1st class

(all for distinct)

P & C

→ $^nC_r = \frac{n!}{r!(n-r)!}$ ↳ selection

$r!$ → arrangement

$^nP_r = {^nC_r} \times r!$ → sel + arr.

→ Law Multiplication & addition :→

↓ dependent

↓ independent

→ 5 l
3 Boxes → 3

5 → variable options for boxes.

↳ constant options for letters.

→ Note → $^nC_r = {^nC_{n-r}}$.

→ Note → $4C_2 \rightarrow \frac{4\times3}{1\times2} = 6$.

↳ ~~till no. of terms i.e 2~~

→ 16 friends :→ 3 jokes (non-similar)

Then, $^nC_3$ → no. of jokes not repeated

↳ i.e total jokes such that no joke gets repeated.

Now, $^nC_3 \geq 16$ and solve by hit & trial.

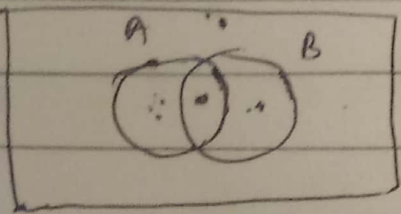
$n=5$

→ n no. of students arranged in $n!$ ways.

→ ^nC_r (Combination) :→

$P(n) \rightarrow U$

Q)



1) In how many ways we can form sets A & B.

2) In how many ways we can form sets A & B such that

a) $n(A \cap B) = 0$

b) $n(A \cap B) = 5$

$^nC_5 \times 3^{n-5}$

c) $A \subset B$ → 3^n

→ We only have to see the no. of options and try to find the constant options for a choice.

→ constant choice.

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→ 1 a) b) c) d)

2

20.

20 questions with –ve marking.

↳ only one option correct.

Ans → 5^{20}.

If he attempts 15 questions, then in how many ways he attempt the examination?

Ans → $^{20}C_{15} \times 4^{15} \times 1^{5}$

→ 15 questions 10 ✓ 5 ✗

Ans → $^{20}C_{15} \times 1^{5} \left({}^{15}C_{10} \times 1^{10} \times 3^{5} \right)$.

$^{15}C_{5}$ is not present as, if we select 10 from 15 which are correct, the rest 5 are auto-ignored.

20 questions.

Q) → multichoice - multi-correct →
(4 options)

1
|
|
|
|
20.

→ ways of exam ?

A) $^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$

1 4 6 4 1 = 16

16^{20} Ans.

Q) 1 =

2 =

3 =

4 =

5 =

ways of exam?

A) 3^5.

Q) In the above ~~eq~~ questions, ways of exam for which he attempts at least 1 question?

A) $\longrightarrow 3^5 - 1^5$

Letters. ★

- without repetition.

Q) AAABBC

$\longrightarrow \dfrac{6!}{3! \times 2!}$

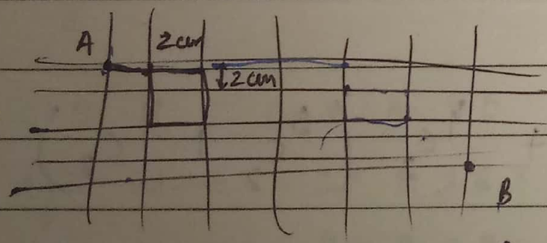
Q) MATHEMATICS

$$\frac{11!}{2!\ 2!\ 2!}$$

8 colums.

Q)

8 rows.



~~total 9 horizon &~~
~~9 vertical lines.~~

1) No. of rectangles $\longrightarrow {}^9C_2 \times {}^9C_2$

$\downarrow$ 2 vertical $\quad \downarrow$ 2 horizon

lines required.

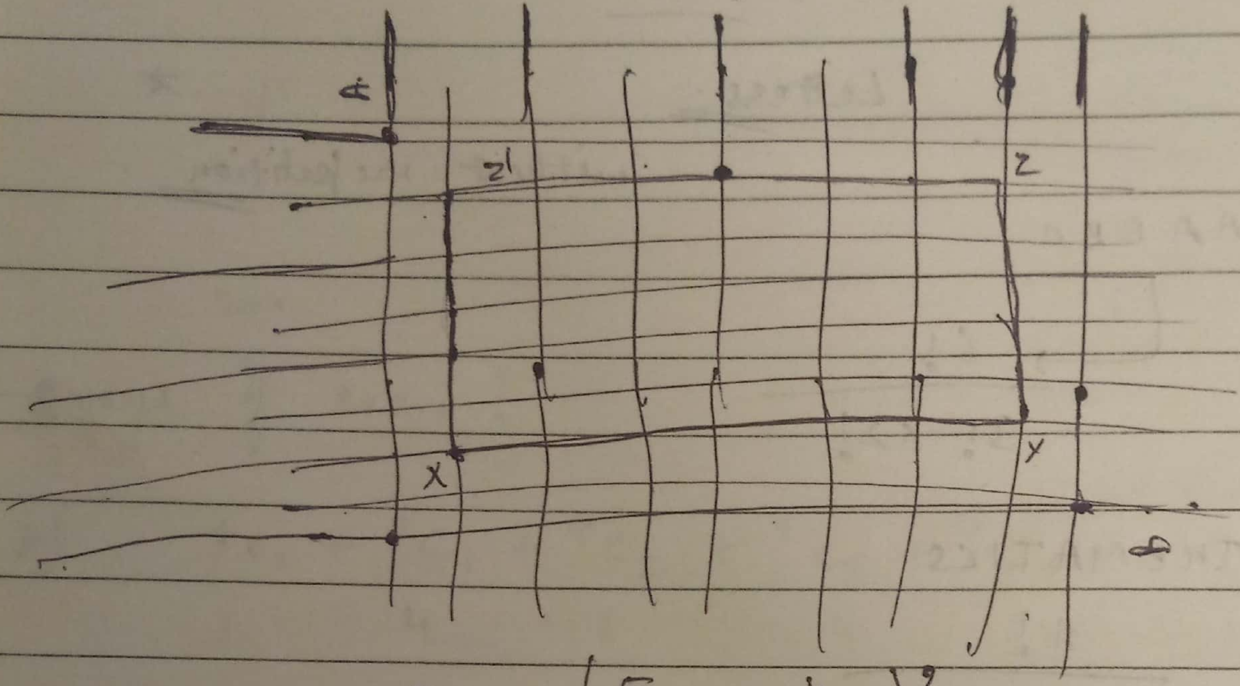
2) ~~which~~ shortest possible paths from A to B?

