1st class

Permutation and Combination

→ ~~Permutat~~ Combination →

It is a grouping or a collection of all or a number of things without any reference to the order of things in which they are arranged.

R, M, S, G

{R}, {M}, {S}, {G} → Combination = 4

{R, M}, {R, S}, {R, G}, {M, S}, {S, G}, {M, G} → Combination → 6

{R, M, S}, {R, M, G}, {M, S, G}, {R, S, G} → combination = 4

{R, M, S, G} → No. of comb. = 1

→ Permutation →

Permutation is an arrangement of all or some of a number of things in a definite order.

or

ordered subset of a set.

Eg →

R, M, S, G

Case ① No. of combination →

$\rightarrow {}^4C_2 = 6$

(R, M) (M, S) (R, S) (R, G) (M, G) (S, G)

chair 2

chair 1

RM	MS	RS	RG	MG	SG
MR	SM	SR	GR	GM	GS

$${}^4P_2 = 2 \times {}^4C_2 = 2 \times 6$$

Case ⑪

RMS, RMG, RSG, MSG $\rightarrow {}^4C_3 = 4$

ch1 ch2 ch3

RMS	RMG	RSG	MSG
RSM	RGM	RGS	MGS
MRS	MRG	SRG	SMG
MSR	MGR	SGR	SGM
SRM	GMR	GRS	GSM
SMR	GRM	GSR	GMS

$${}^4P_3 = 24$$
$$= 6 \times 4$$
$$= 6 \times {}^4C_3$$
$$= 3! \times {}^4C_3$$

from Case - ①

$$^4C_2 = \frac{^4P_2}{2!}$$

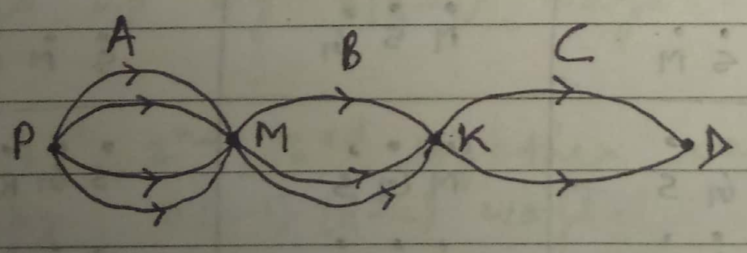
And, from case (iii),

$$^4C_3 = \frac{^4P_3}{3!}$$

$$\therefore \boxed{^nC_r = \frac{^nP_r}{r!}}$$

→ Multiplication → Let a work A is done in m ways and another work B is done in n ways, C is a work which is done when A & B both are done.

* No. of ways in which work C is done $= m \times n$



P – M → 4

P – M – K → 4 × 3 = 12

M – K → 3

M – K – D → 3 × 2 = 6

P – M – K – D → 4 × 3 × 2 = 24

K – D → 2

→ Addition rule →

Let a work A is done in m ways and another work B is done in m ways, C is a work which is done when either A is done or B is done.

* No. of ways ~~when~~ in which work C is done = $m+n$

→ R, M, S, G

$4 \times 3 \times 2$

$\boxed{}$ $\boxed{}$ $\boxed{}$

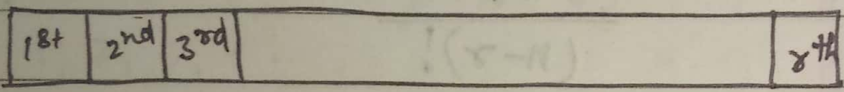
C_1 C_2 C_3

R M S R M G	M R S M R G	S R M S R G	G R M G R S
R S G R S M	M S R M S G	S M R S M G	G M R G M S
R G S R G M	M G S M G R	S G R S G M	G S R G S M

Total no. of permutations

No. of permutation of n distinct things taken r at a time will be same as the no. of ways in which r blank spaces can be filled up with n distinct things.

$a_1, a_2, \ldots\ldots, a_n$



$t_1 = n$ — 1^{st} place can be filled up by any one of $a_1, a_2 \ldots, a_n$ in n ways.

$t_2 = n-1 = \{n-(2-1)\}$ — 2^{nd} place can be filled up by any of the $(n-1)$ remaining things in $(n-1)$ ways.

$t_3 = n-2 = n-(3-1)$ — Now,

1^{st} & 2^{nd} places together can be filled up by $n(n-1)$ ways.

So, 1^{st}, 2^{nd}, 3^{rd} together can be filled up by $n(n-1)(n-2)$ ways.

$t_r = (n-(r-1))$ — $\therefore$ proceeding in this way, r^{th} place can be filled up by $n-(r-1)$ ways.

So, 1^{st}, 2^{nd}, 3^{rd} ……, r^{th} places simultaneously can be filled up by

$n(n-1)(n-2)\ldots\ldots(n-(r-1))$ ways.

$$^{n}P_{r} = n(n-1)(n-2)\ldots\ldots(n-r+1)$$

$$= \frac{n(n-1)(n-2)\ldots\ldots(n-r+1)(n-r)(n-r-1)\ldots\ldots 3\cdot 2\cdot 1}{1\cdot 2\cdot 3\ldots\ldots\ldots(n-r)}$$

$$= \frac{n!}{(n-r)!}$$

Also, $\because$

$$^{n}P_{r} = n(n-1)(n-2)\ldots\ldots(n-r+2)(n-r+1)$$

Put $r = n$

$$\therefore\ ^{n}P_{n} = n(n-1)(n-2)\ldots\ldots(n-n+2)(n-n+1)$$

$$= n(n-1)(n-2)\ldots\ldots 2\cdot 1$$

$$= n!$$

i.e

$$^{n}P_{n} = n!$$

Also,

$$\frac{n!}{0!} = \frac{n!}{1}$$

$$\therefore\ 0! = 1$$

$\rightarrow$ $\because$ $\frac{^{n}P_{r}}{r!} = {}^{n}C_{r}$

$$\therefore\ ^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$\rightarrow$ Ex $\rightarrow$

formed

1, 2, 3,

1,

4

ne

as

beco

int

re

$\rightarrow$ 0, 2, 3,

3

2,3,

$^{3}P_{1}$

filled separately

Tota

3·2·1

→ Ex → Total no. of numbers formed (3-digit) by 4 numbers 1, 2, 3, 4.

1, 2, 3, 4

$4 \times 3 \times 2$

$^4P_3$

Hence, total no. of numbers = 24

Here, $^4C_3$ is not valid as the places automatically become distinct as the interchange of arrangement results in a new number.

→ 0, 2, 3, 4

$3 \times 3 \times 2 \rightarrow 18$.

2,3,4		

$^3P_1$ ways $\quad$ $^3P_2$ ways

filled separately

Total no. of numbers = $^3P_1 \times {}^3P_2$
$= 18$

Repetitions

Of object — Of places.

Q) How many 2 digit numbers can be formed with the digits 1, 2, 3 with repetition of digit allowed?

A) 1st place, 2nd place } Same digit ✓

1st digit, 2nd digit } Same place ✗✗

Distinction b/w repetition of object & places.

Each place can be filled up by any one of 3 digits 1, 2, 3 in 3 ways.

$= 3 \times 3$

$= 3^2$

$\underbrace{3 \times 3}_{\text{options}}$

$= (\text{No. of object})^{\text{no. of place}}$

∵ a ~~place~~ single place cannot be filled by 2 nos. simultaneously.

(Case of repetition of places)

↓

illogical

→ In how many ways 3 persons can stay in two hotels?

A) 1st hotel } same person ✗✗
2nd Hotel

~~1st hotel~~ } ~~S.~~
~~2nd hotel~~ }

1st person } same hotel ✓
2nd person }

More than 1 persons can stay in 1 hotel but one person can be present only at 1 place at a point of time.

y. Each person can stay in any one of the 2 hotels in 2 ways.

Hence,

$H_1 H_2$	$H_1 H_2$	$H_1 H_2$
P_1	P_2	P_3

$2 \times 2 \times 2 \longrightarrow 2^3$

(no. of place) no. of object

A . V	G . P
R, M, S	0
0	R, M, S
R	M, S
M, S	R
M	R, S
R, S	M
S	R, M
R, M	S

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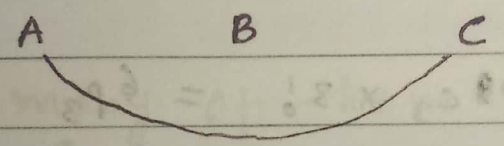
2^{nd} class

Q) There are 6 ~~books~~ roots b/w A & B and 4 roots b/w B and C. In how many ways can one drive from A to C by way of B?

A) $6 \times 4 = 24$

Q) In how many ways can one drive from A to C and back to A passing through B on both trips?

A)



~~24+24=48~~ $24 \times 24 = 24^2$

IMP Q) In how many ways can one drive the circular trip described in question 2 without passing through the same route?

A)

24×15

No. of ways of going from A to C

No. of ways of returning but not by the same route.

~~Q) The no. of ways of going from A to C~~

Q) With repetition not allowed,

a) how many 3 digit nos. can be formed with $\{2, 3, 5, 6, 7, 9\}$?

b) how many less than 400 ?

c) how many are even ?

d) How many odd ?

e) multiples of 5 ?

A) a) $^{6}C_3 \times 3! = {}^{6}P_3$

b)

2,3		

$2 \times 5 \times 4 = 40$

c)

		2,6

$4 \times 5 \times 2 = 40$

(?)

d)

		3,5,7,9

$4 \quad 5 \quad 4$ $= 80$

or $120 - 40 = 80$

e)

		5

$4 \times 5 \times 1 = 20$

Q) Find the no. of 4 letter word that can be formed from the letters of the word

a) 'HISTORY'? (Each letter to be used at most once)

b) How many of them only contain consonants?

c) How many of them begin & end with a consonant?

d) How many of them begin with a vowel?

e) How many contain the letter y?

f) How many begin with 't' & end in a vowel?

g) How many begin with t and also contains s?

h) How many contains both vowels?

A) a) $^{7}C_4 \times 4! = 840$

b) $^{5}C_4 \times 4! = 120$

c) any one of 4 consonants

$4 \times 4 \times 5 \times 5$ = 400

d)

o, i			

2 6 5 4 = 240

e) $^{7}C_4 \times 4! - {}^{6}C_4 \times 4! = 480$

f)

t			o, i

1 5 4 2 = 40

~~t~~

1

~~$^{7}C_4 \times 4! - {}^{5}C_4 \times 4! =$~~

~~$840 - 120 =$~~

g)

T			

$^1C_1 \times {}^3C_1 \times ({}^5C_2 \times 2!) \longrightarrow 60$

for S   for remaining.

h)

$^4P_2 \times {}^5P_2 = 12 \times 20 = 240$

Q) 3 diff. railway passes are alloted to 5 students.
No. of ways this can be done is

A) $^5C_3 \times 3! = 60$

Q) A man has 3 jackets, 10 shirts and 5 pair of socks. If an outfit consists of a jacket, a shirt & a pair of socks, the different no. of outfits can the man make is

A) $^3C_1 \times {}^{10}C_1 \times {}^5C_1$

$= 150$

→ The no. of ways in which 5 letters be mailed if there are 3 diff. mail boxes are available and each letter can be mailed in any mail boxes.

A) 1^{st} letter } same box
2^{nd} letter } (possible)

→ $3 \times 3 \times 3 \times 3 \times 3 = 243$

→ The no. of 4 digit nos. are there which are divisible by 2 is (repetitions allowed)

A) 1000   9999

9	10	10	5

(except 0)

Due to repetition

0, 2, 4, 6, 8

= 4500

→ The no. of natural nos. are there from 1 to 1000 which have none of their digit repeated is

A) $\underset{9}{\boxed{}}\,\underset{9}{\boxed{}}\,\underset{8}{\boxed{}} = 648$

→ 1000 not possible.

$\underset{9}{\boxed{}}\,\underset{9}{\boxed{}} \rightarrow 81$

9×9

except 0

$\boxed{} \rightarrow 9$

${}^{9}C_{1} \times 1!$

Total → 738.

→ (without repetition) No. of natural nos. b/w 100 and 1000 such that at least one of their digit is 7 is

A) $\underset{8}{\boxed{}}\,\underset{8}{\boxed{}}\,\underset{1}{\boxed{7}}$

$\begin{array}{r} 128 \\ 72 \\ \hline 200 \end{array}$

$\underset{8}{\boxed{}}\,\underset{1}{\boxed{7}}\,\underset{8}{\boxed{}}$

$\underset{1}{\boxed{7}}\,\underset{9}{\boxed{}}\,\underset{8}{\boxed{}}$

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No. in which 7 do not occur.

