Magnet Precalculus CD
Polar coordinates

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Contents

Chapter 1

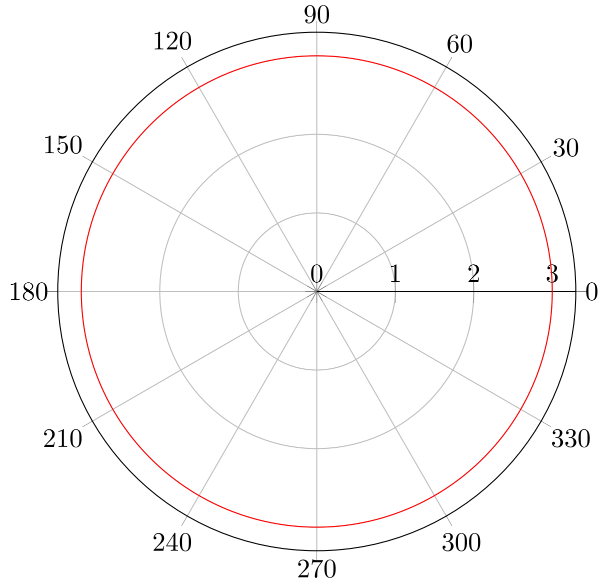
Introduction to Polar Coordinates

Normally, points and functions are represented rectangularly, with an x and a y coordinate. However, they can also be represented in polar form.

Definition 1.0.1: Polar Coordinates

A coordinate represented by a distance from the origin r and an angle from the positive x-axis θ. Polar coordinates are in the form (r, θ).

In polar functions, the dependent variable is generally r. With polar, you can far more easily graph circles and other curves. For instance, take $r = 3$:



Chapter 2

Polar Function Types

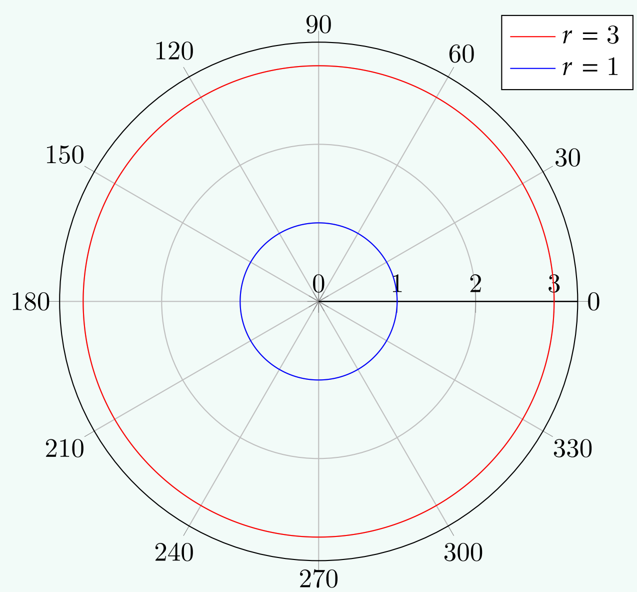
2.1 Circles

Circles in polar form are fairly simple. There are three kinds:

- $r = n$
- $r = n\sin(\theta)$
- $r = n\cos(\theta)$

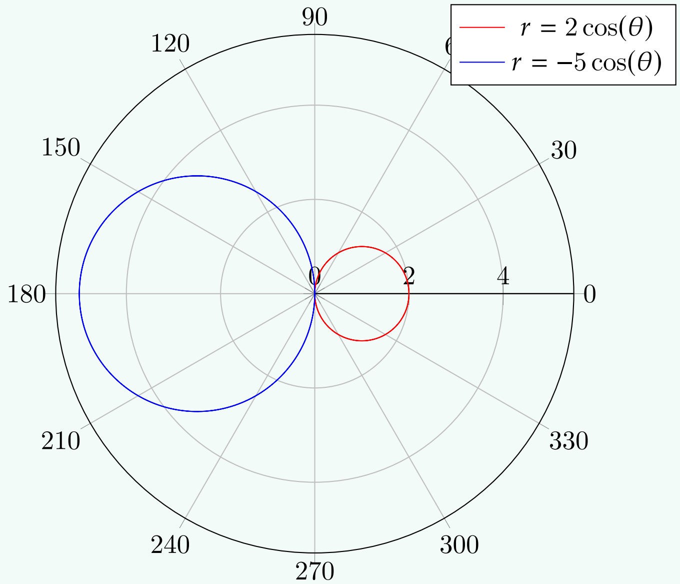
In the case of $r = n$, the circle is centered on the pole (or the origin), and has a radius of $|n|$.

Example 2.1.1 ($r = n$ circle)



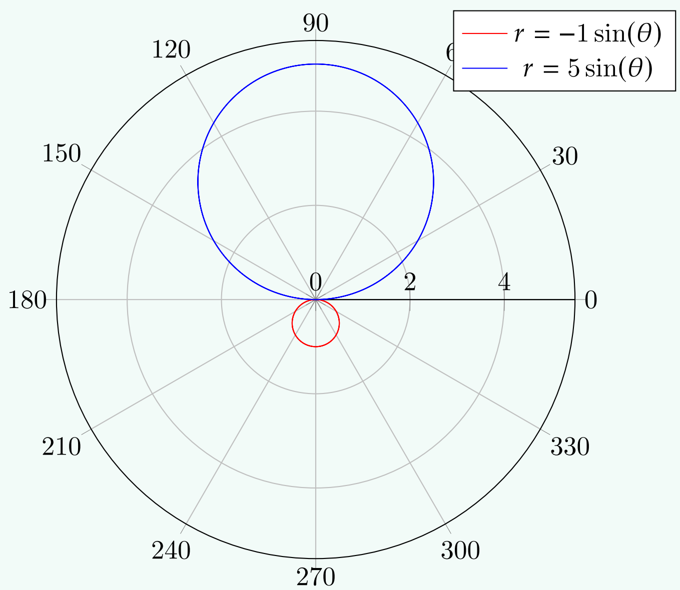
In the case of $r = n\cos(\theta)$, the circle is centered at (polar) $(\frac{n}{2}, 0)$ and has a radius of $\frac{n}{2}$. You can also think of it as being centered on the polar axis (or positive x-axis), tangent to the pole, and tangent to the polar point $(n, 0)$.

Example 2.1.2 ($r = n\cos(\theta)$ circle)



The function $r = n\sin(\theta)$ is the same as $r = n\cos(\theta)$, but instead of being centered on the polar axis, it is centered on the polar line $\theta = \frac{\pi}{2}$ (or the y-axis).

Example 2.1.3 ($r = n\sin(\theta)$ circle)



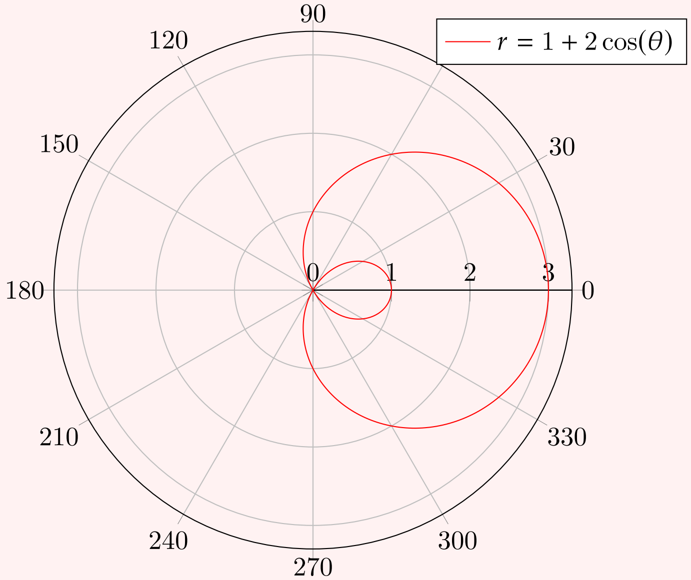
Note:-

This behavior of sin and cos holds true for all functions that stay centered around either the x or y axes. Also, if the trigonometric function is being multiplied by a negative, it is reflected across the axis it isn't centered on (so $-\sin$ would be reflected across the x-axis).

