Magnet Precalculus D
Combinatorics

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Chapter 1

Intro to Combinatorics

1.1 Counting

If decision M can be made in x number of ways and decision N can be made in y number of ways, then the number of ways to make both decisions is $x \cdot y$.

Example 1.1.1 (Counting)

The ice cream shop offers a choice of a 3 cone sizes, 15 flavors, and 8 toppings. How many cones are possible if you can only choose one flavor and one topping?

$$3 \cdot 15 \cdot 8 = 360$$

There are 360 cones possible.

1.2 Factorials

Definition 1.2.1: Factorial

A factorial of some number n, denoted as $n!$, is the product of all natural numbers from 1 to n. For example, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$.

1.3 Permutations

Definition 1.3.1: Permutation

A permutation of objects, denoted as ${}_nP_r$, is an arrangement of r objects chosen from a set of n objects. The number of possible permutations of size r is denoted as ${}_nP_r = \frac{n!}{(n-r)!}$.

Note:-

$0! = 1$. This means that if $n = r$, then ${}_nP_r = n!$, since $n - r = 0$, so $\frac{n!}{(n-r)!} = \frac{n!}{0!} = n!$.

Example 1.3.1 (Permutation)

There are 16 players on the baseball team. How many ways can the coach make a 9-player batting order?

$$n = 16r = 9_nP_r =_{16} P_9 = \frac{16!}{(16-9)!} = \frac{16!}{7!} = \frac{16!}{5040} = \frac{20922789888000}{5040} = 4151347200$$

There are 4,151,347,200 ways to make a 9-player batting order.

1.4 Combinations

Definition 1.4.1: Combination

A combination of objects, denoted as ${}_nC_r$, is a selection of r objects chosen from a set of n objects. The number of possible combinations of size r is denoted as ${}_nC_r = \frac{n!}{r!(n-r)!}$. **This is different from a permutation because you can have multiple of the same object.**

Chapter 2

Theoretical Probability

2.1 Intro to Probability

Probability is the measure of how likely an event is to occur. The set of all possible outcomes is called the **sample space**. For equally likely outcomes, the probability of an event E is given by the formula $P(E) = \frac{\text{number of outcomes in } E}{\text{number of outcomes in the sample space}}$.

Example 2.1.1 (Simple Event)

A jar contains 32 red marbles and 28 blue marbles. The probability that a randomly selected marble is red is $\frac{32}{32+28} = \frac{32}{60} = \frac{8}{15}$.

2.2 Compound Events

A compound event is an event that consists of two or more simple events. There are two kinds of compound events: **independent** and **dependent**. A compound event is independent when the outcome of one event does not affect the outcome of the other event. A compound event is dependent when the outcome of one event does affect the outcome of the other event. In both cases, the probability of a compound event is the product of the probabilities of the simple events.

Example 2.2.1 (Independent Event)

A jar contains 32 red marbles and 28 blue marbles. If you select a marble, replace it, and then select another marble, the probability that both marbles are red is $\frac{32}{60} \cdot \frac{32}{60} = \frac{1024}{3600} = \frac{256}{900}$.

Example 2.2.2 (Dependent Event)

A jar contains 32 red marbles and 28 blue marbles. If you select a marble, do not replace it, and then select another marble, the probability that both marbles are red is $\frac{32}{60} \cdot \frac{31}{59} = \frac{992}{3540} = \frac{248}{885}$.