Another Method →

$^9P_1 \times {}^9P_2 - {}^8P_1 \times {}^8P_2$

Total

$= 200$

Also, with repetition (IMP) → 252

Q) No. of odd nos. of 5 digits can be formed with the digits 0, 1, 2, 3, 4 is

A)

				1,3
3	3	2	1	2

$= 36$

→ No. of exponent of prime p in n! →

$$E_p(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \dots + \left[\frac{n}{p^s}\right]$$

where $p^s \leq n < p^{s+1}$

Eg → $E_2(6!) = \left[\frac{6}{2}\right] + \left[\frac{6}{2^2}\right] = 3+1 = 4$

$6! = 6 \times 5 \times 4 \times 3 \times 2$

$= 2^4 \times 3^2$

Q) Exponent of 2 in 65! $\rightarrow$

A)

$$\left[\frac{65}{2}\right] + \left[\frac{65}{2^2}\right] + \left[\frac{65}{2^3}\right] + \left[\frac{65}{2^4}\right] + \left[\frac{65}{2^5}\right] + \left[\frac{65}{2^6}\right]$$

$$= 32 + 16 + 8 + 4 + 2 + 1$$

$$= 63$$

Q) Exponent of 3 in 100! $\rightarrow$

A)

$$\left[\frac{100}{3}\right] + \left[\frac{100}{9}\right] + \left[\frac{100}{27}\right] + \left[\frac{100}{81}\right]$$

$$33 + 11 + 3 + 1$$

$$= 48$$

Q) Exponent of 2 in $\dfrac{20!}{10!}$

$$\downarrow$$

$$20 \times 19 \times 18 \times \ldots \times 11$$

A)

$$\left[\frac{20}{2}\right] + \left[\frac{20}{4}\right] + \left[\frac{20}{8}\right] + \left[\frac{20}{16}\right] - \left\{\left[\frac{10}{2}\right] + \left[\frac{10}{4}\right] + \left[\frac{10}{8}\right]\right\}$$

$$= (10 + 5 + 2 + 1) - (5 + 2 + 1)$$

$$= 18 - 8 = 10$$

Q) Find the exponent of 12 in 50!

A) $\cancel{\left[\frac{50}{12}\right] + \left[\frac{50}{36}\right] + \left[\frac{50}{48}\right] + \left[\frac{50}{24}\right]}$

$3 \times 2 \times 2 = 12$

$$\left[\frac{50}{3}\right] + \left[\frac{50}{9}\right] + \left[\frac{50}{27}\right] + \left[\frac{50}{81}\right]$$

3^{22}

$$= 16 + 5 + 1$$
$$= 22$$

$$\left[\frac{50}{2}\right] + \left[\frac{50}{4}\right] + \left[\frac{50}{8}\right] + \left[\frac{50}{16}\right] + \left[\frac{50}{32}\right]$$

47

$2^{23}.2$

$=(2^2)^{23}.2$

$=4^{23}.2$

$$= 25 + 12 + 6 + 3 + 1$$
$$= 47$$

Hence, $\underline{\underline{22}}$

Q> No. of zeros at the end of 60!

A> $\left[\frac{60}{2}\right] + \left[\frac{60}{4}\right] + \left[\frac{60}{8}\right] + \left[\frac{60}{16}\right]$
$+ \left[\frac{60}{32}\right]$

$= 30 + 15 + 7 + 3 + 1$
$= 56$

$\left[\frac{60}{5}\right] + \left[\frac{60}{25}\right]$.

$= 12 + 2$
$= 14$

Hence, $\underline{\underline{14}}$.

Q> The no. of prime nos. among the nos. $105! + 2$, $105! + 3$, ..., $105! + 105$

a> 31
b) 32
c) 33
✓ d) None of these

A> $(1 \times 2 \times \ldots \times 105) + 2$
$= 2\,(1 \times 3 \times \ldots \times 105 + 1)$
$(1 \times 2 \times \ldots \times 105) + 105$
$= 105\,(1 \times 2 \times \ldots \times 104 + 1)$

3rd class

→ How many 5 digit nos. can be formed with the digits 0, 1, 2, 3, 4, 5 which is divisible by 3 ? (repetition not allowed)

A) for 1, 2, 3, 4, 5
(excluding 0)

5 × 4 × 3 × 2 × 1 ⟶ 5!

for 0, 1, 2, 4, 5
(excluding 3)

4 4 3 2 1 ⟶ 96

Note ⟶ Only 3 can be excluded for the sum to be divisible by 3

Hence,

Total ⟶ 216
↓
120 + 96

(No other possibility)

→ How many 5 digit nos. can be formed with the digits 0, 1, 2, 3, 4, 5 which is divisible by 6?

A) For 1, 2, 3, 4, 5

				2,4
4 ×	3 ×	2 ×	1	2

→ 48

For 1, 2, 0, 4, 5

1st Case →

				0
4	3	2	1	

→ ending in 0.

→ 24

2nd Case →

				2,4
3	3	2	1	2

→ ending in 2, 4

→ 36

Hence, total → 36 + 24 + 48

= 108

Note →

For divisible by 6,

must be divisible 2, 3.

(∩ of both cases)

* For a no. to be divisible by 4, the last 2 digits must be divisible by 4. If a no. is divisible by 4, it must be divisible by 2.

→ How many 5-digit nos. can be formed [redacted], with the digits 1, 2, 3, 4, 5 which is divisible by 4?

A)

			1, 3, 5	2
3	2	1	3	1

$\to$ odd

$\longrightarrow 18$

* Check in case of repetition.

			2	4
3	2	1	1	1

$\longrightarrow 6$

even

Total nos. = 24.

→ How many 7-digit nos. can be formed with the digits 1, 2, 3, 4, 5, 6, 7 which is divisible by 8?

A)

a	b	c	d	e	f	g
4	3	2	1	$\underbrace{\quad\quad\quad}_{\text{22 ways.}}$		

* For a no. to be divisible by 8, the last 3 digits must be divisible by 8. If a no. is divisible by 8, it is divisible by 4 i.e last 2 digits must be divisible by 4 & hence, the last digit must be even.

* check in case of repetition.

Places e, f, g together can be filled in → 22 ways

Hence, total 7-digit nos. → 24 × 22 = 528

	e		f		g		
	3	×	1	×	1		
	5, 3, 7		1		2	+	10 options
	2	×	1	×	1		
	4, 6		3		2	+	
	3	×	1	×	1		
	1, 3, 7		5		2	+	
	2	×	1	×	1		
	4, 6		7		2		
⊕							
	1	×	1	×	1		
	6		2		4	+	2 options
	1	×	1	×	1		
	2		6		4		
⊕							
	2	×	1	×	1		
	2, 4		1		6	+	10 options
	3	×	1	×	1		
	1, 5, 7		3		6	+	
	2	×	1	×	1		
	2, 4		5		6	+	
	3	×	1	×	1		
	1, 3, 5		7		6		

Totals options = 22

→ How many ~~different~~ nos. less than 1000 can be divisible by 5 from the different digits? (repetition not allowed)

A)

	5
8	1

5
100.

	0
9	

$\underline{\underline{189}}$

* check in case of repetition.

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			5
8	8	1	

64

			0
9	8	1	

72

Total → 154

$$\begin{array}{r} 72 \\ 64 \\ \hline 1316 \end{array}$$

$$\begin{array}{r} 1316 \\ 19 \\ \hline 158 \end{array}$$

→ How many 3 digit nos. can be formed with the digits 1, 2, 3 and also find their sum?
(repetition is not allowed)

A)

${}^3P_3$ ways

No. of Nos. $= 3!$
$= 6$

2, 3

		1

${}^2P_2$ ways

Each of digits 1, 2 & 3 will occur 2 times at unit's, ten's and hundred's place.

Sum of digits at units / tens / hundredths place each will be

$= 2\times1 + 2\times2 + 2\times3$

$= 12$

Req. Sum. $= 12\times1 + 12\times10 + 12\times100$

$= 12(1 + 10 + 10^2)$

$= 1332$

In short,

$$\text{Sum of Nos.} = \frac{\text{No. of Nos. formed}}{\text{No. of digits}} \times (\text{Sum of digits}) \times (1 + 10 + 10^2 + \dots + 10^n)$$

irrespective of it's repetition

given

irrespective of repetition

→ formula is applicable even if repetition of digits is allowed.

→ How many 3 digit nos. can be formed by digits 1, 2, 3 and also find their sum? (repetion allowed)

A)

1, 2, 3	1, 2, 3	1, 2, 3

3 × 3 × 3 ⟶ 27

checking how many times digits occur ⟶

1, 2, 3	1, 2, 3	1

Each of 1, 2, 3 will occur 9 times at unit's, ten's & hundred's place.

122	231	123
212	321	213
312	121	233
132	211	323
232	131	133
322	311	313
222	221	223
332	331	113
112	111	333

Sum of digits at unit's, ten's, hundred's place $= 9 \times 1 + 9 \times 2 + 9 \times 3$
$= 54$

Hence, Sum $= 54 + 54 \times 100 + 54 \times 10 = 5994$

→ How many 4-digit nos. can be formed with the digits 1, 2, 2, 3 and also find their sum?

A)

1 2 2 3

$\frac{^4P_4}{2!} \longrightarrow$ 12 ways.

1 2 3

			2

$^3P_3$ ways

2 will occur 6 times at units, tens, hundred's, ~~place~~ thousand's place

1 2 3 2
1 3 2 2
3 1 2 2
3 2 1 2
2 1 3 2
2 3 1 2
2 2 3 1
2 3 2 1
3 2 2 1
1 2 2 3
2 1 2 3
2 2 1 3

1 2 2

			3

$\frac{^3P_3}{2!}$

each of 1 & 3 will occur 3 times at unit's, ten's, hund's, thousand's place.

Sum of digits at unit - - - place

$= 6 \times 2 + 3 \times 1 + 3 \times 3$

$= 24$

Req. Sum $= 24 \times (1 + 10^2 + 10^3 + 10^4)$

$= 26,664.$

→ In this case also, formula is applicable.
(even if repetition is allowed)

* Check in case of repetition.

→ How many 4-digit can be formed with 4-digits 0, 1, 2, 3 ? (Repetition not allowed) Also, find their sum.

A) | | | | |
|---|---|---|---|

$3 \times 3 \times 2 \times 1 \longrightarrow 18$

1 2 3

			0

$^3P_3$ ways

0 will occur 6 times at unit's, ten's & hundred's place.

			1

0 will not be there.

$^2P_1$ $^2P_2$

each of 1, 2 & 3 will occur 4 times at units, ten's & hundred's place.

Also, each of 1, 2 & 3 will occur 6 times at thousand's place.

Now,

sum of digits at unit's, ten's & hundred's places will be each

$= 6 \times 0 + 4 \times 1 + 4 \times 2 + 4 \times 3$

$= 24$

Sum of digits at thousand's place

$= 6 \times 1 + 6 \times 2 + 6 \times 3$

$= 36$

Req. sum = 24 × 1 + 24 × 10 + 24 × 100
+ 36 × 1000
= 38,664

1230

In short,

1320

$\frac{24}{4}(0+1+2+3)(1+10+10^2+10^3)$

2130

2310

3120

$-\underbrace{\frac{6}{3}(1+2+3)(1+10+10^2)}$

3210

2031

= 38,664

2301

3021

Resembles ex-1 of this type.

3201

1032

* formula gets violated in this case.

1302

3102

3012

1023

1203

2103

2013

Subtracted

0123
0132
0213
0231
0312
0321

4^{th} class

→ The no. of ten digit nos. that can be made with the odd digits so that no 2 consecutive digits are the same.

A) 1, 3, 5, 7, 9

5	4	4	4	4	–		–		–

a) 5×4^9
b) 5^{10}
c) 5^9
d) 5×4^8

Hence, 5×4^9

→ A ~~gentle~~ letter lock consists of 3 rings each marked with 10 different letters. ~~In how~~ The no. of ways it is possible to make an unsuccessful attempt to open the lock.

a) 1000
✓ b) 999 i.e $10^3 - 1$
c) 720
d) 10,000.

→ A telegraph has 5 arms and each capable of ~~ha~~ 4 distinct position including the position of rest. The no. of signals that can be made is

a) 1022
✓ b) 1023
c) 1024
d) 1025

A) 1st arm } same position
2nd arm }
→ Each arm can have any one of 4 positions.

