EXERCISE–I (B)

Q.1 Show that the integral part in each of the following is odd. $n \in N$

(A) $\left(5+2\sqrt{6}\right)^n$ (B) $\left(8+3\sqrt{7}\right)^n$ (C) $\left(6+\sqrt{35}\right)^n$

Q.2 Show that the integral part in each of the following is even. $n \in N$

(A) $\left(3\sqrt{3}+5\right)^{2n+1}$ (B) $\left(5\sqrt{5}+11\right)^{2n+1}$

Q.3 If $\left(7+4\sqrt{3}\right)^n = p+\beta$ where n & p are positive integers and β is a proper fraction show that $(1-\beta)(p+\beta)=1$.

Q.4 If x denotes $\left(2+\sqrt{3}\right)^n$, $n \in N$ & [x] the integral part of x then find the value of : $x - x^2 + x[x]$.

Q.5 If $P=\left(8+3\sqrt{7}\right)^n$ and $f = P - [P]$, where [ ] denotes greatest integer function.
Prove that : $P(1-f)=1$ $(n \in N)$

Q.6 If $\left(6\sqrt{6}+14\right)^{2n+1} = N$ & F be the fractional part of N, prove that $NF = 20^{2n+1}$ $(n \in N)$

Q.7 Prove that if p is a prime number greater than 2, then the difference $\left[\left(2+\sqrt{5}\right)^p\right] - 2^{p+1}$ is divisible by p, where [ ] denotes greatest integer.

Q.8 Prove that the integer next above $\left(\sqrt{3}+1\right)^{2n}$ contains 2^{n+1} as factor $(n \in N)$

Q.9 Let I denotes the integral part & F the proper fractional part of $\left(3+\sqrt{5}\right)^n$ where $n \in N$ and if ρ denotes the rational part and σ the irrational part of the same, show that

$$\rho = \frac{1}{2}(I+1) \text{ and } \sigma = \frac{1}{2}(I+2F-1).$$

Q.10 Prove that $\frac{{}^{2n}C_n}{n+1}$ is an integer, $\forall\, n \in N$.