



Chapter 5

Introduction

Image Processing and Computer Vision

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Sources of noise

① Environmental conditions during image acquisition.

- Light level
- Sensor temperature

② Transmission

- Signal interference
- Quality of transmission channels

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Gaussian probability density function

For each pixel in $I(u, v)$, the noise value z is drawn from a Gaussian probability density function.

$$p(z) = \frac{1}{\sqrt{(2\pi)\sigma^2}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}$$

- ① μ : the mean of noise values, i.e., z
- ② σ : the standard deviation.
- ③ σ^2 : the variance of z

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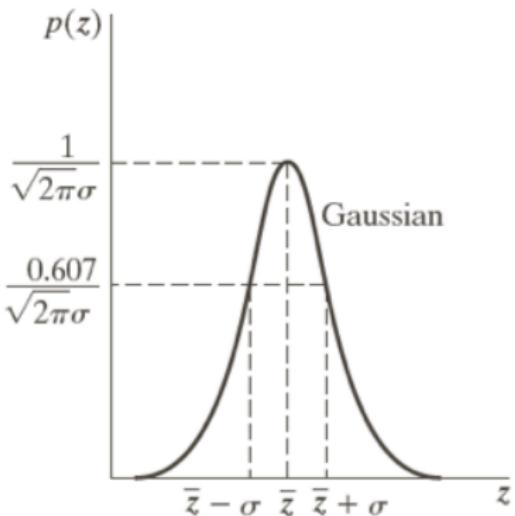
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Gaussian Noise



Properties

- ① $[\mu - \sigma, \mu + \sigma]$: contains approximately 70% of noise values
- ② $[\mu - 2\sigma, \mu + 2\sigma]$: contains approximately 95% of noise values



Rayleigh Noise

Rayleigh probability density function

For each pixel in $I(u, v)$, the noise value z is drawn from a Rayleigh probability density function.

$$p(z) = \begin{cases} \frac{2}{b}(z - a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

① mean of noises:

$$\mu = a + \sqrt{\pi b / 4}$$

② variance of noises:

$$\sigma^2 = \frac{b(4 - \pi)}{4}$$

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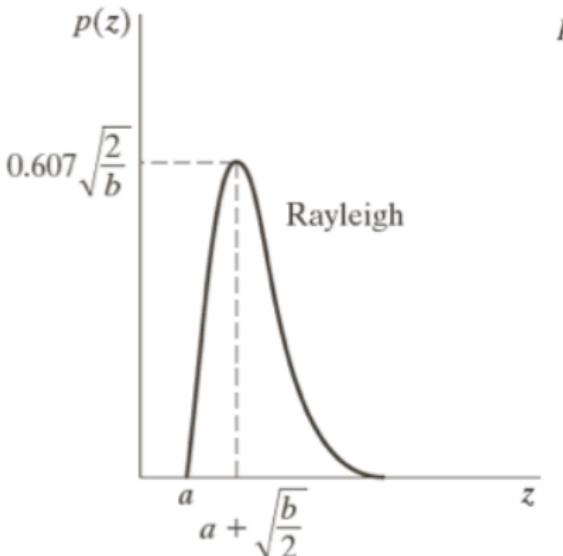
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Properties

- ① The minimum noise value is a
- ② The density is skewed to the right
- ③ $b > 0$



Erlang (Gamma) Noise

Erlang probability density function

For each pixel in $I(u, v)$, the noise value z is drawn from a Erlang probability density function.

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

① mean of noises:

$$\mu = \frac{b}{a}$$

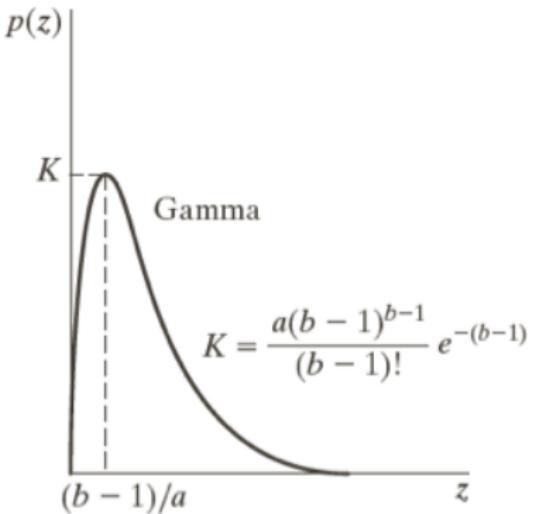
② variance of noises:

$$\sigma^2 = \frac{b}{a^2}$$

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Erlang Noise

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Properties

- ① The minimum noise value is 0
- ② $a > 0$
- ③ b is a positive integer

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Exponential Noise

Exponential probability density function

For each pixel in $I(u, v)$, the noise value z is drawn from a exponential probability density function.

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

① mean of noises:

$$\mu = \frac{1}{a}$$

② variance of noises:

$$\sigma^2 = \frac{1}{a^2}$$

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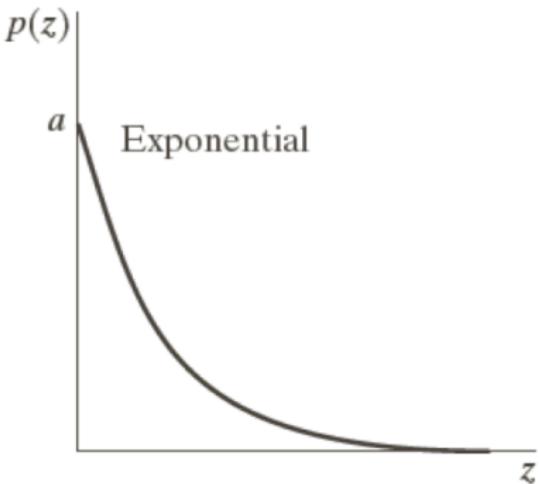
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Exponential Noise



Properties

- ① The minimum noise value is 0
- ② $a > 0$
- ③ Exponential noise is a special case of Erlang noise, with $b = 1$



Uniform Noise

Probability density function of uniform noise

For each pixel in $I(u, v)$, the noise value z is drawn from a uniform probability density function.

$$p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

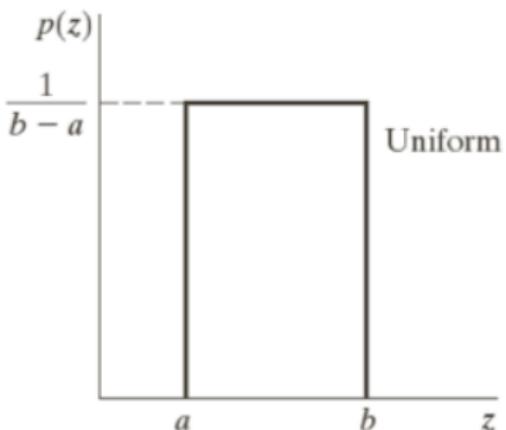
① mean of noises:

$$\mu = \frac{a + b}{2}$$

② variance of noises:

$$\sigma^2 = \frac{(b - a)^2}{12}$$

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Properties

- ① Noise values range from a to b
- ② $a \leq b$
- ③ Probability of noise values are equal ($= \frac{1}{b-a}$).

