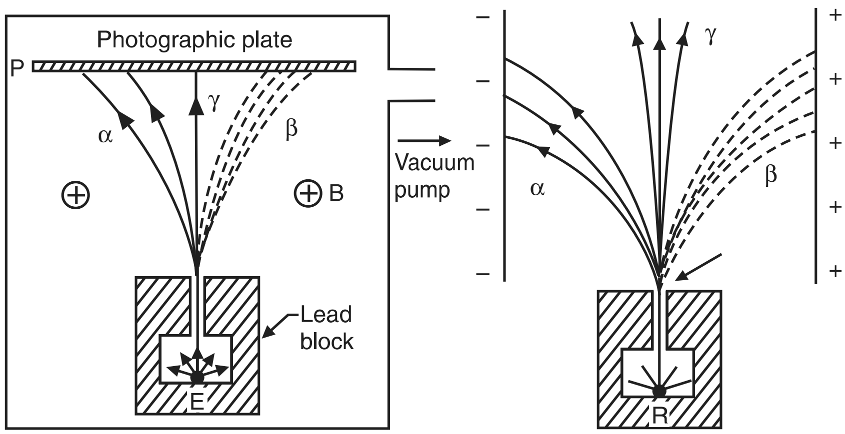
The liquid drop model successfully explained the process of nuclear fission and α-decay.

21.9 NATURAL RADIOACTIVITY



(*a*) (*b*)

Fig. 21.4: Passing the radiation from a radioactive source through (a) a magnetic field or (b) an electric field shows that there are three components

In 1896, Henry Becquerel discovered that uranium salts emit invisible radiation. *The phenomenon of emission of radiation by elements is called* **radioactivity**. The elements, which emit radiations, are called **radioactive** elements. It was found that radioactivity is the property of certain nuclei and hence such nuclei are called **radioactive nuclides**. Investigations on radioactive materials (Fig. 21.4) showed that the emitted radiations consist of three types of radiations, namely, α-rays, β-rays and γ-rays. Soon it was established that α-rays are streams of ${}^{4}_{2}He$ nuclei; β-rays are streams of very fast electrons and γ-rays are electromagnetic radiations which are much more penetrating than X-rays. During the emission of α-rays and β- rays, the composition of the nucleus changes, whereas in γ-rays emission the nuclear composition remains unaltered. γ-rays are emitted whenever the nucleus goes over from a higher energy state to a lower energy state.

Radioactivity may be natural or artificial. The **natural radioactivity** is the radioactivity found in nature and is exhibited by only a very small number of nuclei. Radioactivity exhibited by certain nuclei produced in the laboratory through nuclear reactions is called **induced radioactivity** or **artificial radioactivity**.

21.10 RADIOACTIVE DECAY

The spontaneous transformation occurring in the constitution of radioactive nuclides due to their radioactive property is called **radioactive decay** or **radioactive disintegration**. The transformation accompanied by the emission of α-rays is called **α-decay** and that accompanied by the emission of β-rays is called **β-decay**. The nucleus that undergoes radioactive decay is called the **parent**, the intermediate products are called **daughters** and the final stable nonradioactive nucleus is alled the **end product**.

α-decay: An α-particle is the same as a helium nucleus and consists of two protons and two neutrons. It carries a positive charge of two units. In α-decay, the mass number of the nucleus decreases by four units and the charge on the nucleus by two units. Therefore, the original element is transformed into an element two steps down in the periodic table. For example, α-decay of uranium produces thorium. Thus,

$$ {}^{238}_{92}U \rightarrow {}^{4}_{2}He + {}^{234}_{90}Th $$

β-decay: A beta particle is simply an electron. β-decay of a nucleus leads to a decrease of one negative charge and no change in mass. It produces an element one step higher in the periodic table. For example, Th-234 emits β-particles and transforms into protactinium (Pa).

$$^{234}_{90}Th \;\rightarrow\; ^{0}_{-1}e \;+\; ^{234}_{91}Pa$$

γ-decay: A γ-particle is a quantum of energy. γ-radiation may or may not be emitted simultaneously with α- or β-rays. The nucleus has energy levels similar to the atomic energy levels. A γ-ray is emitted when an excited nucleus jumps to a lower energy level. For example,

$$^{209}_{82}Pb^{*} \;\rightarrow\; ^{209}_{82}Pb \;+\; \gamma$$

Since γ-rays do not involve mass or charge, the emission of a gamma photon does not cause a transformation of species of nucleus.

