^{-11})(10^{-11} + 2[10^{-11}])}}$$

$$f_{0,\max} = 919 \text{ kHz}$$

$$f_{0,\min} = \frac{1}{2\pi(10^4)\sqrt{(10^{-11})(10^{-11} + 2[50 \times 10^{-12}])}}$$

$$f_{0,\min} = 480 \text{ kHz}$$

15.26

$$(a) f_o = \frac{1}{2\pi\sqrt{6}RC} = \frac{1}{2\pi\sqrt{6}(12 \times 10^3)(150 \times 10^{-12})}$$

$$f_o = 36.1 \text{ kHz}, \quad R_2 = 29R = 348 \text{ k}\Omega$$

$$(b) R = \frac{1}{2\pi\sqrt{6}C f_o} = \frac{1}{2\pi\sqrt{6}(0.001 \times 10^{-6})(22 \times 10^3)}$$

$$R = 2.95 \text{ k}\Omega, \quad R_2 = 29R = 85.6 \text{ k}\Omega$$

15.27

$$\frac{\frac{v_0 - v_1}{1}}{sC} = \frac{v_1}{R} + \frac{\frac{v_1 - v_2}{1}}{sC} \quad (1)$$

$$\text{or } (v_0 - v_1)sC = \frac{v_1}{R} + (v_1 - v_2)sC$$

$$\frac{\frac{v_1 - v_2}{1}}{sC} = \frac{v_2}{R} + \frac{\frac{v_2}{1}}{sC} + R \quad (2)$$

$$\text{or } (v_1 - v_2)sC = \frac{v_2}{R} + \frac{v_2(sC)}{1 + sRC}$$

$$\frac{\frac{v_2}{1}}{sC} + R = -\frac{v_0}{R_2} \quad (3)$$

$$\text{or } \frac{v_2 sC}{1 + sRC} = -\frac{v_0}{R_2}$$

so

$$v_2 = \frac{-v_0}{sR_2 C} (1 + sRC)$$

From (2)

$$v_1(sC) = v_2 \left[sC + \frac{1}{R} + \frac{sC}{1 + sRC} \right]$$

or

$$v_1 = -\frac{v_0(1 + sRC)}{sR_2 C} \cdot \left[1 + \frac{1}{sRC} + \frac{1}{1 + sRC} \right]$$

From (1)

$$v_0(sC) = v_1 \left[sC + \frac{1}{R} + sC \right] - v_2(sC)$$

Then

$$v_0 = \left[2 + \frac{1}{sRC} \right] \left[\frac{-v_0(1 + sRC)}{sR_2 C} \right] \times \left[\frac{1 + sRC}{sRC} + \frac{1}{1 + sRC} \right] + \frac{v_0}{sR_2 C} \cdot (1 + sRC)$$

$$-1 = \left[\frac{1 + 2sRC}{sRC} \right] \left[\frac{1 + sRC}{sR_2 C} \right] \left[\frac{(1 + sRC)^2 + sRC}{(sRC)(1 + sRC)} \right] - \frac{1 + sRC}{sR_2 C}$$

$$-1 = \frac{(1 + 2sRC)(1 + 2sRC + s^2 R^2 C^2 + sRC)}{(sRC)^2 (sR_2 C)} - \frac{(1 + sRC)(sRC)^2}{(sRC)^2 (sR_2 C)}$$

$$\text{Set } s = j\omega, \text{ then } -1 = \frac{(1 + 2j\omega RC)(1 + 3j\omega RC + \omega^2 R^2 C^2)}{(-\omega^2 R^2 C^2)(j\omega R_2 C)} - \frac{(1 + j\omega RC)(-\omega^2 R^2 C^2)}{(-\omega^2 R^2 C^2)(j\omega R_2 C)}$$

The real part of the numerator must be zero.

$$1 - \omega^2 R^2 C^2 - 6\omega^2 R^2 C^2 + \omega^2 R^2 C^2 = 0$$

$$6\omega^2 R^2 C^2 = 1$$

so that

$$\omega_0 = \frac{1}{\sqrt{6RC}}$$

Condition for oscillation:

$$-1 = \frac{2j\omega RC + 3j\omega RC - 2j\omega^3 R^3 C^3 + j\omega^3 R^3 C^3}{(-\omega^3 R^2 C^2)(j\omega R_2 C)}$$

$$1 = \frac{5 - \omega^2 R^2 C^2}{(\omega RC)(\omega R_2 C)}$$

But

$$\omega = \omega_0 = \frac{1}{\sqrt{6}RC}$$

Then

$$1 = \frac{\frac{5 - \frac{1}{6}}{(RC)(R_2 C)}}{\frac{6R^2 C^2}{6R^2 C^2}} = \frac{\left(5 - \frac{1}{6}\right)(6R^2 C^2)}{RR_2 C^2}$$

$$1 = \frac{\left(\frac{29}{6}\right)(6R)}{R_2} \text{ or } \frac{R_2}{R} = 29$$

15.28

Let $R_{F1} = R_{F2} = R_{F3} \equiv R_F$

$$v_{o1} = \left(1 + \frac{R_F}{R}\right) \left(\frac{1}{1 + sRC} \right) \cdot v_o$$

$$v_{o2} = \left(1 + \frac{R_F}{R}\right) \left(\frac{1}{1 + sRC} \right) \cdot v_{o1}$$

$$\frac{v_{o3} - v_{o2}}{R} + \frac{v_{o3}}{1/sC} + \frac{v_{o3}}{R} = 0$$

$$v_{o3} \left(\frac{2}{R} + sC \right) = \frac{v_{o2}}{R}$$

$$v_{o3} = \left(\frac{1}{2 + sRC} \right) \cdot v_{o2}$$

$$v_o = -\frac{R_F}{R} \cdot v_{o3}$$

$$v_o = -\frac{R_F}{R} \left(\frac{1}{2 + sRC} \right) \left(1 + \frac{R_F}{R} \right) \left(\frac{1}{1 + sRC} \right) \left(1 + \frac{R_F}{R} \right) \left(\frac{1}{1 + sRC} \right) \cdot v_o$$

$$1 = -\frac{R_F}{R} \left(1 + \frac{R_F}{R} \right)^2 \left(\frac{1}{2 + sRC} \right) \left(\frac{1}{1 + sRC} \right) \left(\frac{1}{1 + sRC} \right)$$

Let $s = j\omega$

$$1 = -\frac{R_F}{R} \left(1 + \frac{R_F}{R} \right)^2 \left(\frac{1}{2 + j\omega RC} \right) \left(\frac{1}{1 + j\omega RC} \right) \left(\frac{1}{1 + j\omega RC} \right)$$

$$= -\frac{R_F}{R} \left(1 + \frac{R_F}{R} \right)^2 \left(\frac{1}{2 + j\omega RC} \right) \left(\frac{1}{1 + 2j\omega RC - \omega^2 R^2 C^2} \right)$$

$$= -\frac{R_F}{R} \left(1 + \frac{R_F}{R} \right)^2 \left(\frac{1}{2 + 4j\omega RC - 2\omega^2 R^2 C^2 + j\omega RC - 2\omega^2 R^2 C^2 - j\omega^3 R^3 C^3} \right)$$

$$1 = -\frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \left[\frac{1}{2 - 4\omega^2 R^2 C^2 + 5j\omega RC - j\omega^3 R^3 C^3} \right]$$

Imaginary Term must be zero

$$5j\omega_0 RC - j\omega_0^3 R^3 C^3 = 0$$

$$5 - j\omega_0^2 R^2 C^2 = 0$$

$$\omega_0 = \frac{\sqrt{5}}{RC}$$

Then

$$1 = -\frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \left[\frac{1}{2 - \frac{4R^2 C^2 - 5}{R^2 C^2}} \right]$$

$$1 = -\frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \left[\frac{1}{2 - 20} \right] = \frac{1}{18} \cdot \frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2$$

$$18 = \frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \Rightarrow \underline{\underline{\frac{R_F}{R} = 2}}$$

15.29

(a) 1st stage:

$$v_{o1} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R_V} \cdot \left(1 + \frac{R_F}{R}\right) \cdot v_o = \left(\frac{1}{1 + j\omega R_V C}\right) \left(1 + \frac{R_F}{R}\right) \cdot v_o$$

2nd stage:

$$v_{o2} = \left(\frac{1}{1 + j\omega RC}\right) \left(1 + \frac{R_F}{R}\right) \cdot v_{o1}$$

At node of C_3 :

$$\frac{v_A - v_{o2}}{R} + v_A(j\omega C) + \frac{v_A}{R} = 0 \Rightarrow v_A = \frac{v_{o2}}{2 + j\omega RC}$$

3rd stage:

$$v_o = -\frac{R_F}{R} \cdot v_A = -\frac{R_F}{R} \cdot \frac{v_{o2}}{(2 + j\omega RC)}$$

Now

$$v_o = -\frac{R_F}{R} \cdot \frac{1}{(2 + j\omega RC)} \left(\frac{1}{1 + j\omega RC} \right) \left(1 + \frac{R_F}{R}\right) \left(\frac{1}{1 + j\omega R_V C} \right) \left(1 + \frac{R_F}{R}\right) \cdot v_o$$

$$1 = -\frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \cdot \frac{1}{(2 + j\omega RC)} \cdot \frac{1}{(1 + j\omega RC)} \cdot \frac{1}{(1 + j\omega R_V C)}$$

From the denominator, we have:

$$(2 + j\omega RC)(1 + j\omega RC)(1 + j\omega R_V C) = (2 + 3j\omega RC - \omega^2 R^2 C^2)(1 + j\omega R_V C)$$

$$[(2 - \omega^2 R^2 C^2) + 3j\omega RC](1 + j\omega R_V C)$$

$$= (2 - \omega^2 R^2 C^2) + 3j\omega RC + j\omega R_V C(2 - \omega^2 R^2 C^2) - 3\omega^2 R R_V C^2$$

For oscillation, imaginary part must be zero.

$$3\omega_o RC + \omega_o R_V C (2 - \omega_o^2 R^2 C^2) = 0$$

$$3RC + 2R_V C - \omega_o^2 R_V R^2 C^3 = 0$$

So that $\omega_o = \frac{1}{RC} \sqrt{\frac{3R+2R_V}{R_V}}$

$$(b) 1 = -\frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \cdot \frac{1}{(2 - \omega_o^2 R^2 C^2 - 3\omega_o^2 R R_V C^2)}$$

Consider the term:

$$2 - \omega_o^2 (R^2 C^2 + 3R R_V C^2) = 2 - \frac{1}{R^2 C^2} \left(\frac{3R+2R_V}{R_V}\right) (R^2 C^2 + 3R R_V C^2)$$

$$= 2 - \left(\frac{3R+2R_V}{R_V}\right) \left(1 + \frac{3R_V}{R}\right) = \frac{2R R_V - (3R+2R_V)(R+3R_V)}{R R_V}$$

$$= \frac{2R R_V - (3R^2 + 9R R_V + 2R R_V + 6R_V^2)}{R R_V} = -\frac{(3R^2 + 9R R_V + 6R_V^2)}{R R_V}$$

Then

$$1 = \frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \cdot \frac{R R_V}{(3R^2 + 9R R_V + 6R_V^2)}$$

For $R = R_V$

$$1 = \frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \cdot \frac{R^2}{18R^2}$$

$$\text{Or } 18 = \frac{R_F}{R} \left(1 + \frac{R_F}{R}\right)^2 \Rightarrow \frac{R_F}{R} = 2$$

(c) For $R_V = 15 \text{ k}\Omega$,

$$f_o = \frac{1}{2\pi(25 \times 10^3)(0.001 \times 10^{-6})} \sqrt{\frac{3(25) + 2(15)}{15}} \Rightarrow f_o = 16.84 \text{ kHz}$$

For $R_V = 30 \text{ k}\Omega$,

$$f_o = \frac{1}{2\pi(25 \times 10^3)(0.001 \times 10^{-6})} \sqrt{\frac{3(25) + 2(30)}{30}} \Rightarrow f_o = 13.5 \text{ kHz}$$

So $13.5 \leq f_o \leq 16.84 \text{ kHz}$

15.30

$$(a) \frac{v_{o1} - v_o}{R} + v_{o1}(j\omega C) + \frac{v_{o1} - v_{o2}}{R} = 0$$

$$v_{o1} \left(\frac{2}{R} + j\omega C \right) = \frac{v_o}{R} + \frac{v_{o2}}{R}$$

$$\text{Then (1)} \quad v_{o1}(2 + j\omega RC) = v_o + v_{o2}$$

$$\frac{v_{o2} - v_{o1}}{R} + v_{o2}(j\omega C) + \frac{v_{o2} - v_{o3}}{R} = 0$$

$$\text{So (2)} \quad v_{o2}(2 + j\omega RC) = v_{o1} + v_{o3}$$

$$\frac{v_{o3} - v_{o2}}{R} + v_{o3}(j\omega C) + \frac{v_{o3}}{R} = 0$$

And (3) $v_{o_3}(2 + j\omega RC) = v_{o_2}$

Also (4) $v_o = -\frac{R_F}{R} \cdot v_{o_3}$

From (1) $v_{o_1}(2 + j\omega RC) = v_o + v_{o_3}(2 + j\omega RC) \Rightarrow v_{o_1} = \frac{v_o}{(2 + j\omega RC)} + v_{o_3}$

From (2) $v_{o_3}(2 + j\omega RC)^2 = v_{o_1} + v_{o_3} \Rightarrow v_{o_3}(2 + j\omega RC)^2 = 2v_{o_3} + \frac{v_o}{(2 + j\omega RC)}$

$v_{o_3}(4 + 4j\omega RC - \omega^2 R^2 C^2 - 2) = \frac{v_o}{(2 + j\omega RC)}$

$v_{o_3} = \frac{v_o}{(2 + j\omega RC)(2 - \omega^2 R^2 C^2 + 4j\omega RC)}$

Using (4) $v_o = -\frac{R_F}{R} \cdot \frac{v_o}{(2 + j\omega RC)(2 - \omega^2 R^2 C^2 + 4j\omega RC)}$

$1 = -\frac{R_F}{R} \cdot \frac{1}{[(2)(2 - \omega^2 R^2 C^2) + 8j\omega RC + j\omega RC(2 - \omega^2 R^2 C^2) - 4\omega^2 R^2 C^2]}$

For oscillation, denominator must be real, so

$8\omega_o RC + \omega_o RC(2 - \omega_o^2 R^2 C^2) = 0 \Rightarrow \omega_o = \frac{\sqrt{10}}{RC}$

(b) $1 = -\frac{R_F}{R} \cdot \frac{1}{[(2)(2 - \omega_o^2 R^2 C^2) - 4\omega_o^2 R^2 C^2]}$

$1 = -\frac{R_F}{R} \cdot \frac{1}{[4 - 2\omega_o^2 R^2 C^2 - 4\omega_o^2 R^2 C^2]} = -\frac{R_F}{R} \cdot \frac{1}{[4 - 6\left(\frac{10}{R^2 C^2}\right)R^2 C^2]} = -\frac{R_F}{R} \cdot \frac{1}{(-56)}$

Or $\frac{R_F}{R} = 56$

(c) $C = \frac{\sqrt{10}}{2\pi f_o R} = \frac{\sqrt{10}}{2\pi(22 \times 10^3)(20 \times 10^3)} \Rightarrow C = 0.00114 \mu F$

$R_F = (56)R \Rightarrow R_F = 1.12 \text{ M}\Omega$

15.31

(a) $\frac{v_{o_1} - v_o}{R_V} + v_{o_1}(j\omega C) + \frac{v_{o_1} - v_{o_2}}{R} = 0$

$v_{o_1}\left(\frac{1}{R_V} + \frac{1}{R} + j\omega C\right) = \frac{v_o}{R_V} + \frac{v_{o_2}}{R}$

$v_{o_1}\left(\frac{1}{R \| R_V} + j\omega C\right) = \frac{v_o}{R_V} + \frac{v_{o_2}}{R}$

Then (1) $v_{o_1}(1 + j\omega(R \| R_V)C) = (R \| R_V)\left(\frac{v_o}{R_V} + \frac{v_{o_2}}{R}\right)$

From Problem 15.30,

(2) $v_{o_2}(2 + j\omega RC) = v_{o_1} + v_{o_3}$

(3) $v_{o_3}(2 + j\omega RC) = v_{o_2}$

$$(4) v_o = -\frac{R_F}{R} \cdot v_{o3}$$

$$\text{From (2), } v_{o3}(2 + j\omega RC)^2 = v_{o1} + v_{o3} \Rightarrow v_{o1} = v_{o3}[(2 + j\omega RC)^2 - 1]$$

$$\text{Then (1) } v_{o3}[(2 + j\omega RC)^2 - 1][1 + j\omega(R\|R_V)C] = (R\|R_V) \left[\frac{v_o}{R_V} + \frac{v_{o3}}{R} (2 + j\omega RC) \right]$$

$$v_{o3} \left\{ [(2 + j\omega RC)^2 - 1][1 + j\omega(R\|R_V)C] - \frac{R\|R_V}{R} (2 + j\omega RC) \right\} = (R\|R_V) \cdot \frac{v_o}{R_V}$$

$$\text{Then, } v_o = -\frac{R_F}{R} \cdot \frac{(R\|R_V)}{R_V} \cdot v_o \cdot \frac{1}{\left\{ [(2 + j\omega RC)^2 - 1][1 + j\omega(R\|R_V)C] - \frac{R\|R_V}{R} (2 + j\omega RC) \right\}}$$

Consider the denominator:

$$[4 + 4j\omega RC - \omega^2 R^2 C^2 - 1][1 + j\omega(R\|R_V)C] - \frac{R\|R_V}{R} (2 + j\omega RC)$$

$$(3 - \omega^2 R^2 C^2 + 4j\omega RC)(1 + j\omega(R\|R_V)C) - \frac{R\|R_V}{R} (2 + j\omega RC)$$

For oscillation, the denominator must be real, so

$$4j\omega_o RC + j\omega_o (R\|R_V)C[3 - \omega_o^2 R^2 C^2] - \frac{R\|R_V}{R} \cdot j\omega_o RC = 0$$

$$4R + (R\|R_V)[3 - \omega_o^2 R^2 C^2] - R\|R_V = 0$$

$$4R + 2(R\|R_V) = \omega_o^2 (R\|R_V) R^2 C^2$$

$$\text{So that } \omega_o = \frac{1}{RC} \sqrt{4 \left(\frac{R}{R\|R_V} \right) + 2}$$

(b) For $R_V = 15 \text{ k}\Omega$,

$$f_o = \frac{1}{2\pi(25 \times 10^3)(0.001 \times 10^{-6})} \sqrt{\frac{4(25)}{25\|15} + 2} \Rightarrow f_o = 22.66 \text{ kHz}$$

For $R_V = 30 \text{ k}\Omega$,

$$f_o = \frac{1}{2\pi(25 \times 10^3)(0.001 \times 10^{-6})} \sqrt{\frac{4(25)}{25\|30} + 2} \Rightarrow f_o = 19.45 \text{ kHz}$$

So $19.45 \leq f_o \leq 22.66 \text{ kHz}$

15.32

a. We can write

$$v_A = \left(\frac{R_1}{R_1 + R_2} \right) v_0 \quad \text{and} \quad v_B = \left(\frac{Z_p}{Z_p + Z_s} \right) v_0$$

$$Z_p = R_B \left| \frac{1}{sC_B} \right. = \frac{R_B}{1 + sR_B C_B}$$

where

$$Z_s = R_A + \frac{1}{sC_A} = \frac{1 + sR_A C_A}{sC_A}$$

and

Setting $v_A = v_B$, we have

$$\frac{R_1}{R_1 + R_2} = \frac{\frac{R_B}{1 + sR_B C_B}}{\frac{R_B}{1 + sR_B C_B} + \frac{1 + sR_A C_A}{sC_A}}$$

$$\frac{R_1}{R_1 + R_2} = \frac{sR_B C_A}{sR_B C_A + (1 + sR_A C_A)(1 + sR_B C_B)} \quad (1)$$

To find the frequency of oscillation, set $s = j\omega$ and set the real part of the denominator on the right side of Equation (1) equal to zero.

The denominator term is

$$j\omega R_B C_A + (1 + j\omega R_A C_A)(1 + j\omega R_B C_B)$$

or

$$j\omega R_B C_A + 1 + j\omega R_A C_A + j\omega R_B C_B - \omega^2 R_A R_B C_A C_B \quad (2)$$

Then from (2), we must have

$$1 - \omega_0^2 R_A R_B C_A C_B = 0$$

or

$$f_0 = \frac{1}{2\pi\sqrt{R_A R_B C_A C_B}}$$

b. To find the condition for sustained oscillation, combine Equations (1) and (2). Then

$$\frac{R_1}{R_1 + R_2} = \frac{j\omega R_B C_A}{j\omega R_B C_A + j\omega R_A C_A + j\omega R_B C_B}$$

or

$$1 + \frac{R_2}{R_1} = 1 + \frac{R_A}{R_B} + \frac{C_B}{C_A}$$

Then

$$\frac{R_2}{R_1} = \frac{R_A}{R_B} + \frac{C_B}{C_A}$$

15.33

a. We can write

$$v_A = \left(\frac{R_1}{R_1 + R_2} \right) v_0$$

and

$$v_B = \left(\frac{R \parallel sL}{R \parallel sL + R + sL} \right) v_0$$

Setting $v_A = v_B$, we have

$$\frac{R_1}{R_1 + R_2} = \left[\frac{\frac{sRL}{R + sL}}{\frac{sRL}{R + sL} + R + sL} \right] \cdot v_0$$

$$\frac{R_1}{R_1 + R_2} = \frac{sRL}{sRL + (R + sL)^2} \quad (1)$$

To find the frequency of oscillation, set $s = j\omega$ and set the real part of the denominator on the right side of Equation (1) equal to zero.

The denominator term is:

$$j\omega RL + (R + j\omega L)^2$$

or

$$j\omega RL + R^2 + 2j\omega RL - \omega^2 L^2 \quad (2)$$

Then

$$R^2 - \omega_0^2 L^2 = 0$$

or

$$f_0 = \frac{1}{2\pi} \cdot \sqrt{\frac{R}{L}}$$

b. To find the condition for sustained oscillations, combine Equations (1) and (2).

$$\frac{R_1}{R_1 + R_2} = \frac{j\omega RL}{j\omega RL + 2j\omega RL} = \frac{1}{3}$$

Then

$$1 + \frac{R_2}{R_1} = 3$$

so that

$$\frac{R_2}{R_1} = 2$$

15.34

$$f_o = \frac{1}{2\pi RC}$$

$$RC = \frac{1}{2\pi f_o} = \frac{1}{2\pi(35 \times 10^3)} = 4.547 \times 10^{-6}$$

Let $C = 0.001 \mu F$, then $R = 4.55 \text{ k}\Omega$

$$\text{Set } \frac{R_2}{R_1} = 2$$

15.35

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.7)(0.8)} = 1.497 \text{ mA/V}$$

$$\frac{C_2}{C_1} = g_m R = (1.497)(2) = 2.993$$

$$C_2 \cong 3(0.02) = 0.06 \mu F$$

$$2\pi f_o = \frac{1}{\sqrt{L \left(\frac{C_1 C_2}{C_1 + C_2} \right)}}$$

$$\text{Then } L \left[\frac{(0.02)(0.06) \times 10^{-6}}{0.02 + 0.06} \right] = \left(\frac{1}{2\pi(350 \times 10^3)} \right)^2 \Rightarrow L = 13.8 \mu H$$

15.36

$$V_\pi = -V_0$$

$$\frac{V_0}{\left(\frac{1}{sC_2}\right)} + \frac{V_0}{R_L} + \frac{V_0 - V_1}{\left(\frac{1}{sC_1}\right)} = g_m V_\pi = -g_m V_0$$

$$V_0 \left[sC_2 + sC_1 + \frac{1}{R_L} + g_m \right] = V_1(sC_1) \quad (1)$$

$$\frac{V_1}{sL} + \frac{V_0 - V_1}{\left(\frac{1}{sC_1}\right)} + g_m V_\pi = 0 \quad (2)$$

$$V_1 \left(\frac{1}{sL} + sC_1 \right) = V_0(sC_1 + g_m)$$

$$V_1 = \frac{V_0(sC_1 + g_m)}{\left(\frac{1}{sL} + sC_1 \right)}$$

Then

$$V_0 \left[s(C_1 + C_2) + \frac{1}{R_L} + g_m \right] = \frac{V_0(sC_1)(sC_1 + g_m)}{\left(\frac{1}{sL} + sC_1 \right)}$$

$$\left[s(C_1 + C_2) + \frac{1}{R_L} + g_m \right] \left(\frac{1}{sL} + sC_1 \right) = sC_1(sC_1 + g_m)$$

$$\frac{C_1 + C_2}{L} + s^2 C_1(C_1 + C_2) + \frac{1}{sR_L L} + \frac{sC_1}{R_L} + sg_m C_1 + \frac{g_m}{sL} = s^2 C_1^2 + sg_m C_1$$

$$\frac{C_1 + C_2}{L} + s^2 C_1 C_2 + \frac{1}{sR_L L} + \frac{sC_1}{R_L} + \frac{g_m}{sL} = 0$$

Set $s = j\omega$

$$\frac{C_1 + C_2}{L} - \omega^2 C_1 C_2 + \frac{1}{j\omega R_L L} + \frac{j\omega C_1}{R_L} + \frac{g_m}{j\omega L} = 0$$

Then

$$\omega^2 = \frac{C_1 + C_2}{C_1 C_2 L} \Rightarrow \omega_0 = \sqrt{\frac{C_1 + C_2}{C_1 C_2 L}}$$

and

$$\frac{g_m}{\omega L} + \frac{1}{\omega R_L L} = \frac{\omega C_1}{R_L}$$

Then

$$\frac{g_m}{L} + \frac{1}{R_L L} = \frac{(C_1 + C_2)C_1}{C_1 C_2 L R_L}$$

$$g_m + \frac{1}{R_L} = \frac{C_1 + C_2}{C_2 R_L}$$

$$g_m R_L + 1 = \frac{C_1}{C_2} + 1 \text{ or } \underline{\underline{\frac{C_1}{C_2} = g_m R_L}}$$

15.37

a.

$$\frac{V_0}{sL_1} + \frac{V_0}{R} + g_m V_\pi + \frac{V_0}{\frac{1}{sC} + sL_2} = 0 \quad (1)$$

$$V_\pi = \left(\frac{sL_2}{\frac{1}{sC} + sL_2} \right) V_0 \quad (2)$$

Then

$$\begin{aligned} V_0 \left\{ \frac{1}{sL_1} + \frac{1}{R} + \frac{sC}{1+s^2 L_2 C} + \frac{g_m (s^2 L_2 C)}{1+s^2 L_2 C} \right\} &= 0 \\ \left\{ \frac{R(1+s^2 L_2 C) + (sL_1)(1+s^2 L_2 C)}{(sRL_1)(1+s^2 L_2 C)} + \frac{s^2 RL_1 C + g_m (sRL_1)(s^2 L_2 C)}{(sRL_1)(1+s^2 L_2 C)} \right\} &= 0 \end{aligned}$$

Set $s = j\omega$. Both real and imaginary parts of the numerator must be zero.

$$R(1-\omega^2 L_2 C) + j\omega L_1(1-\omega^2 L_2 C) - \omega^2 RL_1 C + (j\omega g_m RL_1)(-\omega^2 L_2 C) = 0$$

Real part:

$$R(1-\omega^2 L_2 C) - \omega^2 RL_1 C = 0$$

$$R = \omega^2 RC(L_1 + L_2)$$

or

$$\omega_0 = \frac{1}{\sqrt{C(L_1 + L_2)}}$$

b. Imaginary part:

$$j\omega L_1(1-\omega^2 L_2 C) - j\omega g_m RL_1(\omega^2 L_2 C) = 0$$

$$L_1 = \omega^2 L_1 L_2 C + g_m RL_1(\omega^2 L_2 C)$$

$$\text{Now } \omega^2 = \frac{1}{(L_1 + L_2)}$$

$$1 = \frac{1}{C(L_1 + L_2)} [L_2 C + g_m RL_2 C]$$

$$1 = \frac{L_2}{L_1 + L_2} (1 + g_m R) \Rightarrow \frac{L_1}{L_2} = (1 + g_m R)$$

or

$$\underline{\frac{L_1}{L_2} = g_m R}$$

15.38

$$(a) (1) g_m V_\pi + \frac{V_C}{R} + \frac{V_C}{j\omega L_1} + (V_C - V_\pi)(j\omega C) = 0$$

$$(2) V_\pi = (V_C - V_\pi)(j\omega C)(j\omega L_2)$$

$$V_\pi (1 - \omega^2 CL_2) = -V_C (\omega^2 CL_2)$$

$$V_\pi = \frac{V_C(\omega^2 L_2 C)}{\omega^2 L_2 C - 1}$$

$$\text{Then (1)} V_\pi(g_m - j\omega C) + V_C \left(\frac{1}{R} + \frac{1}{j\omega L_1} + j\omega C \right) = 0$$

$$V_C \cdot \frac{(\omega^2 L_2 C)(g_m - j\omega C)}{\omega^2 L_2 C - 1} + V_C \cdot \left(\frac{1}{R} - \frac{j}{\omega L_1} + j\omega C \right) = 0$$

$$\frac{g_m \omega^2 L_2 C}{\omega^2 L_2 C - 1} + \frac{1}{R} + j \left[\omega C - \frac{1}{\omega L_1} - \frac{\omega C (\omega^2 L_2 C)}{\omega^2 L_2 C - 1} \right] = 0$$

Set the imaginary part equal to zero.

