Magnet Precalculus CDParametric Equations

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Contents

Chapter 1

1.1 Intro to Parametric Equations

We can represent the combination of multiple functions on one 2d plane by defining both x and y in terms of a parameter, often t. These two equations are known as parametric equations. When graphing parametric equations, you should draw arrows between the points you plot, in the direction that t is moving.

1.2 Parametric Equations in Rectangular Form

To put a parametric equation in rectangular form, solve one of the equations for t, substitute the resulting expression into the other equation, and simplify.

Question 1: Write the following pair of parametric equations in rectangular form.

$x = 4t - 1$
$y = 6 - t$

$t = 6 - y$
$x = 4(6 - y) - 1 = 24 - 4y - 1 = 23 - 4y$
$x - 23 = -4y$
$\boxed{y = \dfrac{-x + 23}{4}}$

Question 2: Write the following pair of parametric equations in rectangular form.

$x = \frac{t+2}{t}$
$y = \frac{1}{t}$

$t = \frac{1}{y}$
$x = \frac{\frac{1}{y}+2}{\frac{1}{y}} = \frac{\frac{1}{y}}{\frac{1}{y}} + \frac{2}{\frac{1}{y}} = 1 + \frac{2}{\frac{1}{y}} = 1 + 2y$
$x - 1 = 2y$
$\boxed{y = \dfrac{x - 1}{2}}$

1.3 Polar & Parametric Equations

The graph of a polar equation $r = f(\theta)$ is the same as the graph of the parametric equations $x = f(\theta)\cos(\theta)$ and $y = f(\theta)\sin(\theta)$.

Question 3: Write the following pair of parametric equations in rectangular form

$x = 3\cos(\theta)$
$y = 2\sin(\theta)$

$\cos(\theta) = \frac{x}{3}$
$\cos^2(\theta) = \frac{x^2}{9}$
$\sin(\theta) = \frac{y}{2}$
$\sin^2(\theta) = \frac{y^2}{4}$
$\sin^2(\theta) + \cos^2(\theta) = 1$
$$\boxed{\frac{x^2}{9} + \frac{y^2}{4} = 1}$$

Question 4: Write the following pair of parametric equations in rectangular form

$x = \sin^2(\theta)$
$y = 4\cos(\theta)$

$\cos(\theta) = \frac{y}{4}$
$\cos^2(\theta) = \frac{y^2}{16}$
$$\boxed{x + \frac{y^2}{16} = 1}$$