H H H H H H H H V V V V V V V V

$\dfrac{16!}{8!\,8!}$ $\longrightarrow$ arrangement of H & V decide the root.

3) No. of rect. of side length is odd.
{like (3×1) or (3×1)}

↳ not take 2 cm in account.



A → $\left({}^5C_1 \times {}^4C_1\right)^2$

→ In case of even →

$${}^5C_2\left({}^5C_2 + {}^4C_2\right) + {}^4C_2\left({}^5C_2 + {}^4C_2\right)$$

4) In the above diag, xyzz' is blocked, no. of ways he could take the shortest path?

A) $2\left[1 \times 1 + \left(\frac{8!}{7!}\right)^2\right]$.

*

→ 5 B, 5 G.
ways of arrangement such that no. 2 boys are together.

A) $^6C_5 \times 5! \times 5!$

. G_1 . G_2 . G_3 . G_4 . G_5 .

6 Gaps → $^6C_5$

→ 1 2 3 4 5 6 7 8 9
no. formed.

* 9 digits → $9!$
* 5 digit → $^9P_5$ or $^9C_5 \times 5!$
* 5 digit even no. → $5 \times 6 \times 7 \times 8 \times 4$ ← ↳ check for the constraint.

1 2 3 4 5 6 7 8 9 0

* 10 digit no → $9 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
* 5 digit no. → $9 \times 9 \times 8 \times 7 \times 6$
* 5 ~~odd~~ digit odd → $8 \times 8 \times 7 \times 6 \times 5$

constraint 1

constraint 2.

* 5 digit even →

1st Case → | | | | | 0 |

$9 \times 8 \times 7 \times 6 \times 1$

+

2nd case → | | | | | 2/4/6/8 |

$8 \times 8 \times 7 \times 6 \times 4$

→ ways of arrangement of PERMUTATIONS →

P [ ][ ][ ] S. (P & S can be interchanged)

A) P [ ][ ][ ] S × × × × × × ×

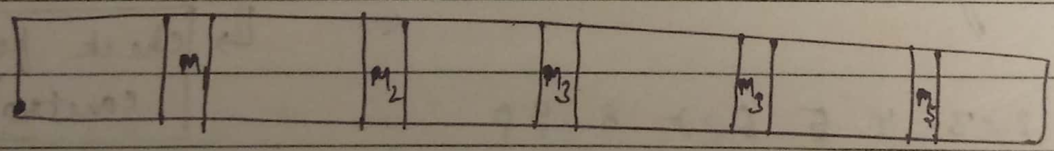
$8 \times 2! \times \frac{10!}{2!}$ or $= \left(\frac{8! \times {}^{10}C_3 \times 3!}{2!} \times 2 \right)$

→ Similarly COMBINATIONS →

(C & S are interchangeable) C [ ][ ][ ][ ] S × . × × × × ×

$7 \times 2! \times \frac{10!}{2! \times 2! \times 2!}$ or $= \left(\frac{7! \times {}^{10}C_4 \times 4!}{2!\, 2!\, 2!} \times 2 \right)$

*



100 different book

ways of arrangement so that order remains the same?

A) $\frac{100!}{5!}$

Q) $A_1, A_2, A_3 \ldots \ldots, A_{10}$

$A_1, A_2 \leftarrow A_{10}$.

$\frac{10!}{3!} \times 2!$ → for $A_1 A_2$ (can be interchanged)

Q) $A_1 \; A_2 \ldots \ldots \; A_{50}$.

→ 50 stations, total no. of possible tickets?

A) $^{50}P_2$ or $^{50}C_2 \times 2$.

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2nd class

(All for distinct)

Circular Permutation.

If clockwise & anticlockwise are taken as diff. $\rightarrow (n-1)!$

& if same $\rightarrow \dfrac{(n-1)!}{2}$

↳ flipping is possible for things.

Q) 20 beads

10 beads form a garlet. No. of ways?

A) $\rightarrow$ $^{20}C_{10} \times \dfrac{9!}{2}$

↳ i.e $\dfrac{(10-1)!}{2}$

Q) 6 B & 6 G

round table such that no. 2 boys and no. 2 girls are consecutive.

No. of ways = ?

A) $5! \times 6!$

↳ $(n-1)!$

Q) 12 diff. persons / 12 seats. No. of ways →

A)

Q) 10 B

circular round table. find no. of ways such that 2 boys do not want to sit together.

A) $9! - 8! \times 2!$

→ Principle of inclusion and exclusion :→

* $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

* $n(A \cup B \cup B) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

↳ alternate +ve & −ve signs.

Q) 1 — 1000

no.s. divisible by $\overbrace{2, 3}^{LC \cdot M}, \underbrace{7\ 5}_{LC \cdot M}$. (or)

A) $500 + 333 + 200 - 166 - 100 - 66 + 33$

Q) a, b, c, d, e, f, g, h

abc def

No. of such words when abc & def are not included.

A) $8! - 6! - 6! + 4!$

Q) 1, 2 $\underbrace{0, 3, 4, 5}$ ×

9 digit no. using 1, 2 at least ones but excluding 0, 3, 4, 5 ? (repetion is allowed)

A) $6^9 - 2 \times 5^9 + 4^9$

Q) 2 Ame
2 Ind
1 chi
1 spa
1 eng

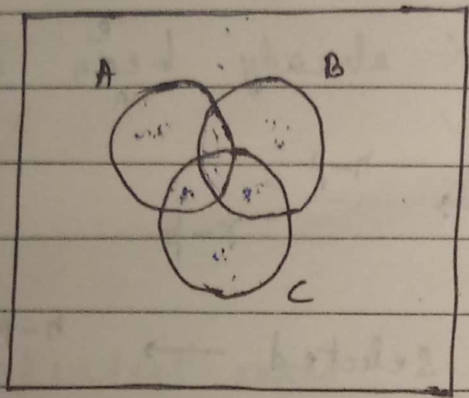
} round table conference

ways in which persons of the same country do not sit together.

A) $6! - 5! - 5! + 4!$

$n(P) = n$

Q)



ways in which A, B, C can be formed such that)

1) $n(A \cap B) = 0$
2) $n(A \cap C) \neq 0$
3) $n(B \cap C) \neq 0$..

Simultaneously

A) $6^n - 2 \times 5^n + 4^n$

P.T.O

On the next page :→

Restricted Selection

$n \rightarrow$ distinct object

$r \rightarrow$ object to be selected

If p object have already been selected.