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Impulse (pepper-and-salt) Noise

Probability density function of uniform noise

For each pixel in $I(u, v)$, the noise value z is drawn from a bipolar impulse probability density function.

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

- ① **Bipolar impulse:** both of P_a and P_b are not zero.
- ② **Unipolar impulse:** either P_a or P_b is zero.
- ③ **Light vs dark dot:** $b > a$, gray-level b will appear as a light dot, gray-level a will appear as a dark dot.

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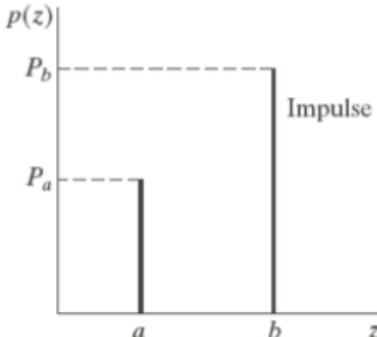
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Properties

- ① Impulse corruption is large; so, impulse noise should be digitalized as extreme (pure black or white)
- ② For signed numbers: negative impulse \rightarrow black; positive impulse \rightarrow white
- ③ For unsigned 8-bit numbers: $a = 0$ (black), $b = 255$ (white)

Images with noise

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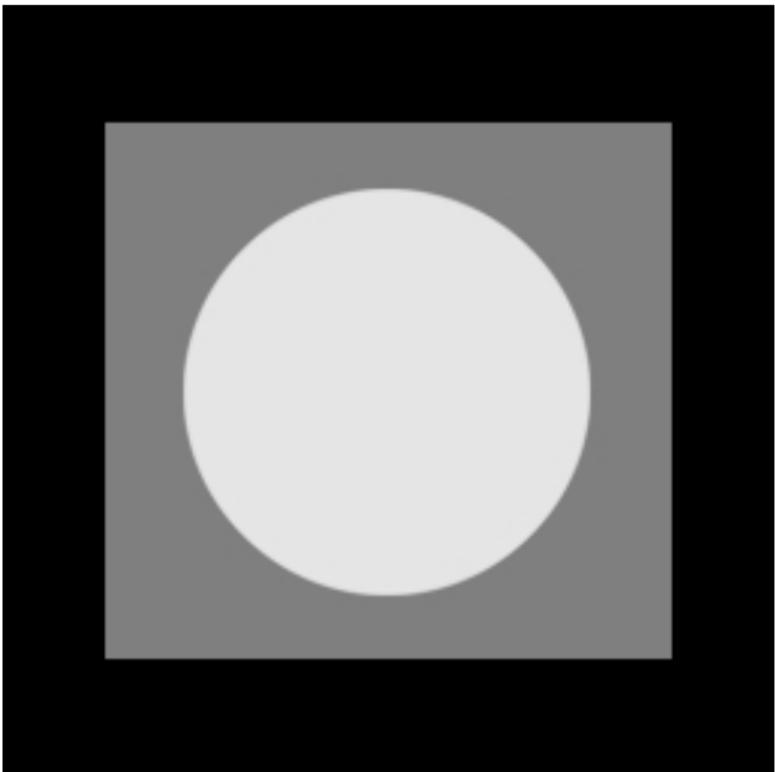
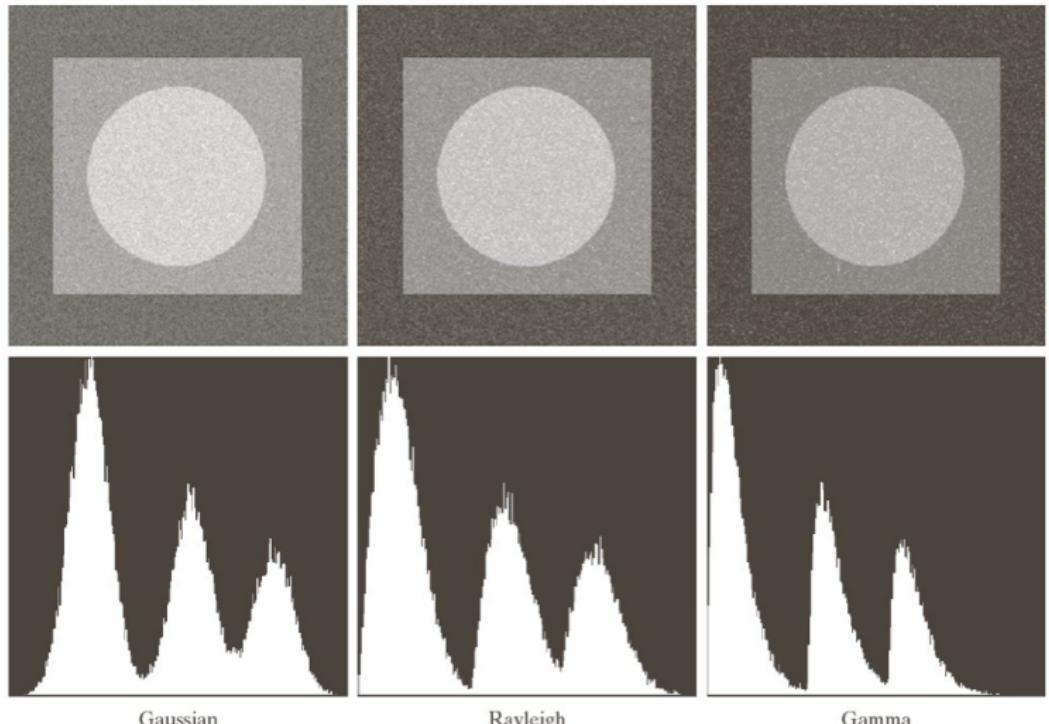
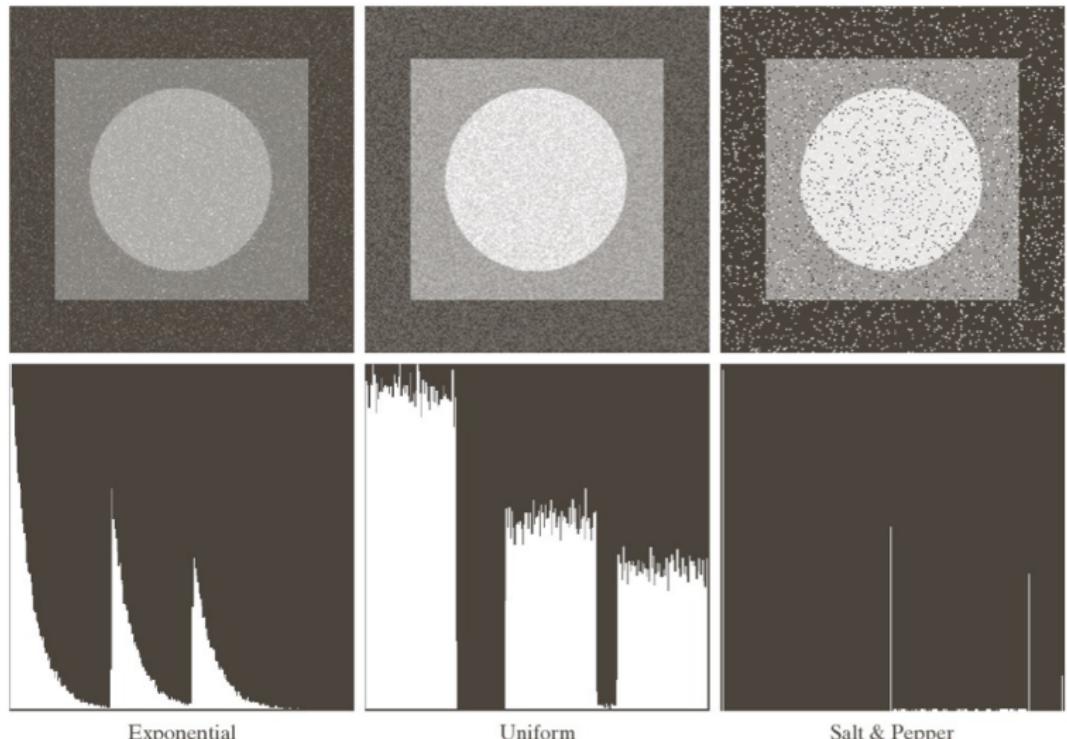