$$\frac{\omega^2 L_1 C - 1}{\omega L_1} = \frac{\omega C (\omega^2 L_2 C)}{\omega^2 L_2 C - 1}$$

$$(\omega^2 L_1 C - 1)(\omega^2 L_2 C - 1) = (\omega^2 L_1 C)(\omega^2 L_2 C)$$

$$\omega^4 L_1 L_2 C^2 - \omega^2 L_2 C - \omega^2 L_1 C + 1 = \omega^4 L_1 L_2 C^2$$

$$\omega_o^2 (L_1 + L_2) C = 1$$

$$\text{So } \omega_o = \frac{1}{\sqrt{(L_1 + L_2)C}}$$

(b) Set the real part equal to zero,

$$\frac{g_m \omega_o^2 L_2 C}{\omega_o^2 L_2 C - 1} + \frac{1}{R} = 0$$

$$g_m R \omega_o^2 L_2 C = 1 - \omega_o^2 L_2 C$$

$$\omega_o^2 L_2 C (g_m R + 1) = 1$$

$$\text{Then } \frac{L_2 C (g_m R + 1)}{(L_1 + L_2)C} = 1 \Rightarrow g_m R + 1 = \frac{(L_1 + L_2)C}{L_2 C}$$

$$\text{We find } g_m R = \frac{L_1}{L_2}$$

$$(c) C = \frac{1}{(2\pi f_o)^2 (L_1 + L_2)} = \frac{1}{[2\pi(750 \times 10^3)]^2 (100 \times 10^{-6})} \Rightarrow C = 450 \text{ pF}$$

$$g_m R = \frac{L_1}{L_2} = 1 \Rightarrow R = \frac{1}{g_m} = \frac{1}{30} \Rightarrow R = 33.3 \Omega$$

15.39

$$\frac{v_0 - v_1}{\left(\frac{1}{sC}\right)} = \frac{v_1}{R} + \frac{v_1 - v_B}{R} \quad (1)$$

and

$$\frac{v_B}{\left(\frac{1}{sC}\right)} + \frac{v_B - v_1}{R} = 0 \quad (2)$$

or

$$v_B \left(sC + \frac{1}{R} \right) = \frac{v_1}{R} \Rightarrow v_1 = v_B (1 + sRC)$$

From (1)

$$v_0 (sC) = v_1 \left(sC + \frac{2}{R} \right) - \frac{v_B}{R}$$

or

$$v_0 (sRC) = v_B (1 + sRC)(2 + sRC) - v_B = v_B [(1 + sRC)(2 + sRC) - 1]$$

Now

$$T(s) = \left(1 + \frac{R_2}{R_1} \right) \left[\frac{sRC}{(1 + sRC)(2 + sRC) - 1} \right] = \left(1 + \frac{R_2}{R_1} \right) \left[\frac{sRC}{2 + 3sRC + s^2 R^2 C^2 - 1} \right]$$

or

$$T(s) = \left(1 + \frac{R_2}{R_1} \right) \left[\frac{sRC}{s^2 R^2 C^2 + 3sRC + 1} \right]$$

$$T(j\omega) = \left(1 + \frac{R_2}{R_1} \right) \left[\frac{j\omega RC}{1 - \omega^2 R^2 C^2 + 3j\omega RC} \right]$$

Frequency of oscillation:

$$\underline{f_0 = \frac{1}{2\pi RC}}$$

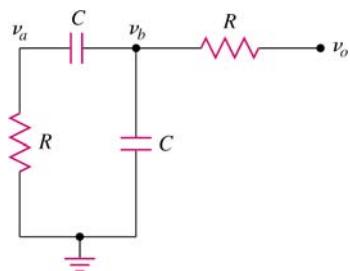
Condition for oscillation:

$$1 = \left(1 + \frac{R_2}{R_1} \right) \left[\frac{j\omega RC}{3j\omega RC} \right]$$

or

$$\underline{\frac{R_2}{R_1} = 2}$$

15.40



$$\begin{aligned}
 & \frac{v_b - v_o}{R} + \frac{v_b}{\frac{1}{sC}} + \frac{v_b - v_a}{\frac{1}{sC}} = 0 \\
 (1) \quad & \frac{v_b - v_o}{R} + 2v_b \cdot sC - v_a \cdot sC = 0 \\
 & \frac{V_a - V_b}{\frac{1}{sC}} + \frac{V_a}{R} = 0 \\
 (2) \quad & V_a \left(sC + \frac{1}{R} \right) = v_b \cdot sC \Rightarrow v_b = v_a \left(\frac{1 + sRC}{sRC} \right)
 \end{aligned}$$

From (1)

$$v_b \left(\frac{1}{R} + 2sC \right) = \frac{v_o}{R} + v_a \cdot sC$$

Substitute (2) into (1)

$$v_a \left(\frac{1 + sRC}{sRC} \right) \left(\frac{1 + 2sRC}{R} \right) = \frac{v_o}{R} + v_a \cdot sC$$

$$v_a \left[\frac{(1 + sRC)(1 + 2sRC)}{(sRC) \cdot R} - sC \right] = \frac{v_o}{R}$$

$$v_a \left[\frac{(1 + sRC)(1 + 2sRC)}{sRC} - sRC \right] = v_o$$

$$\frac{v_o}{v_a} = \frac{(1 + sRC)(1 + 2sRC) - s^2 R^2 C^2}{sRC}$$

$$\frac{v_a}{v_o} = \frac{sRC}{1 + 3sRC + 2(sRC)^2 - s^2 R^2 C^2}$$

$$\frac{v_a}{v_o} = \frac{sRC}{1 + 3sRC + (sRC)^2}$$

$$T(s) = \left(1 + \frac{R_2}{R_1} \right) \cdot \frac{sRC}{1 + 3sRC + (sRC)^2}$$

$$T(j\omega) = \left(1 + \frac{R_2}{R_1} \right) \left[\frac{j\omega RC}{1 - \omega^2 R^2 C^2 + 3j\omega RC} \right]$$

So $1 - \omega_0^2 R^2 C^2 = 0$

$$\text{So } \frac{f_o}{2\pi RC}$$

$$\text{Also } 1 = \left(1 + \frac{R_2}{R_1} \right) \left(\frac{1}{3} \right)$$

$$\text{So } \frac{\frac{R_2}{R_1}}{3} = 2$$

15.41

$$\frac{v_0 - v_1}{sL} = \frac{v_1}{R} + \frac{v_1 - v_B}{R} \quad (1)$$

$$v_B = \left(\frac{sL}{R + sL} \right) v_1 \quad (2)$$

or

$$v_1 = \left(\frac{R + sL}{sL} \right) v_B$$

Then

$$\frac{v_0}{sL} = v_1 \left(\frac{1}{sL} + \frac{2}{R} \right) - \frac{v_B}{R}$$

or

$$\begin{aligned} \frac{v_0}{sL} &= \left(\frac{R + sL}{sL} \right) \left(\frac{1}{sL} + \frac{2}{R} \right) v_B - \frac{v_B}{R} \\ &= v_B \left\{ \left(\frac{R + sL}{sL} \right) \left(\frac{R + 2sL}{sRL} \right) - \frac{1}{R} \right\} \end{aligned} \quad (1)$$

Then

$$v_B = \frac{v_0}{sL} \cdot \frac{1}{\left\{ \frac{(R + sL)(R + 2sL) - (sL)^2}{(sL)(sRL)} \right\}}$$

Now

$$T(s) = \left(1 + \frac{R_2}{R_1} \right) \left(\frac{sRL}{R^2 + 3sRL + 2s^2L^2 - s^2L^2} \right)$$

or

$$T(s) = \left(1 + \frac{R_2}{R_1} \right) \left(\frac{sRL}{s^2L^2 + 3sRL + R^2} \right)$$

And

$$T(j\omega) = \left(1 + \frac{R_2}{R_1} \right) \left(\frac{j\omega RL}{R^2 - \omega^2 L^2 + 3j\omega RL} \right)$$

$$f_0 = \frac{R}{2\pi L}$$

Frequency of oscillation: $\frac{R}{2\pi L}$

Condition for oscillation:

$$1 = \left(1 + \frac{R_2}{R_1} \right) \left(\frac{1}{3} \right)$$

or

$$\frac{R_2}{R_1} = 2$$

15.42

$$v_I = -\frac{R_2}{R_1} \cdot V_{REF} = -\frac{(R_{2f} + R_{2v})}{R_1} \cdot V_{REF}$$

Set $R_{2f} = 20 \text{ k}\Omega$

For $v_I = -2 \text{ V}$, set $R_{2v} = 0$

$$-2 = -\frac{20}{R_1}(5) \Rightarrow R_1 = 50 \text{ k}\Omega$$

Then

$$-4 = -\frac{(20 + R_{2v})}{50}(5) \Rightarrow R_{2v} = 20 \text{ k}\Omega$$

15.43

For $v_O = 10 \text{ V}$, $v_I = -5 \text{ V}$

$$i = 0.2 \text{ mA} = \frac{10 - (-5)}{R_1 + R_2} \Rightarrow R_1 + R_2 = 75 \text{ k}\Omega$$

$$(V_{TH} - V_{TL}) = \frac{R_1}{R_2} (V_H - V_L)$$

$$0.4 = \frac{R_1}{R_2} [10 - (-10)] \Rightarrow R_2 = 50R_1$$

$$R_1 + R_2 = R_1 + 50R_1 = 75 \Rightarrow R_1 = 1.47 \text{ k}\Omega, R_2 = 73.53 \text{ k}\Omega$$

15.44

$$\begin{aligned} \text{(a)} \quad V_{TH} - V_{TL} &= \left(\frac{R_1}{R_1 + R_2} \right) (V_H - V_L) \\ 0.2 &= \left(\frac{R_1}{R_1 + R_2} \right) [9 - (-9)] = 18 \left(\frac{R_1}{R_1 + R_2} \right) \end{aligned}$$

Set $R_1 = 2 \text{ k}\Omega$

$$\text{Then } 2 + R_2 = \frac{(18)(2)}{0.2} \Rightarrow R_2 = 178 \text{ k}\Omega$$

$$\text{(b)} \quad i = \frac{9}{R_1 + R_2} = \frac{9}{2 + 178} \Rightarrow i = 50 \mu\text{A}$$

15.45

$$\text{(a)} \quad V_{TH} = \left(\frac{R_1}{R_1 + R_2} \right) \cdot V_H = \left(\frac{2}{2 + 48} \right) (10) = 0.4 \text{ V}$$

$$V_{TL} = \left(\frac{R_1}{R_1 + R_2} \right) \cdot V_L = \left(\frac{2}{2 + 48} \right) (-10) = -0.4 \text{ V}$$

(b) For $33.33 \leq t \leq 41.67 \text{ ms}$, $\sin[2\pi(60)t] \Rightarrow$ positive half cycle

At $v_I = 0$, $v_O = +10 \text{ V}$

At $v_I = +0.4 = 10 \sin[2\pi(60)t_1] \Rightarrow t_1 = +0.106 + 33.333 = 33.439 \text{ ms}$

So, for $33.333 \leq t \leq 33.439 \text{ ms}$, $v_O = +10 \text{ V}$

$$33.439 \leq t \leq 41.77 \text{ ms}, v_o = -10 \text{ V}$$

$$41.77 \leq t \leq 50 \text{ ms}, v_o = +10 \text{ V}$$

15.46

a. Upper crossover voltage when $v_0 = +V_p$,

Now

$$v_B = \left(\frac{R_1}{R_1 + R_2} \right) (+V_p)$$

and

$$v_A = \left(\frac{R_A}{R_A + R_B} \right) V_{REF} + \left(\frac{R_B}{R_A + R_B} \right) V_{TH}$$

$v_A = v_B$ so that

$$\left(\frac{R_1}{R_1 + R_2} \right) V_p = \left(\frac{R_A}{R_A + R_B} \right) V_{REF} + \left(\frac{R_B}{R_A + R_B} \right) V_{TH}$$

or

$$V_{TH} = \left(\frac{R_A + R_B}{R_1 + R_2} \right) \left(\frac{R_1}{R_B} \right) V_p - \left(\frac{R_A}{R_B} \right) V_{REF}$$

Lower crossover voltage when $v_0 = -V_p$

So

$$V_{TL} = - \left(\frac{R_A + R_B}{R_1 + R_2} \right) \left(\frac{R_1}{R_B} \right) V_p - \left(\frac{R_A}{R_B} \right) V_{REF}$$

b.

$$V_{TH} = \left(\frac{10+20}{5+20} \right) \left(\frac{5}{20} \right) (10) - \left(\frac{10}{20} \right) (2)$$

or

$$\underline{V_{TH} = 2 \text{ V}}$$

and

$$V_{TL} = - \left(\frac{10+20}{5+20} \right) \left(\frac{5}{20} \right) (10) - 1 \Rightarrow \underline{V_{TL} = -4 \text{ V}}$$

15.47

a.

$$\frac{v_B}{R_1} = \frac{V_{REF} - v_B}{R_3} + \frac{v_0 - v_B}{R_2}$$

$$v_B \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) = \frac{V_{REF}}{R_3} + \frac{v_0}{R_2}$$

$$V_{TH} = v_B \text{ when } v_0 = +V_p \text{ and } V_{TL} = v_B \text{ when } v_0 = -V_p$$

So

$$V_{TH} = \frac{\frac{V_{REF}}{R_3} + \frac{V_P}{R_2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)}$$

and

$$V_{TL} = \frac{\frac{V_{REF}}{R_3} - \frac{V_P}{R_2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)}$$

b.

$$V_s = \frac{V_{REF}}{R_3 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)}$$

$$-5 = \frac{-10}{10 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{10} \right)}$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{5} - \frac{1}{10} = 0.10$$

$$\Delta V_T = V_{TH} - V_{TL} = \frac{\frac{2V_P}{R_2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)}$$

$$0.2 = \frac{2(12)}{R_2(0.10 + 0.10)}$$

So $\frac{R_2}{R_1} = 600 \text{ k}\Omega$

Then

$$\frac{1}{R_1} + \frac{1}{R_2} = 0.10$$

$$\frac{1}{R_1} + \frac{1}{600} = 0.10 \Rightarrow R_1 = 10.17 \text{ k}\Omega$$

c.

$$V_{TH} = -5 + 0.1 = -4.9$$

$$V_{TL} = -5 - 0.1 = -5.1$$

15.48

- a. If the saturated output voltage is $|V_p| < 6.2 \text{ V}$, then the circuit behaves as a comparator where $|v_o| < 6.2 \text{ V}$.

If the saturated output voltage is $|V_p| > 6.2 \text{ V}$, the output will flip to either $+V_p$ or $-V_p$ and the input has no control.

- b. Same as part (a) except the curve at $v_i \approx 0$ will have a finite slope.
c. Circuit works as a comparator as long as $v_{o1} < 8.7 \text{ V}$ and $v_{o2} > -3.7 \text{ V}$. Otherwise the input has no control.

15.49

(a) When $v_o = 0$, $v_+ = V_s = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{REF}$

When $v_o = V_H$, $v_+ = V_{TH}$

$$\frac{V_{REF} - V_{TH}}{R_1} = \frac{V_{TH} - V_H}{R_2}$$

$$\frac{V_{REF}}{R_1} + \frac{V_H}{R_2} = V_{TH} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = V_{TH} \left(\frac{R_1 + R_2}{R_1 R_2} \right)$$

$$V_{TH} = V_{REF} \left(\frac{R_2}{R_1 + R_2} \right) + V_H \left(\frac{R_1}{R_1 + R_2} \right) = V_s + V_H \left(\frac{R_1}{R_1 + R_2} \right)$$

$$V_{TL} = V_s + V_L \left(\frac{R_1}{R_1 + R_2} \right)$$

(b) $V_s = -1.75 \text{ V}$, $R_1 = 4 \text{ k}\Omega$

$$V_{TH} = -1.5 = -1.75 + \left(12 \left(\frac{4}{4 + R_2} \right) \right) \Rightarrow R_2 = 188 \text{ k}\Omega$$

$$-1.75 = \left(\frac{188}{188 + 4} \right) \cdot V_{REF} \Rightarrow V_{REF} = -1.787 \text{ V}$$

(c) (i) For $v_o = 12 \text{ V}$, $|i| = \frac{12 - (-1.787)}{R_1 + R_2} = \frac{13.787}{4 + 188} \Rightarrow |i| = 71.8 \mu \text{A}$

(ii) For $v_o = -12 \text{ V}$, $|i| = \frac{12 - 1.787}{192} \Rightarrow |i| = 53.2 \mu \text{A}$

15.50

a. Switching point when $v_0 = 0$.

Now

$$v_+ = V_{REF} = \left(\frac{R_2}{R_1 + R_2} \right) v_I \quad \text{where } v_I = V_s.$$

Then

$$V_s = \left(\frac{R_1 + R_2}{R_2} \right) V_{REF} = \left(1 + \frac{R_1}{R_2} \right) V_{REF}$$

Now upper crossover voltage for v_1 occurs when $v_0 = V_L$ and $v_+ = V_{REF}$. Then

$$\frac{V_{TH} - V_{REF}}{R_1} = \frac{V_{REF} - V_L}{R_2}$$

$$\text{or } V_{TH} = -\frac{R_1}{R_2} \cdot V_L + V_{REF} \left(1 + \frac{R_1}{R_2} \right)$$

$$\text{or } V_{TH} = V_s - \frac{R_1}{R_2} \cdot V_L$$

Lower crossover voltage for v_I occurs when $v_0 = V_H$ and $v_I = V_{REF}$. Then

$$\frac{V_H - V_{REF}}{R_2} = \frac{V_{REF} - V_{TL}}{R_1}$$

$$\text{or } V_{TL} = -\frac{R_1}{R_2} \cdot V_H + V_{REF} \left(1 + \frac{R_1}{R_2} \right)$$

$$\text{or } V_{TL} = V_s - \frac{R_1}{R_2} \cdot V_H$$

$$\text{b. For } V_{TH} = -1 \text{ and } V_{TL} = -2, V_s = -1.5 \text{ V. Then } V_{TL} = V_s - \frac{R_1}{R_2} \cdot V_H \Rightarrow -2 = -1.5 - \frac{R_1}{20} (12)$$

$$\text{so that } \frac{R_1 = 0.833 \text{ k}\Omega}{}$$

Now

$$V_s = \left(1 + \frac{R_1}{R_2} \right) V_{REF}$$

$$-1.5 = \left(1 + \frac{0.833}{20} \right) V_{REF}$$

which gives

$$\underline{V_{REF} = -1.44 \text{ V}}$$

15.51

$$(a) V_{TH} = \left(\frac{R_1}{R_1 + R_3} \right) \cdot V_\gamma = \left(\frac{25}{25 + 75} \right) (0.7) = 0.175 \text{ V}$$

$$V_{TL} = \left(\frac{R_1}{R_1 + R_3} \right) (-V_\gamma) = -0.175 \text{ V}$$

$$(c) (i) v_I = 2 \text{ V}, v_O = -0.7 \text{ V}$$

$$I_{D1} = 0$$

$$I_{R2} = -\left(\frac{-0.7 - (-10)}{20} \right) = -0.465 \text{ mA}$$

$$I_{R3} = +\left(\frac{-0.175 - (-0.7)}{75} \right) \Rightarrow I_{R3} = 7 \mu \text{ A}$$

$$I_{R3} + I_{R2} + I_{D2} = 0 \Rightarrow I_{D2} = -I_{R3} - I_{R2} = -0.007 - (-0.465) = 0.458 \text{ mA}$$

$$(ii) v_I = -2 \text{ V}, v_O = +0.7 \text{ V}$$

$$I_{D2} = 0$$

$$I_{R2} = \left(\frac{10 - 0.7}{20} \right) = 0.465 \text{ mA}$$

$$I_{R3} = \left(\frac{0.175 - (0.7)}{75} \right) \Rightarrow I_{R3} = -7 \mu \text{ A}$$

$$I_{D1} = I_{R2} + I_{R3} = 0.465 - 0.007 = 0.458 \text{ mA}$$

15.52

(a) $V_\gamma + V_Z = 0.7 + 5.6 = 6.3 \text{ V}$

$$\frac{V_{TH}}{R_1} = \frac{0 - (-6.3)}{R_2} \Rightarrow V_{TH} = \frac{R_1}{R_2}(6.3)$$

$$V_{TL} = -\frac{R_1}{R_2}(6.3)$$

$$V_{TH} - V_{TL} = 0.6 = \frac{R_1}{R_2}[6.3 - (-6.3)] = (12.6)\frac{R_1}{R_2}$$

$$\text{Then } \frac{R_2}{R_1} = 21, \text{ Set } R_1 = 4 \text{ k}\Omega, \text{ then } R_2 = 84 \text{ k}\Omega$$

(b) Maximum current in R_2 ,

$$i_{R2} = \frac{6.3}{R_2} = \frac{6.3}{84} = 0.075 \text{ mA}$$

$$i_R = \frac{10 - 6.3}{R} = 0.8 + 0.075 = 0.875 \text{ mA}$$

$$R = \frac{10 - 6.3}{0.875} = 4.23 \text{ k}\Omega$$

15.53

a.

$$v_0 = V_{REF} + 2V_\gamma$$

$$5 = V_{REF} + 2(0.7)$$

or

$$\underline{V_{REF} = 3.6 \text{ V}}$$

b.

$$V_{TH} = \left(\frac{R_1}{R_1 + R_2} \right) (V_{REF} + 2V_\gamma)$$

$$0.5 = \left(\frac{R_1}{R_1 + R_2} \right) (5)$$

$$\text{or } 1 + \frac{R_2}{R_1} = 10 \Rightarrow \frac{R_2}{R_1} = 9$$

For example, let $R_2 = 90 \text{ k}\Omega$ and $R_1 = 10 \text{ k}\Omega$

c. For $v_I = 10 \text{ V}$, and v_0 is in its low state. D_1 is on and D_2 is off.

$$\frac{v_1 - (v_1 + 0.7)}{100} + \frac{V_{REF} - v_1}{1} = \frac{v_1 - v_0}{1}$$

For $v_1 = -0.7$, then

$$\frac{10 - 0}{100} + \frac{3.6 - (-0.7)}{1} = \frac{-0.7 - v_0}{1}$$

or

$$\underline{v_0 = -5.1 \text{ V}}$$

15.54

For $v_0 = \text{High} = (V_{REF} + 2V_\gamma)$. Then switching point is when.

$$v_I = v_B = \left(\frac{R_1}{R_1 + R_2} \right) v_0$$

$$\text{or } V_{TH} = \left(\frac{R_1}{R_1 + R_2} \right) (V_{REF} + 2V_\gamma)$$

Lower switching point is when

$$v_I = v_B = \left(\frac{R_1}{R_1 + R_2} \right) v_0 \quad \text{and} \quad v_0 = -(V_{REF} + 2V_\gamma)$$

so

$$V_{TL} = -\left(\frac{R_1}{R_1 + R_2} \right) (V_{REF} + 2V_\gamma)$$

15.55

By symmetry, inverting terminal switches about zero.

Now, for v_0 low, upper diode is on.

$$V_{REF} - v_1 = v_1 - v_0$$

$$v_0 = 2v_1 - V_{REF} \quad \text{where } v_1 = -V_\gamma$$

so

$$\underline{v_0 = -(V_{REF} + 2V_\gamma)}$$

Similarly, in the high state

$$\underline{v_0 = (V_{REF} + 2V_\gamma)}$$

Switching occurs when non-inverting terminal is zero.

So for v_0 low.

$$\frac{V_{TH} - 0}{R_1} = \frac{0 - [-(V_{REF} + 2V_\gamma)]}{R_2}$$

$$\text{or } V_{TH} = \frac{R_1}{R_2} \cdot (V_{REF} + 2V_\gamma)$$

By symmetry

$$V_{TL} = -\frac{R_1}{R_2} \cdot (V_{REF} + 2V_\gamma)$$

15.56

$$(a) \quad v_+ = \left(\frac{R_1}{R_1 + R_2} \right) \cdot v_o = \left(\frac{10}{10 + 20} \right) (5) = 1.667 \text{ V}$$

$$v_x = 5 + (-1.667 - 5) \exp\left(-\frac{t}{\tau_x}\right) = 5 - 6.667 \exp\left(-\frac{t}{\tau_x}\right), \text{ for } 0 < t < t_1$$

$$v_x = -5 + (1.667 - (-5)) \exp\left(-\left(\frac{t-t_1}{\tau_x}\right)\right) = -5 + 6.667 \exp\left(-\frac{(t-t_1)}{\tau_x}\right), \text{ for } t_1 < t < T$$

$$(b) \quad 1.667 = 5 - 6.667 \exp\left(-\frac{t_1}{\tau_x}\right)$$

$$6.667 \exp\left(-\frac{t_1}{\tau_x}\right) = 3.333$$

$$\exp\left(+\frac{t_1}{\tau_x}\right) = 2 \Rightarrow t_1 = \tau_x \ln(2)$$

By symmetry, $T - t_1 = \tau_x \ln(2)$

Then $T = 2\tau_x \ln(2)$

$$f = \frac{1}{T} = \frac{1}{2\tau_x \ln(2)}, \quad \tau_x = R_x C_x = (40 \times 10^3)(0.02 \times 10^{-6}) = 8 \times 10^{-4} \text{ s}$$

$$f = \frac{1}{2(8 \times 10^{-4}) \ln(2)} = 902 \text{ Hz}$$

Duty cycle = 50%

15.57

$$(a) \quad v_+(+) = \left(\frac{R_1}{R_1 + R_2}\right)(5) = \left(\frac{10}{10 + 20}\right)(5) = 1.667 \text{ V}$$

$$v_+(-) = \left(\frac{R_1}{R_1 + R_2}\right)(-10) = \left(\frac{10}{10 + 20}\right)(-10) = -3.333 \text{ V}$$

$$v_x = 5 + (-3.333 - 5) \exp\left(-\frac{t}{\tau_x}\right) = 5 - 8.333 \exp\left(-\frac{t}{\tau_x}\right), \quad \text{for } 0 < t < t_1$$

$$v_x = -10 + (1.667 - (-10)) \exp\left(-\frac{(t-t_1)}{\tau_x}\right) = -10 + 11.667 \exp\left(-\frac{(t-t_1)}{\tau_x}\right), \quad \text{for } t_1 < t < T$$

$$(b) \quad 1.667 = 5 - 8.333 \exp\left(-\frac{t_1}{\tau_x}\right)$$

$$8.333 \exp\left(\frac{-t_1}{\tau_x}\right) = 3.333 \Rightarrow \exp\left(+\frac{t_1}{\tau_x}\right) = 2.5$$

$$t_1 = \tau_x \ln(2.5)$$

$$\text{Also } -3.333 = -10 + 11.667 \exp\left(-\frac{(T-t_1)}{\tau_x}\right)$$

$$11.667 \exp\left(-\frac{(T-t_1)}{\tau_x}\right) = 6.667 \Rightarrow \exp\left(+\frac{(T-t_1)}{\tau_x}\right) = 1.75$$

$$T - t_1 = \tau_x \ln(1.75)$$

$$\text{Now } T = \tau_x [\ln(2.5) + \ln(1.75)] = \tau_x \ln[(2.5)(1.75)]$$

$$f = \frac{1}{T}, \quad \tau_x = R_x C_x = 8 \times 10^{-4} \text{ s}$$

$$f = \frac{1}{(8 \times 10^{-4}) \ln[(2.5)(1.75)]} = 847 \text{ Hz}$$

15.58

$$f = \frac{1}{2.2R_x C_x}$$

$$R_x C_x = \frac{1}{2.2f} = \frac{1}{(2.2)(12 \times 10^3)}$$

$$R_x C_x = 3.788 \times 10^{-5}$$

$$R_x = 56 \text{ K}$$

For example, Let $C_x = 680 \text{ pF}$

Within $\frac{1}{2}$ of 1% of design specification.

15.59

$$t_1 = 1.1R_x C_x = (1.1)(10^4)(0.1 \times 10^{-6}) \Rightarrow t_1 = 1.1 \text{ ms}$$

$$0 < t < t_1, v_Y = 10(1 - e^{-t/\tau_Y})$$

$$\begin{aligned} \tau_Y &= R_Y C_Y = (2 \times 10^3)(0.02 \times 10^{-6}) \\ &= 4 \times 10^{-5} \text{ s} \end{aligned}$$

$$\text{Now } \frac{t_1}{\tau_Y} = 2.75$$

$\Rightarrow C_Y$ completely charges during each cycle.

