Magnet Precalculus C
Semester Exam Review

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Contents

Chapter 1

1.1 Solving Polynomials

Question 1: 1a

Solve for x where $2x^3 = -3x^2 + 2x$.

Solution: Subtract $-3x^2 + 2x$ from both sides to find that $2x^3 + 3x^2 - 2x = 0$. Factor x from that identity to find that $x(2x^2 + 3x - 2) = 0$. Multiplying 2 by -2 tells us that we need to find two numbers which sum to 3 and produce -4. These numbers are -1 and 4. $2x + 4$ can be simplified to $x + 2$. We now know that the factors of $2x^2 + 3x - 2$ are $x + 2$ and $2x - 1$. Therefore, $x(2x^2 + 3x - 2) = x(x + 2)(2x - 1)$. The values of x are 0, -2, and $\frac{1}{2}$.

Question 2: 1b

Solve for x where $x^2 = 3x - 1$.

Solution: Subtract $3x - 1$ from both sides to find that $x^2 - 3x + 1 = 0$. There are no two numbers which sum to -3 and produce 1, so we must use the quadratic formula. $x = \frac{3 \pm \sqrt{9-4(1*1)}}{2} = \underbrace{\boxed{\frac{3 \pm \sqrt{5}}{2}}}_{Solution}$.

1.2 Domain and Range of Functions

Question 3: 2a

Find the domain and range of the function $f(x) = x^2 + sqrtx - 3$.

Solution: The more restrictive function is the square root function, so we must look there to find our domain restriction. We can see that the square root is translated 3 units to the right, so the domain is $[3, \infty)$. At $x = 3$, $y = 3^2 + sqrt3 - 3 = 3^2 = 9$, so the point at which the curve ends is $(3, 9)$. The range is $[9, \infty)$

Question 4: 2b

Find the domain and range of the function $f(x) = \frac{x-5}{x^2-x-20}$.

Solution: $x^2 - x - 20$ can be factored into $(x + 4)(x - 5)$, and since $x - 5$ is in the numerator and denominator, it can be removed. We are left with $f(x) = \frac{1}{x+4}$. This is a reciprocal function, translated 4 units left. This means that its domain is $\mathbb{R}; x \neq -4$, and its range is $\mathbb{R}; y \neq 0$, since its asymptotes are at $y = 0$ and $x = -4$.

1.3 Increasing, Decreasing, and Constant Intervals

Question 5: 3a

Determine the intervals over which the function $f(x) = (x^2 - 4)^2$ is increasing, decreasing, or constant.

Solution: $(x^2-4)^2$ can be factored into $(x^2-4)(x^2-4)$, which can be further factored into $(x+2)(x-2)(x+2)(x-2) = (x+2)^2(x-2)^2$. The degree of the function is 4, which is positive and even, so the end behavior of the function is $\lim_{x\to\infty} f(x) = \infty$; $\lim_{x\to-\infty} f(x) = \infty$. The zeros -2 and 2 both have a multiplicity of 2, so they serve as relative minimums/maximums. $f(x)$ decreases from $-\infty$, so $(-2, 0)$ is a relative minimum. $f(x)$ increases to ∞, so $(2, 0)$ must also be a relative minimum. This means there must be a relative maximum somewhere in the middle between these two points. $2-(-2) = 4$; $\frac{4}{2} = 2$; $-2+2 = 0$, therefore the x-value of the relative maximum must be 0. $f(0) = (0^2-4)^2 = (-4)^2 = 16$, therefore the relative maximum is located at $(0, 16)$. With our 2 relative minimums and relative maximum located, we can find the increasing and decreasing intervals. Increasing: $(-2, 0)u(2, \infty)$. Decreasing: $(-\infty, -2)u(0, 2)$.

Question 6: 3b

Determine the intervals over which the function $f(x) = |x - 1| + 1$ is increasing, decreasing, or constant.

Solution: This absolute value function is translated 1 unit up and 1 unit to the right. Therefore, its increasing interval is $(1, \infty)$, and its decreasing interval is $(-\infty, 1)$

Question 7: 3c

Determine the intervals over which the function $g(x) = \frac{x^2}{x^2-4}$ is increasing, decreasing, or constant.

Solution: In this reciprocal function, the degree of the numerator is equal to that of the denominator, so the horizontal asymptote is at $y = 1$, since the quotient of the two leading coefficients is $\frac{1}{1}$. There are two vertical asymptotes, since the denominator has a degree of 2. $x^2 - 4$ can be factored into $(x + 2)(x - 2)$, making the zeros of the expression 2 and -2. Therefore, the vertical asymptotes are at $x = 2$ and $x = -2$. We can determine direction by calculating a few values. $g(-3) = \frac{(-3)^2}{(-3)^2-4} = \frac{9}{9-4} = \frac{9}{5} > 1$, so on the interval $(-\infty, -2)$, $g(x)$ increases. $g(-1) = \frac{(-1)^2}{(-1)^2-4} = \frac{1}{1-4} = \frac{1}{-3} < 1$, so on the interval $(-2, 0)$, $g(x)$ increases. In order to follow the vertical asymptote $x = 2$, $g(x)$ must decrease on the interval $(0, 2)$. $g(3) = \frac{3^2}{3^2-4} = \frac{9}{9-4} = \frac{9}{5} > 1$, therefore on the interval $(2, \infty)$, $g(x)$ decreases.

Question 8: 3d

Determine the intervals over which the function $h(x) = \sqrt[3]{x + 1}$ is increasing, decreasing, or constant.

Solution: This third root function is translated 1 unit to the left. However, it isn't reflected, so it is always increasing.

1.4 Even and Odd Functions

Question 9: 4a

Determine whether the function $f(x) = x^5 + 3x - 4$ is even, odd, or neither