$\rightarrow {}^{n-p}C_{r-p}$

p-object never get selected $\rightarrow {}^{n-p}C_r$

Q) 50 stu. 30 select for picnic

$A_1 A_2$ $A_3 \times A_4$

must be together otherwise not go.

ways in which selection is made?

A) $\left[\left({}^{46}C_{27} \times 2\right) + {}^{46}C_{28}\right] + \left[\left({}^{46}C_{29} \times 2\right) + {}^{46}C_{30}\right]$

Q) 50 stu. 30 (selection)

$A_1 A_2$ $A_3 A_4 A_5$ $A_6 \times A_7$

A) On the next page $\rightarrow$

A)

$A_1 A_2$	$A_3 A_4 A_5$	$A_6 \times A_7$
✓	✓	$^{43}C_{24} \times 2 + {}^{43}C_{25}$
✓	✗	$^{43}C_{27} \times 2 + {}^{43}C_{28}$
✗	✓	$^{43}C_{26} \times 2 + {}^{43}C_{27}$
✗	✗	$^{43}C_{29} \times 2 + {}^{43}C_{30}$

All are independent cases and thus are added to get the answer.

Q) AAA BCD
no. of arrange such that no 2 a's are consecutive?

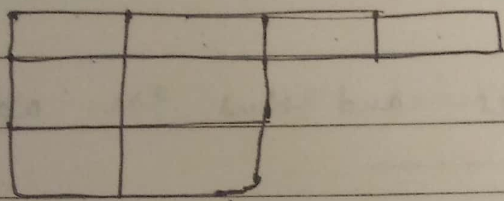
A) $^4C_3 \times 3! \times 1$

$\cdot$ B $\cdot$ C $\cdot$ D $\cdot$

selecting any 3 spaces of 4.
and BCD can be arranged among themselves in 3! ways.
$\because$ A A A are identical,
$\frac{3!}{3!} = 1$

→ If we throw a dice 100 times, no. of outcomes such that at least 1 five should occur.

A) $6^{100} - 5^{100}$

→ 

ways in which a, b, c, d, e, f can be filled in the above boxes such that no row is empty.

A) ${}^{8}C_{6} \times 6! - 2 \times {}^{6}C_{6} \times 6!$

Q) → B A R U N

↓

A B N R U (alphabetical order)

Rank of the above word →

$1 \times 4! + 1 \times 2! + 1! + 1$

↓

as rank is always 1 more than the position.

Q) Rank of the word ~~KASHISH~~ →

→ ~~AHIKSS~~

$$= 2 \times 4! + 2 \times 3! + 1 \times 1! + 1$$

Q) (For repetition)

Eg :→ Rank of ~~CHANDAN~~

~~A~~ A ~~C~~ ~~D~~ ~~H~~ ~~N~~ ~~N~~

↳ counted only 1 times
but ignored is only 1

$$= \frac{6!}{2!} + \frac{5!}{2!} + \frac{5!}{2!\,2!} + 2 \times \frac{3!}{2!} + 2! + 1$$

---

* Q) ~~RITTJEE~~ (Rank)

~~EERITJT~~

$$= \frac{6!}{2!} + \frac{5!}{2!} + 4! + 3! + \frac{3!}{2!} + 2! + 1$$

Q) ~~K~~ ~~A~~ ~~N~~ K A R B A G H (Rank)

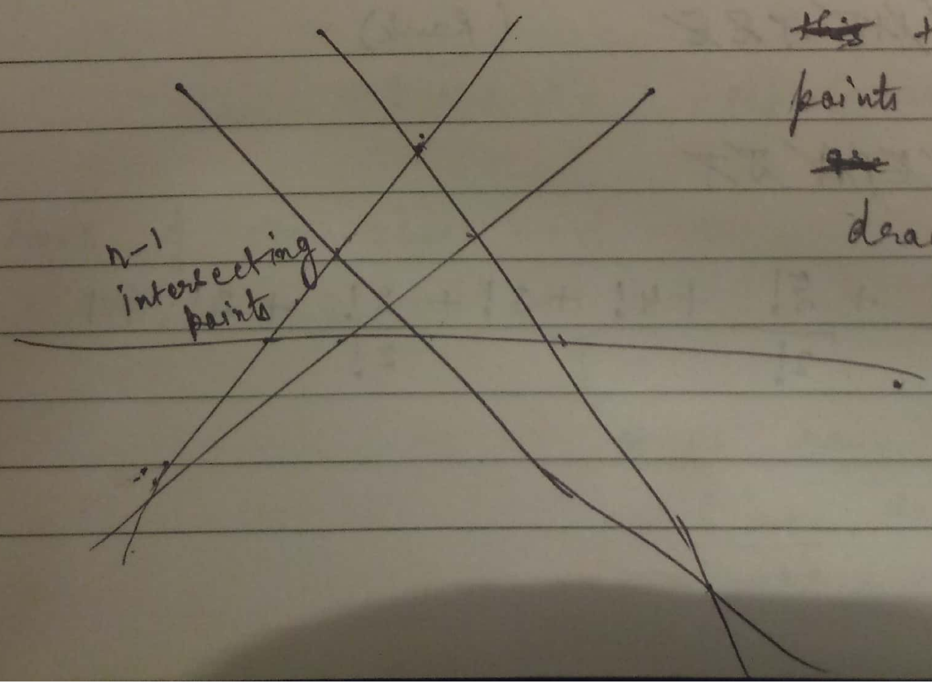
~~A~~ A A B G H ~~K~~ K A N R

A) $\frac{9!}{2!2!} + 3 \times \frac{9!}{2!3!} + 7! + 4 \times \frac{7!}{2!} + 6!$

$+ 3 \times \frac{6!}{2!} + 4 \times 4! + 3! + 1$

Q) n Lines are given where, no 2 lines are parallel, no 3 lines are concurrent All the intersecting points are joined. Find all the fresh lines introduced.

A) $^{^nC_2}C_2 - n \; {}^{n-1}C_2$ → subtracted as ~~this~~ through such points fresh lines ~~are~~ cannot be drawn.



→ Identical → (To be continued).

→ 100 identical balls has 101 ways to buy.

→ Q)

R	Bl	Green	W	Grey	P	Br
100	200	300	1	1	1	1

at least 1 ball is brought.

A) $101 \times 201 \times 301 \times 2 \times 2 \times 2 \times 2 - 1$

Q) (distinct)

	Maths	Phy	Che
different books	(7)	(6)	(5)
at least to buy.	4	3	2

find no of ways.

