Convolution

Matrix computation

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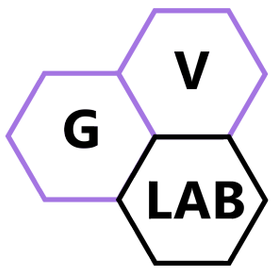

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Convolution

im2col

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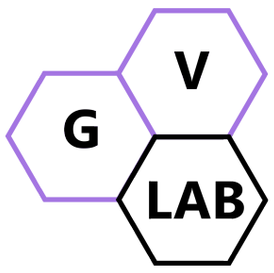

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4 im2col

- **Notation:**

$\mathbf{X}$: input image or feature map

$\mathbb{X}_{conv}$: Matrix created by collecting all the values of X corresponding to the kernel W into columns of $\mathbb{X}_{conv}$ (illustrated in next slide)

$\frac{\partial b}{}$

$\mathbf{W}$: Filter's kernel

5 im2col

x_{11}	x_{12}	x_{13}
x_{21}	x_{22}	x_{23}
x_{31}	x_{32}	x_{33}

∂b

X

Input image

w_{11}	w_{12}
w_{21}	w_{22}

W

Filter's kernel

w_{22}	w_{21}
w_{12}	w_{11}

Rot180^0(W)

Rotated kernel

im2col

How to create matrix $\mathbb{X}_{conv}$

$w_{22}x_{11}$	$w_{21}x_{12}$	x_{13}
$w_{12}x_{21}$	$w_{11}x_{22}$	x_{23}
x_{31}	x_{32}	x_{33}