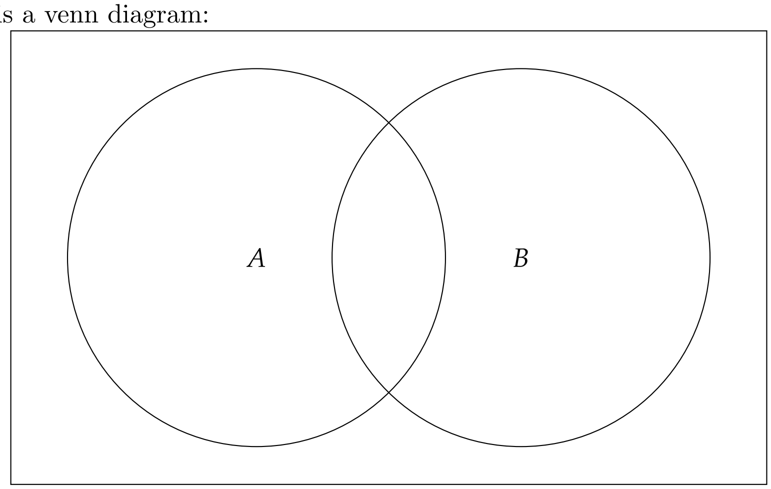
Note:-

In the dependent event example, the effect of the first event is that the sample space is reduced by one.

Chapter 3

Venn Diagrams

This is a venn diagram:

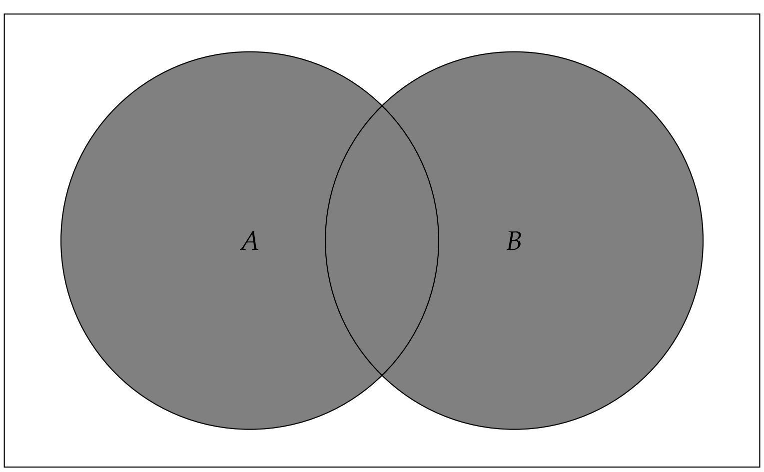


You can specify many different areas within the diagram, such as only where A and B overlap, or everywhere other than A. There is standard mathematical notation to represent these areas.

3.1 Basic Venn Diagram Notation

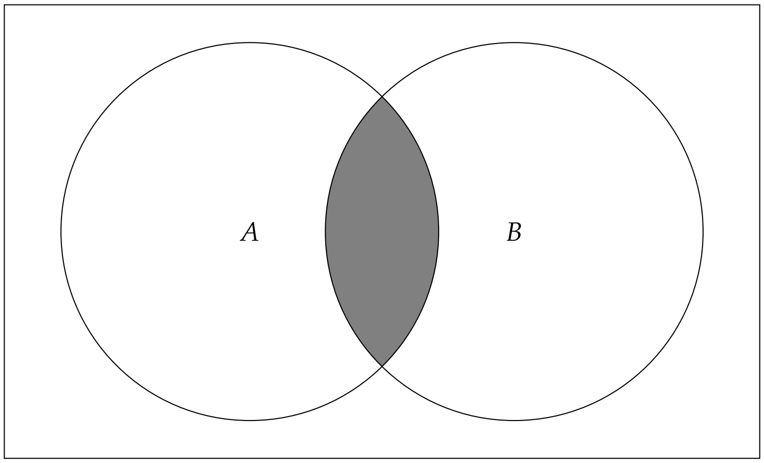
First, $\cup$, the **or** operator. This is the union of two sets, or the area where either A or B is true.

