COMP344 Digital Image Processing
Fall 2007
Final Examination

Time allowed: 3 hours

Name	Student ID	Email

Question 1	Question 2	Question 3	Question 4
Question 5	**Question 6**	**Question 7**	**Question 8**

Total ________

With model answer

HK University of Science and Technology

Course Code : COMP344 Section No.: ALL Time Allowed : 3 Hour(s)

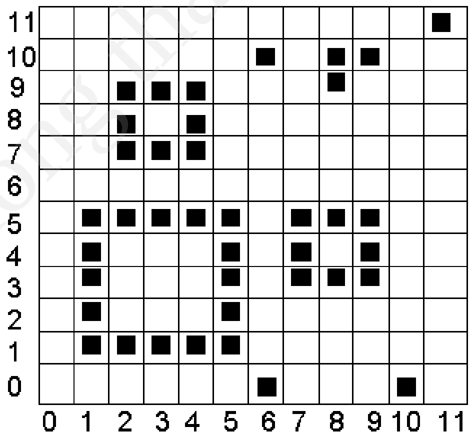
Course Title : Digital Image Processing Total Number of Pages : 19

Time allowed: **3 hours**
Answer all questions.
The total mark for this quiz is 100.
This is a closed-book quiz.

1. a) In the Hough Transform, a point (x_0, y_0) in the xy-plane is mapped into a curve in the (ρ, θ)-parameter space. Write down the equation of the curve. (2 marks)

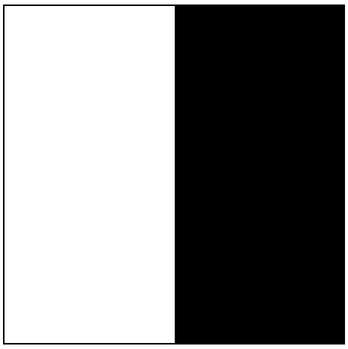
1. b) If we apply the Hough transform on the image below, what would be the maximum value for the accumulator cell in the (ρ, θ) space? What is the corresponding (ρ, θ) value? (5 marks)

Note: each black square denotes a point and the numbers are the coordinates.

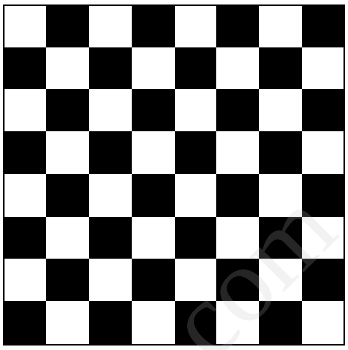


Answer to question 1:
1. a) $x\cos(\theta) + y\sin(\theta) = \rho$
1. b) The maximum value is 8, corresponding to the horizontal line with vertical coordinate 5, because there are 8 points lying on this line. This line has the parameter $\rho = 5, \theta = \pi/2$.

2) The two texture images shown below are quite different, but their histograms are identical. Both images have size 80×80, with black (0) and white (1) pixels.



a



b

2. a) Suppose that both images are blurred with a 3×3 smoothing mask. Would the resultant histograms still be the same? Draw the two histograms and explain your answer. (5 marks)

