Agenda

1. Percentiles and Quartiles ✓
2. 5 Number Summary {Outliers} ✓
3. Box plot ✓
4. Covariance And Correlation }.
5. Probability distribution function
6. Different types of distributions

1. Percentiles And Quartiles [GATE, CAT]

Percentage : 1, 2, 3, 4, 5, 6

% of numbers that are odd = $\frac{3}{6}$ = $\frac{\text{No. of odd numbers}}{\text{No. of total no}}$

$= \frac{1}{2} = 50\%$

Percentiles :

Defn : A percentile is a value below which a certain percentage of data points lie.

$n = 15$

$X = \{2, 3, 3, 4, 6, 6, 6, 7, 8, 8, 9, 9, \boxed{10}, 11, 12\}$

Percentile Rank of (value) 10 = $\frac{\#\text{ of values below } 10}{n} * 100$

$= \frac{12}{15} \times 100 = 80$ percentile

80 percentile = 80% of the distribution fall below the value of 10 //

② What value exists at 25 percentile ?

$$\text{value} = \frac{\text{Percentile}}{100} * (n+1)$$

$$= \frac{25}{100} * 14 = 4^{th} \text{ Element} = 4$$

$$X = \{2, 3, 3, 4, 6, 6, 6, 7, 8, 8, 9, 9, 10, 11, 12\}$$

$\frac{4+4}{2} = 5$

4.5

Quartiles

① Q1 → 25 percentile

Q2 → Median → 50 percentile

Q3 → 75 percentile

② 5 Number Summary

(1) Minimum
(2) First Quartile (25 percentile) (Q1)
(3) Median (Q2)
(4) Third Quartile (75 percentile) (Q3)
(5) Maximum

Remove the Outliers

$$X = \{1, 2, 2, 2, 3, 3, 4, 5, 5, 5, 6, 6, 6, 6, 7, 8, 8, 9, 29\}.$$

[Lower Fence ⟷ Higher Fence]

Lower Fence = Q1 − 1.5(IQR)

Inter Quartile Range = Q3 − Q1

Higher Fence = Q3 + 1.5(IQR)

$X = \{1, 2, 2, 2, 3, 3, 4, 5, 5, 5, 6, 6, 6, 6, 7, 8, 8, 9, 29\}$ (29 is the outlier)

$Q1 = 25^{th}$ percentile $= \frac{25}{100} * 20 = 5^{th}$ value $= 3$

$Q3 = 75^{th}$ percentile $= \frac{75}{100} * 20 = 15^{th}$ value $= 7$

$IQR = 7 - 3 = 4$

Lower Fence $= Q_1 - 1.5(IQR)$
$= 3 - 1.5(4)$
$= 3 - 6$
$= -3$

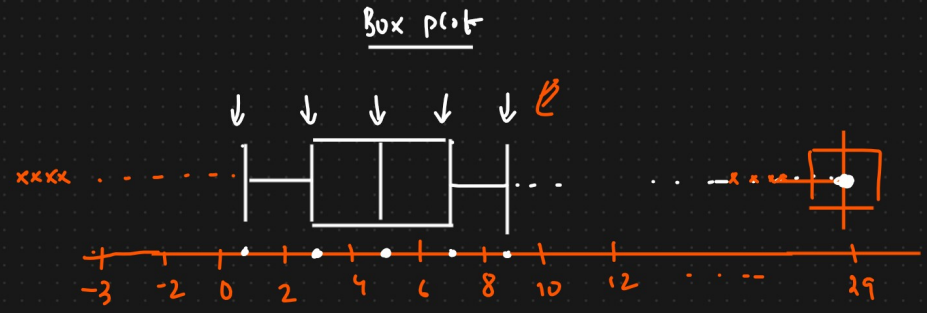
Higher Fence $= Q3 + 1.5(IQR)$
$= 7 + 1.5(4)$
$= 7 + 6$
$= 13$

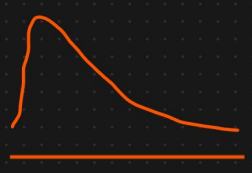
$[-3, 13]$.