A) $({}^7C_4 + {}^7C_5 + {}^7C_6 + {}^7C_7) \times$

$({}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6) \times$

$({}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5)$, Class - 3

→ Identicals :→

Q) $N = 2^3 \times 3^4 \times 5^2$

1) T.f = $4 \times 5 \times 3$
2) E.f = $3 \times 5 \times 3$
3) factors of the form $(4n+2)$ → even but not divisible by 4. = $1 \times 5 \times 3$.

4) Sum of all factors :→
$= (2^0 + 2^1 + 2^2 + 2^3)(3^0 + 3^1 + 3^2 + 3^3 + 3^4)(5^0 + 5^1 + 5^2)$

5) factors (sum of all) divisible by 60 :→
↳ $2^2 \times 3^1 \times 5^1$

Ans → $(2^2 + 2^3)(3^1 + 3^2 + 3^3 + 3^4)(5^1 + 5^2)$

6) factors that are not prime → 60 − 3

* N identicals r distinct groups

a) $1) + \underbrace{1 + 1 + 1 + 1)} + \underbrace{1 + 1 + 1 + \dots + 1)} = 20$

${}^{19}C_2$

↳ Q) $x + y + z = 20$, a) $x, y, z \geq 1$
b) $x, y, z \geq 0$

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b) Ans $\longrightarrow$ $^{22}C_2$.

---

* 1) $x_1 + x_2 + x_3 + \dots\dots + x_r = n$

a) $x_1, x_2 \dots\dots x_r \geq 1$

$\hookrightarrow$ $^{n-1}C_{r-1}$

b) $x_1, x_2 \dots\dots x_r \geq 0$

$\hookrightarrow$ $^{n+r-1}C_{r-1}$

Q) R G W

100 200 300 identical

ways in 20 balls to buy?

Ans $\rightarrow$

$R + G + W = 20$

$^{20+3-1}C_{3-1} = {}^{22}C_2$

Q) 100 identical dice - Total no. of outcomes.

distinct?

$\hookrightarrow$ Set does not occur even twice.

Ans)

$\rightarrow$ no. of 1's.

$$\underset{1}{\square} + \underset{2}{\square} + \underset{3}{\square} + \underset{4}{\square} + \underset{5}{\square} + \underset{6}{\square} = 100$$

$^{105}C_5$

→ find integral sol^n of equation :→

$x + y + z = 30$

$x \geq 2$, $y \geq 3$, $z \geq 5$

A) $x - 1 \geq 1$, $y - 2 \geq 1$, $z - 4 \geq 1$

α β γ

$\alpha + 1 + \beta + 2 + \gamma + 4 = 30$

$\Rightarrow \alpha + \beta + \gamma = 23$

$\hookrightarrow {}^{22}C_2$

→ n- distinct r- distinct

arrangement

used when the arrangement of objects inside the box is important.

$= {}^{n-1}C_{r-1} \times n!$

$= {}^{n+r-1}C_{r-1} \times n!$

Q) 100 stu-

Crick. Volley. Foot.

compulsory to part in at least 1 game!

A) $^{n+r-1}C_{r-1} \times n! = {}^{102}C_2 \times 100!$

Q) 10 rings
5 fingers
Ans → $^{14}C_4 \times 10!$

→ ~~Dist~~ Division and distribution of groups :→

* If $a+b+c=n$ (for only distinct groups)
↳ distributed among 3 places.

we have,

$^nC_a \times {}^{n-a}C_b \times {}^{n-a-b}C_c$

$$= \frac{n!}{a!\,(n-a)!} \times \frac{(n-a)!}{b!\,(n-a-b)!} \times \frac{(n-a-b)!}{(n-a-b)!\,c!}$$

$$= \frac{n!}{a!\,b!\,c!}$$

→ for distinct groups →

$$\frac{n!}{a!\,a!\,a!\,b!\,b!\,c!} \times \frac{1}{3!\times 2!}$$

→ for persons (distinct) →

$$\frac{n!}{a!\,b!\,c!} \times 3!$$

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Q) 52 total cards.

10 12 14 16 placed.

ways of arrangement in different cases?

A) $\dfrac{52!}{10!\,12!\,14!\,16!}$

For Persons $\longrightarrow$ $\dfrac{52!}{10!\,12!\,14!\,16!} \times 4!$

For identical groups $\rightarrow$ $\dfrac{52!}{13!\,13!\,13!\,13!} \times \dfrac{1}{4!}$

Q) M W

100 100

ways of arrangement of mixed doubles?

A) $\underbrace{\left(\dfrac{100!}{(1!)^{100}} \times \dfrac{1}{100!}\right)}_{\text{Men's}} \times \underbrace{\left(\dfrac{100!}{(1!)^{100}} \times \dfrac{1}{100!}\right)}_{\text{Women's}} \times 100!$

Q) 5 B 5 G

Car 1	Car 2	Car 3	
3	3	3	Max^{m} cap.

ways of dropping to school?

A) $\frac{8!}{3!\,3!\,2!} \times \frac{1}{2!} \times 3!$

If sitting arrangment is considered.
Then,

$$\frac{8!}{3!\,3!\,2!} \times \frac{1}{2!} \times 3! \times (3!)^2 \times 2!$$

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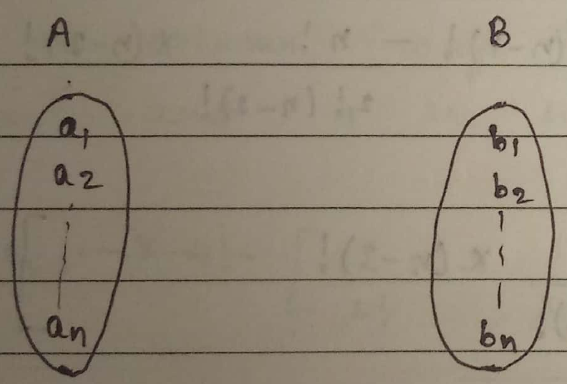
* **Dearrangement Theorem :** $\rightarrow$



Now, ways of dearrangement
$\rightarrow$ $n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + (-1)^n \frac{1}{n!}\right]$

Proof $\rightarrow$ E_1 a_2 is mapped with b_1
E_2 a_2 — — b_2
⋮
E_n a_n — — b_n

we have to find →

$n(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 \cap \ldots \cap \bar{E}_n)$

$= n(\overline{E_1 \cap E_2 \cap E_3 \cap \ldots \cap E_n})$

$\cancel{= n! - n(E_1 \cap E_2 \cap E_3}$

$= n! - n(E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n)$

$= n! - [\Sigma n(E_1) - \Sigma n(E_1 \cap E_2) + \Sigma n(E_1 \cap E_2 \cap E_3) - \ldots]$

$= n! - [{}^nc_1 (n-1)! - {}^nc_2 (n-2)! + {}^nc_3 (n-3)! \ldots]$

$$= n! - \left[\frac{n!}{1!(n-1)!} * (n-1)! - \frac{n!}{2!(n-2)!} \times (n-2)! + \frac{n!}{3!(n-3)!} \times (n-3)! - \ldots \right]$$

$$= n! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots + (-1)^n \frac{1}{n!}\right]$$ Proved

Q)

	A	B	C
Houses	2	1	1

ways in which they go in wrong houses ?