$$A \cup B$$



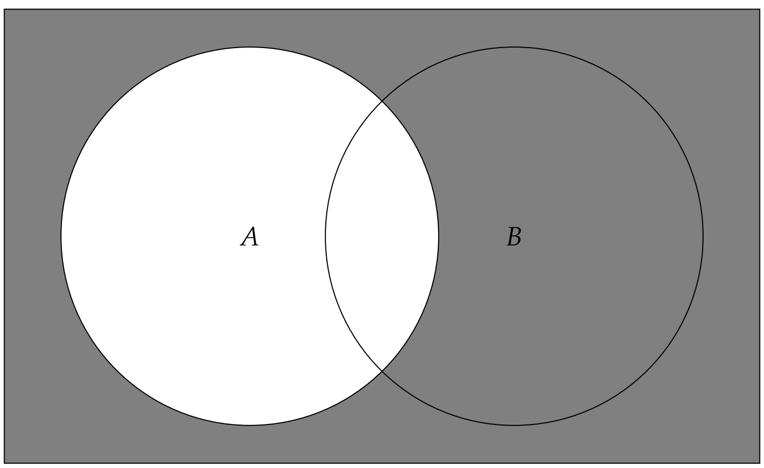
Second is $\cap$, the **and** operator. This is the intersection of two sets, or the area where both A and B are true.

$A \cap B$



Finally, A^c, also written as A', the **complement** of A. This is also known as the **not** operator. This is the area where A is false.

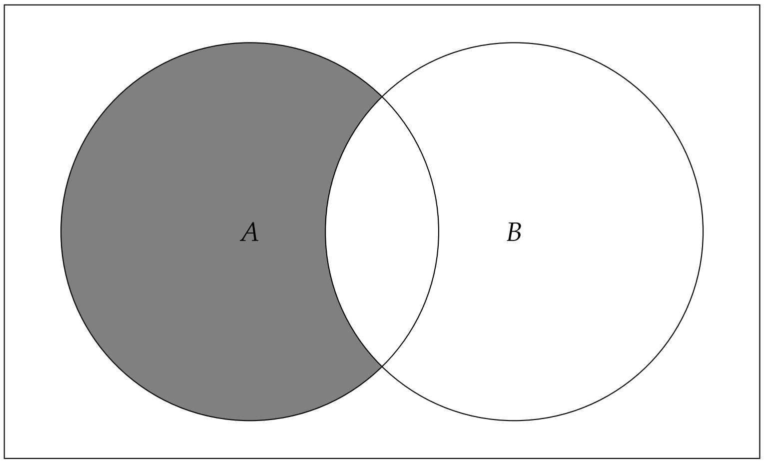
A^c



3.2 Combining Venn Diagram Notation

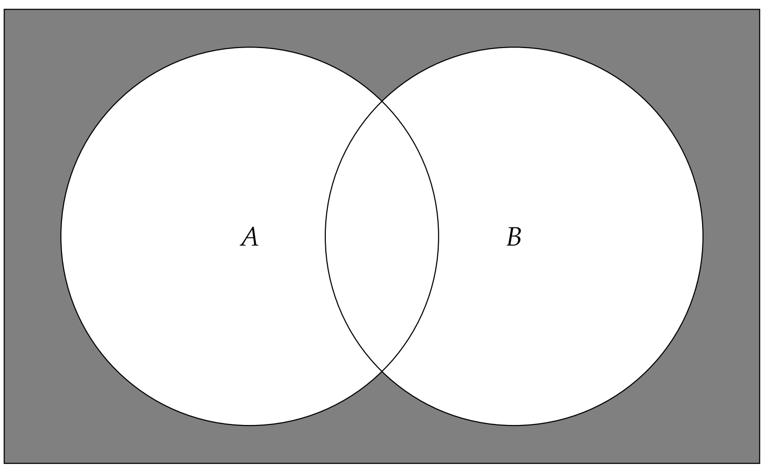
You can combine these operators to create more complex areas. For example, $A \cap B^c$ is the area where A is true and B is false.