∂b

X

flattening

x_{22}			
x_{21}			
x_{12}			
x_{11}			

$\mathbb{X}_{conv}$

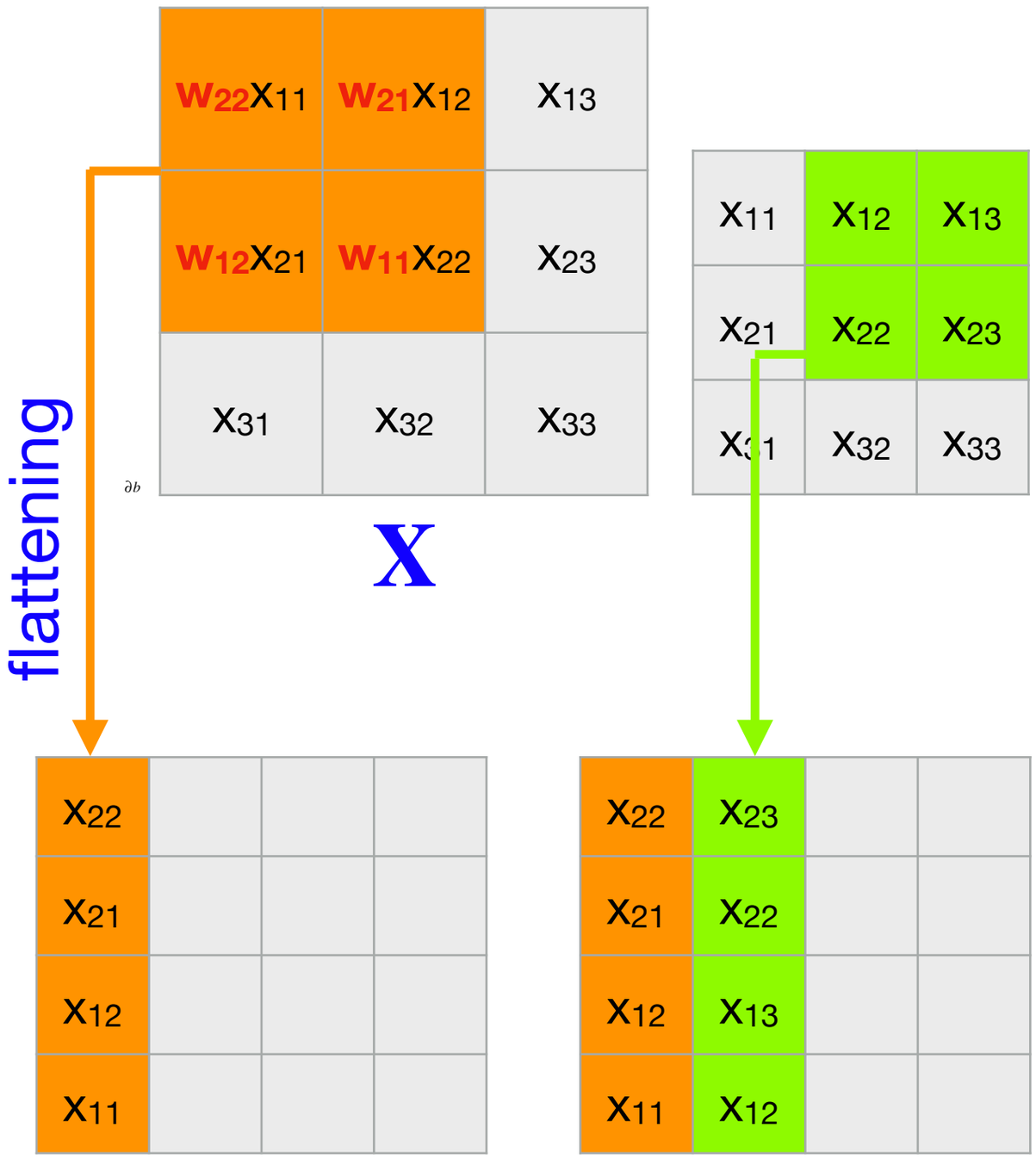
w_{22}	w_{21}
w_{12}	w_{11}

Rotate180^0(W)