Figure: Pattern without noise

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a b c
d e f

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

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Exponential

Uniform

Salt & Pepper

g	h	i
j	k	l

FIGURE 5.4 (Continued) Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3

Periodic Noise

- ① **Cause:** during image acquisition
- ② **Type:** This is spatially dependent noise

Example

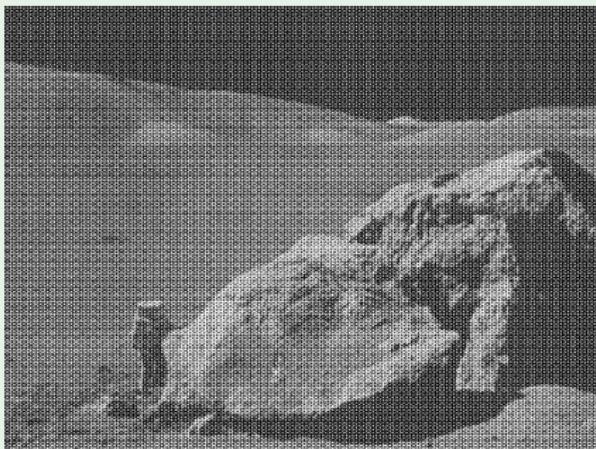


Figure: Example of periodic noise

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Example

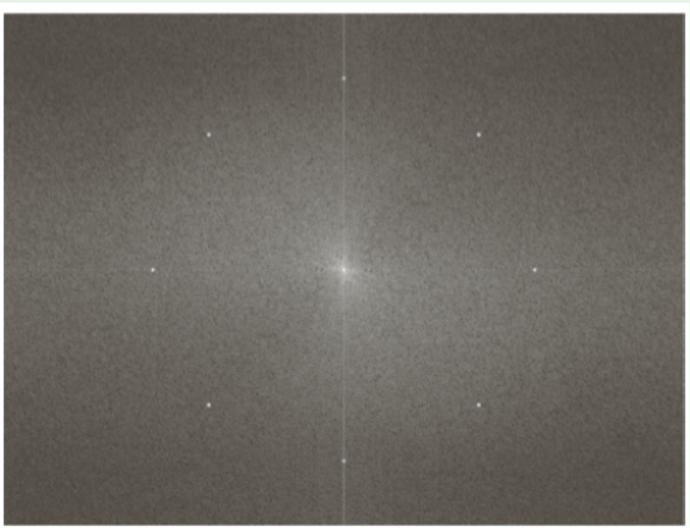


Figure: Frequency spectrum of previous image

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Properties

- ① Image shows periodic noise (sinusoidal) spatially.
- ② So, frequency spectrum has some bright dot at some frequencies.
- ③ Therefore, this noise can be removed effectively in frequency domain.

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Why do we need to generate noise?

- Noise is unwanted
- However, we need to generate them and to model them for research purpose.

Problem

Assume that we have following

- ① $p_x(x)$: a probability density function of a random variable x
- ② $p_z(z)$: a probability density function of a random variable z

Can we generate variable z if we have x , $p_x(x)$, and $p_z(z)$?

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Noise generation

Let $c_x(x)$ and $c_z(z)$ be distribution function of variables x and z .

$c_x(x)$ and $c_z(z)$ are cumulative density functions of x and z .
These functions can be computed as follows:

$$c_x(x) = \sum_{i=-\infty}^x p_x(i)$$

$$c_z(z) = \sum_{i=-\infty}^z p_z(i)$$



Noise generation

Let $z = f(x)$ be a function that map one-to-one between x and z . We have to discover this function.

Discovering $z = f(x)$

- Assume that we have $z_1 = f(x_1)$
- Then, $c_z(z_1) = c_x(x_1)$. This is because of one-one mapping
 - A value $x < x_1$ will be mapped into $z < z_1$
- Define $w \equiv c_x(x_1)$, i.e., $w \equiv c_z(z_1) \equiv c_x(x_1)$

Therefore,

- $z_1 = c_z^{-1}(w) \equiv c_z^{-1}(c_x(x_1))$

Mapping function $f(x)$

$$z = f(x) = c_z^{-1}(c_x(x))$$

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Noise generation

Mapping function $f(x)$: Meaning

$$z = f(x) = c_z^{-1}(c_x(x))$$

- We can generate noise value z distributed with PDF $p_z(z)$, if we has input value x distributed with PDF $p_x(x)$

Why?

Because,

- If we know PDF $p_x(x)$, we can compute $c_x(x)$.
- If we know PDF $p_z(z)$, we can compute $c_z(z)$.
- From $c_z(z)$, we can obtain $c_z^{-1}(z)$ in term of closed-form or in term of a look-up table.

This idea is also applied to Histogram equalization and matching



Some random number generators

Almost programming languages provide function **rand()** and **randn()** to generate number distributed with uniform or Gaussian probability density function respectively.

Therefore, We can use these function to generate x and then from x to generate noise z with other kinds distributions.

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Noise generation

How do we compute generate y in practice?