Ans

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A) $4! - \Big[\overbrace{2\times3\times2 + 1\times3\times2 + 1\times3\times2}^{\Sigma n(E_1)} \underbrace{- 2\times2 - 1\times2 - 2\times2}_{\Sigma n(E_1 \cap E_2)} + \underbrace{2}_{\Sigma n(E_1 \cap E_2 \cap E_3)}\Big]$

Q)

A	B
1	1
2	2
.	.
.	.
.	.
10	10

Match the column, ways in which 6 are correct and 4 are wrong.

Ans) $^{10}C_6 \times 4! \left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right]$

---

* n-distinct objects → r-distinct groups.

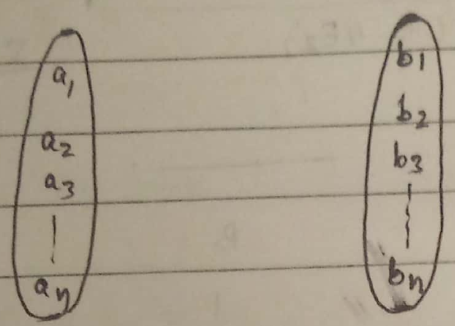
– distribute when each group must have min 1.

↓

$r^n - {}^rC_1 (r-1)^n + {}^rC_2 (r-2)^n - \ldots$

Proof $\longrightarrow$

Letters A   Postbox B



E_1 b_1 is ~~empty~~ not mapped / empty.

E_2 b_2 "   "   "   "

|

E_n b_r ------------

To find $\longrightarrow$ $n(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 \cap \cdots \cap \bar{E}_n)$

$$= n(\overline{E_1 \cap E_2 \cap E_3 \cap \cdots \cap E_n})$$

$$= r^n - n(E_1 \cup E_2 \cup E_3 \cup \cdots \cup E_n)$$

$$= r^n - [\Sigma n(E_1) - \Sigma n(E_1 \cap E_2) + \Sigma n(E_1 \cap E_2 \cap E_3) \; \text{------}]$$

$$\cancel{= r^n \; [{}^nC_1 (r-1)^n - {}^nC_2}$$

$$= r^n - [{}^rC_1 (r-1)^n - {}^rC_2 (r-2)^n + {}^rC_3 (r-3)^n \; \text{------}]$$

$$= r^n - {}^rC_1 (r-1)^n + {}^rC_2 (r-2)^n - {}^rC_3 (r-3)^n + \text{------}$$

Proved.

Q) $n = 5$, $r = 3$

ways of placement such that each postbox has at least 1 letter?

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Ans) $3^5 - {}^3C_1(3-1)^5 + {}^3C_2$
$= 243 - 96 + 3$
$= 150.$

Q) ① 1 2 3 4 5 6 7 8 9

$\square\square\square\square\square$

$S = (1+2+3+4+5+6+7+8+9)\,{}^8C_4 \times 4!\,(10^4 + 10^3 + 10^2 + 10^1 + 10^0).$

If 0 is involved, [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
We have,
$S_2 = S_1 - (1+2+3+\dots+9)\,{}^8C_3 \times 3!\,(10^3 + 10^2 + 10^1 + 10^0)$

Q) 1 2 3 4 0
5-digit nos. (Sum of all)

A) $\cancel{(1+2+3+4)\,{}^5C_4 \times 4!}$
$(0+1+2+3+4)\,4!\,(10^0 + 10^1 + 10^2 + 10^3 + 10^4)$
$- (0+1+2+3+4)\,3!\,(10^0 + 10 + 10^2 + 10^3).$

① $S_1 = (0+1+2+3+\dots+9)\,{}^9C_4 \times 4!\,(10^4 + 10^3 + \dots 10^0).$

→ Numerical Standings

→ Dummy Variable : → (proxy variables)

$^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ (works on)

also known as indicator variable, Boolean indicator, qualitative variable etc.
It is a variable that takes the value 0 or 1 to indicate the absence or presence of some categorial effects that may be expected to shift the outcome.

Q) 1 2 3 4 5 6 7 8 9 P_1 P_2 P_3

$a \leq b \leq c \leq d$

abcd

How many 4 digit no. can be formed?

A) can be solved without using dummy variable.

Ans → $^9C_4 + 3 \times {}^9C_3 + 3 \times {}^9C_2 + {}^9C_1 + {}^{10}C_4$
$+ 2 \times {}^{10}C_3 + {}^{10}C_2 + {}^{11}C_4 + {}^{11}C_3$

→ $^{12}C_4$.

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Q.) n_1 n_2

5 digit no 5 digit no.

$n_1 + n_2 = n_3$ (also a 5 digit no.)

ways such that the above is satisfied and there is no carry over? (Repetition allowed)

A)
$$\begin{array}{cccccc} & a_1 & a_2 & a_3 & a_4 & a_5 \\ + & b_1 & b_2 & b_3 & b_4 & b_5 \\ \hline \end{array}$$

Now, $a_5 + b_5 \leq 9$

$a_5 + b_5 + x = 9$

a_5 and b_5 can be selected in ${}^{11}C_2$

$({}^{11}C_2)^4$

for $a_4, b_4, a_3, b_3, a_2, b_2$.

Now, $a_1 + b_1 \leq 9$, $a_1, b_1 \geq 1$

$a_1 + b_1 + y = 9$

$(x+y+1) \geq 1$

$a_1 + b_1 + x = 0$ $x \geq 0$ → y

→ ${}^9C_2$

∴ Ans →

$${}^9C_2 \times ({}^{11}C_2)^4.$$

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Q) In the above question,

$n_1 - n_2 = n_3 \geq 0$

↳ not compulsory a 5 digit no.

A) Now,

$a_5 \geq b_5$

Then,

$0, 1, 2, \ldots 9, D_1$

a_5 & b_5 can be chosen in ${}^{11}C_2$ ways.

