113-2 Calculus Midterm Exam – 1 - Solution

Chapter : 7-3, 7-5, 7-6, 7-7, 7-8, 8-2, 8-3

1. Find $f'(3)$ if $f(x) = e^{\int_3^x \frac{t}{1+t^5} dt}$. (10 pts)

Set $g(x) = \int_3^x \frac{t}{1+t^5} dt \to f(x) = e^{g(x)} \to f'(x) = e^{g(x)} g'(x)$

Find $f'(3) = e^{g(3)} g'(3)$

$$g(3) = \int_3^3 \frac{t}{1+t^5} dt = 0 \to e^{g(3)} = 1$$

$$g'(x) = \frac{x}{1+x^5} \to g'(3) = \frac{3}{1+3^5} = \frac{3}{244}$$

$$\therefore f'(3) = \frac{3}{244}$$

2. Find the following limits. (10 pts)

$a.$ $\lim_{x\to0^+} (1+sin6x)^{\cot x}$ (5 pts)

$b.$ $\lim_{x\to0} \frac{x-\tan^{-1}x}{\sin^{-1}x - x}$ (5 pts)

$$a.\ \lim_{x\to0^+} (1+sin6x)^{\cot x} = \lim_{x\to0^+} e^{cotx \ln(1+sin6x)}\ \left(\frac{0}{0}\right) \to \lim_{x\to0^+} e^{\frac{\ln(1+sin6x)}{tanx}} \to$$

$$\to \lim_{x\to0^+} e^{\frac{\frac{6cos6x}{1+sin6x}}{sec^2x}} \to \lim_{x\to0^+} e^{\frac{\frac{6\cdot1}{1+0}}{1}} = e^6$$

$$b.\ \lim_{x\to0} \frac{x-\tan^{-1}x}{\sin^{-1}x - x} = \lim_{x\to0} \frac{1-\frac{1}{1+x^2}}{\frac{1}{\sqrt{1-x^2}}-1}$$

$$\to \lim_{x\to0} \frac{\frac{2x}{(1+x^2)^2}}{-\frac{1}{2}\cdot\frac{-2x}{(\sqrt{1-x^2})^3}} \to \lim_{x\to0} \frac{\frac{2}{(1+x^2)^2}}{\frac{1}{(\sqrt{1-x^2})^3}} \to \lim_{x\to0} \frac{2(\sqrt{1-x^2})^3}{(1+x^2)^2} = 2$$

3. Evaluate the integral $\int \frac{e^{cosx}(sinx)}{\sqrt{e^{2cosx}-9}}\,dx$. (10 pts)

$$\int \frac{e^{cosx}(sinx)}{\sqrt{e^{2cosx}-9}}dx \to u = e^{cosx} \to du = e^{cosx}\cdot(-sinx)dx \to dx = \frac{du}{(-sinx)e^{cosx}}$$

$$\int \frac{-1}{\sqrt{u^2-3^2}}du = -\cosh^{-1}\left(\frac{u}{3}\right) + C = -\cosh^{-1}\left(\frac{e^{cosx}}{3}\right) + C$$

4. Evaluate the following integral. (15 pts)

a. $\int (tan^4x)(sec^4x)\,dx$. (5 pts)

b. $\int sec^3\theta\,d\theta$. (10 pts)

$$a. \int (tan^4x)(sec^4x)\,dx \to \int (tan^4x)(1+tan^2x)(sec^2x)dx$$

$$\to \int (tan^4x + tan^6x)(sec^2x)dx$$

$$\to \int (tan^4x)(sec^2x)dx + \int (tan^6x)(sec^2x)dx$$

$$\to u = tanx \to du = sec^2x\,dx$$

$$\to \int u^4du + \int u^6du$$

$$\to \frac{u^5}{5} + \frac{u^7}{7} + C \to \frac{tan^5x}{5} + \frac{tan^7x}{7} + C$$

$$b. \int sec^3\theta\,d\theta$$

$$\to u = \sec\theta\ , dv = sec^2\theta\,d\theta \to du = sec\theta tan\theta d\theta\ , v = tan\theta$$

$$\to I = \int sec^3\theta\,d\theta = \sec\theta\,tan\theta - \int tan\theta(sec\theta tan\theta)\,d\theta$$

$$= \sec\theta\,tan\theta - \int tan^2\theta sec\theta\,d\theta = \sec\theta\,tan\theta - \int (sec^2\theta - 1)\cdot sec\theta\,d\theta$$

$$= \sec\theta\,tan\theta - \int (sec^3\theta - sec\theta)\,d\theta$$

$$\to 2I = \sec\theta\,tan\theta + \int sec\theta\,d\theta \to 2I = \sec\theta\,tan\theta + \ln|sec\theta + tan\theta| + C$$

$$\to I = \int sec^3\theta\,d\theta = \frac{\sec\theta\,tan\theta + \ln|sec\theta + tan\theta|}{2} + C$$

5. The region between the curve $y = sin(x)$ and the x-axis from $x = 0$ to π is revolved about the x-axis to generate a solid. Find the volume of the solid. (10 pts)
$V = \int_0^{\pi} \pi\, sin^2\mathrm{x}\, \mathrm{dx} \to \pi \int_0^{\pi} \frac{1-\cos(2x)}{2} dx \to \frac{\pi}{2}\int_0^{\pi} 1-\cos(2x)\, dx$ $\to \frac{\pi}{2}[\pi - \frac{\sin(2\pi)}{2}] \to \frac{\pi^2}{2}$