2.2 Limaçon Curves

Definition 2.2.1: Limaçon Curve

A limaçon curve function takes the form of $r = a \pm b\sin(\theta)$ or $r = a \pm b\cos(\theta)$.



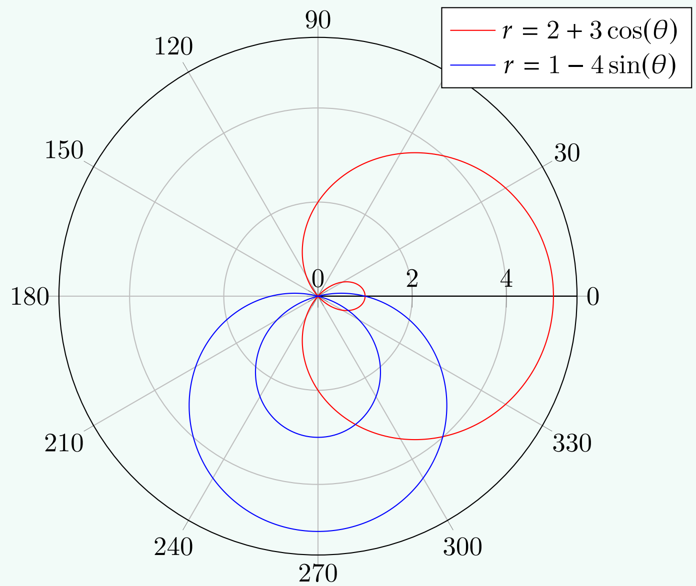
4 points can be derived from a given limaçon function:

- In the case of cos:
  - Its 2 x-intercepts are at the rectangular points $(a + b, 0)$ and $(b - a, 0)$
  - its 2 y-intercepts are at the rectangular points $(0, a)$ and $(0, -a)$
- In the case of sin:
  - Its 2 y-intercepts are at the rectangular points $(a + b, 0)$ and $(b - a, 0)$
  - its 2 x-intercepts are at the rectangular points $(0, a)$ and $(0, -a)$

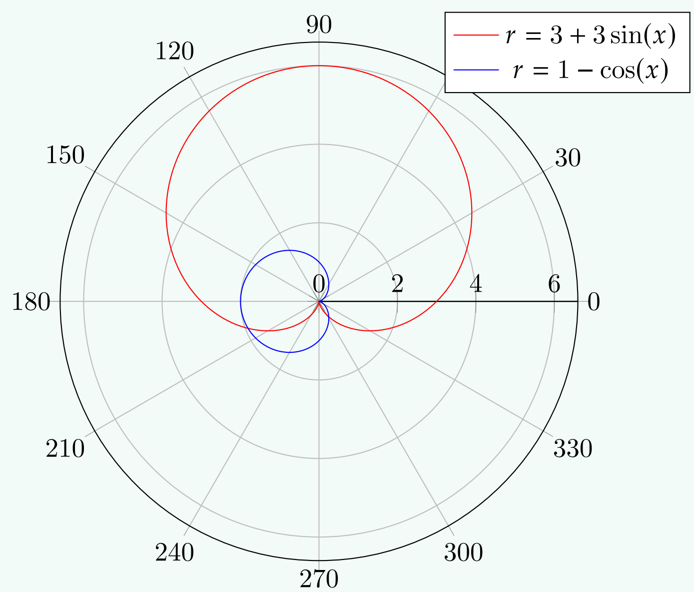
This behavior creates 3 types of limaçon curve:

- Innerloop limaçon curves, where $a < b$
- Cardioid curves, where $a = b$
- Convex limaçon curves, where $a > b$

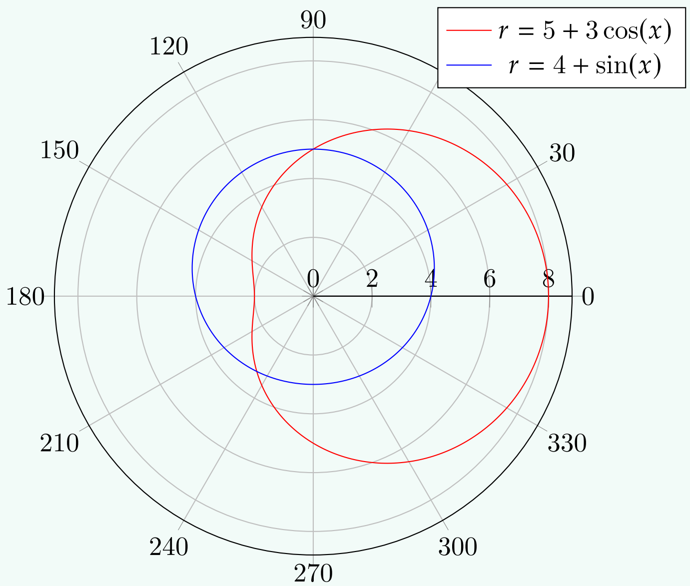
Example 2.2.1 (Innerloop Limaçon Curve)



Example 2.2.2 (Cardioid curve)



Example 2.2.3 (Convex Limaçon Curve)

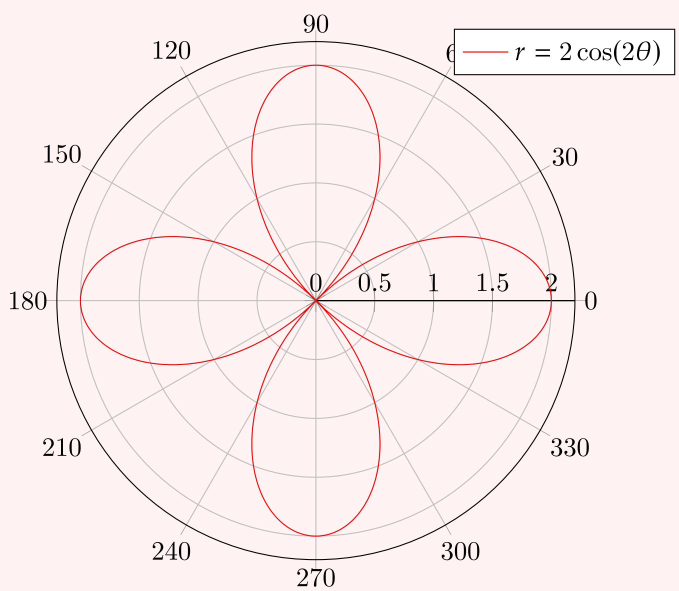


These can look a good deal like circles sometimes, but it's important to recognize that they aren't.

