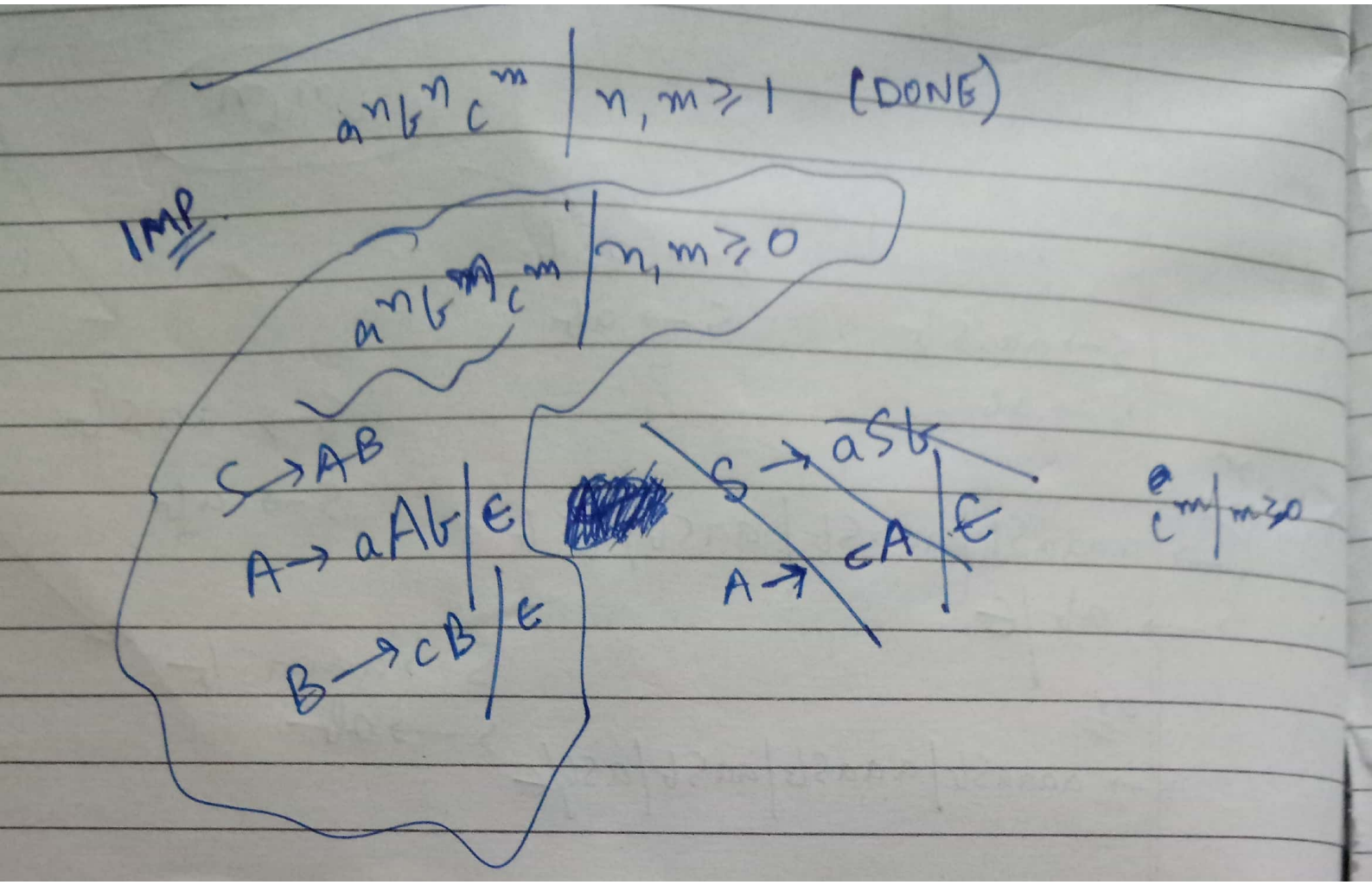


IMP

$a^n c^m b^n / n, m \geq 0$

$S \to aSb / aAb / A$

$A \to cA / \epsilon$

for inserting c in b/w

$S \to aAb$

$a^n c^m b^n / n, m \geq 1$

$S \to aSb / aAb$

$A \to cA / c$

$a^n c^m b^n / n \geq 0, m \geq 0$

$S \to aSb / aAb$

$A \to cA / \epsilon$

$a^n c^m b^n / n \geq 0, m \geq 1$

VVI

$S \to aSb / aAb / A$

$A \to cA / c$

$a^n b^m c^m / n \geq 0, m \geq 1$

$S \to AB$

$A \to aAb / \epsilon$

$B \to cB / c$

$a^n b^n c^m / n \geq 1, m \geq 0$

$S \to AB$

$A \to aAb / ab$

$B \to cB / \epsilon$

$L = \{ a^n b^{2n} c^m / m, n >= 0 \}$

$= \{ a^n b^n b^n c^m / m, n \geq 0 \}$

$A \quad B$

$S \to AB$

$A \to aAb / \epsilon$

$B \to bD / \epsilon$

$D \to bE / \epsilon$

$E \to cE / \epsilon$

$L = \{a^n b^m c^m d^m \mid n, m \geq 1\}$

$A \to aAb \mid ab$

$B \to cBd \mid cd$

$S \to AB$

$L = \{a^n b^m c^m d^m \mid n, m \geq 0\}$

$A \to aAb \mid \epsilon$

$B \to cBd \mid \epsilon$

$S \to AB$

$L = \{a^n b^{2n} \mid n \geq 1\}$

$S \to aSbb \mid abb$

$L = \{a^n b^{2n} \mid n \geq 0\}$

$S \to aSbb \mid \epsilon$

$L = \{a^n b^m \mid n = m-1\}$

$\Rightarrow m = n+1$

$L = \{a^n b^{n+1}\}$

$L = \{a^n b^n b\}$

$L = \{a^n b^n\} b$

$S \to Ab$

$A \to aAb \mid \epsilon$