$A \cap B^c$

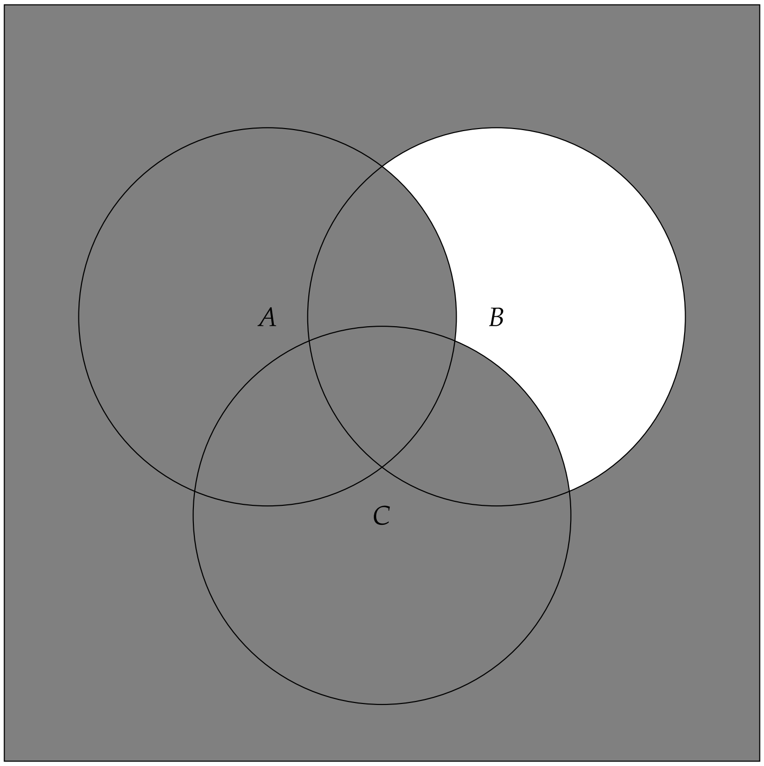


3.3 Venn Diagram Examples

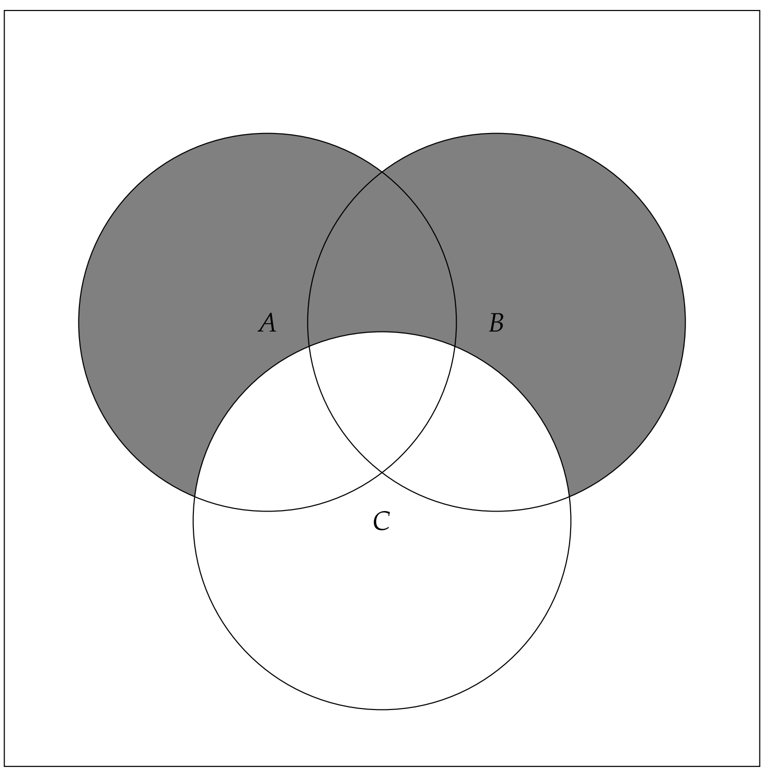
$(A \cap B)^c$



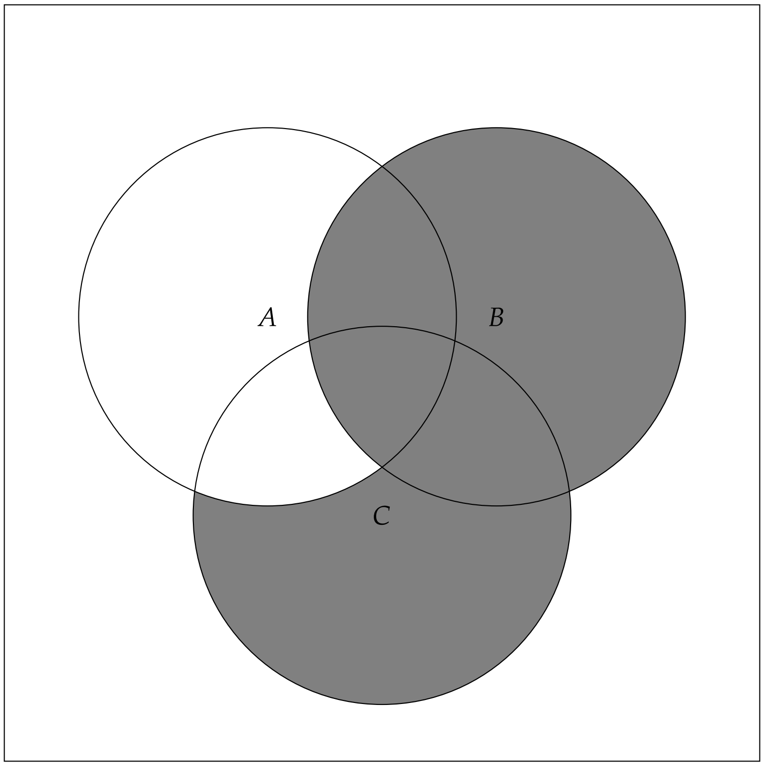
