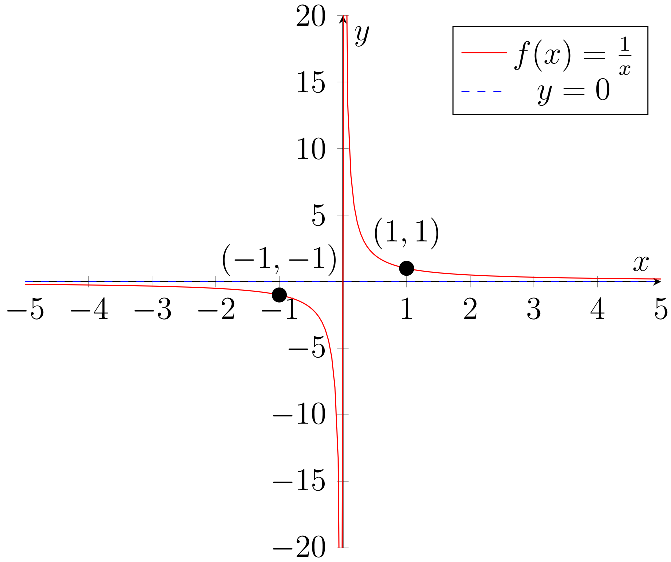
Reciprocal functions

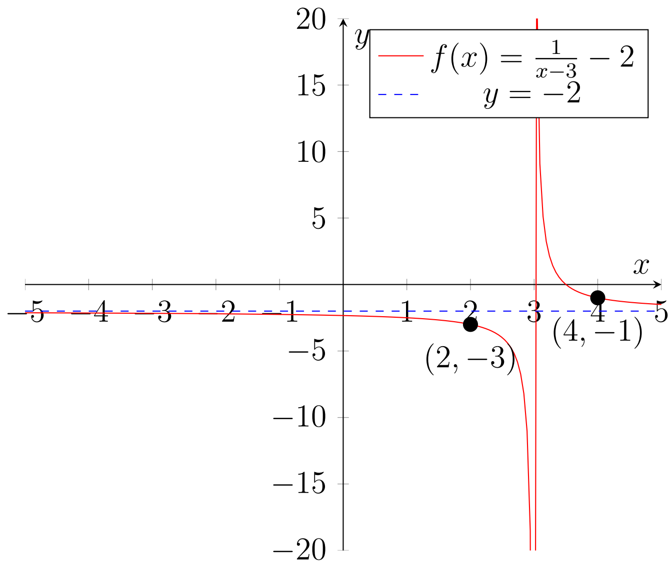
Parent function: $f(x) = \frac{1}{x}$
Standard form: $f(x) = \frac{a}{x-h} + k$



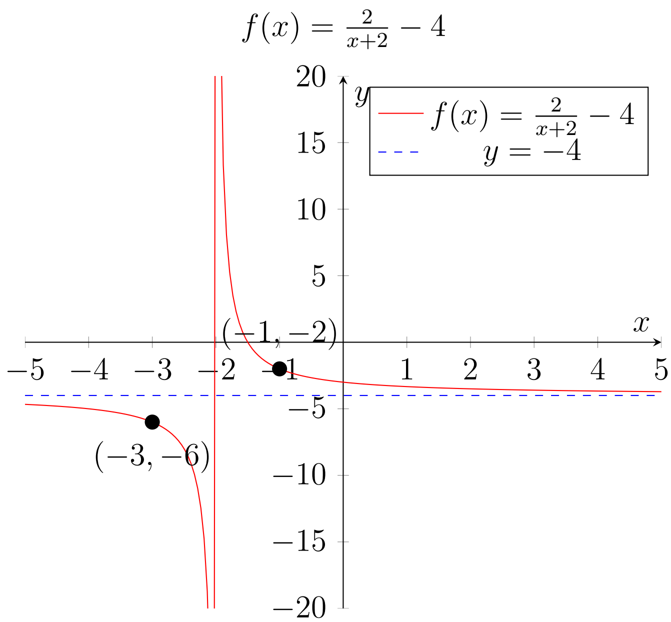
Reciprocal functions always have 2 or more asymptotes: 1 or more vertical asymptote, and a horizontal **or** slant asymptote. Should a remain a constant, the horizontal asymptote is at $y = k$, and the vertical asymptotes are at $x = n$, where n is all values of x that make the expression on the denominator equal 0.