$\forall n, m \geq 0$ → This page

$L = \{a^n b^m \mid n = m+3\}$

$\Rightarrow L = \{a^{m+3} b^m\}$

$= a^3 a^m b^m$

concatenate

i.e. →

$S \to aaaA$

$A \to aAb \mid \epsilon$

2nd Approach.

How to generate grammar for union of languages? ??

$L = \{a^n b^m \mid n \leq m+3\}$

$\Rightarrow L = \{a^3 a^m b^m\} \cup \{a^2 a^m b^m\}$

$\cup \{a\, a^m b^m\} \cup \{a^m b^m\}$

$\cup \{a^m b^{m-1}\} \cup \{\ldots\}.$

$n = m+3$

$n = m+2$

$n = m+1$

$n = m$

$n = m-1$

$n = m-2$

$n = 0.$

$L = \{a^n b^m c^{n+2m} \mid n, m \geq 1\}$

$= \{a^n b^m c^m c^n \mid n, m \geq 1\}$

$= \{a^n b^m c^{m'} c^n \mid m, m' \geq 1\}$

$= \{a^n b^m c^{m+m'} c^n \mid m, m' \geq 1\}$

$= \{a^n b^m c^{m} c^n \mid m \geq 1\}$ ①

check!

① $B \to Acc$

$A \to bAc \mid bc$

$S \to aSc \mid aBc$

$S \to aSd \mid aAd$

$A \to bAc \mid bc$

$L = \{a^n b^m c^m d^n \mid n, m \geq 1\}$ → Smallest string abcd.

$S \to$

~~$B \to aBd \mid ad$~~

$A \to bSc \mid bc$

$L = \{a^n b^m c^m d^n \mid n, m \geq 0\}$ → Smallest string ad, bc, abcd

$S \to aAd \mid aAd \mid A$

$A \to bAc \mid \epsilon$

$L = \{a^{m+n} b^m c^n \mid m, n \geq 1\}$

$\Rightarrow L = \{a^n a^m b^m c^n \mid m, n \geq 1\}$

$S \to aSc \mid aAc$

$A \to aAb \mid ab$

Similarly,

$L = \{a^n b^m c^{n+m} \mid n, m \geq 1\}$.