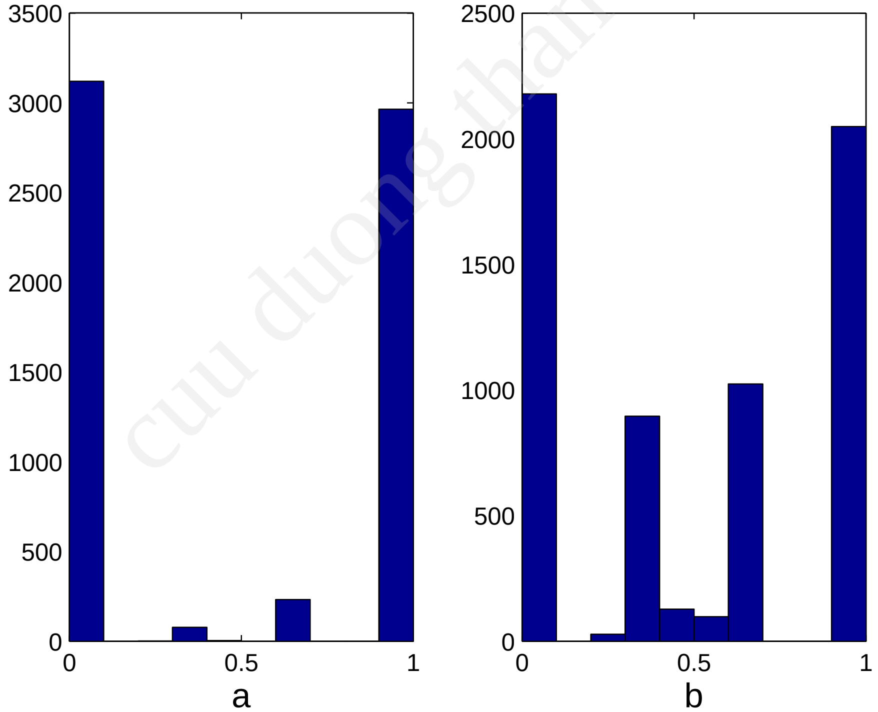
Note: the black lines are used to signify the boundaries of the two images but not part of them.

2. b) For image (b), compute the co-occurrence matrix corresponding to the property "a pixel to the right". Each grid (black or white) in image (b) is of size 10×10.

(5 marks)

Answer to question 2:

2. a) the histograms will no longer be the same. They are plotted as follows

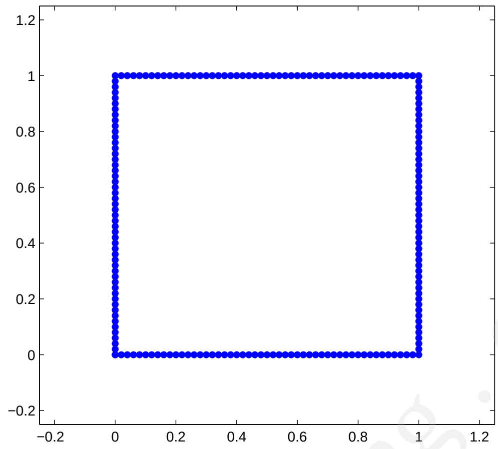


2. b) The co-occurrence matrix is $\begin{bmatrix} 2880 & 280 \\ 280 & 2880 \end{bmatrix}$.

3. a) Given a sequence of $\{(x_i, y_i)\}_{i=1}^{n}$ lying on the object boundary, briefly describe how to obtain its Fourier descriptor? Why is the Fourier descriptor capable of dealing with noisy boundaries? (8 marks)

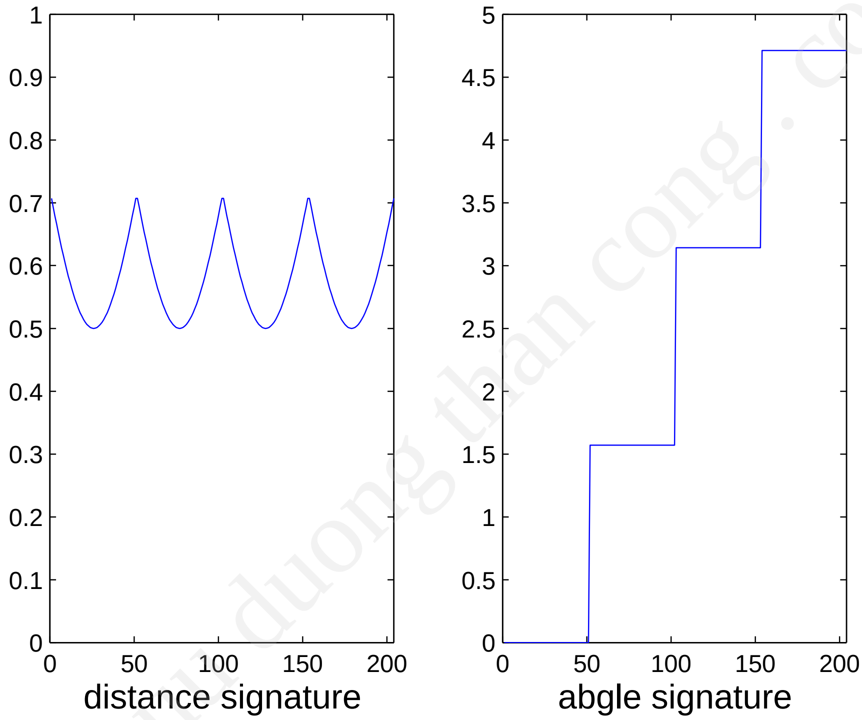
3. b) As discussed in the lecture notes, one can obtain two types of signatures based on the object boundary. One of them is based on distances and the other is based on angles. Describe how to obtain them. (5 marks)