21.11 RADIOACTIVE SERIES

All the naturally occurring radioactive elements lie in the range of atomic numbers from $Z = 81$ to $Z = 92$. The nuclei of these elements are unstable and disintegrate by emitting either α or β particles. Sometimes γ-rays accompany the emission of these particles. A radioactive nucleus often decays to another radioactive nucleus. This daughter nucleus decays to a third nucleus, which is also radioactive. The chain of radioactive decays continues until finally a stable nucleus forms. It is known that all natural disintegration processes end with the formation of stable lead atoms. The chain of successive radioactive decays is said to form a **radioactive series**.

There are three series of naturally occurring radioactive elements and a fourth one produced artificially. The three series of naturally occurring radioactive elements are

1. Uranium series
2. Actinium series
3. Thorium series

The series of artificially produced radio-active elements is Neptunium series.

Each series is named after the name of the parent nucleus from which the series starts. These series follow a decay pattern similar to that of the uranium series. In each radioactive series, each nuclide transforms into the next through a chain of α- and β-decays and ultimately a stable nucleus forms at the end. Thus, the uranium series terminates in the $^{206}_{82}Pb$ nucleus, the actinium series in $^{207}_{82}Pb$ nucleus and the thorium series in $^{208}_{82}Pb$ nucleus. The uranium radioactive decay series is shown in Fig. 21.5. The neptunium series terminates in the $^{209}_{83}Bi$ nucleus.

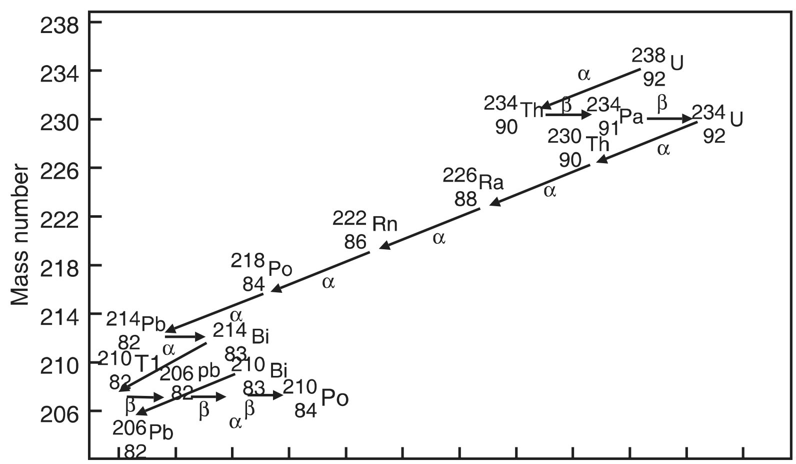


Fig. 21.5

21.12 LAW OF RADIOACTIVE DECAY

The number of parent nuclei in a radioactive material decreases with time because of radioactive disintegration. The disintegration is a statistical and random process. Which nucleus in the material disintegrates first is only a matter of chance. Assuming that each nucleus has the same probability of decaying in one second, we can determine how many nuclei in a sample will decay over a given period of time.

The number of nuclei that disintegrate per second is called *rate of radioactive decay*. The rate of decay is proportional to the number of nuclei that have not yet disintegrated at any instant.

Let there be N untransformed nuclei present in a radioactive sample at time t and let dN be the number of decays in a short duration between $t+dt$. Then, dN, the number of nuclei disintegrating in the interval dt will be proportional to N and dt. That is,

$$dN \propto N\,dt$$

or

$$dN = -\lambda N\,dt \tag{21.7}$$

where λ is constant of proportionality and is known as **decay constant** or **disintegration constant**. It is characteristic of the nuclear species. The minus sign in the above equation (21.7) indicates that N decreases with time. The above relation implies that during a longer interval of time, a greater number of nuclei disintegrate and the number of nuclei undergoing decay per unit time will be greater with a larger sample.