Flattened in the order corresponding to w_{11}, w_{12}, w_{21}, w_{22}

im2col

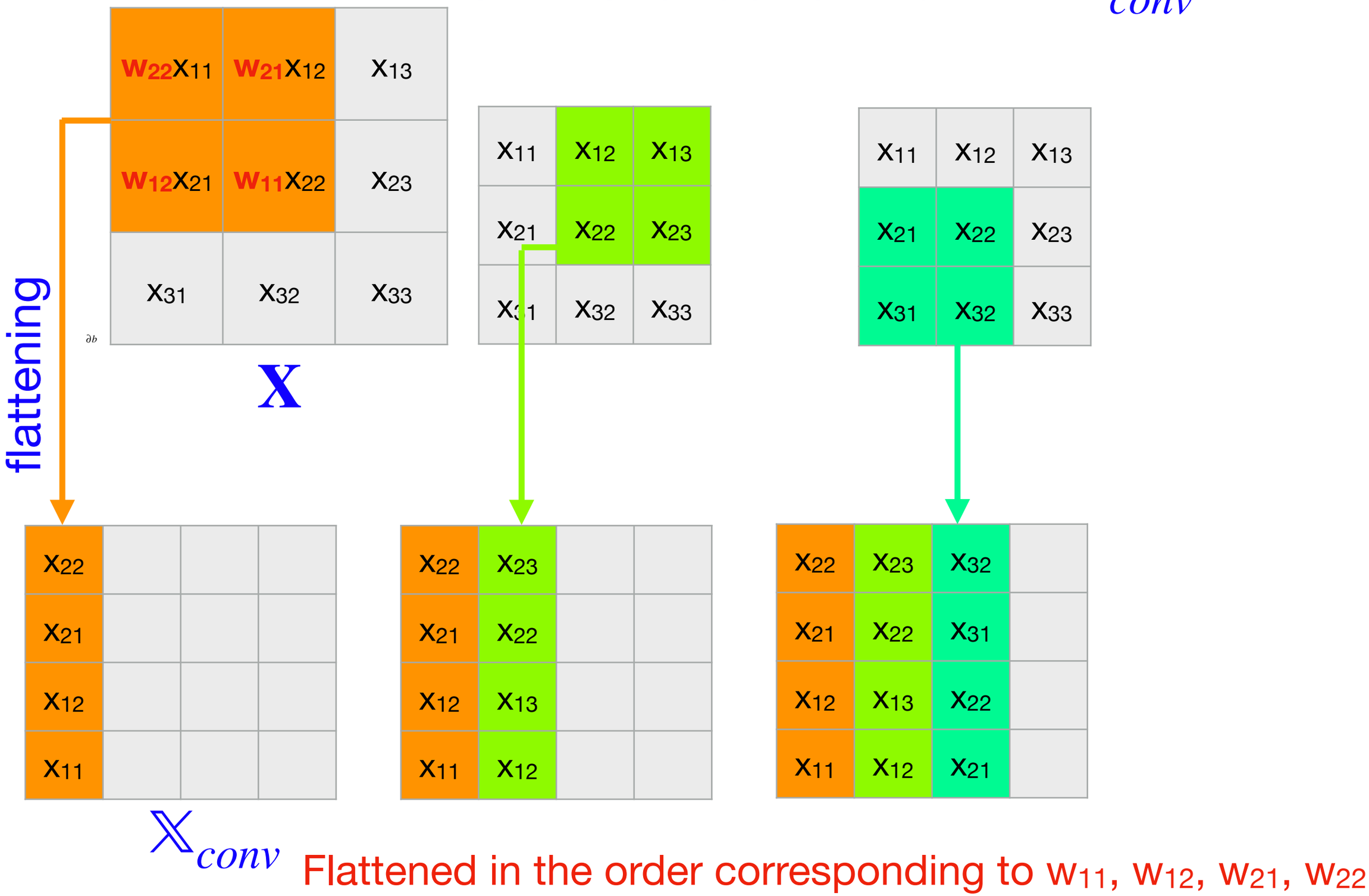
How to create matrix $\mathbb{X}_{conv}$