$L = \{a^n b^m c^{n+m} \mid n, m \geq 0\}$.

~~for~~

Q $L = \{a^n b^{m+n} c^m \mid n, m \geq 1\}$

$= \{a^n b^n b^m c^m \mid n, m \geq 1\}$

$= \{a^n b^n \; b^m c^m\}$

$S \rightarrow AB$

$A \rightarrow aAb \mid ab$

$B \rightarrow bBc \mid bc$

Pending.

$L = \{a^n b^m c^{n+2m}, n, m \geq 0\}$

$L = \{a^n b^{m+n} c^m \mid n, m \geq 0\}$

$= \{a^n b^n \; b^m c^m \mid n, m \geq 0\}$

$S \rightarrow AB$

$A \rightarrow aAb \mid \epsilon$

$B \rightarrow bBc \mid \epsilon$

$S \rightarrow$

$B \rightarrow$

$A \rightarrow bAc \mid \epsilon$

$L = \{a^{m+n} b^m c^n \mid n, m \geq 0\}$

$= \{a^n \; a^m b^m \; c^n \mid n, m \geq 0\}$

$S \rightarrow aSc \mid aAc \mid A$

$A \rightarrow aAb \mid \epsilon$

ϵ

ab

ac

} Smallest strings

$\forall m, n \geq 1$

$L = \{a^n b^m c^m cc\, c^n\}$

$= \{a^n b^m c^{m+n+2}\}$

$S \to aSc \mid aBc$

$A \to bAc \mid bc$

$B \to Acc$

check (DONE)

$L = \{a^n b^m c^{n+2m} \mid n, m \geq 1\}$

$= \{a^n b^m c^m c^n \mid n, m \geq 1\}$

order doesn't matter ? true

$S \to aScB \mid aAcB$

$A \to bAc \mid bc$

$B \to cB \mid c$

smallest string

$abccc$

$L = \{c^{2n} \mid n \geq 0\}$

$A \to ccA \mid \epsilon$

$L = \{\epsilon, cc, cccc, \ldots\}$

$L = (cc)^*$

next step:

PENDING

$L = \{a^n b^m c^{n+2m} \mid n, m \geq 0\}$

$= \{a^n b^m c^m c^n c^m \mid n, m \geq 0\}$

$S \to aScB \mid aAcB \mid \epsilon$

$A \to bAc \mid \epsilon$

$B \to cB \mid \epsilon$

Smallest strings:-

ϵ

ac,--

bcc,

↳ when $n=0$.

↳ $\{ab^m c^{2m}\}$.