2.3 Rose Curves

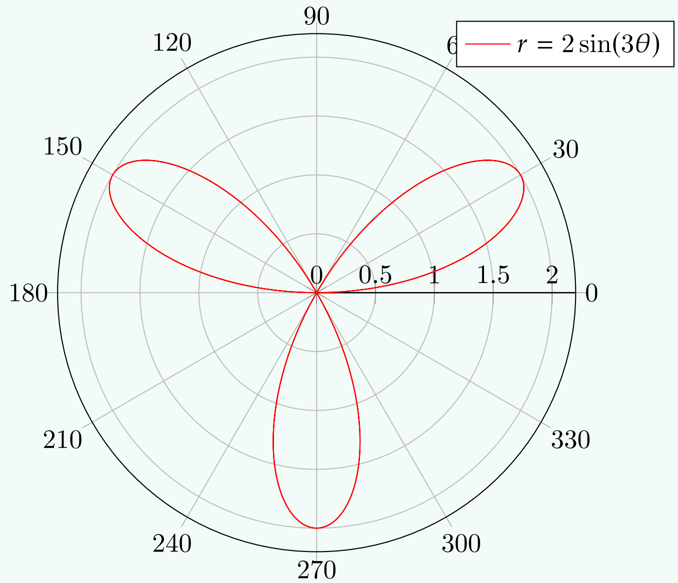
Definition 2.3.1: Rose Curve

A rose curve function takes the form of $r = b\sin(n\theta)$ or $r = b\cos(n\theta)$.



Rose curves appear like a flower, hence their name. The number of petals is determined by n. Should n be even, the number of petals is $2n$. Should n be odd, the number of petals is n. The length of the petals is always equal to b. The petals are always separated by an angle $\frac{2\pi}{n_p}$, where n_p is the number of petals. In the case of cos, the first pedal is always at the angle 0. In the case of sin, however, the first petal is located at $\frac{90°}{n_p}$ where, once again, n_p is the number of petals.

Example 2.3.1 (sin Rose Curve)

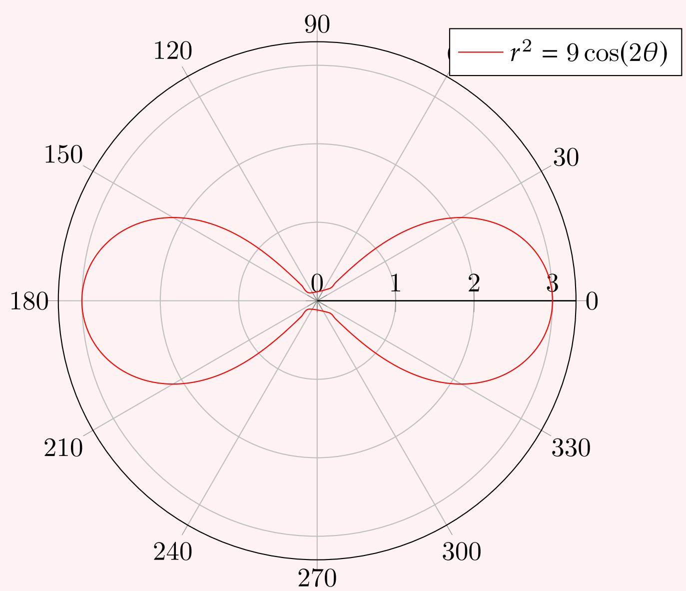


2.4 Lemniscate Curves

Well, how does one graph a rose with 2 petals? Using lemniscate curves, that's how!

Definition 2.4.1: Lemniscate Curve

A lemniscate curve function takes the form of $r^2 = \pm n^2 \sin(2\theta)$ or $r^2 = \pm n^2 \cos(2\theta)$.