- ① Generate random variable x by uniform distribution.
Existing function $rand()$ in many programming language can do this task
- ② Compute $w = c_x(x)$. Please note that $c_x(x)$ is uniform distribution function.
- ③ If we have closed-form of $z = c_z^{-1}(w)$ then can use this function to determine z .
- ④ If we do not have closed-form of $c_z^{-1}(z)$
 - Create a lookup table at the beginning for mapping $z \rightarrow c_z^{-1}(p)$, for discrete values p . We can do this because we know $c_z(z)$ in advance. This task is equal to rasterize $p = c_z(z)$ and store pairs into lookup table
 - Determine z according to the lookup table.

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Noise generation: some popular PDF and CDF functions



TABLE 5.1 Generation of random variables.

Name	PDF	Mean and Variance	CDF	Generator [†]
Uniform	$p_z(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$	$m = \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12}$	$F_z(z) = \begin{cases} 0 & z < a \\ \frac{z-a}{b-a} & a \leq z \leq b \\ 1 & z > b \end{cases}$	MATLAB function <code>rand</code>
Gaussian	$p_z(z) = \frac{1}{\sqrt{2\pi}b} e^{-(z-a)^2/2b^2}$ $-\infty < z < \infty$	$m = a, \sigma^2 = b^2$	$F_z(z) = \int_{-\infty}^z p_z(v) dv$	MATLAB function <code>randn</code>
Salt & Pepper	$p_z(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$ $b > a$	$m = aP_a + bP_b$ $\sigma^2 = (a-m)^2P_a + (b-m)^2P_b$	$F_z(z) = \begin{cases} 0 & \text{for } z < a \\ P_a & \text{for } a \leq z < b \\ P_a + P_b & \text{for } b \leq z \end{cases}$	MATLAB function <code>rand</code> with some additional logic
Lognormal	$p_z(z) = \frac{1}{\sqrt{2\pi}bz} e^{-[\ln(z)-a]^2/2b^2}$ $z > 0$	$m = e^{a+(b^2/2)}, \sigma^2 = [e^{b^2} - 1]e^{2a+b^2}$	$F_z(z) = \int_0^z p_z(v) dv$	$z = ae^{bN(0,1)}$
Rayleigh	$p_z(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & z \geq a \\ 0 & z < a \end{cases}$	$m = a + \sqrt{\pi b/4}, \sigma^2 = \frac{b(4-\pi)}{4}$	$F_z(z) = \begin{cases} 1 - e^{-(z-a)^2/b} & z \geq a \\ 0 & z < a \end{cases}$	$z = a + \sqrt{-b \ln[1 - U(0,1)]}$
Exponential	$p_z(z) = \begin{cases} ae^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$	$m = \frac{1}{a}, \sigma^2 = \frac{1}{a^2}$	$F_z(z) = \begin{cases} 1 - e^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$	$z = -\frac{1}{a} \ln[1 - U(0,1)]$
Erlang	$p_z(z) = \frac{a^b z^{b-1}}{(b-1)!} e^{-az}$ $z \geq 0$	$m = \frac{b}{a}, \sigma^2 = \frac{b}{a^2}$	$F_z(z) = \left[1 - e^{-az} \sum_{n=0}^{b-1} \frac{(az)^n}{n!} \right]$ $z \geq 0$	$z = E_1 + E_2 + \dots + E_b$ (The E 's are exponential random numbers with parameter a .)

[†] $N(0, 1)$ denotes normal (Gaussian) random numbers with mean 0 and a variance of 1. $U(0, 1)$ denotes uniform random numbers in the range $(0, 1)$.

Integration model

① Additive Noise

- Noise values will be **added** to free-noise image to create image with noise.
- Model: $g(x, y) = f(x, y) + \eta(x, y)$

② Multiplicative Noise

- Noise values will be **multiplied** to free-noise image to create image with noise.
- Model: $g(x, y) = f(x, y) \times \eta(x, y)$

Each pixel in noise image $z = \eta(x, y)$ is generated according to previous algorithm to follow PDF $p_z(z)$

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Exercise

- ① Write a program to add noise (additive and multiplicative) to input image

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Reasons of estimation

- Noise will be removed more effectively, if we know noise model in advance.
- So, noise reduction needs estimation of noise model

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Method of estimating noise model

- ① Select a small strip (called S) in input images. This strip contains reasonably constant gray level.
 - "Reasonably" means gray levels are not too dark or bright
- ② Compute histogram of values inside of the strip
- ③ Observe the histogram to determine noise model
- ④ Estimate the mean and the variance of PDF of noise model.
- ⑤ Use the estimated mean and variance to estimate other parameters, for examples, a and b in other types of models different to Gaussian.

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Input data

- ① S : a small strip in input image
- ② $p(z_i)$: normalized histogram computed from S



Mean and Variance Estimation

① Mean:

$$\mu = \sum_{z_i \in S} z_i p(z_i)$$

② Variance:

$$\sigma^2 = \sum_{z_i \in S} (z_i - \mu)^2 p(z_i)$$

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Example



a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

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Mean filters: Arithmetic mean filter

Mathematical model

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t)$$

Properties

- ① Smooth local variations in an image.
- ② Can reduce the following noises
 - Additive Gaussian noise with zero mean
 - Additive Uniform noise with zero mean
- ③ Result blurred image, especially, at edges, with large S_{xy} .

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Mean filters: Geometric mean filter

Mathematical model

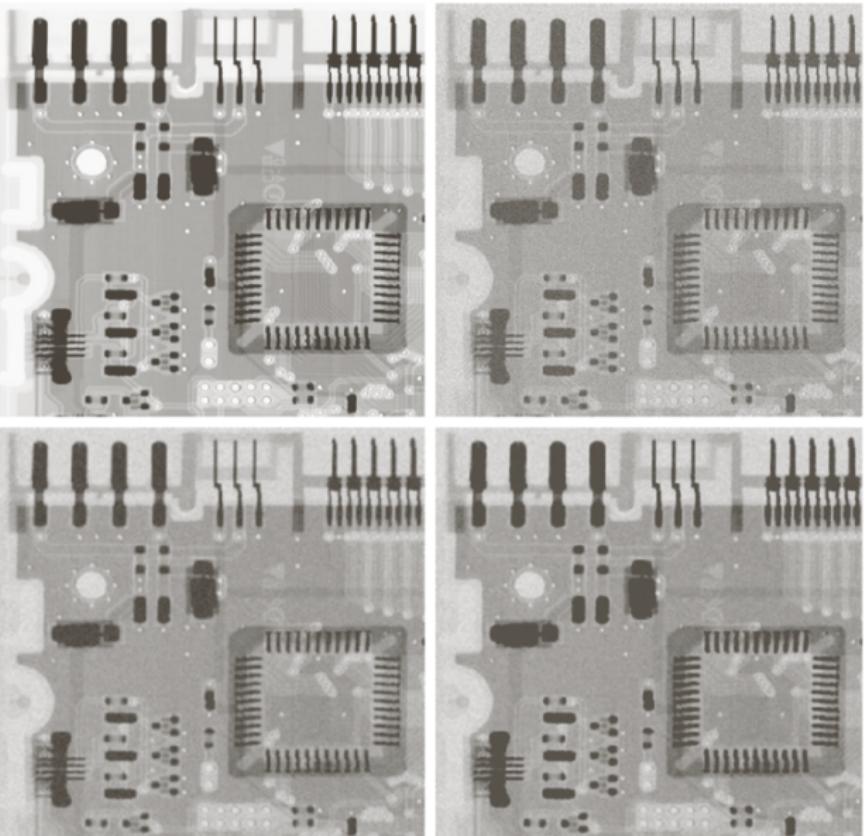
$$\hat{f}(x, y) = \left[\prod_{(s,t) \in S_{xy}} g(s, t) \right]^{\frac{1}{mn}}$$

