# Magnet Precalculus CD
# Parametric Equations

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# Contents

# Chapter 1

## 1.1 Intro to Parametric Equations

We can represent the combination of multiple functions on one 2d plane by defining both $x$ and $y$ in terms of a parameter, often $t$. These two equations are known as parametric equations. When graphing parametric equations, you should draw arrows between the points you plot, in the direction that $t$ is moving.

## 1.2 Parametric Equations in Rectangular Form

To put a parametric equation in rectangular form, solve one of the equations for t, substitute the resulting expression into the other equation, and simplify.

**Question 1: Write the following pair of parametric equations in rectangular form.**

$x = 4t - 1$
$y = 6 - t$

$t = 6 - y$
$x = 4(6 - y) - 1 = 24 - 4y - 1 = 23 - 4y$
$x - 23 = -4y$
$$\boxed{y = \frac{-x + 23}{4}}$$

**Question 2: Write the following pair of parametric equations in rectangular form.**

$x = \frac{t+2}{t}$
$y = \frac{1}{t}$

$t = \frac{1}{y}$
$x = \frac{\frac{1}{y}+2}{\frac{1}{y}} = \frac{\frac{1}{y}}{\frac{1}{y}} + \frac{2}{\frac{1}{y}} = 1 + \frac{2}{\frac{1}{y}} = 1 + 2y$
$x - 1 = 2y$
$$\boxed{y = \frac{x - 1}{2}}$$

## 1.3 Polar & Parametric Equations

The graph of a polar equation $r = f(\theta)$ is the same as the graph of the parametric equations $x = f(\theta)\cos(\theta)$ and $y = f(\theta)\sin(\theta)$.

**Question 3: Write the following pair of parametric equations in rectangular form**

$x = 3\cos(\theta)$
$y = 2\sin(\theta)$

$\cos(\theta) = \frac{x}{3}$
$\cos^2(\theta) = \frac{x^2}{9}$
$\sin(\theta) = \frac{y}{2}$
$\sin^2(\theta) = \frac{y^2}{4}$
$\sin^2(\theta) + \cos^2(\theta) = 1$

$$\boxed{\frac{x^2}{9} + \frac{y^2}{4} = 1}$$

**Question 4: Write the following pair of parametric equations in rectangular form**

$x = \sin^2(\theta)$
$y = 4\cos(\theta)$

$\cos(\theta) = \frac{y}{4}$
$\cos^2(\theta) = \frac{y^2}{16}$

$$\boxed{x + \frac{y^2}{16} = 1}$$

## 1.4 Modeling Projectile Motion With Parametric Equations

Projectile motion can be modeled with parametric equations.

**Example 1.4.1** (Tennis Serve)

A baseball player in the outfield throws a ball towards home plate. The ball leaves his hand at a height of 6.5 feet at an initial velocity of 110 feet per second.

**Question 5**

The ball is caught by the player at home plate after 1.8 seconds. If the ball traveled a horizontal distance of 191 feet, find the angle of elevation at which the ball was thrown.

$191 = 1.8 \cdot 110\cos(\theta)$
$\cos(\theta) = \frac{191}{1.8 \cdot 110} = \frac{191}{198}$
$\theta = \cos^{-1}(\frac{191}{198}) \approx 15.28$

**Question 6**

What was the height of the ball when it was caught by the player at home plate?

$x = 110\cos(\theta)t$
$y = 110\sin(\theta)t - \frac{gt^2}{2}$
$t = \frac{x}{110\cos(\theta)}$

$$y = 110\sin(\theta)\frac{x}{110\cos(\theta)} - \frac{\frac{gx^2}{12100cos^2(\theta)}}{2} = \frac{110\sin(\theta)x}{110\cos(\theta)} - \frac{gx^2}{24200\cos^2(\theta)} = x\tan(\theta) - \frac{gx^2}{24200\cos^2(\theta)}$$