$((A \cup C)^c \cap B)^c$



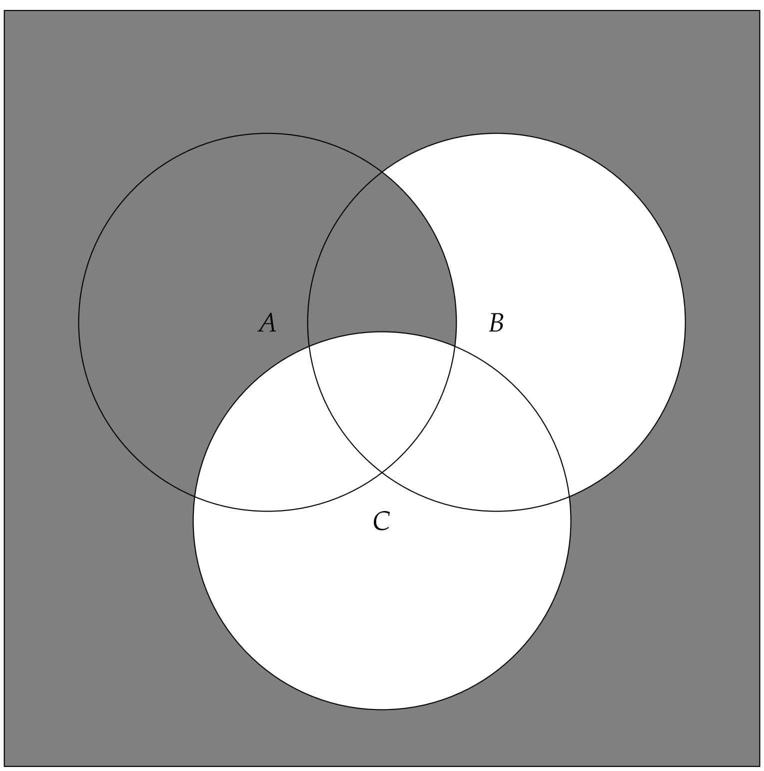
$C^c \cap (A \cup B)$



$(C \cap A^c) \cup B$



$(A \cap C^c) \cup (B \cup C)^c$



$$((A \cap B) \cup (B \cap C))^c$$