Hence, No. of signals = 4^5 (including rest)

Now,

Total signals = $4^5 - 1$

→ A servant has to post 5 letters and there are 4 letter boxes. The no. of ways he can post the letters is ____

a) 5^4
b) $5^4 - 1$
c) $4^5 - 1$
✓ d) 4^5

→ There are m men and n monkeys ($m<n$). If a man may have any no. of monkeys, the no. of ways in which every monkey has a master

a) n^m
b) $n^m - 1$
✓ c) m^n
d) $m^n - 1$

1st monkey } same
2nd monkey } master

$m \times m \times m \ldots = m^n$

→ ~~In how m~~ The no. of diff. ways, the following 5 prizes be distributed among 10 students, 1st & 2nd in mathematics, 1st & 2nd in physics & 1st in Hindi is

a) 8000
b) 80,000
c) 82,000
✓ d) 81,000

A) $\underbrace{{}^{10}C_2 \times 2!}_{} \times \underbrace{{}^{10}C_2 \times 2!}_{} \times \underbrace{{}^{10}C_1}_{}$.

→ There are stalls of 12 animals in a ship. The no. of ways, ship load can be made if there are cows, calves and horses to be transported. Each kind being not less than 12 is

a) 3^{12}

→ When certain things occur or do not occur together →

Q) In how many ways 4 boys & 3 girls sit ~~around~~ in a row so that

1. there is no restriction
2. all the girls sit together
3. all the girls do not sit together.
4. No 2 girls sit together.

A)

a) when there is no restriction, we have,

$(4+3) = 7$ persons

the no. of arrangement of 7 per.

$= 7!$

b) R M S G (BLV)

when all the girls regarding as 1 girl, then, we have,

$(4+1) = 5$ persons

No. of arrangement of 5 persons

$= 5!$

And, all the girls can be arranged among themselves in $3!$ ways.

when all the girls come together no. of arrangement

$= 5! \times 3!$

c) No. of ways in which all the 3 girls do not sit together

$= 7! - 5! \times 3!$

d) $\times \quad \overset{B_1}{\bullet} \quad \times \quad \overset{B_2}{\bullet} \quad \times \quad \overset{B_3}{\bullet} \quad \times \quad \overset{B_4}{\bullet} \quad \times$

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4 boys can be arranged among themselves in 4! ways. If the girls sit at the places indicated by X, no 2 girl will sit together. There are 5 X places for 3 girls, so, 3 cross places can be selected for 5 cross places and all the 3 girls can be arranged among themselves in 3! ways.

Hence, Req. no. of ways

$$= 4! \times {}^5C_3 \times 3!$$

$$= 4! \times {}^5P_3.$$

→ In how many ways, 4 boys and 4 girls can sit along a row so that boys & girls are alternate?

A) when arrangement begins with a boy →

$$B_1 \times B_2 \times B_2 \times B_4 \times$$

$$4! \times 4!$$

beginning with Girls →

$$\times B_1 \times B_2 \times B_3 \times B_4$$

$$4! \times 4!$$

Hence, total ways $= 2 \times 4! \times 4!$

→ In how many ways 4 boys & 3 girls can be arranged in a row so that boys & girls are alternate?

A) $B_1 \times B_2 \times B_3 \times B_4$

$4! \times 3!$

→ There are 2 books each of 3 volumes and 2 books each of 2 volumes. In how many ways can the 10 books be arranged on a table so that vol. of the same books are not separate?

A)

A	B	C	D
3	3	2	2

when vol. of the same book regarding as 1 book, then, we have,

4 different books (sets)

when vol. of the same book regarding as a 1 book, then we have, No. of arrangement of 4 diff. books = 4!

And, vol. of the same book can be arranged among themselves, hence,

req. no. of ways

$= 4! \times 3! \times 3! \times 2! \times 2!$

→ A Lib. has 5 copies of 1 book, 4 copies each of 2 books, 6 copies each of 3 books & single copy of 8 books. In how many ways all books be arranged so that copies of the same book are always together?

A) When all the copies of the same book regarding as a one book, then, we have,

14 diff. sets of copies for books.

No. of arrangement of these sets

$= 14!$

And, copies of the same book can be arranged in 1 way.

Hence, no. of ways = 14!

A A A A A

B B B B

C C C C

P P P P P P

V V V V V V

L L L L L L

M

N

O

P

Q

R

S

T

→ you have six balls of diff. colors black, white, red, green, violet & yellow. In how many ways, you can arrange them in a row, so that black & white never come together?

A) Method – I →

$\overset{\times}{B}$ G r $\overset{\times}{W}$ V Y

$6! - 5! \times 2!$

Date:
Page:

Method - 2 →

× × × × ×

$4! \times {}^5C_2 \times 2!$

→ In how many ways 18 white and 19 black balls be arranged in a row so that no two white balls may be together if

i> balls of same color are distinct.
ii> all the white balls are identical.
iii> all the black balls are identical.
iv> balls of the same color are identical.

A) i> $19! \times {}^{20}C_{18} \times 18!$
ii> $19! \times {}^{20}C_{18} \times 1$

→ when balls of same color are distinct. Arrangement of 19 black balls → 19!. Now, 18 balls (white) can be accomodated in 20 × places.

→ when white balls are identical, no. of arrangement of white balls is 1.

iii) $1 \times {}^{20}C_{18} \times 18!$

iv) $1 \times {}^{20}C_{18} \times 1$

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5^{th} class

→ In how many ways 3 Rasogulle & 2 gulabjamun can be selected?

A) 0 or more Rasogulle can be selected out of 3 Rasogulle in (3+1) = 4 ways

ϕ, {R}, {R, R}, {R, R, R}

Zero or more gulabjamuns can be selected out of 2 gulabjamuns in (2+1) = 3 ways.

ϕ, {G}, {G, G}

Total Selection = 3 × 4
= 12 ways.

ϕ, {G}, {R}, {G, R}, {R, R}, {G, G},
{G, G, R}, {R, R, G}, {R, R, R},
{R, R, G, G}, {R, R, R, G}, {R, R, R, G, G}

→ In how many ways 3 Rasogulle & 2 gulabjamun can be selected so that in each of the selection, 1 Rasogulle must be present?

A> 1 or more Rasogulle can be selected out of 3 Rasogulle in 3 ways.

{R}, {R,R}, {R,R,R}

0 or more Gulabjamun can be selected out of 2 Gulabjamuns in 3 ways.

ϕ, G, G,G.

Hence, total no. of ways
= 3 × 3
= 9

{R}, {G,R}, {R,R}, {G,G,R}, {R,R,G}, {R,R,R}

→ In how many ways 3 Rasogulle & 2 gulabjamun can be selected so that 1 gulabjamun must be present?

A> zero or more Rasogulle can be selected out of 3 Rasogulle in 4 ways.

{R}, {R,R}, {R,R,R}

One or more gulabjamuns can be selected out of 2 Gulabjamuns in 2 ways.

Hence, Total ways = 4 × 2 = 8.

→ In how many ways 3 Rasogulla and 2 gulabjamuns can be selected so that 1 Rasogulla & 1 Gulabjamun must be present?

A) One or more Rasogulla can be selected out of 3 Rasogulla in 3 ways.

One or more Gulabjamuns can be selected out of 2 Gulabjamuns in 2 ways.

Hence, Total ways = 6.

→ Find the no. of divisor of 180.

A) $180 \rightarrow 2^2 \times 3^2 \times 5$

Zero or more 2 can be selected out of 2 twos in $(2+1)$ ways

Zero or more 3 can be selected out of 2 threes in $(2+1)$ ways.

Zero or more 5 can be selected out out 1 5 in $(1+1)$ ways.

Hence, total ways
$= 3 \times 3 \times 2$
$= 18$ ways

i.e

null set ← 1, {2}, {3}, {5}, {2,2}, {3,3}, {2,5}, {3,5}, {2,3}, {2,2,3}, {2,2,5}, {3,3,2}, {3,3,5}, {2,3,5}, {2,2,3,3}, {2,2,3,5}, {3,3,2,5}, {2,2,3,3,5}

→ No. of divisor of 180 excluding 1 or the no. itself → 18 − 1
$= 17$

No. of proper divisor of 180
$= 18 - 2 = 16$

→ How many even divisor of 180?

A) $180 = 2^2 \times 3^2 \times 5$

2^2 ↓ one or more 2 can be selected in 2 ways.

3^2 → 3 ways

5 → 2 ways.

Hence, total even divisors
$= 6 \times 2 = 12$ ways.

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→ No. of odd divisor of 180 →

A) One Method

→ $18 - 12 = 6$

Another Method →

$180 = 2^2 \times 3^2 \times 5$

2^2 → neglected

3^2 → 3 ways

5 → 2 ways

Hence, total odd divisors = $\underline{\underline{6}}$

→ find the sum of divisors of 180 →

A) from the set of divisors,

Sum →

$(2^0 + 2^1 + 2^2) + 3 + 2 \times 3 + 2^2 \times 3$
$+ 3^2 + 2 \times 3^2 + 2^2 \times 3^2 + 5 + 2 \times 5 + 2^2 \times 5$
$+ 3 \times 5 + 2 \times 3 \times 5 + 2^2 \times 3 \times 5 + 3^2 \times 5$
$+ 2 \times 3^2 \times 5 + 2^2 \times 3^2 \times 5$

$= (1 + 2 + 2^2) + 3(1 + 2 + 2^2)$
$+ 3^2 (1 + 2 + 2^2) + 5(1 + 2 + 2^2)$
$+ 3 \times 5 (1 + 2 + 2^2) + 3^2 \times 5 (1 + 2 + 2^2)$

$= (1 + 2 + 2^2)(1 + 3 + 3^2 + 5 + 3 \times 5 + 3^2 \times 5)$

$= \underbrace{(1 + 2 + 2^2)(1 + 3 + 3^2)(1 + 5)}$

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$$= \left(\frac{2^3-1}{2-1}\right) \times \left(\frac{3^3-1}{3-1}\right) \times \left(\frac{5^2-1}{5-1}\right)$$

* Sum of divisors (S_1)

$$= (2^0+2^1+2^2)(3^0+3^1+3^2)(5^0+5^1)$$

Now,

Sum of proper divisors

$$= S_1 - (1+180)$$

→ Selection for distinct objects →

R, M, S   G, B

Individual selection

ϕ, {R} = 2

ϕ, {M} = 2

ϕ, {S} = 2

ϕ, {G} = 2

ϕ, {B} = 2

Total Selection $= 2^5$

$${}^5C_0 + {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 2^5$$

OR

$${}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3 \longrightarrow \text{for R, M, S}$$

$${}^2C_0 + {}^2C_1 + {}^2C_2 \longrightarrow \text{for G, B}$$

Total selection done together

$$\rightarrow ({}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3)({}^2C_0 + {}^2C_1 + {}^2C_2)$$

$$= 2^3 \times 2^2 = 2^5 = 32$$

→ In how many ways 3 boys and 2 girls can be selected?

A) 2^5 (previous problem)

Set →

ϕ, {R}, {M}, {S}, {G}, {B}

→ In how many ways 3 boys and 2 girls can be selected?

A) Zero or more boys can be selected out of 3 boys in $^3C_0 + {^3C_1} + {^3C_2} + {^3C_3}$ * $\cancel{^3C_4 * {^3C_6}}$. $= 2^3$ ways

{ϕ}, {R}, {M}, {S},

Zero or more girls can be selected out of 2 girls in $^2C_0 + {^2C_1} + {^2C_2} = 2^2$ ways.

Hence, total selection
i.e when they are selected together $= 2^3 \times 2^2$
$= 2^5$ ways.

→ In how many ways 3 boys & 2 girls can be selected so that in each of the selection at least one boy must be present?

A) $({}^3C_1 + {}^3C_2 + {}^3C_3) \times ({}^2C_0 + {}^2C_1 + {}^2C_2)$
$= (2^3 - 1) \times 2^2 = 28$ ways.

→ In how many ways 3 boys & 2 girls can be selected so that in each of the selection, one girl must be present?