15.60

a. Switching voltage

$$v_X = \left(\frac{R_1 + R_3}{R_1 + R_3 + R_2} \right) \cdot V_P = \left(\frac{10 + 10}{10 + 10 + 10} \right) (\pm 10)$$

$$\text{So } v_X = \pm 6.667 \text{ V}$$

Using Equation (15.83(a))

$$v_X = V_P + \left(-\frac{2}{3}V_P - V_P \right) e^{-t_1/\tau_X} = \frac{2}{3}V_P$$

$$\text{Then } 1 - \frac{5}{3} \cdot e^{-t_1/\tau_X} = \frac{2}{3}$$

$$\frac{1}{3} = \frac{5}{3} \cdot e^{-t_1/\tau_X} \text{ or } t_1 = \tau_X \ln(5)$$

$$t_1 = \frac{T}{2} = \frac{1}{2f} = \frac{1}{2(500)} \Rightarrow t_1 = 0.001 \text{ s}$$

$$10^{-3} = \tau_X \ln(5) \Rightarrow \tau_X = 6.21 \times 10^{-4} = R_X (0.01 \times 10^{-6})$$

$$\text{So } R_X = 62.1 \text{ k}\Omega$$

b. Switching voltage

$$\begin{aligned} v_X &= \left(\frac{R_1}{R_1 + R_3 + R_2} \right) (\pm V_P) \\ &= \left(\frac{10}{10 + 10 + 10} \right) (\pm V_P) = \frac{1}{3} \cdot (\pm V_P) \end{aligned}$$

Using Equation (15.83(a))

$$v_x = V_p + \left(-\frac{1}{3}V_p - V_p \right) e^{-t_1/\tau_x} = \frac{1}{3}V_p$$

Then $1 - \frac{4}{3}e^{-t_1/\tau_x} = \frac{1}{3}$

$$\frac{2}{3} = \frac{4}{3} \cdot e^{-t_1/\tau_x}$$

$$t_1 = \tau_x \ln(2) = (6.21 \times 10^{-4}) \ln(2) = 4.30 \times 10^{-4} \text{ s}$$

$$T = 2t_1 = 8.6 \times 10^{-4} \text{ s}$$

$$f = \frac{1}{T} \Rightarrow f = 1.16 \text{ kHz}$$

15.61

(a) $\beta = \frac{R_1}{R_1 + R_2} = \frac{20}{20+12} = 0.625$

$$T = 250 \times 10^{-6} = \tau_x \ln \left[\frac{1 + \frac{0.7}{10}}{1 - 0.625} \right] = \tau_x (1.0485)$$

So $\tau_x = 2.384 \times 10^{-4} = R_x C_x$

Set $C_x = 0.01 \mu\text{F}$, then $R_x = 23.84 \text{ k}\Omega$

(b) For $t < 0$, $v_x = 0.7 \text{ V}$

So v_y must be $< v_x \Rightarrow v_i < 0$

(c) $v_x = V_\gamma = 10 + [-\beta(10) - 10] \exp \left(-\frac{(T' - T)}{\tau_x} \right)$

$$0.7 = 10 - 16.25 \exp \left(-\frac{(T' - T)}{\tau_x} \right) \Rightarrow 16.26 \exp \left(-\frac{(T' - T)}{\tau_x} \right) = 9.3$$

$$\exp \left(+\frac{(T' - T)}{\tau_x} \right) = 1.747$$

$$T' - T = \tau_x \ln(1.747) = 2.384 \times 10^{-4} \ln(1.747)$$

$$T' - T = 133 \mu\text{s}$$

15.62

(a) $T = \tau_x \ln \left[\frac{1 + \frac{V_\gamma}{V_p}}{1 - \beta} \right]$, $V_\gamma = 0.7 \text{ V}$, $V_p = 5 \text{ V}$, $\beta = 0.5$

$$\tau_x = R_x C_x = (20 \times 10^3)(1.2 \times 10^{-6}) = 2.4 \times 10^{-2} \text{ s}$$

$$T = (2.4 \times 10^{-2}) \ln \left[\frac{1 + \frac{0.7}{5}}{1 - 0.5} \right] \Rightarrow T = 19.78 \text{ ms}$$

(b) $T' - T \cong 0.4\tau_x = 9.6 \text{ ms}$

15.63

a. From Equation (15.95)

$$T = 1.1 RC$$

For $T = 60 \text{ s} = 1.1 RC$

then $RC = 54.55 \text{ s}$

For example, let

$$C = 50 \mu\text{F} \text{ and } R = 1.09 \text{ M}\Omega$$

b. Recovery time: capacitor is discharged by current through the discharge transistor.

$$\text{If } V^+ = 5 \text{ V, then } I_B \cong \frac{5 - 0.7}{100} = 0.043 \text{ mA}$$

If $\beta = 100$, $I_C = 4.3 \text{ mA}$

$$V_C = \frac{1}{C} \int I_C dt = \frac{I_C}{C} \cdot t$$

$$\frac{2}{3} \cdot V^+ = 3.33 \text{ V}$$

Capacitor has charged to $\frac{2}{3}$

$$t = \frac{V_C \cdot C}{I_C} = \frac{(3.33)(50 \times 10^{-6})}{4.3 \times 10^{-3}}$$

So that

So recovery time $t \approx 38.7 \text{ ms}$

15.64

$$T = 1.1 RC$$

$$5 \times 10^{-6} = 1.1 RC$$

so $RC = 4.545 \times 10^{-6} \text{ s}$

For example, let

$$C = 100 \text{ pF} \text{ and } R = 45.5 \text{ k}\Omega$$

From Problem (15.63), recovery time

$$t \cong \frac{V_C \cdot C}{I_C} = \frac{(3.33)(100 \times 10^{-12})}{4.3 \times 10^{-3}}$$

or

$$t = 77.4 \text{ ns}$$

15.65

$$\text{Duty cycle} = 60\% = \frac{R_A + R_B}{R_A + 2R_B} \times 100\%$$

$$0.60 = \frac{25 + R_B}{25 + 2R_B} \Rightarrow R_B = 50 \text{ k}\Omega$$

$$f = 80 \times 10^3 = \frac{1}{(0.693)(25 + 2(50)) \times 10^3 C} \Rightarrow C = 144.3 \text{ pF}$$

15.66

$$f = \frac{1}{(0.693)(R_A + 2R_B)C}$$

$$R_A = R_1 = 10 \text{ k}\Omega, R_B = R_2 + xR_3$$

$$\text{So } 10 \text{ k}\Omega \leq R_B \leq 110 \text{ k}\Omega$$

$$f_{\min} = \frac{1}{(0.693)(10 + 2(110)) \times 10^3 \times (0.01 \times 10^{-6})} = 627 \text{ kHz}$$

$$f_{\max} = \frac{1}{(0.693)(10 + 2(10)) \times 10^3 \times (0.01 \times 10^{-6})} = 4.81 \text{ kHz}$$

$$\text{So } 627 \text{ Hz} \leq f \leq 4.81 \text{ kHz}$$

$$\text{Duty cycle} = \frac{R_A + R_B}{R_A + 2R_B} \times 100\%$$

Now

$$\frac{10 + 10}{10 + 2(10)} \times 100\% = \underline{\underline{66.7\%}}$$

and

$$\frac{10 + 110}{10 + 2(110)} \times 100\% = \underline{\underline{52.2\%}}$$

$$\text{So } 52.2\% \leq \text{Duty cycle} \leq 66.7\%$$

15.67

$$1 \text{ k}\Omega \leq R_A \leq 51 \text{ k}\Omega$$

$$1 \text{ k}\Omega \leq R_B \leq 51 \text{ k}\Omega$$

$$f_{\min} = \frac{1}{(0.693)(1 + 2(51)) \times 10^3 \times (0.01 \times 10^{-6})} = 1.40 \text{ Hz}$$

$$f_{\max} = \frac{1}{(0.693)(51 + 2(1)) \times 10^3 \times (0.01 \times 10^{-6})} = 2.72 \text{ kHz}$$

$$\text{or } 1.40 \text{ kHz} \leq f \leq 2.72 \text{ kHz}$$

$$\text{Duty cycle} = \frac{R_A + R_B}{R_A + 2R_B} \times 100\%$$

$$\frac{1 + 51}{1 + 2(51)} \times 100\% = 50.5\%$$

or

$$\frac{51 + 1}{51 + 2(1)} \times 100\% = \underline{\underline{98.1\%}}$$

or

$$50.5\% \leq \text{Duty cycle} \leq 98.1\%$$

15.68

a.

$$I_{E3} = I_{E4} = \frac{V^+ - 3V_{EB}}{R_{1A} + R_{1B}}$$

Assume $V_{EB} = 0.7$

$$I_{E3} = I_{E4} = \frac{22 - 3(0.7)}{25 + 25} = 0.398 \text{ mA}$$

Now

$$I_{C3} = I_{C4} = I_{C5} = I_{C6} = \left(\frac{20}{21}\right)(0.398)$$

$$\underline{I_{C3} = I_{C4} = I_{C5} = I_{C6} = 0.379 \text{ mA}}$$

$$I_{C1} = I_{C2} = \frac{0.398}{21} \left(\frac{20}{21}\right) \Rightarrow \underline{I_{C1} = I_{C2} = 0.018 \text{ mA}}$$

b. $\underline{I_D = 0.398 \text{ mA}}$, current in D_1 and D_2

$$V_{BB} = 2V_D = 2V_T \ln\left(\frac{I_D}{I_S}\right) = 2(0.026) \ln\left(\frac{0.398 \times 10^{-3}}{10^{-13}}\right)$$

or

$$V_{BB} = 1.149 \text{ V} = V_{BE7} + V_{EB8}$$

Now

$$I_{C7} \approx I_{C4} + I_{C9} + I_{E8}$$

$$I_{C4} = 0.379 \text{ mA}$$

$$I_{B9} = I_{C8} = \left(\frac{20}{21}\right)I_{E8}$$

So

$$I_{E8} = 1.05I_{B9} = 1.05\left(\frac{I_{C9}}{100}\right)$$

$$I_{C7} = I_{C4} + \left(\frac{100}{1.05}\right)I_{E8} + I_{E8} = I_{C4} + (96.24)\left(\frac{21}{20}\right)I_{C8}$$

So $I_{C7} = 0.379 \text{ mA} + 101I_{C8}$

and

$$V_{BE7} = V_T \ln\left(\frac{I_{C7}}{I_S}\right); V_{EB8} = V_T \ln\left(\frac{I_{C8}}{I_S}\right)$$

Then

$$\begin{aligned}
 1.149 &= 0.026 \left[\ln \left(\frac{I_{C7}}{I_s} \right) + \ln \left(\frac{I_{C8}}{I_s} \right) \right] \\
 44.19 &= \ln \left[\frac{I_{C8}(0.379 \times 10^{-3}) + 101I_{C8}}{(10^{-13})^2} \right] \\
 (10^{-13})^2 \exp(44.19) &= 101I_{C8}^2 + 3.79 \times 10^{-4} I_{C8} \\
 1.554 \times 10^{-7} &= 101I_{C8}^2 + 3.79 \times 10^{-4} I_{C8} \\
 I_{C8} &= \frac{-3.79 \times 10^{-4} \pm \sqrt{(3.79 \times 10^{-4})^2 + 4(101)(1.554 \times 10^{-7})}}{2(101)} \\
 I_{C8} &= 37.4 \mu\text{A} \\
 I_{C7} &= 0.379 + 101(0.0374) \Rightarrow I_{C7} = 4.16 \text{ mA} \\
 I_{C9} &= 4.16 - 0.379 - 0.0374 \left(\frac{21}{20} \right) \\
 I_{C9} &= 3.74 \text{ mA} \\
 \text{c. } P &= (0.398 + 0.398 + 4.16)(22) \Rightarrow P = 109 \text{ mW}
 \end{aligned}$$

15.69

a. From Figure 15.44, 3.7 W to the load

b. $V^+ \approx 19 \text{ V}$

c. $\bar{P} = \frac{1}{2} \frac{V_p^2}{R_L}$

or

$$V_p = \sqrt{2R_L \bar{P}} = \sqrt{2(10)(3.7)} \Rightarrow V_p = 8.6 \text{ V}$$

15.70

(a) $\left(1 + \frac{R_2}{R_1} \right) = 12 \Rightarrow \frac{R_2}{R_1} = 11$, and $\frac{R_4}{R_3} = 12$

(b) $\bar{P}_L = \frac{V_L^2}{2R_L} \Rightarrow V_L = \sqrt{2R_L \bar{P}_L} = \sqrt{2(12)(15)} = 18.97 \text{ V}$

$|v_{o1}|_{\max} = |v_{o2}|_{\max} = 9.49 \text{ V}$

$I_L = \frac{V_L}{R_L} = \frac{18.97}{12} = 1.58 \text{ A}$

(c) (i) $\bar{P}_L = \left(\frac{I_L}{\sqrt{2}} \right) \left(\frac{V_L}{\sqrt{2}} \right) = \left(\frac{0.8}{\sqrt{2}} \right) \left(\frac{24}{\sqrt{2}} \right) = 9.6 \text{ W}$

(ii) $R_L = \frac{V_L}{I_L} = \frac{24}{0.8} = 30 \Omega$

15.71

a. $v_{01} = iR_2 + v_I$ where $i = \frac{v_I}{R_l}$

Then

$$v_{01} = v_I \left(1 + \frac{R_2}{R_l} \right)$$

Now

$$v_{02} = -iR_3 = -v_I \left(\frac{R_3}{R_l} \right)$$

So

$$v_L = v_{01} - v_{02} = v_I \left(1 + \frac{R_2}{R_l} \right) - \left[-v_I \left(\frac{R_3}{R_l} \right) \right]$$

$$\underline{A_v = \frac{v_L}{v_I} = 1 + \frac{R_2}{R_l} + \frac{R_3}{R_l}}$$

$$A_v = 10 \Rightarrow \frac{R_2}{R_l} + \frac{R_3}{R_l} = 9$$

b. Want

$$\left(1 + \frac{R_2}{R_l} \right) = \frac{R_3}{R_l}$$

Also want $\frac{R_2}{R_l} + \left(1 + \frac{R_2}{R_l} \right) = 9$ so $\frac{R_2}{R_l} = 4$

Then $\frac{R_2}{R_l} + \left(1 + \frac{R_2}{R_l} \right) = 9$ so $\frac{R_2}{R_l} = 4$

For $R_l = 50 \text{ k}\Omega$, $\underline{R_2 = 200 \text{ k}\Omega}$

and

$$\frac{R_3}{R_l} = 5 \quad \text{so} \quad \underline{R_3 = 250 \text{ k}\Omega}$$

$$\bar{P} = \frac{1}{2} \frac{V_p^2}{R_L}$$

c.

or

$$V_p = \sqrt{2R_L \bar{P}} = \sqrt{2(20)(10)} = 20 \text{ V}$$

So peak values of output voltages are

$$|v_{01}| = |v_{02}| = 10 \text{ V}$$

$$\text{Peak load current} = \frac{20}{20} = 1 \text{ A}$$

15.72

(a) $v_{01} = \left(1 + \frac{R_2}{R_1} \right) \cdot v_I$

$$v_{02} = -\frac{R_4}{R_3} \cdot v_{01} = -\frac{R_4}{R_3} \left(1 + \frac{R_2}{R_1} \right) \cdot v_I$$

$$v_L = v_{01} - v_{02}$$

$$A_v = \frac{v_L}{v_I} = \left(1 + \frac{R_2}{R_1} \right) \left(1 + \frac{R_4}{R_3} \right)$$

$$(b) \left(1 + \frac{R_2}{R_1}\right) = 12 \Rightarrow \frac{R_2}{R_1} = 11, \text{ Set } R_2 = 120 \text{ k}\Omega, \text{ then } R_1 = 10.9 \text{ k}\Omega$$

$$\frac{R_4}{R_3} = 1, \text{ Set } R_3 = R_4 = 120 \text{ k}\Omega$$

$$(c) (i) \bar{P}_L = \frac{V_L^2}{2R_L} = \frac{(16)^2}{2(25)} = 5.12 \text{ W}$$

$$(ii) I_L = \frac{V_L}{R_L} = \frac{16}{25} = 0.64 \text{ A}$$

15.73

(a) From Problem 15.72

$$v_{o2} = -\frac{R_4}{R_3} \cdot v_{o1}, \text{ Set } \frac{R_4}{R_3} = 1$$

$$A_v = \left(1 + \frac{R_2}{R_1}\right) \left(1 + \frac{R_4}{R_3}\right)$$

$$25 = \left(1 + \frac{R_2}{R_1}\right)(2) \Rightarrow \frac{R_2}{R_1} = 11.5$$

Set $R_2 = R_3 = R_4 = 100 \text{ k}\Omega$, then $R_1 = 8.69 \text{ k}\Omega$

$$(b) (i) \bar{P}_L = \left(\frac{V_L}{\sqrt{2}}\right) \left(\frac{I_L}{\sqrt{2}}\right) = \left(\frac{24}{\sqrt{2}}\right) \left(\frac{1.2}{\sqrt{2}}\right) = 14.4 \text{ W}$$

$$(ii) R_L = \frac{V_L}{I_L} = \frac{24}{1.2} = 20 \Omega$$

$$(c) \bar{P}_L = \frac{V_L^2}{2R'_L} = \frac{(24)^2}{2(40)} = 7.2 \text{ W}$$

15.74

$$\text{Line regulation} = \frac{\Delta V_0}{\Delta V^+}$$

Now

$$\Delta I = \frac{\Delta V^+}{R_1} \quad \text{and} \quad \Delta V_Z = r_Z \cdot \Delta I \quad \text{and} \quad \Delta V_0 = 10 \Delta V_Z$$

So

$$\Delta V_0 = 10 \cdot r_Z \cdot \frac{\Delta V^+}{R_1}$$

So

$$\text{Line regulation} = \frac{\Delta V_0}{\Delta V^+} = \frac{10(15)}{9300} \Rightarrow 1.61\%$$

15.75

$$(a) R_{of} = \frac{|\Delta V_o|}{|\Delta I_o|} = \frac{8}{2} \Rightarrow R_{of} = 4 \text{ m}\Omega$$

$$(b) \Delta V_o = R_{of} \cdot \Delta I_o = (10)(1.2) \Rightarrow \Delta V_o = 12 \text{ mV}$$

15.76

For $V_0 = 8 \text{ V}$

$$V^+(\min) = V_0 + I_0(\max)R_{11} + V_{BE11} + V_{BE10} + V_{EB5}$$

This assumes $V_{BC5} = 0$.

Then

$$V^+(\min) = 8 + (0.1)(1.9) + 0.6 + 0.6 + 0.6$$

$$\underline{V^+(\min) = 9.99 \text{ V}}$$

15.77

a.

$$I_{C3} = I_{C5} = \frac{V_Z - 3V_{BE}(\text{npn})}{R_1 + R_2 + R_3}$$

$$I_{C3} = I_{C5} = \frac{6.3 - 3(0.6)}{0.576 + 3.4 + 3.9} = 0.571 \text{ mA}$$

$$I_{C8} = \frac{1}{2} \left(\frac{0.6}{2.84} \right) = 0.106 \text{ mA}$$

Neglecting current in Q_9 , total collector current and emitter current in Q_5 is
 $0.571 + 0.106 = 0.677$

Now

$$I_{Z2}R_4 + V_{EB4} = V_{EB5}$$

$$V_{EB4} = V_T \ln \left(\frac{I_{Z2}}{I_S} \right)$$

$$V_{EB5} = V_T \ln \left(\frac{I_{C5}}{2I_S} \right)$$

$$I_{Z2}R_4 = V_T \ln \left(\frac{I_{C5}}{2I_{Z2}} \right)$$

Then

$$R_4 = \frac{0.026}{0.25} \cdot \ln \left(\frac{0.677}{2(0.25)} \right)$$

or

$$\underline{R_4 = 31.5 \Omega}$$

b. From Example 15.16, $V_{B7} = 3.43$ V. Then

$$\left(\frac{R_{13}}{R_{12} + R_{13}} \right) V_0 = V_{B8} = V_{B7}$$

or

$$\left(\frac{2.23}{2.23 + R_{12}} \right) (12) = 3.43$$

$$3.43(2.23 + R_{12}) = (2.23)(12)$$

which yields

$$\underline{R_{12} = 5.57 \text{ k}\Omega}$$

15.78

$$\text{Line regulation} = \frac{\Delta V_0}{\Delta V^+}$$

Now

$$\Delta V_{B7} = \Delta I_{C3} \cdot R_l$$

$$\text{and } \left(\frac{R_{13}}{R_{12} + R_{13}} \right) (\Delta V_0) = \Delta V_{B7} = \Delta I_{C3} R_l$$

$$\text{and } \Delta I_{C3} = \frac{\Delta V_Z}{R_1 + R_2 + R_3} = \frac{\Delta I_Z \cdot r_z}{R_1 + R_2 + R_3}$$

$$\text{and } \Delta I_Z = \frac{\Delta V^+}{r_0} \quad \text{where } r_0 = \frac{V_A}{I_Z}$$

Then

$$(0.4288)(\Delta V_0) = \Delta I_{C3}(3.9) = (3.9)\Delta I_Z \left(\frac{0.015}{7.876} \right)$$

$$r_0 = \frac{50}{0.571} = 87.6 \text{ k}\Omega$$

Then

$$(0.4288)(\Delta V_0) = (0.00743) \left(\frac{\Delta V^+}{87.6} \right)$$

So

$$\underline{\frac{\Delta V_0}{\Delta V^+} = 0.0198\%}$$

15.79

$$(a) R_l + r_z = \frac{V^+}{I_Z} = \frac{25}{12} = 2.0833 \text{ k}\Omega$$

$$R_l = 2.0833 - 0.012 = 2.0713 \text{ k}\Omega$$

$$(b) V^+ = 5.6 + (0.012)(12) = 5.744 \text{ V}$$

For $x = 0$,

$$5.744 = \left(\frac{R_3 + R_4}{R_2 + R_3 + R_4} \right) \cdot V_o \Rightarrow V_o = \left(\frac{1+2+1}{2+1} \right) (5.744) = 7.659 \text{ V}$$

For $x = 1$,

$$5.744 = \left(\frac{R_4}{R_2 + R_3 + R_4} \right) \cdot V_o \Rightarrow V_o = \left(\frac{1+2+1}{1} \right) (5.744) = 22.976 \text{ V}$$

So $7.659 \leq V_o \leq 22.976 \text{ V}$

(c) For $x = 1$, $V_1 = \frac{1}{4}V_o$

$$v_d = V_{REF} - V_1 = V_{REF} - \frac{1}{4}V_o$$

$$V_o = A_{OL}v_d - V_{BE} = A_{OL}v_d - V_T \ln\left(\frac{I_o}{I_s}\right)$$

$$V_o = A_{OL}\left(V_{REF} - \frac{1}{4}V_o\right) - V_T \ln\left(\frac{I_o}{I_s}\right)$$

$$V_o \left(1 + \frac{1}{4}A_{OL}\right) = A_{OL}V_{REF} - V_T \ln\left(\frac{I_o}{I_s}\right)$$

$$V_o = \frac{A_{OL}V_{REF} - V_T \ln\left(\frac{I_o}{I_s}\right)}{1 + \frac{1}{4}A_{OL}}$$

$$\text{Load regulation} = \frac{V_o(NL) - V_o(FL)}{V_o(NL)}$$

$$= \frac{\frac{A_{OL}V_{REF} - V_T \ln[I_o(NL)/I_s]}{1 + (1/4)A_{OL}} - \left[\frac{A_{OL}V_{REF} - V_T \ln[I_o(FL)/I_s]}{1 + (1/4)A_{OL}} \right]}{\frac{A_{OL}V_{REF} - V_T \ln[I_o(NL)/I_s]}{1 + (1/4)A_{OL}}}$$

$$= \frac{V_T \left[\ln\left(\frac{I_o(FL)}{I_s}\right) - \ln\left(\frac{I_o(NL)}{I_s}\right) \right]}{A_{OL}V_{REF} - V_T \ln\left(\frac{I_o(NL)}{I_s}\right)} = \frac{V_T \ln\left(\frac{I_o(FL)}{I_o(NL)}\right)}{A_{OL}V_{REF} - V_T \ln\left(\frac{I_o(NL)}{I_s}\right)}$$

$$\text{Let } I_o(FL) = 5 \text{ A}, \quad I_o(NL) = 1\% = 0.05 \text{ A}$$

$$\text{Let } V_T \ln\left(\frac{I_o(NL)}{I_s}\right) \approx 0.7 \text{ V}$$

$$\text{Load regulation} = \frac{(0.026)\ln\left(\frac{5}{0.05}\right)}{(5 \times 10^3)(5.744) - 0.7} = \frac{0.11973}{2.8719 \times 10^4}$$

$$\text{Load regulation} = 4.169 \times 10^{-4} \%$$

15.80

$$(a) \quad I_E = \frac{V_Z}{R_2} = \frac{6.8}{5} = 1.36 \text{ mA}$$

$$I_O = \left(\frac{\beta}{1 + \beta} \right) \cdot I_E = \left(\frac{80}{81} \right) (1.36) = 1.343 \text{ mA}$$

For $V_{BC} = 0$, $V_O = 20 - V_Z - V_{EB} = 20 - 6.8 - 0.6 = 12.6 \text{ V}$

$$R_L(\max) = \frac{V_O}{I_O} = \frac{12.6}{1.343} = 9.38 \text{ k}\Omega$$

So $0 \leq R_L \leq 9.38 \text{ k}\Omega$

$$(b) \quad V_+ = \left(\frac{R_1}{R_1 + r_Z} \right) (V^+ - V_{Z0}) = \left(\frac{10^4}{10^4 + 20} \right) (V^+ - 6.8)$$

$$\text{For } V^+ = 20 \text{ V}, \quad V_+ = \left(\frac{10^4}{10^4 + 20} \right) (13.2) = 13.17365 \text{ V}$$

$$I_O = \left(\frac{80}{81} \right) \left(\frac{20 - 13.17365}{5} \right) = 1.3484 \text{ mA}$$

$$\text{For } V^+ = 16 \text{ V}, \quad V_+ = \left(\frac{10^4}{10^4 + 20} \right) (16 - 6.8) = 9.181637 \text{ V}$$

$$I_O = \left(\frac{80}{81} \right) \left(\frac{16 - 9.181637}{5} \right) = 1.3468 \text{ mA}$$

So $1.3468 \leq I_O \leq 1.3484 \text{ mA}$

Chapter 16

16.1

$$(a) v_o = V_{DD} - \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)R_D [2(v_I - V_{TN})v_o - v_o^2]$$

$$0.1 = 3.3 - \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(40)[2(3.3 - 0.5)(0.1) - (0.1)^2]$$

$$0.1 = 3.3 - \left(\frac{W}{L}\right)(1.1) \Rightarrow \left(\frac{W}{L}\right) = 2.91$$

$$(b) \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)R_D V_{ot}^2 + V_{ot} - V_{DD} = 0$$

$$\left(\frac{0.1}{2}\right)(2.91)(40)V_{ot}^2 + V_{ot} - 3.3 = 0$$

$$5.82V_{ot}^2 + V_{ot} - 3.3 = 0 \Rightarrow V_{ot} = 0.672 \text{ V}$$

$$V_{ot} = V_{It} - V_{TN} \Rightarrow V_{It} = 1.172 \text{ V}$$

$$(c) i_{D,\max} = \frac{3.3 - 0.1}{40} \Rightarrow i_{D,\max} = 80 \mu \text{A}$$

$$P_{D,\max} = (80)(3.3) = 264 \mu \text{W}$$

16.2

$$(a) (i) K_n R_D V_{ot}^2 + V_{ot} - V_{DD} = 0$$

$$(0.05)(100)V_{ot}^2 + V_{ot} - 3.3 = 0 \Rightarrow V_{ot} = 0.7185 \text{ V}$$

$$\Rightarrow V_{It} = 1.219 \text{ V}$$

$$(ii) v_o = 3.3 - (0.05)(100)[2(3.3 - 0.5)v_o - v_o^2]$$

We find $5v_o^2 - 29v_o + 3.3 = 0 \Rightarrow v_o = 0.116 \text{ V}$

$$(b) (i) (0.05)(30)V_{ot}^2 + V_{ot} - 3.3 = 0 \Rightarrow V_{ot} = 1.187 \text{ V}$$

$$\Rightarrow V_{It} = 1.687 \text{ V}$$

$$(ii) v_o = 3.3 - (0.05)(30)[2(3.3 - 0.5)v_o - v_o^2]$$

Or $1.5v_o^2 - 9.4v_o + 3.3 = 0 \Rightarrow v_o = 0.373 \text{ V}$

$$(c) (i) (0.05)(5)V_{ot}^2 + V_{ot} - 3.3 = 0 \Rightarrow V_{ot} = 2.147 \text{ V}$$

$$\Rightarrow V_{It} = 2.647 \text{ V}$$

$$(ii) v_o = 3.3 - (0.05)(5)[2(3.3 - 0.5)v_o - v_o^2]$$

Or $0.25v_o^2 - 2.4v_o + 3.3 = 0 \Rightarrow v_o = 1.663 \text{ V}$

16.3

(a) $P = IV$
 $0.25 = I(3.3) \Rightarrow I = 75.76 \mu A$

$$R = \frac{3.3 - 0.15}{0.07576} = 41.6 \text{ k}\Omega$$

$$I = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) [2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$$

$$75.76 = \left(\frac{100}{2} \right) \left(\frac{W}{L} \right) [2(3.3 - 0.5)(0.15) - (0.15)^2] \Rightarrow \left(\frac{W}{L} \right) = 1.85$$

(b) $I_D = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) (V_{GS} - V_{TN})^2 = \frac{V_{DD} - V_{DS}(\text{sat})}{R}$
 $\left(\frac{0.1}{2} \right) (1.85) V_{DS}^2(\text{sat}) = \frac{3.3 - V_{DS}(\text{sat})}{41.6}$

Or $3.848 V_{DS}^2(\text{sat}) + V_{DS}(\text{sat}) - 3.3 = 0 \Rightarrow V_{DS}(\text{sat}) = 0.805 \text{ V}$

$$V_{DS}(\text{sat}) = V_{GS} - V_{TN}$$

$$0.805 = V_{GS} - 0.5 \Rightarrow V_{GS} = 1.305 \text{ V}$$

Then $0.5 \leq V_{GS} \leq 1.305 \text{ V}$

16.4

(a) From Equation (16.21)

$$\frac{K_D}{K_L} = \frac{(W/L)_D}{(W/L)_L} = \frac{(V_{DD} - v_o - V_{TNL})^2}{2(v_I - V_{TND})v_o - v_o^2}$$

$$= \frac{(1.8 - 0.08 - 0.4)^2}{2(1.4 - 0.4)(0.08) - (0.08)^2} = \frac{1.7424}{0.1536}$$

$$\frac{(W/L)_D}{(W/L)_L} = 11.34$$

$$P_{D,\max} = i_{D,\max} \cdot V_{DD}$$

$$0.3 = i_{D,\max}(1.8) \Rightarrow i_{D,\max} = 0.1667 \text{ mA}$$

$$i_{D,\max} = 0.1667 = \left(\frac{0.1}{2} \right) \left(\frac{W}{L} \right)_D [2(1.4 - 0.4)(0.08) - (0.08)^2]$$