$X = \{1, 2, 2, 2, 3, 3, 4, 5, 5, 5, 6, 6, 6, 6, 7, 8, 8, 9, 29\}$ (29 is the outlier)

Box plot [To visualize Outliers]

1. Minimum value = 1
2. Q1 = 3
3. Median Q2 = 5
4. Q3 = 7
5. Maximum = 9





mean > median > mode

$Q3 - Q2 > Q2 - Q1$

Internal Assignment

$\frac{-5+3}{2} = -1$

$Y = \{-13, -12, -6, -5, 3, 4, 5, 6, 7, 7, 8, 10, 10, 11, 24, 55\}$.

[Lower Fence ⟷ Higher Fence]

$Q1 = \frac{25}{100} \times 17 = 4.25$

$Q3 = \frac{75}{100} \times 17 = 12.75$

Lower Fence $= -1 - 1.5(10+1)$

$= -1 - 1.5(11)$

$= -17.5$

Higher Fence $= 10 + 1.5(10+1)$

$= 10 + 16.5$

$= 26.5$

$[-17.5, 26.5]$.

⇓

55 is an outlier

$Z = \{1, 2, 4, 6, 7, 12, 18, 34, 77, 66, 108, 99, 14\}$.

$\{1, 2, 4, 6, 7, 12, 14, 18, 34, 66, 77, 99, 108\}$

$Q1 = 5$

$Q3 = 71.5$

$[-94.75, 171.25]$

$Q3 = \frac{75}{100} \times 14 = \frac{42}{4} \; 2 + 10.5$

$Q3 = \frac{66+77}{2} = 71.5$

Covariance And Correlation

[Relationship between X and Y]

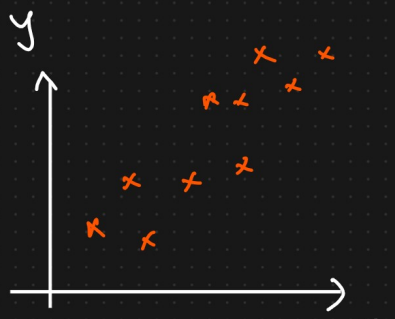
X	Y
2	3
4	5
6	7

→	X↑	Y↑
→	X↓	Y↑
→	X↑	Y↓

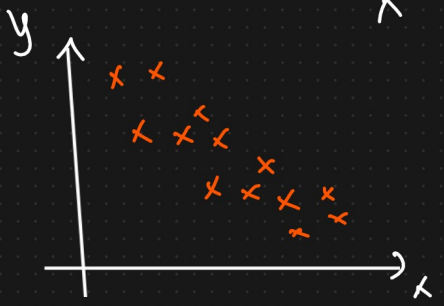
IS Haunted

Size of House → Price

8 9 → | x↓ y↓ |



x↑ y↑
x↓ y↓



x↑ y↓
x↓ y↑

① Covariance

$$Cov(x,y) = \sum_{i=1}^{n} \frac{(x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

$$Var(x) = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1} = \sum_{i=1}^{n} \frac{(x_i - \bar{x}) * (x_i - \bar{x})}{n-1}$$

$Cov(x,y)$ → | x↑ y↑ / x↓ y↓ | ⇒ +ve Covariance

| x↑ y↓ / x↓ y↑ | ⇒ -ve Covariance.

$\Downarrow$

$Var(x) \Leftarrow Cov(x,x)$

x	y
→ 2	3
→ 4	5
6	7
$\bar{x} = 4$	$\bar{y} = 5$

$$Cov(x,y) = \sum_{i=m}^{n} \frac{(x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