Flattened in the order corresponding to w_{11}, w_{12}, w_{21}, w_{22}

im2col

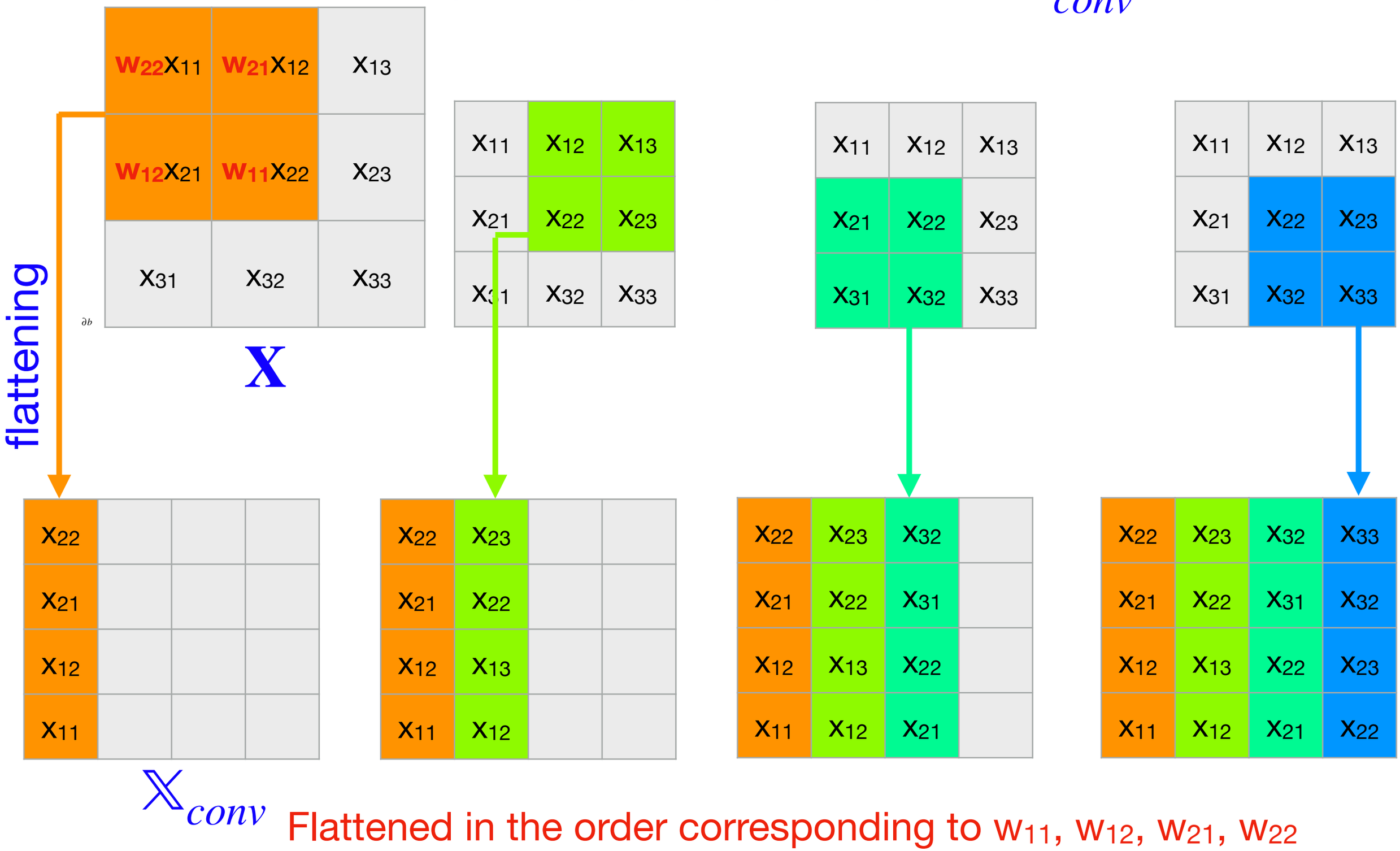
How to create matrix $\mathbb{X}_{conv}$



Flattened in the order corresponding to w_{11}, w_{12}, w_{21}, w_{22}

im2col

How to create matrix $\mathbb{X}_{conv}$



Flattened in the order corresponding to w_{11}, w_{12}, w_{21}, w_{22}

Convolution

kernel2row

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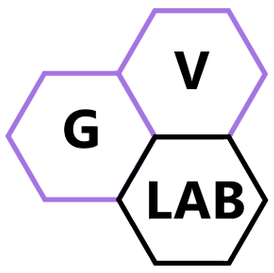

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kernel2row

- **Notation:**

$\mathbf{X}$: input image or feature map

$\mathbb{X}_{conv}$: Matrix created by collecting all the values of X corresponding to the kernel W into columns of $\mathbb{X}_{conv}$ (illustrated in next slide)

$\frac{\partial b}{\partial}$

$\mathbf{W}$: Filter's kernel

$\mathbb{W}_{conv}$: Matrix created by rearranging values in the kernel into row of a matrix (illustrated in next slide)

kernel2row

x_{11}	x_{12}	x_{13}
x_{21}	x_{22}	x_{23}
x_{31}	x_{32}	x_{33}

∂b

X

Input image

w_{11}	w_{12}
w_{21}	w_{22}

W

Filter's kernel

w_{22}	w_{21}
w_{12}	w_{11}

Rot180^0(W)

Rotated kernel

kernel2row

How to create matrix $\mathbb{W}_{conv}$

$w_{22}x_{11}$	$w_{21}x_{12}$	$0^{*}x_{13}$
$w_{12}x_{21}$	$w_{11}x_{22}$	$0^{*}x_{23}$
$0^{*}x_{31}$	$0^{*}x_{32}$	$0^{*}x_{33}$