Which yields $\left(\frac{W}{L} \right)_D = 21.7$ and $\left(\frac{W}{L} \right)_L = 1.91$

(b) $V_{Ii} = \frac{1.8 - 0.4 + (0.4)\sqrt{1+11.34}}{1+\sqrt{11.34}} = 0.7206 \text{ V}$

$$0.4 \leq V_I \leq 0.7206 \text{ V}$$

16.5

(a) $\frac{K_D}{K_L} [2(3 - 0.5)(0.25) - (0.25)^2] = [3 - 0.25 - 0.5]^2$

$$\frac{K_D}{K_L} (1.1875) = (5.0625) \Rightarrow \frac{K_D}{K_L} = 4.26$$

$$(b) \frac{K_D}{K_L} [2(2.5 - 0.5)(0.25) - (0.25)^2] = [3 - 0.25 - 0.5]^2$$

$$\frac{K_D}{K_L} (0.9375) = (5.0625) \Rightarrow \frac{K_D}{K_L} = 5.4$$

$$(c) i_D = K_L (V_{GSL} - V_{TN})^2 = \left(\frac{0.1}{2}\right) (1) (3 - 0.25 - 0.5)^2 = 0.253 \text{ mA}$$

$$P = i_D \cdot V_{DD} = (0.253)(3) = 0.759 \text{ mW}$$

16.6

$$(a) \frac{K_D}{K_L} = \frac{(V_{DD} - v_O - V_{TNL})^2}{2(v_I - V_{TND})v_O - v_O^2}$$

$$v_I = \text{Logic 1} = V_{DD} - V_{TNL} = 3 - 0.5 = 2.5 \text{ V}$$

$$\frac{K_D}{K_L} = \frac{(W/L)_D}{(W/L)_L} = \frac{(3 - 0.1 - 0.5)^2}{2(2.5 - 0.5)(0.1) - (0.1)^2} = \frac{5.76}{0.39}$$

$$\frac{(W/L)_D}{(W/L)_L} = 14.77$$

$$P_{D,\max} = i_{D,\max} \cdot V_{DD}$$

$$0.4 = i_{D,\max}(3) \Rightarrow i_{D,\max} = 0.1333 \text{ mA}$$

$$i_{D,\max} = 0.1333 = \left(\frac{0.1}{2}\right) \left(\frac{W}{L}\right)_D [2(2.5 - 0.5)(0.1) - (0.1)^2]$$

$$\text{Which yields } \left(\frac{W}{L}\right)_D = 6.84 \text{ and } \left(\frac{W}{L}\right)_L = 0.463$$

$$(b) V_{I_t} = \frac{3 - 0.5 + (0.5)(1 + \sqrt{14.77})}{1 + \sqrt{14.77}} = 1.016 \text{ V}$$

$$V_{O_t} = 1.016 - 0.5 = 0.516 \text{ V}$$

16.7

We have

$$\frac{K_D}{K_L} [2(v_I - V_{TND})v_O - v_O^2] = (V_{DD} - v_O - V_{TNL})^2$$

$$\frac{(W/L)_D}{(W/L)_L} [2(V_{DD} - V_{TN} - V_{TN})(0.08V_{DD}) - (0.08V_{DD})^2] = (V_{DD} - 0.08V_{DD} - V_{TN})^2$$

$$\frac{(W/L)_D}{(W/L)_L} [2(V_{DD} - 2(0.2)V_{DD})(0.08V_{DD}) - 0.0064V_{DD}^2] = [(0.92 - 0.2)V_{DD}]^2 = 0.5184V_{DD}^2$$

$$\frac{(W/L)_D}{(W/L)_L} [0.096] = 0.5184 \Rightarrow \frac{(W/L)_D}{(W/L)_L} = 5.4$$

16.8

$$V_{OH} = V_B - V_{TN} = \text{Logic 1}$$

So

- (a) $V_B = 4 \text{ V} \Rightarrow V_{OH} = 3 \text{ V}$
- (b) $V_B = 5 \text{ V} \Rightarrow V_{OH} = 4 \text{ V}$
- (c) $V_B = 6 \text{ V} \Rightarrow V_{OH} = 5 \text{ V}$
- (d) $V_B = 7 \text{ V} \Rightarrow V_{OH} = 5 \text{ V}, \text{ since } V_{DS} = 0$

For $v_I = V_{OH}$

$$K_D [2(v_I - V_T)v_O - v_O^2] = K_L [V_B - v_O - V_T]^2$$

Then

- (a) $(1)[2(3-1)V_{OL} - V_{OL}^2] = (0.4)[4 - V_{OL} - 1]^2 \Rightarrow V_{OL} = 0.657 \text{ V}$
- (b) $(1)[2(4-1)V_{OL} - V_{OL}^2] = (0.4)[5 - V_{OL} - 1]^2 \Rightarrow V_{OL} = 0.791 \text{ V}$
- (c) $(1)[2(5-1)V_{OL} - V_{OL}^2] = (0.4)[6 - V_{OL} - 1]^2 \Rightarrow V_{OL} = 0.935 \text{ V}$
- (d) Load in non-sat region

$$i_{DD} = i_{OL}$$

$$\begin{aligned} (1) [2(5-1)V_{OL} - V_{OL}^2] &= (0.4) [2(7 - V_{OL} - 1)(5 - V_{OL}) - (5 - V_{OL})^2] \\ 8V_{OL} - V_{OL}^2 &= (0.4) [2(6 - V_{OL})(5 - V_{OL}) - (25 - 10V_{OL} + V_{OL}^2)] \\ &= (0.4) [2(30 - 11V_{OL} + V_{OL}^2) - 25 + 10V_{OL} - V_{OL}^2] \\ &= (0.4) [60 - 22V_{OL} + 2V_{OL}^2 - 25 + 10V_{OL} - V_{OL}^2] \end{aligned}$$

$$8V_{OL} - V_{OL}^2 = 14 - 4.8V_{OL} + 0.4V_{OL}^2$$

$$1.4V_{OL}^2 - 12.8V_{OL} + 14 = 0$$

$$V_{OL} = \frac{12.8 \pm \sqrt{163.84 - 4(1.4)(14)}}{2(1.4)}$$

$$V_{OL} = 1.27 \text{ V}$$

For load

$$V_{DS}(\text{sat}) = 7 - 1.27 - 1 = 4.73 \text{ V}$$

$$V_{DS} = 5 - 1.27 = 3.73 \text{ V, non-sat}$$

16.9

$$(a) \sqrt{\frac{K_D}{K_L}} (V_{I_t} - V_{TND}) = -V_{TNL}$$

$$\sqrt{\frac{500}{100}} (V_{I_t} - 0.5) = -(-0.8) \Rightarrow V_{I_t} = 0.8578 \text{ V}$$

$$\text{For Driver: } V_{O_t} = V_{I_t} - V_{TND} = 0.8578 - 0.5 = 0.3578 \text{ V}$$

$$\text{For Load: } V_{O_t} = V_{DD} + V_{TNL} = 3.3 + (-0.8) = 2.5 \text{ V}$$

$$(b) \frac{K_D}{K_L} [2(v_I - V_{TND})v_O - v_O^2] = (-V_{TNL})^2$$

$$\left(\frac{500}{100}\right) [2(3.3 - 0.5)v_O - v_O^2] = [-(-0.8)]^2$$

$$\text{We find } 5v_O^2 - 28v_O + 0.64 = 0 \Rightarrow v_O = 0.0230 \text{ V}$$

$$(c) \quad i_{D,\max} = K_L (-V_{TNL})^2 = 100[-(-0.8)]^2 = 64 \mu A$$

$$P_{D,\max} = i_{D,\max} \cdot V_{DD} = (64)(3.3) = 211 \mu W$$

16.10

$$\left(\frac{500}{50}\right) \left[2(3 - 0.5)(0.1) - (0.1)^2 \right] = (-V_{TNL})^2$$

So

$$(-V_{TNL})^2 = 4.9 \Rightarrow V_{TNL} = -2.21V$$

16.11

$$(a) \quad P_{D,\max} = i_{D,\max} \cdot V_{DD}$$

$$80 = i_{D,\max}(1.8) \Rightarrow i_{D,\max} = 44.44 \mu A$$

$$i_{D,\max} = 44.44 = \left(\frac{100}{2}\right) \left(\frac{W}{L}\right)_L [-(-0.6)]^2 \Rightarrow \left(\frac{W}{L}\right)_L = 2.47$$

$$\frac{K_D}{K_L} \left[2(1.8 - 0.3)(0.06) - (0.06)^2 \right] = [-(-0.6)]^2$$

$$\frac{K_D}{K_L} (0.1764) = (0.36) \Rightarrow \frac{K_D}{K_L} = 2.04 = \frac{(W/L)_D}{(W/L)_L}$$

$$\text{Then } \left(\frac{W}{L}\right)_D = 5.04$$

$$(b) \quad \sqrt{\frac{K_D}{K_L}} (V_{I_t} - V_{TND}) = -V_{TNL}$$

$$\sqrt{\frac{5.04}{2.47}} (V_{I_t} - 0.3) = [-(-0.6)] \Rightarrow V_{I_t} = 0.720 V$$

$$\text{For Driver: } V_{O_t} = 0.720 - 0.3 = 0.420 V$$

$$\text{For Load: } V_{O_t} = 1.8 - 0.6 = 1.2 V$$

$$(c) \quad P_{D,\max} = 80 \mu W$$

$$2 \left(\frac{5.04}{2.47}\right) \left[2(1.8 - 0.3)v_o - v_o^2 \right] = [-(-0.6)]^2$$

$$\text{We find } 4.08v_o^2 - 12.24v_o + 0.36 = 0 \Rightarrow v_o = 0.0297 V$$

16.12

a. From Equation (16.27(b)):

$$\left(\frac{W}{L}\right)_D \left[2(2.5 - 0.5)(0.05) - (0.05)^2 \right] = \left(\frac{W}{L}\right)_L [-(-1)]^2$$

$$\left(\frac{W}{L}\right)_L = 1$$

$$\left(\frac{W}{L}\right)_D = 5.06$$

Then $\left(\frac{W}{L}\right)_D = 5.06$

b. $i_D = \left(\frac{80}{2}\right)(1)\left[-(-1)\right]^2$
or $\underline{i_D = 40 \mu A}$
 $P = i_D \cdot V_{DD} = (40)(2.5) \Rightarrow \underline{P = 100 \mu W}$

16.13

a. i. $v_I = 0.5 \text{ V} \Rightarrow i_D = 0 \Rightarrow \underline{P = 0}$
ii. $v_I = 5 \text{ V}$, From Equation (16.11),
 $v_0 = 5 - (0.1)(20)\left[2(5 - 1.5)v_0 - v_0^2\right]$
 $2v_0^2 - 15v_0 + 5 = 0$
 $v_0 = \frac{15 \pm \sqrt{(15)^2 - 4(2)(5)}}{2(2)} \Rightarrow \underline{v_0 = 0.35 \text{ V}}$
 $i_D = \frac{5 - 0.35}{20} = 0.2325 \text{ mA}$
 $P = i_D \cdot V_{DD} = (0.2325)(5) \Rightarrow \underline{P = 1.16 \text{ mW}}$

b. i. $v_I = 0.25 \text{ V} \Rightarrow i_D = 0 \Rightarrow \underline{P = 0}$
ii. $v_I = 4.3 \text{ V}$, From Equation (16.21),
 $100\left[2(4.3 - 0.7)v_0 - v_0^2\right] = 10\left[5 - v_0 - 0.7\right]^2$
 $10\left[7.2v_0 - v_0^2\right] = 18.49 - 8.6v_0 + v_0^2$

Then

$$11v_0^2 - 80.6v_0 + 18.49 = 0$$
 $v_0 = \frac{80.6 \pm \sqrt{(80.6)^2 - 4(11)(18.49)}}{2(11)} \Rightarrow \underline{v_0 = 0.237 \text{ V}}$

Then

$$i_D = 10\left[5 - 0.237 - 0.7\right]^2 = 165 \mu A$$
 $P = i_D \cdot V_{DD} = (165)(5) \Rightarrow \underline{P = 825 \mu W}$

c. i. $v_I = 0.03 \text{ V} \Rightarrow i_D = 0 \Rightarrow \underline{P = 0}$
ii. $v_I = 5 \text{ V}$
 $i_D = K_L (-V_{TNL})^2 = (10)\left[-(-2)\right]^2 = 40 \mu A$
 $P = i_D \cdot V_{DD} = (40)(5) \Rightarrow \underline{P = 200 \mu W}$

16.14

(a) $v_I = V_{DD} - V_{TNL} = 5 - 0.5 = 4.5 \text{ V}$
 $K_D [2(v_I - V_{TND})v_{O1} - v_{O1}^2] = K_L [V_{DD} - v_{O1} - V_{TNL}]^2$
 $10[2(4.5 - 0.5)v_{O1} - v_{O1}^2] = (1)[5 - v_{O1} - 0.5]^2$
 We find $11v_{O1}^2 - 89v_{O1} + 20.25 = 0 \Rightarrow v_{O1} = 0.234 \text{ V}$
 $v_{O2} = V_{DD} - V_{TNL} = 4.5 \text{ V}$

(b) $v_I = 0.234 \text{ V}, \Rightarrow v_{O1} = 4.5 \text{ V}$
 From part (a), $v_{O2} = 0.234 \text{ V}$

16.15

$$(a) \frac{K_D}{K_L} [2(v_I - V_{TND})v_O - v_O^2] = (-V_{TNL})^2$$

$$\left(\frac{4}{1}\right) [2(5 - 0.8)v_{O1} - v_{O1}^2] = [-(-1.2)]^2$$

$$\text{We find } 4v_{O1}^2 - 33.6v_{O1} + 1.44 = 0 \Rightarrow v_{O1} = 0.0431 \text{ V}$$

$$\Rightarrow v_{O2} = 5 \text{ V}$$

$$(b) \text{ For } v_I = 0.0431 \text{ V}, \quad v_{O1} = 5 \text{ V}$$

$$\text{From part (a), } v_{O2} = 0.0431 \text{ V}$$

16.16

$$(a) v_O = V_{DD} - V_{TNLO} = 2.5 - 0.5 = 2.0 \text{ V}$$

$$(b) v_O = V_{DD} - [V_{TNLO} + \gamma (\sqrt{2\phi_{fp} + v_O} - \sqrt{2\phi_{fp}})]$$

$$v_O = 2.5 - [0.5 + 0.25 (\sqrt{0.7 + v_O} - \sqrt{0.7})]$$

$$v_O - 2.209 = -0.25 \sqrt{0.7 + v_O}$$

$$v_O^2 - 4.418v_O + 4.88 = 0.0625(0.7 + v_O)$$

$$v_O^2 - 4.4805v_O + 4.836 = 0 \Rightarrow v_O = 1.81 \text{ V}$$

16.17

$$(a) \sqrt{\frac{K_D}{K_L}} (v_I - V_{TND}) = (-V_{TNL})$$

$$\text{Or } v_I = [-(V_{TNL})] \sqrt{\frac{K_L}{K_D}} + V_{TND} = [-(-0.6)] \sqrt{\frac{20}{100}} + 0.4 = 0.6683 \text{ V}$$

$$(b) V_{TNL} = V_{TNLO} + \gamma (\sqrt{2\phi_{fp} + v_O} - \sqrt{2\phi_{fp}})$$

$$= -0.6 + 0.25 (\sqrt{0.7 + 1.25} - \sqrt{0.7}) = -0.460 \text{ V}$$

$$v_I = [-(-0.460)] \sqrt{\frac{20}{100}} + 0.4 = 0.6057 \text{ V}$$

6.18

$$(a) \frac{K_D}{K_L} [2(v_I - V_{TND})v_O - v_O^2] = (-V_{TNL})^2$$

$$\frac{K_D}{K_L} [2(1.8 - 0.4)(0.1) - (0.1)^2] = [-(-0.6)]^2$$

$$\frac{K_D}{K_L} (0.27) = 0.36 \Rightarrow \frac{K_D}{K_L} = 1.33$$

$$(b) \quad 2\left(\frac{K_D}{K_L}\right)\left[2(v_I - V_{TND})v_o - v_o^2\right] = [-V_{TNL}]^2$$

$$2(1.333)\left[2(1.8 - 0.4)v_o - v_o^2\right] = [-(-0.6)]^2$$

We find $2.667v_o^2 - 7.467v_o + 0.36 = 0 \Rightarrow v_o = 49.1 \text{ mV}$

$$(c) \quad i_{D,\max} = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_L (-V_{TNL})^2 = \left(\frac{0.1}{2}\right)(1)[-(-0.6)]^2 \Rightarrow i_{D,\max} = 18 \mu \text{A}$$

$$P = i_{D,\max} \cdot V_{DD} = (18)(1.8) = 32.4 \mu \text{W}$$

6.19

(a) One input high,

$$\frac{K_D}{K_L}\left[2(3 - 0.5)(0.1) - (0.1)^2\right] = [-(-1)]^2 \Rightarrow \frac{K_D}{K_L} = 2.04$$

$$(b) \quad P = i_D \cdot V_{DD}$$

$$0.1 = i_D(3) \Rightarrow i_D = 33.33 \mu \text{A}$$

$$i_D = \left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_L [-V_{TNL}]^2$$

$$33.33 = \left(\frac{100}{2}\right)\left(\frac{W}{L}\right)_L (1) \Rightarrow \left(\frac{W}{L}\right)_L = 0.667$$

$$\text{Then } \left(\frac{W}{L}\right)_D = (2.04)(0.667) = 1.36$$

$$(c) \quad 3(2.04)\left[2(3 - 0.5)v_o - v_o^2\right] = [-(-1)]^2$$

$$6.12v_o^2 - 30.6v_o + 1 = 0 \Rightarrow v_o = 0.0329 \text{ V}$$

16.20

$$(a) \quad \frac{K_D}{K_L}\left[2(2.5 - 0.4)(0.05) - (0.05)^2\right] = [-(-0.6)]^2$$

$$\frac{K_D}{K_L}(0.2075) = 0.36 \Rightarrow \frac{K_D}{K_L} = 1.735$$

$$(b) \quad P = i_{D,\max} \cdot V_{DD}$$

$$50 = i_{D,\max}(2.5) \Rightarrow i_{D,\max} = 20 \mu \text{A}$$

$$i_{D,\max} = 20 = \left(\frac{100}{2}\right)\left(\frac{W}{L}\right)_L [-(-0.6)]^2$$

$$\text{We find } \left(\frac{W}{L}\right)_L = 1.11 \text{ and } \left(\frac{W}{L}\right)_D = 1.93$$

$$(c) \quad (i) \quad 2(1.735)\left[2(2.5 - 0.4)v_o - v_o^2\right] = 0.36$$

$$3.47v_o^2 - 14.574v_o + 0.36 = 0 \Rightarrow v_o = 24.9 \text{ mV}$$

$$(ii) \quad 3(1.735)\left[2(2.5 - 0.4)v_o - v_o^2\right] = 0.36$$

$$5.205v_o^2 - 21.861v_o + 0.36 = 0 \Rightarrow v_o = 16.5 \text{ mV}$$

$$(iii) \quad 4(1.735)[2(2.5 - 0.4)v_o - v_o^2] = 0.36 \\ 6.94v_o^2 - 29.148v_o + 0.36 = 0 \Rightarrow v_o = 12.4 \text{ mV}$$

16.21

a.

$$P = i_D \cdot V_{DD} \\ 250 = i_D (5) \Rightarrow i_D = 50 \mu\text{A}$$

$$i_D = \left(\frac{k_n}{2}\right) \left(\frac{W}{L}\right)_{ML1} [-V_{T_{NL1}}]^2$$

$$50 = \left(\frac{60}{2}\right) \left(\frac{W}{L}\right)_{ML1} [-(-2)]^2$$

$$\left(\frac{W}{L}\right)_{ML1} = 0.417$$

So that $\frac{K_D}{K_L} = 3.23$

$$\frac{K_D}{K_L} [2(v_i - V_{TN})v_o - v_o^2] = [-V_{T_{NL}}]^2$$

$$\frac{K_D}{K_L} [2(5 - 0.8)(0.15) - (0.15)^2] = [-(-2)]^2$$

$$\text{or } \frac{K_D}{K_L} = 3.23 \Rightarrow \left(\frac{W}{L}\right)_{MD1} = 1.35$$

b. For $v_x = v_r = 0 \Rightarrow v_{o1} = 5$ and $v_{o3} = 4.2$

Then

$$K_{D2} [2(v_{o1} - V_{TN})v_{o2} - v_{o2}^2] + K_{D3} [2(v_{o3} - V_{TN})v_{o2} - v_{o2}^2] = K_{L2} [-V_{T_{NL2}}]^2$$

$$K_{D2} \propto 8, K_{D3} \propto 8, K_{L2} \propto 1$$

$$8[2(5 - 0.8)v_{o2} - v_{o2}^2] + 8[2(4.2 - 0.8)v_{o2} - v_{o2}^2] = (1)[-(-2)]^2$$

$$67.2v_{o2} - 8v_{o2}^2 + 54.4v_{o2} - 8v_{o2}^2 = 4$$

Then

$$16v_{o2}^2 - 121.6v_{o2} + 4 = 0$$

$$v_{o2} = \frac{121.6 \pm \sqrt{(121.6)^2 - 4(16)(4)}}{2(16)}$$

So $v_{o2} = 0.0330 \text{ V}$

16.22

$$(a) \quad i_{D,\max} = \left(\frac{100}{2}\right)(1)[V_{DD} - v_o - V_{TN}]^2 = 50[3.3 - v_o - 0.4]^2 = 50[2.9 - v_o]^2$$

The output is small, so neglect v_o^2 .

$$\text{Then } i_{D,\max} \cong 50[8.41 - 5.8v_o] \quad (\text{Eq. 1})$$

Also,

$$i_{D,\max} = \left(\frac{100}{2}\right)(12)[2(v_{GSX} - V_{TN})v_{DSX} - v_{DSX}^2] \\ = 600[2(2.9 - 0.4)v_{DSX} - v_{DSX}^2] \cong 3000v_{DSX} \quad (\text{Eq. 2})$$

And

$$i_{D,\max} = \left(\frac{100}{2}\right)(12)\left[2(v_{GSY} - V_{TN})v_{DSY} - v_{DSY}^2\right]$$

We have $v_{GSY} = v_Y - v_{DSX}$; $v_{DSY} = v_O - v_{DSX}$

$$\begin{aligned} i_{D,\max} &= 600\left[2(2.9 - v_{DSX} - 0.4)(v_O - v_{DSX}) - (v_O - v_{DSX})^2\right] \\ &\cong 600\left[2(2.5 - v_{DSX})(v_O - v_{DSX})\right] \quad (\text{Eq.3}) \end{aligned}$$

Now $3000v_{DSX} = 50[8.41 - 5.8v_O]$

$$v_{DSX} = 0.016667[8.41 - 5.8v_O] \quad (\text{Eq. 4})$$

Also $3000v_{DSX} = 600[2(2.5 - v_{DSX})(v_O - v_{DSX})] \cong 1200[2.5v_O - 2.5v_{DSX}]$

$$2.5v_{DSX} = 2.5v_O - 2.5v_{DSX}$$

Or $v_{DSX} \cong 0.5v_O$

Then from Eq. 4,

$$0.5v_O = 0.140 - 0.09667v_O \Rightarrow v_O \cong 0.2346 \text{ V}$$

(b) $v_{DSX} \cong 0.5v_O \cong 0.1173 \text{ V}$

$$v_{GSX} = 2.9 \text{ V}$$

$$v_{GSY} \cong 2.9 - 0.1173 \cong 2.783 \text{ V}$$

$$v_{DSY} = v_O - v_{DSX} \cong 0.1173 \text{ V}$$

16.23

a. We can write

$$K_x \left[2(v_x - V_{TNx})v_{DSX} - v_{DSX}^2 \right] = K_y \left[2(v_y - v_{DSX} - V_{TNY})v_{DSY} - v_{DSY}^2 \right] = K_L \left[-V_{TNL} \right]^2$$

From the first and third terms, (neglect v_{DSX}^2),

$$4[2(5 - 0.8)v_{DSX}] = (1)[-(-1.5)]^2$$

or $v_{DSX} = 0.067 \text{ V}$

From the second and third terms, (neglect v_{DSY}^2),

$$4[2(5 - 0.067 - 0.8)v_{DSY}] = (1)[-(-1.5)]^2$$

or $v_{DSY} = 0.068 \text{ V}$

Now

$$v_{GSX} = 5, v_{GSY} = 5 - 0.067 \Rightarrow v_{GSY} = 4.933 \text{ V}$$

and $v_0 = v_{DSX} + v_{DSY} \Rightarrow v_0 = 0.135 \text{ V}$

Since v_0 is close to ground potential, the body-effect has little effect on the results.

16.24

(a) $\frac{1}{4} \left(\frac{K_D}{K_L} \right) [2(v_I - V_{TND})v_O - v_O^2] = (-V_{TNL})^2$

$$\frac{1}{4} \left(\frac{K_D}{K_L} \right) [2(3.3 - 0.4)(0.1) - (0.1)^2] = [-(-0.6)]^2$$

$$\left(\frac{K_D}{K_L} \right) (0.1425) = 0.36 \Rightarrow \left(\frac{K_D}{K_L} \right) = 2.53$$

(b) $P = i_{D,\max} \cdot V_{DD}$
 $100 = i_{D,\max} (3.3) \Rightarrow i_{D,\max} = 30.3 \mu\text{A}$
 $i_{D,\max} = 30.3 = \left(\frac{100}{2}\right) \left(\frac{W}{L}\right)_L [-(-0.6)]^2$
 Which yields $\left(\frac{W}{L}\right)_L = 1.68$ and $\left(\frac{W}{L}\right)_D = 4.26$

16.25

$$\bar{Y} = [A \text{ OR } (B \text{ AND } C)] \text{ AND } D$$

16.26

Considering a truth table, we find

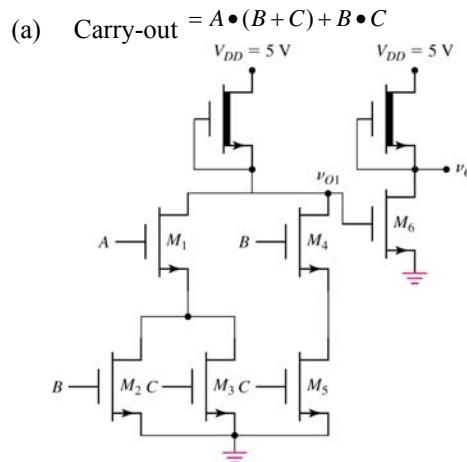
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

which shows that the circuit performs the exclusive-OR function.

