1 Sumrizations

$$\sum_{i=1}^{n} i^6 = \frac{1}{42} n(n+1)(2n+1)(3n^4+6n^3-3n+1)$$

$$\sum_{i=1}^{n} i^7 = \frac{1}{24} n^2(n+1)^2(3n^4+6n^3-n^2-4n+2)$$

$$1-2+3-4+\cdots+(-1)^{n+1}n = \begin{cases} \frac{1}{2}(n+1), & n \equiv 1 \mod 2 \\ -\frac{n}{2}, & n \equiv 0 \mod 2 \end{cases}$$

$$1^2-2^2+3^2-4^2+\cdots+(-1)^{n+1}n^2 = (-1)^{n+1}\frac{1}{2}n(n+1)$$

$$1^3-2^3+3^3-4^3+\cdots+(-1)^{n+1}n^3 = \begin{cases} \frac{1}{4}(2n-1)(n+1)^2, & n \equiv 1 \mod 2 \\ -\frac{1}{4}n^2(2n+3), & n \equiv 0 \mod 2 \end{cases}$$

$$1^4-2^4+3^4-4^4+\cdots+(-1)^{n+1}n^4 = (-1)^{n+1}\frac{1}{2}n(n+1)(n^2+n-1)$$

$$\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\frac{1}{3\cdot 4\cdot 5}+\cdots+\frac{1}{n(n+1)(n+2)} = \frac{1}{4}-\frac{1}{2(n+1)(n+2)}$$

$$\sum_{i=1}^{n} \frac{1}{i(i+1)(i+2)(i+3)} = \frac{1}{18}-\frac{1}{3(n+1)(n+2)(n+3)}$$

2 Inequalities

$$\sin x < x < \tan x, (\theta < x < \frac{\pi}{2})$$

$$\cos x < \frac{\sin x}{x} < 1, (0 < x < \pi)$$

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$$\frac{\sin x}{x} > \frac{2}{\pi}, (-\frac{\pi}{2}, \frac{\pi}{2}), \quad \text{Holder}$$

$$\cos x \geq 1-\frac{1}{2}x^2, (\mathbb{R}), \quad \text{Minkowski}$$

$$\sin x > x-\frac{1}{6}x^3, (\mathbb{R}^+)$$

$$\tan x > x+\frac{1}{3}, (0, \frac{\pi}{2})$$

$$x < \frac{\sin \pi x}{x(1-x)} < 4, (0, \frac{1}{2}) \cup (\frac{1}{2}, 1), \quad \text{Jansen}$$

$$e^x \geq 1+x, (\mathbb{R})$$

$$e^x < \frac{1}{1-x}, (-\infty, 0) \cup (0, 1)$$

$$e^{\frac{x}{1-x}} > \frac{1}{x}, (x \neq 0)$$
$$\frac{x}{1+x} < \ln(1+x) < x, (-1,0) \cup (0,+\infty)$$
$$\frac{1}{n+1} < \ln(1+\frac{1}{n}) < \frac{1}{n}, (\mathbb{N}_+)$$
$$\ln x \leq x-1, (\mathbb{R}_+)$$
$$x < -\ln(1-x) < \frac{x}{1-x}, (-\infty,0) \cup (0,1)$$

$\forall z \in \mathbb{C}$ **in following inequalities:**

$$|\cos z| < z, (|z| < 1)$$
$$|\sin z| < \frac{6}{5}|z|, (0 < |z| < 1)$$
$$|e^z - 1| < e^{|z|} - 1 < |z|e^{|z|}, (z \neq 0)$$
$$\frac{1}{4}|z| < |e^z - 1| < \frac{7}{4}|z|, (0 < |z| < 1)$$
$$|\ln(1+z)| < -\ln(1-|z|)$$