aSc

aAc

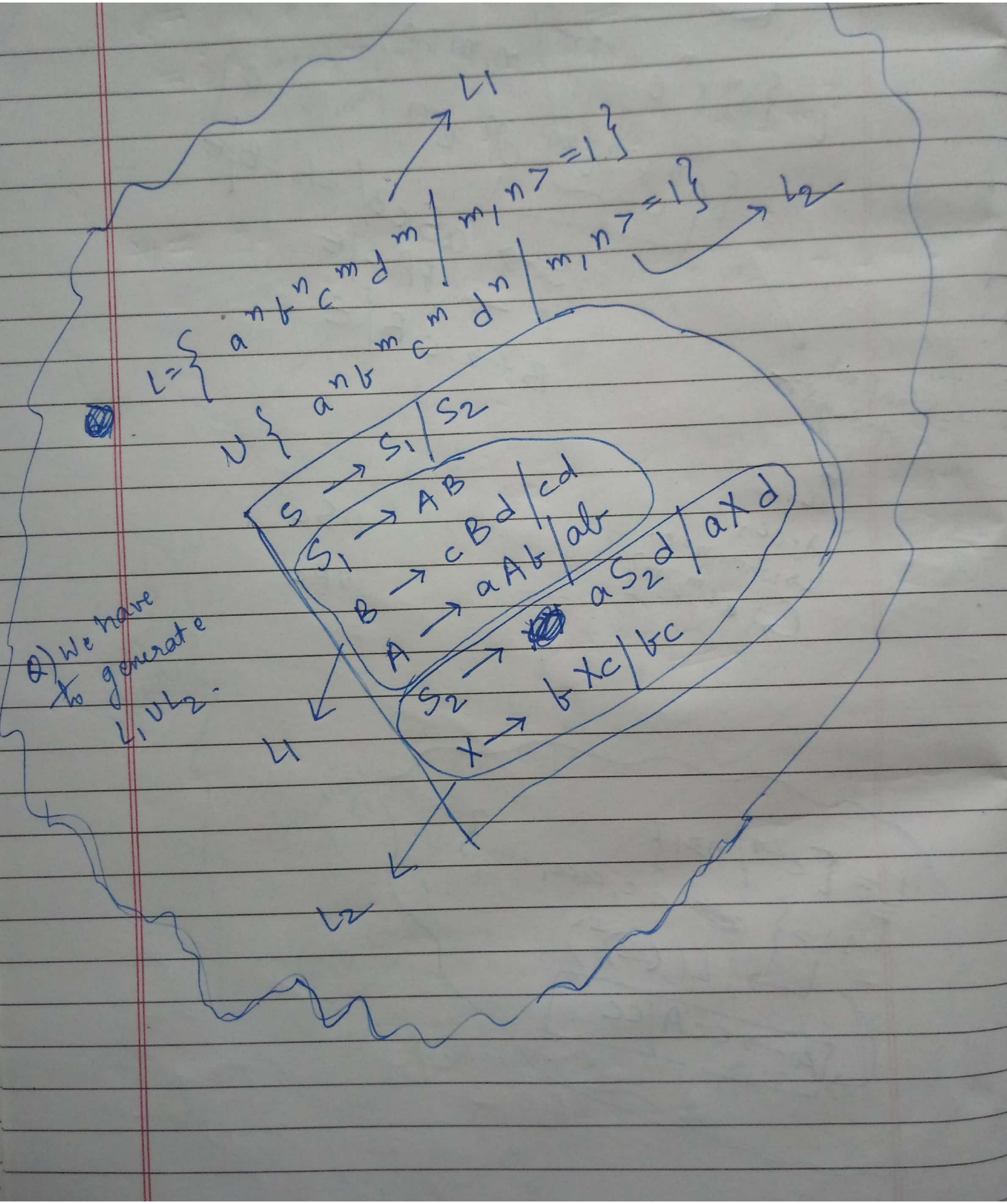
$L = \{c^{2n} \mid n \geq 1\}$

$= \{cc, cccc, \text{---}\}$

↳ $L = \{$ $cc, cccc, \text{---}\}$

↳ $L((cc)^+)$

$A \to ccA \mid cc$



$$L = \{a^l b^m c^n \mid l = m \text{ or } m = n\}$$

$$= \{a^l b^m c^n \mid l = m\} \cup \{a^l b^m c^n \mid m = n\}$$

$$= \{\underbrace{a^m b^m}_{L_1} c^n\} \cup \{a^l \underbrace{b^m c^m}_{L_2}\}.$$

In our problem, if $l, m, n \geq 1$

then,

$S \to S_1 \mid S_2$

$S_1 \to AB \mid ab$ → L_1

$A \to aAb \mid ab$

$B \to cB \mid c$

$S_2 \to CD$ → L_2

$C \to aC \mid a$

$D \to bDc \mid bc$

& if $l, m, n \geq 0$

$S \to S_1 \mid S_2$

$S_1 \to AB$

$A \to aAb \mid \epsilon$

$B \to cB \mid \epsilon$

$S_2 \to CD$

$C \to aC \mid \epsilon$

$D \to bDc \mid \epsilon$

* Conditions on n & m →

PENDING

$L = \{ a^l b^m c^n \mid n \le l \text{ or } n \le m \}$

$= \{ a^l b^m c^n \mid n \le l \} \cup \{ a^l b^m c^n \mid n \le m \}$

* If $l, m, n \ge 1$,

$L = \{ a^l b^m c^n \mid n \le l \} \cup \{ a^l b^m c^n \mid n \le m \}$

$= \{ a^l b^m c^n \mid l \ge 1, n \ge 1, n \le l, m \ge 1 \} \cup \{ a^l b^m c^n \mid n \ge 1, n \le m, m \ge 1, l \ge 1 \}$