16.27

$$\overline{(A+B)(C+D)}$$

16.28



(b) For $v_{O1} = \text{Low} = 0.2 \text{ V}$

$$\frac{K_D}{K_L} [2(5 - 0.8)(0.2) - (0.2)^2] = [-(-1.5)]^2 \Rightarrow$$

$$\text{For } \left(\frac{W}{L}\right)_L = 1, \text{ then } \left(\frac{W}{L}\right)_D = 1.37$$

$$\text{So, for } M_6 : \left(\frac{W}{L}\right)_6 = 1.37$$

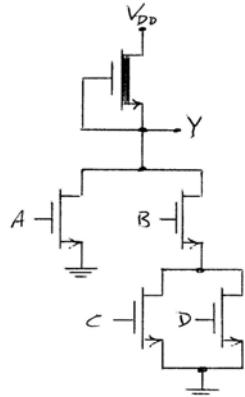
To achieve the required composite conduction parameter,

$$M_1 - M_5 : \frac{W}{L}_{1-5} = 2.74$$

For

16.29

(a)



$$(b) \frac{(W/L)_A}{1} [2(2.5 - 0.4)(0.05) - (0.05)^2] = [-(-0.6)]^2$$

$$\left(\frac{W}{L}\right)_A (0.2075) = 0.36 \Rightarrow \left(\frac{W}{L}\right)_A = 1.735$$

$$\Rightarrow \left(\frac{W}{L}\right)_{B,C,D} = 2 \left(\frac{W}{L}\right)_A = 3.47$$

16.30 Design Problem

16.31

(a) By symmetry, $V_{It} = \frac{V_{DD}}{2} = \frac{2.5}{2} = 1.25$ V

$$V_{Op_t} = 1.25 - (-0.4) = 1.65$$
 V

$$V_{On_t} = 1.25 - 0.4 = 0.85$$
 V

(c) For $v_I = 1.1$ V, NMOS in saturation, PMOS in nonsaturation

$$K_n [v_I - V_{TN}]^2 = K_p [2(V_{DD} - v_I + V_{TP})(V_{DD} - v_O) - (V_{DD} - v_O)^2]$$

$$(1.1 - 0.4)^2 = 2(2.5 - 1.1 - 0.4)(V_{DD} - v_O) - (V_{DD} - v_O)^2$$

$$(V_{DD} - v_O)^2 - 2(V_{DD} - v_O) + 0.49 = 0 \Rightarrow (V_{DD} - v_O) = 0.2859$$
 V

Or $v_O = 2.5 - 0.2859 = 2.214$ V

For $v_I = 1.4$ V, NMOS in nonsaturation, PMOS in saturation

$$K_n [2(v_I - V_{TN})v_O - v_O^2] = K_p (V_{DD} - v_I + V_{TP})^2$$

$$2(1.4 - 0.4)v_O - v_O^2 = (2.5 - 1.4 - 0.4)^2$$

$$v_O^2 - 2v_O + 0.49 = 0 \Rightarrow v_O = 0.286$$
 V

16.32

(a) $K_n = \left(\frac{100}{2}\right)(2) = 100 \mu A/V^2$; $K_p = \left(\frac{40}{2}\right)(5) = 100 \mu A/V^2$

(i) By symmetry

$$V_{It} = \frac{V_{DD}}{2} = \frac{3.3}{2} = 1.65 \text{ V}$$

$$\Rightarrow V_{Op_t} = 1.65 - (-0.4) = 2.05 \text{ V}$$

$$\Rightarrow V_{On_t} = 1.65 - 0.4 = 1.25 \text{ V}$$

(iii) For $v_o = 0.25 \text{ V}$; NMOS in nonsaturation, PMOS in saturation

$$2(v_I - V_{TN})v_o - v_o^2 = (V_{DD} - v_I + V_{TP})^2$$

$$2(v_I - 0.4)(0.25) - (0.25)^2 = (3.3 - v_I - 0.4)^2$$

$$0.5v_I - 0.2 - 0.0625 = 8.41 - 5.8v_I + v_I^2$$

$$v_I^2 - 6.3v_I + 8.6725 = 0 \Rightarrow v_I = 2.03 \text{ V}$$

For $v_o = 3.05 \text{ V}$; NMOS in saturation, PMOS in nonsaturation

$$(v_I - V_{TN})^2 = 2(V_{DD} - v_I + V_{TP})(V_{DD} - v_o) - (V_{DD} - v_o)^2$$

$$(v_I - 0.4)^2 = 2(3.3 - v_I - 0.4)(3.3 - 3.05) - (3.3 - 3.05)^2$$

$$v_I^2 - 0.8v_I + 0.16 = 0.5(2.9 - v_I) - 0.0625$$

$$v_I^2 - 0.3v_I - 1.2275 = 0 \Rightarrow v_I = 1.27 \text{ V}$$

(b) $K_n = \left(\frac{100}{2}\right)(4) = 200 \mu A/V^2$; $K_p = \left(\frac{40}{2}\right)(5) = 100 \mu A/V^2$

(i) $V_{It} = \frac{3.3 - 0.4 + \sqrt{\frac{200}{100}(0.4)}}{1 + \sqrt{\frac{200}{100}}} = 1.436 \text{ V}$

$$\Rightarrow V_{Op_t} = 1.436 + 0.4 = 1.836 \text{ V}$$

$$\Rightarrow V_{On_t} = 1.436 - 0.4 = 1.036 \text{ V}$$

(iii) For $v_o = 0.25 \text{ V}$; NMOS in nonsaturation, PMOS in saturation

$$200[2(v_I - 0.4)(0.25) - (0.25)^2] = 100(3.3 - v_I - 0.4)^2$$

$$2(0.5v_I - 0.2 - 0.0625) = 8.41 - 5.8v_I + v_I^2$$

$$v_I^2 - 6.8v_I + 8.935 = 0 \Rightarrow v_I = 1.78 \text{ V}$$

For $v_o = 3.05 \text{ V}$; NMOS in saturation, PMOS in nonsaturation

$$200(v_I - 0.4)^2 = 100[2(3.3 - v_I - 0.4)(3.3 - 3.05) - (3.3 - 3.05)^2]$$

$$2(v_I^2 - 0.8v_I + 0.16) = 0.5(2.9 - v_I) - 0.0625$$

$$2v_I^2 - 1.1v_I - 1.0675 = 0 \Rightarrow v_I = 1.06 \text{ V}$$

16.33

(a) $K_n = \left(\frac{100}{2}\right)(4) = 200 \mu A/V^2$; $K_p = \left(\frac{40}{2}\right)(12) = 240 \mu A/V^2$

$$(i) \quad V_{I_t} = \frac{3.3 - 0.4 + \sqrt{\frac{200}{240}}(0.4)}{1 + \sqrt{\frac{200}{240}}} = 1.707 \text{ V}$$

$$\Rightarrow V_{O_{Pt}} = 1.707 + 0.4 = 2.107 \text{ V}$$

$$\Rightarrow V_{O_{Nt}} = 1.707 - 0.4 = 1.307 \text{ V}$$

(ii) For $v_o = 3.1 \text{ V}$; NMOS in saturation, PMOS in nonsaturation

$$200(v_I - 0.4)^2 = 240[2(3.3 - v_I - 0.4)(3.3 - 3.1) - (3.3 - 3.1)^2]$$

$$v_I^2 - 0.8v_I + 0.16 = 1.2[0.4(2.9 - v_I) - 0.04]$$

$$v_I^2 - 0.32v_I - 1.184 = 0 \Rightarrow v_I = 1.26 \text{ V}$$

(iii) For $v_o = 0.2 \text{ V}$; NMOS in nonsaturation, PMOS in saturation

$$200[2(v_I - 0.4)(0.2) - (0.2)^2] = 240(3.3 - v_I - 0.4)^2$$

$$0.4v_I - 0.2 = 1.2(8.41 - 5.8v_I + v_I^2)$$

$$1.2v_I^2 - 7.36v_I + 10.292 = 0 \Rightarrow v_I = 2.157 \text{ V}$$

$$(b) \quad K_n = \left(\frac{100}{2}\right)(6) = 300 \mu \text{A/V}^2 ; \quad K_p = \left(\frac{40}{2}\right)(4) = 80 \mu \text{A/V}^2$$

$$(i) \quad V_{I_t} = \frac{3.3 - 0.4 + \sqrt{\frac{300}{80}}(0.4)}{1 + \sqrt{\frac{300}{80}}} = 1.25 \text{ V}$$

(ii) For $v_o = 3.1 \text{ V}$; NMOS in saturation, PMOS in nonsaturation

$$300(v_I - 0.4)^2 = 80[2(3.3 - v_I - 0.4)(3.3 - 3.1) - (3.3 - 3.1)^2]$$

$$3.75(v_I^2 - 0.8v_I + 0.16) = 0.4(2.9 - v_I) - 0.04$$

$$3.75v_I^2 - 2.6v_I - 0.52 = 0 \Rightarrow v_I = 0.855 \text{ V}$$

(iii) For $v_o = 0.2 \text{ V}$; NMOS in nonsaturation, PMOS in saturation

$$300[2(v_I - 0.4)(0.2) - (0.2)^2] = 80(3.3 - v_I - 0.4)^2$$

$$3.75(0.4v_I - 0.2) = 8.41 - 5.8v_I + v_I^2$$

$$v_I^2 - 7.3v_I + 9.16 = 0 \Rightarrow v_I = 1.61 \text{ V}$$

16.34

a. For $v_{o1} = 0.6 < V_{TN} \Rightarrow v_{o2} = 5 \text{ V}$

N_1 in nonsaturation and P_1 in saturation. From Equation (16.43),

$$[2(v_I - 0.8)(0.6) - (0.6)^2] = [5 - v_I - 0.8]^2$$

$$1.2v_I - 1.32 = 17.64 - 8.4v_I + v_I^2$$

or

$$v_I^2 - 9.6v_I + 18.96 = 0$$

$$v_I = \frac{9.6 \pm \sqrt{(9.6)^2 - 4(1)(18.96)}}{2}$$

or

$v_I = 2.78 \text{ V}$

b. $V_{0Nt} \leq v_{02} \leq V_{0Pt}$

From symmetry, $V_t = 2.5 \text{ V}$

$V_{0Pt} = 2.5 + 0.8 = 3.3 \text{ V}$

and $V_{0Nt} = 2.5 - 0.8 = 1.7 \text{ V}$

So $\frac{1.7 \leq v_{02} \leq 3.3 \text{ V}}{}$

16.35

a. $V_{0Nt} \leq v_{01} \leq V_{0Pt}$

By symmetry, $V_t = 2.5 \text{ V}$

$V_{0Pt} = 2.5 + 0.8 = 3.3 \text{ V}$

and $V_{0Nt} = 2.5 - 0.8 = 1.7 \text{ V}$

So $\frac{1.7 \leq v_{01} \leq 3.3 \text{ V}}{}$

b. For $v_{02} = 0.6 < V_{TN} \Rightarrow v_{03} = 5 \text{ V}$

N_2 in nonsaturation and P_2 in saturation. From Equation (16.43),

$$\left[2(v_{I2} - 0.8)(0.6) - (0.6)^2 \right] = [5 - v_{I2} - 0.8]^2$$

$$1.2v_{I2} - 1.32 = 17.64 - 8.4v_{I2} + v_{I2}^2$$

or

$$v_{I2}^2 - 9.6v_{I2} + 18.96 = 0$$

So $v_{I2} = \underline{v_{01} = 2.78 \text{ V}}$

For $v_{01} = 2.78$, both N_1 and P_1 in saturation. Then

$v_I = 2.5 \text{ V}$

16.36

(a) $V_{I1} = \frac{2.5}{2} = 1.25 \text{ V}$

For $0.4 \leq v_I \leq 1.25 \text{ V}; i_D = 120(v_I - 0.4)^2 \mu \text{A}$

$$i_{D,peak} = 120(1.25 - 0.4)^2 = 86.7 \mu \text{A}$$

For $1.25 \leq v_I \leq 2.1 \text{ V}; i_D = 120(2.5 - v_I - 0.4)^2 \mu \text{A}$

(b) $V_{I1} = \left(\frac{1.8}{2} \right) = 0.9 \text{ V}$

For $0.4 \leq v_I \leq 0.9 \text{ V}; i_D = 120(v_I - 0.4)^2 \mu \text{A}$

$$i_{D,peak} = 120(0.9 - 0.4)^2 = 30 \mu \text{A}$$

For $0.9 \leq v_I \leq 1.4 \text{ V}; i_D = 120(1.8 - v_I - 0.4)^2 \mu \text{A}$

16.37

(a) $K_n = \left(\frac{80}{2} \right)(2) = 80 \mu \text{A/V}^2, K_p = \left(\frac{40}{2} \right)(4) = 80 \mu \text{A/V}^2$

$$V_{It} = \frac{1.8}{2} = 0.9 \text{ V}$$

$$i_{D,peak} = 80(0.9 - 0.35)^2 = 24.2 \mu \text{A}$$

$$(b) \quad K_n = \left(\frac{80}{2}\right)(2) = 80 \mu \text{A/V}^2, \quad K_p = \left(\frac{40}{2}\right)(6) = 120 \mu \text{A/V}^2$$

$$V_{It} = \frac{1.8 - 0.35 + \sqrt{\frac{80}{120}}(0.35)}{1 + \sqrt{\frac{80}{120}}} = 0.9556 \text{ V}$$

$$i_{D,peak} = 80(0.9556 - 0.35)^2 = 29.34 \mu \text{A}$$

$$(c) \quad K_n = \left(\frac{80}{2}\right)(4) = 160 \mu \text{A/V}^2, \quad K_p = \left(\frac{40}{2}\right)(4) = 80 \mu \text{A/V}^2$$

$$V_{It} = \frac{1.8 - 0.35 + \sqrt{\frac{160}{80}}(0.35)}{1 + \sqrt{\frac{160}{80}}} = 0.8057 \text{ V}$$

$$i_{D,peak} = 160(0.8057 - 0.35)^2 = 33.23 \mu \text{A}$$

16.38

$$(a) \quad K_n = \left(\frac{100}{2}\right)(3) = 150 \mu \text{A/V}^2, \quad K_p = \left(\frac{40}{2}\right)(7.5) = 150 \mu \text{A/V}^2$$

$$V_{It} = \frac{3.3}{2} = 1.65 \text{ V}$$

$$i_{D,peak} = 150(1.65 - 0.4)^2 = 234 \mu \text{A}$$

$$(b) \quad K_n = \left(\frac{100}{2}\right)(4) = 200 \mu \text{A/V}^2, \quad K_p = \left(\frac{40}{2}\right)(4) = 80 \mu \text{A/V}^2$$

$$V_{It} = \frac{3.3 - 0.4 + \sqrt{\frac{200}{80}}(0.4)}{1 + \sqrt{\frac{200}{80}}} = 1.369 \text{ V}$$

$$i_{D,peak} = 200(1.369 - 0.4)^2 = 188 \mu \text{A}$$

$$(c) \quad K_n = \left(\frac{100}{2}\right)(3) = 150 \mu \text{A/V}^2, \quad K_p = \left(\frac{40}{2}\right)(12) = 240 \mu \text{A/V}^2$$

$$V_{It} = \frac{3.3 - 0.4 + \sqrt{\frac{150}{240}}(0.4)}{1 + \sqrt{\frac{150}{240}}} = 1.796 \text{ V}$$

$$i_{D,peak} = 150(1.796 - 0.4)^2 = 292 \mu \text{A}$$

16.39

(a) $P = fC_L V_{DD}^2 = (10^7)(0.2 \times 10^{-12})(2.5)^2 \Rightarrow P = 12.5 \mu \text{W}$

(b) $P = fC_L V_{DD}^2 = (10^7)(0.2 \times 10^{-12})(1.8)^2 \Rightarrow P = 6.48 \mu \text{W}$

16.40

(a) $P = (4 \times 10^6)(150 \times 10^6)(0.12 \times 10^{-12})(1.8)^2 \Rightarrow P = 233 \text{ W}$

(b) $P = 233.28 = (4 \times 10^6)(300 \times 10^6)(0.12 \times 10^{-12})V_{DD}^2 \Rightarrow V_{DD} = 1.27 \text{ V}$

16.41

(a) $P = \frac{3}{10^7} = \underline{3 \times 10^{-7} \text{ W}}$

(b) $P = fC_L V_{DD}^2 \Rightarrow C_L = \frac{P}{fV_{DD}^2}$

(i) $C_L = \frac{3 \times 10^{-7}}{(5 \times 10^6)(5)^2} \Rightarrow \underline{C_L = 0.0024 \text{ pF}}$

(ii) $C_L = \frac{3 \times 10^{-7}}{(5 \times 10^6)(3.3)^2} \Rightarrow \underline{C_L = 0.00551 \text{ pF}}$

(iii) $C_L = \frac{3 \times 10^{-7}}{(5 \times 10^6)(1.5)^2} \Rightarrow \underline{C_L = 0.0267 \text{ pF}}$

16.42

(a) $P = \frac{10}{5 \times 10^6} = 2 \times 10^{-6} \text{ W}$

(b) $C_L = \frac{P}{fV_{DD}^2}$

(i) $C_L = \frac{2 \times 10^{-6}}{(8 \times 10^6)(5)^2} \Rightarrow \underline{C_L = 0.01 \text{ pF}}$

(ii) $C_L = \frac{2 \times 10^{-6}}{(8 \times 10^6)(3.3)^2} \Rightarrow \underline{C_L = 0.023 \text{ pF}}$

(iii) $C_L = \frac{2 \times 10^{-6}}{(8 \times 10^6)(1.5)^2} \Rightarrow \underline{C_L = 0.111 \text{ pF}}$

16.43

(a) For $v_I \approx V_{DD}$, NMOS in nonsaturation

$i_D = K_n [2(v_I - V_{TN})v_{DS} - v_{DS}^2]$ and v_{DS} very small

So $\frac{1}{r_{ds}} = \frac{di_D}{dv_{DS}} \approx K_n [2(v_I - V_{TN})]$

$$\text{Or } r_{ds} \approx \frac{1}{\left(\frac{k'_n}{2}\right)\left(\frac{W}{L}\right)_n [2(V_{DD} - V_{TN})]} = \frac{1}{k'_n \left(\frac{W}{L}\right)_n (V_{DD} - V_{TN})}$$

For $v_I \approx 0$, PMOS in nonsaturation

$$i_D = K_p [2(V_{DD} - v_I + V_{TP})v_{SD} - v_{SD}^2] \text{ and } v_{SD} \text{ very small}$$

$$\text{So } \frac{1}{r_{sd}} = \frac{di_D}{dv_{SD}} = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_p [2(V_{DD} - v_I + V_{TP})] \approx k'_p \left(\frac{W}{L}\right)_p (V_{DD} + V_{TP})$$

$$\text{Or } r_{sd} = \frac{1}{k'_p \left(\frac{W}{L}\right)_p (V_{DD} + V_{TP})}$$

$$(b) \text{ Let } \left(\frac{W}{L}\right)_n = 2, \left(\frac{W}{L}\right)_p = 4, \text{ and } V_{DD} = 5 \text{ V}$$

For NMOS:

$$r_{ds} = \frac{1}{(0.1)(2)(5-0.5)} \Rightarrow r_{ds} = 1.11 \text{ k}\Omega$$

$$i_d = \frac{v_{ds}}{r_{ds}} = \frac{0.5}{1.11} = 0.45 \text{ mA}$$

For PMOS:

$$r_{sd} = \frac{1}{(0.04)(4)(5-0.5)} \Rightarrow r_{sd} = 1.39 \text{ k}\Omega$$

$$i_d = \frac{v_{sd}}{r_{sd}} = \frac{0.5}{1.39} = 0.36 \text{ mA}$$

16.44

$$(a) V_{IL} = V_{TN} + \frac{3}{8}(V_{DD} + V_{TP} - V_{TN}) = 0.5 + \frac{3}{8}(3.3 - 0.5 - 0.5)$$

$$\Rightarrow V_{IL} = 1.3625 \text{ V}$$

$$V_{IH} = V_{TN} + \frac{5}{8}(V_{DD} + V_{TP} - V_{TN}) = 0.5 + \frac{5}{8}(3.3 - 0.5 - 0.5)$$

$$\Rightarrow V_{IH} = 1.9375 \text{ V}$$

$$(b) V_{OHU} = \frac{1}{2}[2(1.3625) + 3.3 - 0.5 + 0.5] = 3.0125 \text{ V}$$

$$V_{OLU} = \frac{1}{2}[2(1.9375) - 3.3 - 0.5 + 0.5] = 0.2875 \text{ V}$$

$$\text{Then } NM_L = 1.3625 - 0.2875 = 1.075 \text{ V}$$

$$NM_H = 3.0125 - 1.9375 = 1.075 \text{ V}$$

16.45

$$(a) \frac{K_n}{K_p} = \frac{100}{50} = 2$$

$$V_{IL} = 0.35 + \frac{(2.5 - 0.35 - 0.35)}{(2-1)} \left[2\sqrt{\frac{2}{2+3}} - 1 \right]$$

$$\Rightarrow V_{IL} = 0.8268 \text{ V}$$

$$V_{IH} = 0.35 + \frac{(2.5 - 0.35 - 0.35)}{(2-1)} \left[\frac{2(2)}{\sqrt{3(2)+1}} - 1 \right]$$

$$\Rightarrow V_{IH} = 1.2713 \text{ V}$$

$$(b) \quad V_{OHU} = \frac{1}{2} [(1+2)(0.8268) + 2.5 - (2)(0.35) + 0.35]$$

$$\Rightarrow V_{OHU} = 2.3152 \text{ V}$$

$$V_{OLU} = \frac{(1.2713)(1+2) - 2.5 - (2)(0.35) + 0.35}{2(2)}$$

$$\Rightarrow V_{OLU} = 0.2410 \text{ V}$$

$$\text{Then } NM_L = 0.8268 - 0.2410 = 0.5858 \text{ V}$$

$$NM_H = 2.3152 - 1.2713 = 1.0439 \text{ V}$$

16.46

$$(a) \quad K_n = \left(\frac{100}{2} \right)(2) = 100 \mu \text{A/V}^2, \quad K_p = \left(\frac{40}{2} \right)(5) = 100 \mu \text{A/V}^2$$

$$V_{IL} = 0.4 + \frac{3}{8}(3.3 - 0.4 - 0.4) = 1.3375 \text{ V}$$

$$V_{IH} = 0.4 + \frac{5}{8}(3.3 - 0.4 - 0.4) = 1.9625 \text{ V}$$

$$V_{OHU} = \frac{1}{2}[2(1.3375) + 3.3 - 0.4 + 0.4] = 2.9875 \text{ V}$$

$$V_{OLU} = \frac{1}{2}[2(1.9625) - 3.3 - 0.4 + 0.4] = 0.3125 \text{ V}$$

$$\text{Then } NM_L = 1.3375 - 0.3125 = 1.025 \text{ V}$$

$$NM_H = 2.9875 - 1.9625 = 1.025 \text{ V}$$

$$(b) \quad K_n = \left(\frac{100}{2} \right)(4) = 200 \mu \text{A/V}^2, \quad K_p = \left(\frac{40}{2} \right)(12) = 240 \mu \text{A/V}^2$$

$$\frac{K_n}{K_p} = \frac{200}{240} = 0.8333$$

$$V_{IL} = 0.4 + \frac{(3.3 - 0.4 - 0.4)}{(0.8333 - 1)} \left[2\sqrt{\frac{0.8333}{3.8333}} - 1 \right] = 1.4127 \text{ V}$$

$$V_{IH} = 0.4 + \frac{(3.3 - 0.4 - 0.4)}{(0.8333 - 1)} \left[\frac{2(0.8333)}{\sqrt{3(0.8333)+1}} - 1 \right] = 2.0370 \text{ V}$$

$$\text{Now } V_{OHU} = \frac{1}{2}[(1+0.8333)(1.4127) + 3.3 - (0.8333)(0.4) + 0.4]$$

$$\Rightarrow V_{OHU} = 2.9783 \text{ V}$$

$$V_{OLU} = \frac{2.0370(1+0.8333) - 3.3 - (0.8333)(0.4) + 0.4}{2(0.8333)}$$

$$\Rightarrow V_{OLU} = 0.3007 \text{ V}$$

Then $NM_L = 1.4127 - 0.3007 = 1.1112 \text{ V}$
 $NM_H = 2.9783 - 2.0370 = 0.9413 \text{ V}$

16.47

a. $v_A = v_B = 5 \text{ V}$

N_1 and N_2 on, so $v_{DS1} \approx v_{DS2} \approx 0 \text{ V}$

P_1 and P_2 off

So we have a $P_3 - N_3$ CMOS inverter. By symmetry, $v_c = 2.5 \text{ V}$ (Transition Point).

b. For $v_A = v_B = v_C \equiv v_I$

Want $K_{n,eff} = K_{p,eff}$

$$\frac{k'_n}{2} \cdot \left(\frac{W}{3L}\right)_n = \frac{k'_p}{2} \cdot \left(\frac{3W}{L}\right)_p$$

With $k'_n = 2k'_p$, then

$$\frac{2}{2} \cdot \frac{1}{3} \cdot \left(\frac{W}{L}\right)_n = \frac{1}{2} \cdot 3 \cdot \left(\frac{W}{L}\right)_p$$

$$\text{Or } \frac{\left(\frac{W}{L}\right)_n}{\left(\frac{W}{L}\right)_p} = \frac{9}{2} \cdot \left(\frac{W}{L}\right)_p$$

c. We have

$$K_n = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_n = \left(\frac{2k'_p}{2}\right) \left(\frac{9}{2}\right) \left(\frac{W}{L}\right)_p$$

$$K_p = \left(\frac{k'_p}{2}\right) \left(\frac{W}{L}\right)_p$$

Then from Equation (16.41)

$$V_h = \frac{5 + (-0.8) + \sqrt{\frac{K_n}{K_p}} \cdot (0.8)}{1 + \sqrt{\frac{K_n}{K_p}}}$$

Now

$$\frac{K_n}{K_p} = (2) \left(\frac{9}{2}\right) = 9$$

Then

$$V_h = \frac{5 + (-0.8) + 3(0.8)}{1 + 3} \Rightarrow V_h = 1.65 \text{ V}$$

16.48

By definition, NMOS is on if gate voltage is 5 V and is off if gate voltage is 0 V.

State	$\underline{N_1}$	$\underline{N_2}$	$\underline{N_3}$	$\underline{N_4}$	$\underline{N_5}$	$\underline{\frac{v_0}{0}}$
1	off	on	off	on	on	0

2	off	off	on	on	off	0
3	on	on	off	off	on	5
4	on	on	off	on	on	0

Logic function $(v_x \text{ OR } v_y) \otimes (v_x \text{ AND } v_z)$

Exclusive OR of $(v_x \text{ OR } v_y)$ with $(v_x \text{ AND } v_z)$

16.49

NMOS in Parallel $\Rightarrow \left(\frac{W}{L}\right)_n = 2$

4-PMOS in series $\Rightarrow \left(\frac{W}{L}\right)_p = 4(4) = 16$

(b) C_L doubles \Rightarrow current must double to maintain switching speed.

$\Rightarrow \left(\frac{W}{L}\right)_n = 4$

$\left(\frac{W}{L}\right)_p = 32$

16.50

4-NMOS in series $\left(\frac{W}{L}\right)_n = 4(2) = 8$

4-PMOS in parallel $\left(\frac{W}{L}\right)_p = 4$

$\left(\frac{W}{L}\right)_n = 16$

(b) $\left(\frac{W}{L}\right)_p = 8$

16.51

(a) NMOS in parallel $\Rightarrow \left(\frac{W}{L}\right)_n = 2$

3-PMOS in series $\Rightarrow \left(\frac{W}{L}\right)_p = 3(4) = 12$

$\left(\frac{W}{L}\right)_n = 4$

(b) $\left(\frac{W}{L}\right)_p = 24$

16.52

(a) 3-NMOS in series $\left(\frac{W}{L}\right)_n = 3(2) = 6$

3-PMOS in parallel $\left(\frac{W}{L}\right)_p = 4$

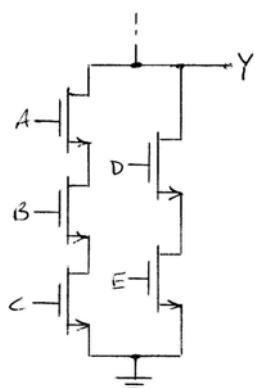
$\left(\frac{W}{L}\right)_n = 12$
 $\left(\frac{W}{L}\right)_p = 8$

(b)

16.53

(a) $Y = \overline{ABC + DE}$

(b)



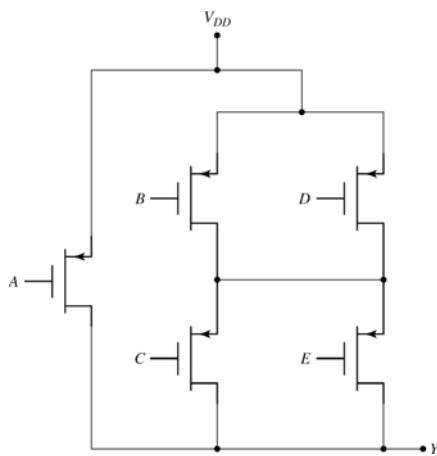
(c) $(W/L)_{An,Bn,Cn} = 6$, $(W/L)_{Dn,E_n} = 4$

All $(W/L)_p = 8$

16.54

(a) $Y = \overline{A(BD + CE)}$

(b)



(c) NMOS: 3 transistors in series for pull down mode.

$$\left(\frac{W}{L}\right)_r = 2(3)(2) = 12$$

For twice the speed:

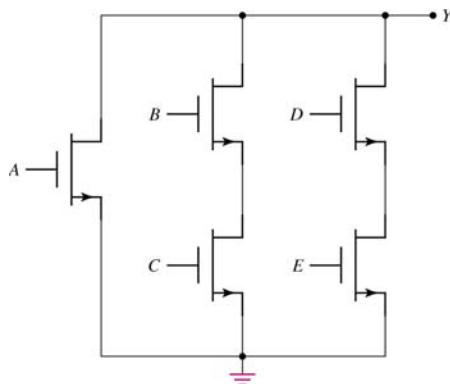
$$\text{PMOS: } \left(\frac{W}{L}\right)_{P,A} = 2(4) = 8$$

$$\left(\frac{W}{L}\right)_{P,B,C,D,E} = 2(2)(4) = 16$$

16.55

$$(a) \quad Y = \overline{A + BC + DE}$$

(a)
(b)



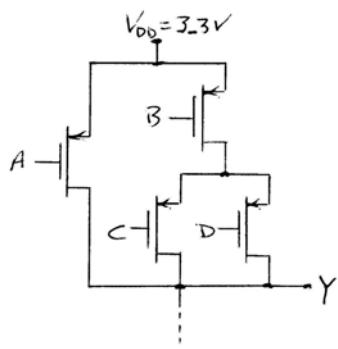
$$(c) \quad \text{NMOS: } \left(\frac{W}{L} \right)_{n,A} = 2 \quad \left(\frac{W}{L} \right)_{n,B,C,D,E} = 4$$

PMOS: 3 transistors in series for the pull-up mode

$$\left(\frac{W}{L}\right)_p = 3(4) = 12$$

16.56

$$(a) \quad Y = \overline{A[B + CD]}$$



(b)

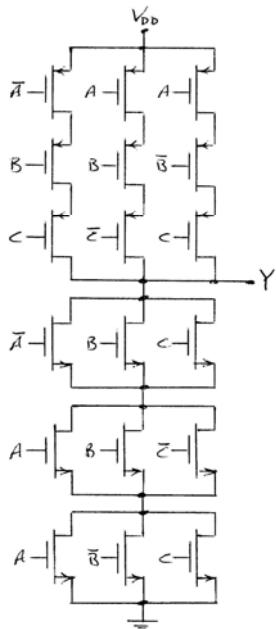
$$(c) \quad (W/L)_{An,Bn} = 4, \quad (W/L)_{Cn,Dn} = 8$$

$$(W/L)_{Ap} = 4, \quad (W/L)_{Bp,Cp,Dp} = 8$$

16.57

(a) For $Y = A\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC$

We have $\bar{Y} = \overline{A\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC} = (\bar{A} + B + C)(A + B + \bar{C})(A + \bar{B} + C)$