Properties

- ① Smooth local variations in an image. Do smoothing comparable to the arithmetic mean filter
- ② Tend to lose less image detail
- ③ Can reduce the following noises
 - Additive Gaussian noise with zero mean
 - Additive Uniform noise with zero mean
- ④ Result blurred image, especially, at edges, with large S_{xy} .

a
b
c
d**FIGURE 5.7**

(a) X-ray image.
(b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3 . (d) Result of filtering with a geometric mean filter of the same size.
(Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

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Mathematical model

$$\hat{f}(x, y) = \frac{mn}{\sum_{(s,t) \in S_{xy}} \frac{1}{g(s, t)}}$$

Properties

- ① Can reduce the following noises
 - Additive Gaussian noise with zero mean
 - Additive Uniform noise with zero mean
 - Salt noise
- ② Can not reduce pepper noise (black)

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Mean filters: Contraharmonic mean filter

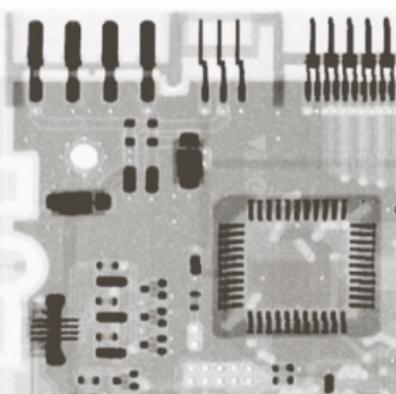
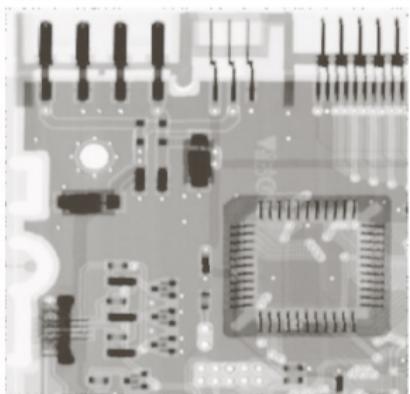
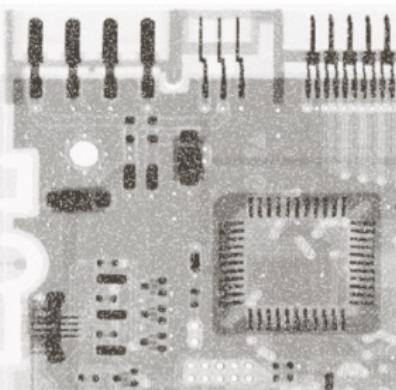
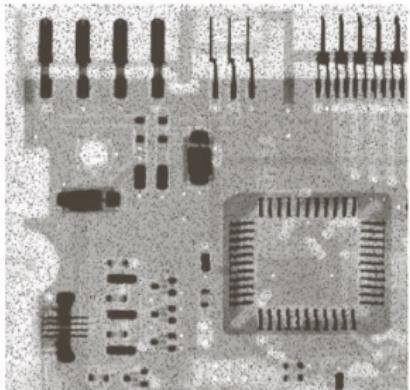
Mathematical model

$$\hat{f}(x, y) = \frac{\sum_{(s,t) \in S_{xy}} g(s, t)^{Q+1}}{\sum_{(s,t) \in S_{xy}} g(s, t)^Q}$$

- Q : order of the filter
- $Q = 0$: Contraharmonic \rightarrow Arithmetic
- $Q = 1$: Contraharmonic \rightarrow Harmonic

Properties

- ① Can reduce pepper-and-salt noise
 - $Q > 0$: reduce pepper noise
 - $Q < 0$: reduce salt noise
- ② Can not reduce pepper and salt noise simultaneously



a
b
c
d

FIGURE 5.8

(a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

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a b

FIGURE 5.9

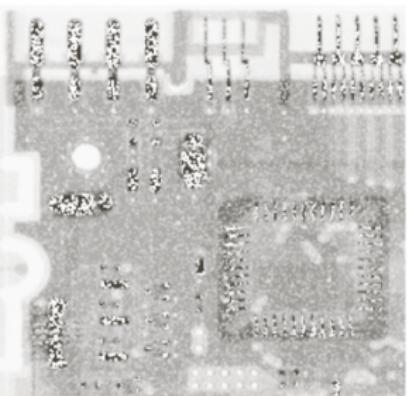
Results of selecting the wrong sign in contraharmonic filtering.

(a) Result of filtering

Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$.

(b) Result of filtering 5.8(b)

with $Q = 1.5$.



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Mathematical model

$$\hat{f}(x, y) = \underset{(s, t) \in S_{xy}}{\text{median}}\{g(s, t)\}$$

- Assign to the output image $\hat{f}(x, y)$ the median value of gray levels in the neighborhood of (x, y)

Properties

- Effectively reduce both of bipolar and unipolar impulse noise, i.e., salt-and-pepper noise
- Produce less blurring images compared to linear
- Can not work with Gaussian noise