A) $({}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3) \times ({}^2C_1 + {}^2C_2)$
$= (2^3)(2^2 - 1)$
$= \underline{\underline{24}}$

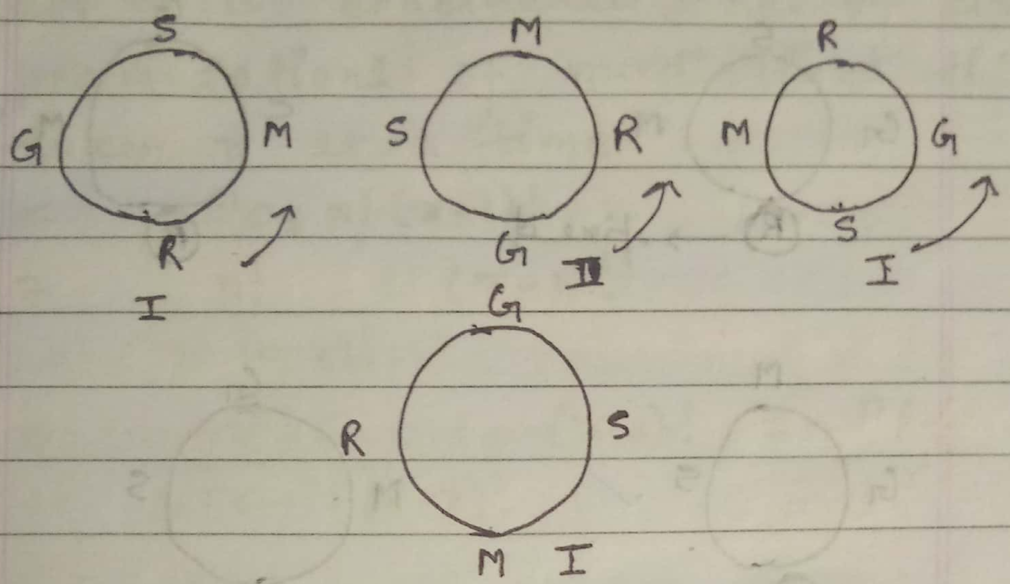
→ In how many ways a committee of ~~2 boy & 1 girl~~ 3 can be formed so that in each of the committee, at least must be present?

× A) $({}^3C_1 + {}^3C_2)({}^2C_1 + {}^2C_2)$

→ 5 stations P, M, K, D, A. Total possible tickets?

→ 5 persons handshake

→ Circular permutations →



RMSG GRMS SGRM MSGR	for 1 circular permutation, no. of linear permutation $= 4$

Let no. of circular permutation of n-distinct things taken all at a time when anti-clockwise or clockwise dir$^{\underline{n}}$ is considered $= x$

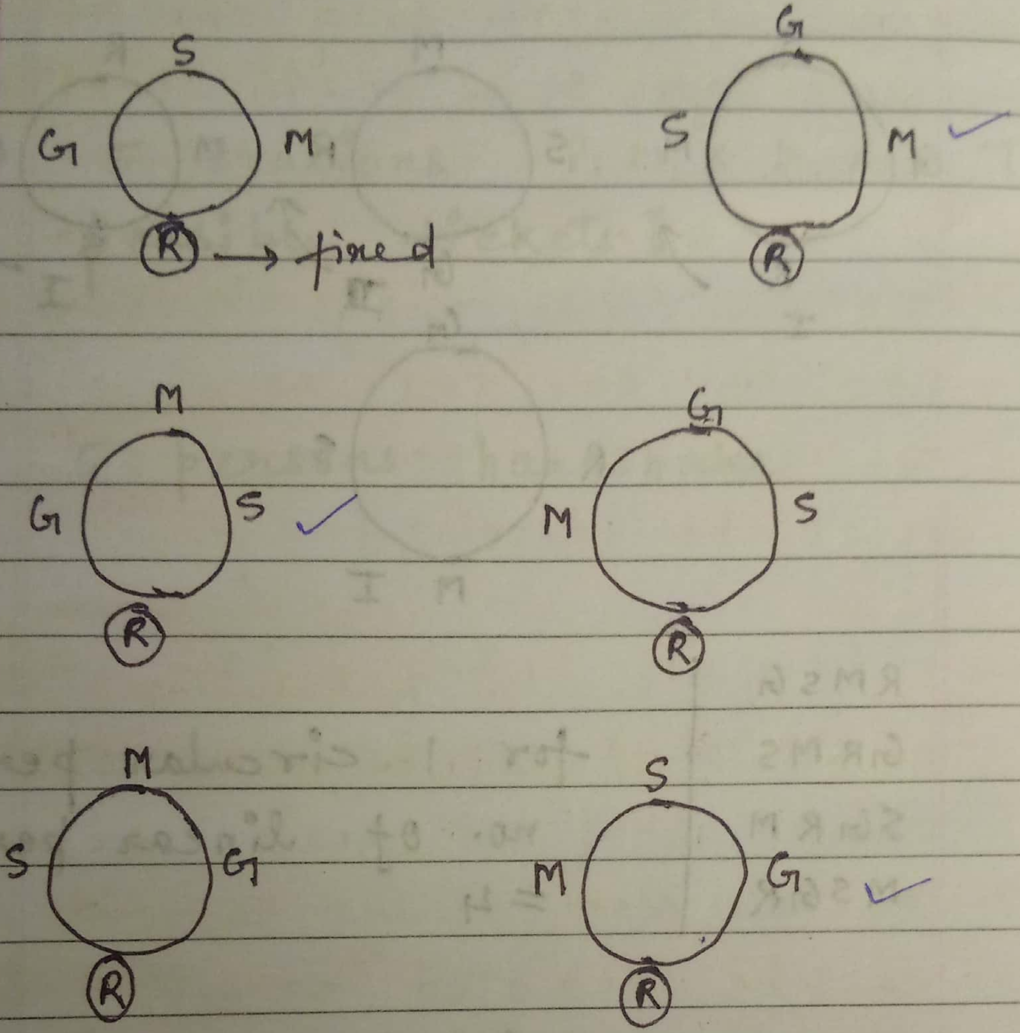
$\therefore$ for x circular permutation, no. of linear permutation $= nx$

NOW,

$$4x = 4!$$

$$\Rightarrow x = \frac{4!}{4} = 3! = 6.$$

Permutations →



→ No. of circular permutation of n-distinct things taken all at a time when clockwise and anti-clockwise dir^{n} is considered $= (n-1)!$ (in general)

→ No. of circular permutation of n-distinct things taken all at a time when clockwise and anti-clockwise dir^{n} is not considered
$= (n-1)!/2$

→ when clockwise & anti-clockwise dir^n is considered, no. of circular permutations of n-distinct things taken r at a time

$$= {}^nC_r \times (r-1)!$$

$$= \frac{n!}{r!\,(n-r)!} (r-1)!$$

$$= \frac{n!}{r\,(r-1)!\,(n-r)!} (r-1)! = \frac{{}^nP_r}{r}$$

→ No. of circular permutations of n-distinct things taken r at a time, when clockwise or anti-clo.

Q) A round-table conference is to be held b/w 20 delegates of 20 countries. No. of ways in which they can be seated if two particular delegates is always to seat together is ____

a) $19! \times 2!$

✓ b) $18! \times 2!$

c) $18!$

d) $19!$

when 2 particular delegates regarding as one person, we have $(18+1) = 19$ persons.

No. of circular permutations of 19 persons $= 18!$

2 particular delegates can be arranged among themselves in $2!$ ways.

Hence, total ways $= 18! \times 2!$

Q) The no. of ways in which 5 men, 5 women & 12 children can sit around a circular table so that children are always together

A) a) $4! \times 4! \times 12!$

b) $11! \times 12!$

✓c) $10! \times 12!$

d) $(12!)^2$

Q) 20 persons are invited for a party. The diff. no. of ways in which they can be seat at a circular table with 2 particular persons seated on either side of the host is

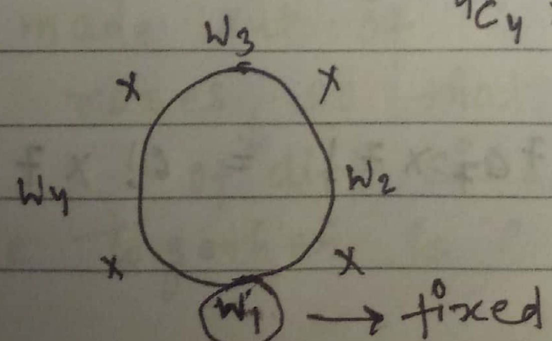
A)
when 2 particular person and the host regarding as one person, we have 19 persons.

No. of circular permutations = 18!
2 particular persons can be arranged on either side of the host in 2! ways.
Ans → 18! × 2!

Q) No. of ways in which 4 men and 4 women sit for a dinner at round table so that no 2 men are to sit together is

A) 4 women can sit around a table in 3! ways.

4 men can be accomodated on 4 cross places in 4! ways.
↓
$^4C_4 \times 4!$



Ans → 3! × 4!

IMP Q) No. of ways in which 7 men can sit around a table so that all shall will not have the same neighbours in any arrangement is

✓ a) 360
b) 720
c) 700
d) 300

A) Hence, $\frac{6!}{2} = \frac{720}{2} = 360$

Q) 7 men and 7 women are to be seated around a circular table so that there is a man on either side of every woman. No. of ways is

a) $(7!)^2$
b) $(6!)^2$
✓ c) $6! \times 7!$
d) $7!$

A) $6! \times {}^7C_7 \times 7! = 6! \times 7!$

Q) The no. of circular permutation of 7 dissimilar things taken 5 at a time is

a) 2550
b) 1260
c) 500
✓ d) 504

A) $\frac{{}^7P_5}{5} = 12 \times 42 = 504$

Q) The no. of ways in which 8 red roses and 5 white roses of diff. sizes can be made to form garlet so that no. 2 white roses come together is

A) $\frac{7!}{2} \times {}^8C_5 \times 5!$

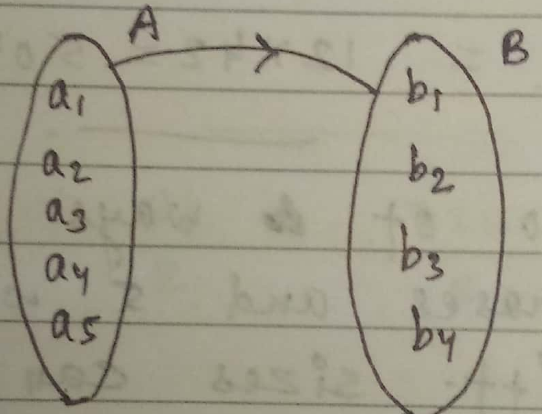
$\because$ clockwise & anti-clockwise are same.

Q) The no. of ways that a garlet can be made out of 4 white and 6 red roses, so that all white roses come together is
of diff. sizes

A) $\frac{6!}{2} \times 4! = 360 \times 24 = 8640$

→ The no. of f^n from set A containing 5 elements to set B containing 4 elements is

A) 4^5



1^{st} element } same
2^{nd} element } image

→ The no. of one-one f^n that can be defined from A to B is
where, $A = \{a, b, c\}$
$B = \{1, 2, 3, 4, 5\}$

A) ${}^5P_3$

→ The no. of many-one f^n from A to B where, $A \equiv \{1, 2, 3\}$
$B \equiv \{a, b, c, d\}$

is

A) $4^3 - {}^4P_3 = 64 - 24 = 40$.

$\underset{\text{total}}{4^3}$ $\underset{\text{no. of one-one } f^n}{{}^4P_3}$

a) 64
b) 24
✓ c) 40
d) None

→ The no. of f^n that can be defined from A to B where, $A \rightarrow \{a, b, c, d, e\}$
$B \rightarrow \{1, 2\}$

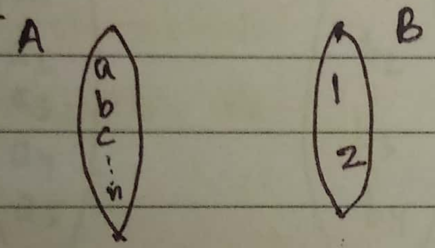
a) 16
✓ b) 32
c) 64
d) None.

→ The no. of one-one onto f^n that can be defined from A to B where $A \to \{a, b, c, d\}$, $B \to \{1, 2, 3, 4\}$

A) 24

→ No. of constant mapping from $A \equiv \{1, 2, 3, \ldots, n\}$, $B = \{a, b\}$ is

A) 2



→ Derangement theorem →
The no. of ways in which n distinct things or n-distinct letters can be put into n- wrong envelope is

$$= n!\left\{1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots + \frac{(-1)^n}{n!}\right\}$$

* Eg →

R	M	S
↓	↓	↓
P	D	K
D	K	P
	P	

A) Proof →

$n(A_1' \cap A_2' \cap A_3' \cap \ldots \cap A_n')$

$= n! - n(A_1 \cup A_2 \cup \ldots \cup A_n)$

$= n! - \{ \sum n(A_1) - \sum n(A_1 \cap A_2)$
$+ \sum n(A_1 \cap A_2 \cap A_3)$
$- \sum n(A_1 \cap A_2 \cap A_3 \cap A_4)$
$+ \ldots + (-1)^{n-1} n(A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n) \}$

$= n! - \{ {}^nC_1 (n-1)! - {}^nC_2 (n-2)!$
$+ {}^nC_3 (n-3)! - \ldots$
$\ldots + (-1)^{n-1} \, {}^nC_1 \cdot 1 \}$

---

* Another way →

a_r → no. of ways in which r letters can be put into r right envelopes $= 1$

U_r → no. of ways in which r letters can be put into r wrong envelopes out of n envelopes.

