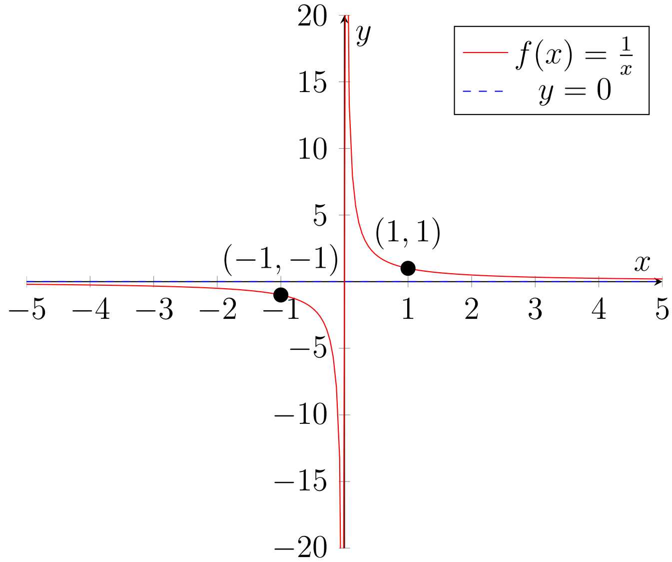
Reciprocal Functions

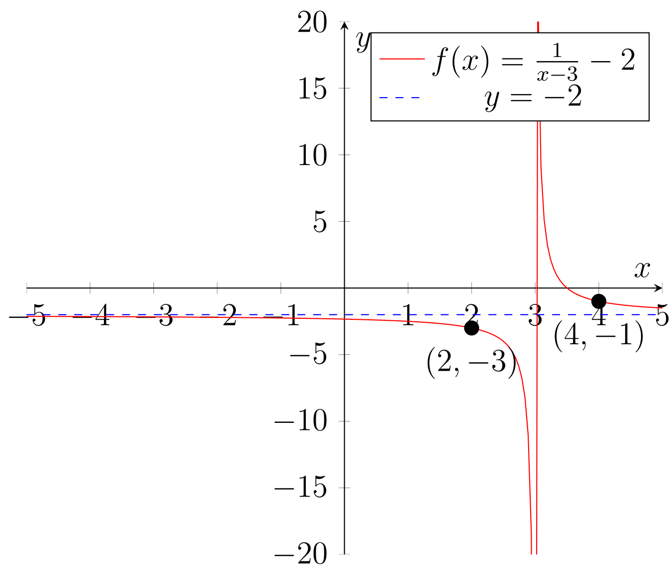
Parent function: $f(x) = \frac{1}{x}$
Standard form: $f(x) = \frac{a}{x-h} + k$



Reciprocal functions always have 2 or more asymptotes: 1 or more vertical asymptote, and a horizontal **or** slant asymptote. Should a remain a constant, the horizontal asymptote is at $y = k$, and the vertical asymptotes are at $x = n$, where n is all values of x that make the expression on the denominator equal 0.

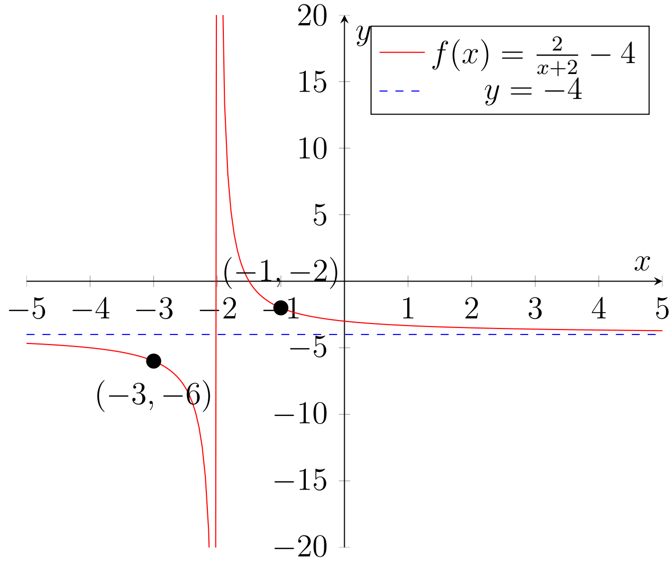
Some examples:

$$f(x) = \frac{1}{x-3} - 2$$



Here, $h = 3$, so the function is translated 3 units right, so the vertical asymptote is at $x = 3$. Similarly, $k = -2$, so the function is translated 2 units down, so the horizontal asymptote is at $y = -2$.

