Q.1 Find the number of ways in which 3 distinct numbers can be selected from the set $\{3^1, 3^2, 3^3, ... 3^{100}, 3^{101}\}$ so that they form a G.P.

Q.2 Let n & k be positive integers such that $n \geq \frac{k(k+1)}{2}$. Find the number of solutions $(x_1, x_2, ..., x_k)$, $x_1 \geq 1$, $x_2 \geq 2$, ..., $x_k \geq k$, all integers, satisfying $x_1 + x_2 + ... + x_k = n$.

Q.3 There are counters available in 7 different colours. Counters are all alike except for the colour and they are atleast ten of each colour. Find the number of ways in which an arrangement of 10 counters can be made. How many of these will have counters of each colour.

Q.4 For each positive integer k, let S_k denote the increasing arithmetic sequence of integers whose first term is 1 and whose common difference is k. For example, S_3 is the sequence 1, 4, 7, 10...... Find the number of values of k for which S_k contain the term 361.

Q.5 Find the number of 7 lettered words each consisting of 3 vowels and 4 consonants which can be formed using the letters of the word "DIFFERENTIATION".

Q.6 A shop sells 6 different flavours of ice-cream. In how many ways can a customer choose 4 ice-cream cones if

(i) they are all of different flavours

(ii) they are non necessarily of different flavours

(iii) they contain only 3 different flavours

(iv) they contain only 2 or 3 different flavours?

Q.7 6 white & 6 black balls of the same size are distributed among 10 different urns. Balls are alike except for the colour & each urn can hold any number of balls. Find the number of different distribution of the balls so that there is atleast 1 ball in each urn.

Q.8 There are 2n guests at a dinner party. Supposing that the master and mistress of the house have fixed seats opposite one another, and that there are two specified guests who must not be placed next to one another. Show that the number of ways in which the company can be placed is $(2n-2)!.(4n^2-6n+4)$.

Q.9 Each of 3 committees has 1 vacancy which is to be filled from a group of 6 people. Find the number of ways the 3 vacancies can be filled if ;

(i) Each person can serve on atmost 1 committee.

(ii) There is no restriction on the number of committees on which a person can serve.

(iii) Each person can serve on atmost 2 committees.

Q.10 How many 15 letter arrangements of 5 A's, 5 B's and 5 C's have no A's in the first 5 letters, no B's in the next 5 letters, and no C's in the last 5 letters.

Q.11 5 balls are to be placed in 3 boxes. Each box can hold all 5 balls. In how many different ways can we place the balls so that no box remains empty if,

(i) balls & boxes are different

(ii) balls are identical but boxes are different

(iii) balls are different but boxes are identical

(iv) balls as well as boxes are identical

(v) balls as well as boxes are identical but boxes are kept in a row.

Q.12 In how many other ways can the letters of the word **MULTIPLE** be arranged;

(i) without changing the order of the vowels

(ii) keeping the position of each vowel fixed &

(iii) without changing the relative order/position of vowels & consonants.

Q.13 Find the number of ways in which the number 30 can be partitioned into three unequal parts, each part being a natural number. What this number would be if equal parts are also included.

Q.14 In an election for the managing committee of a reputed club, the number of candidates contesting elections exceeds the number of members to be elected by r (r > 0). If a voter can vote in 967 different ways to elect the managing committee by voting atleast 1 of them & can vote in 55 different ways to elect (r – 1) candidates by voting in the same manner. Find the number of candidates contesting the elections & the number of candidates losing the elections.

Q.15 Find the number of three digits numbers from 100 to 999 inclusive which have any one digit that is the average of the other two.

Q.16 Prove by combinatorial argument that :

(a) $^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$

(b) $^{n+m}C_r = {}^nC_0 \cdot {}^mC_r + {}^nC_1 \cdot {}^mC_{r-1} + {}^nC_2 \cdot {}^mC_{r-2} + \ldots\ldots + {}^nC_r \cdot {}^mC_0.$

Q.17 A man has 3 friends. In how many ways he can invite one friend everyday for dinner on 6 successive nights so that no friend is invited more than 3 times.

Q.18 12 persons are to be seated at a square table, three on each side. 2 persons wish to sit on the north side and two wish to sit on the east side. One other person insists on occupying the middle seat (which may be on any side). Find the number of ways they can be seated.

Q.19 There are 15 rowing clubs; two of the clubs have each 3 boats on the river; five others have each 2 and the remaining eight have each 1; find the number of ways in which a list can be formed of the order of the 24 boats, observing that the second boat of a club cannot be above the first and the third above the second. How many ways are there in which a boat of the club having single boat on the river is at the third place in the list formed above?

Q.20 25 passengers arrive at a railway station & proceed to the neighbouring village. At the station there are 2 coaches accommodating 4 each & 3 carts accommodating 3 each. Find the number of ways in which they can proceed to the village assuming that the conveyances are always fully occupied & that the conveyances are all distinguishable from each other.