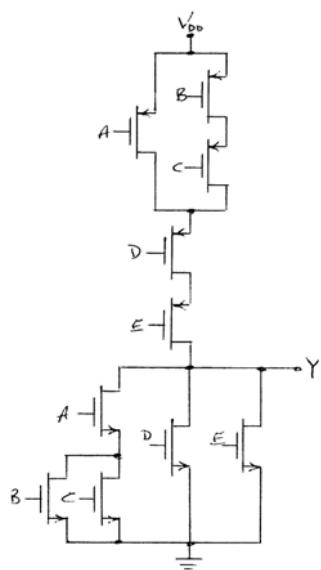


(b) All $(W/L)_n = 3$

All $(W/L)_p = 6$

16.58

(a)



(b) $(W/L)_{An,Bn,Cn} = 2$, $(W/L)_{Dn,En} = 1$

$(W/L)_{Ap,Dp,Ep} = 6$, $(W/L)_{Bp,Cp} = 12$

16.59

- (a) $\bar{Y} = C(A + B)$
 (b) All $(W/L)_n = 4$
 $(W/L)_{Cp} = 4, (W/L)_{Ap,Bp} = 8$

16.60

- (a) All $(W/L)_n = 2$
 All $(W/L)_p = 20$
 (b) All $(W/L)_n = 10$
 All $(W/L)_p = 4$

16.61

By definition:

NMOS off if gate voltage = 0
 NMOS on if gate voltage = 5 V
 PMOS off if gate voltage = 5 V
 PMOS on if gate voltage = 0

State	N_1	P_1	N_A	N_B	N_C	v_{01}	N_2	P_2	v_{02}
1	off	on	off	off	off	5	on	off	0
2	on	off	on	off	off	5	on	off	0
3	off	on	off	off	off	5	on	off	0
4	on	off	off	off	on	5	on	off	0
5	off	on	off	off	off	5	on	off	0
6	on	off	off	on	on	0	off	on	5

Logic function is

$$v_{02} = (v_A \text{ OR } v_B) \text{ AND } v_C$$

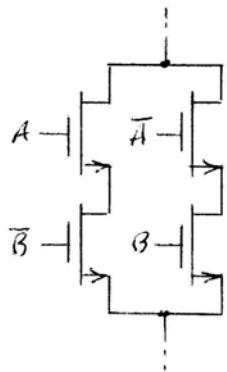
16.62

State	v_{01}	v_{02}	v_{03}
1	5	5	0
2	0	0	5
3	5	5	0
4	5	0	5
5	5	5	0
6	0	5	0

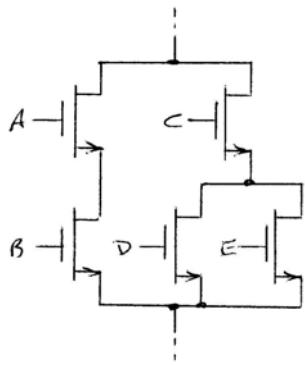
Logic function:

$$v_{03} = (v_X \text{ OR } v_Z) \text{ AND } v_Y$$

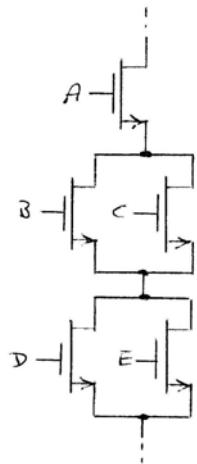
16.63



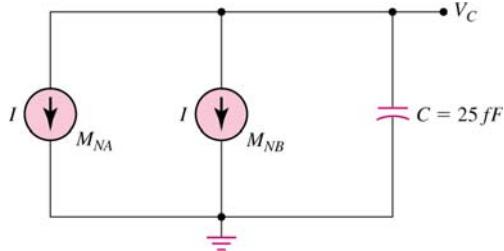
16.64



16.65



16.66



$$2I = -C \frac{dV_C}{dt}$$

So

$$\Delta V_C = -\frac{1}{C}(2I) \cdot t$$

For $\Delta V_C = -0.5 \text{ V}$

$$-0.5 = -\frac{2(2 \times 10^{-12}) \cdot t}{25 \times 10^{-15}} \Rightarrow t = \underline{\underline{3.125 \text{ ms}}}$$

16.67

(a) (i) $v_o = 0$

(ii) $v_o = \phi - V_{TN} = 3.3 - 0.4 = 2.9 \text{ V}$

(iii) $v_o = 2.5 \text{ V}$

(b) (i) $v_o = 0$

(ii) $v_o = \phi - V_{TN} = 1.8 - 0.4 = 1.4 \text{ V}$

(iii) $v_o = \phi - V_{TN} = 1.8 - 0.4 = 1.4 \text{ V}$

16.68

(a) (i) $v_o = 0$

(ii) $v_o = \phi - V_{TN} = 2.5 - 0.5 = 2 \text{ V}$

(iii) $v_o = 1.8 \text{ V}$

(b) (i) $v_o = 0$

(ii) $v_o = \phi - V_{TN} = 2 - 0.5 = 1.5 \text{ V}$

(iii) $v_o = \phi - V_{TN} = 2 - 0.5 = 1.5 \text{ V}$

16.69

(a) $v_{o1} = 2.5 - 0.4 = 2.1 \text{ V}$

$v_{o2} = 2.5 \text{ V}$

(b) $v'_{I1} = 2.5 - 0.4 = 2.1 \text{ V}$

$$\frac{(W/L)_1}{(W/L)_2} [2(2.1 - 0.4)(0.1) - (0.1)^2] = (2.5 - 0.1 - 0.4)^2$$

$$(W/L)_1[0.33] = 4 \Rightarrow \left(\frac{W}{L}\right)_1 = 12.1$$

$$v'_{I3} = 2.1 \text{ V}$$

$$\frac{(W/L)_3}{(W/L)_4} [2(2.1 - 0.4)(0.1) - (0.1)^2] = [-(-0.6)]^2$$

$$(W/L)_3[0.33] = 0.36 \Rightarrow \left(\frac{W}{L}\right)_3 = 1.09$$

16.70

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0.1 \Rightarrow indeterminate

Without the top transistor, the circuit performs the exclusive-NOR function.

16.71

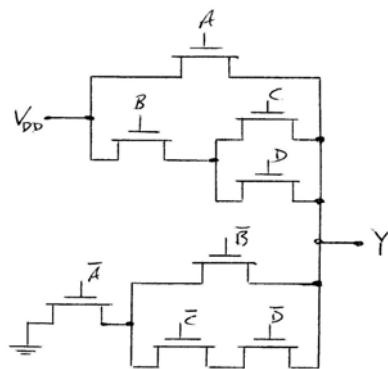
A	\bar{A}	B	\bar{B}	Y	Z
0	1	0	1	0	1
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1	0	1	0

$$Y = A + \bar{A}B = A + B$$

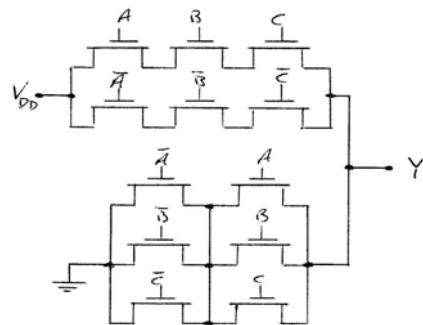
$$Z = \bar{Y} \text{ or } Z = \bar{A}\bar{B}$$

16.72

(a)



(b)



16.73

- (a) (i) $Y = 0$
 (ii) $Y = 2.5 \text{ V}$
 (iii) $Y = 0$
 (iv) $Y = 2.5 \text{ V}$

(b) (i) $Y = 0$
 (ii) $Y = 0$
 (iii) $Y = 2.5 \text{ V}$
 (iv) $Y = 2.5 \text{ V}$

(c) For $\phi = 1, \bar{\phi} = 0$; then $Y = B$
 For $\phi = 0, \bar{\phi} = 1$; then $Y = A$
 A multiplexer

16.74

$$Y = AC + B\bar{C}$$

16.75

- (a) (i) $Y = 0$
 (ii) $Y = 2.5 \text{ V}$
 (iii) $Y = 2.5 \text{ V}$
 (iv) $Y = 0$

(b) $Y = A\bar{B} + \bar{A}B = A \otimes B$

16.76

<u>A</u>	<u>B</u>	<u>Y</u>
0	0	0
1	0	1
0	1	1
1	1	0

Exclusive-OR function.

16.77

This circuit is referred to as a two-phase ratioed circuit. The same width-to-length ratios between the driver and load transistors must be maintained as discussed previously with the enhancement load inverter.

When ϕ_1 is high, v_{o1} becomes the complement of v_I . When ϕ_2 goes high, then v_0 becomes the complement of v_{o1} or is the same as v_I . The circuit is a shift register.

16.78

Want \bar{Q} to be the transition point of $M_5 - M_6$.

From Equation (16.26(b)),

$$\sqrt{\frac{K_5}{K_6}}(V_{I_t} - V_{TN5}) = -V_{TN6}$$

$$\sqrt{\frac{100}{40}}(V_{I_t} - 0.4) = [-(-0.6)] \Rightarrow V_{I_t} = \bar{Q} = 0.7795 \text{ V}$$

This is region where M_1 and M_3 are biased in the saturation region.

$$\sqrt{\frac{K_1}{K_3}}(V_{I_t} - V_{TN1}) = -V_{TN3}$$

$$\sqrt{\frac{150}{40}}(V_{I_t} - 0.4) = [-(-0.6)] \Rightarrow V_{I_t} = S = 0.7098 \text{ V}$$

This analysis neglected the effect of M_2 starting to conduct.

16.79

$$v_I = \frac{3.3 + (-0.4) + 0.5}{1+1} = 1.7 \text{ V}$$

$v_I = 1.5 \text{ V}$ NMOS Sat; PMOS Non Sat

$$(v_I - 0.5)^2 = [2(3.3 - v_I - 0.4)(3.3 - v_{o1}) - (3.3 - v_{o1})^2] \Rightarrow v_{o1} = 2.88 \text{ V}$$

$$v_I = 1.6 \text{ V} \quad v_{o1} = 2.693 \text{ V}$$

$v_I = 1.7 \text{ V}$ v_{o1} = variable (switching region)

$v_I = 1.8 \text{ V}$ NMOS Non Sat; PMOS Sat

$$(3.3 - V_I - 0.4)^2 = [2(v_I - 0.5)v_{o1} - v_{o1}^2] \Rightarrow v_{o1} = 0.607 \text{ V}$$

Now

$$v_I = 1.5 \text{ V}, v_{o1} = 2.88 \text{ V} \Rightarrow v_o \approx 0 \text{ V}$$

$$v_I = 1.6 \text{ V}, v_{o1} = 2.693 \text{ V}$$

NMOS Non Sat; PMOS Sat

$$(3.3 - v_{o1} - 0.4)^2 = [2(v_{o1} - 0.5)v_o - v_o^2]$$

$$v_o = 0.00979 \text{ V}$$

$v_I = 1.7 \text{ V}$, v_{o1} = Switching Mode $\Rightarrow v_0$ = Switching Mode.

$v_I = 1.8 \text{ V}$, $v_{o1} = 0.607 \text{ V}$ NMOS Sat; PMOS Non Sat

$$(v_{o1} - 0.5)^2 = [2(3.3 - v_{o1} - 0.4)(3.3 - v_0) - (3.3 - v_0)^2] \Rightarrow v_0 = 3.298 \text{ V}$$

16.80

For $R = \phi = V_{DD}$ and $S = 0 \Rightarrow Q = 0, \bar{Q} = 1$

For $S = \phi = V_{DD}$ and $R = 0 \Rightarrow Q = 1, \bar{Q} = 0$

The signal ϕ is a clock signal.

For $\phi = 0$, The output signals will remain in their previous state.

16.81

a. Positive edge triggered flip-flop when $CLK = 1$, output of first inverter is \bar{D} and then $Q = \bar{\bar{D}} = D$.

b. For example, put a CMOS transmission gate between the output and the gate of M_1 driven by a \overline{CLK} pulse.

16.82

For $J = 1, K = 0$, and $CLK = 1$; this makes $Q = 1$ and $\bar{Q} = 0$.

For $J = 0, K = 1$, and $CLK = 1$, and if $Q = 1$, then the circuit is driven so that $Q = 0$ and $\bar{Q} = 1$.

If initially, $Q = 0$, then the circuit is driven so that there is no change and $Q = 0$ and $\bar{Q} = 1$.

$J = 1, K = 1$, and $CLK = 1$, and if $Q = 1$, then the circuit is driven so that $Q = 0$.

If initially, $Q = 0$, then the circuit is driven so that $Q = 1$.

So if $J = K = 1$, the output changes state.

16.83

For $J = v_x = 1, K = v_y = 0$, and $CLK = v_z = 1$, then $v_0 = 0$.

For $J = v_x = 0, K = v_y = 1$, and $CLK = v_z = 1$, then $v_0 = 1$.

Now consider $J = K = CLK = 1$. With $v_x = v_z = 1$, the output is always $v_0 = 0$. So the output does not change state when $J = K = CLK = 1$. This is not actually a $J - K$ flip-flop.

16.84

(a) $256K \Rightarrow 262,144 \text{ cells} \Rightarrow 512 \times 512$

Each decoder $\Rightarrow 9$ inputs

(b) (i) Row 52, address = 000110011

(ii) Row 129, address = 010000000

(iii) Row 241, address = 011110000

(c) (i) Column 24, address = 000010111

(ii) Column 165, address = 010100100

(iii) Column 203, address = 011001010

16.85

1-Megabit memory \Rightarrow

(a) $= 1,048,576 \Rightarrow 1024 \times 1024$

Number of input row and column decodes lines necessary $= 10$

(b) $250K \times 4 \text{ bits} \Rightarrow 262,144 \times 4 \text{ bits} \Rightarrow 512 \times 512$

For 512 lines $\Rightarrow 9$ row and column decoder lines necessary.

16.86

32 rows \times 16 columns
Each column contains 8 bits

16.87

Assume the address line is initially uncharged, then

$$I = C \frac{dV_C}{dt} \quad \text{or} \quad V_C = \frac{1}{C} \int I dt = \frac{I}{C} \cdot t$$

$$t = \frac{V_C \cdot C}{I} = \frac{(2.7)(5.8 \times 10^{-12})}{250 \times 10^{-6}} \Rightarrow$$

Then

$$\underline{t = 6.26 \times 10^{-8} s \Rightarrow 62.6 \text{ ns}}$$

16.88

$$(a) \frac{V_{DD} - Q}{R} = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right)_D [2(V_{GSD} - V_{TN})Q - Q^2]$$

$$\frac{2.5 - 0.02}{1} = \left(\frac{80}{2} \right) \left(\frac{W}{L} \right)_D [2(2.5 - 0.4)(0.02) - (0.02)^2]$$

$$0.062 = \left(\frac{W}{L} \right)_D (0.0836) \Rightarrow \left(\frac{W}{L} \right)_D = 0.74$$

(b) 16K \Rightarrow 16,384 cells

$$i_D \cong \frac{V_{DD}}{2} = \frac{1.2}{1} = 1.2 \mu A$$

$$P = (1.2)(1.2)(16,384) \Rightarrow P = 23.6 \text{ mW}$$

16.89

16 K \Rightarrow 16,384 cells

$$P_T = 200 \text{ mW} \Rightarrow \text{Power per cell} = \frac{200}{16,384} \Rightarrow 12.2 \mu W$$

$$i_D = \frac{P}{V_{DD}} = \frac{12.2}{2.5} = 4.88 \mu A \cong \frac{V_{DD}}{R} = \frac{2.5}{R} \Rightarrow R = 0.512 M\Omega$$

If we want $v_o = 0.1 V$ for a logic 0, then

$$i_D = \left(\frac{k'_n}{2} \right) \left(\frac{W}{L} \right) [2(V_{DD} - V_{TN})v_o - v_o^2]$$

$$4.88 = \left(\frac{35}{2} \right) \left(\frac{W}{L} \right) [2(2.5 - 0.7)(0.1) - (0.1)^2]$$

$$\text{So } \underline{\left(\frac{W}{L} \right) = 0.797}$$

16.90

$$\overline{D} = V_{DD} = 2.5 \text{ V}$$

Assume M_{P3} in saturation; M_{NA} , M_{N1} in nonsaturation

$$I_{DP3} = I_{DNA} = I_{DN1}$$

$$I_{DP3} = \left(\frac{k'_p}{2} \right) \left(\frac{W}{L} \right)_3 (V_{SG3} + V_{TP})^2 = \left(\frac{35}{2} \right) (1)(2.5 - 0.4)^2 = 77.175 \mu A$$

$$I_{DP3} = I_{DN1}$$

$$77.175 = \left(\frac{80}{2} \right) (2)[2(2.5 - 0.4)Q - Q^2]$$

We find $Q^2 - 4.2Q + 0.9647 = 0 \Rightarrow Q = 0.244 \text{ V}$

$$I_{DP3} = I_{DNA}$$

$$77.175 = \left(\frac{80}{2} \right) (1)[2(2.5 - 0.244 - 0.4)(D - 0.244) - (D - 0.244)^2]$$

$$1.9294 = [3.712(D - 0.244) - (D - 0.244)^2]$$

$$= 3.712D - 0.9057 - D^2 + 0.488D - 0.05954$$

We find $D^2 - 4.2D + 2.895 = 0 \Rightarrow D = 0.869 \text{ V}$

2nd approximation, M_{P3} in nonsaturation

Assume $V_{DS} = V_{DD} - D \approx 2.5 - 0.869 = 1.631 \text{ V}$

$$I_{DP3} = \left(\frac{35}{2} \right) (1)[2(2.5 - 0.4)(1.631) - (1.631)^2] = 73.33 \mu A$$

$$I_{DP3} = I_{DN1}$$

$$73.33 = \left(\frac{80}{2} \right) (2)[2(2.5 - 0.4)Q - Q^2]$$

$$Q^2 - 4.2Q + 0.9166 = 0 \Rightarrow Q = 0.231 \text{ V}$$

$$I_{DP3} = I_{DNA}$$

$$73.33 = \left(\frac{80}{2} \right) (1)[2(2.5 - 0.231 - 0.4)(D - 0.231) - (D - 0.231)^2]$$

$$1.833 = [3.738(D - 0.231) - (D^2 - 0.462D + 0.05336)]$$

We find $D^2 - 4.2D + 2.75 = 0 \Rightarrow D = 0.812 \text{ V}$

16.91

Approximation, M_{N2} cutoff

$$I_{DP2} = I_{DB}, \text{ assume } \bar{D} = 0$$

Both M_{P2} and M_B in nonsaturation

$$\left(\frac{35}{2} \right) (4)[2(2.5 - 0.4)(2.5 - \bar{Q}) - (2.5 - \bar{Q})^2]$$

$$= \left(\frac{80}{2} \right) (1)[2(2.5 - 0.4)\bar{Q} - \bar{Q}^2]$$

$$70[4.2(2.5 - \bar{Q}) - (6.25 - 5\bar{Q} + \bar{Q}^2)] = 40[4.2\bar{Q} - \bar{Q}^2]$$

We find $0.75\bar{Q}^2 + 2.8\bar{Q} - 7.4375 = 0 \Rightarrow \bar{Q} = 1.794 \text{ V}$

Approximation, M_{P1} cutoff; assume $D = 2.5 \text{ V}$

$$I_{DNA} = I_{DN1}$$

Both M_A and M_{N1} in nonsaturation

$$\begin{aligned} & \left(\frac{80}{2} \right) (1) [2(2.5 - Q - 0.4)(2.5 - Q) - (2.5 - Q)^2] \\ &= \left(\frac{80}{2} \right) (2) [2(2.5 - 0.4)Q - Q^2] \\ & 2(2.1 - Q)(2.5 - Q) - (2.5 - Q)^2 = 2(4.2Q - Q^2) \\ & 2(5.25 - 2.1Q - 2.5Q + Q^2) - (6.25 - 5Q + Q^2) = 8.4Q - 2Q^2 \\ & \text{We find } 3Q^2 - 12.6Q + 4.25 = 0 \Rightarrow Q = 0.370 \text{ V} \end{aligned}$$

16.92

For Logic 1, v_1 :

$$(5)(0.05) + (4)(1) = (1 + 0.05)v_1 \Rightarrow v_1 = 4.0476 \text{ V}$$

v_2 :

$$(5)(0.025) + (4)(1) = (1 + 0.025)v_2 \Rightarrow v_2 = 4.0244 \text{ V}$$

For Logic 0, v_1 :

$$(0)(0.05) + (4)(1) = (1 + 0.05)v_1 \Rightarrow v_1 = 3.8095 \text{ V}$$

v_2 :

$$(0)(0.025) + (4)(1) = (1 + 0.025)v_2 \Rightarrow v_2 = 3.9024 \text{ V}$$

16.93

Design Problem

16.94

Design Problem

16.95

Design Problem

16.96

(a) Quantization error $= \frac{1}{2} LSB \leq 1\% \leq 0.05 \text{ V}$

Or $LSB \leq 0.10 \text{ V}$

For a 6-bit word, $LSB = \frac{5}{64} = 0.078125 \text{ V}$

(b) $1 - LSB = \frac{5}{64} = 0.078125 \text{ V}$

(c) $\frac{3.5424}{5} \times 64 = 45.34 \Rightarrow n = 45$

Digital Output = 101101

$$\frac{45 \times 5}{64} = 3.515625$$

$$\Delta = 3.5424 - 3.515625 = 0.026775 < \frac{1}{2} LSB.$$

16.97

(a) Quantization error = $\frac{1}{2}$ LSB $\leq 0.5\% \leq 0.0165$ V

$1 - \text{LSB} = 0.033$ V

For a 7-bit word, LSB = $\frac{3.3}{128} = 0.0258$ V

(b) $1 - \text{LSB} = 0.02578125$ V

(c) $\frac{2.5321}{3.3} \times 128 = 98.2 \Rightarrow n = 98$

Digital output = 1100010

Now $\frac{(98)(3.3)}{128} = 2.5265625$ V

$\Delta = 2.5321 - 2.5265625 = 0.0055375$ V $< \frac{1}{2}$ LSB

16.98

(a) $v_o = \left(\frac{0}{2} + \frac{1}{4} + \frac{1}{8} + \frac{0}{16} \right)(5) = 1.875$ V

(b) $v_o = \left(\frac{1}{2} + \frac{0}{4} + \frac{0}{8} + \frac{1}{16} \right)(5) = 2.8125$ V

16.99

LSB = $\left(\frac{1}{16} \right)(5) = 0.3125$ V

(a) $\frac{1}{2}$ LSB = 0.15625 V

$v_o = \left(\frac{10}{20 + \Delta R_l} \right)(5)$

Now $v_o = 2.5 + 0.15625 = 2.65625$ V

For $v_o = 2.5 - 0.15625 = 2.34375$ V

$20 + \Delta R_l = \frac{(10)(5)}{2.65625} \Rightarrow \Delta R_l = -1.176$ K

For $v_o = 2.5 - 0.15625 = 2.34375$ V

$20 + \Delta R_l = \frac{(10)(5)}{2.34375}$ V

$\Delta R_l = +1.333$ K

For $|\Delta R_l| = 1.176$ K $\Rightarrow \Delta R_l = 5.88\%$

(b) For $R_4: v_o = \left(\frac{10}{160 + \Delta R_4} \right)(5)$

$$v_o = 0.3125 + 0.15625 = 0.46875 \text{ V}$$

$$160 + \Delta R_4 = \frac{(10)(5)}{0.46875} \Rightarrow \Delta R_4 = -53.33 \text{ K}$$

Or $v_o = 0.3125 - 0.15625 = 0.15625 \text{ V}$

$$160 + \Delta R_4 = \frac{(10)(5)}{0.15625} \Rightarrow \Delta R_4 = 160 \text{ K}$$

For $|\Delta R_4| = 53.33 \text{ K} \Rightarrow \underline{\Delta R_4 = 33.33\%}$

16.100

$$R_5 = 320 \text{ k}\Omega$$

$$R_6 = 640 \text{ k}\Omega$$

$$R_7 = 1280 \text{ k}\Omega$$

(a) $R_8 = 2560 \text{ k}\Omega$

(b) $v_o = \left(\frac{10}{2560} \right)(5) = 0.01953125 \text{ V}$

16.101

(a)

$$I_1 = \frac{V_{REF}}{2R} = \frac{-5}{10} \Rightarrow I_1 = -0.50 \text{ mA}$$

$$I_2 = \frac{I_1}{2} = -0.25 \text{ mA}$$

$$I_3 = \frac{I_2}{2} = -0.125 \text{ mA}$$

$$I_4 = \frac{I_3}{2} = -0.0625 \text{ mA}$$

$$I_5 = \frac{I_4}{2} = -0.03125 \text{ mA}$$

$$I_6 = \frac{I_5}{2} = -0.015625 \text{ mA}$$

$$\Delta v_o = |I_6| R_F = (0.015625)(5)$$

(b) $\Delta v_o = 0.078125 \text{ V}$

$$v_o = -[I_2 + I_5 + I_6] R_F = [0.25 + 0.03125 + 0.015625](5)$$

(c) $\underline{v_o = 1.484375 \text{ V}}$

(d) For 101010; $v_o = (0.50 + 0.125 + 0.03125)(5) = 3.28125 \text{ V}$
For 010101; $v_o = (0.25 + 0.0625 + 0.015625)(5) = 1.640625 \text{ V}$
 $\underline{\Delta v_o = 1.640625 \text{ V}}$

16.102

$$\frac{1}{2} \text{ LSB} = \left(\frac{V_{REF}}{8R} \right) \left(\frac{R}{2} \right) = \frac{3.3}{16} = 0.20625 \text{ V}$$

Ideal

$$v_A \text{ for 101} \Rightarrow \left(\frac{5V_{REF}}{8R} \right)(R) = \frac{5}{8}(3.3) = 2.0625 \text{ V}$$

$$\text{Range of } v_A = 2.0625 \pm \frac{1}{2} \text{ LSB}$$

$$1.85625 \leq v_A \leq 2.26875 \text{ V}$$

16.103

$$6\text{-bits} \Rightarrow 2^6 = 64 \text{ resistors}$$

$$2^6 - 1 = 63 \text{ comparators}$$

16.104

(a) 10-bit output $\Rightarrow 1024$ clock periods

$$\frac{1}{f} = \frac{1}{10^6} = 1 \mu\text{s}$$

1 clock period

$$\text{Maximum conversion time} = 1024 \mu\text{s} = 1.024 \text{ mS}$$

(b)

$$\frac{1}{2} \text{ LSB} = \frac{1}{2} \left(\frac{5}{1024} \right) = 0.002441406 \text{ V}$$

$$v'_A = (128 + 16 + 2) \left(\frac{5}{1024} \right) = 0.712890625 \text{ V}$$

$$\text{So range of } v_A = v'_A \pm \frac{1}{2} \text{ LSB}$$

$$0.710449219 \leq v_A \leq 0.715332031 \text{ V}$$

(c) 0100100100 $\Rightarrow 256 + 32 + 4 = 292$ clock pulses

16.105

$$(a) 3.125 = \frac{N \times 5}{1024} \Rightarrow N = 640 \Rightarrow 512 + 128$$

$$\text{Output} = \underline{1010000000}$$

(b)

$$1.8613 = \frac{N \times 5}{1024} \Rightarrow N = 381.19 \Rightarrow N = 381 \Rightarrow 256 + 64 + 32 + 16 + 8 + 4 + 1$$

$$\text{Output} = \underline{0101111101}$$

Chapter 17

17.1

(a) $R_C = \frac{0 - (-0.2)}{0.1} = 2 \text{ k}\Omega$

(b) (i) $v_1 = -1 \text{ V}$, Q_1 off, Q_2 on

$$v_{O2} = 0 - (0.2)(2) = -0.4 \text{ V}$$

$$v_{O1} = 0$$

(ii) $v_1 = -0.4 \text{ V}$, Q_1 on, Q_2 off

$$v_{O1} = 0 - (0.2)(2) = -0.4 \text{ V}$$

$$v_{O2} = 0$$

(c) For (i) and (ii)

$$P = (I_Q)(0 - V^-) = (0.2)(1.8) = 0.36 \text{ mW}$$

17.2

(a) $i_E = \frac{-1 - 0.7 - (-2.5)}{R_E} = 0.08 \text{ mA}$, $\Rightarrow R_E = 10 \text{ k}\Omega$

$$R_C = \frac{0 - (-0.25)}{0.04} = 6.25 \text{ k}\Omega$$

(b) (i) $v_1 = -1.3 \text{ V}$, Q_1 off, Q_2 on

$$i_E = \frac{-1 - 0.7 - (-2.5)}{10} = 0.08 \text{ mA}$$

$$v_{O2} = 0 - (0.08)(6.25) = -0.50 \text{ V}$$

$$v_{O1} = 0$$

(ii) $v_1 = -0.7 \text{ V}$, Q_1 on, Q_2 off

$$i_E = \frac{-0.7 - 0.7 - (-2.5)}{10} = 0.11 \text{ mA}$$

$$v_{O1} = 0 - (0.11)(6.25) = -0.6875 \text{ V}$$

$$v_{O2} = 0$$

(c) (i) $i_E = 0.08 \text{ mA}$, $P = (0.08)(2.5) = 0.2 \text{ mW}$

(ii) $i_E = 0.11 \text{ mA}$, $P = (0.11)(2.5) = 0.275 \text{ mW}$

17.3

(a) $i_{C2} = I_Q = 0.5 = \frac{3 - 0}{R_{C2}} \Rightarrow R_{C2} = 6 \text{ K}$

(b) $i_{C1} = I_Q = 0.5 = \frac{3 - 1}{R_{C1}} \Rightarrow R_{C1} = 4 \text{ K}$

$$\frac{i_{C1}}{I_Q} = \frac{I_S \exp\left(\frac{V_{BE1}}{V_T}\right)}{I_S \left[\exp\left(\frac{V_{BE1}}{V_T}\right) + \exp\left(\frac{V_{BE2}}{V_T}\right) \right]} \\ = \frac{1}{1 + \exp\left(\frac{V_{BE2} - V_{BE1}}{V_T}\right)}$$