Similarly,

for the 4 nos. before,

We have,

$({}^{11}C_2)^4$

And, for a_1 & b_1,

$a_1 \geq b_1$

$0, 1, 2, 3, 4 - \text{———} 9, D_1$

a_1 & b_1 selected.

${}^{10}C_2 \times ({}^{11}C_2)^4$ → ways of nos. arrangement.

→ n-distinct object   r-distinct group.

Distribution.

↳ r^n.

Q) $100! = 2^{\alpha} \cdot 3^{\beta} \times 5^{\gamma} \ldots\ldots$

$$\alpha = \left[\frac{100}{2}\right] + \left[\frac{100}{4}\right] + \left[\frac{100}{8}\right] + \left[\frac{100}{16}\right] + \left[\frac{100}{32}\right] + \left[\frac{100}{64}\right]$$

$\gamma = \ldots\ldots$ and so on.

Q) ${}^{100}C_{50} = 3^{\alpha} \ldots$ (max^m α)

↳ $\frac{100!}{(50!)^2} = 3^{\alpha}$, $\alpha = 3$

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Q) 100 lines ; No 2 lines are || and no 3 lines are concurrent. How many regions are formed ?

A)

1 2 3 4 5 6 7 - - -

2 4 7 11 16 22 29

2 3 4 5 6 7

1 1 1 1 1

$T_n = an^2 + bn + c$

$T_2 = 4a + 2b + c$ —— (i)

$T_4 = 16a + 4b + c$ —— (ii)

$T_7 = 49a + 7b + c$ —— (iii)

After solving, we have, $a, b = \frac{1}{2}$, $c = 1$

$\therefore \quad T_n = \frac{n(n+1)}{2} + 1$

$= {}^{n+1}C_2 + 1$

After putting $n = 100$, we get the answer.

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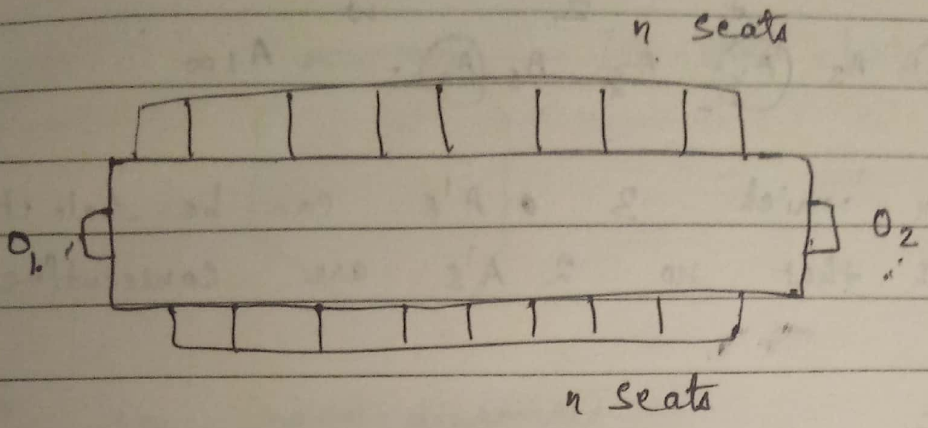
Q)



2 organisers
2n guests.
2 persons who do not want to sit together. Ways in which party can be organised.

A) $\underbrace{2n!\ 2!}_{\text{Total}} - \dfrac{(2n-2)!}{n!\,(n-2)!} \times (n-1)! \times 2! \times n! \times 2!$

Q) After 20 balls, his score is 100.
He only plays dot, four or six. no wicket
ways of playing the game?

A)

0	4	6
0	10	10
1	7	12
2	4	14
3	1	16

Ans $\longrightarrow \dfrac{20!}{10!\,10!} + \dfrac{20!}{7!\,12!} + \dfrac{20!}{2!\,4!\,14!} + \dfrac{20!}{3!\,16!}$

Q) $\overset{x}{A_1}$ A_2 A_3 $\overset{y}{A_4}$ $\underset{2}{A_5}$ $\overset{z}{A_6}$ A_7 . — — $\overset{w}{A_{100}}$

ways in which 3 A's can be selected such that no 2 A's are consecutive

A) $^{98}C_3$

2nd Method

$x \geq 0 \quad y, z \geq 1 \quad w \geq 0$

$\Rightarrow x+1 \geq 1 \quad y, z \geq 1 \quad w+1 \geq 1$

$x_1 \leftarrow$ $\quad x+y+z+w = 97 \quad \hookrightarrow w_1$

$\downarrow$

$\Rightarrow x_1+y+z+w_1 = 99$

Ans $\rightarrow$ $^{98}C_3$.

Q) In the above question, ways of selecting 3 A's such that every 2 A has 2 A in the middle?

A) $^{96}C_3$.

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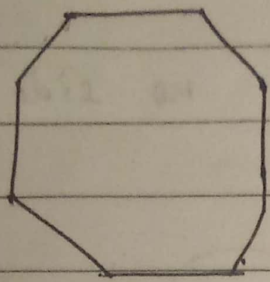
* Q) 100 sided polygon.



A) $^{100}C_2 - 100$ (no. of diagonal)

Q) In the ~~ways~~ above question; all the diagonals are joined & no 3 diagonals are concurrent. Find the no. of intersecting points inside the polygon?

A) $^{100}C_4 \times 1$, ∵ every 4 point has only only 1 point inside.

- Only see the min^m condition.

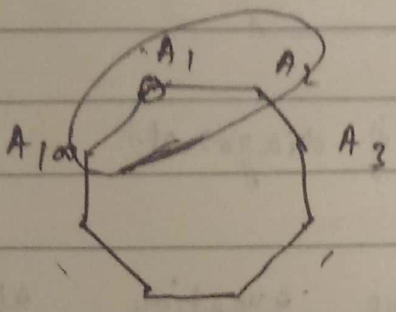
Q) In the above question, Δs are formed using vertices of the polygon, no. of Δs whose 2 sides are common with the polygon?

A) 100.

Q) only side common with the polygon?
A) 96×100.

* 100 sided polygon.
No. of Δs with no side common with the polygon?

A)



A_{100} & A_2 cannot be taken.

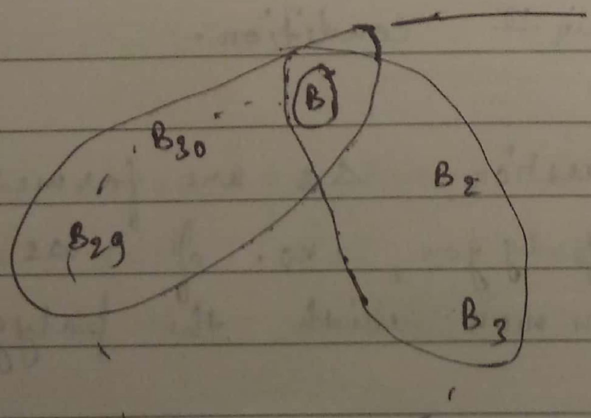
$B_1\ B_2\ B_3 \ldots B_{97}$.