Q.21 An 8 oared boat is to be manned by a crew chosen from 14 men of which 4 can only steer but can not row & the rest can row but cannot steer. Of those who can row, 2 can row on the bow side. In how many ways can the crew be arranged.

Q.22 How many 6 digits odd numbers greater than 60,0000 can be formed from the digits 5, 6, 7, 8, 9, 0 if
(i) repetitions are not allowed (ii) repetitions are allowed.

Q.23 Find the sum of all numbers greater than 10000 formed by using the digits 0, 1, 2, 4, 5 no digit being repeated in any number.

Q.24 The members of a chess club took part in a round robin competition in which each plays every one else once. All members scored the same number of points, except four juniors whose total score were 17.5. How many members were there in the club? Assume that for each win a player scores 1 point, for draw 1/2 point and zero for losing.

Q.25 There are 3 cars of different make available to transport 3 girsls and 5 boys on a field trip. Each car can hold up to 3 children. Find

(a) the number of ways in which they can be accomodated.

(b) the numbers of ways in which they can be accomodated if 2 or 3 girls are assigned to one of the cars.

In both the cars internal arrangement of childrent inside the car is to be considered as immaterial.

Q.26 Six faces of an ordinary cubical die marked with alphabets A, B, C, D, E and F is thrown n times and the list of n alphabets showing up are noted. Find the total number of ways in which among the alphabets A, B, C, D, E and F only three of them appear in the list.

Q.27 Find the number of integer betwen 1 and 10000 with at least one 8 and at least one 9 as digits.

Q.28 The number of combinations n together of 3n letters of which n are 'a' and n are 'b' and the rest unlike is $(n+2).\ 2^{n-1}$.

Q.29 In Indo–Pak one day International cricket match at Sharjah, India needs 14 runs to win just before the start of the final over. Find the number of ways in which India just manages to win the match (i.e. scores exactly 14 runs), assuming that all the runs are made off the bat & the batsman can not score more than 4 runs off any ball.

Q.30 A man goes in for an examination in which there are 4 papers with a maximum of m marks for each paper; show that the number of ways of getting 2m marks on the whole is $\frac{1}{3}(m+1)(2m^2+4m+3)$.

EXERCISE–I

Q.1 ${}^{m+n+k}C_3 - ({}^mC_3 + {}^nC_3 + {}^kC_3)$ Q.2 744 Q.3 13, 156

Q.4 4316527 Q.5 43200 Q.6 145 Q.7 420

Q.8 960 Q.9 ${}^{24}C_2 \cdot {}^{15}C_3$ Q.10 1106 Q.11 $12!\,;\dfrac{11!\,.\,4!}{(3!)^4\,2!}$

Q.13 $\dfrac{52!}{(13!)^4}\,;\,\dfrac{52!}{3!(17!)^3}$ Q.14 5400 Q.15 ${}^{45}C_6$ Q.17 1638

Q.18 22100, 52 Q.19 2111 Q.20 576

Q.21 (a) $5\cdot(6!)$, (b) $3!\cdot 4!$, (c) 12 Q.23 (a) 72 ; 78120 ; (b) 23 ; (c) 32

Q.24 143 Q.25 440 Q.26 ${}^{n+5}C_5$ Q.27 3888

Q.28 $\dfrac{(14)!}{5!9!}$ Q.29 (ii) 792 ; (iii) $\dfrac{143}{4025}$; (v) $r=3$

EXERCISE–II

Q.1 2500 Q.2 ${}^mC_{k-1}$ where $m=(1/2)(2n-k^2+k-2)$

Q.3 7^{10} ; $\binom{49}{6}\ 10!$ Q.4 24

Q.5 532770 Q.6 (i) 15, (ii) 126, (iii) 60, (iv) 105 Q.7 26250

Q.9 120, 216, 210 Q.10 2252 Q.11 (i) 150 ; (ii) 6 ; (iii) 25 ; (iv) 2 ; (v) 6

Q.12 (i) 3359 ; (ii) 59 ; (iii) 359 Q.13 61, 75 Q.14 10, 3

Q.15 121 Q.17 510 Q.18 2!3!8!

Q.19 $\dfrac{24!}{(3!)^2(2!)^5}\,;\ {}^8C_1\cdot\dfrac{23!}{(3!)^2(2!)^5}$ Q.20 $\dfrac{(25)!}{(3!)^3\,(4!)^4\,.\,4}$ Q.21 $4\,.\,(4!)^2\,.\,{}^8C_4\,.\,{}^6C_2$

Q.22 240, 15552 Q.23 3119976 Q.24 27

Q.25 (a) 1680; (b) 1140 Q.26 ${}^6C_3[3^n - {}^3C_1(2^n-2) - {}^3C_2]$

Q.27 974 Q.29 1506