$a_1 = 1$ $\qquad u_0 = 1$

$a_2 = 1$ $\qquad u_1 = 0$

$a_3 = 1$ $\qquad u_2 = 2! - {}^2c_1\, a_1 u_1 - {}^2c_2\, a_2 u_0$

$\vdots$ $\qquad = 2! - 1$

$a_n = 1$ $\qquad = 1$

$u_3 = 3! - {}^3c_1\, a_1 u_2 - {}^3c_2\, a_2 u_1 - {}^3c_3\, a_3 u_0$

$= 6 - 3 - 1 = 2$

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Q) The no. of triangles formed by joining the angular points of a polygon of n sides

a) ${}^nC_2$
b) ${}^nC_4$
✓ c) ${}^nC_3$
d) None

A) A polygon of n side has n vertices. By joining any 3 of them, a Δ will be formed.
Hence, no. of $\Delta = {}^nC_3$.

Q) A man has 8 children to take them into zoo. If he take 3 of them at a time to the zoo as often as he can without taking same 3 children more than once. The no. of times will he have to go to zoo

a) 54
✓ b) 56
c) 58
d) 60

→ The no. of times a particular children will go

✓a) 21

b) 23

c) 25

d) 27

A) $^7C_2$

→ On a new year day, every student of a class sends a card to every other student. Total cards = 600.

A) $^nC_2 \times 2! = 600$

$\Rightarrow {^nC_2} = 300$

$\Rightarrow \frac{n!}{2!(n-2)!} = 300$

$\Rightarrow n(n-1) = 600$

$\Rightarrow n^2 - n - 600 = 0$

$\Rightarrow n^2 - 25n + 24n - 600 = 0$

$\Rightarrow n = 25$

→ A polygon of m sides ~~to~~. The no. of diagonals by joining any 2 of them is?

A> $^mC_2 - m$

→ Out of 6 gentlemen and 4 ladies, a committe of 5 is to be formed. In how many ways can this be done so as to include 1 lady in each committee?

A> $^6C_1 \times {}^4C_4 +$
$^6C_2 \times {}^4C_3 +$
$^6C_3 \times {}^4C_2 +$
$^6C_4 \times {}^4C_1$
$= \underline{\underline{246}}$

→ There are 10 points in a plane. Of these 10 points, 4 points are in a straight line. With the exception of lines, no other lines are concurrent.
find

(1) No. of triangles formed
(2) No. of straight lines formed
(3) No. of quadrilaterals formed.

A) (1) for a triangle, we need 3 pts,

$$\begin{aligned} & {}^6C_3 \times {}^4C_0 \\ &+ {}^6C_2 \times {}^4C_1 \\ &+ {}^6C_1 \times {}^4C_2 \\ &+ 0 \end{aligned}$$

Another method,

$${}^{10}C_3 - {}^4C_3$$

(ii)

$$\left.\begin{aligned} & {}^6C_2 \times {}^4C_0 \\ &+ {}^6C_1 \times {}^4C_1 \\ &+ {}^6C_0 \times {}^4C_2 \\ & \text{Another method} \rightarrow \\ & {}^{10}C_2 - {}^4C_2 + 1 \\ &= 40 \end{aligned}\right\} \text{no. of lines.}$$

No. of segments $= {}^{10}C_2$

(iii)

$$\begin{aligned} & {}^6C_4 \times {}^4C_0 \\ &+ {}^6C_3 \times {}^4C_1 \\ &+ {}^6C_2 \times {}^4C_2 \\ &= 185 \end{aligned}$$

Another Method →

$${}^{10}C_4 - ({}^4C_4 \times {}^6C_0 + {}^4C_3 \times {}^6C_1)$$
$$= 185$$

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→ In an examination, a min^m is to be secured in each of 5 subjects for a pass. The no. of ways can a student fail is

a) 31

b) 32

c) 33

d) 34

A) $^5C_1 + {^5C_2} + {^5C_3} + {^5C_4} + {^5C_5}$

$= 2^5 - 1$

$= 31$

→ Distribution of things among persons or sets :→

* (m+n+p) distinct things can be distributed among 3 groups or sets containing m, n & p things

$$^{m+n+p}C_m \times {^{n+p}C_n} \times {^pC_p} = \frac{(m+n+p)!}{m!\,n!\,p!}$$

ways.

Q) The no. of ways in which 5 players can be distributed among 2 countries 1 get 2 players and another 3 players is

A) $\frac{5!}{2!\,3!} = 10$

Aus	Eng
S, R, G	K, J
S, K, G	J, R
S, K, J	R, G
K, J, R	S, G
K, J, G	R, S
K, S	G, R, J
K, G	S, R, J
K, R	S, G, J
J, S	R, G, K
J, G	R, S, K

distinct

→ (m+n+p) things can be distributed among 3 persons containing m, n, p things in

$$^{m+n+p}C_m \times {}^{n+p}C_n \times {}^{p}C_p = \frac{(m+n+p)!}{m!\,n!\,p!} \times 3!$$

Q) In how manys ways can 5 books be distributed among 2 persons, one containing 3 books and other 2 books is

A) $\frac{5!}{2!\,3!}(2!) = 20$

Akhilesh	Primlesh
A, V, C	T, D
T, D	A, V, C
A, V, T	C, D
C, D	A, V, T
A, V	T, C, D
T, C, D	A, V
A, T	V, C, D
V, C, D	A, T
V, T	A, C, D
A, C, D	V, T

V, C	A, T, D
A, T, D	V, C
A, C	V, T, D
V, T, D	A, C
C, T	A, V, D
A, V, D	C, T
A, D	V, C, T
V, C, T	A, D
A, C, T	V, D
V, D	A, C, T

→ 3m distinct things can be distributed equally among 3 sets each containing m things in

$$\frac{{}^{3m}C_m \times {}^{2m}C_m \times {}^{m}C_m}{3!} = \frac{(3m)!}{3!\,(m!)^3}$$

Q) The no. of ways in which 4 players can be distributed among 2 countries is

A) $$\frac{{}^{4}C_2 \times {}^{2}C_2}{2!} = \frac{6}{2} = 3$$

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Aus	Eng
S, D	K, J
S, K	D, J
S, J	D, K

→ 3m distinct things can be distributed equally among 3 persons each containing m things in

$$\frac{{}^{3m}C_m \times {}^{2m}C_m \times {}^{m}C_m \times 3!}{3!}$$

$$= (3m)! / (m!)^3$$

Q) The no. of ways in which 4 books can be distributed equally among 2 persons is

A)

Akhilesh	Brimlesh
A, V	T, C
T, C	A, V
A, T	V, C
V, C	A, T
A, C	V, T
V, T	A, C

6 ways

→ (3m+p) distinct things can be distributed among 4 sets each of 3 gets m things and 4th gets 1 thing in

$$\frac{{}^{3m+p}C_m \times {}^{2m+p}C_m \times {}^{m+p}C_m \times {}^{p}C_p}{3!}$$

$$= \frac{(3m+p)!}{(m!)^3\, 3!\, p!}$$

Q) In how manys ways 5 players be distributed among 3 countries each of 2 get 2 players and 3rd gets 1 player?

A) $$\frac{{}^5C_2 \times {}^4C_2 \times {}^3C_1}{2!} = \frac{10 \times 3}{2} = 15$$

Aus	Eng	Pakistan
R, G	J, K	S
R, J	K, G	S
R, K	J, G	S
S, K	G, J	R
S, J	K, G	R

S, G	K, J	R
S, R	J, K	G
S, K	R, J	G
S, J	K, R	G
S, D	G, K	J
S, G	K, D	J
S, K	D, G	J
S, D	G, J	K
S, G	D, J	K
D, G	S, J	K

→ $(3m+p)$ distinct things can be distributed among 4 persons, each of 3 gets m things & 4 th p things in

Q) In how many ways 5 books can be distributed among 3 persons each of 2 gets 2 books and third gets 1 book?

A) $\dfrac{{}^5C_2 \times {}^3C_2 \times {}^1C_1}{2!} \times 3! = 15 \times 6 = 90.$

→ n identical things can be distributed among r persons when any person may get any no. of things in ${}^{n+r-1}C_{r-1}$ ways.

Coefficient of x^n in
$(x^0+x^1+x^2+\ldots+x^n)(x^0+x^1+\cdots+x^n)$
---- to r factors

$=$ coefficient of x^n in
$(1+x+x^2+\cdots+x^n)^r$

$=$ co-efficient of x^n in
$\left(\frac{1-x^{n+1}}{1-x}\right)^r$

$=$ co-efficient of x^n in $\left(\frac{1}{1-x}\right)^r$

$=$ co-efficient of x^n in $(1-x)^{-r}$

$=$ co-efficient of x^n in
$\left(1+{}^{r}C_1 x+{}^{r+1}C_2 x^2+\cdots\right.$
$\left.\cdots+{}^{n+r-1}C_n x^n+\cdots\right)$

$= {}^{n+r-1}C_n$

$= {}^{n+r-1}C_{r-1}$

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$r \quad (r-1)$

* Eg→

No. of integral sol^n of $x+y+z=5$
(+ve)

Ans→ $^{5+3-1}C_{3-1} = {}^{7}C_2 = 21$

0 0 0 | 0 0 | 0 0 | 0 0 0

0 0 | 0 0 | | 0 0 0 | 0 0

(n) · $\quad \underline{1,1+1,1+1} = 5$

$(r-1)$

AABBBCC

AAAAAAA . . n

BBB . . $(r-1)$

AAAB AAAAB $\quad \dfrac{(n+r-1)!}{n!\,(r-1)!} = {}^{n+r-1}C_{r-1}$

→ n-identical things can be distributed among r persons when every person get at least one thing is coefficient of

x^n in $(x^1 + x^2 + x^3 + \dots + x^{n-(r-1)})^r$

i.e co-efficient of x^n in
$x^r (1\cancel{x} + x^2 + x^3 + \dots + x^{n-r})^r$

i.e co-efficient of x^{n-r} in
$(\cancel{x}\ 1 + x + x^2 + \dots + x^{n-r})^r$

i.e co-efficient of x^{n-r} in
$\left(\dfrac{1}{1-x}\right)^r$

$=$ co-eff. of x^{n-r} in $(1-x)^{-r}$

$=$ Coeff. of x^{n-r} in
$\left(1 + {}^{r}C_1 x + {}^{r+1}C_2 x^2 + \dots\dots \right.$
$\left. \dots + {}^{n-1}C_{r+1}\, x^{n-r} + \dots\right)$

$= {}^{n-1}C_{r-1}$

* Eg → No. of integral sol$^{\underline{n}}$ for $x + y + z = 5$
$x \geq 1, y \geq 1, z \geq 1$
is ${}^{n-1}C_{r-1} = {}^{5-1}C_{3-1} = 6$

Another method →

$(x-1)+(y-1)+(z-r)=2$

$\Rightarrow \quad X+Y+Z=2$

$X \geq 0, \quad Y \geq 0, \quad Z \geq 0$

No. of $sol^n = {}^4C_2 = 6$

Q) No. of ways of dividing 80 cards into 5 equal groups of 16 each is

a) $\dfrac{80!}{(16!)^5}$

b) $\dfrac{80!}{(5!)^{16}}$

c) $\dfrac{80!}{(5!)^5}$

✓ d) $\dfrac{80!}{(16!)^5} \times 5!$

A) $\dfrac{{}^{80}C_{16} \times {}^{64}C_{16} \times {}^{48}C_{16} \times {}^{32}C_{16} \times {}^{16}C_{16}}{5!}$

$= \dfrac{80!}{(160!)^5 \times 5!}$

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Date :
Page :

Q) The no. of ways in which 52 cards can be divided among 4 players so that each may have 13 cards.