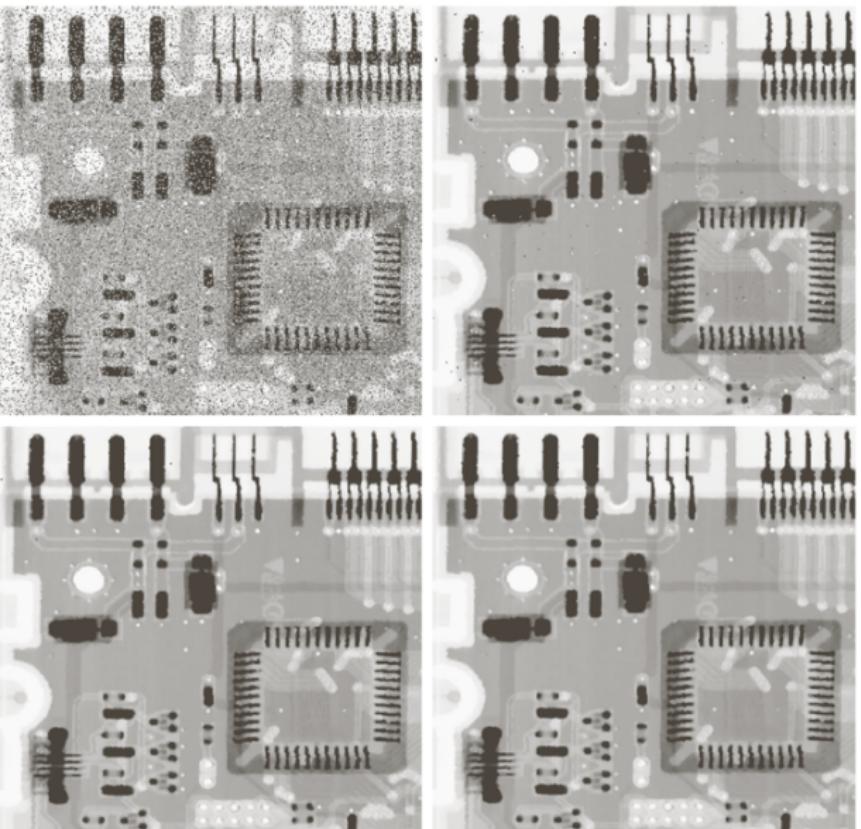
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a
b
c
d

FIGURE 5.10

- (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$.
(b) Result of one pass with a median filter of size 3×3 .
(c) Result of processing (b) with this filter.
(d) Result of processing (c) with the same filter.



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Mathematical model

$$\hat{f}(x, y) = \max_{(s,t) \in S_{xy}} \{g(s, t)\}$$

$$\hat{f}(x, y) = \min_{(s,t) \in S_{xy}} \{g(s, t)\}$$

- **Max filter:** Assign to the output image $\hat{f}(x, y)$ the maximum value of gray levels in the neighborhood of (x, y)
- **Min filter:** Assign to the output image $\hat{f}(x, y)$ the minimum value of gray levels in the neighborhood of (x, y)

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Mathematical model

$$\hat{f}(x, y) = \max_{(s,t) \in S_{xy}} \{g(s, t)\}$$

$$\hat{f}(x, y) = \min_{(s,t) \in S_{xy}} \{g(s, t)\}$$

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Properties

① Max filter:

- Finds the brightest points in image
- Remove pepper noise

② Min filter:

- Finds the darkest points in image
- Remove salt noise

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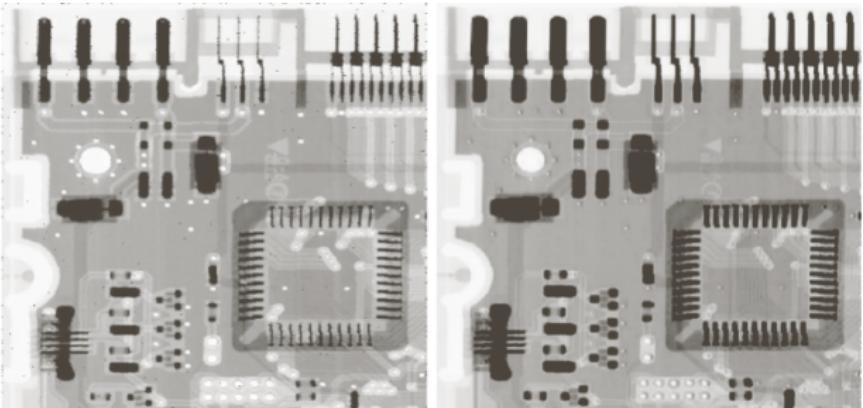
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a b

FIGURE 5.11

(a) Result of filtering Fig. 5.8(a) with a max filter of size 3×3 . (b) Result of filtering 5.8(b) with a min filter of the same size.



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Mathematical model

$$\hat{f}(x, y) = \frac{1}{2} \times \left(\max_{(s,t) \in S_{xy}} \{g(s, t)\} + \min_{(s,t) \in S_{xy}} \{g(s, t)\} \right)$$

Properties

- ① Can reduce randomly distributed noise
 - Additive Gaussian noise with zero mean
 - Additive uniform noise with zero mean

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Order-Statistics Filters: Alpha-trimmed mean filter

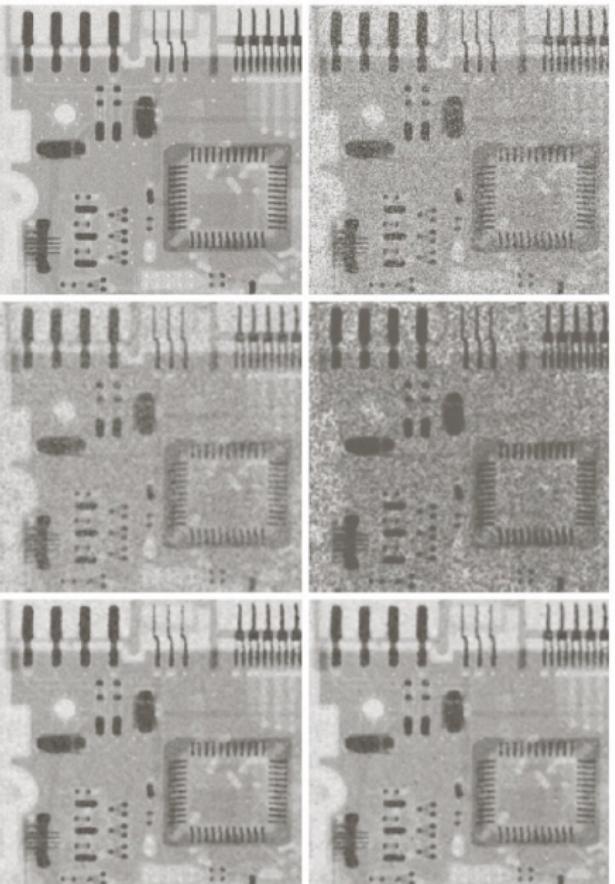
Mathematical model

$$\hat{f}(x, y) = \frac{1}{mn - d} \sum_{(s,t) \in S_{xy}} g_r(s, t)$$

- Delete $d/2$ lowest and $d/2$ highest gray values in neighborhood of (x, y) to obtain $g_r(s, t)$ of $mn - d$ gray values.
- Assign the average of $g_r(s, t)$ to $\hat{f}(x, y)$
- $d = 0$: Alpha-trimmed \rightarrow Arithmetic mean filter
- $d = (mn - 1)/2$: Alpha-trimmed \rightarrow Median filter

Properties

- ① Useful in situations involving multiple types of noise, in combination of salt-and-pepper and Gaussian noise.



a
b
c
d
e
f

FIGURE 5.12

(a) Image corrupted by additive uniform noise.
 (b) Image additionally corrupted by additive salt-and-pepper noise.
 Image (b) filtered with a 5×5 ;
 (c) arithmetic mean filter;
 (d) geometric mean filter;
 (e) median filter;
 and (f) alpha-trimmed mean filter with $d = 5$.