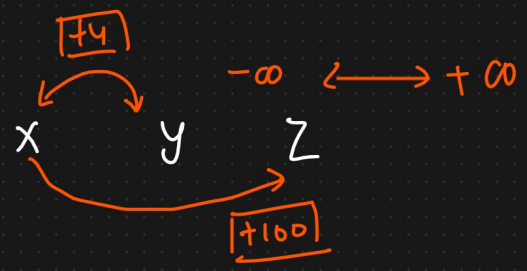
$$= \frac{[(2-4)(3-5) + (4-4)(5-5) + (6-4)*(7-5)]}{2}$$

$$= \frac{4+0+4}{2} = \frac{8}{2} = 4 \text{ +ve covariance}$$

X and y are having a positive covariance

Advantages

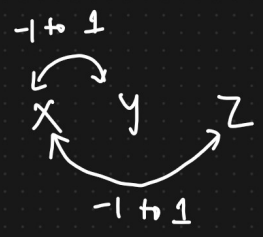
① Relation ship between X & y



Disadvantage

① Covariance does not have a specific limit value

① Pearson Correlation Coefficient [-1 to 1]



$$\rho_{x,y} = \frac{Cov(x,y)}{\sigma_x \cdot \sigma_y}$$

④ The more the value towards +1 the more +ve correlated it is

② The more the " " -1 " " -ve " " "

X	y	0.6
X	Z	0.7

② Spearman Rank correlation

$$r_s = \frac{Cov(R(x), R(y))}{\sigma_{R(x)} * \sigma_{R(y)}}$$

X	y	R(x)	R(y)
5	6	3	1
7	4	2	2
8	3	1	3
1	1	5	5
2	2	4	4

Feature Selection

+ve	+ve	+ve	≈0	-ve	O/P
Size of house	No. of Rooms	Location	~~No. of people Staying~~	Haunted	Price ↑

Probability Distribution Function And Probability Density Function

Probability Mass function

Probability Distribution Function

① Probability density fn
② Probability mass fn ✓
③ Cummulative distribute fn.

① PMF

① Discrete Random Variable

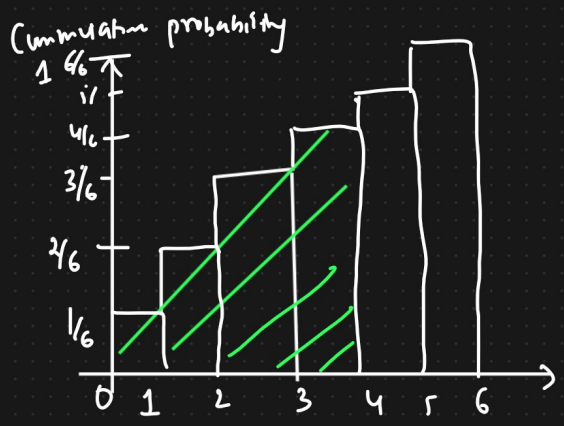
Eg: Rolling a dice {1,2,3,4,5,6}

$Pr(1) = 1/6$

$Pr(2) = 1/6$

$Pr(1 \text{ or } 2) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

Cummulative distribution function (cdf)

