

Magnet Precalculus CD  
Complex Coordinates

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## 1.1

Polar coordinates on the complex plane can be represented as a complex number. Any polar point  $(r, \theta)$  can be represented as  $r(\cos(\theta) + i\sin(\theta))$ , or  $r \operatorname{cis}(\theta)$ . Complex numbers are generally written as  $z = x + iy$ .

The multiplication rule says that for any two complex points,  $z_1$  and  $z_2$ , written as  $z = r(\operatorname{cis}(\theta))$ ,  $z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$ . Conversely,  $\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$ .

DeMoivre's Theorem is that  $z^n = (r \operatorname{cis}(\theta))^n = r^n \operatorname{cis}(n\theta)$ .

If a complex number  $z$  satisfies the equation  $z^n = w$ , we say that  $z$  is a complex  $n^{\text{th}}$  root of  $w$ . We can use DeMoivre's Theorem to find these distinct  $n^{\text{th}}$  roots. Let  $w = r(\cos(\theta) + i\sin(\theta))$  be a complex number in polar form. If  $w \neq 0$ ,  $w$  has  $n$  distinct roots given by the formula below, where  $k = 0, 1, 2, 3, \dots, n-1$ .

$$z_k = \sqrt[n]{r} \operatorname{cis}\left(\frac{\theta + 2\pi k}{n}\right)$$