Q.1 If $(1+x)^{15} = C_0 + C_1 . x + C_2 . x^2 + ... + C_{15} . x^{15}$, then find the value of :
$C_2 + 2C_3 + 3C_4 + ... + 14C_{15}$

Q.2 If $(1 + x + x^2 + ... + x^p)^n = a_0 + a_1 x + a_2 x^2 + ... + a_{np} . x^{np}$, then find the value of :
$a_1 + 2a_2 + 3a_3 + ... + np . a_{np}$

Q.3 $1^2 . C_0 + 2^2 . C_1 + 3^2 . C_2 + 4^2 . C_3 + ... + (n+1)^2 C_n = 2^{n-2} (n+1)(n+4)$.

Q.4 $\sum_{r=0}^{n} r^2 . C_r = n(n+1) 2^{n-2}$

Q.5 Given $p+q=1$, show that $\sum_{r=0}^{n} r^2 . {}^nC_r . p^r . q^{n-r} = np[(n-1)p+1]$

Q.6 Show that $\sum_{r=0}^{n} C_r (2r-n)^2 = n.2^n$ where C_r denotes the combinatorial coeff. in the expansion of $(1+x)^n$.

Q.7 $C_0 + \frac{C_1}{2}x + \frac{C_2}{3}x^2 + \frac{C_3}{4}x^3 + ... + \frac{C_n}{n+1} . x^n = \frac{(1+x)^{n+1}-1}{(n+1)x}$

Q.8 Prove that, $2 . C_0 + \frac{2^2}{2} . C_1 + \frac{2^3}{3} . C_2 + ... + \frac{2^{11}}{11} . C_{10} = \frac{3^{11}-1}{11}$

Q.9 If $(1+x)^n = \sum_{r=0}^{n} C_r . x^r$ then prove that ;

$$\frac{2^2 . C_0}{1.2} + \frac{2^3 . C_1}{2.3} + \frac{2^4 . C_2}{3.4} + ... + \frac{2^{n+2} . C_n}{(n+1)(n+2)} = \frac{3^{n+2}-2n-5}{(n+1)(n+2)}$$

Q.10 $\frac{C_0}{1} + \frac{C_2}{3} + \frac{C_4}{5} + ... = \frac{2^n}{n+1}$

Q.11 $\frac{C_0}{1} - \frac{C_1}{5} + \frac{C_2}{9} - \frac{C_3}{13} + ...(-1)^n \frac{C_n}{4n+1} = \frac{4^n . n!}{1.5.9.13.....(4n-3)(4n+1)}$

Q.12 $\frac{C_0}{2} + \frac{C_1}{3} + \frac{C_2}{4} + \frac{C_3}{5} + ... + \frac{C_n}{n+2} = \frac{1+n.2^{n+1}}{(n+1)(n+2)}$

Q.13 $\frac{C_0}{2} - \frac{C_1}{3} + \frac{C_2}{4} - \frac{C_3}{5} + ... + (-1)^n . \frac{C_n}{n+2} = \frac{1}{(n+1)(n+2)}$

Q.14 $\frac{C_1}{1} - \frac{C_2}{2} + \frac{C_3}{3} - \frac{C_4}{4} + ... + (-1)^{n-1} . \frac{C_n}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... + \frac{1}{n}$

Q.15 If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then show that :

$$C_1(1-x) - \frac{C_2}{2}(1-x)^2 + \frac{C_3}{3}(1-x)^3 - ....+ (-1)^{n-1}\frac{1}{n}(1-x)^n = (1-x) + \frac{1}{2}(1-x^2) + \frac{1}{3}(1-x^3) + ...+ \frac{1}{n}(1-x^n)$$

Q.16 Prove that, $\frac{1}{2}{}^nC_1 - \frac{2}{3}{}^nC_2 + \frac{3}{4}{}^nC_3 - \frac{4}{5}{}^nC_4 + ... + \frac{(-1)^{n+1} n}{n+1} . {}^nC_n = \frac{1}{n+1}$