$^{100}C_1 \longrightarrow$ one vertex selected.

$^{96}C_2 \longrightarrow$ 2nd vertex selected.

Ans $\rightarrow$ $\dfrac{^{100}C_1 \times {}^{96}C_2}{3}$

Q)



30 stu.
ways of arrangement such that 3 is selected such that every 2 has 2 gaps?

A) $\dfrac{{}^{30}C_1 \times {}^{23}C_2}{3}$

---

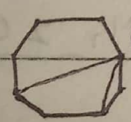
100 sides polygon.

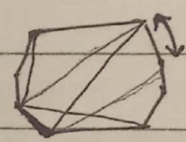
Q) No. of Quadrilateral such that 3 side of the quadrilateral is common with the polygon?

A) 100

Q) In the above question, such that 2 side of the quadrilateral is common with the polygon?

A) $\underbrace{95 \times 100}_{1^{st}\ \text{case}} + \underbrace{\dfrac{95 \times 100}{2}}_{2^{nd}\ \text{case}}$.





Q) aaabbb

~~wat~~ No. of words formed such that (ab) does not occur?

A) Answer must be 1.

Now, ~~(ab) aabb~~

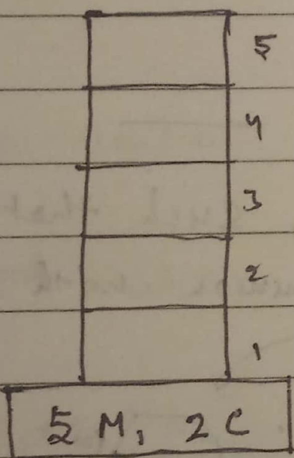
$\dfrac{6!}{3!\,3!} - \dfrac{5!}{2!\,2!} + \dfrac{4!}{2!} - 1$

Q) abc abc abc

Ways of arrangement such that abc pattern does not occur?

A) $\frac{9!}{(3!)^3} - \frac{7!}{(2!)^3} + \frac{5!}{2!} - 1$

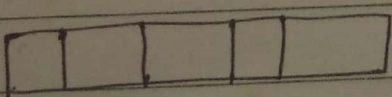
Q)



→ Ways of arrangement such that 2 children are not alone?

A) $5^7 - \left[1^5 \cdot 4^2 + (2^5 - 1^5) \cdot 3^2 + (3^5 - 2^5) \cdot 2^2 + (4^5 - 3^5)\, 1^2\right]$

Q)



How many 5 digit nos. can be formed such that ~~at least~~ one of the digits is repeated once? (0 is not ignored)

A) $^4C_2 \times 9 \times 8 \times 7$ → 1st case $\boxed{00}$

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2nd Case → 11, 22, 33 . . . 99

$$^9c_1 \times [\,^4c_2 \times 8 \times 8 \times 7 + 4 \times 9 \times 8 \times 7]$$

Ans is obtained by adding the above 2 independent cases.

Q) AAAABCD
No. of arrangement such that no 3 A's are consecutive?

A) $\frac{7!}{4!} - 5! + 4!$

Q) FIITJEE → II EE F T J
4 letter words?

4 left if 1 gets selected

A)

all different	1 1 1 1	$^5c_4 \times 4!$	+
	2 1 1	$^2c_1 \times \,^4c_2 \times \frac{4!}{2!}$	+
repeated	2 2	$\underbrace{^2c_2}_{1\ way} \times \frac{4!}{2!2!}$	

Q) MATHEMATICS → AA MM TT C E H I S
4 letter words?

A) $\left(^8c_4 \times 4!\right) + \left(^3c_1 \times \,^7c_2 \times \frac{4!}{2!}\right) + \left(^3c_2 + \frac{4!}{2!2!}\right)$

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Multinomial Theorem : →

$$(1+x)^{-n} = 1 - {}^{n}C_1 x + {}^{n+1}C_2 x^2 + {}^{n+2}C_3 x^3 + {}^{n+3}C_4 x^4 \text{ ------- } \infty$$

$$(1-x)^{-n} = 1 + {}^{n}C_1 x + {}^{n+1}C_2 x^2 + {}^{n+2}C_3 x^3 + {}^{n+3}C_4 x^4 + \text{ - - - - - } \infty.$$

To be proved later on.

Q) $x+y+z = 20$ $\quad x, y, z \geq 1$

~~Ans~~ Total no. of co-efficient of t^{20} in the expression

$$(\overset{\text{denotes the value taken}}{t^1} + t^2 + t^3 + \text{ - - - - }) (t^1 + t^2 + t^3 + \text{ - - - }) (t^1 + t^2 + t^3 + \text{ - - })$$

Now, $\left(\dfrac{t}{1-t}\right)^3$

Now, coefficient of t^{20} in $\left(\dfrac{t}{1-t}\right)^3$

coefficient of t^{17} in $(1-t)^{-3}$

$$= 1 + {}^{3}C_1 + {}^{4}C_2 t^2 + {}^{5}C_3 t^3 + \text{ ---- } + {}^{19}C_{17} t^{17} + \text{ .---}$$

$\therefore$ Ans → ${}^{19}C_{17} = {}^{19}C_2$.

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Q) For the above question in the interval,

$0 \leq x \leq 8$, $y, z \geq 1$

↳ upper limit (used by multinomial theorem).

A)

coefficient of t^{20} in

$(t^0 + t^1 + t^2 + t^3 + t^4 + t^5 + t^6 + t^7 + t^8)(t^1 + t^2 + t^3 - \ldots)^2$

$\hookrightarrow \left(\frac{1-t^9}{1-t}\right)\left(\frac{t}{1-t}\right)^2$

Now,

coefficient of t^{20} in $\frac{t^2(1-t^9)}{(1-t)^3}$ or $t^9(1-t^9)(1-t)^3$

Now, coefficient of t^{18} in $(1-t^9)(1 + {}^3c_1 t + {}^4c_2 t^2 + \ldots \infty)$

↳ we can get this when $1 \times t^{18}$ & $t^9 \times t^9$

∴ Ans → ${}^{20}c_{18} - {}^{11}c_9$.