INDEPENDENT VARIABLE

$l, m, n \geq 0$

$L = \{ a^l b^m c^n \mid n \leq l,\ n \geq 0,\ l \geq 0,\ m \geq 1 \}$ } independent variable

$l \geq n$

when $n = 0$, l takes the value (1)

Hence, Minimum value l can take is 1. (i.e when $n=0$)

$\Rightarrow L = \{ a^l b^m c^n \mid l \geq 1, n \geq 0, m \geq 1 \}$

~~$S \rightarrow aS_1 c / \epsilon$~~

$A \rightarrow bA / b$

$S \rightarrow aS_1 c / \epsilon$

~~$S \rightarrow S_1 A S_1$~~

Hence, Answer for L_1

$S \rightarrow S_1 aSc / S_1 aAc / ab$

$A \rightarrow bA / b$

$S_1 \rightarrow aS_1 / a$

$S \rightarrow S_1 aSc / S_1 aAc$

$A \rightarrow bA / b$

$S_1 \rightarrow aS_1 / a$

beech mein 'b' kodalna hai

side se 'a' aur 'c'

Remember

:: not included

for increasing the no. of a's. (by at least 1)

Smallest string $\rightarrow ab$

$aabc$

$aabbc$

$L_2 = \{a^l b^m c^n / n \geq 0, n \leq m, m \geq 0\}$

$l \geq 1$ → independent variable.

$m \geq n$

$m \geq 0$
$n \geq 0$

→ Here, Min^m value m takes is 1
i.e when $n = 0$

Hence,

$L_2 = \{a^l b^m c^n / m \geq 1, n \geq 0, l \geq 1\}$ (Use S_2 & b for this part)

$B \rightarrow aB/a$.

$S \rightarrow BS_2$

$S_2 \rightarrow MS_3$

$S_3 \rightarrow bS_3c/\epsilon$

$M \rightarrow bM/b$

$B \rightarrow aB/a$.

↳ Answer for L_2.

Now,

S_2 is for

$\{b^m c^n / m \geq 1, n \geq 0\}$

$S_2 \rightarrow MS_3$

$S_3 \rightarrow bS_3c/\epsilon$

$M \rightarrow bM/b$

↓

Using it in our question.

$L = L_1 \cup L_2$

Now,

$S \rightarrow S_1aSc/S_1aAc/S_1S_2/ab$

$S_1 \rightarrow aS_1/a$

$A \rightarrow bA/b$

$S_2 \rightarrow AS_3$

$S_3 \rightarrow bS_3c/\epsilon$

Solution

Problem:- Grammar for an infinite union of languages?

$m, n \geq 0$

→ for the time being, convert it to PDA & then to grammar

(??)

$L = \{a^n b^m \mid n \geq m\}$

$= \{a^n b^n, n \geq 0\} \cup \{$

$n \geq 0 \quad m \geq 0$

$n = m$

$n = m+1$

$n = m+2$

$\vdots$

$n, k \geq 0$

$L = \{a^n b^n c^k \mid k \geq 3\}$

$\Rightarrow L = \{a^n b^n c^k \mid n \geq 0 \text{ & } k \geq 3\}$

$L = \{a^n, n \geq 3\}$

$L = \{aaa, aaaa, \ldots\}$

R.E = aaa(a*)

S → aaaA

A → aA | ∈

S → AB

A → aAb | ∈

B → ccc X

X → cX | ∈

$n, m, k \geq 0$

* $L = \{a^n b^m c^k \mid k = |n-m|\}$

$n, m \geq 0$

if $m=0$, then $(n=1)$ → min n value n can take when $n > m$.