Q.17 If $n \in N$; show that $\frac{{}^nC_0}{x} - \frac{{}^nC_1}{x+1} + \frac{{}^nC_2}{x+2} - ...... + (-1)^n \frac{{}^nC_n}{x+n} = \frac{n!}{x(x+1)(x+2)....(x+n)}$

Q.18 Prove that, $({}^{2n}C_1)^2 + 2 . ({}^{2n}C_2)^2 + 3 . ({}^{2n}C_3)^2 + ... + 2n . ({}^{2n}C_{2n})^2 = \frac{(4n-1)!}{[(2n-1)!]^2}$

Q.19 If $(1+x+x^2)^n = \sum_{r=0}^{2n} a_r x^r$, $n \in N$, then prove that

$$(r+1)\, a_{r+1} = (n-r)\, a_r + (2n-r+1)\, a_{r-1}. \qquad (0<r<2n)$$

Q.20 Prove that the sum to $(n+1)$ terms of $\frac{C_0}{n(n+1)} - \frac{C_1}{(n+1)(n+2)} + \frac{C_2}{(n+2)(n+3)} - ....$ equals $\int_0^1 x^{n-1} . (1-x)^{n+1} . dx$ & evaluate the integral.

FIITJEE CPP *MATHEMATICS*

ANSWER KEY

EXERCISE–I (A)

Q 1. (i) ${}^{11}C_5 \frac{a^6}{b^5}$ (ii) ${}^{11}C_6 \frac{a^5}{b^6}$ (iii) $ab = 1$ **Q 2.** $r = 6$ **Q 3.** $r = 5$ or 9 **Q 4.** (a) $\frac{5}{12}$ (b) $T_6 = 7$

Q 5. $\frac{(2^{mn}-1)}{(2^n-1)(2^{mn})}$ **Q 7.** (i) 3^n (ii) 1, (iii) a_n **Q 9.** $x = 0$ or 1 **Q 10.** $x = 0$ or 2

Q 11. (a)101^{50} (Prove that $101^{50} - 99^{50} = 100^{50}$ + some +ive qty) **Q 12.** $1 + \sum_{k=1}^{5} {}^{11}C_{2k} \cdot {}^{2k}C_k\ 7^k$

Q 14. (i) 990 (ii) 3660 **Q 15.** (i) $T_7 = \frac{7.3^{13}}{2}$ (ii) 455×3^{12} **Q 18.** $\frac{17}{54}$

Q.19 T_8 **Q.20** $n = 2$ or 3 or 4 **Q.24** (a) $\frac{n^2+n+2}{2}$

Q 25. (a) $84b^6c^3 + 630ab^4c^4 + 756a^2b^2c^5 + 84a^3c^6$; (b) $-1260 \,.\, a^2b^3c^4$; (c) -12600

Q 28. ${}^nC_r\,(3^{n-r} - 2^{n-r})$ **Q 29.** (a) $n = 12$ (b) $\frac{5}{8} < x < \frac{20}{21}$ **Q.32** $\frac{n-k}{n}$

EXERCISE–I(B)

Q.4 1

EXERCISE–III (B)

Q 1. divide expansion of $(1+x)^{15}$ both sides by x & diff. w.r.t.x , put $x = 1$ to get 212993

Q 2. Differentiate the given expn. & put $x = 1$ to get the result $\frac{np}{2}(p+1)^n$

Q 9. Integrate the expn. of $(1+x)^n$. Determine the value of constant of integration by putting $x = 0$. Integrate the result again between 0 & 2 to get the result.

Q 10. Consider $\frac{1}{2}[(1+x)^n + (1-x)^n] = C_0 + C_2x^2 + C_4x^4 + ...$ Integrate between 0 & 1.

Q 12. Multiply both sides by x the expn. $(1+x)^n$. Integrate both sides between 0 & 1.

Q 14. Note that $\frac{(1-x)^n - 1}{x} = -C_1 + C_2x - C_3x^2 + ... + C_n.\ x^{n-1}$. Integrate between 1 & 0

Q 20. $\frac{(n-1)!\,(n+1)!}{(2n+1)!}$