∂b

X

flattening

w_{22}	w_{21}
w_{12}	w_{11}

Rotate180^{0}(W)

w_{22}	w_{21}	0	w_{12}	w_{11}	0	0	0	0

$\mathbb{W}_{conv}$

kernel2row

How to create matrix $\mathbb{W}_{conv}$

$0*x_{11}$	$w_{22}x_{12}$	$w_{21}x_{13}$
$0*x_{21}$	$w_{12}x_{22}$	$w_{11}x_{23}$
$0*x_{31}$	$0*x_{32}$	$0*x_{33}$

∂b

X

w_{22}	w_{21}
w_{12}	w_{11}

Rotate180^0(W)

w_{22}	w_{21}	0	w_{12}	w_{11}	0	0	0	0
0	w_{22}	w_{21}	0	w_{12}	w_{11}	0	0	0

$\mathbb{W}_{conv}$

kernel2row

How to create matrix $\mathbb{W}_{conv}$

$0*x_{11}$	$0*x_{12}$	$0*x_{13}$
$w_{22}x_{21}$	$w_{21}x_{22}$	$0*x_{23}$
$w_{12}x_{31}$	$w_{11}x_{32}$	$0*x_{33}$

∂b

X

w_{22}	w_{21}
w_{12}	w_{11}

Rotate180^0(W)

w_{22}	w_{21}	0	w_{12}	w_{11}	0	0	0	0
0	w_{22}	w_{21}	0	w_{12}	w_{11}	0	0	0
0	0	0	w_{22}	w_{21}	0	w_{12}	w_{11}	0

$\mathbb{W}_{conv}$

kernel2row

How to create matrix $\mathbb{W}_{conv}$

$0*x_{11}$	$0*x_{12}$	$0*x_{13}$
$0*x_{21}$	$w_{22}x_{22}$	$w_{21}x_{23}$
$0*x_{31}$	$w_{12}x_{32}$	$w_{11}x_{33}$

∂b

X

w_{22}	w_{21}
w_{12}	w_{11}

Rotate180^0(W)

w_{22}	w_{21}	0	w_{12}	w_{11}	0	0	0	0
0	w_{22}	w_{21}	0	w_{12}	w_{11}	0	0	0
0	0	0	w_{22}	w_{21}	0	w_{12}	w_{11}	0
0	0	0	0	w_{22}	w_{21}	0	w_{12}	w_{11}

$\mathbb{W}_{conv}$

Notation

x_{11}	x_{12}	x_{13}
x_{21}	x_{22}	x_{23}
x_{31}	x_{32}	x_{33}

X

$\mathbb{X}_{flat}$

x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}

$\mathbb{X}_{conv}$

x_{22}	x_{23}	x_{32}	x_{33}
x_{21}	x_{22}	x_{31}	x_{32}
x_{12}	x_{13}	x_{22}	x_{23}
x_{11}	x_{12}	x_{21}	x_{22}

w_{11}	w_{12}
w_{21}	w_{22}

W

$\mathbb{W}_{flat}$

w_{11}	w_{12}	w_{21}	w_{22}

$\mathbb{W}_{conv}$

w_{22}	w_{21}	0	w_{12}	w_{11}	0	0	0	0
0	w_{22}	w_{21}	0	w_{12}	w_{11}	0	0	0
0	0	0	w_{22}	w_{21}	0	w_{12}	w_{11}	0
0	0	0	0	w_{22}	w_{21}	0	w_{12}	w_{11}

Convolution as matrix multiplication

$$\mathbf{X} * \mathbf{W} = \mathbf{Reshape}_{o_1 \times o_2}(\mathbb{W}_{flat} \times \mathbb{X}_{conv})$$
$$= \mathbf{Reshape}_{o_1 \times o_2}(\mathbb{W}_{conv} \times \mathbb{X}_{flat})$$

$$\frac{\partial b}{\partial b}$$

$$\Delta\mathbf{W} = \mathbf{Reshape}_{k_1 \times k_2}(\Delta\mathbb{Y}_{flat} \times \mathbb{X}_{conv}^{T})$$

$$\Delta\mathbf{X} = \mathbf{Reshape}_{i_1 \times i_2}(\mathbb{W}_{conv}^{T} \times \Delta\mathbb{Y}_{flat})$$