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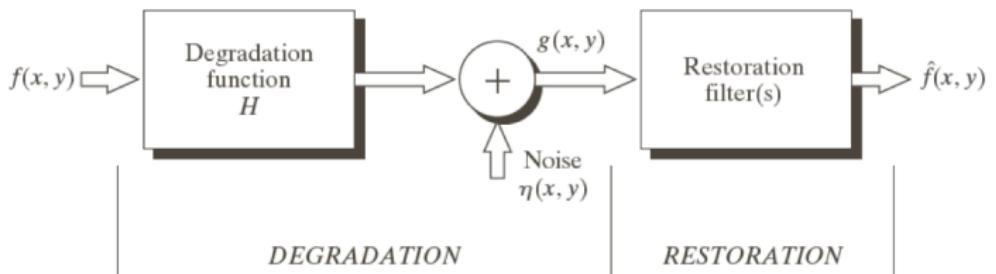


Figure: Model of degradation and restoration process

- $f(x, y)$: input image
- $\eta(x, y)$: noise at (x, y)
- $\hat{f}(x, y)$: restored image

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Linear, Position-Invariant Degradation

- ① Model in space domain:

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

- ① Model in frequency domain:

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

$G(u, v)$, $H(u, v)$, $F(u, v)$, and $N(u, v)$ are **Fourier transforms** of $g(x, y)$, $h(x, y)$, $f(x, y)$ and $\eta(x, y)$ respectively.

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- Let $\hat{F}(u, v)$ be estimate of Fourier transforms of $f(x, y)$

Ideal Restoration

$$\begin{aligned}\hat{F}(u, v) &= \frac{G(u, v)}{H(u, v)} \\ &= F(u, v) + \frac{N(u, v)}{H(u, v)}\end{aligned}$$

Problems

- Problem 1:** even you know $H(u, v)$, you can not recover $f(x, y)$ exactly because you do not know $N(u, v)$.
- Problem 2:** At some (u, v) , $H(u, v) = 0$ will cause $N(u, v)/H(u, v)$ to dominate $\hat{F}(u, v)$.



Inverse Filtering

Inverse Filtering

$$\hat{F}(u, v) = \frac{G(u, v)}{H(u, v)}$$

Tools for existing problems

① Problem 1: Assume that there is no noise.

② Problem 2: At some (u, v) , $H(u, v) = 0$:

① Replacement: Replace $H(u, v) = 0$ at (u, v) where $H(u, v) = 0$:

② Cut-off: Filter $G(u, v)/H(u, v)$ with Butterworth lowpass function of some order, e.g., order = 10, with some radius, e.g., 40, 70, etc - dependent on image size.

③ Finding radius: Go from the origin to outside radially, find the first (u, v) that $H(u, v) = 0$. Limit the filter frequencies from the origin to this radius.

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(a) Original image



(b) Cut-off, $R=\text{full}$

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(a) Cut-off, $R = 40$



(b) Cut-off, $R = 85$

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Figure: Cut-off, $R = 70$

Exercise

- ① Write a program to simulate the atmospheric turbulence phenomenon, modeled by $H(u, v)$ in frequency domain as in the following. Take a look at Gonzalez's Book, page 258.

$$H(u, v) = e^{-k(u^2+v^2)^5/6}$$

- Some k : $k = 0.001, 0.0025, 0.00025$
- ② Write a program to remove the atmospheric turbulence phenomenon from images (generated from previous question.)

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Names and Wiener Filter's objective

- Other name: Minimum Mean Square Error Filter
- Objective: to minimize the mean square error between uncorrupted image f and its estimate \hat{f} .
- \equiv Minimize

$$e^2 = E\{(f - \hat{f})^2\}$$

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Mathematical Model

$$\hat{F}(u, v) = \left[\frac{1}{H(u, v)} \times \frac{|H(u, v)|^2}{|H(u, v)|^2 + S_\eta(u, v)/S_f(u, v)} \right] G(u, v)$$

- $H(u, v)$: degradation function (assume that it has been estimated).
- $G(u, v)$: Fourier transforms of degraded image $g(x, y)$, can be computed.
- $|H(u, v)|^2 = H^*(u, v)H(u, v)$: power spectrum of degradation function, can be computed
- $S_\eta(u, v) = |N(u, v)|^2$: power spectrum of noise.
- $S_f(u, v) = |F(u, v)|^2$: power spectrum of uncorrupted image.
This seldom is known.



Mathematical Model

$$\hat{F}(u, v) = \left[\frac{1}{H(u, v)} \times \frac{|H(u, v)|^2}{|H(u, v)|^2 + S_\eta(u, v)/S_f(u, v)} \right] G(u, v)$$

Wiener filter to Inverse filter

Sepcial case: There is no noise. $N(u, v) = 0$.

Wiener filtering becomes Inverse filtering.

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Wiener Filtering

A special case - White noise

We do not know:

- ① $S_\eta(u, v)$: power spectrum of noise
- ② $S_f(u, v)$: power spectrum of input signal

For white noise, we hope that, at a specific frequency (u, v) , the power of noise is proportional to the power of input signal. It means $S_\eta(u, v) = K \times S_f(u, v)$

$$\frac{S_\eta(u, v)}{S_f(u, v)} = K$$

Remind

White noise is a type of noise that affects on all frequencies.

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Mathematical Model for white noise

$$\hat{F}(u, v) = \left[\frac{1}{H(u, v)} \times \frac{|H(u, v)|^2}{|H(u, v)|^2 + K} \right] G(u, v)$$

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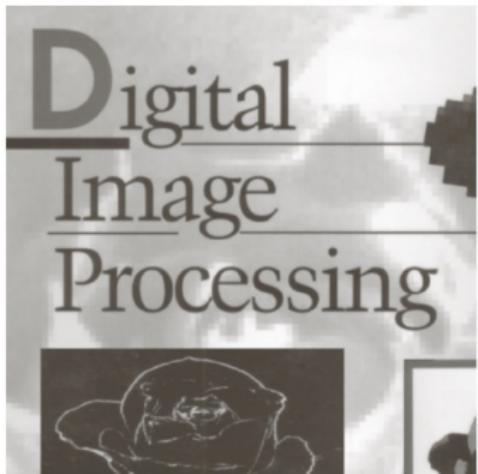
Demonstration's model of degradation

The demonstration for Wiener filtering in some consecutive slides uses **motion blur** degradation model, as shown below.