A) $\underline{{}^{52}C_{13} \times {}^{39}C_{13} \times {}^{26}C_{13} \times {}^{13}C_{13} \times 4!}$

Q) The no. of ways 52 cards can be divided among 4 players in 4 sets, 3 of them having 17 card each and 4th just 1 card

✓ a) $\dfrac{52!}{(17!)^3} \times 4$

b) $\dfrac{52!}{3 \times (17!)^3}$

c) $\dfrac{52!}{3! \times (17!)^3}$

d) $\dfrac{52!}{(3!)^3 \times (17!)}$

A) ${}^{52}C_{17}$

Q) At an election, 3 wards of a town are convased by 4, 5 & 3 men respectively. If 20 men volunteer. The no. of ways they can be alloted to the different wards is

a) $\frac{20!}{3!\,4!\,5!}$

b) $\frac{12!}{3!\,4!\,5!}$

✓ c) $\frac{20!}{3!\,4!\,5!\times 8!}$

d) $\frac{12!}{3!\,4!\,5!\times 8!}$

A) ${}^{20}C_4 \times {}^{16}C_5 \times {}^{11}C_3$

$= \frac{20!}{3!\,4!\,5!\,8!}$

Q) The no. of ways in which 5 things (distinct.) can be divided b/w A and B so that each receive at least 1 thing is

✓ a) 30

b) 60

c) 20

d) 80

$= {}^5C_1 \times {}^4C_4 + {}^5C_2 \times {}^3C_3$
$+ {}^5C_3\, {}^2C_2 + {}^5C_4 \times {}^1C_1$

$= 30$

→ No. of ways in which 12 balls can be divided b/w 2 friends, one receive 8 and other 4 is

A) $\dfrac{12!}{8!\,4!}$

✓ B) $\dfrac{12! \times 2!}{8! \times 4!}$

c) $\dfrac{12!}{8!\,4!\,2!}$

d) $\dfrac{12!}{4!}$

A) ${}^{12}C_4 \times {}^8C_8 \times 2!$

→ The no. of ways in which 13 gold coins can be distributed among 3 persons such that each one get at least 2 gold coin is

A) $x+y+z=13$

$x \geq 2,\ y \geq 2,\ z \geq 2$

$\Rightarrow (x-2)+(y-2)+(z-2)=7$

$\Rightarrow X+Y+Z=7$

$X \geq 0,\ Y \geq 0,\ Z \geq 0$

$^{n+r-1}C_{r-1}$

$= {}^{7+3-1}C_{3-1} = {}^{9}C_{2} = 36$

→ No. of ways in which can 3 sovereigns be given away when there are 4 applicants and any applicants may have either 0, 1, 2 or 3 sovereigns is

a) 15

✓ b) 20

c) 24

d) 48

A) $x+y+z+w=3$

$x \geq 0,\ y \geq 0,\ z \geq 0,\ w \geq 0$

$^{n+r-1}C_{r-1}$

$= {}^{3+4-1}C_{4-1} = {}^{6}C_{3} = 20$

Q) The no. of Integral sol^n of the eq^n $x+y+z+t=29$

$x \geq 1,\ y \geq 2,\ z \geq 3,\ t \geq 0$

is

A) $(x-1)+(y-2)+(z-3)+t=23$

$X+Y+Z+T=23$

$X \geq 0,\ Y \geq 0,\ Z \geq 0,\ T \geq 0$

$^{23+4-1}C_{4-1}$

$^{26}C_3$

$$\frac{\overset{13}{\cancel{26}} \times 25 \times \overset{8}{\cancel{24}}}{\cancel{3} \times \cancel{2}} = 2600$$

Q) The no. of ways in which 12 identical things can go into 5 purses, no purses should be empty is

A) $a+b+c+d+e=12$

$^{12-1}C_{5-1} = {}^{11}C_4$

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IMP → The no. of rational numbers P/q where, $P, q \in \{1,2,3,4,5,6\}$ is

✓ A> 23

B> 32

C> 36

D> 63

A> ~~[illegible]~~

(1,1), (1,2), (1,3), (1,4)

. . . .

Q> A chess tournament where the participants get to play 1 game with another. 2 chess players fell ill having played 3 games each. If the total no. of games played is 84. The no. of ~~ways~~ participants at the beginning was

✓ a> 15

b> 16

c> 20

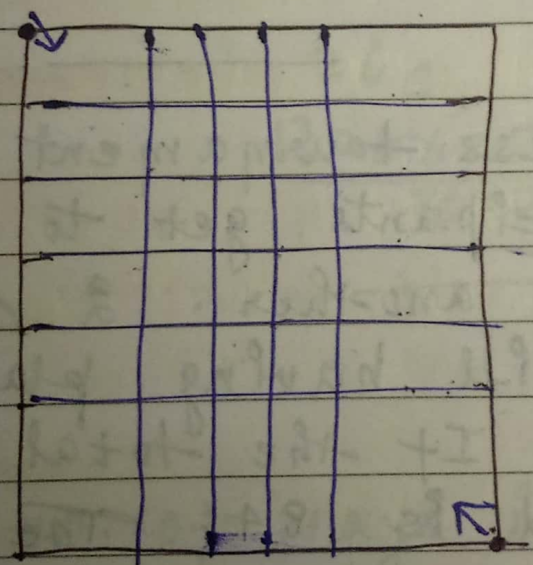
d> 21

A> $^{n-2}C_2 = 84 - 6$

$n = 15$

Q) Streets of a city are arranged like the lines of a chess-board. There are 5 streets running north & south and 6 east & west. The no. of ways in which a man can travel from ~~the~~ northwest to south-east corner going with shortest possible distance is

A)



In order to reach from N-W to S-E, he can take 5 steps in horizontal dir^n and 4 steps in vertical dir^n out of (5+4) steps.

is $\quad {}^9C_5 \times {}^4C_4$

$$= \frac{9!}{4!\,5!}$$

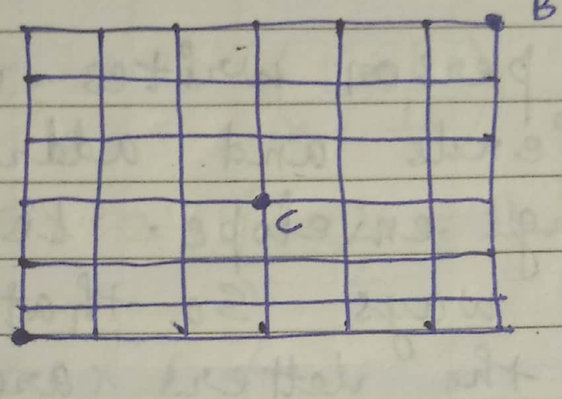
A) $^6C_3 \times {}^3C_3 \times {}^6C_3 \times {}^3C_3$

or

$$\frac{6!}{3!\,3!} \times \frac{6!}{3!\,3!}$$

6x6 square grid.

Q)



ways of going from A to B via C

Q) $f: A \to A$, $A = \{a_1, a_2, a_3, \ldots, a_5\}$ then, no. of one-one f^n so that $f(x_i) \neq x_i$, $x_i \in A$ is

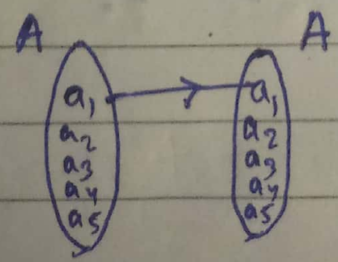
✓ i) 44

ii) 88

iii) 22

iv) 20

A)



Use dearrangement,

$$= 5!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right)$$

$= 44$

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→ The no. of ways that all the letters of the word 'SWORD' such that no letter is in it's original position is

A) 44

→ A person writes letters to 6 friends and addresses co-responding envelope. Let x be the no. of ways so that at least 2 of the letters are in the wrong envelope and y be the no. of ways so that all the letters are in wrong envelope, then, $x-y=?$

A) 716
✓ B) ~~4~~54
C) 264
D) 0

A) $y = 6!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\frac{1}{6!}\right)$

$= 265$

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Another Method

$$x = {}^6C_2 \times 4! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right)$$

$$+ {}^6C_3 \times 3! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right)$$

$$+ {}^6C_4 \times 2! \left(1 - \frac{1}{1!} + \frac{1}{2!}\right)$$

$$+ {}^6C_5 \times 1! \left(1 - \frac{1}{1!}\right)$$

$$= 15 \times 24 \left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right)$$

$$+ 20 \times 6 \,(\quad$$

$x = 6! - 1$

$= 719$

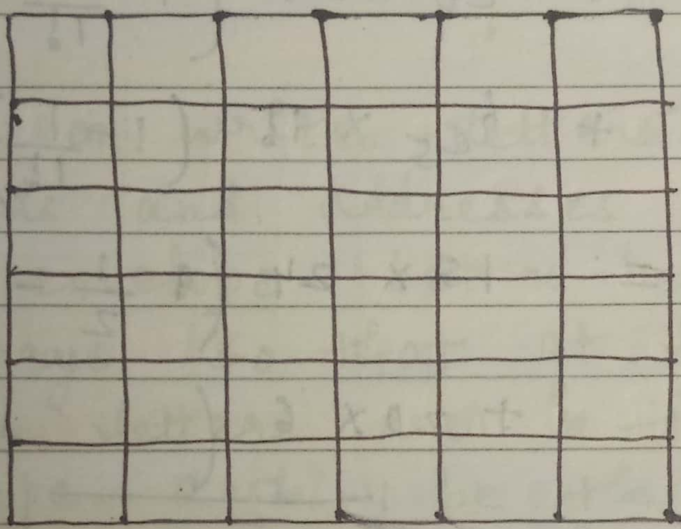
Now, $x - y = 719 - 265$

$= 454$

→ No. of squares of any size in a square of size $n \times n$

$$= \sum_{r=1}^{n} r^2$$

$$= \frac{n(n+1)(2n+1)}{6}$$



horizontal & vertical lines

No. of sides = 7 (horizontally)
(vertically)

No. of 1×1 squares $= (7-1)(7-1)$
$= 6 \times 6$
$= 36$

No. of squares of size 2×2
$= 5^2 = 25$

No. of squares of size 3×3
$= 4^2 = 16$

No. of squares of size 4×4
$= 3^2 = 9$

No. of squares of size 5×5
$= 4$

No. of squares of size 6 × 6

$= 1$

Now,

Total no. of squares

$= 1^2 + 2^2 + 3^2 + \dots + 6^2$

$= \dfrac{6(6+1)(2 \times 6+1)}{6}$

$= 7 \times 13$

$= 91$

* No. of rectangles similarly

$= \sum r^3$

$= \left\{ \dfrac{n(n+1)}{2} \right\}^2$

Proof → ${}^7C_2 \times {}^7C_2$ ⟶ total possible rectangles.

$= \left(\dfrac{7(7+1)}{2} \right)^2$

* No. of rectangle of any size in a rectangle of size $n \times p$

$= {}^{n+1}C_2 \times {}^{p+1}C_2$

$= \dfrac{np}{4} (n+1)(p+1)$

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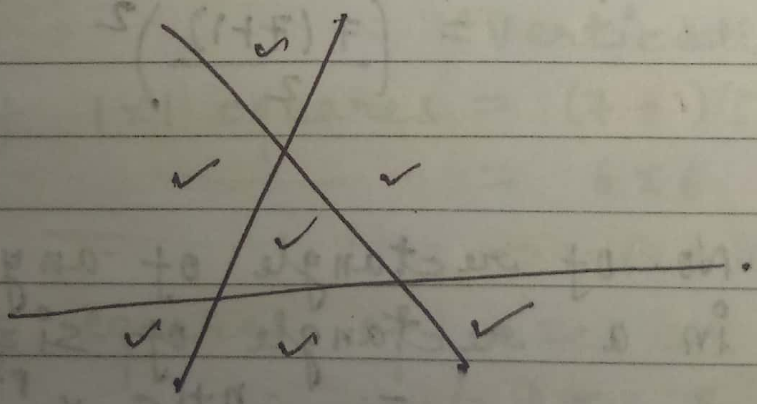
→ No. of squares of any size in a rectangle of size $n \times p$

$$= \sum_{r=1}^{p} (n+1-r)(p+1-r)$$

for $p > n$

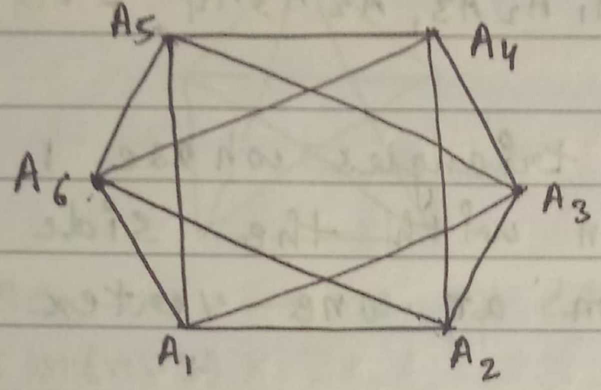
→ n straight lines are drawn in a plane such that no 2 lines are parallel to each other and no 3 of them are concurrent. Then, the no. of parts in which these straight lines divide the plane is $1 + \frac{n(n+1)}{2}$.



$$1 + \frac{3(4)}{2}$$

→ The no. of triangles whose angular points are at the angular points of a given polygon of n sides but none of 2 sides are sides of the polygon is $\frac{1}{6} n(n-4)(n-5)$.