The fraction of nuclei decaying in a time dt is given by

$$\frac{dN}{N} = -\lambda\,dt$$

Assuming that there were N_o nuclei at time $t = 0$, and integrating the above equation, we can find out the nature of the decay.

$$\int_{N_o}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

$$ln[N/N_o] = -\lambda t$$

Taking exponential on both the sides of the above equation, we obtain

$$\frac{N}{N_o} = e^{-\lambda t}$$

or

$$N = N_o e^{-\lambda t} \tag{21.8}$$

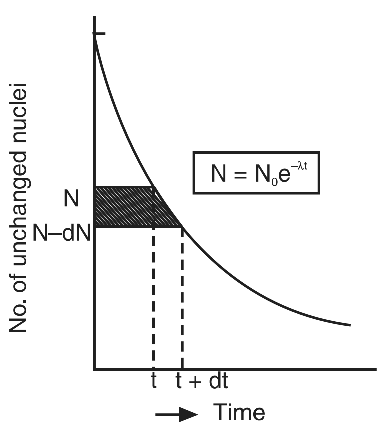


Fig. 21.6

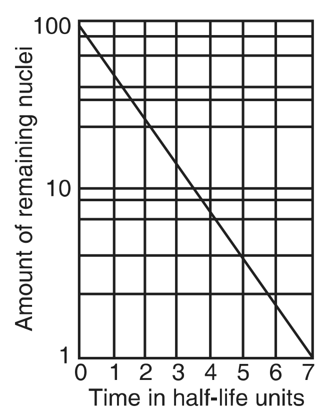


Fig. 21.7

The above equation (21.8) is known as the **law of radioactive decay**. This relation shows that the number of nuclei of a given species decreases exponentially with time provided no new nuclei are added. The decay occurs rapidly initially and then becomes slower and slower as depicted in Fig. 21.6.

A plot of *ln* (N/N_o) versus time is plotted in Fig. 21.7. It is a straight line and the slope of the line gives the value of λ. Equ.(21.7) may be rearranged as

$$\lambda = -\frac{(dN/N)}{dt} \tag{21.9}$$

λ may be defined as the fractional decrease in the number of nuclei decaying per unit time.

21.13 ACTIVITY

In the decay process, we are generally interested in the number of disintegrations per second. This is called the **activity**, A of the sample. Thus,

$$A = \left|\frac{dN}{dt}\right| = \lambda N_o e^{-\lambda t} = \lambda N \tag{21.10}$$

It is obvious from equ.(21.10) that the activity at $t = 0$ is given by $A_o = \lambda N_o$. Therefore,

$$A = A_o e^{-\lambda t} \tag{21.11}$$

The plot of activity as a function of time is shown in Fig. 21.8.

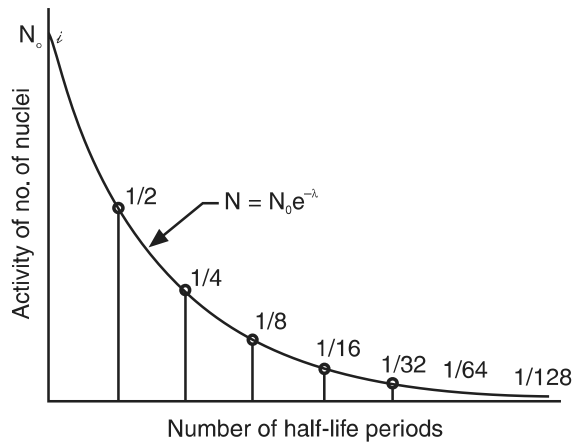


Fig. 21.8

21.14 HALF-LIFE

The rate of decay or activity of a radioactive element is measured in terms of a characteristic time called the half-life. The **half-life** of an element is defined as the time taken by half of the original quantity to undergo decay. If N_o are the number of nuclei present at $t = 0$, then the time in which $N_o/2$ nuclei decay will be the half-life. In other words, half-life is the time after which half of the original number of nuclei remains untransformed. Half-life can also be defined as the time required for the initial activity A_o to decrease to $A_o/2$. Half-life is denoted by $T_{1/2}$.