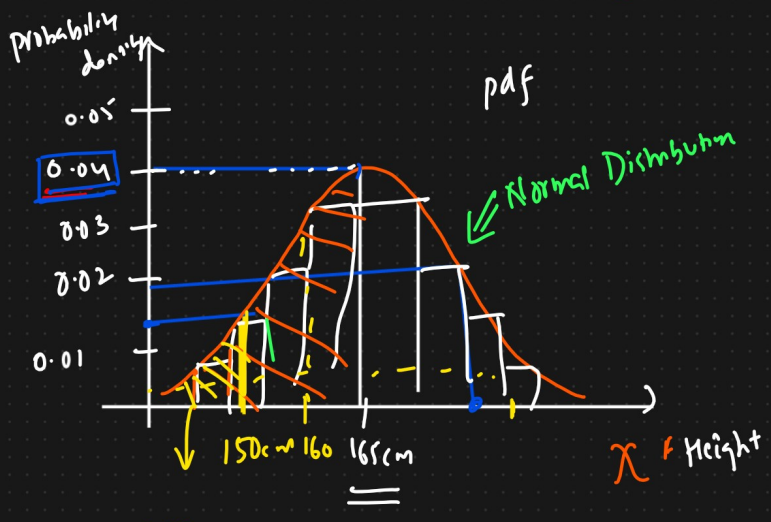


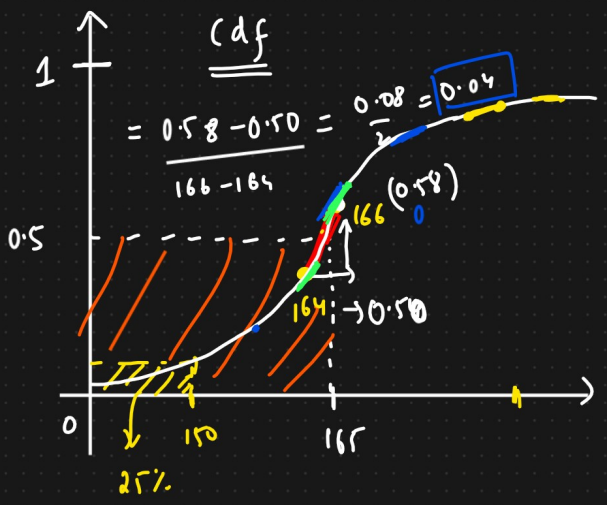
$Pr(X \leq 4) = Pr(X=1) + Pr(X=2) + Pr(X=3) + Pr(X=4)$

$= O/P$

② Probability Density Function

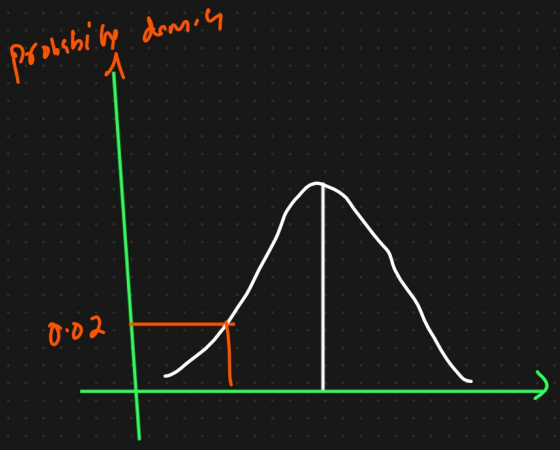
① Distribution of continuous Random Variable

