Magnet Precalculus CD
Vectors

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Chapter 1

Geometric Representation of Vectors

1.1 Combination

1.2 Magnitude of Vectors

The magnitude (length) of a vector $v = \langle a, b \rangle$ is $||v|| = \sqrt{a^2 + b^2}$. If vector v is represented by the arrow from (x_1, y_1) to (x_2, y_2), then $||v|| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

1.3 Unit Vector

Definition 1.3.1: Unit Vector

A vector u for which $||u|| = 1$ is called a **unit vector**.

$$u = \frac{v}{||v||}$$
$$i = \langle 1, 0 \rangle \text{ and } j = \langle 0, 1 \rangle$$

1.4 Components of a Vector

Let v be a vector with magnitude $||v||$ and direction θ. Then, $v = \langle a, b \rangle = ai + bj$ where $a = ||v|| \cos(\theta)$ and $b = ||v|| \sin(\theta)$. Therefore, we can express v as $v = ||v|| \cos(\theta) i + ||v|| \sin(\theta) j$

Chapter 2

The Language of Vectors

2.1 The Dot Product

Definition 2.1.1: Dot Product

A **dot product** of $u = \langle a_1, b_1 \rangle$ and $v = \langle a_2, b_2 \rangle$ is $u \bullet v = a_1 a_2 + b_1 b_2$

Another form of the dot product is $u \bullet v = ||u||||v|| \cos(\theta)$, where θ is the angle between u and v.

To find the angle between two vectors, the dot product can be used:
Let θ be the angle between two vectors, u and v. Where $0 \leq \theta \leq \pi$, $\cos(\theta) = \frac{u \bullet v}{||u||||v||}$

The dot product can be used to find the magnitude of a vector. $||u|| = u \bullet u = u^2$

Definition 2.1.2: Orthogonal Vectors

When two angles are orthogonal (a.k.a. perpendicular or normal), their dot product is 0.

Question 1

Find the angle between the vectors $u = 2i - j$ and $v = 6i + 4j$

Solution: $\cos(\theta) = \frac{(2 \bullet 6) + (-1 \bullet 4)}{\sqrt{2^2 + (-1)^2} \bullet \sqrt{6^2 + 4^2}} = \frac{12-4}{\sqrt{5} \bullet \sqrt{52}} = \frac{8}{\sqrt{260}} = \frac{8}{2\sqrt{65}}$; $\theta = \cos^{-1}(\frac{8}{1\sqrt{65}}) \approx \boxed{60.255°}$