3. c) A square boundary in the domain $[0,1] \times [0,1]$ is shown below. Draw the two signatures obtained in Question 3b. (8 marks)



Answer to question 3:

3. a) Given a set of 2-d coordinates (x_k, y_k)s on a boundary. Start from arbitrary point, traverse the boundary, and obtain a sequence (x1,y1), (x2,y2),... Deem this a s a sequence of complex numbers, S. Compute DFT of F = DFT(S). Then coefficients in F are Fourier descriptors. Usually the high frequency component in F have small magnitude and will be removed. By doing this the small fluctuations or high-variation parts in the boundary will be smoothed, achieving the noise reduction effect.
3. b) Method 1: plot the distance from the centroid to the boundary as a function of angle. Method 2: traverse the boundary corresponding to each point on the boundary, plot the angle between the line tangent to the boundary at that point and a reference line.
3. c) The two signatures are plotted as follows



4. a) Discuss the advantages and disadvantages of global and local histogram equalization. (5 marks)

4. b) The following figure shows
(a) a 3-bit image of size 5-by-5 image in the square, with x and y coordinates specified,

(b) a Laplacian filter and

(c) a low-pass filter.

y \ x	0	1	2	3	4
0	3	7	6	2	0
1	2	4	6	1	1
2	4	7	2	5	4
3	3	0	6	2	1
4	5	7	5	1	2

Laplacian filter

0	1	0
1	-4	1
0	1	0

Low pass filter

0.01	0.1	0.01
0.10	0.56	0.10
0.01	0.1	0.01

Compute the following:

(a) The output of a 3×3 mean filter at (2,2). (1 marks)

(b) The output of a 3×3 median filter at (2,2). (1 marks)

(c) The output of the 3×3 Laplacian filter shown above at (2,2). (1 marks)

(d) The output of the 3×3 low-pass filter shown above at (2,2). (1 marks)

(e) The histogram of the whole image. (3 marks)

(f) The result of histogram equalization at the point (2,2). Show steps in obtaining your solution. (6 marks)

Answer to question 4:

4. a) Global equalization may greatly enhance noise, and may fail to recover detailed structures that are hidden due to low contrast. However it is faster than the local one.
Local histogram equalization can reveal more detailed structures, being adaptive to local image statistics. So it is suited for images with varying illumination. However, local equalization may not produce a histogram that is as flat as the global one. It is slow and may cause some discontinuities, because two pixels that are of initially the same intensity may be transformed to different intensities.

4. b)

(a) 3.67; (b) 4; (c) 16; (d) 3.59; (e) $\begin{array}{cccccccccc} frequency & 2 & 4 & 5 & 2 & 3 & 3 & 3 & 3 \\ intensity & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array}$ (f) 3.08

5. a) Explain the image degradation model described in the lecture notes, and how to use inverse filters for image restoration. (10 marks)

5. b) Inverse filters may encounter numerical problem in practice. Provide one solution to overcome this instability. (5 marks)

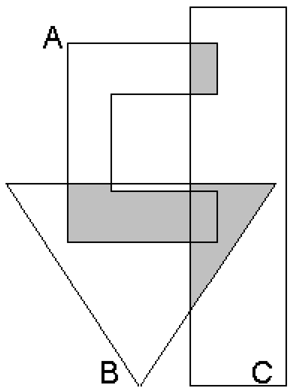
Answer to question 5:

5. a) In the spatial domain, the model is $g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$, where $g(x,y)$ is the observed image at position (x,y), $f(x,y)$ is the original image, $\eta(x,y)$ is the spatial noise, and the convolution $h(x,y) * f(x,y)$ corresponds to the image degradation process, where $h(x,y)$ is the spatial representation of the degradation operator.