$$N = \frac{N_o}{2} \quad \text{at } t = T_{1/2}$$

Using these values into the exponential decay law (21.8), we get

$$\frac{N_o}{2} = N_o e^{-\lambda T_{1/2}}$$

or $$e^{\lambda T_{1/2}} = 2$$

∴ $$\lambda T_{1/2} = \ln 2 = 0.693$$

∴ $$T_{1/2} = \frac{0.693}{\lambda} \tag{21.12}$$

Eq.(21.12) suggests that the longer the half-life of an element, the slower it decays. The half-lives of natural radioactive nuclei vary between wide limits. For uranium, it is 4500 million years, for radium 1620 years, and for radon it is 3.8 days only.

21.15 AVERAGE LIFE TIME

The atoms of radioactive substances are continuously disintegrating. Therefore, the life of each nucleus is different. The nuclei, which disintegrate earlier, have shorter life whereas those that disintegrate at the end have a longer life. It is difficult to specify why some atoms have shorter existence while others remain untransformed for a long time. Therefore, it becomes necessary to specify the **mean lifetime** or **average lifetime** of a radioactive species. The average lifetime, τ of a nuclide of a radioactive substance is defined as the ratio of the sum of lifetimes of all nuclei present in the substance to the total number of nuclei. That is,

$$\tau = \frac{\text{Sum of the life times of all nuclei}}{\text{Total number of nuclei}}$$

Out of the original N_o nuclei, let dN_1 nuclei live for time t_1, dN_2 nuclei live for time t_2, and so on. Then,

$$\tau = \frac{t_1 dN_1 + t_2 dN_2 + \dots\dots\dots}{dN_1 + dN_2 + \dots\dots.}$$

The above equation may be written in the integral form as

$$\tau = \frac{\int_{N_o}^{0} t \cdot dN}{\int_{N_o}^{0} dN} = \frac{\int_{0}^{N_o} t \cdot dN}{\int_{0}^{N_o} dN}$$

As, $N = N_o e^{-\lambda t}$, $dN = -\lambda N_o e^{-\lambda t}\, dt$.

$$\therefore \qquad \tau = \frac{\int_{\infty}^{0} (-\lambda) t N_o e^{-\lambda t} dt}{N_o} \qquad \because \int_{0}^{N_o} dN = N_o$$

$$= \lambda \int_{0}^{\infty} t e^{-\lambda t} dt$$

$$= \lambda \left[\frac{e^{-\lambda t}}{-\lambda} t - \frac{e^{-\lambda t}}{-\lambda^2} \right]_0^{\infty} = -\frac{1}{\lambda} \left[(\lambda t + 1) e^{-\lambda t} \right]_0^{\infty} = \frac{1}{\lambda}$$

$$\therefore \qquad \tau = \frac{1}{\lambda} \qquad (21.13)$$

Thus, the mean lifetime of a radioactive species is equal to the reciprocal of the decay constant.

Using equ. (21.13) into equ. (21.12), we obtain

$$T_{1/2} = 0.693\ \tau \qquad (21.14)$$

21.16 UNITS OF ACTIVITY

The **activity** of a radioactive source is defined as the number of disintegrations that occur per second. The traditional unit of activity is **curie** (Ci). It is defined as

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ disintegrations per second.}$$

This definition is based on the activity of 1 gm of radium. Curie is rather a large unit, so millicuries (*m Ci*) and microcuries (μ Ci) are commonly used.

The *SI* unit of radioactivity is the Becquerel (*Bq*). It is defined as

$$1 \text{ Bq} = 1 \text{ decay per second}$$

Example 21.5. *A certain radioactive substance has a disintegration constant $\lambda = 1.44 \times 10^{-3}$ per hour. In what time will 75% of the initial number of atoms disintegrate?*

Solution. $$N = N_o - \tfrac{3}{4}N_o = \tfrac{1}{4}N_o$$

But $$N = N_o e^{-\lambda t} \qquad \therefore \quad \frac{1}{4} = e^{-\lambda t} \quad \text{or} \quad e^{\lambda t} = 4$$

$$\therefore \qquad t = \frac{\log_e 4}{\lambda} = \frac{2.3026 \times 0.6021}{1.44 \times 10^{-3}} \text{hours} = \mathbf{962.9 \ hrs.}$$

