**Parametrically synthesized fast orthogonal transforms**

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**Abstract**

A method of fast orthogonal linear transform algorithm synthesis by a single arbitrary vector is introduced. The

method is based on factorization of a matrix and using the arbitrary input vector to define nonzero terms of the factors for building fast linear transform algorithms.

**Introduction**

A direct linear transform is a dot product of a transformation matrix by an input vector:

(1)

Here

- is an orthogonal normalized matrix,

- is an input vector, and

- is an output vector.

Our goal is to build orthogonal matrix and its inverse providing desired properties of vector for a given arbitrary vector . Another goal is to build fast transform algorithms with low computational complexity.

**1. Matrix factorization**

Let's find such a matrix that for an arbitrary input vector the output vector will have one and only one nonzero element situated at the first position and all other elements equal zero

(2)

and is the energy of the input vector :

(3)

Superscript symbols 'T' and '\*' depict transpose and complex conjugate operators correspondingly.

The inverse of (1) can be written as

(4)

Matrix is orthogonal and normalized and therefore its inverse is equal to its conjugate transposition

(5)

and equation (4) becomes

(6)

which is equivalent to the system of scalar equations

(7)

Let's assume that matrix is defined as a product of sparsely populated matrices

(8)

where , and

(9)

Nonzero elements of matrix form a set of generalized spectral cores represented by matrices:

(10)

Let's represent matrix element indices in binary format as follows:

(11)

The elements of the matrix can be represented by products of the corresponding spectral core components:

(12)

where

(13)

(14)

Since matrix is orthogonal and normalized, the spectral core elements (10) are related to each other as follows [1,2]:

(15)

Let's here and below define

(16)

and

(17)

Then (10) and (15) become

(18)

and nonzero terms of the matrices according to (9) will be:

(19)

where

(20)

and

(21)

Now (7) becomes

(22)

2. **Transform synthesis**

Using well known trigonometric identities [1,2], the solution of the system of equations (22) can be easily found:

(23)

Similar derivation can be done for vector defined not as in (2) where its nonzero term was in the first position with index 0, but having its nonzero term at position :

(24)

In this case (23) becomes

(25)

Only half of all spectral cores are defined in (23) or (25). The other half can be chosen arbitrarily. These spectral core parameter sets define a unique pair of direct and inverse fast orthogonal transforms.

The direct transform is

(26)

and the inverse transform is

(27)

where

and were defined in (20) and (21),

- intermitting vector ( ),

- input vector.

**Conclusion**

Transform pair (26) and (27) require each two-argument multiplications and two-argument additions and has computational complexity similar to fast Fourier transform algorithm.

**References**

[1] P. Dourbal, V. Shabalov. "Generator of Basis Functions" USSR Patent 1319013.

[2] V. Grigoriev, P. Dourbal, V. Shabalov. Generator of Basis Functions" USSR Patent 1413615.