A)



$A_1 A_2 A_3$
$A_2 A_3 A_4$
$A_3 A_4 A_5$
$A_4 A_5 A_6$
$A_5 A_6 A_1$
$A_6 A_1 A_2$
↳ 2 sides common

$\left.\begin{matrix} A_1 A_3 A_4 \\ A_1 A_4 A_5 \end{matrix}\right\}$ for 1 vertex

$6 \times 2 = 12$

Hence, req. no. of triangles
$= {}^6C_3 - 6 - 12$
$= 2.$

A polygon of n side has n vertices, by joining any 3 of them, a $\triangle$ will be formed.
Total no. of $\triangle$'s formed by joining the angular points is ${}^nC_3$.

No. of triangles whose 2 sides are common with the sides of a polygon $= n$

i.e

$(A_1 A_2 A_3, A_2 A_3 A_4, \ldots, A_n A_1 A_2)$

No. of triangles whose 1 side is common with the side of the polygon at one vertex $= n-4$

Total no. of triangles with 1 side common $= n(n-4)$

No. of triangles whose no side is common with the side of the polygon $= {}^nC_3 - n(n-4) - n$

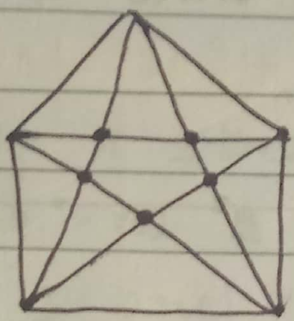
$$= \frac{n(n-1)(n-2)}{6} - n(n-4) - n$$

$$= \frac{n\{n^2-2n-n+2-6n+24-6\}}{6}$$

$$= \frac{n\{n^2-9n+20\}}{6}$$

$$= \frac{n(n-4)(n-5)}{6}$$

→ No. of point of intersection of diagonals of a polygon of n sides which lie completely inside the polygon are ${}^nC_4$.



${}^nC_2 - n \rightarrow$ no. of diagonals

$= \dfrac{n(n-3)}{2}$

${}^5C_2 - 5$

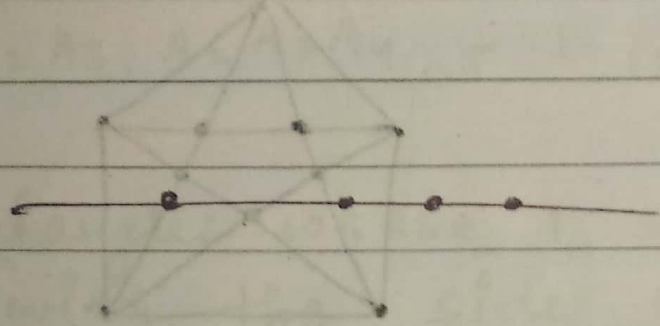
Now,

${}^5C_4 = 5$

↓ no. of point O. I of diagonals.

→ There are n straight lines in a plane no 2 of which are parallel and no 3 of which are concurrent. Their P. O. I are joint, then, the no. of fresh lines thus introduced is $\frac{1}{8} n(n-1)(n-2)(n-3)$.

A) $\underbrace{({}^{n}C_2)C_2}_{\text{Total lines.}} - n\,\underbrace{{}^{n-1}C_2}_{\text{Common lines.}}$



If $n=5$

Hence, $\frac{1}{8}\times 5\times 4\times 3\times 2$

$= 15$

Total lines $= ({}^{5}C_2)C_2 = {}^{10}C_2$

$= 45$

Common lines $= 5\cdot{}^{4}C_2$

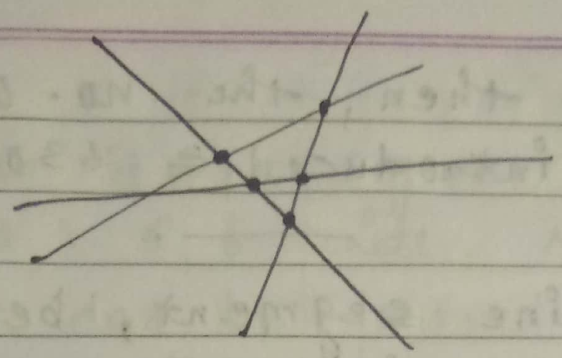
$= 5\cdot\frac{4\times 3}{2}$

$= 30$

Hence, req. no. of lines

$= 45-30$

$= 15$



P.O.I of n straight lines in which no 2 ~~of~~ are parallel and no 3 are concurrent

$$= {}^{n}C_2 = m \text{ (Let)}$$

On each of the n given lines, there are $(n-1)$ points of intersection. By joining any 2 of them, only 1 given line is to be obtained. So, no. of fresh line other than point of intersection

$$= {}^{m}C_2 - n\,{}^{n-1}C_2$$

$$= {}^{({}^{n}C_2)}C_2 - n\,{}^{n-1}C_2$$

→ There are 10 straight lines in a plane, no 2 of which are parallel & no 3 of which are concurrent. The P.O.I are joined, then, the no. of fresh lines introduced = 630

Q> If a line segment, be cut at n points, then the no. of line segments thus formed is

a> $n(n+3)$

b> $\frac{n(n-3)}{2}$

✓ c> $\frac{(n+1)(n+2)}{2}$

d> n

A) 

$^{n+2}C_2$ Total points = $n+2$

Q> There are 15 lines terminating at a point. The no. of angles formed is

✓ a> 105 $^{15}C_2$

b> 120

c> 125

d> None

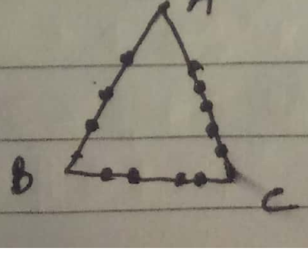
Q) given 5 line segments of length 2, 3, 4, 5, 6 units. No. of triangles that can be formed by joining these points is

a) $^5C_3 - 3$
b) $^5C_3 - 1$
c) $^5C_3$
d) $^5C_3 - 2$

A) $^5C_3 - 3$
$\downarrow$
check by arranging.

Q) The sides AB, BC, CA of a Δ has 3, 4, 5 interior points respectively. No. of Δs formed using these pts as vertices is

A) 205
B) 220
C) 225
D) 230



A) | Selection of pts from AB | Sel. of pts from BC | Sel. of pts from AC |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 1 | 0 |
| 1 | 2 | 0 |
| 0 | 2 | 1 |
| 0 | 1 | 2 |
| 1 | 0 | 2 |
| 2 | 0 | 1 |

$$\left.\begin{aligned}
& {}^3C_1 \times {}^4C_1 \times {}^5C_1 \\
+ \; & {}^3C_2 \times {}^4C_1 \times {}^5C_0 \\
+ \; & {}^3C_1 \times {}^4C_2 \times {}^5C_0 \\
+ \; & {}^3C_0 \times {}^4C_2 \times {}^5C_1 \\
+ \; & {}^3C_0 \times {}^4C_1 \times {}^5C_2 \\
+ \; & {}^3C_1 \times {}^4C_0 \times {}^5C_2 \\
+ \; & {}^3C_2 \times {}^4C_0 \times {}^5C_1
\end{aligned}\right\} \therefore = 405$$

or

$${}^{12}C_3 - ({}^5C_3 + {}^4C_3 + {}^5C_3)$$

→ The no. of triangles where vertices are the vertices of the octagon but none of the sides happen to ~~be~~ come from the sides of the octagon are

A) $\frac{1}{6} \times 8(8-4)(8-5)$

$= \frac{1}{\cancel{6}_{2}} \times 8 \times \cancel{4}^{2} \times \cancel{3} = 16$

→ There are p pts in a space of which ~~two~~ q pts are coplanar, then, no. of planes formed is

A) ${}^{p}C_3 - {}^{q}C_3 + 1$

→ How many +ve unequal integral solⁿ of $a+b+c+d=20$?

A) $a+b+c+d=20$ —— ①

$a \geq 01,\ b \geq 01,\ c \geq 01,\ d \geq 01$

$a \neq b \neq c \neq d$

1, 2, 3, 4, 5, 6, 7, 8, 9

20 19 18 17

a b c d

Ans → 552

~~11 6 2 8 0~~

Let $(a<b<c<d)$

And,

$x = a,\ y = b-a$

$z = c-b,\ t = d-c$

Now,

$y = b-x$	$z = c-(x+y)$	$t = d-(x+y+z)$
$x+y=b$	$c = x+y+z$	$d = x+y+z+t$

Putting the value in ①,

we have,

$$4x+3y+2z+t=20$$

$x \geq 1,\ y \geq 1,\ z \geq 1,\ t \geq 1$

$1 \leq t \leq 11$

Co-efficient of x^{20} in

$$(x^4+x^8+x^{12}+x^{16})(x^3+x^6+x^9+x^{12}+\dots) + \dots$$

$$(x^2+x^4+x^6+x^8+\dots)(x+x^2+x^3+x^4+\dots)$$

$=$ Coefficient of x^{10} in

$$\{(1+x^4+x^8+\dots)(1+x^3+x^6+x^9+\dots)\}$$
$$\{(1+x^2+x^4+x^6+x^8+\dots)(1+x+x^2+x^3+\dots)\}$$

$=$ Coefficient of x^{10} in

$$\{1+x^3+x^6+x^4+x^7+x^{10}+x^8+\dots\}$$
$$\{1+x+\underline{x^2}+\underline{x^3}+\underline{x^4}+\underline{x^5}+\underline{x^6}+\underline{x^7}+\underline{x^8}+\underline{x^9}$$
$$+x^{10}+\underline{x^2}+\underline{x^3}+\underline{x^4}+\underline{x^5}+\underline{x^6}$$
$$+\underline{x^7}+\underline{x^8}+\underline{x^9}+x^{10}$$
$$+\underline{x^4}+\underline{x^5}+\underline{x^6}+\underline{x^7}+\underline{x^8}$$
$$+\underline{x^6}+\underline{x^7}+\underline{x^8}+\underline{x^9}+x^{10}$$
$$+\underline{x^8}+\underline{x^9}+x^{10}+x^{10}+\dots\}$$

$=$ co-efficient of x^{10} in

$$\{1+x^3+x^4+x^6+x^8+x^9+x^7+x^{10}+\dots\}$$
$$\{1+x+2x^2+2x^3+3x^4+3x^5+4x^6+4x^7$$
$$+5x^8+5x^9+5x^{10}+\dots\}$$

$=$ Co-efficient of x^{10} is 23.

Q) Find the total no. of ways of selecting 5 letters and arranging from the letters of the word

'INDEPENDENT'.

A) $\underbrace{I}\ \underbrace{NNN}\ \underbrace{DD}\ \underbrace{EEE}\ P\ T$

Possibility	No. of Combination / arrangement	No. of permutation
3 identical, 1 pair	$^2C_1 \times {}^2C_1 = 4$	$4 \times 5!/3!2!$
2 Pair, 1 distinct	$^3C_2 \times {}^4C_1 = 12$	$12 \times 5!/2!2!$
1 pair, 3 distinct	$^3C_1 \times {}^5C_3 = 30$	$30 \times 5!/2!$
3 identical, 2 distinct	$^2C_1 \times {}^5C_2 = 20$	$20 \times 5!/3!$
All distinct	$^6C_5 = 6$	
	$\cancel{66}$ 72 ↓ Total Combinations.	3320 ↓ Total

NNN EE

NNN DD

EEE NN

EEE DD

→ find the no. of permutations and combinations of the word 'PARALLEL' taken 4 at a time.

A) PARALLEL

E R P AA LLL

Possibility	No. of combination or arrangement	No. of Permutations
3 identical, 1 distinct	${}^1C_1 \times {}^3C_1 = 3$	$2 \times \frac{5!}{3!\,2!}$

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* Another Method :→

co-efficient of x^4 in $4! \left(x^0 + \frac{x^1}{\lfloor 1}\right)$

$$\left(x^0 + \frac{x^1}{\lfloor 1} + \frac{x^2}{\lfloor 2}\right)\left(x^0 + \frac{x^1}{\lfloor 1}\right)$$

$$\left(x^0 + \frac{x^1}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3}\right)$$

$$\left(x^0 + \frac{x^1}{\lfloor 1}\right)$$

= co-efficient of x^4 in $24(1+x)^3$

$$\left(1 + x + \frac{x^2}{2}\right)$$

$$\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right)$$

$=$ Coefficient of x^4 in

$$24(1+x^3+3x^2+3x)\frac{(2+2x+x^2)}{2}\frac{(6+6x+3x^2+x^3)}{6}$$

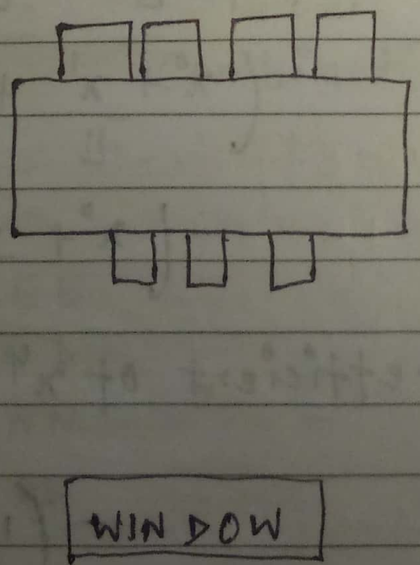
$=$ Co-efficient of x^4 in

$$2(1+x^3+3x^2+3x)(2+2x+x^2)(6+6x+3x^2+x^3)$$

$=$ Coefficient of x^4 is $\underline{286}$

Q) A table has 7 seats, 4 being on 1 side facing the window and 3 being on the opposite side. In how many ways can 7 people be seated at the table, if 3 people x, y & z must sit on the side facing the window?

