

Magnet Precalculus CD
Semester 1 Notes

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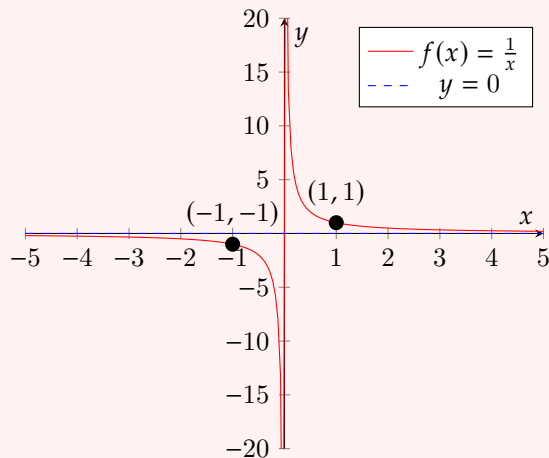
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Chapter 1

1.1 The Reciprocal Function

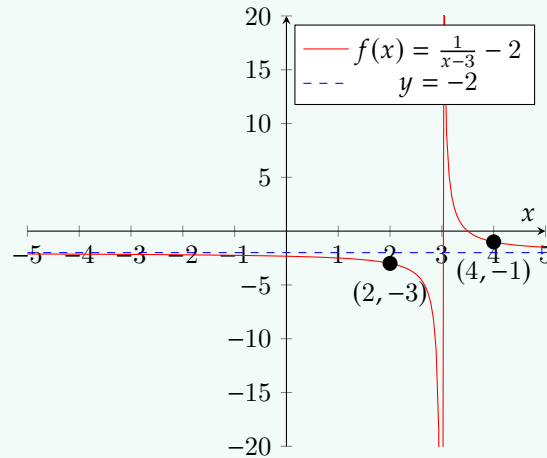
Definition 1.1.1: Reciprocal function

Any function where an expression is being divided by x . The parent reciprocal function is $f(x) = \frac{1}{x}$, and the standard form of a reciprocal function is $f(x) = \frac{a}{x+h} + k$



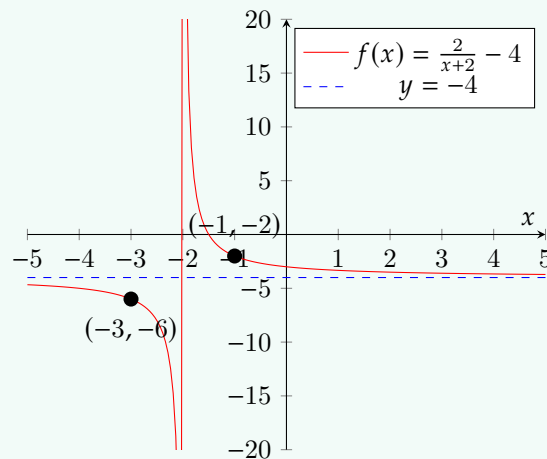
Reciprocal functions always have 2 or more asymptotes: 1 or more vertical asymptote, and a horizontal **or** slant asymptote. Should a remain a constant, the horizontal asymptote is at $y = k$, and the vertical asymptotes are at $x = n$, where n is all values of x that make the expression on the denominator equal 0.

Example 1.1.1 (Translation 3 units right and 2 units down)



Here, $h = 3$, so the function is translated 3 units right, so the vertical asymptote is at $x = 3$. Similarly, $k = -2$, so the function is translated 2 units down, so the horizontal asymptote is at $y = -2$.

Example 1.1.2 (Translation 2 units left and 4 units down)



Here, $h = -2$, so the function is translated 2 units left, so the vertical asymptote is at $x = -2$. Similarly, $k = -4$, so the function is translated 4 units down, so the horizontal asymptote is at $y = -4$.

Note:-

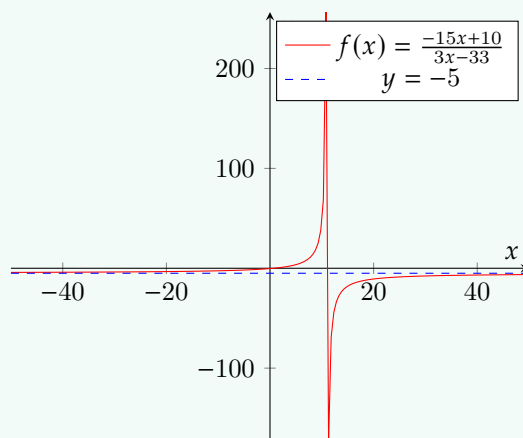
There are a few special cases for the horizontal asymptote where a is a polynomial rather than a constant. **Should the degree in the numerator be less than in the denominator**, the horizontal asymptote will be at $y = 0$

Should the degree in the numerator be equal to that in the denominator, the horizontal asymptote will be at $y = n$, where n is the quotient of the greatest-degree coefficients of the numerator and denominator.

Should the degree in the numerator be greater than in the denominator, there is not a horizontal asymptote, but a slant asymptote. The slant asymptote will be at $y = n$, where n is the result of long-dividing the numerator and denominator and excluding the remainder.

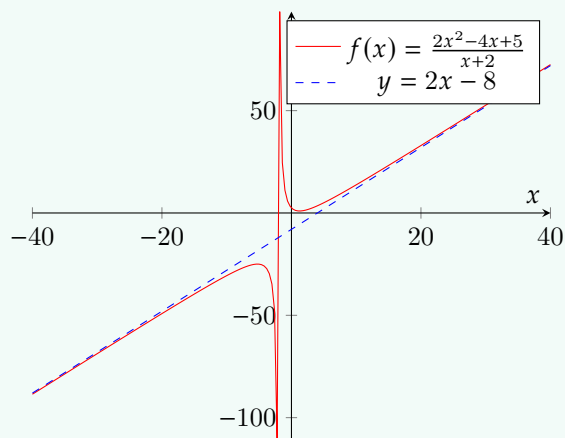
Example 1.1.3 (Equal degree polynomials)

Given the function $f(x) = \frac{-15x+10}{3x-33}$, we can divide 6 by 3 to find that the horizontal asymptote of $f(x)$ is at $y = -5$.



Example 1.1.4 (Greater degree numerator)

Given the function $f(x) = \frac{2x^2-4x+5}{x+2}$, we can do long division to find that $\frac{2x^2-4x+5}{x+2} = 2x - 8 + \frac{21}{x+2}$. From this, we can derive that the slant asymptote of $f(x)$ is $y = 2x - 8$.



1.2 The Trigonometric Identities

The angle sum identities are as follows:

- $\sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \sin(\beta) \cos(\alpha)$
- $\cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta)$
- $\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha) \tan(\beta)}$

These “double” identities can be derived from the sum identities:

- $\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$
- $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2 \cos^2(\theta) - 1 = 1 - 2 \sin^2(\theta)$
- $\tan(2\theta) = \frac{2 \tan(\theta)}{1 - \tan^2(\theta)}$

There are also half-angle identities:

- $\sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos(\theta)}{2}}$

- $\cos(\frac{\theta}{2}) = \pm \sqrt{\frac{1+\cos(\theta)}{2}}$
- $\tan(\frac{\theta}{2}) = \pm \sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}} = \frac{1-\cos(\theta)}{\sin(\theta)} = \frac{\sin(\theta)}{1+\cos(\theta)}$

Some other crucial identities:

- $\sin^2(\theta) + \cos^2(\theta) = 1$
- $\sec^2(\theta) = 1 + \tan^2(\theta)$
- $\csc^2(\theta) = 1 + \cot^2(\theta)$