Magnet Precalculus CD
Matrices

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Contents

Chapter 1

1.1 Introduction to Matrices

Definition 1.1.1: Matrix

A **matrix** is a rectangular array of variables or constants in rows or columns, usually enclosed in brackets. These constants or variables are known as **elements**.

Definition 1.1.2: Element

An element is an individual value within a matrix. Given a matrix A, a given element in side of A is notated as A_{xy}, where x is the row and y is the column in which the element is located.

Note:-

If either the width or height of a matrix is more than one digit, x and y in element notation are generally separated by a dash (e.g. A_{10-4})

Example 1.1.1 (Find an Element of a Matrix)

$$A = \begin{bmatrix} -8 & 40 & 0 & -1 & 21 \\ 27 & 32 & -29 & 6 & -2 \\ 5 & -7 & 14 & 52 & -35 \end{bmatrix}$$

$A_{12} = 40$

$A_{34} = 52$

A matrix with m rows and n columns is known as an "m by n" matrix, written as mxn. These are its **dimensions**.

Example 1.1.2 (Dimensions of a matrix)

Consider some matrix $A = \begin{bmatrix} 1 & -8 \\ -4 & 13 \\ -6 & -2 \\ 28 & 0 \end{bmatrix}$. A has 4 rows and 2 columns, so its dimensions are 4x2.

1.2 Summation of Matrices

Matrices can be summed **only if their dimensions are the same**. The process is as simple as summing all corresponding elements.

Example 1.2.1 (Sum of Two Matrices)

$$W = \begin{bmatrix} -1 & 9 \\ -11 & 15 \\ 8 & -20 \end{bmatrix}$$
$$Z = \begin{bmatrix} -3 & -2 \\ -16 & 0 \\ 12 & 9 \end{bmatrix}$$
$$W + Z = \begin{bmatrix} -4 & 7 \\ -27 & 15 \\ 20 & -11 \end{bmatrix}$$
$$W - Z = \begin{bmatrix} 2 & 11 \\ 5 & 15 \\ -4 & -29 \end{bmatrix}$$

1.3 Multiplication of Matrices

Before we get to the method of matrix multiplication, there is a very important condition that must be met.

Consider two matrices, A and B. **They can only be multiplied if A has the same number of columns as B has rows**. In other words, if A had dimensions $m_1 \text{x} n_1$ and B had $m_2 \text{x} n_2$, they could only be multiplied if $n_1 = m_2$. The dimensions of the product matrix are $m_1 \text{x} n_2$

Element $AB_{hk} = A_{h1}B_{1k} + A_{h2}B_{2k} + A_{h3}B_{3k} + ... + A_{hn_2}B_{n_2k}$. So, if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$, then $AB = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + ch \end{bmatrix}$

Example 1.3.1 (Multiplication of Matrices)

$$A = \begin{bmatrix} 9 & -5 \\ -2 & 4 \end{bmatrix}$$
$$B = \begin{bmatrix} -3 & 13 & -5 \\ -1 & -7 & 2 \end{bmatrix}$$
$$AB = \begin{bmatrix} (9 \cdot -3) + (-5 \cdot -1) & (9 \cdot 13) + (-5 \cdot -7) & (9 \cdot -5) + (-5 \cdot 2) \\ (-2 \cdot -3) + (4 \cdot -1) & (-2 \cdot 13) + (4 \cdot -7) & (-2 \cdot -5) + (4 \cdot 2) \end{bmatrix} = \begin{bmatrix} -27 + 5 & 117 + 35 & -45 - 10 \\ 6 - 4 & -26 - 28 & -10 + 8 \end{bmatrix} = \begin{bmatrix} -22 & 152 & -55 \\ 2 & -54 & -2 \end{bmatrix}$$