$*$: Convolution, NOT matrix multiplication

$\times$: Matrix multiplication

o_1 x o_2: size of **Y** = **X*****W**

k_2 x k_2: size of **W**

i_1 x i_2: size of **X**

Convolution as matrix multiplication

$$\mathbb{X}_{conv} = \begin{bmatrix} x_{22} & x_{23} & x_{32} & x_{33} \\ x_{21} & x_{22} & x_{31} & x_{32} \\ x_{12} & x_{13} & x_{22} & x_{23} \\ x_{11} & x_{12} & x_{21} & x_{22} \end{bmatrix} \qquad \mathbb{W}_{flat} = \begin{bmatrix} w_{11} & w_{12} & w_{21} & w_{22} \end{bmatrix}$$

$$\mathbb{Y}_{flat} = \mathbb{W}_{flat} * \mathbb{X}_{conv} = \begin{bmatrix} y_{11} & y_{12} & y_{21} & y_{22} \end{bmatrix}$$

$$y_{11} = w_{22}x_{11} + w_{21}x_{12} + w_{12}x_{21} + w_{11}x_{22}$$

$$y_{12} = w_{22}x_{12} + w_{21}x_{13} + w_{12}x_{22} + w_{11}x_{23}$$

$$y_{21} = w_{22}x_{21} + w_{21}x_{22} + w_{12}x_{31} + w_{11}x_{32}$$

$$y_{22} = w_{22}x_{22} + w_{21}x_{23} + w_{12}x_{32} + w_{11}x_{33}$$

Reshape to 2x2: $\mathbf{Y} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$

Convolution as matrix multiplication

$$\mathbb{X}^T = \begin{array}{|c|c|c|c|} \hline x_{22} & x_{21} & x_{12} & x_{11} \\ \hline x_{23} & x_{22} & x_{13} & x_{12} \\ \hline x_{32} & x_{31} & x_{22} & x_{21} \\ \hline x_{33} & x_{32} & x_{23} & x_{22} \\ \hline \end{array}$$

$$\Delta\mathbb{Y}_{flat} = \begin{array}{|c|c|c|c|} \hline \delta y_{11} & \delta y_{12} & \delta y_{21} & \delta y_{22} \\ \hline \end{array}$$

$$\frac{\partial b}{\partial w_{11}} = \delta y_{11}x_{22} + \delta y_{12}x_{23} + \delta y_{21}x_{32} + \delta y_{22}x_{33}$$

$$\frac{\partial b}{\partial w_{12}} = \delta y_{11}x_{21} + \delta y_{12}x_{22} + \delta y_{21}x_{31} + \delta y_{22}x_{32}$$

$$\frac{\partial b}{\partial w_{21}} = \delta y_{11}x_{12} + \delta y_{12}x_{13} + \delta y_{21}x_{22} + \delta y_{22}x_{23}$$

$$\frac{\partial b}{\partial w_{22}} = \delta y_{11}x_{11} + \delta y_{12}x_{12} + \delta y_{21}x_{21} + \delta y_{22}x_{22}$$

$$\Delta\mathbb{W}_{flat} = \Delta\mathbb{Y}_{flat} \times \mathbb{X}^T :$$

Reshape to 2x2:

$$\Delta\mathbf{W} = \begin{array}{|c|c|} \hline \delta y_{11} & \delta y_{12} \\ \hline \delta y_{21} & \delta y_{22} \\ \hline \end{array}$$