In the frequency domain, the model becomes $G(u,v) = H(u,v)F(u,v) + N(u,v)$. Suppose the noise is zero, then we have $\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$, which is called inverse filter and can be used to estimate the Frequency response of the original image hence restoring the original image.

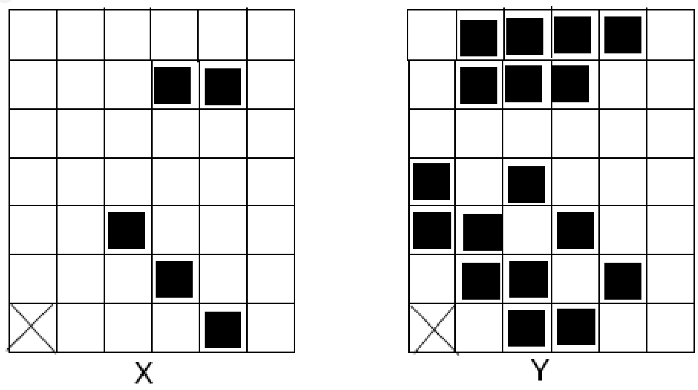
5. b) In practice the denominator $H(u,v)$ may have too small magnitude in the high frequency part, making the inverse filter highly unstable. To prevent this, we can (1) confine the inverse filter operation only to the low frequency part; or (2) add a small constant in the denominator of the inverse filter process, similar to Wiener filter.

6. a) The following figure shows three closed sets A, B and C. Give an expression using $\cap$, $\cup$ and $\bar{\cdot}$ for the shadowed part. (3 marks)



6. b) Images X and Y are shown below with the crossing in the left bottom showing their correspondence. Design an appropriate structure element such that Y can be obtained after dilating X. (black for 1 and white for 0) (10 marks)

Note: mark the center of the structure element clearly.



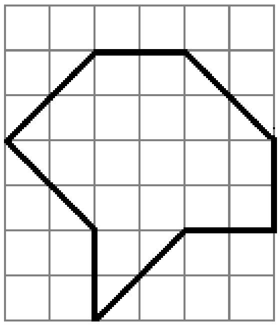
Answer to question 6:
6. a) $(A \cap C) \cup (A \cap B) \cup (C \cap B)$

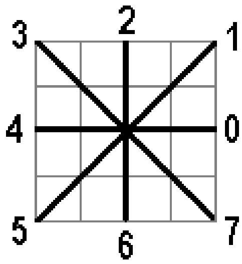
6. b) B is a 2-by-3 matrix $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & \times \end{bmatrix}$, with $\times$ being the origin (and is a **zero**).

The mask that is being shifted through all pixels of image X (or the reflection of B) is $\begin{bmatrix} \times & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$.

If you give the reflection of B as your answer without specifying it is the reflection, 4 marks will be deducted.

7) For the following boundary (in counterclockwise direction)





compute the chain code that is:

7. a) invariant to starting point; (3 marks)

7. b) invariant to both starting point and rotation. (5 marks)

Answer to question 7:
7. a) To achieve invariance w.r.t. starting point, we can simply regard the code as a circular number and find the lowest number when the chain code is read like a ordinary number; to make it invariant to rotation, do not keep track of the absolute direction in which we are traveling, but only the changes in direction (in counterclockwise direction) that separate two adjacent elements of the code.
7. b) 02345761 or 0022334455776611
7. c) 11127372 or 0101010207030702

8) Given a gray image I, we want to find all its pixels that are in the domain $[a, b]$. Design a matlab function F = f(I, a, b) that returns a binary matrix F that is of the same size as I, where 1 for pixels satisfying the domain condition and 0 otherwise. (8 marks)

Answer to question 8:

```
function F = f(I, a, b);
[m, n] = size(I);
F = zeros(m,n);
dex = find(I >=a & I <=b);
F(dex) = 1;
```