A)



$^4C_3 \times 3! \times {}^4C_4 \times 4!$

$24 \times 6 \times 4$

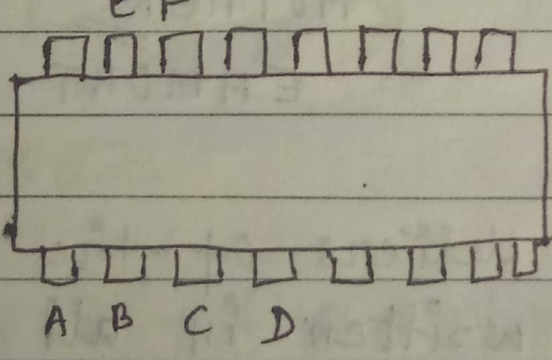
$= 24 \times 24$

$= 576$

3 persons X, Y, Z who wish to sit on the side facing the window may be accomodated on 4 chair in $^4C_3 \times 3!$ ways.

Remaining 4 persons may be accomodated on remaining 4 chairs in $^4C_4 \times 4!$ ways.

Q) A tea party is arranged for 16 people along 2 sides of a long table with 8 chairs on each side. 4 men wish to sit on one particular side & other 2 on the other side. In how many ways they can be seated?

A) 

$$^8C_4 \times 4! \times {}^8C_2 \times 2! \times {}^{10}C_{10} \times 10!$$
$$= {}^{10}C_4 \times {}^6C_6 \times 8! \times 8!$$

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Q) 8 chairs are numbered 1 to 8, two women ~~choose the c~~ and 3 men wish to occupy 1 chair each. First, the 2 women chose the chairs from the chairs marked 1 to 4. Then, men select the chairs from the remaining chairs. Find the no. of possible arrangements.

A) $^4C_2 \times 2! \times {}^6C_3 \times 3!$
$= 12 \times 20 \times 6$
$= 1440.$

Q) Find the rank of 'MOTHER' when dictionary is made from the letters of the word.

A) MOTHER   Ans → 309

EHMORT

Q) The letters of the word 'SURITI' are written in all possible orders as in a dictionary. Find the rank of the word 'SURITI'?

A) IIRSTU $\frac{5!}{2!}$

I $\quad 5! = 120$

II $\quad$ ~~5! = 120~~

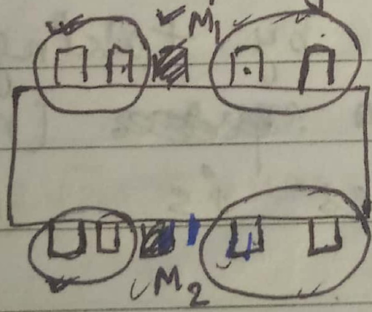
R $\quad \frac{4!}{2!} = 12$

Ans → 236

S $\quad \frac{4!}{2!} = 12$

Q) There are 8 guests at a dinner party. If the master and the mistress of the house have a fixed seat opposite to one another and there are 2 specific guests who must not be placed next to one another. Find the no. of ways in which company can be placed.

A)



$$8! - ({}^4C_1 \times 2! \times 6!)$$

$$= \lfloor 6 \times 48$$

Ans → 48

---

* Greatest.

IMP Q) $3x + y + z = 24$

$3k + y + z = 24$

$\Rightarrow \quad y + z = 24 - 3k$

No. of non-negative integral sol^n $= {}^{24-3k+2-1}C_{2-1} = 25 - 3k$

No. of non-negative integral sol^n $= \sum_{r=0}^{8} 25 - 3\sum_{r=0}^{8} k = 225 - 108$
$= 117$

IMP * A square of n units by n units is divided into n^2 squares each of area 1 unit^2. The no. of ways by taking 4 points out of $(n+1)^2$ vertices of n squares is $\frac{n^2(n+1)}{2}$

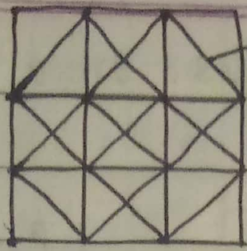
Q) No. of squares by taking 4 points as a vertices of a square in a square of size 3×3 is

a) 17

✓ b) 18

c) 19

d) 20

 → included.

i.e 1×1 square + 2×2 square + 1×1 square + $\underbrace{\sqrt{2}\ \text{unit}^2 \text{ area square}}_{\text{IMP}}$

→ By taking 4 points, from a square of size 4×4 to form a square is

A) 38
B) 39
✓ C) 40
D) 41

(check $2\sqrt{2}$ squares)

→ The no. of rectangles excluding squares from a rectangle of size 9×6 is

A) $\underbrace{{}^{10}C_2 \times {}^{7}C_2}_{\text{Total no. of squares + rectangles}} - \underbrace{\sum_{r=1}^{6} (6-r)(9-r)}_{\text{No. of squares}}$

Ans → 791

→ No. of rectangles on a chess board is

a) 1296
b) 204
c) 1292
✓d) 1092

$$({}^{9}C_2)^2 = 36 \times 36$$

$$({}^{9}C_2)^2 - \{(9-1)(9-1) + (9-2)(9-2) + (9-3)(9-3) + (9-4)(9-4) + (9-5)(9-5) + (9-6)(9-6) + (9-7)(9-7) + (9-8)(9-8)\}$$

$$\frac{9 \times 8^{4}}{2}$$

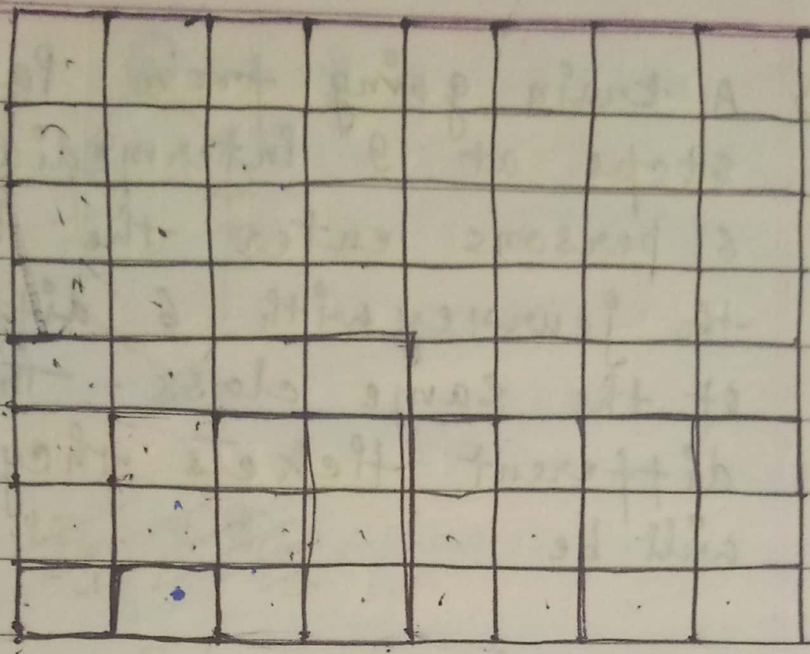
$$= (36 \times 36) - \{64 + 49 + 36 + 25 + 16 + 9 + 4 + 1\}$$

Q) The no. of rectangles on a chess board which are not squares is (above sol^n)

IMP Q) The no. of ways of choosing 2 squares from a chess board such that they have a side in common is

of 1×1

a) 224 b) 112 c) 56 d) 68



Try counting for different types of squares.

$7\times7 + 5\times5 + 3\times3 +$

for 1×1 for 2×2 for 3×3

→ The no. of products that can be formed with 8 prime nos. is

✓a) 247

b) 25[illegible]2

c) 5

d) 248

$${}^8C_2 + {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8$$

$$= 2^8 - 9$$

$$= 247$$

Q) A train going from Patna to Delhi stops at 9 intermediate stations. 6 persons enter the train during the journey with 6 different tickets of the same class. The no. of different tickets they may have will be

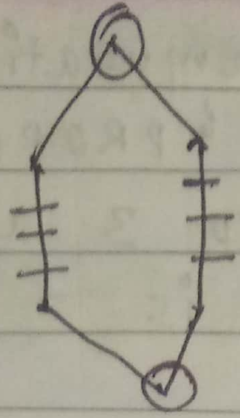
a) ${}^{4}C_6$
✓ b) ${}^{45}C_6$
c) ${}^{9}C_6$
d) ${}^{10}C_6$

P x x x x x x x x x D

$${}^{10}C_2 \; C_2 = {}^{45}C_2$$

+ Q) A crew of an 8 oar baat is to be chosen of 12 men of whom 3 can row in stroke side only. The no. of crew can be arranged is.

a) ${}^{9}C_4 \cdot \times {}^{8}C_4$
b) ${}^{9}C_4 \times {}^{8}C_4 \times 4! \times 4!$
c) ${}^{12}C_8 \times {}^{9}C_4 \times 4! \times 4!$
d) ${}^{12}C_4 \times {}^{8}C_4 \times$



$\cancel{{}^{12}C_8 \times {}^{8}C_6}$

$${}^{9}C_4 \times {}^{5}C_4 \times 4! \times 4!$$

Q) A boat crew consists of 8 men 3 of whom can only row on 1 side and 2 only on the other. find the no. of ways in which crew can be arranged?

A) [illegible] $\;{}^{3}C_1 \times {}^{2}C_2 \times 4! \times 4!$

$= 3 \times 24 \times 24$

$= 3 \times 576$

Q) A boat crew consists of 8 men 3 of whom can only row on one side & 2 on the other side. find the no.

→ The no. of permutation of the letters of the word 'PROPORTION' taken 4 at a time so 3 are alike and 1 is different is

A) PP RR OOO T I N

3 identical , 1 different

1 $\quad$ $5C_1$

$1 \times 5C_1 \rightarrow$ Combination

$\frac{5C_1}{1} \times \frac{4!}{3!} = 20$

→ Permutation

→ 3^r divides $\lfloor 50$, then, max^m value of r = ?

a) 27 b) 25 c) 23 ✓d) 22

A) $\left[\frac{50}{3}\right] + \left[\frac{50}{9}\right] + \left[\frac{50}{27}\right]$

$= 16 + 5 + 1$

$= 22$

→ The no. of ways in which 5 identical balls can be kept in 10 identical boxes if not more than 1 can go into a box is

a) $^{10}P_5$ b) $^{10}C_5$ c) 5 ✓d) 1

Q) The no. of dividers of 3630 which have a remainder of 1 ~~and~~ when divisible by 4 :→

(5)

A) ~~[illegible]~~

(4m+1)

3) 3630 (1210

$$\begin{array}{r|l} 3 & 3630 \\ 2 & 1210 \\ 11 & 605 \\ 11 & 55 \\ & 5 \end{array}$$

~~[3630/3] + [3630/9]~~

~~[illegible]~~

$3630 = 3 \times 2 \times 5 \times 11^2$

5 ; (11, 3) ; (11, 11) ;

~~(11, 3, 5) ; (11, 11, 5)~~

$^{x}C_2 = 84 - 3$

6

~~$^{x-2}C_2 = 81$~~

Q) There are 10 bags $B_1, B_2, \ldots, B_{10}$ which contains 21, 22, ..., 30 diff. articles. The total no. of ways to bring out 10 articles from a bag is

$$\frac{(x-2)!}{2!\,(x-4)!} = 81$$

$(x-2)(x-3) = 162$

$x^2 - 5x + 6 = 162$

a) $^{31}C_{20} - {}^{21}C_{10}$

b) $^{31}C_{21}$

c) $^{31}C_{20}$

d) $^{32}C_{20}$

(use ~~$^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$~~)

A) $^{21}C_{10} + {}^{22}C_{10} + \ldots + {}^{30}C_{10}$

$= {}^{31}C_{11} - {}^{21}C_{10}$

$= {}^{31}C_{20} - {}^{21}C_{10}$