Magnet Precalculus CD
Vectors

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Chapter 1

Geometric Representation of Vectors

1.1 Combination

1.2 Magnitude of Vectors

The magnitude (length) of a vector $v = \langle a, b\rangle$ is $||v|| = \sqrt{a^2 + b^2}$. If vector v is represented by the arrow from (x_1, y_1) to (x_2, y_2), then $||v|| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

1.3 Unit Vector

Definition 1.3.1: Unit Vector

A vector u for which $||u|| = 1$ is called a **unit vector**.

$$u = \frac{v}{||v||}$$
$$i = \langle 1, 0\rangle \text{ and } j = \langle 0, 1\rangle$$

1.4 Components of a Vector

Let v be a vector with magnitude $||v||$ and direction θ. Then, $v = \langle a, b\rangle = ai + bj$ where $a = ||v|| \cos(\theta)$ and $b = ||v|| \sin(\theta)$. Therefore, we can express v as $v = ||v|| \cos(\theta)i + ||v|| \sin(\theta)j$

Chapter 2

The Language of Vectors

2.1 The Dot Product

Definition 2.1.1: Dot Product

A **dot product** of $u = \langle a_1, b_1 \rangle$ and $v = \langle a_2, b_2 \rangle$ is $u \bullet v = a_1 a_2 + b_1 b_2$

Another form of the dot product is $u \bullet v = ||u||||v|| \cos(\theta)$, where θ is the angle between u and v.

To find the angle between two vectors, the dot product can be used:
Let θ be the angle between two vectors, u and v. Where $0 \leq \theta \leq \pi$, $\cos(\theta) = \frac{u \bullet v}{||u||||v||}$

The dot product can be used to find the magnitude of a vector:
$u = \langle a, b \rangle \,; ||u|| = \sqrt{a^2 + b^2} = \sqrt{(a \bullet a) + (b \bullet b)} = \sqrt{u \bullet u} = \sqrt{u^2}$

Definition 2.1.2: Orthogonal Vectors

When two angles are orthogonal (a.k.a. perpendicular or normal), their dot product is 0.

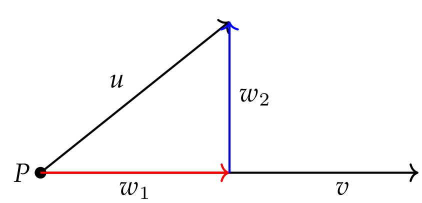
Question 1

Find the angle between the vectors $u = 2i - j$ and $v = 6i + 4j$

Solution: $\cos(\theta) = \frac{(2 \bullet 6)+(-1 \bullet 4)}{\sqrt{2^2+(-1)^2} \bullet \sqrt{6^2+4^2}} = \frac{12-4}{\sqrt{5} \bullet \sqrt{52}} = \frac{8}{\sqrt{260}} = \frac{8}{2\sqrt{65}}; \theta = \cos^{-1}(\frac{8}{1\sqrt{65}}) \approx \boxed{60.255°}$

2.2 Projection of a Vector Onto Another

The projection of one vector onto another can be thought of as finding the shadow that one casts on the other. Consider the following scenario:



Here, vectors u and v start at the same point P. The projection of u onto v is w_1. If there were a light source pointing straight down, w_1 would the the shadow cast by u on v. w_2 is defined as $u - w_1$, but by the nature of projection, it is also orthogonal to v.

Projection is represtented mathematically as $\text{proj}_v(u)$, where u is being projected onto v. $\text{proj}_v\, u = (\frac{u \bullet v}{v^2})v$

Question 2

Find $\text{proj}_v(u)$ where $u = \langle 4, 2 \rangle$ and $v = \langle 1, 2 \rangle$

Solution: $\text{proj}_v(u) = (\frac{u \bullet v}{v^2})v = \frac{(4 \bullet 1)+(2 \bullet 2)}{1^2+2^2} \langle 1, 2 \rangle = \frac{4+4}{1+4} \langle 1, 2 \rangle = \frac{8}{5} \langle 1, 2 \rangle = \left\langle \frac{8}{5}, \frac{16}{5} \right\rangle$