(c) $v_I = V_{BE1} - V_{BE2}$

So

$$\frac{i_{C1}}{I_Q} = \frac{1}{1 + \exp\left(\frac{-v_I}{V_T}\right)}$$

$$\frac{0.1}{0.5} = \frac{1}{1 + \exp\left(\frac{-v_I}{V_T}\right)} = 0.2$$

$$\exp\left(\frac{-v_I}{V_T}\right) = \frac{1}{0.2} - 1 = 4$$

$$(-v_I) = (0.026) \ln(4)$$

$$\underline{v_I = -0.0360 \text{ V}}$$

17.4

(a) $v_I = 0.5 \text{ V}, Q_1 \text{ on}, Q_2 \text{ off} \Rightarrow v_{o2} = 3 \text{ V}$

$$v_{o1} = 3 - (1)(0.5) = 2.5 \text{ V}$$

(b) $v_I = -0.5 \text{ V} Q_1 \text{ off}, Q_2 \text{ on} \Rightarrow v_{o1} = 3 \text{ V}$

$$v_{o2} = 3 - (1)(0.5) = 2.5 \text{ V}$$

17.5

(a) $Q_2 \text{ on}, v_E = -1.2 - 0.7 = -1.9 \text{ V}$

$$i_E = i_{C2} = \frac{-1.9 - (-5.2)}{2.5} = 1.32 \text{ mA}$$

$$v_2 = -1 \text{ V} = -i_{C2} R_{C2} = -(1.32)(R_{C2})$$

$$\underline{R_{C2} = 0.758 \text{ k}\Omega}$$

(b) $Q_1 \text{ on}, v_E = -0.7 - 0.7 = -1.40 \text{ V}$

$$i_E = i_{C1} = \frac{-1.4 - (-5.2)}{2.5} = 1.52 \text{ mA}$$

$$v_1 = -1 \text{ V} = -i_{C1} R_{C1} = -(1.52)(R_{C1})$$

$$\underline{R_{C1} = 0.658 \text{ k}\Omega}$$

(c) For $v_{in} = -0.7 \text{ V}, Q_1 \text{ on}, Q_2 \text{ off}$

$$\Rightarrow v_{o1} = -0.70 \text{ V}$$

$$v_{o2} = -1 - 0.7 \Rightarrow \underline{v_{o2} = -1.7 \text{ V}}$$

For $v_{in} = -1.7 \text{ V}, Q_1 \text{ off}, Q_2 \text{ on}$

$$\Rightarrow v_{o2} = -0.7 \text{ V}$$

$$v_{o1} = -1 - 0.7 \Rightarrow \underline{v_{o1} = -1.7 \text{ V}}$$

(d) (i) For $v_{in} = -0.7V$, $i_E = 1.52 \text{ mA}$

$$i_{C4} = \frac{-1.7 - (-5.2)}{3} = 1.17 \text{ mA}$$

$$i_{C3} = \frac{-0.7 - (-5.2)}{3} = 1.5 \text{ mA}$$

$$P = (i_E + i_{C4} + i_{C3})(5.2) = (1.52 + 1.17 + 1.5)(5.2)$$

or $\underline{P = 21.8 \text{ mW}}$

(ii) For $v_{in} = -1.7V$, $i_E = 1.32 \text{ mA}$

$$i_{C4} = \frac{-0.7 - (-5.2)}{3} = 1.5 \text{ mA}$$

$$i_{C3} = \frac{-1.7 - (-5.2)}{3} = 1.17 \text{ mA}$$

$$P = (1.32 + 1.5 + 1.17)(5.2)$$

or $\underline{P = 20.7 \text{ mW}}$

17.6

a. $I_3 = \frac{3.7 - 0.7}{0.67 + 1.33} = 1.5 \text{ mA}$

$$V_R = I_3 R_4 + V_\gamma = (1.5)(1.33) + 0.7$$

or

$$\underline{V_R = 2.70 \text{ V}}$$

b. logic 1 level $= 3.7 - 0.7 \Rightarrow \underline{3.0 \text{ V}}$

For $v_X = v_Y = \text{logic 1}$.

$$i_E = \frac{3 - 0.7}{0.8} = 2.875 \text{ mA} = i_{RC1}$$

$$v_{B3} = 3.7 - (2.875)(0.21) = 3.10 \text{ V}$$

$$\Rightarrow \underline{v_{01}(\text{logic 0}) = 2.4 \text{ V}}$$

For $v_X = v_Y = \text{logic 0}$, Q_R on

$$i_E = \frac{2.7 - 0.7}{0.8} = 2.5 \text{ mA} = i_{RC2}$$

$$v_{B4} = 3.7 - (2.5)(0.24) = 3.1 \text{ V}$$

$$\Rightarrow \underline{v_{02}(\text{logic 0}) = 2.4 \text{ V}}$$

17.7

(a) $R_1 = \frac{0.7 - 0.7 - (-2.1)}{0.20} = 10.5 \text{ k}\Omega$

(b) $R_5 = R_6 = \frac{0 - (-2.1)}{0.12} = 17.5 \text{ k}\Omega$

(c) $I_Q = I_{REF} = 0.20 \text{ mA}$

$$v_{O1} = -0.7 \text{ V}, v_{C2} = 0$$

$$R_{C1} = \frac{0.7 - 0}{0.2} = 3.5 \text{ k}\Omega$$

$$(d) \quad I_Q = I_{REF} = 0.20 \text{ mA}$$

$$v_{O2} = -0.7 \text{ V}, \quad v_{CR} = 0$$

$$R_{C2} = \frac{0.7 - 0}{0.20} = 3.5 \text{ k}\Omega$$

17.8

$$V_R = -0.5 \text{ V}$$

$$i_E = \frac{-0.5 - 0.7 - (-3)}{R_E} = 0.4 \text{ mA}, \quad R_E = 4.5 \text{ k}\Omega$$

$$R_5 = \frac{-0.5 - (-3)}{0.4} = 6.25 \text{ k}\Omega$$

$$V_{B2} = -0.5 + 0.7 = 0.2 \text{ V}, \quad R_1 = \frac{0.7 - 0.2}{0.4} = 1.25 \text{ k}\Omega$$

$$R_2 = \frac{0.2 - 0.7 - (-3)}{0.4} = 4.5 \text{ k}\Omega$$

$$R_3 = R_4 = \frac{0 - (-3)}{0.8} = 3.75 \text{ k}\Omega$$

$$v_{OR} = -1 \text{ V}, \Rightarrow v_{CR} = -0.3 \text{ V}$$

$$R_{C2} = \frac{0.7 - (-0.3)}{0.4} = 2.5 \text{ k}\Omega$$

$$v_I = 0, \quad i_E = \frac{-0.7 - (-3)}{4.5} = 0.511 \text{ mA}$$

$$v_{C1} = -1 + 0.7 = -0.3 \text{ V}$$

$$R_{C1} = \frac{0.7 - (-0.3)}{0.511} = 1.957 \text{ k}\Omega$$

17.9

$$v_O = \text{logic 1} = 1.8 \text{ V}, \quad \text{logic 0} = 1.2 \text{ V}$$

For $v_I = \text{logic 1} = 1.8 \text{ V}$

$$i_E = 0.8 = \frac{1.8 - 0.7}{R_E} \Rightarrow R_E = 1.375 \text{ k}\Omega$$

$$v_{C1} = 1.2 + 0.7 = 1.9 \text{ V}, \Rightarrow R_{C1} = \frac{2.5 - 1.9}{0.8} = 0.75 \text{ k}\Omega$$

$$\text{For } v_I = \text{logic 0}, \quad Q_R \text{ on; } i_E = \frac{1.5 - 0.7}{R_E} = \frac{0.8}{1.375} = 0.5818 \text{ mA}$$

$$v_{CR} = 1.2 + 0.7 = 1.9 \text{ V}$$

$$R_{C2} = \frac{2.5 - 1.9}{0.5818} = 1.031 \text{ k}\Omega$$

$$R_2 = R_3 = \frac{1.8}{0.8} = 2.25 \text{ k}\Omega$$

17.10

Neglecting base currents:

(a) $I_{E1} = 0, I_{E3} = 0$

$$I_{E5} = \frac{5 - 0.7}{2.5} \Rightarrow I_{E5} = 1.72 \text{ mA}$$

$$Y = 0.7 \text{ V}$$

$$I_{E1} = \frac{5 - 0.7}{18} \Rightarrow I_{E1} = 0.239 \text{ mA}$$

$$I_{E3} = 0$$

$$I_{E5} = \frac{5 - 0.7}{2.5} \Rightarrow I_{E5} = 1.72 \text{ mA}$$

(b) $Y = 0.7 \text{ V}$

(c) $I_{E1} = I_{E3} = \frac{5 - 0.7}{18} \Rightarrow I_{E1} = I_{E3} = 0.239 \text{ mA}$

$$I_{E5} = 0, Y = 5 \text{ V}$$

(d) Same as (c).

17.11

(a) $V_R = -(1)(1) - 0.7 \Rightarrow V_R = -1.7 \text{ V}$

(b) Q_R off, then $v_{O1} = \text{Logic 1} = -0.7 \text{ V}$

$$Q_R \text{ on, then } v_{O1} = -(1)(2) - 0.7 \Rightarrow$$

$$v_{O1} = \text{Logic 0} = -2.7 \text{ V}$$

$$Q_A/Q_B\text{-off, then } v_{O2} = \text{Logic 1} = -0.7 \text{ V}$$

$$Q_A/Q_B\text{-on, then } v_{O2} = -(1)(2) - 0.7 \Rightarrow$$

$$v_{O2} = \text{Logic 0} = -2.7 \text{ V}$$

$$A = B = \text{Logic 0} = -2.7 \text{ V}, Q_R \text{ on,}$$

(c) $V_E = -1.7 - 0.7 \Rightarrow V_E = -2.4 \text{ V}$

$$A = B = \text{Logic 1} = -0.7 \text{ V}, Q_A/Q_B \text{ on,}$$

$$V_E = -0.7 - 0.7 \Rightarrow V_E = -1.4 \text{ V}$$

(d) $A = B = \text{Logic 1} = -0.7 \text{ V}, Q_A/Q_B \text{ on,}$

$$i_{C3} = \frac{-2.7 - (-5.2)}{1.5} = 1.67 \text{ mA}$$

$$i_{C2} = \frac{-0.7 - (-5.2)}{1.5} = 3 \text{ mA}$$

$$P = (1.67 + 1 + 1 + 1 + 3)(5.2) \Rightarrow P = 39.9 \text{ mW}$$

$$A = B = \text{Logic 0} = -2.7 \text{ V}$$

$$i_{C3} = 3 \text{ mA}, i_{C2} = 1.67 \text{ mA}$$

$$P = 39.9 \text{ mW}$$

17.12

- a. AND logic function
logic 0 = 0 V

$$Q_3 \text{ on}, i = \frac{5 - (1.6 + 0.7)}{1.2} = 2.25 \text{ mA}$$

$$V_2 = (2.25)(0.8) \Rightarrow \underline{\text{logic 1} = 1.8 \text{ V}}$$

$$i_{E1} = \frac{5 - 0.7}{2.6} \Rightarrow \underline{i_{E1} = 1.65 \text{ mA}}$$

$$i_{E2} = \frac{5 - (0.7 + 0.7)}{1.2} \Rightarrow \underline{i_{E2} = 3 \text{ mA}}$$

$$\underline{i_{C3} = 0, \quad i_{C2} = i_{E2} = 3 \text{ mA}}$$

c. $\underline{V_2 = 0}$

$$i_{E1} = \frac{5 - (1.8 + 0.7)}{2.6} \Rightarrow \underline{i_{E1} = 0.962 \text{ mA}}$$

$$i_{E2} = \frac{5 - (1.6 + 0.7)}{1.2} \Rightarrow \underline{i_{E2} = 2.25 \text{ mA}}$$

$$\underline{i_{C2} = 0, \quad i_{C3} = i_{E2} = 2.25 \text{ mA}}$$

d. $\underline{V_2 = 1.8 \text{ V}}$

17.13

(a) $v_R = (\text{logic 0} + \text{logic 1})/2 - 0.7 = \frac{3.5 + 3.1}{2} - 0.7 = 2.6 \text{ V}$

(b) For $v_X = v_Y = \text{logic 1} = 3.5 \text{ V}$

$$v_{E1} = 3.5 - 0.7 - 0.7 = 2.1 \text{ V}$$

$$i_{E1} = \frac{2.1 - 0}{12} = 0.175 \text{ mA}$$

$$i_{RC1} = \frac{1}{3} \cdot i_{E1} = 0.05833 \text{ mA} = \frac{0.4}{R_{C1}} \Rightarrow R_{C1} = 6.86 \text{ k}\Omega$$

(c) For $v_X = v_Y = \text{logic 0} = 3.1 \text{ V}$

$$v_{E1} = V_R - 0.7 = 2.6 - 0.7 = 1.9 \text{ V}$$

$$i_{E2} = \frac{1.9}{12} = 0.1583 \text{ mA}$$

$$i_{RC2} = \frac{1}{3} \cdot i_{E2} = 0.05277 \text{ mA} = \frac{0.4}{R_{C2}} \Rightarrow R_{C2} = 7.58 \text{ k}\Omega$$

(d) For $v_X = v_Y = \text{logic 0} = 3.1 \text{ V}$

$$i_E = 0.1583 \text{ mA}$$

$$i_{R1} = \frac{3.1 - 0.7}{8} = 0.3 \text{ mA}$$

$$P = (0.1583 + 0.3)(3.5) = 1.60 \text{ mW}$$

17.14

Assume $V_\gamma = 0.4$ V

(a) Logic 1 = 0.2 V, Logic 0 = -0.2 V

$$(b) i_E = \frac{0 - 0.7 - (-3.10)}{R_E} = 0.25 \Rightarrow R_E = 9.6 \text{ k}\Omega$$

$$(c) i_{D1} + i_{R1} = i_E$$

$$2i_{R1} + i_{R1} = 3i_{R1} = 0.25 \Rightarrow i_{R1} = 0.08333 \text{ mA}$$

$$R_1 = \frac{0.4}{0.08333} = 4.8 \text{ k}\Omega$$

$$(d) i_E = \frac{0.2 - 0.7 - (-3.10)}{9.6} = 0.2708 \text{ mA}$$

$$i_{R2} = \frac{0.4}{R_2} = \frac{0.4}{4.8} = 0.0833 \text{ mA}$$

$$i_{D2} = i_E - i_{R2} = 0.2708 - 0.0833 = 0.1875 \text{ mA}$$

$$(e) i_E = 0.2708 \text{ mA}$$

$$i_{R4} = \frac{-0.2 - (-3.10)}{3.3} = 0.8788 \text{ mA}$$

$$i_{R3} = \frac{0.2 - (-3.10)}{3.3} = 1.0 \text{ mA}$$

$$P = (i_E + i_{R3} + i_{R4}) [0.9 - (-3.10)] = (0.2708 + 1.0 + 0.8788)(4) = 8.6 \text{ mW}$$

17.15

a.

$$i_1 = \frac{(-0.9 - 0.7) - (-3)}{1} \Rightarrow i_1 = 1.4 \text{ mA}$$

$$i_3 = \frac{(-0.2 - 0.7) - (-3)}{15} \Rightarrow i_3 = 0.14 \text{ mA}$$

$$i_4 = \frac{(-0.2 - 0.7) - (-3)}{15} \Rightarrow i_4 = 0.14 \text{ mA}$$

$$i_2 + i_D = i_1 + i_3 = 1.4 + 0.14 = 1.54 \text{ mA}$$

$$i_2 = \frac{0.4}{0.5} \Rightarrow i_2 = 0.8 \text{ mA}$$

$$\underline{i_D = 0.74 \text{ mA}}$$

$$\underline{v_0 = -0.4 \text{ V}}$$

$$b. \quad \underline{i_1 = 1.4 \text{ mA}}$$

$$i_3 = \frac{(0 - 0.7) - (-3)}{15} \Rightarrow \underline{i_3 = 0.153 \text{ mA}}$$

$$i_4 = i_3 \Rightarrow \underline{i_4 = 0.153 \text{ mA}}$$

$$i_2 + i_D = i_4 \Rightarrow \underline{i_2 = 0.153 \text{ mA}}$$

$$\underline{i_D = 0}$$

$$v_0 = -(0.153)(0.5) \Rightarrow \underline{v_0 = -0.0765 \text{ V}}$$

c. $i_1 = \frac{(0 - 0.7 - 0.7) - (-3)}{1} \Rightarrow i_1 = 1.6 \text{ mA}$

$$i_3 = \frac{(-0.2 - 0.7) - (-3)}{15} \Rightarrow i_3 = 0.14 \text{ mA}$$

$$i_4 = i_3 \Rightarrow i_4 = 0.14 \text{ mA}$$

$$i_2 + i_D = i_3 \Rightarrow i_2 = 0.14 \text{ mA}$$

$$i_D = 0.0$$

$$v_0 = -(0.14)(0.5) \Rightarrow v_0 = -0.07 \text{ V}$$

d. $i_1 = \frac{(0 - 0.7 - 0.7) - (-3)}{1} \Rightarrow i_1 = 1.6 \text{ mA}$

$$i_3 = \frac{(0 - 0.7) - (-3)}{15} \Rightarrow i_3 = 0.153 \text{ mA}$$

$$i_4 = i_3 \Rightarrow i_4 = 0.153 \text{ mA}$$

$$i_2 + i_D = i_1 + i_4 = 1.6 + 0.153 = 1.753 \text{ mA}$$

$$i_2 = \frac{0.4}{0.5} \Rightarrow i_2 = 0.8 \text{ mA}$$

$$i_D = 0.953 \text{ mA}$$

$$v_0 = -0.40 \text{ V}$$

17.16

(a) (i) $A = B = C = D = 0 \text{ V}$, Q_1, Q_2, Q_3, Q_4 off; Q_5, Q_6 on

$$2.5 = i_{R1}(2) + 0.7 + \frac{1}{2} \left(\frac{i_{R1}}{91} \right) (15)$$

$$1.8 = i_{R1}(2 + 0.0824) \Rightarrow i_{R1} = 0.8644 \text{ mA}$$

$$Y = 2.5 - i_{R1}(2) \Rightarrow Y = 0.771 \text{ V}$$

(ii) $A = B = 0 \text{ V}$, $C = D = 2.5 \text{ V}$; Q_1, Q_2, Q_6 off; Q_3, Q_4, Q_5 on

$$2.5 = i_{R1}(2) + 0.7 + \left(\frac{i_{R1}}{91} \right) (15)$$

$$1.8 = i_{R1}(2 + 0.1648) \Rightarrow i_{R1} = 0.8315 \text{ mA}$$

$$Y = 2.5 - i_{R1}(2) \Rightarrow Y = 0.837 \text{ V}$$

(iii) $A = C = 2.5 \text{ V}$, $B = D = 0 \text{ V}$; Q_1, Q_3 on, Q_5, Q_6 off

$$i_{R1} = 0 \Rightarrow Y = 2.5 \text{ V}$$

(b) $Y = (A \text{ OR } B) \text{ AND } (C \text{ OR } D)$

(c) (i) $P = i_{R1}(2.5) = (0.8644)(2.5) = 2.16 \text{ mW}$

(ii) $i_{R3} = \frac{2.5 - 0.7}{15} = 0.12 \text{ mA}$

$$P = (i_{R1} + i_{R3})(2.5) = (0.8315 + 0.12)(2.5) = 2.38 \text{ mW}$$

(iii) $i_{R2} = i_{R3} = \frac{2.5 - 0.7}{15} = 0.12 \text{ mA}$

$$P = (i_{R2} + i_{R3})(2.5) = (0.24)(2.5) = 0.60 \text{ mW}$$

17.17

- a. logic $1 = 0 \text{ V}$
 logic $0 = -0.4 \text{ V}$
- b. $v_{01} = \overline{A \text{ OR } B}$
 $v_{02} = \overline{C \text{ OR } D}$
 $v_{03} = v_{01} \text{ OR } v_{02}$
 or
 $v_{03} = (A \text{ OR } B) \text{ AND } (C \text{ OR } D)$
-

17.18

- a. For CLOCK = high, I_{DC} flows through the left side of the circuit.. If D is high, I_{DC} flows through the left R resistor pulling \bar{Q} low. If D is low, I_{DC} flows through the right R resistor pulling Q low.
 For CLOCK = low, I_{DC} flows through the right side of the circuit maintaining Q and \bar{Q} in their previous state.

b. $P = (I_{DC} + 0.5I_{DC} + 0.1I_{DC} + 0.1I_{DC})(3)$
 $P = 1.7I_{DC}(3) = (1.7)(50)(3) \Rightarrow P = 255 \mu\text{W}$

17.19

- (a) (i) For $v_I = 0.1 \text{ V}$
 $v_1 = 0.8 \text{ V}$
 $i_1 = i_2 = \frac{2.5 - 0.8}{12} = 0.1417 \text{ mA}$
 $i_2 = i_3 = 0, v_O = 2.5 \text{ V}$
- (ii) For $v_I = 2.5 \text{ V}$
 $v_1 = 0.7 + 0.8 = 1.5 \text{ V}$
 $i_1 = i_2 = \frac{2.5 - 1.5}{12} = 0.0833 \text{ mA}$
 $v_O = 0.1 \text{ V}$
 $i_3 = \frac{2.5 - 0.1}{12} = 0.20 \text{ mA}$
- (b) (i) $v_1 = 0.7 + 0.7 = 1.4 \text{ V}$
 $v_I = v_1 - 0.7 = 0.7 \text{ V}$
- (ii) $v_1 = 0.7 + 0.8 = 1.5 \text{ V}$
 $v_I = v_1 - 0.7 = 0.8 \text{ V}$
-

17.20

(a)

$$v_I = 0 \Rightarrow V_1 = 0.7 \text{ V}$$

$$i_1 = \frac{3.3 - 0.7}{6} = 0.433 \text{ mA}$$

$$i_B = i_C = 0$$

$$v_o = 3.3 \text{ V}$$

(b)

$$v_I = 3.3 \text{ V} \quad v_1 = 0.7 + 0.8 = 1.5 \text{ V}$$

$$i_1 = \frac{3.3 - 1.5}{6} = 0.3 \text{ mA}$$

$$i_R = \frac{0.8}{20} = 0.04 \text{ mA}$$

$$i_B = 0.3 - 0.04 = 0.26 \text{ mA}$$

$$i_C = \frac{3.3 - 0.1}{4} = 0.8 \text{ mA}$$

$$v_o = 0.1 \text{ V}$$

17.21

i. For $v_X = v_Y = 0.1 \text{ V} \Rightarrow v' = 0.8 \text{ V}$

$$i_1 = \frac{5 - 0.8}{8} \Rightarrow i_1 = 0.525 \text{ mA}$$

$$\underline{i_3 = i_4 = 0}$$

ii. For $v_X = v_Y = 5 \text{ V}$,

$$v' = 0.8 + 0.7 + 0.7 \Rightarrow v' = 2.2 \text{ V}$$

$$i_1 = \frac{5 - 2.2}{8} \Rightarrow i_1 = 0.35 \text{ mA}$$

$$i_4 = i_1 - \frac{0.8}{15} \Rightarrow i_4 = 0.297 \text{ mA}$$

$$i_3 = \frac{5 - 0.1}{2.4} \Rightarrow i_3 = 2.04 \text{ mA}$$

17.22

(i) $v_X = v_Y = 0.1 \text{ V}$

$$v' = 0.8 \text{ V}$$

$$i_1 = \frac{3.3 - 0.8}{8} = 0.3125 \text{ mA}$$

$$\underline{i_3 = i_4 = 0}$$

(ii) $v_X = v_Y = 3.3 \text{ V}$

$$v' = 0.8 + 0.7 + 0.7 = 2.2 \text{ V}$$

$$i_1 = \frac{3.3 - 2.2}{8} = 0.1375 \text{ mA}$$

$$i_4 = 0.1375 - \frac{0.8}{15} = 0.08417 \text{ mA}$$

$$i_3 = \frac{3.3 - 0.1}{2.4} = 1.333 \text{ mA}$$

17.23

a. For $v_x = v_r = 5 \text{ V}$, both Q_1 and Q_2 driven into saturation.

$$v_1 = 0.8 + 0.7 + 0.8 \Rightarrow v_1 = 2.3 \text{ V}$$

$$i_1 = \frac{5 - 2.3}{4} \Rightarrow i_1 = i_{B1} = 0.675 \text{ mA}$$

$$i_2 = \frac{5 - (0.8 + 0.7 + 0.1)}{2} \Rightarrow i_2 = 1.7 \text{ mA}$$

$$i_4 = i_{B1} + i_2 \Rightarrow i_4 = 2.375 \text{ mA}$$

$$i_5 = \frac{0.8}{10} \Rightarrow i_5 = 0.08 \text{ mA}$$

$$i_{B2} = i_4 - i_5 \Rightarrow i_{B2} = 2.295 \text{ mA}$$

$$i_3 = \frac{5 - 0.1}{4} \Rightarrow i_3 = 1.225 \text{ mA}$$

$$v_0 = 0.1 \text{ V}$$

$$i'_L = \frac{5 - (0.1 + 0.7)}{4} = 1.05 \text{ mA}$$

b. $i_C(\max) = \beta i_{B2} = N i'_L + i_3$

$$(20)(2.295) = N(1.05) + 1.225$$

So

$$\underline{N = 42}$$

17.24

(a) $v_1 = 0.8 + 0.7 + 0.8 = 2.3 \text{ V}$

$$i_1 = \frac{3.3 - 2.3}{4} = 0.25 \text{ mA} = i_{B1}$$

$$v_{C1} = 0.8 + 0.7 + 0.1 = 1.6 \text{ V}$$

$$i_2 = \frac{3.3 - 1.6}{2} = 0.85 \text{ mA}$$

$$i_4 = i_{B1} + i_2 = 0.25 + 0.85 = 1.10 \text{ mA}$$

$$i_5 = \frac{0.8}{10} = 0.08 \text{ mA}$$

$$i_{B2} = i_4 - i_5 = 1.10 - 0.08 = 1.02 \text{ mA}$$

$$i_3 = \frac{3.3 - 0.1}{4} = 0.80 \text{ mA}$$

(b) $i_{Co}(\max) = \beta i_{B2} = i_3 + N i'_L$

$$i'_L = \frac{3.3 - (0.1 + 0.7)}{4} = 0.625 \text{ mA}$$

$$(20)(1.02) = 0.8 + N(0.625) \Rightarrow N = 31.36 \Rightarrow N = 31$$

17.25

D_x and D_y off, Q_1 forward active mode

$$v_i = 0.8 + 0.7 + 0.7 = 2.2V$$

$$5 = i_1 R_1 + i_2 R_2 + v_i \text{ and } i_1 = (1 + \beta) i_2$$

$$\text{So } 5 - 2.2 = i_2 [(1 + \beta) R_1 + R_2]$$

Assume $\beta = 25$

$$i_2 = \frac{5 - 2.2}{(26)(1.75) + 2} \Rightarrow i_2 = 0.0589 \text{ mA}$$

$$i_1 = (1 + \beta) i_2 = (26)(0.05895) \Rightarrow i_1 = 1.53 \text{ mA}$$

$$i_3 = \beta i_2 \Rightarrow i_3 = 1.47 \text{ mA}$$

$$i_{Bo} = i_2 + i_3 - \frac{0.8}{5} = 0.0589 + 1.47 - 0.16 \Rightarrow$$

$$\frac{i_{Bo}}{Q_o} = 1.37 \text{ mA}$$

Q_o in saturation

$$i_{Co} = \frac{5 - 0.1}{6} \Rightarrow i_{Co} = 0.817 \text{ mA}$$

17.26

(a) $v_i = 0 \text{ V}$, Q_1 forward active

$$i_B = \frac{5 - 0.7}{6} = 0.717 \text{ mA}$$

$$i_C = (25)(0.71667) = 17.9 \text{ mA}$$

$$i_E = (26)(0.71667) = 18.6 \text{ mA}$$

(b) $V_i = 0.8 \text{ V}$

$$i_B = \frac{5 - (0.8 + 0.7)}{6} = 0.583 \text{ mA}$$

Because of the relative doping levels of the Emitter and collector, and because of the difference in B-C and B-E areas, we have $-i_C \approx i_B = 0.583 \text{ mA}$ and $i_E = \text{small value}$.