Convolution as matrix multiplication

$$\mathbb{W}_{conv} =$$

w_{22}	w_{21}	**0**	w_{12}	w_{11}	0	0	0	0
0	w_{22}	w_{21}	0	w_{12}	w_{11}	0	0	0
0	0	0	w_{22}	w_{21}	**0**	w_{12}	w_{11}	0
0	0	0	0	w_{22}	w_{21}	0	w_{12}	w_{11}

$$\mathbb{W}_{conv}^{T} =$$

w_{22}	0	0	0
w_{21}	w_{22}	0	0
0	w_{21}	0	0
w_{12}	0	w_{22}	0
w_{11}	w_{12}	w_{21}	w_{22}
0	w_{11}	0	w_{21}
0	0	w_{12}	0
0	0	w_{11}	w_{12}
0	0	0	w_{11}

∂b

Convolution as matrix multiplication

$$W^T_{conv} =$$

w_{22}	0	0	0
w_{21}	w_{22}	0	0
0	w_{21}	0	0
w_{12}	0	w_{22}	0
w_{11}	w_{12}	w_{21}	w_{22}
0	w_{11}	0	w_{21}
0	0	w_{12}	0
0	0	w_{11}	w_{12}
0	0	0	w_{11}

$$\Delta Y_{flat} =$$

δy_{11}	δy_{12}	δy_{21}	δy_{22}

Convolution as matrix multiplication

$$\mathbb{W}^T_{conv} =$$

w_{22}	0	0	0
w_{21}	w_{22}	0	0
0	w_{21}	0	0
w_{12}	0	w_{22}	0
w_{11}	w_{12}	w_{21}	w_{22}
0	w_{11}	0	w_{21}
0	0	w_{12}	0
0	0	w_{11}	w_{12}
0	0	0	w_{11}

$$\Delta\mathbb{Y}_{flat} = \begin{array}{|c|c|c|c|}\hline \delta y_{11} & \delta y_{12} & \delta y_{21} & \delta y_{22} \\ \hline \end{array}$$

$$\Delta\mathbb{X}_{flat} = \mathbb{W}^T_{conv} \times \Delta\mathbb{Y}_{flat} :$$

$$\delta x_{11} = \delta y_{11} w_{22}$$

$$\delta x_{12} = \delta y_{11} w_{21} + \delta y_{12} w_{22}$$

$$\delta x_{13} = \delta y_{12} w_{21}$$

$$\delta x_{21} = \delta y_{11} w_{12} + \delta y_{21} w_{22}$$

$$\delta x_{22} = \delta y_{11} w_{11} + \delta y_{12} w_{12} + \delta y_{21} w_{21} + \delta y_{11} w_{22}$$

$$\delta x_{23} = \delta y_{12} w_{11} + \delta y_{22} w_{21}$$

$$\delta x_{31} = \delta y_{21} w_{12}$$

$$\delta x_{32} = \delta y_{21} w_{11} + \delta y_{22} w_{12}$$

$$\delta x_{33} = \delta y_{22} w_{11}$$

Convolution as matrix multiplication

$$\Delta \mathbb{X}_{flat} = \mathbb{W}^{T}_{conv} \times \Delta \mathbb{Y}_{flat} :$$

$$\delta x_{11} = \delta y_{11} w_{22}$$

$$\delta x_{12} = \delta y_{11} w_{21} + \delta y_{12} w_{22}$$

$$\delta x_{13} = \delta y_{12} w_{21}$$

$$\delta x_{21} = \delta y_{11} w_{12} + \delta y_{21} w_{22}$$

$$\delta x_{22} = \delta y_{11} w_{11} + \delta y_{12} w_{12} + \delta y_{21} w_{21} + \delta y_{11} w_{22}$$

$$\delta x_{23} = \delta y_{12} w_{11} + \delta y_{22} w_{21}$$

$$\delta x_{31} = \delta y_{21} w_{12}$$

$$\delta x_{32} = \delta y_{21} w_{11} + \delta y_{22} w_{12}$$

$$\delta x_{33} = \delta y_{22} w_{11}$$

Reshape: 4x4

δx_{11}	δx_{12}	δx_{13}
δx_{21}	δx_{22}	δx_{23}
δx_{31}	δx_{31}	δx_{33}

ΔX