→ $L = \{a^n b^m / n \neq m\}$ ..... ab initio

L_1 → $n > m$

L_2 → $n < m$ (Similarly)

① a) We first generate an equal no. of a's & b's

b) & then, add extra a's on the left.

by analogy

$S \to S_1 B$

$S_1 \to a S_1 b / \epsilon$

$B \to bB / b$ → this can be right linear or left linear

$L = \{a^n b^m / n \geq m\}$

$S \to AS_1$ ✓

$S_1 \to a S_1 b / \epsilon$

$A \to aA / a$ ✓

↳ a min m of 1a is required before it.

doesn't matter

$L = \{a^n b^m / m \geq 0, n \geq 1\}$ (VVI).

Now,

$S \to A S_1$

↓

generates equal no. of a & bs. i.e $a^n b^n$ } ∵ $S_1 \to a S_1 b / \epsilon$

Hence,

$A \to aA / a$

↓

for adding extra a's

then,

$\{a \ldots\} a^n b^n$

↓

extra a's must be added before it.

GENERATING GRAMMARS FOR INFINITE LANGUAGES (1st Example)

Q) Construct a CFG for language in which there are no consecutive b's, the string may or may not have consecutive a.

A)

Now, $\because L = \{ a^n b^m, n > m \} \cup \{ a^n b^m, m > n \}$

$= L_1 \cup L_2$.

Hence, we have,

$S \rightarrow A S_1 / S_1 B$

$S_1 \rightarrow aS_1b / \epsilon$

$A \rightarrow aA / a$

$B \rightarrow bB / b$.

In our previous problem, What if?

$n, m \geq 1$

then, for $n > m$

min. value n can take is 2

$\because$

for $m = 1$

$n = 2, 3, 4, \ldots$

Hence,

for,

$$L_1 = \{ a^n b^m \mid n > m \}$$

$$= \{ a^n b^m \mid n \geq 2 \text{ & } m \geq 1 \}$$

Hence,

$$S \rightarrow AS_1$$
$$S_1 \rightarrow aS_1b \mid \epsilon$$
$$A \rightarrow aA \mid aa$$

$\rightarrow L_1$

& $L_2 = \{ a^n b^m \mid m > n \}$

$= \{ a^n b^m \mid m \geq 2 \text{ & } n \geq 1 \}$

$$S \rightarrow S_1B$$
$$S_1 \rightarrow aS_1b \mid \epsilon$$
$$B \rightarrow bB \mid bb$$

L_2

$L = L_1 \cup L_2$

$\downarrow$

$$S \rightarrow AS_1 \mid S_1B$$
$$S_1 \rightarrow aS_1b \mid \epsilon$$
$$B \rightarrow bB \mid bb$$
$$A \rightarrow aA \mid aa$$

Also,
note that,
while taking union for grammars,

2 common productions are taken once

↓

Union of productions.

But,
while taking Union of 2 sets of productions

↓

(which is required while taking union of 2 languages).

↓

We take the starting Variable as common & use '/' symbol to separate all RHS.

↓

Then, after that →

Also,

We could also have written it like this :-

$S \rightarrow S_3 / S_4$

$S_3 \rightarrow A S_1$

$S_1 \rightarrow a S_1 b / \epsilon$

$A \rightarrow aA / a$

$S_4 \rightarrow S_1 B$

$S_1 \rightarrow a S_1 b / \epsilon$

$B \rightarrow bB / b$

Redundant productions.

for $L = \{ a^n b^m / n \neq m \}$

Unit productions:

$S \rightarrow S_3 / S_4$

Useless productions:

$S_3 \rightarrow A S_1$

$S_4 \rightarrow S_1 B$

$S_1 \rightarrow a S_1 b / \epsilon$

$A \rightarrow aA / a$

$B \rightarrow bB / b$.

(Redundant productions eliminated).