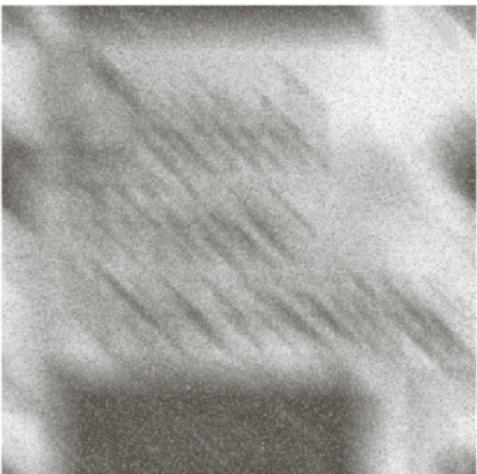
$$H(u, v) = \frac{T}{\pi(ua + vb)} \sin [\pi(ua + vb)] e^{-j\pi(ua + vb)}$$

- $a = b = 0.1$
- $T = 1$

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(a) Original image



(b) Degraded image

Degradation method

- ① Blurring the original with motion model
- ② Corrupting heavily with additive Gaussian noise, zeros mean, variance of 650

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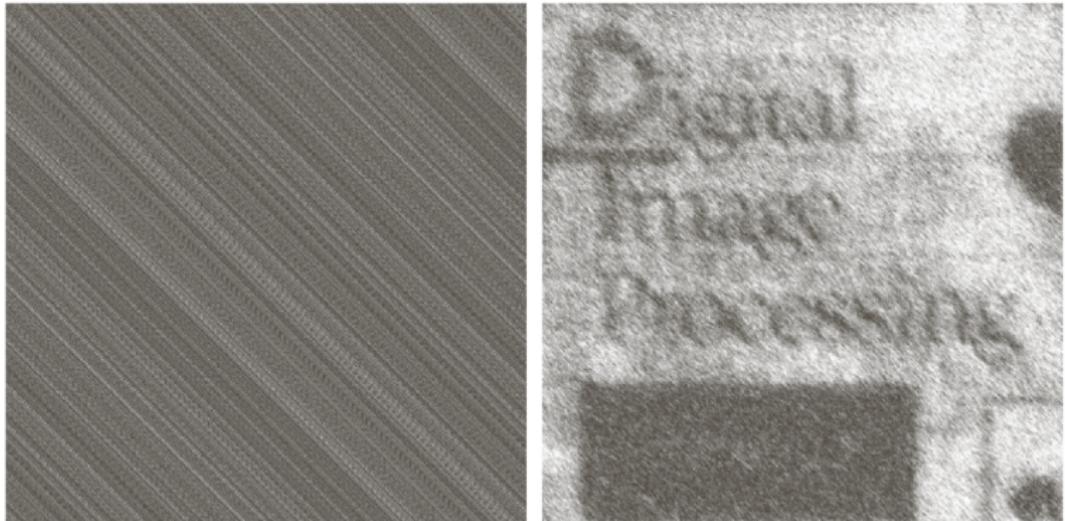


Figure: Restored images. Left: Inverse Filtering, Right: Wiener Filtering, K is selected to for best result

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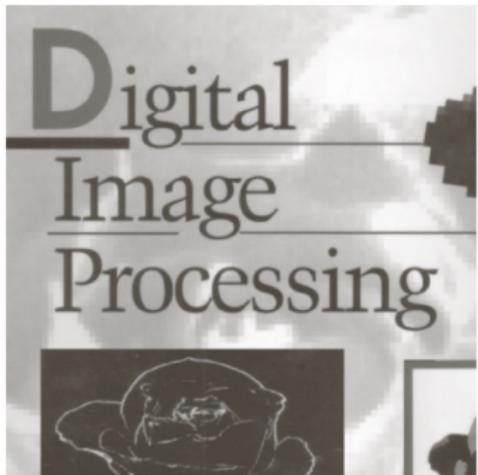
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(a) Original image



(b) Degraded image

Degradation method

- ① Blurring the original with motion model
- ② Corrupting additive Gaussian noise with smaller variance compared to the previous case, zeros mean.

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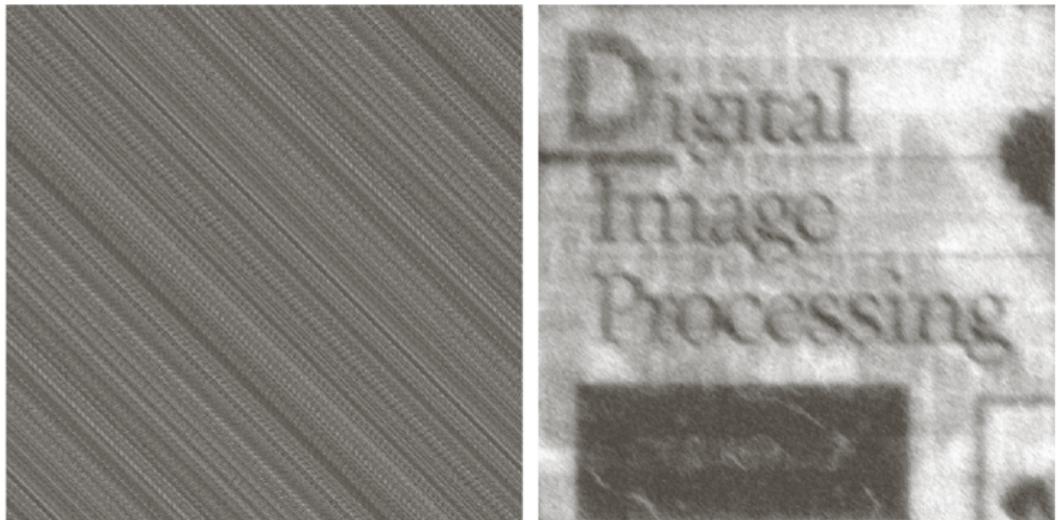


Figure: Restored images. Left: Inverse Filtering, Right: Wiener Filtering, K is selected to for best result

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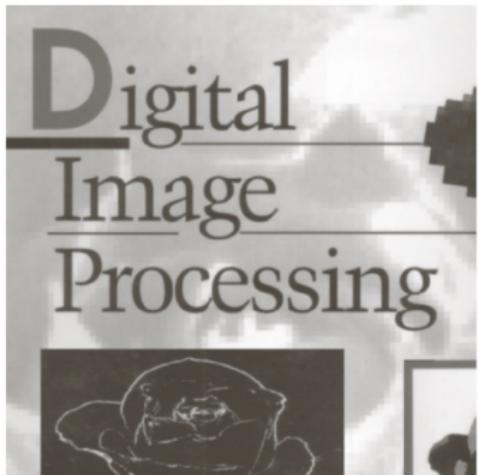
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(a) Original image



(b) Degraded image

Degradation method

- ① Blurring the original with motion model
- ② Corrupting additive Gaussian noise with smaller variance compared to the previous case, zeros mean.

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Figure: Restored images. Left: [Inverse Filtering](#), Right: [Wiener Filtering](#), K is selected to for best result

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- The result of Wiener filter in this case shows excellent quality!



Exercise

- ① Write a program to degrade input images with motion blur model and additive Gaussian noise with zeros mean.
- ② Write a program to restore degraded images with Wiener and Inverse filtering

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