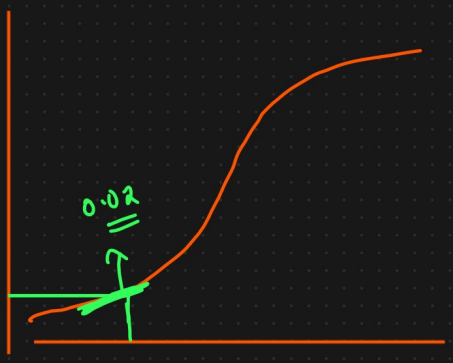




(A) 0.25

Probability Density ⇒ Gradient of Cumulative Curve





Different types of Distribution

① Normal / Gaussian Distribution → pdf

② Standard Normal Distribution → pdf

③ Log Normal Distribution → pdf

④ Power Law Distribution → pdf

⑤ Bernoulli Distribution → pmf

⑥ Binomial Distribution → pmf

⑦ Poisson Distribution → pmf

⑧ Uniform Distribution → Discrete → pmf
→ Continuous → pdf

⑨ Exponential Distribution. → pdf

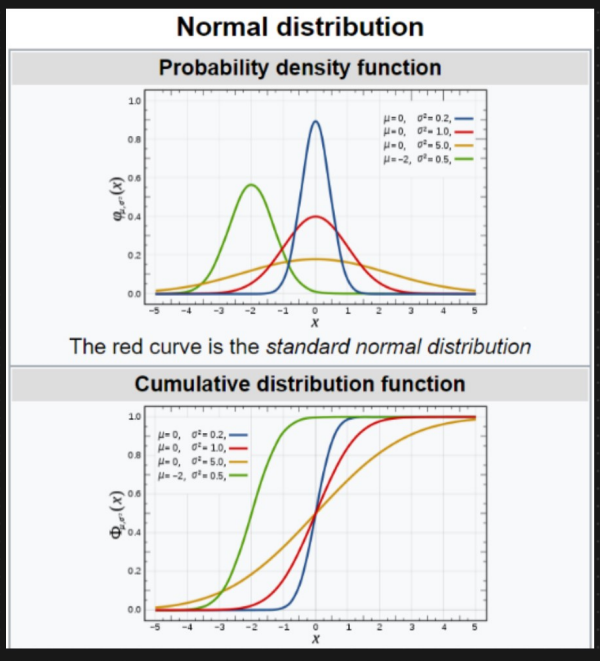
⑩ CHI SQUARE Distribution → pdf

⑪ F Distribution → pdf.

① Normal/Gaussian Distribution

X :) Continuous Random Variable.

Eg: Height, weight, age, IRIS dataset



$X \sim N(\mu, \sigma^2)$

Support parameters μ = mean

σ^2 = variance

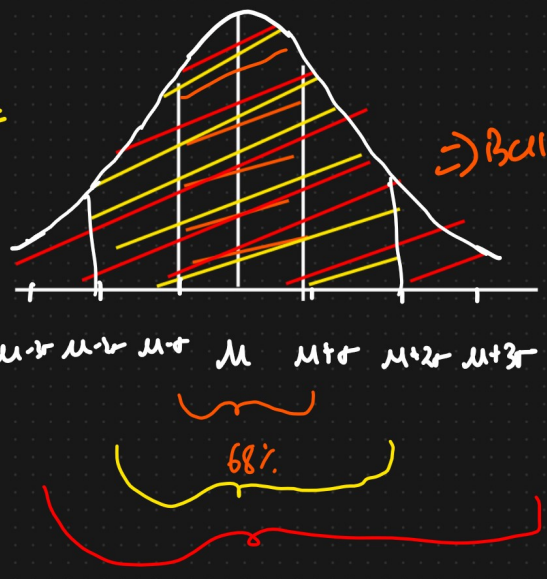
$$PDF = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Empirical Rule 68 - 95 - 99.7% Rule

100 datapoint

$X = \{ \quad \}$

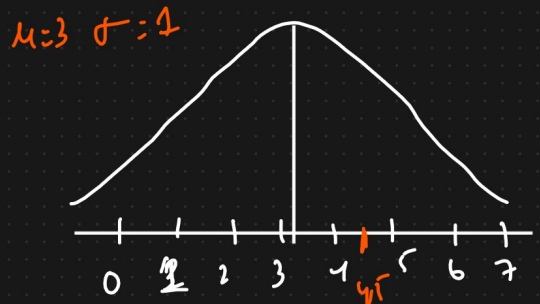
mean : mode : median



⇒ Bell Curve

Standard Normal Distribution

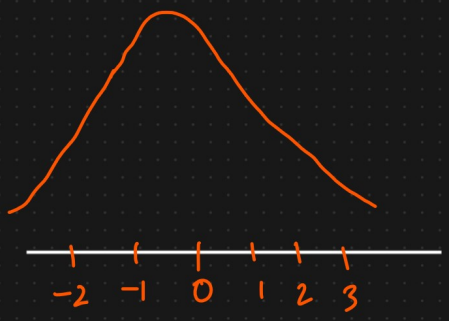
$X \quad \mu = 3 \quad \sigma = 1$



Transformation ⇒

$\mu = 0, \sigma = 1$

↓ Standard Normal Distribution



$$Z\text{-score} = \frac{x_i - \mu}{\sigma}$$

$$\frac{3-3}{1} = 0$$

$$= \frac{1-3}{1} = -2$$

$$= \frac{2-3}{1} = -1$$

$$\boxed{4.5} \Rightarrow \frac{4.5-3}{1} = 1.5$$

Z-score tells you about a value how many standard deviation away from the mean

X $\mu = 4$ $\sigma = 1$

What is the percentage of scores lies above 4.5?

$\Rightarrow$ z table

$$z\text{-score} = \frac{4.5 - 4}{1} = 0.5 \Leftarrow$$

Area under curve $= 1 - 0.6914$

$= 0.3086 = 30.86\%$