Q) $x+y+z+w = 30$, $2 \leq x \leq 12$, $y \geq 1$, $z \geq 2$, $3 \leq w \leq 15$

A) Coefficient of t^{30} in

$(t^2 + t^3 + t^4 + \ldots t^{12})(t^1 + t^2 + t^3 + \ldots \infty)(t^2 + t^3 + \ldots)$

$(t^3 + t^4 + \ldots + t^{15})$.

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Now,

the above expression becomes $\longrightarrow$

$\cancel{t(1-t^2)}$

$$t^2 \left(\frac{1-t^{11}}{1-t}\right)\left(\frac{t}{1-t}\right)\left(\frac{t^2}{1-t}\right) t^3 \left(\frac{1-t^{13}}{1-t}\right)$$

$$= t^8 (1-t^{11})(1-t^{13})(1-t)^{-4}$$

Now, coefficient of t^{22} in

$$(1-t^{11}-t^{13}+\underbrace{t^{24}}_{\text{(does not give } t^{22})})(1+{}^4c_1+{}^5c_2 t^2+ \cdots)$$

Ans $\longrightarrow$ ${}^{25}c_{22} - {}^{14}c_{11} - {}^{12}c_9$

$\longrightarrow$ find integral sol^n of the eq^n

$x+y+z=30$

in the interval $1 \leq x \leq 12$, $2 \leq y \leq 15$, $z \geq 0$.

A) coefficient of t^{30} in

$$(t^1+t^2+ \cdots +t^{12})(t^2+t^3+t^4+\cdots+t^{15})(t^0+t^1+\cdots \infty)$$

$$= t\left(\frac{1-t^{12}}{1-t}\right) t^2 \left(\frac{1-t^{14}}{1-t}\right)\left(\frac{1}{1-t}\right)$$

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Q> $x + 2y + z = 30$, $x, y, z \ge 0$

$2y \to m$

coefficient of t^{30} in

$$(t^0 + t^1 + \ldots \infty)^2 (t^0 + t^2 + t^4 + \ldots \infty)$$

$$= \left(\frac{1}{1-t^2}\right)\left(\frac{1}{1-t}\right)^2 = (1-t)^{-3}(1+t)^{-1}$$

$$(1 + {}^3c_1 t + {}^4c_2 t^2 + \ldots)$$
$$(1 - t + t^2 \ldots)$$

Now,

Ans $\longrightarrow$ $1 - {}^3c_1 + {}^4c_2 - \ldots + {}^{32}c_{30}$.

Q) ABCDEFEF

No. 2 e's come together (arr)

$$\frac{8!}{2!\,2!} - \frac{7!}{2!}$$

ways of arrangement such that EF does not occur?

A) $\frac{8!}{2!\,2!} - 7! + \frac{6!}{2!}$

Q) 10 stu. A | 20 stu B

3 | 5

$A_1 \times A_2$ | B_1 goes only when B_2 goes.

A) $\left({}^{8}C_3 + 2 \times {}^{8}C_2\right)\left({}^{18}C_3 + {}^{18}C_4 + {}^{18}C_5\right)$

Q) 10 M 7 P 6 C

(different)

(at least) 7 5 4

A) $\left({}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}\right)\left({}^{7}C_5 + {}^{7}C_6 + {}^{7}C_7\right)$
$\left({}^{6}C_4 + {}^{6}C_5 + {}^{6}C_6\right)$.

Q) I (8) II (7)

Min attempt 3 3

Total Questions to be attempted $\longrightarrow$ 8 (overall).
ways of examination?

A)

Q) Different 5 Red stripes (different)
4 Blue stripes.
6 Green stripes.

ways in which flag can be formed?

A) ${}^5C_1\ {}^4C_1\ {}^6C_1 \times 3!$

Q) find +ve integral solⁿ of the eqⁿ $abc = 45$

$45 = 3^2 \times 5^1$

Now, $(3^{\alpha_1} \times 5^{\beta_1})(3^{\alpha_2} \times 5^{\beta_2})(3^{\alpha_3}\, 5^{\beta_3}) = 3^2 \times 5^1$

Now, $\alpha_1 + \alpha_2 + \alpha_3 = 2$

$\beta_1 + \beta_2 + \beta_3 = 1$

${}^4C_2 \times {}^3C_2 = 18.$

for integral solⁿs. →

we have,

$18\,(1 + {}^3C_2).$

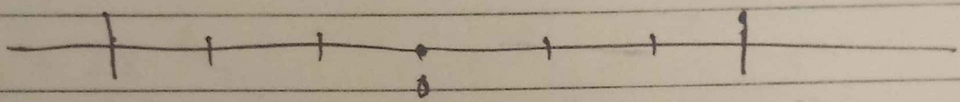
Q) $abcde = 120 \longrightarrow 5 \times 3 \times 2^3$

find integral solⁿ of the eqⁿ.

A) ${}^7C_4 \times {}^5C_4 \times {}^5C_4\ (1 + {}^5C_2 + {}^5C_4)$

Q) A man moves either right or left only. He took a total of 13 steps. After that, he was 3 steps away from the origin. Ways in which this can be done?

A)



L L L L L L L L R R R R R

$$\frac{18!}{8!\,5!} \times 2!$$

Q) FIITJEE

No. of words such that 4 letter word possible.

A) Coefficent of t^4 in

$$4!\left(t^0 + \frac{t^1}{1!}\right)^3 \left(t^0 + \underbrace{\frac{t^1}{1!}}_{\text{one selected}} + \frac{t^2}{2!}\right)^2$$

$\rightarrow$ both I & E

can be rejected

both selected.

$\underbrace{F\,T\,J}$, $\underbrace{I\,I \quad E\,E}$

$\hookrightarrow$ has only 2 options to be selected or rejected.

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Now, the above expression becomes,

$$4!\,(1+t)^3\left[(1+t)^2 + \frac{t^4}{4} + (1+t)t^2\right]$$

$$= 4!\left[(1+t)^5 + \frac{t^4(1+t)^3}{4} + (1+t)^4 t^2\right].$$

further can be done.

Q) ~~m — men~~

~~n — women.~~ ~~(n>m)~~

~~ways of arrangement such that there is~~

Q) ways in which 100 can be written?

Like for $3 \longrightarrow 1+1+1$

$\hookrightarrow {}^2C_0 + {}^2C_1 + {}^2C_2$

(ways of selecting + sign)

A) $^{99}C_0 + {}^{99}C_1 + {}^{99}C_2 + \cdots + {}^{99}C_{99}$

$\hookrightarrow 2^{99}$.

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