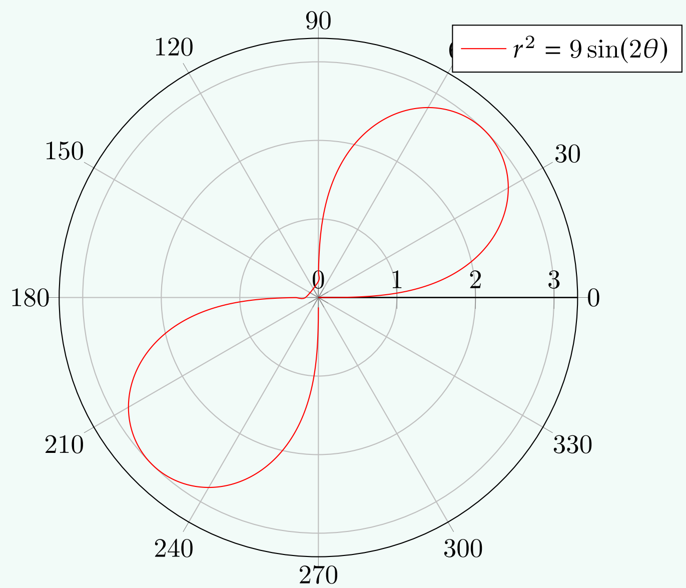
Note:-

Lemniscates cross the pole. It just takes... infinite graphing precision to represent that. I'd rather spare my computer the grief of calculating more than 2,000 samples for this one graph, though.

2 very important pieces of information can be derived from a given lemniscate curve function: the length of each petal, and the angle of the curve. The length of the petals is n. In the case of cos, the curve is entirely

horizontal. In the case of sin, the curve is at 45°. It gets flipped if the function is negative.

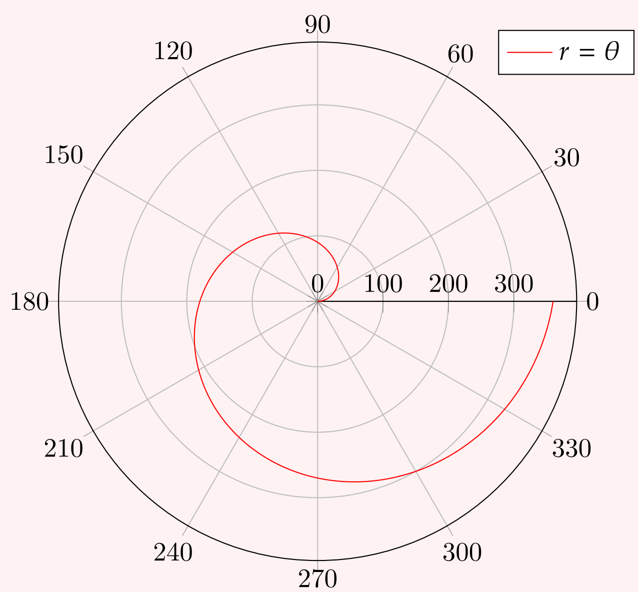
Example 2.4.1 (sin Case of a Lemniscate)



2.5 Spirals of Archimedes

Definition 2.5.1: Spiral of Archimedes

A spiral of Archimedes function takes the form of $r = n\theta$.



Spirals are fairly simple. I don't think it takes much of an explanation.

2.6 Symmetry of Polar Functions

There are 3 kinds of symmetry polar functions can have:

- Symmetry across the line $\theta = \frac{\pi}{2}$
- Symmetry across the polar axis
- Symmetry about the pole

In order to determine if a function has each kind of symmetry, substitute (r, θ) for the following expressions and find whether or not the new equation is equivalent to the original function.

- For symmetry across the line $\theta = \frac{\pi}{2}$, use $(-r, -\theta)$ or $(r, \pi - \theta)$
- For symmetry across the polar axis, use $(r, -\theta)$ or $(-r, \pi - \theta)$
- For symmetry about the pole, use $(-r, \theta)$ or $(r, \pi + \theta)$

Example 2.6.1 (Symmetry of a Polar Function)

Question 1

Find the symmetry of the function $r^2 = 9\cos(2\theta)$

Solution:

- Symmetry across the line $\theta = \frac{\pi}{2}$
  $(-r)^2 = 9\cos(-2\theta)$
  $r^2 = 9\cos(2\theta)$
  Yes
- Symmetry across the polar axis
  $r^2 = 9\cos(-2\theta)$
  $r^2 = 9\cos(2\theta)$
  Yes
- Symmetry about the origin
  $(-r)^2 = 9\cos(2\theta)$
  $r^2 = 9\cos(2\theta)$
  Yes

Looking at the graph below, we can see that all of these symmetries are correct.