(c) $v_i = 3.6$ Q_1 inverse active.

$$i_B = \frac{5 - (0.8 + 0.7)}{6} = 0.583 \text{ mA}$$

$$i_E = -\beta_R i_E = -(0.5)(0.583) = -0.292 \text{ mA}$$

$$i_C = -i_B - i_E = -0.583 - 0.292 \Rightarrow i_C = -0.875 \text{ mA}$$

17.27

(a) (i) For $v_I = 0.1 \text{ V}$, $v_1 = 0.1 + 0.8 = 0.9 \text{ V}$, and $v_O = 2.5 \text{ V}$

$$i_1 = \frac{2.5 - 0.9}{12} = 0.1333 \text{ mA}$$

$$i_2 = i_3 = 0$$

(ii) For $v_I = 2.5 \text{ V}$, $v_1 = 0.8 + 0.7 = 1.5 \text{ V}$, and $v_O = 0.1 \text{ V}$

$$i_1 = \frac{2.5 - 1.5}{12} = 0.0833 \text{ mA}$$

$$i_2 = i_1(1 + 0.1) = (0.0833)(1.1) = 0.09167 \text{ mA}$$

$$i_3 = \frac{2.5 - 0.1}{12} = 0.20 \text{ mA}$$

(b) (i) $v_1 = 0.7 + 0.7 = 1.4 \text{ V}$

$$v_I = 1.4 - 0.8 = 0.6 \text{ V}$$

(ii) $v_1 = 0.8 + 0.7 = 1.5 \text{ V}$

$$v_I = 1.5 - 0.8 = 0.7 \text{ V}$$

17.28

a. i. $v_X = v_Y = 0.1 \text{ V}$, so \mathcal{Q}_1 in saturation.

$$i_1 = \frac{5 - (0.1 + 0.8)}{6} \Rightarrow \underline{i_1 = 0.683 \text{ mA}}$$

$$\Rightarrow \underline{i_{B2} = i_2 = i_4 = i_{B3} = i_3 = 0}$$

ii. $v_X = v_Y = 5 \text{ V}$, so \mathcal{Q}_1 in inverse active mode.

Assume \mathcal{Q}_2 and \mathcal{Q}_3 in saturation.

$$i_1 = \frac{5 - (0.8 + 0.8 + 0.7)}{6} \Rightarrow \underline{i_1 = i_{B2} = 0.45 \text{ mA}}$$

$$i_2 = \frac{5 - (0.8 + 0.1)}{2} \Rightarrow \underline{i_2 = 2.05 \text{ mA}}$$

$$i_4 = \frac{0.8}{1.5} \Rightarrow \underline{i_4 = 0.533 \text{ mA}}$$

$$i_{B3} = (i_{B2} + i_2) - i_4 = 0.45 + 2.05 - 0.533$$

or

$$\underline{i_{B3} = 1.97 \text{ mA}}$$

$$i_3 = \frac{5 - 0.1}{2.2} \Rightarrow \underline{i_3 = 2.23 \text{ mA}}$$

b. For \mathcal{Q}_3 :

$$\frac{i_3}{i_{B3}} = \frac{2.23}{1.97} = 1.13 < \beta$$

For \mathcal{Q}_2 :

$$\frac{i_2}{i_{B2}} = \frac{2.05}{0.45} = 4.56 < \beta$$

Since $(I_C / I_B) < \beta$, then each transistor is in saturation.

17.29

(a) (i) $v_X = 0.1 \text{ V}$, $v_Y = 3.3 \text{ V}$

$$v' = 0.1 + 0.7 = 0.8 \text{ V}$$

$$i_i = \frac{3.3 - 0.8}{16} = 0.156 \text{ mA}$$

$$i_3 = i_4 = 0$$

(ii) $v_X = v_Y = 3.3 \text{ V}$

$$v' = 0.8 + 0.7 + 0.7 = 2.2 \text{ V}$$

$$i_1 = \frac{3.3 - 2.2}{16} = 0.06875 \text{ mA}$$

$$i_4 = 0.06875 - \frac{0.8}{20} = 0.02875 \text{ mA}$$

$$i_3 = \frac{3.3 - 0.1}{6} = 0.5333 \text{ mA}$$

(b) $i_{C1}(\max) = \beta i_4 = i_3 + N i'_L$

$$i'_L = \frac{3.3 - (0.1 + 0.7)}{16} = 0.15625 \text{ mA}$$

$$(50)(0.02875) = 0.5333 + N(0.15625) \Rightarrow N = 5.8 \Rightarrow N = 5$$

(c) $i_{C1}(\max) = \beta i_4 = (50)(0.02875) = 1.44 \text{ mA} < 5 \text{ mA}, \Rightarrow N = 5$

17.30

a. For $v_X = v_Y = 5 \text{ V}$, Q , in inverse active mode.

$$i_{B1} = \frac{5 - (0.8 + 0.8 + 0.7)}{6} = 0.45 \text{ mA}$$

$$i_{B2} = i_{B1} + 2\beta_R i_{B1} = 0.45(1 + 2[0.1]) = 0.54 \text{ mA}$$

$$i_{C2} = \frac{5 - (0.8 + 0.1)}{2} = 2.05 \text{ mA}$$

$$i_{B3} = (i_{B2} + i_{C2}) - \frac{0.8}{1.5} = 0.54 + 2.05 - 0.533$$

or

$$\underline{i_{B3} = 2.06 \text{ mA}}$$

Now

$$i'_L = \frac{5 - (0.1 + 0.8)}{6} = 0.683 \text{ mA}$$

Then

$$i_{C3}(\max) = \beta_F i_{B3} = N i'_L$$

$$\text{or } (20)(2.06) = N(0.683)$$

$$\Rightarrow \underline{N = 60}$$

b. From above, for v_0 high, $I'_L = (0.1)(0.45) = 0.045 \text{ mA}$. Now

$$I'_L(\max) = (1 + \beta_F) \left(\frac{5 - 4.9}{R_2} \right) = \frac{(21)(0.1)}{2} \\ = 1.05 \text{ mA}$$

So

$$I_L(\max) = N I'_L$$

$$\text{or } 1.05 = N(0.045)$$

$$\Rightarrow \underline{N = 23}$$

17.31

(a) (i) $V_{in} = 0.1 \text{ V}$

$$i_{RB} = \frac{5 - (0.1 + 0.8)}{4} = 1.025 \text{ mA}$$

$$i_{RCP} = i_{Bo} = 0, \quad V_{out} = 5 \text{ V}$$

(ii) $V_{in} = 5 \text{ V}$

$$i_{RB} = \frac{5 - (0.7 + 0.8 + 0.7)}{4} = 0.70 \text{ mA}$$

$$V_{out} = 0.7 + 0.1 = 0.8 \text{ V}$$

$$i_{RCP} = \frac{5 - 0.8}{1} = 4.2 \text{ mA}$$

$$i_{BS} = (1.1)(0.7) = 0.77 \text{ mA}$$

$$i_{Co} = \beta i_{Bo}, \quad i_{CS} = 4.2 - \beta i_{Bo}, \quad i_{ES} = 0.77 + (4.2 - \beta i_{Bo})$$

$$i_{Bo} = i_{ES} - \frac{0.7}{1}$$

$$i_{Bo} = 0.77 + 4.2 - \beta i_{Bo} - 0.7$$

$$(1 + \beta)i_{Bo} = 4.27 \Rightarrow i_{Bo} = \frac{4.27}{51} = 0.0837 \text{ mA}$$

(b) (i) $V_{in} = 0.1 \text{ V}, \quad V_{out} = \text{High}$,

$$i'_L = 5\beta_R i_{RB} = 5(0.1)(0.7) = 0.35 \text{ mA}$$

$$V_{out} = 5 - (0.35)(1) = 4.65 \text{ V}$$

$$P = (5 - 0.1)(1.025) + (0.35)(5 - 4.65) = 5.145 \text{ mW}$$

$$(ii) \quad i_L = 5(1.025) = 5.125 \text{ mA}$$

$$P = (0.77 + 4.2)(5) + (5.125)(0.1) = 25.4 \text{ mW}$$

17.32

a. $v_X = v_Y = v_Z = 0.1 \text{ V}$

$$i_{B1} = \frac{5 - (0.1 + 0.8)}{3.9} \Rightarrow i_{B1} = 1.05 \text{ mA}$$

Then

$$\underline{i_{C1} = i_{B2} = i_{C2} = i_{B3} = i_{C3} = 0}$$

b. $v_X = v_Y = v_Z = 5 \text{ V}$

$$i_{B1} = \frac{5 - (0.8 + 0.8 + 0.7)}{3.9} \Rightarrow i_{B1} = 0.692 \text{ mA}$$

Then

$$i_{C1} = i_{B2} = i_{B1}(1 + 3\beta_R) = (0.692)(1 + 3[0.5])$$

$$\Rightarrow \underline{i_{C1} = i_{B2} = 1.73 \text{ mA}}$$

$$i_{C2} = \frac{5 - (0.1 + 0.8)}{2} \Rightarrow \underline{i_{C2} = 2.05 \text{ mA}}$$

$$i_{B3} = i_{B2} + i_{C2} - \frac{0.8}{0.8} = 1.73 + 2.05 - 1.0$$

$$\Rightarrow \underline{i_{B3} = 2.78 \text{ mA}}$$

$$i_{R3} = \frac{5 - 0.1}{2.4} = 2.04 \text{ mA}$$

$$i'_L = \frac{5 - (0.1 + 0.8)}{3.9} = 1.05 \text{ mA}$$

$$i_{C3} = i_{R3} + 5i'_L = 2.04 + (5)(1.05)$$

$$\Rightarrow \underline{i_{C3} = 7.29 \text{ mA}}$$

17.33

(a) (i) $I'_L = 5 \mu\text{A}$, $i_{B4} = \frac{5}{51} = 0.098 \mu\text{A}$

$$V_{B5} = 5 - (0.000098)(2) \cong 5 \text{ V}$$

$$v_O = 3.6 \text{ V}$$

(ii) $I'_L = 5 \text{ mA}$, $i_{B4} = \frac{5}{51} = 0.098 \text{ mA}$

$$V_{B5} = 5 - (0.098)(2) = 4.804 \text{ V}$$

$$v_O = 4.804 - 1.4 = 3.404 \text{ V}$$

(iii) Q_4 in saturation

$$I_B = \frac{5 - V_B}{2} = \frac{5 - (V_E + 0.8)}{2}$$

$$I_C = \frac{5 - V_C}{0.13} = \frac{5 - (V_E + 0.1)}{0.13}$$

$$I_B + I_C = I_E = I_L = 25 = \frac{5 - (V_E + 0.7)}{2} + \frac{5 - (V_E + 0.1)}{0.13}$$

$$25 = 2.10 - V_E(0.5) + 37.69 - V_E(7.692) \Rightarrow V_E = 1.81 \text{ V}$$

$$v_O = 1.81 - 0.7 = 1.11 \text{ V}$$

(b) $V_{B4} = 0.7 + 0.8 = 1.5 \text{ V}$

$$I_{B4} = \frac{5 - 1.5}{2} = 1.75 \text{ mA}$$

$$V_{C4} = 0.7 + 0.1 = 0.8 \text{ V}, \quad I_{C4} = \frac{5 - 0.8}{0.13} = 32.31 \text{ mA}$$

$$I_L = 1.75 + 32.31 = 34.06 \text{ mA}$$

17.34

a. $v_X = v_Y = v_Z = 2.8 \text{ V}$, Q_1 biased in the inverse active mode.

$$i_{B1} = \frac{2.8 - (0.8 + 0.8 + 0.7)}{2} \Rightarrow i_{B1} = 0.25 \text{ mA}$$

$$i_{B2} = i_{B1}(1 + 3\beta_R) = 0.25(1 + 3[0.3])$$

$$\Rightarrow i_{B2} = 0.475 \text{ mA}$$

$$v_{C2} = 0.8 + 0.1 = 0.9 \text{ V}$$

$$i_{B4} = \frac{0.9 - (0.7 + 0.1)}{(1 + \beta_F)(0.5)} = \frac{0.1}{(101)(0.5)}$$

= 0.00198 mA (Negligible)

$$i_{R2} = \frac{5 - 0.9}{0.9} = 4.56 \text{ mA}$$

$$\Rightarrow i_{C2} = 4.56 \text{ mA}$$

$$i_{B3} = i_{B2} + i_{C2} - \frac{0.8}{1} = 0.475 + 4.56 - 0.8$$

$$\Rightarrow i_{B3} = 4.235 \text{ mA}$$

b. $v_X = v_Y = v_Z = 0.1 \text{ V}$

$$i_{B1} = \frac{5 - (0.1 + 0.8)}{2} \Rightarrow i_{B1} = 2.05 \text{ mA}$$

From part (a),

$$i'_L = \beta_R \cdot i_{B1} = (0.3)(0.25) = 0.075 \text{ mA}$$

Then

$$i_{B4} = \frac{5i'_L}{1 + \beta_F} = \frac{5(0.075)}{101} \Rightarrow i_{B4} = 0.00371 \text{ mA}$$

17.35

a. $v_X = v_Y = v_Z = 0.1 \text{ V}$

$$i_{B1} = \frac{2 - (0.1 + 0.8)}{R_{B1}} + i_{B3}$$

where

$$i_{B3} = \frac{(2 - 0.7) - (0.9)}{R_{B2}} = \frac{0.4}{1}$$

$$\Rightarrow i_{B3} = 0.4 \text{ mA}$$

Then

$$i_{B1} = \frac{1.1}{1} + 0.4 \Rightarrow i_{B1} = 1.5 \text{ mA}$$

$$i_{B2} = 0 = i_{C2}$$

Q_3 in saturation $i_{C3} = 5i'_L$ For v_0 high,

$$v'_{B1} = 0.8 + 0.7 = 1.5 \text{ V} \Rightarrow Q'_3 \text{ off}$$

$$i'_{B1} = \frac{2 - 1.5}{1} = 0.5 \text{ mA}$$

$$i'_L = \beta_R i'_{B1} = (0.2)(0.5) = 0.1 \text{ mA}$$

Then

$$i_{C3} = 0.5 \text{ mA}$$

b. $v_x = v_y = v_z = 2 \text{ V}$

From part (a),

$$\Rightarrow i_{B1} = 0.5 \text{ mA}$$

$$\underline{i_{B3} = 0 = i_{C3}}$$

$$i_{B2} = i_{B1}(1 + 3\beta_R) = (0.5)(1 + 3[0.2])$$

$$\underline{i_{B2} = 0.8 \text{ mA}}$$

$$i_{C2} = 5i'_L, \text{ and from part (a), } i'_L = 1.5 \text{ mA}$$

So

$$\underline{i_{C2} = 7.5 \text{ mA}}$$

17.36

(a) $I_B + I_D = \frac{5.8 - 0.7}{10} = 0.51 \text{ mA}$

$$I_C - I_D = \frac{5 - (0.7 - 0.3)}{1} = 4.6 \text{ mA}$$

Now

$$I_D = 0.51 - I_B = 0.51 - \frac{I_C}{\beta} = 0.51 - \frac{I_C}{50}$$

Then

$$I_C - I_D = I_C - \left(0.51 - \frac{I_C}{50}\right) = I_C \left(1 + \frac{1}{50}\right) - 0.51 = 4.6$$

So $\underline{\frac{I_C = 5.01 \text{ mA}}{}}$

$$I_B = \frac{I_C}{\beta} = \frac{5.01}{50} \Rightarrow \underline{I_B = 0.1002 \text{ mA}}$$

$$I_D = 0.51 - 0.1002 \Rightarrow \underline{I_D = 0.4098 \text{ mA}}$$

$$\underline{V_{CE} = 0.4 \text{ V}}$$

(b) $\underline{I_D = 0, V_{CE} = V_{CE}(\text{sat}) = 0.1 \text{ V}}$

$$I_B = \frac{5.8 - 0.8}{10} \Rightarrow \underline{I_B = 0.5 \text{ mA}}$$

$$I_C = \frac{5 - 0.1}{1} \Rightarrow \underline{I_C = 4.9 \text{ mA}}$$

17.37

(a) (i) $v_I = 0, v_1 = 0.3 \text{ V}$

$$i_1 = \frac{1.5 - 0.3}{1} = 1.2 \text{ mA}$$

$$i_B = i_C = 0, v_O = 1.5 \text{ V}$$

(ii) $v_I = 1.5 \text{ V}, v_1 = 0.7 + 0.3 = 1.0 \text{ V}$

$$i_1 = \frac{1.5 - 1.0}{1} = 0.5 \text{ mA}$$

$$i_B = 0.5 - \frac{0.7}{20} = 0.465 \text{ mA}$$

$$i_C = \frac{1.5 - 0.4}{1.2} = 0.9167 \text{ mA}, \quad v_O = 0.4 \text{ V}$$

(b) (i) $v_1 = 0.7 + 0.3 = 1.0 \text{ V}, \quad v_I = 0.7 \text{ V}$

$$i_B = i_C = 0$$

(ii) $v_1 = 1.0 \text{ V}, \quad v_I = 0.7 \text{ V}$

$$i_C = \frac{1.5 - 0.4}{1.2} = 0.9167 \text{ mA}$$

$$i_B = \frac{i_C}{\beta} = \frac{0.9167}{25} = 0.03667 \text{ mA}$$

(c) $i_{Co}(\max) = \beta i_B = i_C + N i'_L$

$$i'_L = \frac{1.5 - (0.4 + 0.3)}{1} - \frac{0.4}{20} = 0.78 \text{ mA}$$

$$(25)(0.465) = (0.9167) + N(0.78) \Rightarrow N = 13.7 \Rightarrow N = 13$$

17.38

a. $v_X = v_Y = 0.4 \text{ V}$

$$v_{B1} = 0.4 + 0.7 \Rightarrow v_{B1} = 1.1 \text{ V}$$

$$i_{B1} = \frac{5 - 1.1}{2.8} \Rightarrow i_{B1} = 1.39 \text{ mA}$$

$$v_{B2} = 0.4 + 0.4 \Rightarrow v_{B2} = 0.8 \text{ V}$$

$$\begin{aligned} i_{B2} &= i_{C2} = i_{B0} = i_{C0} = i_{B5} = i_{C5} \\ &= i_{B3} = i_{C3} = 0 \quad (\text{No load}) \end{aligned}$$

$$5 = i_{B4} R_2 + V_{BE} + (1 + \beta) i_{B4} R_4$$

$$i_{B4} = \frac{5 - 0.7}{0.76 + (31)(3.5)} \Rightarrow i_{B4} = 0.0394 \text{ mA}$$

$$i_{C4} = \beta_F i_{B4} \Rightarrow i_{C4} = 1.18 \text{ mA}$$

$$v_{B4} = 5 - (0.0394)(0.76) \Rightarrow v_{B4} = 4.97 \text{ V}$$

b. $v_X = v_Y = 3.6 \text{ V}$

$$v_{B1} = 0.7 + 0.7 + 0.3 \Rightarrow v_{B1} = 1.7 \text{ V}$$

$$v_{B2} = 1.4 \text{ V}$$

$$v_{B0} = 0.7 \text{ V}$$

$$v_{C2} = 1.1 \text{ V}$$

$$i_{B1} = \frac{5 - 1.7}{2.8} \Rightarrow i_{B1} = 1.1786 \text{ mA}$$

$$i_{B2} = i_{B1}(1 + 2\beta_R) = 1.18(1 + 2[0.1])$$

$$i_{B2} = 1.41 \text{ mA}$$

$$i_{B4} = \frac{1.1 - 0.7}{(31)(3.5)} \Rightarrow i_{B4} = 0.00369 \text{ mA}$$

$$i_{R2} = \frac{5 - 1.1}{0.76} = 5.13 \text{ mA} \Rightarrow i_{C2} \approx 5.13 \text{ mA}$$

$$i_{B0} \approx i_{B2} + i_{C2}$$

$$i_{B0} = 6.54 \text{ mA}$$

17.39

(a) For the load, $i_{RB1} = \frac{2.5 - (0.4 + 0.3)}{R_{B1}} = \frac{1}{2}(0.2) \Rightarrow R_{B1} = 18 \text{ k}\Omega$

For $v_X = v_Y = v_Z = \text{logic 1}$

$$i_{RB1} = \frac{2.5 - (0.7 + 0.8)}{18} = 0.05556 \text{ mA}$$

$$i_{RC1} = \frac{2.5 - (0.7 + 0.1)}{R_{C1}}$$

$$i_{B2} = 0.1 = 0.05556 + \frac{1.7}{R_{C1}} - \frac{0.7}{0.7} \Rightarrow R_{C1} = 1.63 \text{ k}\Omega$$

(b) $v_X = 0.4 \text{ V}, v_{B1} = 0.7 \text{ V}, v_{B2} = 0$

$$v_O \cong 2.5 - 0.7 = 1.8 \text{ V}$$

All $i_B = 0$, All $i_C = 0$

(c) $v_{B1} = 1.5 \text{ V}, v_{B2} = 0.7 \text{ V}$

$$i_{B1} = \frac{2.5 - 1.5}{18} = 0.0556 \text{ mA}$$

$$i_{C1} = \frac{2.5 - (0.7 + 0.1)}{1.63} = 1.043 \text{ mA}$$

$$i_{B2} = 0.10 \text{ mA}$$

$$i_{C2} = \frac{4[2.5 - (0.4 + 0.3)]}{18} = 0.40 \text{ mA}$$

$$v_O = 0.4 \text{ V}$$

(d) $i_{C2}(\max) = \beta i_{B2} = N i'_L$

$$i'_L = 0.1 \text{ mA}$$

$$(20)(0.1) = N(0.1) \Rightarrow N = 20$$

17.40

a. For $v_X = v_Y = 3.6 \text{ V}$

$$v_{B1} = 3(0.7) = 2.1 \Rightarrow i_{B1} = \frac{5 - 2.1}{10} = 0.29 \text{ mA}$$

$$v_{C1} = 0.7 + 0.7 + 0.4 = 1.8 \text{ V} \Rightarrow i_{C1} = \frac{5 - 1.8}{10} = 0.32 \text{ mA}$$

$$i_{B2} = i_{B1} + i_{C1} - \frac{1.4}{15} = 0.29 + 0.32 - 0.0933$$

So

$$i_{B2} = 0.517 \text{ mA}$$

$$v_{C2} = 0.7 + 0.4 = 1.1 \text{ V}$$

$$i_{C2} = \frac{5 - 1.1}{4.1} = 0.951 \text{ mA}$$

$$i_{B5} = i_{B2} + i_{C2} - \frac{0.7}{4} = 0.517 + 0.951 - 0.175$$

or $i_{B5} = 1.293 \text{ mA}$

For $v_0 = 0.4 \text{ V}, v'_{B1} = 0.4 + 0.7 = 1.1 \text{ V}$

Then

$$i'_{B1} = \frac{1.1 - 0.7}{(31)(15)} = 0.00086 \text{ mA}$$

$$i'_L = \frac{5 - 1.1}{10} - 0.00086 \text{ or } i'_L \approx 0.39 \text{ mA}$$

$$\text{So } i_{C5}(\max) = \beta i_{B5} = N i'_L$$

$$(30)(1.293) = N(0.39) \Rightarrow N = 99$$

b. $P = (0.29 + 0.32 + 0.951)(5) + (99)(0.39)(0.4)$

$$P = 7.805 + 15.444 \text{ or } P = 23.2 \text{ mW}$$

(Assumming 99 load circuits which is unreasonably large.)

17.41

a. Assume no load. For $v_x = \text{logic 0} = 0.4 \text{ V}$

$$i_{E1} = \frac{5 - (0.4 + 0.7)}{40} = 0.0975 \text{ mA}$$

Essentially all of this current goes to ground from V_{CC} .

$$P = i_{E1} \cdot V_{CC} = (0.0975)(5) \Rightarrow P = 0.4875 \text{ mW}$$

b. $i_{R1} = \frac{5 - (3)(0.7)}{40} = 0.0725 \text{ mA}$

$$i_{R2} = \frac{5 - (0.7 + 0.7 + 0.4)}{50} = 0.064 \text{ mA}$$

$$i_{R3} = \frac{5 - (0.7 + 0.4)}{15} = 0.26 \text{ mA}$$

$$P = (0.0725 + 0.064 + 0.26)(5)$$

$$P = 1.98 \text{ mW}$$

c. For $v_0 = 0$, $v_{C7} = 0.7 + 0.4 = 1.1 \text{ V}$

$$i_{R7} = \frac{5 - 1.1}{0.050} \Rightarrow i_{R7} = 78 \text{ mA} \approx i_{SC}$$

17.42

(a) $i_E = \frac{3 - (0.7 + 0.3)}{2} = 1.0 \text{ mA}$

(b) $i_E = \frac{2.4 - 0.7}{2} = 0.85 \text{ mA}$

$$R_C = \frac{3 - 2.4}{0.85} = 0.706 \text{ k}\Omega$$

(c) (i) $P = (1.0)(3) = 3.0 \text{ mW}$

(ii) $P = (0.85)(3) = 2.55 \text{ mW}$

17.43

(a) $v_I = v_O = 2.5 \text{ V}$; A transient situation

$$v_{DS}(M_N) = 2.5 - 0.7 = 1.8 \text{ V}$$

$$v_{GS}(M_N) = 2.5 - 0.7 = 1.8 \text{ V} \Rightarrow M_N \text{ in saturation}$$

$$v_{SD}(M_P) = 5 - (2.5 + 0.7) = 1.8 \text{ V}$$

$$v_{SG}(M_P) = 5 - 2.5 = 2.5 \text{ V} \Rightarrow M_P \text{ in saturation}$$

$$i_{DN} = K_n(v_{GSN} - V_{TN})^2 = (0.1)(1.8 - 0.8)^2 \Rightarrow i_{DN} = 0.1 \text{ mA}$$

$$i_{DP} = K_P(v_{SGP} + V_{TP})^2 = (0.1)(2.5 - 0.8)^2 \Rightarrow i_{DP} = 0.289 \text{ mA}$$

$$i_{C1} = \beta i_{DP} = (50)(0.289) \Rightarrow i_{C1} = 14.45 \text{ mA}$$

$$i_{C2} = \beta i_{DN} = (50)(0.1) \Rightarrow i_{C2} = 5 \text{ mA}$$

Difference between i_{E1} and $i_{DN} + i_{C2}$ is a load current.

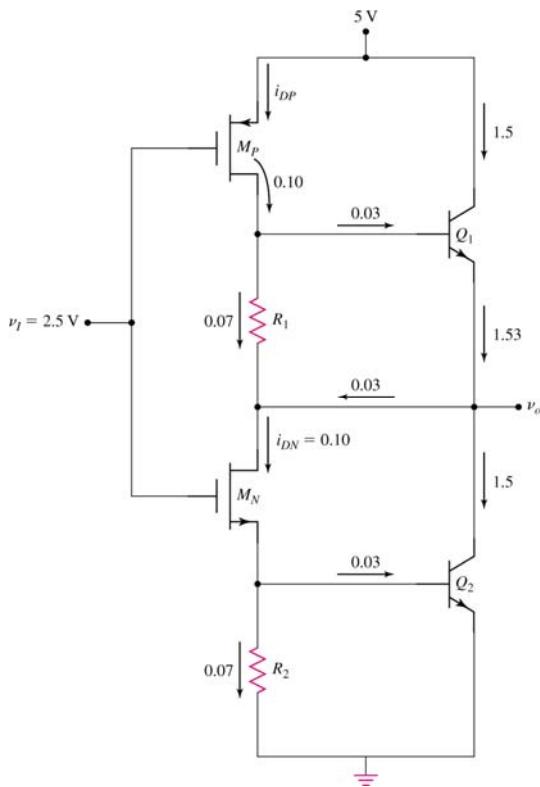
(b) Assume $i_{C1} = 14.45 \text{ mA}$ is a constant

$$V_C = \frac{1}{C} \int i_{C1} dt = \frac{i_{C1} \cdot t}{C} \Rightarrow t = \frac{(V_C)(C)}{i_{C1}}$$

$$t = \frac{(5)(15 \times 10^{-12})}{14.45 \times 10^{-3}} \Rightarrow t = 5.19 \text{ ns}$$

$$(c) t = \frac{(5)(15 \times 10^{-12})}{0.289 \times 10^{-3}} \Rightarrow t = 260 \text{ ns}$$

17.44



(a) Assume $R_1 = R_2 = 10 \text{ k}\Omega$; $\beta = 50$

$$i_{R1} = i_{R2} = \frac{0.7}{10} = 0.07 \text{ mA}$$

Then

NMOS in saturation region; $v_{GSN} = 2.5 - 0.7 = 1.8 \text{ V}$

$$i_{DN} = K_n (v_{GSN} - V_{TN})^2 = (0.1)(1.8 - 0.8)^2$$

$$i_{DN} = 0.10 \text{ mA}$$

Then $i_{B2} = 0.03 \Rightarrow i_{C2} = (50)(0.03) = 1.5 \text{ mA}$

$$i_{E1} = 1.53 \text{ mA} \Rightarrow i_{B1} = 0.03 \text{ mA} \Rightarrow i_{C1} = 1.5 \text{ mA}$$

So $i_{DP} = 0.10 \text{ mA}$

Now, M_p biased in non-saturation region

$$v_{SGP} = 2.5 \text{ V}$$

$$i_{DP} = 0.10 = 0.10 [2(2.5 - 0.8)v_{SD} - v_{SD}^2]$$

$$0.10v_{SD}^2 - 0.34v_{SD} + 0.10 = 0$$

$$v_{SD} = \frac{0.34 \pm \sqrt{(0.34)^2 - 4(0.10)(0.10)}}{2(0.10)}$$

Or

$$v_{SD} = 0.325 \text{ V}$$

Then $v_o = 5 - 0.325 - 0.7$

$$v_o = 3.975 \text{ V}$$

$$(b) \quad v = \frac{1}{C} \int idt = \frac{i \cdot t}{C}$$

$$t = \frac{Cv}{i} = \frac{(15 \times 10^{-12})(5)}{1.53 \times 10^{-3}}$$

$$t = 49 \text{ ns}$$

(c)

$$t = \frac{Cv}{i} = \frac{(15 \times 10^{-12})(5)}{0.1 \times 10^{-3}}$$

$$t = 0.75 \mu\text{s}$$