Some examples:

$f(x) = \frac{1}{x-3} - 2$



Here, $h = 3$, so the function is translated 3 units right, so the vertical asymptote is at $x = 3$. Similarly, $k = -2$, so the function is translated 2 units down, so the horizontal asymptote is at $y = -2$.

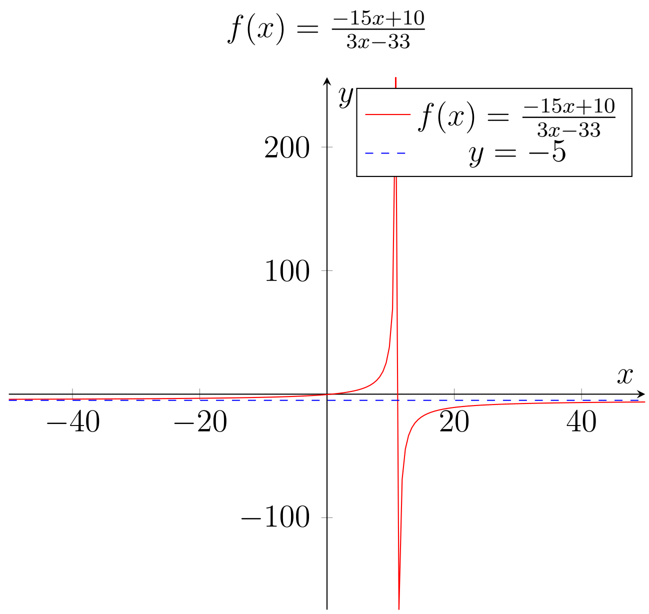


Here, $h = -2$, so the function is translated 2 units left, so the vertical asymptote is at $x = -2$. Similarly, $k = -4$, so the function is translated 4 units down, so the horizontal asymptote is at $y = -4$.

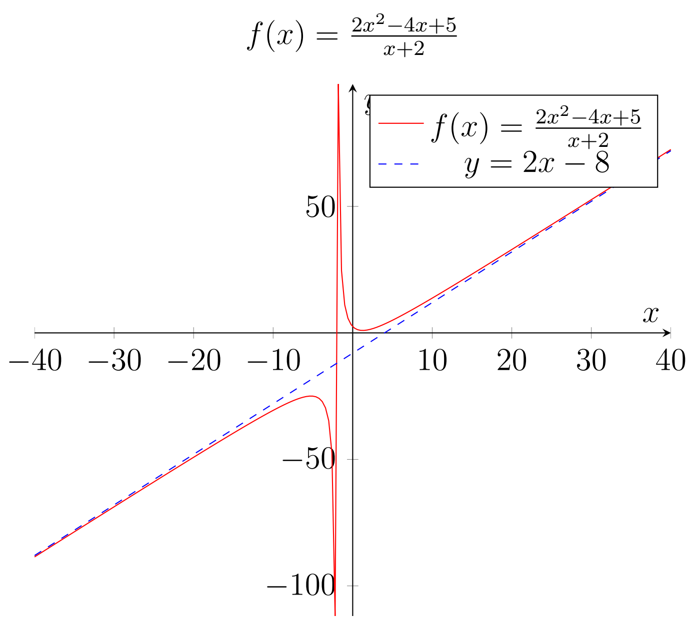
There are a few special cases for the horizontal asymptote where a is a polynomial rather than a constant.

Should the degree in the numerator be less than in the denominator, the horizontal asymptote will be at $y = 0$

Should the degree in the numerator be equal to that in the denominator, the horizontal asymptote will be at $y = n$, where n is the quotient of the greatest-degree coefficients of the numerator and denominator.
For example, given the function $f(x) = \frac{-15x+10}{3x-33}$, we can divide 6 by 3 to find that the horizontal asymptote of $f(x)$ is at $y = -5$.



Should the degree in the numerator be greater than in the denominator, there is not a horizontal asymptote, but a slant asymptote. The slant asymptote will be at $y = nx$, where n is the result of long-dividing the numerator and denominator and excluding the remainder.
For example, given the function $f(x) = \frac{2x^2-4x+5}{x+2}$, we can do long division to find that $\frac{2x^2-4x+5}{x+2} = 2x - 8 + \frac{21}{x+2}$. From this, we can derive that the slant asymptote of $f(x)$ is $y = 2x - 8$.



And here we can see that this is correct.