$f(x) = \frac{2}{x+2} - 4$



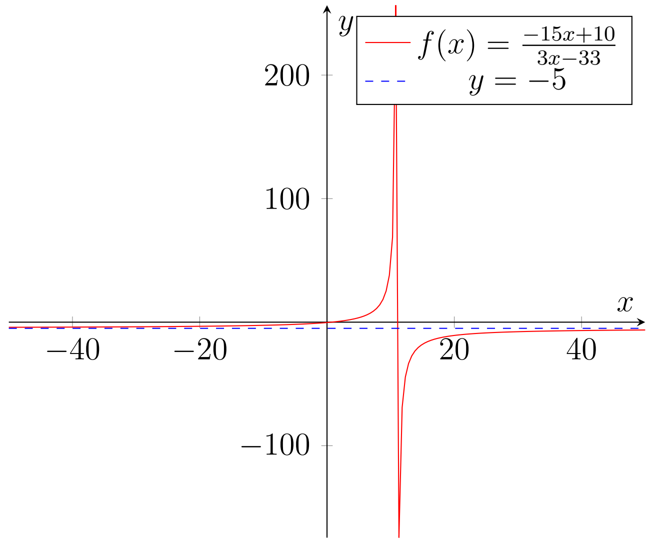
Here, $h = -2$, so the function is translated 2 units left, so the vertical asymptote is at $x = -2$. Similarly, $k = -4$, so the function is translated 4 units down, so the horizontal asymptote is at $y = -4$.

There are a few special cases for the horizontal asymptote where a is a polynomial rather than a constant.

Should the degree in the numerator be less than in the denominator, the horizontal asymptote will be at $y = 0$

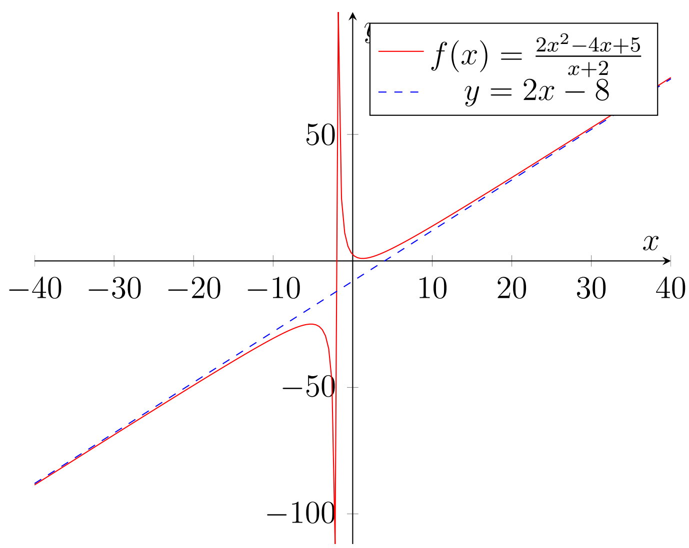
Should the degree in the numerator be equal to that in the denominator, the horizontal asymptote will be at $y = n$, where n is the quotient of the greatest-degree coefficients of the numerator and denominator.
For example, given the function $f(x) = \frac{-15x+10}{3x-33}$, we can divide 6 by 3 to find that the horizontal asymptote of $f(x)$ is at $y = -5$.

$f(x) = \frac{-15x+10}{3x-33}$



Should the degree in the numerator be greater than in the denominator, there is not a horizontal asymptote, but a slant asymptote. The slant asymptote will be at $y = nx$, where n is the result of long-dividing the numerator and denominator and excluding the remainder.
For example, given the function $f(x) = \frac{2x^2-4x+5}{x+2}$, we can do long division to find that $\frac{2x^2-4x+5}{x+2} = 2x - 8 + \frac{21}{x+2}$. From this, we can derive that the slant asymptote of $f(x)$ is $y = 2x - 8$.

$f(x) = \frac{2x^2-4x+5}{x+2}$



And here we can see that this is correct.

The Trigonometric Identities

The angle sum identities are as follows:

- $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \sin(\beta)\cos(\alpha)$
- $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha) sin(\beta)$
- $\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$

These "double" identities can be derived from the sum identities:

- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- $\cos(2\theta) = cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$
- $\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$

There are also half-angle identities:

- $\sin(\frac{\theta}{2}) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$
- $\cos(\frac{\theta}{2}) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$
- $\tan(\frac{\theta}{2}) = \pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}} = \frac{1-\cos(\theta)}{\sin(\theta)} = \frac{\sin(\theta)}{1+\cos(\theta)}$

Some other crucial ones:

- $\sin^2(\theta) + \cos^2(\theta) = 1$
- $\sec^2(\theta) = 1 + \tan^2(\theta)$
- $\csc^2(\theta) = 1 + \cot^2(\theta)$