FIITJEE CPP *MATHEMATICS*

EXERCISE–I (A)

Q.1 Find the coefficients : (i) x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$ (ii) x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11}$

(iii) Find the relation between a & b, so that these coefficients are equal.

Q.2 If the coefficients of $(2r+4)^{th}$, $(r-2)^{th}$ terms in the expansion of $(1+x)^{18}$ are equal, find r.

Q.3 If the coefficients of the r^{th}, $(r+1)^{th}$ & $(r+2)^{th}$ terms in the expansion of $(1+x)^{14}$ are in AP, find r.

Q.4 Find the term independent of x in the expansion of (a) $\left[\sqrt{\frac{x}{3}}+\frac{\sqrt{3}}{2x^2}\right]^{10}$ (b) $\left[\frac{1}{2}x^{1/3}+x^{-1/5}\right]^{8}$

Q.5 Find the sum of the series $\sum_{r=0}^{n}(-1)^r.{}^nC_r\left[\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\frac{15^r}{2^{4r}}+\text{.....up to m terms}\right]$

Q.6 If the coefficients of 2^{nd}, 3^{rd} & 4^{th} terms in the expansion of $(1+x)^{2n}$ are in AP, show that $2n^2-9n+7=0$.

Q.7 Given that $(1+x+x^2)^n=a_0+a_1x+a_2x^2+....+a_{2n}x^{2n}$, find the values of :
(i) $a_0+a_1+a_2+.....+a_{2n}$; (ii) $a_0-a_1+a_2-a_3.....+a_{2n}$; (iii) $a_0^2-a_1^2+a_2^2-a_3^2+.....+a_{2n}^2$

Q.8 If a, b, c & d are the coefficients of any four consecutive terms in the expansion of $(1+x)^n, n\in N$, prove that $\frac{a}{a+b}+\frac{c}{c+d}=\frac{2b}{b+c}$.

Q.9 Find the value of x for which the fourth term in the expansion, $\left(5^{\frac{2}{5}\log_5\sqrt{4^x+44}}+\frac{1}{5^{\log_5\sqrt[3]{2^{x-1}+7}}}\right)^8$ is 336.

Q.10 Prove that : ${}^{n-1}C_r+{}^{n-2}C_r+{}^{n-3}C_r+....+{}^rC_r={}^nC_{r+1}$.

Q.11 (a) Which is larger : $(99^{50}+100^{50})$ or $(101)^{50}$.

(b) Show that ${}^{2n-2}C_{n-2}+2.{}^{2n-2}C_{n-1}+{}^{2n-2}C_n>\frac{4n}{n+1}$, $n\in N$, $n>2$

Q.12 In the expansion of $\left(1+x+\frac{7}{x}\right)^{11}$ find the term not containing x.

Q.13 Show that coefficient of x^5 in the expansion of $(1+x^2)^5.(1+x)^4$ is 60.

Q.14 Find the coefficient of x^4 in the expansion of :
(i) $(1+x+x^2+x^3)^{11}$ (ii) $(2-x+3x^2)^6$

Q.15 Find numerically the greatest term in the expansion of :
(i) $(2+3x)^9$ when $x=\frac{3}{2}$ (ii) $(3-5x)^{15}$ when $x=\frac{1}{5}$

Q.16 Given $s_n=1+q+q^2+.....+q^n$ & $S_n=1+\frac{q+1}{2}+\left(\frac{q+1}{2}\right)^2+....+\left(\frac{q+1}{2}\right)^n$, $q\neq 1$,
prove that ${}^{n+1}C_1+{}^{n+1}C_2.s_1+{}^{n+1}C_3.s_2+....+{}^{n+1}C_{n+1}.s_n=2^n.S_n$.

Q.17 Prove that the ratio of the coefficient of x^{10} in $(1-x^2)^{10}$ & the term independent of x in

$\left(x-\frac{2}{x}\right)^{10}$ is 1 : 32 .

Q.18 Find the term independent of x in the expansion of $(1+x+2x^3)\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$.

Q.19 In the expansion of the expression $(x+a)^{15}$, if the eleventh term is the geometric mean of the eighth and twelfth terms, which term in the expansion is the greatest ?

Q.20 Let $(1+x^2)^2 . (1+x)^n = \sum_{K=0}^{n+4} a_K . x^K$. If a_1, a_2 & a_3 are in AP, find n.

Q.21 If the coefficient of a^{r-1}, a^r, a^{r+1} in the expansion of $(1+a)^n$ are in arithmetic progression, prove that $n^2 - n(4r+1) + 4r^2 - 2 = 0$.

Q.22 If $^nJ_r = \dfrac{(1-x^n)(1-x^{n-1})(1-x^{n-2})..................(1-x^{n-r+1})}{(1-x)(1-x^2)(1-x^3)..................(1-x^r)}$, prove that $^nJ_{n-r} = {}^nJ_r$.

Q.23 Prove that $\sum_{K=0}^{n} {}^nC_K \sin Kx.\cos(n-K)x = 2^{n-1}\sin nx$.

Q.24 The expressions $1+x$, $1+x+x^2$, $1+x+x^2+x^3$,............ $1+x+x^2+$........... $+x^n$ are multiplied together and the terms of the product thus obtained are arranged in increasing powers of x in the form of $a_0 + a_1x + a_2x^2 +$................., then,

(a) how many terms are there in the product.

(b) show that the coefficients of the terms in the product, equidistant from the beginning and end are equal.

(c) show that the sum of the odd coefficients = the sum of the even coefficients = $\dfrac{(n+1)!}{2}$

Q.25 Find the coeff. of

(a) x^6 in the expansion of $(ax^2+bx+c)^9$.

(b) $x^2y^3z^4$ in the expansion of $(ax-by+cz)^9$.

(c) $a^2b^3c^4d$ in the expansion of $(a-b-c+d)^{10}$.

Q.26 If $\sum_{r=0}^{2n} a_r(x-2)^r = \sum_{r=0}^{2n} b_r(x-3)^r$ & $a_k = 1$ for all $k \ge n$, then show that $b_n = {}^{2n+1}C_{n+1}$.

Q.27 If $P_k(x) = \sum_{i=0}^{i=k-1} x^i$ then prove that, $\sum_{k=1}^{n} {}^nC_k P_k(x) = 2^{n-1} \cdot P_n\left(\frac{1+x}{2}\right)$

Q.28 Find the coefficient of x^r in the expression of :
$(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}(x+2)^2 + ... + (x+2)^{n-1}$

Q.29(a) Find the index n of the binomial $\left(\frac{x}{5}+\frac{2}{5}\right)^n$ if the 9th term of the expansion has numerically the greatest coefficient $(n \in N)$.

(b) For which positive values of x is the fourth term in the expansion of $(5+3x)^{10}$ is the greatest.

Q.30 Prove that $\dfrac{(72)!}{(36!)^2} - 1$ is divisible by 73.

Q.31 If the $3^{rd}, 4^{th}, 5^{th}$ & 6^{th} terms in the expansion of $(x+y)^n$ be respectively a, b, c & d then prove that

$$\frac{b^2-ac}{c^2-bd}=\frac{5a}{3c}.$$

Q.32 Find x for which the $(k+1)^{th}$ term of the expansion of $(x+y)^n$ is the greatest if $x+y=1$ and $x>0,\ y>0$.