Example 21.6: *Calculate the activity of 1 mg sample of $^{90}_{38}Sr$ whose half life is 28 years.*

Solution: $$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{28 \text{ yrs}} = \frac{0.693}{28 \times 365 \times 24 \times 3600\text{s}}$$

$$= 7.85 \times 10^{-10}/\text{s}.$$

Number of nuclei in one mg sample,

$$N = \frac{mN_A}{M} = \frac{10^{-3} \times 6.02 \times 10^{26}}{90} = 6.69 \times 10^{21}$$

Activity of the sample $A = \lambda N = 7.85 \times 10^{-10}/\text{s} \times 6.69 \times 10^{21}$

$$= \mathbf{5.25 \times 10^{12} \ disintegrations/s}.$$

Example 21.7: *One gm of a radioactive material having a half-life of 2 years is kept in store for a duration of 4 years. Calculate how much of the material remains unchanged.*

Solution: $$T_{1/2} = \frac{0.6931}{\lambda} = 2 \ yrs$$

$$\therefore \qquad \lambda = \frac{0.6931}{2 \ yrs.}$$

As $$N = N_0 e^{-\lambda t}, \ t = \frac{1}{\lambda} ln \frac{N_0}{N}$$

$$\therefore \qquad 4 \ yrs. = \left(\frac{2 \ yrs.}{0.6931}\right) ln \frac{N_0}{N}$$

$$\therefore \qquad ln \frac{N_0}{N} = \frac{4 \times 0.6931}{2} = 1.386$$

$$\therefore \qquad \frac{N_0}{N} = 4$$

or $$N = \frac{N_0}{4} = \frac{1 gm}{4} = \mathbf{0.25\ gm}$$

The material that remains unchanged after 4 years is 0.25 gm.

Example 21.8. *5 gm of radium is reduced by 10.5 mg in 5 years. Calculate the half-life of radium.*

Solution. The initial quantity of radium $N_o = 5\ gm$.

Quantity of radium present $N = 5 - 10.5 \times 10^3\ gm = 4.9895\ gm$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\therefore \quad \frac{4.9895\, gm}{5\, gm} = e^{-\lambda t}$$

$$\therefore \quad \lambda t = ln\left[\frac{5}{4.9895}\right]$$

As t = 5 years, $$\lambda = \frac{ln\left(\frac{5}{4.9895}\right)}{5\, years} = 4.2 \times 10^{-4} \text{ dis./year}$$

$$T_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.693}{4.2 \times 10^{-4}} \text{ yrs.} = 1650 \text{ years}$$

21.17 INDUCED RADIOACTIVITY

Joliot-Curie and Frederic discovered in 1934 the phenomenon of **induced** or **artificial radioactivity**. They discovered that aluminium, boron and magnesium become radioactive when irradiated with α-particles. The irradiation of aluminium induces the nuclear reaction

$$^{27}_{13}Al \quad + \quad ^{4}_{2}He \quad \rightarrow \quad ^{30}_{15}P^* \quad + \quad ^{1}_{0}n$$

The phosphorous isotope $^{30}_{15}P*$ is radioactive. It turns into a stable silicon isotope after emitting a positron.

$$^{30}_{15}P^* \quad \rightarrow \quad ^{30}_{14}Si \quad + \quad ^{0}_{+1}e$$

The discovery of induced radioactivity is important on two counts. It was for the first time that radioactive materials were synthesized and secondly, it proved that not only heavy nuclei have radioactive isotopes but light nuclei also have radioactive isotopes. Subsequent works demonstrated that radioactive isotopes of all elements could be synthesized. Radioactive isotopes are produced by irradiating the nuclei with α-particles, protons, deuterons or high energy γ-rays.

Fermi studied radioactivity induced by neutrons. At present the most widely used method for producing radioactive isotopes is by neutron irradiation. All nuclei except $^{4}_{2}He$ absorb neutrons and transform into β-active isotopes.

21.18 APPLICATIONS OF RADIOACTIVITY

Radioactivity is widely used in many areas of science and technology.